11.3. Resonance States

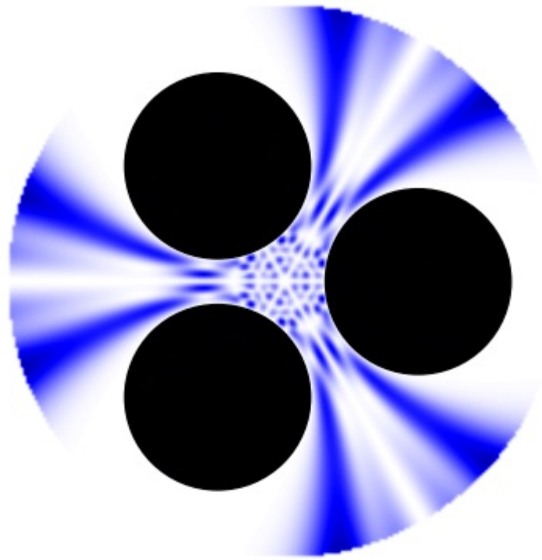
Solutions of

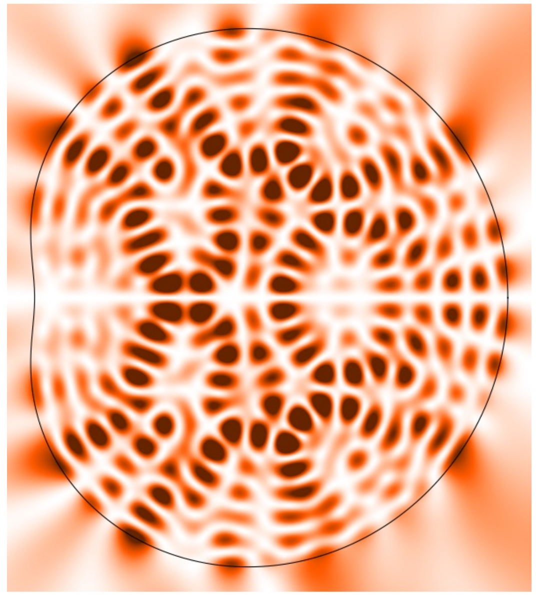
- Schrödinger equation
- boundary condition
- without incoming wave

$\Rightarrow$ outgoing solution decaying in time

$\Rightarrow$ resonance poles at discrete $E_n \in \mathbb{C}$

with $\mathrm{Im}(E_n) < 0$





$\Rightarrow$ decay of norm:

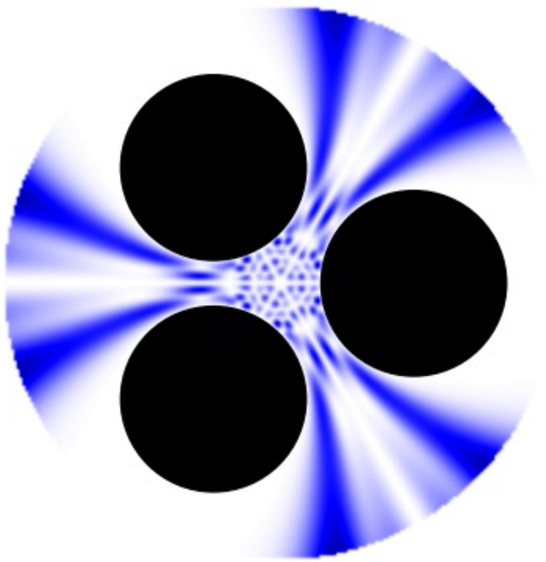
$$\left| e^{-\frac{i}{\hbar}Et} \right|^2 = e^{-\frac{i}{\hbar}\overbrace{(E-E^*)}^{2i\,\mathrm{Im}(E)}t} = e^{\frac{2}{\hbar}\mathrm{Im}(E)\,t}$$

$$\overset{!}{=} e^{-\gamma t} \qquad \Rightarrow \text{ decay rate } \gamma = -\mathrm{Im}(E)\cdot\frac{2}{\hbar}$$

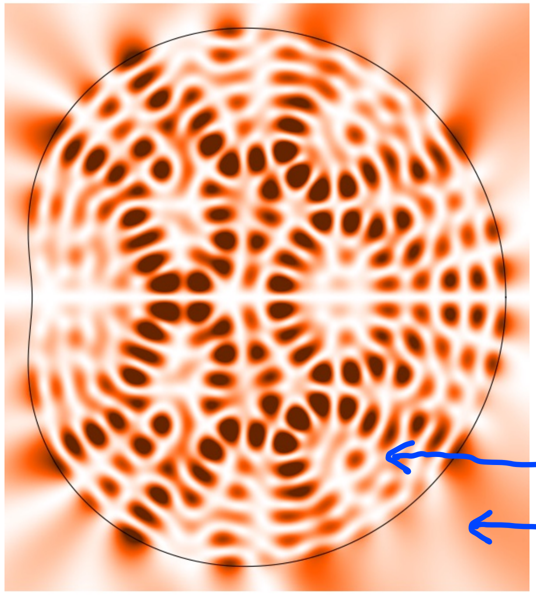
Chaotic systems:

- distribution/statistics of poles in $\mathbb{C}$ → fractal Weyl law
- structure of resonance states ergodic?

full escape



partial escape



Quantum maps with partial / full escape

$$\mathcal{U}_{open} = \mathcal{U}_{closed} \; P$$

full escape

P: projector on non-escaping ph. sp. region

$$\mathcal{U}_{open} = \mathcal{U}_{closed} \; R$$

partial escape

$\mathcal{U}_{open}$: subunitary

$\mathcal{U}_{closed}$: unitary

R: reflectivity operator on ph. sp.

$\Rightarrow$ eigenvalue problem $\quad \mathcal{U}_{open} \psi = \lambda \psi \quad$ with $|\lambda| < 1$

$$|\mathcal{U}_{open} \psi|^2 = |\lambda|^2 \cdot |\psi|^2 \quad \Rightarrow \quad |\lambda|^2 = e^{-\gamma}$$

γ: decay rate