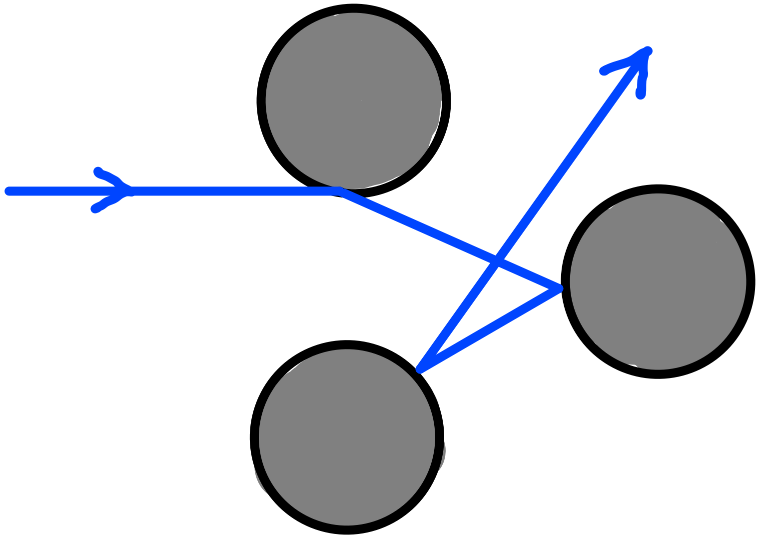
11. Chaotic Scattering



Chaos defined for $t \to \infty$

Scattering takes finite time

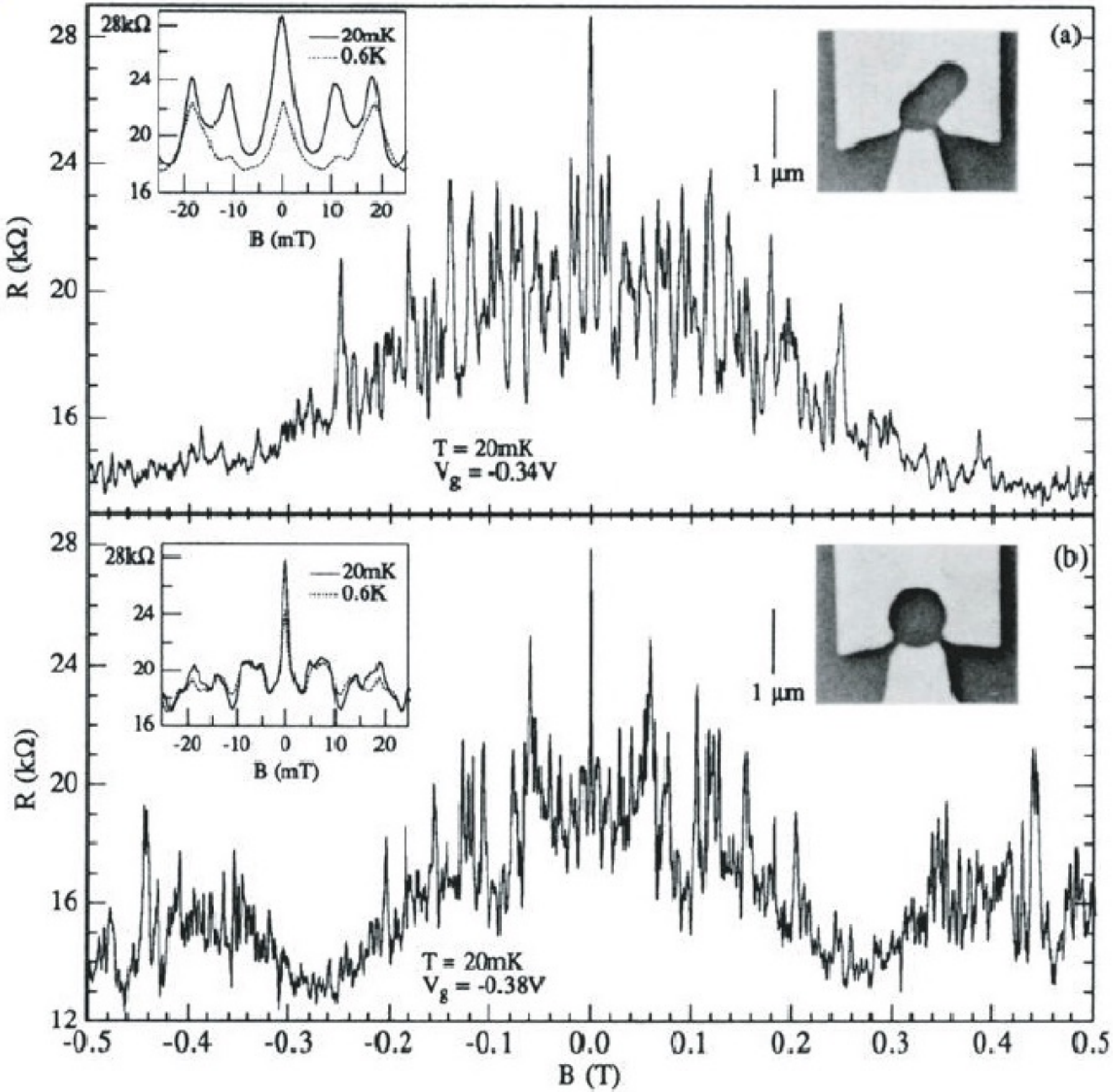
How to measure from outside whether system is chaotic?

