1. Motivation

1.1. What is chaos ?

- everyday life
- mathematics and physics :

  chaos is a dynamical property

  sensitive dependence on initial conditions

  → butterfly effect on weather

  Lorenz 1960s

  „Does the flap of a butterfly's wing in Brazil set off a Tornado in Texas ?"

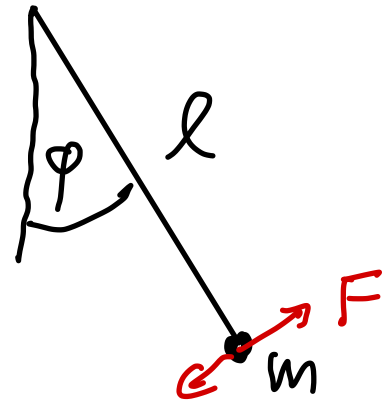
→ Atmospheric Physics, Kantz

→ Network Dynamics and Research on Complex Systems, Timme

here : Hamiltonian systems – classical and quantum

Example: Double pendulum

Example: Driven pendulum



force balance in direction of motion:

acc. force — gravitation — ~~dissipation~~ — driving

$$m \ell \ddot{\varphi} = -mg \sin\varphi \; \cancel{- \eta \ell \dot{\varphi}} \; + F \sin \omega t$$

$$\ddot{\varphi} + \frac{g}{\ell} \sin\varphi = \frac{F}{m\ell} \sin \omega t$$

dimensionless units (important for numerics!)

$$\tilde{t} = \frac{t}{\sqrt{\frac{\ell}{g}}} \quad \Rightarrow \quad \ddot{\varphi} = \frac{d^2\varphi}{dt^2} = \frac{d^2\varphi}{d\tilde{t}^2} \cdot \frac{g}{\ell}$$

$$\Rightarrow \quad \frac{d^2\varphi}{d\tilde{t}^2} + \sin\varphi = \underbrace{\frac{F}{mg}}_{=:\tilde{A}} \sin \underbrace{\omega\sqrt{\frac{\ell}{g}}}_{=:\tilde{\omega}} \tilde{t}$$

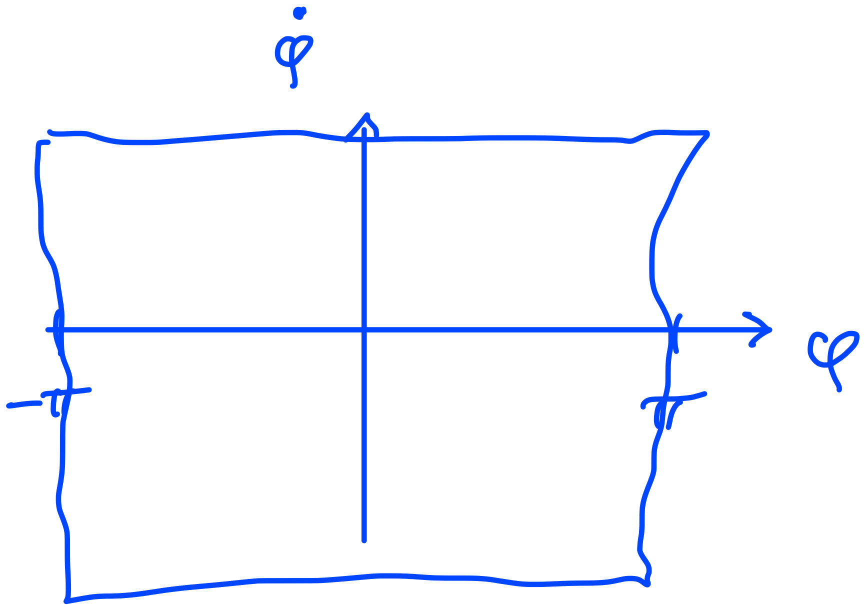
dim. less frequency

amplitude

without tilde:

$$\boxed{\ddot{\varphi} + \sin\varphi = A \sin \omega t}$$

phase space:



Presented examples for Hamiltonian chaos

- Double pendulum
- Driven pendulum
- Billiards
- Kicked rotor – standard map

$\longrightarrow$ regular and chaotic dynamics

1.2. What is quantum chaos?

classical mechanics

deterministic: Hamilton eq.

chaos ⇒ practically non-deterministic

quantum mechanics

deterministic: Schrödinger eq.

non-deterministic:

- probability for result of measurement
- Heisenberg uncertainty relation

← correspondence principle $\hbar \to 0$

related?

independent?

contradictory?

questions:

- Is there chaos in quantum mechanics?
- Does q.m. care about chaos?
- How to get q.m. properties (spectrum, eigenfunctions) from classical information?

  Bohr-Sommerfeld quantization $\oint p\,dq = (n+\frac{1}{2})\hbar$ for chaotic dynamics?

  (Einstein 1917)
- Can one understand q.m. experiments by knowing about chaotic dynamics?
- New q.m. phenomena due to chaos?

names for the field:

- quantum chaology
- quantum signatures of chaos
- quantum chaos

1.3 Contents

classical chaos:

- dynamical systems, fixed points, dynamics near fixed points
- integrable systems, action-angle variables, dynamics on tori
- cat map
- kicked rotor / standard map
- billiards
- KAM theorem, Poincaré-Birkhoff theorem
- homoclinic tangle
- Smale's horseshoe
- ergodicity, entropy, Lyapunov-exponent
- transport in phase space: diffusion, anomalous diffusion, Cantori, turnstiles

quantum chaos:

- level statistics + eigenfunction statistics
- random matrix theory
- Husimi representation
- Floquet theory
- qm kicked rotor
- dynamical localization (related to Anderson localization in disordered systems)
- semiclassics, Gutzwiller trace formula
- orbit bunching and spectral statistics
- chaotic scattering, conductance fluctuations
- experimental systems

Literature:

- quantum chaos: Haake, Stöckmann, (Gutzwiller), Reichl, Wimberger
- classical: Ozorio de Almeida, Tabor (+ quantum chaos), Lichtenberg/Liebermann
- math.: Arrowsmith/Place, Arnold
- nonlinear dynamics: Schuster, Ott