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Problem Set 3

1. **Interacting electron gas with screened Coulomb potential** (3+5 points)

Consider electrons in 3D which interact via a screened Coulomb (Yukawa) potential

$$V(\mathbf{r}-\mathbf{r}') = \frac{A}{|\mathbf{r}-\mathbf{r}'|} e^{-\lambda|\mathbf{r}-\mathbf{r}'|} \,.$$

(a) Derive the Fourier transform of the screened Coulomb potential and write down the second-quantized Hamiltonian in momentum space.

(b) Use the perturbation theory to derive the Hartree-Fock correction to the total energy of the electrons.

2. **Density-density correlation function** (4+4+2 points)

Consider the Fermi sphere of a *spinless* Fermi gas in 3D

$$|\mathrm{FS}\rangle = \prod_{|\mathbf{k}|<k_{\mathrm{F}}} c^\dagger_{\mathbf{k}} |0\rangle \,.$$

(a) Calculate

$$G(\mathbf{r}_1, \mathbf{r}_2) = \langle \psi^\dagger(\mathbf{r}_1) \psi(\mathbf{r}_2) \rangle \,,$$

where $\psi(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$ and $\langle \cdots \rangle$ is the expectation value with respect to $|\mathrm{FS}\rangle$. Here $G(\mathbf{r}_1, \mathbf{r}_2)$ is often referred to as the one-particle reduced density matrix or correlation matrix.

(b) Calculate the density-density correlation function

$$C(\mathbf{r}_1, \mathbf{r}_2) = \langle n(\mathbf{r}_1) n(\mathbf{r}_2) \rangle - \langle n(\mathbf{r}_1) \rangle \langle n(\mathbf{r}_2) \rangle \,,$$

where $n(\mathbf{r}) = \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$ is the particle-density operator.

(c) Plot $C(\mathbf{r}_1, \mathbf{r}_2)$ as a function of the distance $|\mathbf{r}_1 - \mathbf{r}_2|$ and interpret the results (e.g. short/large distance behaviors and a characteristic oscillation known as "Friedel oscillation").