classical? quantum?

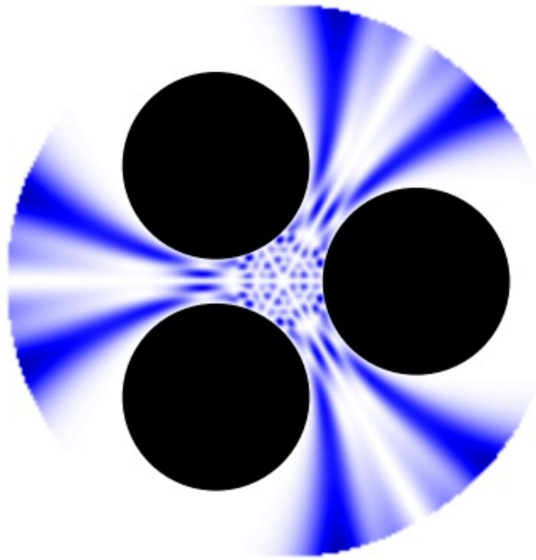
11.1. 11.2. S-Matrix fluctuations

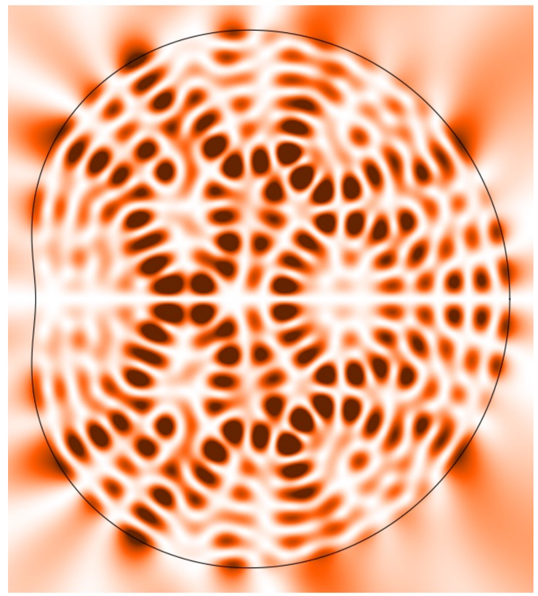
11.3. Resonance states

conductance fluctuations



resonance states

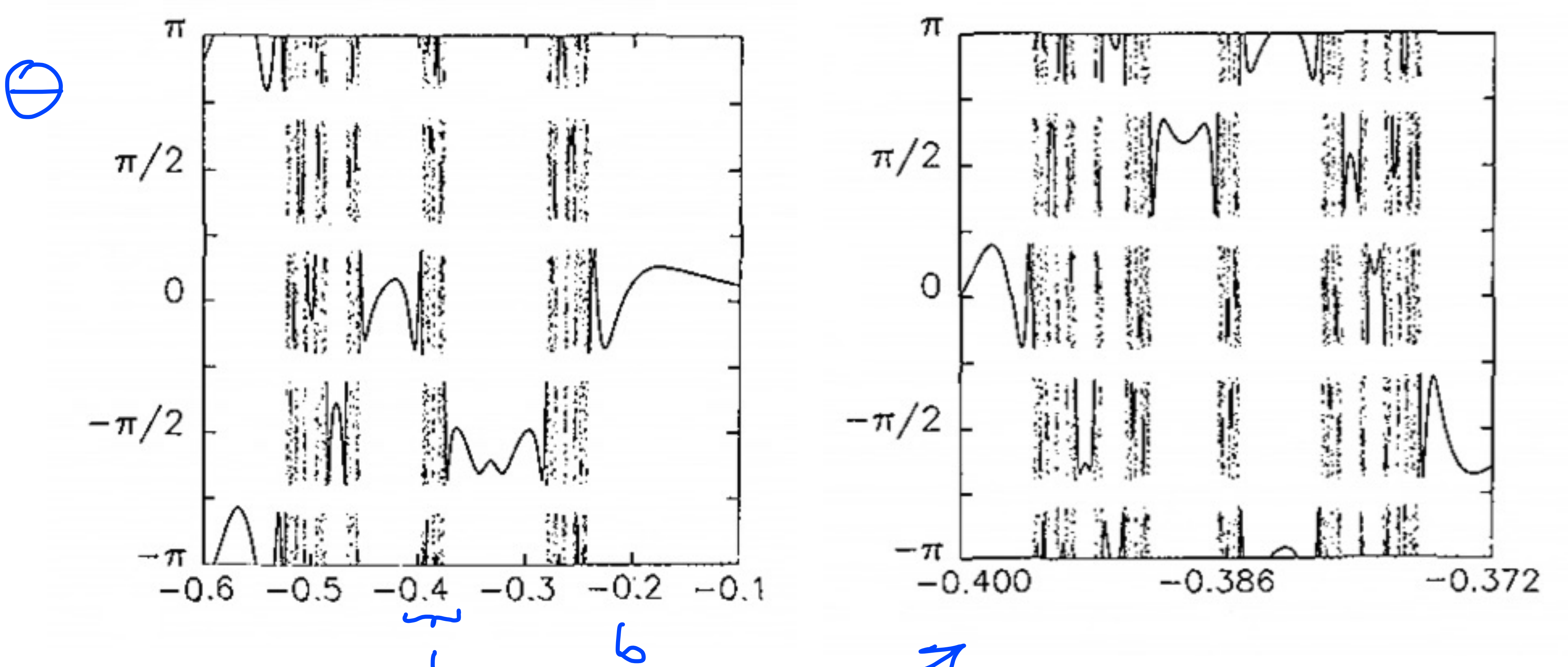




11.1. Classical Chaotic Scattering

