Topological Kondo Effect in Transport through a Superconducting Wire with Multiple Majorana End States

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(Received 11 July 2014; published 16 March 2015)

We investigate a system of multiple Majorana states at the end of a topological superconducting wire coupled to a normal lead. For a minimum of three Majorana fermions at the interface, we find nontrivial renormalization physics. Interface tunneling processes can be classified in terms of spin-1/2 and spin-3/2 irreducible representations of the SU(2) group. We show that the renormalization of the tunneling amplitudes belonging to different representations is completely different in that one type is suppressed, whereas the other is enhanced, depending on the sign of the Kondo-type interaction coupling. This results in distinct temperature dependencies of the tunneling current through the interface and different spin polarizations of this current.

DOI: 10.1103/PhysRevLett.114.116801 PACS numbers: 73.63.-b, 73.21.La, 74.45.+c, 75.20.Hr

Introduction.—Majorana fermions were first proposed as hypothetical elementary particles that are their own antiparticles [1]. The possibility of Majorana states at the surfaces of triplet superconductors has been discussed for a long time [2–7]. The realization that they are related to topological properties of the system [8] has generated a lot of interest in Majorana states at the surfaces of topological superconductors (TSs) [7,9–15].

The first signatures of Majorana states at the ends of a TS wire were found in transport measurements involving the interface between the wire and a normal (N) lead [16,17]. These experiments have so far been compared to a model with a single Majorana state coupled to the normal lead [10,11,18], which cannot contain any interaction between the Majorana state and the lead because the (single) Majorana operator γ squares to unity. The theory has already advanced to more sophisticated noninteracting systems, such as Josephson junctions between TSs, where the tunneling takes place between Majorana states [19], and setups with one or several quantum dots mediating the electron transfer between the leads and the TS [20–22]. The study of interaction processes in such systems is of interest since interactions generically lead to strong renormalizations in low dimensions. However, so far only on-dot interactions have been studied for these setups [23,24]. The implementation of Majorana-lead interactions requires the presence of several Majorana modes. Multiple Majorana states and the renormalization of interaction couplings have been studied in Refs. [25–28]. Each Majorana end state is either coupled by a tunneling term to its own normal lead or is not coupled at all [25–28]. We consider a different situation: Multiple Majorana states hybridizing with a single lead.

Our goal is to understand the interplay between multiple tunneling channels and the electron-Majorana interaction, which we find to induce strong renormalization. This research is meant to help in interpreting, regardless of microscopic details, the results of transport measurements by studying the temperature dependence and spin polarization of the current. We show that these observables exhibit clear signatures of the presence of Majorana fermions and of their coupling to the leads. A TS wire coupled to a normal lead is modeled by N Majorana fermions localized at one end of the wire and a Fermi sea of spinful electrons, coupled by general tunneling and interaction terms. The minimal nontrivial case of $N = 2$ gives nothing new because the two Majorana states make up a spinless fermion and the interaction in the system is equivalent to the one in the interacting resonant-level model, leading to the same renormalization flow, which has been studied extensively [29,30]. Systems with $N \geq 3$ are fundamentally different: Unlike the $N = 2$ system, their interaction couplings get strongly renormalized, similarly to the Kondo model [28]. Here, we will demonstrate that interesting renormalization physics occurs already for $N = 3$. The predictions made in this work are unique for this system, which supports both Kondo and tunneling couplings, whose interplay leads to the nontrivial discrimination of the tunneling processes depending on the sign of the interaction.

In the general case of N Majorana states, the sets of N Majorana operators before and after some symmetry transformation are related by $\gamma_i{}' = \sum_{j=1}^{N} R_{ij}\gamma_j$, where R is a real (since $\gamma_i^\dagger = \gamma_i$) orthogonal matrix belonging to the group SO(N). A candidate for this symmetry transformation is the electron spin rotation. In this case Majorana states transform into each other according to a representation of the SU(2) group, which also has to be a subgroup of SO(N). The case of three Majorana states is particularly interesting since the whole SO(3) group is equivalent to the spin-1 representation of SU(2). An experimental realization of a set of three Majorana states transforming under SO(3) is still unknown, but there is already a proposal assuming the existence of such sets in vortex cores in TS [31]. As we shall see, for the N-TS interface the tunneling terms inevitably break the SU(2) spin symmetry.

