SS2020: Solid State Theory
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Problem Set 7

1. **Chiral edge states** (3+3+3 points)

The following Hamiltonian describes the quantum Hall system discretized on a square lattice:

$$H = t\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}(e^{-in\alpha}c^\dagger_{m,n}c_{m+1,n} + e^{in\alpha}c^\dagger_{m+1,n}c_{m,n}) + t\sum_{m=1}^{N_x}\sum_{n=1}^{N_y-1}(c^\dagger_{m,n}c_{m,n+1} + c^\dagger_{m,n+1}c_{m,n}),$$

where $c^\dagger_{m,n}$ ($c_{m,n}$) is the electron creation (annihilation) operator at site $\mathbf{r} = (ma, na)$ (a: lattice spacing). The boundary condition is periodic in x-direction with $c_{m+N_x,n} = c_{m,n}$ and open in y-direction.

(a) Show that the Hamiltonian is reduced to

$$H = \sum_{k_x}\sum_{n,n'=1}^{N_y} c^\dagger_{k_x,n}[\mathcal{H}(k_x)]_{n,n'}c_{k_x,n'}$$

with

$$[\mathcal{H}(k_x)]_{n,n'} = t(\delta_{n',n+1} + \delta_{n',n-1}) + 2t\cos(k_x a - n\alpha)\delta_{n',n}$$

by using the Fourier transformation $c_{m,n} = \frac{1}{\sqrt{N_x}}\sum_{k_x} c_{k_x,n}e^{ik_x ma}$, where $k_x = 0, \pm\frac{2\pi}{N_x a}, \ldots, \frac{\pi}{a}$.

(b) Consider a finite-size lattice with $N_x = 100$, $N_y = 30$ and $a = 1$. The parameters in the Hamiltonian may be taken as $t = 5$ and $\alpha = 0.2$. Diagonalize the matrix $\mathcal{H}(k_x)$ numerically for each k_x to determine the single-particle energies $E_q(k_x)$,

$$\sum_{n,n'=1}^{N_y}[U(k_x)]_{q,n}[\mathcal{H}(k_x)]_{n,n'}[U(k_x)]^\dagger_{n',q'} = E_q(k_x)\delta_{q,q'},$$

where $U(k_x)$ is the unitary matrix encoding the first-quantized single-electron wave function (why?). Plot $E_q(k_x)$ as a function of k_x.

Hint: The Hamiltonian is diagonalized as

$$H = \sum_{k_x}\sum_{q=1}^{N_y}E_q(k_x)d^\dagger_{k_x,q}d_{k_x,q},$$

where $d_{k_x,q} = \sum_{n=1}^{N_y}[U(k_x)]_{q,n}c_{k_x,n}$.

(c) Pick up several single-electron wave functions (labeled by k_x and q) in the lowest Landau level. Can you distinguish which ones correspond to chiral edge states and which ones are "bulk" states? Verify your results by plotting $|[U(k_x)]_{q,n}|^2$ (probability of finding the electron in the n-th row of the lattice) as a function of n.