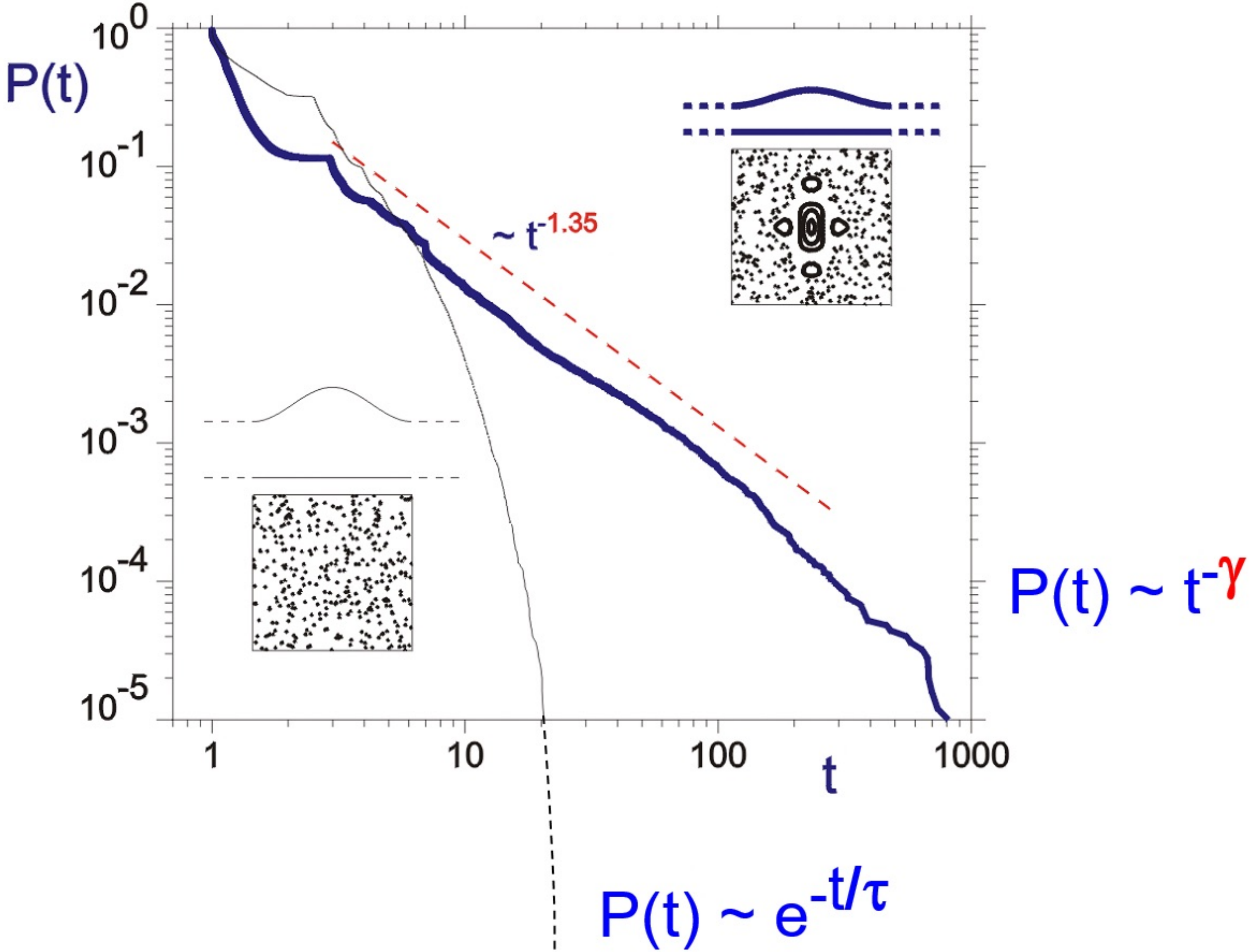
Some phenomena:

- self-similarity of scattering function $\Theta_{final}(b)$



$V(x,y) = x^2 y^2 e^{-(x^2+y^2)}$ Ott, Tél 1993

• distribution of delay times $\stackrel{1}{=}$ Poincaré recurrence time distrib.