In this Letter, we derive the renormalization-group (RG) flow equations for the electron-Majorana interaction strengths and tunneling amplitudes within the framework of poor man's scaling for arbitrary N. We solve the RG equations for the simplest nontrivial case $N = 3$, and demonstrate that the tunneling amplitudes can be classified according to the irreducible representations of the SU(2) group and that the components belonging to different representations obey different RG equations. In practice, this means that starting from arbitrary tunneling parameters, the interaction will lead to the suppression of one set of parameters and the enhancement of the other. Moreover, depending on the initial value of the interaction, a different tunneling type will dominate in the scaling limit, leading to a different temperature dependence of the current through the interface.

Model.—The investigated system consists of a noninteracting normal lead with a Fermi sea of electrons coupled to three Majorana states localized at the same end of a TS wire, which are described by the Hermitian fermionic operators γ_i. The Hamiltonian of the lead is $H_L = \sum_{\alpha\mathbf{p}}\epsilon_{\mathbf{p}} a^\dagger_{\alpha\mathbf{p}} a_{\alpha\mathbf{p}}$, where $a^\dagger_{\alpha\mathbf{p}}$, $a_{\alpha\mathbf{p}}$ are creation and annihilation operators of electrons with spin $\alpha = \uparrow, \downarrow$ and momentum $\mathbf{p}$. It is assumed that the electronic band with the dispersion relation $\epsilon_{\mathbf{p}}$ approximately covers the energy interval $[-D, D]$ and has a constant normalized density of states $\rho(E) \equiv \mathcal{N}^{-1}\sum_{\mathbf{p}}\delta(E - \epsilon_{\mathbf{p}}) \approx \nu$ for $E \ll D$ (here $\mathcal{N}$ is a total number of states in the lead). Henceforth, we take $\hbar = k_B = 1$.

The couplings between the states localized at opposite ends of the wire are exponentially suppressed with the distance between them. If the SO(N) symmetry of the Majorana states γ_i at the same end is broken, a coupling of the form $H_D = i\sum_{ij}E_{ij}\gamma_i\gamma_j$ is allowed. However, as we will discuss later, H_D does not affect the RG equations as long as the flow parameter satisfies $\Lambda \gg |E_{ij}|$.

The N-TS coupling consists of a bilinear tunneling part and an interaction part. Assuming that the coupling is local in real space, the most general tunneling term is

$$H_T = \sum_{i\alpha} t_{i\alpha}\gamma_i a^\dagger_\alpha + \text{H.c.}, \tag{1}$$

where $a_\alpha = \mathcal{N}^{-1/2}\sum_{\mathbf{p}} a_{\alpha\mathbf{p}}$ and $t_{i\alpha}$ are tunneling amplitudes. The leading interaction terms are of fourth order in fermionic operators. We focus on terms that are quadratic in Majorana operators [32]. Because of the anticommutation relation $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$ only $N(N-1)/2$ combinations exist. Thus, the most general local biquadratic interaction term reads

$$H_V = \frac{1}{2}\sum_{ij\alpha\beta} V^{ij}_{\alpha\beta}\gamma_i\gamma_j a^\dagger_\alpha a_\beta, \tag{2}$$

where $V^{ij}_{\alpha\beta} = -V^{ji}_{\alpha\beta}$ are coupling parameters. If there is any interaction between the TS and the leads, we expect an expansion in the order of vertices to generate H_V. While the direct Coulomb interaction vanishes for the neutral Majorana fermions, an exchange-type interaction emerges naturally since the zero-energy Majorana surface states of nodal TSs with strong spin-orbit coupling typically carry a large spin [15,33,34]. An interaction H_V can also be realized in a small superconducting island with large charging energy hybridized with normal leads [25,27]. H_V is here obtained by integrating out charge fluctuations, which removes the tunneling term H_T. More generally, the coupling of Marojana states and normal electrons to any additional modes, such as phonons, will typically introduce an effective interaction of this form when these modes are integrated out.

