Technische Universität Dresden
Fakultät Physik, Institut für Theoretische Physik
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Chaos and Quantum Chaos

Summer term 2021

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2. Exercise

(to be presented and discussed on Monday April 26)

Repetition

1. What is the definition of a fixed point of a dynamical system (time-continuous and time-discrete)?
2. What kind of dynamics is possible near a fixed point of a Hamiltonian system?
3. When are N frequencies incommensurate?
4. What is the definition of ergodicity?

2.1 Eigenvalues of dynamical matrix

Show that the dynamical matrix A near a fixed point of a hamiltonian system has eigenvalues occurring in pairs $\lambda, -\lambda$.
Hint: Use $A = SM$, $M^T = M$, $S^T = -S$, and that S is unitary ($S^\dagger S = SS^\dagger = \mathbb{1}$).

2.2 Integrable dynamics on N-torus

Give a proof for the following statements:

1. The integrable dynamics $\vec{\varphi}(t) = \vec{\varphi}_0 + \vec{\omega}t$ on a N-torus with incommensurate frequencies $\vec{\omega}$ is ergodic.

   Hint: Write an arbitrary function $f(\vec{\varphi})$ as a Fourier series.

2. From 1. follows that a trajectory is dense on the torus.

   Hint: Choose a convenient $f(\vec{\varphi})$.

3. From 1. follows that a trajectory spents on average a time t in a region D that is proportional to the volume V_D of region D.

   Hint: Choose a convenient $f(\vec{\varphi})$.

2.3 Cat map

a) Find the periodic points of the cat map.

   Hint: Consider points on a lattice. Try first to find an upper bound for the period. Are there other periodic points?

b) Determine whether the periodic points are elliptic or hyperbolic.

c) Determine the period of the periodic points. *(difficult!)*

d) Which lattice size ≈ 1000 gives periodic points with a small period $p = 16$? *(difficult!)*