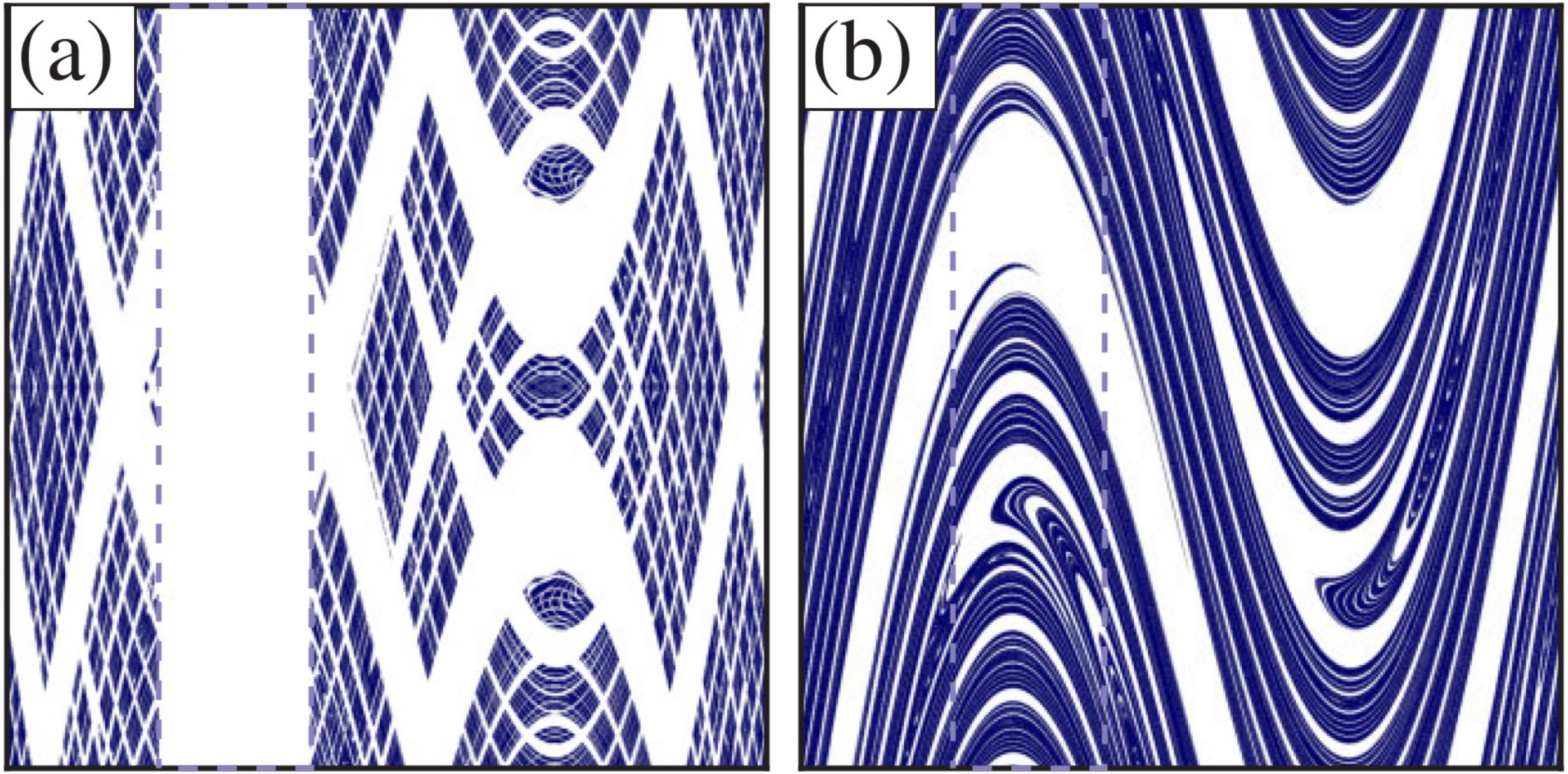
· points never leaving system

· under backw. + forw. iteration

· under backward iteration

chaotic saddle

backward trapped set



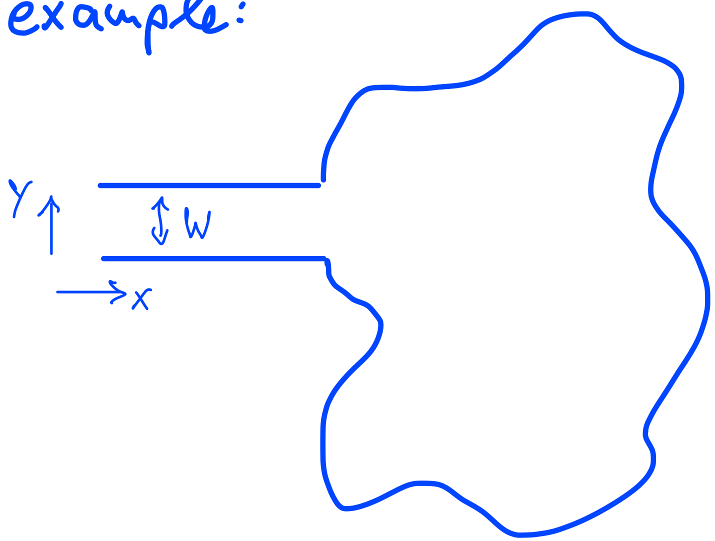
standard map K=10 (half-kick)

11.2. S-Matrix fluctuations

S-Matrix: $\psi = \psi_m^{in} + \sum_n S_{mn} \psi_n^{out}$

solution of S.E. — incoming wave m — outgoing wave n

example:



$$\psi_n^{\pm} = e^{\pm i k_x^n x} \sin k_y^n y \qquad \begin{array}{l} + \text{ incoming} \\ - \text{ outgoing} \end{array}$$

$$\left.\begin{array}{l} k_y^n = \dfrac{n\pi}{W} \qquad n = 1, 2, \dots, N \\ k_x^n = \sqrt{\dfrac{2mE}{\hbar^2} - (k_y^n)^2} \end{array}\right\} \quad E = \frac{\hbar^2}{2m}\left((k_x^n)^2 + (k_y^n)^2\right)$$