RG and symmetry analysis.—To study the renormalization effects, we employ the poor man's scaling approach [35,36]: The RG flow parameter Λ denotes the maximal energy of the electron modes, $|\epsilon_{\mathbf{p}}| < \Lambda$; the electron modes are divided into *fast* modes $a_{\alpha\mathbf{k}}$ with energies in the thin shell $\Lambda - \Delta\Lambda < |\epsilon_{\mathbf{k}}| < \Lambda$ and *slow* modes $a_{\alpha\mathbf{p}'}$ with $|\epsilon_{\mathbf{p}'}| < \Lambda - \Delta\Lambda$; integration over the fast modes results in corrections to the slow-mode terms in the Hamiltonian. Repeating this step, we integrate out all electron degrees of freedom, obtaining an effective low-energy Hamiltonian. Taking the N-TS coupling as the perturbation and $H_0 = H_L + H_D$ as the bare Hamiltonian, the correction to the interaction for excitations with small energy E, from a single RG step, reads

$$\Delta H_V \approx \langle H_V(E - H_0)^{-1}H_V\rangle = -\frac{1}{4\mathcal{N}^2}\sum_{\substack{ii'jj',\alpha\beta\eta \\ \mathbf{p}'\mathbf{q}';\mathbf{k}}} V^{ij}_{\alpha\eta}V^{i'j'}_{\eta\beta} \times\left(\gamma_i\gamma_j\gamma_{i'}\gamma_{j'}\frac{1-n_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} + \gamma_{i'}\gamma_{j'}\gamma_i\gamma_j\frac{n_{\mathbf{k}}}{\epsilon_{\mathbf{k}}}\right)a^\dagger_{\alpha\mathbf{p}'}a_{\beta\mathbf{q}'}, \tag{3}$$

where $\mathbf{p}'$, $\mathbf{q}'$ denote slow modes, $\mathbf{k}$ refers to a fast mode, angular brackets denote the integration over the fast modes only, $\Delta H \equiv H(\Lambda - \Delta\Lambda) - H(\Lambda)$ is the difference between the values after and before the RG step, and $n_{\mathbf{k}} \equiv n_F(\epsilon_{\mathbf{k}})$ is a Fermi distribution function. Λ is of the order of the band width, which is assumed to be large compared to the other energy scales of the problem, in particular the energy E and the inter-Majorana couplings E_{ij}. Therefore, these terms do not affect the RG flow to leading order and can be neglected. The terms relevant for the RG flow decay as Λ^{-1}. For the assumed constant and symmetric density of states we drop the sum $\mathcal{N}^{-1}\sum_{\mathbf{k}}1/\epsilon_{\mathbf{k}}$ and approximate $\mathcal{N}^{-1}\sum_{\mathbf{k}}(1/2 - n_{\mathbf{k}})/\epsilon_{\mathbf{k}} \approx \nu\Delta\Lambda/\Lambda$, and find the RG equation

$$\frac{dV^{ij}_{\alpha\beta}}{d\Lambda} = \frac{2\nu}{\Lambda}\sum_{l,\eta}(V^{il}_{\alpha\eta}V^{lj}_{\eta\beta} - V^{jl}_{\alpha\eta}V^{li}_{\eta\beta}). \tag{4}$$

