Technische Universität Dresden
Fakultät Physik, Institut für Theoretische Physik
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Chaos and Quantum Chaos

Summer term 2021

Homework - Classical Chaos (2 out of 3 required)

H1 Time Evolution of Phase-Space Region (Due: Monday 10.05.2021, 10:00)

Determine the time evolution for various phase space regions of the standard map on the torus with $K = 2.5$. Use a circular initial region (represented by a sufficiently large number of initial points) with radius 0.1 and centered on the point (Θ, p) with $p = 0$ and

a) $\Theta = 2.8$ (regular)

b) $\Theta = 2.0$ (period 4 resonance chain)

c) $\Theta = 2.38$ (period 16 resonance chain)

d) $\Theta = 1.0$ (chaotic).

Hand in four figures showing the 0. and the 10. iterate of the initial region. Add some of the relevant regular tori to the background of the figure (choose $\Theta \in [0, 2\pi]$, $p \in [-\pi, \pi]$).

H2 Homoclinic tangle (Due: Monday 17.05.2021, 10:00)

Determine the stable and unstable manifolds of the hyperbolic fixed point at $p = 0$ of the standard map on the torus with $K = 2$.

a) Analytical: Determine the eigenvectors of the linearized map at the fixed point.

b) Numerical: Spread 10^4 points along these eigenvectors (close enough to the fixed point) and iterate them forward and backward in time, such that a few (not too many!) homoclinic points and loops develop.

Hand in the calculation for a) and a figure (with colors and labels) for b).

H3 Poincaré Recurrence Time Statistics (Due: Monday 7.06.2021, 10:00)

Determine the Poincaré recurrence time distribution for the standard map at different kicking strengths. Consider as initial and return region a part of the chaotic sea, which contains e.g. 1% of the total phase space volume. Iterate the initial points (with $N(0)$ at least 10^7 better 10^9) until they return to the initial region the first time (t_{return}) and compute the fraction of orbits, which have not yet returned

$$R(t) = \frac{N(t)}{N(0)} = \frac{\text{number of orbits with} \quad t_{\text{return}} > t}{\text{number of started orbits}}.$$

Consider $K = 10$ (where almost the whole phase space is chaotic) and $K = 2.5$ (where regular islands do exist).

a) Compare the results for $R(t)$ in one figure on a double logarithmic scale. Include a (roughly) fitted exponential decay for $K = 10$ and a (roughly) fitted power-law decay for $K = 2.5$ at large times.

b) Same data as in a) but on a semilogarithmic scale.

c) Show all iterations of the orbit with the longest return time for $K = 2.5$. Where does it get trapped?