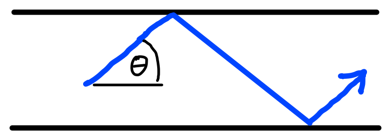
S_{mn} is unitary $N \times N$ matrix

Semiclassical approximation of matrix element S_{mn}

$$S_{mn}(E) = \sum_j \sqrt{p_j}\, e^{\frac{i}{\hbar} S_j(E) - i \nu_j \frac{\pi}{2}}$$

j: classical trajectories corresponding to incoming mode m and outgoing mode n

$$|\tan \theta_m| \approx \frac{k_y^m}{k_x^m}, \quad \text{position arbitrary}$$



p_j: probability

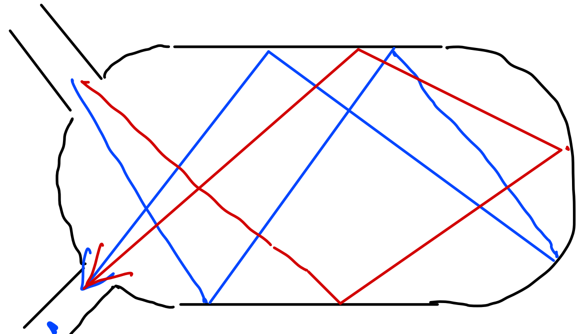
S_j: action

ν_j: # of conj. points

Energy dependence of $S_{mn}(E)$ contains information on chaos!

example: billiard

$$S_j = \int_j \vec{p}\, d\vec{q}_j = p\underbrace{l_j}_{\frac{p}{m}t_j} = \frac{p^2}{m}t_j = 2E\,t_j$$



$$\Rightarrow S_{mn}(E) = \sum_j \sqrt{p_j}\, e^{\frac{i}{\hbar}2E\,t_j}$$

variation of $E \Rightarrow$ phases change individually, depending on t_j

$\Rightarrow$ fluctuations of $S_{mn}(E)$

(correlated to classical fluctuations $\Theta_f(b)$)

aim: quantify fluctuations $\tilde{S} := S - \langle S \rangle_E$

Def.: auto correlation

$$C_{mn}(\Delta E) = \left\langle \tilde{S}^*_{mn}(E)\, \tilde{S}_{mn}(E+\Delta E) \right\rangle_E$$

semicl. approx.:

$$C_{mn}(\Delta E) = \left\langle \sum_{j,j'} \sqrt{p_j p_{j'}}\; e^{\frac{i}{\hbar}\left(S_j(E+\Delta E) - S_{j'}(E)\right)} \right\rangle_E$$

$$= \underbrace{\left\langle \sum_j p_j\, e^{\frac{i}{\hbar}\Delta E \frac{\partial S_j}{\partial E}} \right\rangle_E}_{\text{diagonal}} + \underbrace{\left\langle \sum_{\substack{j,j' \\ j\neq j'}} \dots \right\rangle}_{\text{off-diagonal}}$$

- diagonal approx.: $\sum_{j\neq j'} \approx 0$, as classical actions $S_j, S_{j'} \gg \hbar$ uncorrelated

use $\frac{\partial S_j}{\partial E} = t_j$

$$\Rightarrow \quad C_{mn}(\Delta E) = \int_0^\infty dt \; p(t) \; e^{\frac{i}{\hbar}\Delta E t}$$

Blümel, Smilansky 1988

quantum

classical

a) fully chaotic : $p(t) \sim e^{-\gamma t} \quad \Rightarrow \quad |C(\Delta E)| = C(0) - \text{const.}\,(\Delta E)^2$

b) mixed : $p(t) \sim t^{-\gamma-1} \quad \Rightarrow \quad |C(\Delta E)| = C(0) - \text{const.}\,(\Delta E)^\gamma$