The corresponding correction to the tunneling term is

$$\Delta H_T \approx \langle H_V (E-H_0)^{-1} H_T \rangle + [T \leftrightarrow V] = -\frac{1}{2\mathcal{N}^{3/2}} \times \sum_{\substack{ijj',\eta\beta \\ \mathbf{p}';\mathbf{k}}} V^{ij}_{\alpha\beta} t_{j'\beta} \left(\gamma_i \gamma_j \gamma_{j'} \frac{1-n_\mathbf{k}}{\epsilon_\mathbf{k}} + \gamma_{j'} \gamma_i \gamma_j \frac{n_\mathbf{k}}{\epsilon_\mathbf{k}} \right) a^\dagger_{\alpha\mathbf{p}'} + \text{H.c.} \quad (5)$$

Keeping only the RG-relevant contribution, we obtain

$$\frac{dt_{i\alpha}}{d\Lambda} = \frac{2\nu}{\Lambda} \sum_{j,\beta} V^{ij}_{\alpha\beta} t_{j\beta}. \quad (6)$$

The obtained equations couple $2N(N-1)$ quantities $V^{ij}_{\alpha\beta}$ and $2N$ quantities $t_{i\alpha}$. To simplify the analysis but preserve the interesting renormalization physics, we restrict ourselves to $N=3$.

The case of three Majorana states.—The special feature of the SO(3) group is that its irreducible representations are equivalent to integer-spin representations of SU(2). This feature allows us to classify the elements $V^{ij}_{\alpha\beta}$ and $t_{i\alpha}$ in terms of the irreducible representations of SU(2). The products of two Majorana operators, which form vectors belonging to the spin-1 representation Γ_1, can be split into the irreducible representations $\Gamma_1 \otimes \Gamma_1 \cong \Gamma_0 \oplus \Gamma_1 \oplus \Gamma_2$. Since expressions belonging to the scalar (Γ_0) representation, $\sum_i \gamma_i^2 = 3$, and to the spin-2 (Γ_2) representation, $\gamma_i\gamma_j + \gamma_j\gamma_i = 0$, are just numbers, the only nontrivial combination is the Majorana pseudospin operator $s^M_i = -(i/2)\sum_{jj'} \epsilon_{ijj'} \gamma_j \gamma_{j'}$ (here $\epsilon_{ijj'}$ is the three-dimensional Levi-Civita tensor), which belongs to the Γ_1 representation of SU(2). The operators s^M_i play the role of pseudospin components; they satisfy the algebra $[s^M_j, s^M_{j'}] = 2i\sum_i \epsilon_{ijj'} s^M_i$ and $[s^M_j, \gamma_{j'}] = 2i\sum_i \epsilon_{ijj'}\gamma_i$. Expressed in these terms, the interaction term in Eq. (2) takes the form

$$H_V = \sum_i M_i s^M_i n^L + \sum_{ij} V_{ij} s^M_i s^L_j, \quad (7)$$

where $n^L = \mathcal{N}^{-1} \sum_{\alpha,\mathbf{pq}} a^\dagger_{\alpha\mathbf{p}} a_{\alpha\mathbf{q}}$ is the local lead-electron number operator and $s^L_i = \mathcal{N}^{-1} \sum_{\alpha\beta,\mathbf{pq}} a^\dagger_{\alpha\mathbf{p}} \sigma^i_{\alpha\beta} a_{\beta\mathbf{q}}/2$ the corresponding spin operator, where σ^i are Pauli matrices. The first term, when substituted into Eq. (4), is not renormalized and just leads to a renormalization of the tunneling amplitudes through Eq. (6), similar to the interacting resonant-level model [30]. Setting the vector M_i to $(0,0,M_z)$ by choosing an appropriate basis, we find that the z component of the tunneling amplitude does not change, $t_{\alpha z}(\Lambda) = t_{\alpha z}$, while the others are renormalized as $t_{\alpha,\pm}(\Lambda) = t_{\alpha,\pm}(D/\Lambda)^{\pm 2\nu M_z}$, where $t_{\alpha,\pm} = t_{\alpha x} \pm i t_{\alpha y}$ [26]. The second term in Eq. (7) contains the product of two vectors, so it can be decomposed as $V_{ij} = \delta_{ij} J + \sum_k \epsilon_{ijk} J^k + J^{ij}$, where J is a scalar (Γ_0), which describes the Kondo-type interaction between the lead electrons and the effective Majorana spin, J^i is a vector (Γ_1), and the symmetric matrix J^{ij} with zero trace corresponds to the spin-2 representation Γ_2.

