

§4. Transport properties of solids

* Classical theory (Drude's theory)

Q: How does the solid react in the presence of an external perturbation?

electric field \vec{E}

temperature gradient $\vec{\nabla}T$

- DC electrical conductivity

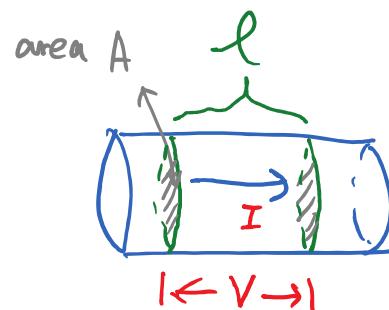
↳ direct current

Ohm's law:

$$V = IR$$

potential difference
↓
current
resistance

$$R = \rho \frac{l}{A} \rightarrow \begin{array}{l} \text{length} \\ \text{cross-sectional area} \\ \text{resistivity} \end{array}$$



$$J = \frac{I}{A}$$

↳ current density

$$V = El$$

↳ electric field

$$\Rightarrow El = JA \cdot \rho \frac{l}{A}$$

$$E = \rho J$$

$$\rightarrow \vec{E} = \rho \vec{J}$$

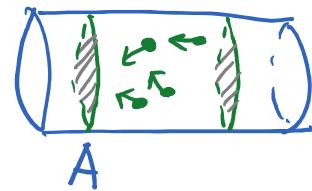
resistivity (assumed
to be isotropic)

$$\vec{J} = \sigma \vec{E}$$

conductivity
 $\sigma = \frac{1}{\rho}$

electron density (more precisely, "carrier" density)

$$n = \frac{N}{AL} \quad (N: \# \text{ of electrons within volume } AL)$$



velocity v $\xrightarrow{\text{after time } dt}$ distance $v dt$

$\Rightarrow n \times (v dt) \times A$ electrons cross the area A
 \downarrow charge $-e$ ($e > 0$)

current $I = \frac{dQ}{dt} = \frac{-e n(v dt)A}{dt} = -envA$

current density $J = \frac{I}{A} = -env$
 \downarrow
 $\overline{J} = -env$

No electric field, \vec{v} averaged to zero \Rightarrow no charge current

With electric field,

$$\vec{v}_0 \xrightarrow{\text{time } \tau} \vec{v}_0 + \frac{(-e)\vec{E}}{m} \tau \quad \begin{array}{l} \text{relaxation time} \\ (\text{collision/scattering}) \end{array}$$

$$\Rightarrow \vec{v}_{\text{avg}} \approx -\frac{e\vec{E}}{m}\tau$$

$$\overline{J} = -en\vec{v}_{\text{avg}} = \underbrace{\frac{ne^2\tau}{m}\vec{E}}$$

$$\tau = \frac{\sigma m}{ne^2} \simeq 10^{-14} \text{ s} \quad \parallel \sigma : \text{conductivity}$$

(room temperature)

Set up E.O.M.:

$$\vec{V}(t) = \frac{\vec{P}(t)}{m} \quad \text{momentum per electron}$$

$$\vec{P}(t+dt) \begin{cases} \xrightarrow{\text{collision}} \text{probability } \frac{dt}{\tau} \\ \xrightarrow{\text{no collision}} \text{probability } 1 - \frac{dt}{\tau} \end{cases}$$

$$\vec{P}(t+dt) = \vec{P}(t) + \vec{f}(t) dt \quad \begin{matrix} \vec{f}(t) dt \\ \text{random average} \end{matrix}$$

Averaging over two possibilities:

$$\begin{aligned} \vec{P}(t+dt) &= \frac{dt}{\tau} \cdot \vec{f}(t) dt + \left(1 - \frac{dt}{\tau}\right) (\vec{P}(t) + \vec{f}(t) dt) \\ &= \vec{P}(t) - \frac{\vec{P}(t)}{\tau} dt + f(t) dt + O(dt^2) \end{aligned}$$

$$\vec{P}(t+dt) - \vec{P}(t) = \frac{d\vec{P}}{dt}$$

$$\Rightarrow \underbrace{\frac{d\vec{P}}{dt}}_{-\frac{\vec{P}}{\tau}} = -\frac{\vec{P}}{\tau} + f(t) \quad \begin{matrix} \text{classical E.O.M.} \\ \text{under the relaxation} \\ \text{time assumption} \end{matrix}$$