$\gamma \approx 1.3 - 1.9$ Lai et al 1992

look at $S_{mn}(E)$: $\Delta S := |S(E+\Delta E)|^2 - |S(E)|^2$

$$\Rightarrow \left.\begin{array}{l} \Delta S \text{ Gaussian} \\ \langle \Delta S \rangle = 0 \\ \langle (\Delta S)^2 \rangle \sim (\Delta E)^\gamma \end{array}\right\} \text{Fractional Brownian Motion}$$

$$D = 2 - \frac{\gamma}{2} \qquad \gamma \in (1,2)$$

fractional Brownian motion

Mandelbrot 1968

stochastic process $x(t)$ with Gaussian increments:

$$< x(t+\Delta t) - x(t) > = 0$$

$$<[x(t+\Delta t) - x(t)]^2> \sim (\Delta t)^{\gamma} \quad \text{with } \gamma \in [0,2]$$

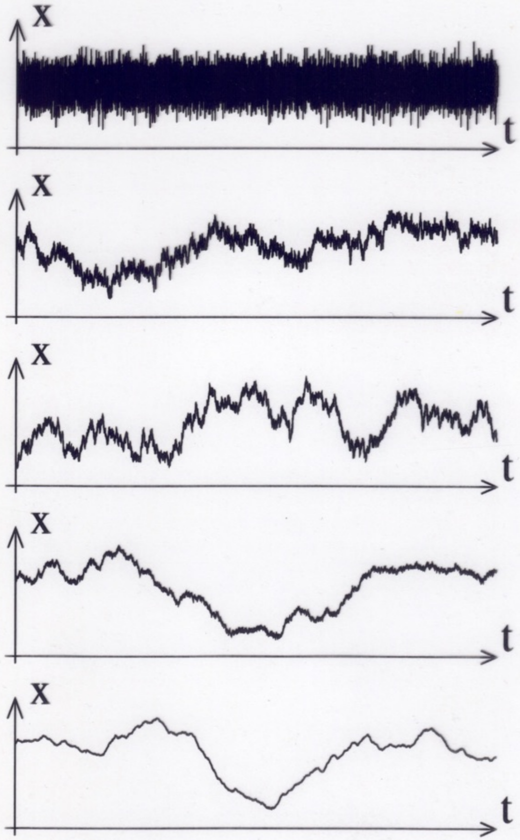
$\gamma = 0$ white noise — D=2

$\gamma = 0.5$ — D=1.75

$\gamma = 1$ Brownian motion — D=1.5

$\gamma = 1.5$ — D=1.25

$\gamma = 2$ — D=1



fractal dimension

$$D = 2 - \gamma/2$$

experiments with semiconductor billiards

$\Rightarrow$ vary magnetic field B (not energy E):