Since the main goal of this Letter is to demonstrate the possibility of interesting renormalization physics, we restrict ourselves to the simplest case with unbroken SU(2) symmetry in the interaction between normal lead and TS, choosing $V_{ij} = \delta_{ij} J$. Then Eq. (4) leads to the well-known RG flow equation for the Kondo coupling [25,26,35],

$$\frac{dJ}{d\Lambda} = -\frac{2\nu J^2}{\Lambda}. \quad (8)$$

The solution depends on the sign of the initial unrenormalized coupling J_0 (we denote initial values by a subscript 0): The coupling is enhanced for $J_0 > 0$ and suppressed for $J_0 < 0$, depending on Λ as

$$J = \frac{1}{2\nu \ln(\Lambda/T_K)}, \quad (9)$$

where $T_K = De^{-1/2\nu J_0}$ is the Kondo temperature. The poor man's scaling approach, however, breaks down when Λ reaches the largest of the low-energy scales of the problem, Λ_c, which plays the role of an infrared cutoff. For the antiferromagnetic case ($J > 0$) this means that J actually saturates and does not diverge at $\Lambda = T_K$, as Eq. (9) would predict [30,35,37]. In the context of a possible implementation utilizing spin-polarized Majorana surface states, it is plausible that either sign of J can be realized since model calculations find Majorana states in pairs with opposite spin expectation value [15,34].

The tunneling term in Eq. (1) contains a product of a vector and a spinor. Thus the tunneling amplitudes can be classified by the irreducible representations of SU(2), $\Gamma_1 \otimes \Gamma_{1/2} \cong \Gamma_{1/2} \oplus \Gamma_{3/2}$, and split into spin-1/2 and spin-3/2 terms according to $t_{i\alpha} = \sum_{S,m} t^{S,m} \tau^{S,m}_{i\alpha}$, where $m = \pm 1/2$ for $S = 1/2$ and $m = \pm 1/2, \pm 3/2$ for $S = 3/2$. The Clebsch-Gordan coefficients for $S = 1/2$ read

$$\tau^{1/2,+1/2}_{i\alpha} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 1 \\ 1 & i & 0 \end{pmatrix}_{\alpha i}, \quad \tau^{1/2,-1/2}_{i\alpha} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & -i & 0 \\ 0 & 0 & -1 \end{pmatrix}_{\alpha i}, \quad (10)$$

which are basically Pauli matrices with swapped indices, $\tau^{1/2,+1/2}_{i\alpha} = \sigma^i_{\alpha,\uparrow}$, $\tau^{1/2,-1/2}_{i\alpha} = \sigma^i_{\alpha,\downarrow}$. The Clebsch-Gordan coefficients for $S = 3/2$ are

$$\tau_{i\alpha}^{3/2,+3/2} = \begin{pmatrix} \frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\alpha i},$$
$$\tau_{i\alpha}^{3/2,+1/2} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & -1 \\ \frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}_{\alpha i},$$
$$\tau_{i\alpha}^{3/2,-1/2} = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}_{\alpha i},$$
$$\tau_{i\alpha}^{3/2,-3/2} = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}_{\alpha i}. \tag{11}$$

The absence of the scalar representation for the tunneling amplitudes between spin-$1/2$ electrons and triplets of Majorana states signifies that the SU(2) is always broken, as mentioned in the Introduction.