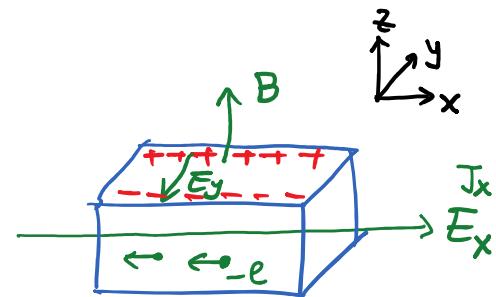
$-\frac{\vec{P}}{\tau}$: damping term

$$[\text{If } f(t) = 0, \quad \vec{P}(t) = \vec{P}_0 e^{-t/\tau}]$$

- Hall effect (E.H. Hall, 1879)

$$\frac{d\vec{p}}{dt} = -\frac{\vec{P}}{\tau} - e(\vec{E} + \frac{\vec{P}}{m} \times \vec{B})$$

Lorentz force



Steady state: $\frac{dP_x}{dt} = \frac{dP_y}{dt} = 0$

$$\Rightarrow \left\{ \begin{array}{l} 0 = -\frac{P_x}{\tau} - eE_x - e\frac{P_y}{m} B \\ 0 = -\frac{P_y}{\tau} - eE_y + e\frac{P_x}{m} B \end{array} \right. \quad (1)$$

$$\left. \begin{array}{l} 0 = -\frac{P_y}{\tau} - eE_y + e\frac{P_x}{m} B \end{array} \right. \quad (2)$$

$P_y = 0$: balance of E_y and B

$$(1) \Rightarrow E_x = -\frac{P_x}{e\tau}$$

$$J_x = -nev_x = -ne\frac{P_x}{m}$$

$$\Rightarrow P_x = \frac{E_x}{J_x} = \frac{m}{ne^2\tau} \quad \left. \begin{array}{l} \\ \text{(no } B \text{ dependence)} \end{array} \right\}$$

$$(2) \Rightarrow E_y = \frac{B}{m} P_x$$

$$J_x = -nev_x = -ne\frac{P_x}{m} \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\Rightarrow \frac{E_y}{J_x} = -\frac{B}{ne}$$

Hall coefficient: $R_H = \frac{E_y}{BJ_x} = -\frac{1}{ne}$

$$\text{Hall coefficient} \quad - \frac{l}{R_H ne}$$

Li	0.8	{	1 valence electron as "carrier"
Na	1.2		
K	1.1		
Cu	1.5		
Ag	1.3		

Be	-0.2	{	2 valence electrons	?!
Mg	-0.4			
Al	-0.3			

(It's better to think of "holes" with positive charge as carriers! \Rightarrow Quantum theory needed)

- AC electrical conductivity
 \hookrightarrow alternating current

$$\vec{E}(t) = \operatorname{Re}(\vec{E}(\omega)e^{-i\omega t})$$



$$\begin{aligned}\frac{d\vec{P}}{dt} &= -\frac{\vec{P}}{\tau} - e\vec{E}(t) \\ &= -\frac{\vec{P}}{\tau} - e\operatorname{Re}(\vec{E}(\omega)e^{-i\omega t})\end{aligned}$$

steady state: $\vec{P}(t) = \operatorname{Re}(\vec{P}(\omega)e^{-i\omega t})$

$$\left\{ \begin{array}{l} \vec{E}(\omega) = \operatorname{Re}E(\omega) + i\operatorname{Im}E(\omega) \\ \vec{P}(\omega) = \operatorname{Re}\vec{P}(\omega) + i\operatorname{Im}\vec{P}(\omega) \end{array} \right.$$

$$\text{E.O.M.} \Rightarrow -i\omega \vec{P}(\omega) = -\frac{\vec{P}(\omega)}{\tau} - e \vec{E}(\omega)$$

$$(i\omega - \frac{1}{\tau}) \vec{P}(\omega) = e \vec{E}(\omega)$$

Alternating current:

$$\begin{aligned}\vec{J}(t) &= -ne \frac{\vec{P}(t)}{m} = -\frac{ne}{m} \operatorname{Re} [\vec{P}(\omega) e^{-i\omega t}] \\ &= \operatorname{Re} [\vec{J}(\omega) e^{-i\omega t}]\end{aligned}$$

$$\begin{aligned}\Rightarrow \vec{J}(\omega) &= -\frac{ne}{m} \vec{P}(\omega) \\ &= -\frac{ne}{m} \frac{e}{i\omega - \frac{1}{\tau}} \vec{E}(\omega) \\ &\quad \text{~~~~~} \approx \\ &\quad \sigma(\omega) : \text{AC conductivity}\end{aligned}$$

$$\sigma(\omega) = \frac{ne^2 \tau}{m(1-i\omega\tau)} = \frac{\sigma_0}{1-i\omega\tau}$$

$$\sigma_0 = \frac{ne^2 \tau}{m} : \text{DC Drude conductivity}$$

$$\sigma(\omega \rightarrow 0) = \sigma_0 \checkmark$$