$$S_j(B) = S_j(B_0) + \frac{(B-B_0)\cdot A_j}{h/e}$$

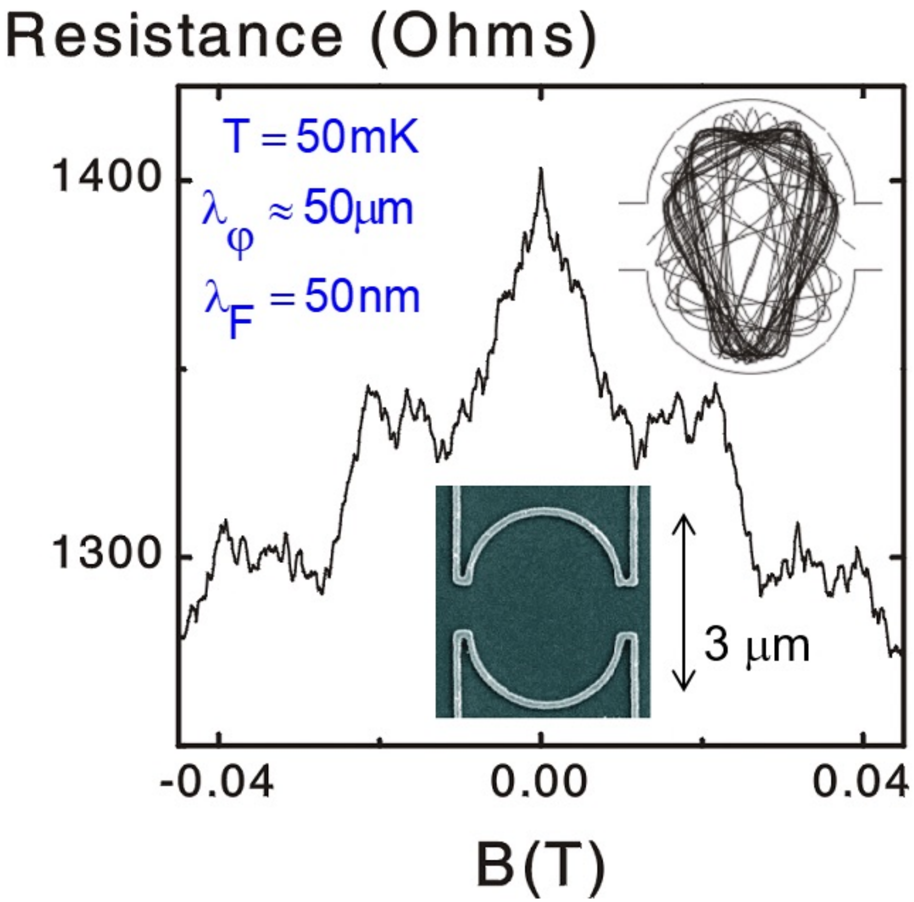
area enclosed by trajectory

a) fully chaotic: $p(A) \sim e^{-\alpha |A|}$ $\Rightarrow$ $C(\Delta B) = \dfrac{C(0)}{\left[1+\left(\dfrac{\Delta B}{\alpha\, h/e}\right)^2\right]^2}$

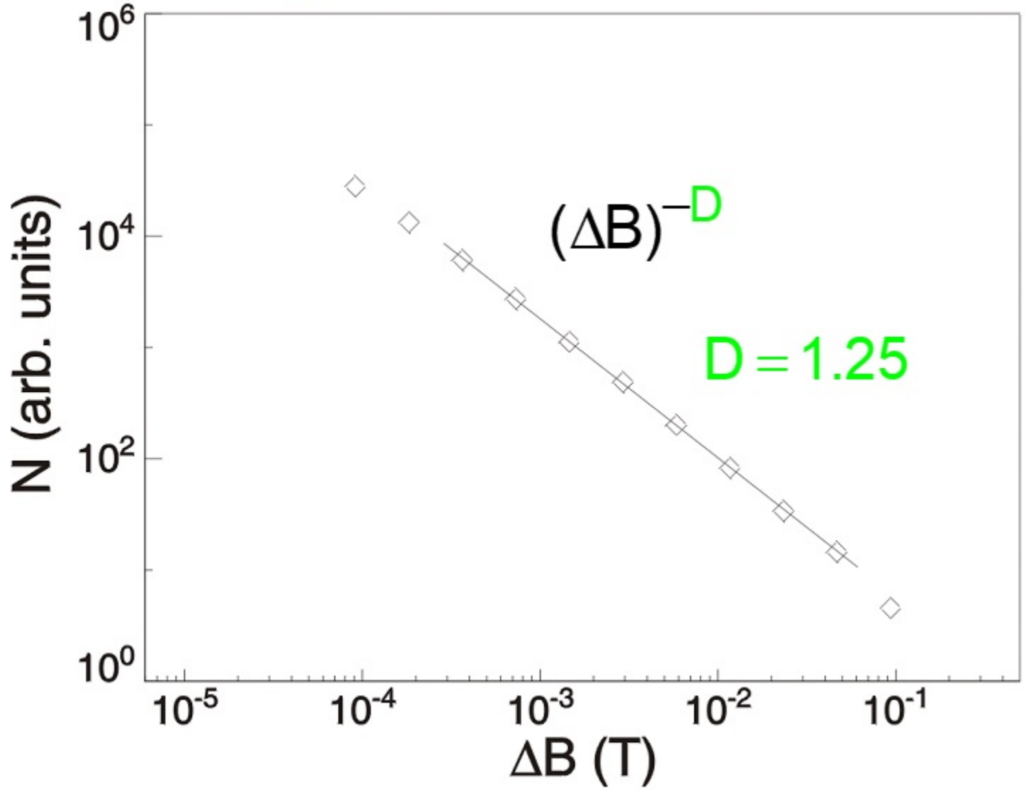
Jalabert, Baranger, Stone 1990

b) mixed: $p(A) \sim A^{-\gamma-1}$ $\Rightarrow$ Fractional Brownian motion

$D = 2 - \frac{\gamma}{2}$



fractal analysis:



Sachrajda et al. (1998)