According to Eq. (6), the tunneling coefficients obey the RG equations

$$\frac{dt^{1/2,m}}{d\Lambda} = -\frac{4\nu J t^{1/2,m}}{\Lambda}, \qquad \frac{dt^{3/2,m}}{d\Lambda} = \frac{2\nu J t^{3/2,m}}{\Lambda}. \tag{12}$$

Together with Eq. (9), the solutions read

$$t^{1/2,m} = t_0^{1/2,m} \frac{J^2}{J_0^2}, \qquad t^{3/2,m} = t_0^{3/2,m} \frac{J_0}{J}. \tag{13}$$

For antiferromagnetic coupling, spin-$3/2$ tunneling is suppressed, whereas spin-$1/2$ tunneling rapidly increases as Λ approaches T_K, together with the Kondo coupling J. Ferromagnetic coupling leads to the opposite behavior: Spin-$3/2$ tunneling increases, while spin-$1/2$ tunneling decreases. The physical values of the renormalized parameters are obtained at the end of the RG flow. Although Eqs. (9) and (13) generally break down at the infrared cutoff Λ_c, if the temperature T is much larger than all other low-energy scales (but still much smaller than ultraviolet cutoff D), the renormalized coupling parameters can be obtained by substituting the flow parameter by the temperature, $\Lambda = T$.

Results and discussion.—The results in Eq. (13) demonstrate that antiferromagnetic coupling at the interface enhances the transport with smaller total spin, while ferromagnetic coupling enhances the tunneling transport with larger total spin. In the general case when the initial Hamiltonian contains all possible tunneling amplitudes, the presence of a Kondo interaction leads to a strong renormalization, which manifests itself by an instability of the tunneling amplitudes. Independently of the coupling sign, the total tunneling probability is enhanced. However, if the initial interaction is antiferromagnetic the system is dominated by spin-$1/2$ tunneling, while for ferromagnetic interaction it is dominated by spin-$3/2$ tunneling. The type of coupling thus manifests itself in transport processes. One of its signatures is the temperature dependence of the current through the N-TS interface. For voltages U much larger than the temperature but smaller than the superconducting gap, the current I is proportional to the tunneling probability, $I \propto |t|^2$ [18]. According to Eqs. (9) and (13), the current thus depends on temperature as $I \propto \ln^{-4}(T/T_K)$ for the antiferromagnetic case and as $I \propto \ln^2(T_K/T)$ for the ferromagnetic case. This provides us with a criterion for the detection of multiple Majorana states and for determining the type of interaction between normal lead and TS.

The dominant renormalized spin-S tunneling also leads to a distinctive spin dependence of the current through the interface. For spin-$1/2$ tunneling (the antiferromagnetic case), two of the three Majorana fermions can be combined into one conventional (Dirac) fermion $d = \frac{1}{2}(\gamma_x + i\gamma_y)$ so that the tunneling Hamiltonian becomes

$$\begin{aligned} H_T &= \sum_{i\alpha m} t^{1/2,m} \tau_{i\alpha}^{1/2,m} \gamma_i a_\alpha^\dagger + \text{H.c.} \\ &= t_1'(-\gamma_z a_\downarrow^\dagger + 2d^\dagger a_\uparrow^\dagger) + t_2'(\gamma_z a_\uparrow^\dagger + 2d a_\downarrow^\dagger) + \text{H.c.}, \end{aligned} \tag{14}$$

