SS2020: Solid State Theory
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Problem Set 2

1. **Debye model in two dimensions** (2+3+3 points)

Consider acoustic phonons in a two-dimensional crystal described by

$$H = \sum_{\mathbf{q}} \sum_{\nu=1,2} \hbar\omega_{\mathbf{q},\nu}(a^\dagger_{\mathbf{q},\nu} a_{\mathbf{q},\nu} + \frac{1}{2}),$$

where $\omega_{\mathbf{q},\nu} \approx v_s|\mathbf{q}|$ in the long wavelength limit.

(a) Calculate the internal energy U and heat capacity C at temperature T.

(b) Show that the heat capacity approaches a constant at high temperature.

(c) Show that the heat capacity behaves as $C \propto T^2$ at low temperature.

2. **Free electron gas in low dimensions** (2+2+2+2+2 points)

Consider free electron gases confined in 1D and 2D boxes with "volume" V. The total number of electrons is N_e.

(a) Calculate the density of states (DOS) $D(\varepsilon)$, plot them and compare with the 3D result.

(b) Determine the Fermi energies ε_F by using the DOS obtained in (a). What are the corresponding Fermi wave vectors k_F?

(c) Calculate the total energies of the Fermi gas at zero temperature. Compare your results with the 3D case ($E_{3\mathrm{D}}/N_e = \frac{3}{5}\varepsilon_\mathrm{F}$).

(d) Now consider the low temperature regime $T \ll T_\mathrm{F} = \varepsilon_\mathrm{F}/k_\mathrm{B}$ and calculate the internal energies of the 1D and 2D free electron gases

$$U(T) = \int_{-\infty}^{\infty} d\varepsilon\, \varepsilon D(\varepsilon) f(\varepsilon, \mu, T),$$

where $f(\varepsilon, \mu, T) = 1/(e^{\beta(\varepsilon-\mu)} + 1)$ is the Fermi-Dirac distribution function.

Hint: You may use the Sommerfeld expansion to approximate the integral.

(e) Calculate the specific heat based on the results obtained in (d). How does the specific heat behave when $T \to 0$?