where $t_1' \equiv t^{1/2,-1/2}/\sqrt{3}$, $t_2' \equiv t^{1/2,1/2}/\sqrt{3}$. The tunneling amplitudes $t^{1/2,\pm 1/2}$ form a spinor, so their component values depend on the choice of basis in spin space. By an appropriate choice one can always set one of the elements $t_n{}'$ to zero. Upon setting $t_1{}' = 0$, the system decomposes into two noninteracting parts. The first one consists of spin-up electrons bound to the γ_z Majorana state, while the second is a resonant-level model made up of spin-down electrons and the additional fermion d. We now discuss the contributions of the two parts to the tunneling current under a bias voltage. The first part allows a nonzero stationary current, as we can see as follows: The Majorana operator can be expressed in terms of Dirac operators as $\gamma_z = d' + (d')^\dagger$. Thus the combined particle number $a_\uparrow^\dagger a_\uparrow + (d')^\dagger d'$ is not conserved. If we assume, to be specific, a positive bias voltage to be applied to the TS, spin-up electrons will tunnel into the TS alternatingly creating and annihilating the d' fermion. Physically, this represents Andreev tunneling [18]; the charge conservation is restored by the creation of Cooper pairs in the superconducting condensate. On the other hand, the second part of the model does conserve the combined particle number $a_\downarrow^\dagger a_\downarrow + d^\dagger d$ and the d fermion is not connected to any other lead. Thus the stationary current for the spin-down electrons vanishes. In conclusion, the spin-$1/2$ coupling results in a fully spin-polarized current in the basis defined by the tunneling-amplitude spinor.

For spin-$3/2$ tunneling (the ferromagnetic case), the tunneling Hamiltonian can analogously be written as

$$\begin{aligned} H_T = &-t_1'' d^\dagger a_\downarrow^\dagger - t_2''(d^\dagger a_\uparrow^\dagger + \gamma_z a_\downarrow^\dagger) \\ &+ t_3''(-\gamma_z a_\uparrow^\dagger + d a_\downarrow^\dagger) + t_4'' d a_\uparrow^\dagger + \text{H.c.}, \end{aligned} \tag{15}$$

where $t_1'' \equiv t^{3/2,-3/2}$, $t_2'' \equiv t^{3/2,-1/2}/\sqrt{3}$, $t_3'' \equiv t^{3/2,1/2}/\sqrt{3}$, $t_4'' \equiv t^{3/2,3/2}$. The tunneling amplitudes $t^{3/2,m}$ form a spin-$3/2$

spinor. By an appropriate choice of spin basis we can again set one of the $t^{3/2,m}$ (and thus the corresponding t''_n) to zero. However, no matter which tunneling amplitude is set to zero, both spin channels remain coupled through the d fermion and thus, directly or indirectly, to the Majorana fermion γ_z. Under a bias, the current is nonzero for all electron-spin states. Therefore, in general the current for spin-3/2 tunneling can only be partially spin polarized.

Summary.—The presence of the Kondo interaction between the electrons in the normal lead and Majorana fermions at the ends of a TS wire results in a strong renormalization of the tunneling processes through the interface. The tunneling amplitudes can be classified according to the irreducible representations of the SU(2) group. The amplitudes belonging to different representations obey different scaling laws. Depending on the sign of the interaction, one component is enhanced, while the other is suppressed, so that only one type of tunneling survives. Ferromagnetic interaction favors a spin-3/2 tunneling with parallel electron spin and Majorana pseudospin, whereas antiferromagnetic coupling enhances spin-1/2 tunneling with opposite spin and pseudospin. The temperature dependence and spin polarization of the current through the N-TS interface reflects the presence of multiple Majorana states and the type of interaction, and therefore can be used as a tool for the search of a topological system with multiple edge states and for the determination of their interaction type.

Financial support by the Deutsche Forschungsgemeinschaft through Research Training Group GRK 1621 is gratefully acknowledged. We would like to thank M. Vojta for useful discussions.

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[32] In principle, there are also N-TS interaction terms of the form $\gamma_i a_\uparrow a_\downarrow a_\alpha^\dagger + \mathrm{H.c.}$ and $\gamma_i \gamma_j \gamma_k a_\alpha^\dagger + \mathrm{H.c.}$, which can be interpreted as interaction corrections to tunneling, and $\gamma_i \gamma_j a_\uparrow a_\downarrow + \mathrm{H.c.}$, which is an Andreev tunneling term. These more exotic terms can be treated in the same manner. We find that these terms are generally renormalized and produce corrections to the tunneling and the biquadratic interaction, but we assume these couplings to be small for simplicity.
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