TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

Solid State Theory (SS2020)

Lecture 14: Classical electron transport theory

Hong-Hao Tu (*ITP, TU Dresden*)

Email: hong-hao.tu@tu-dresden.de
Zoom: tuhonghao@gmail.com

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§ 4.1 Drude theory

- Q: How does the solid react in the presence of external perturbations?

electric field $\vec{E}$

temperature gradient $\vec{\nabla} T$

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- Direct-current (DC) conductivity:

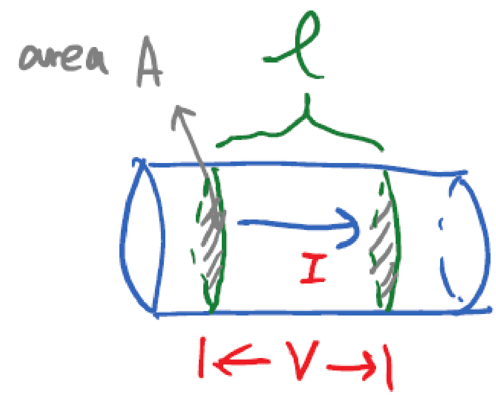
Ohm's law:

$$V = I R$$

V: potential difference

I: current

R: resistence



§ 4.1 Drude theory

- Direct-current (DC) conductivity:

Ohm's law:

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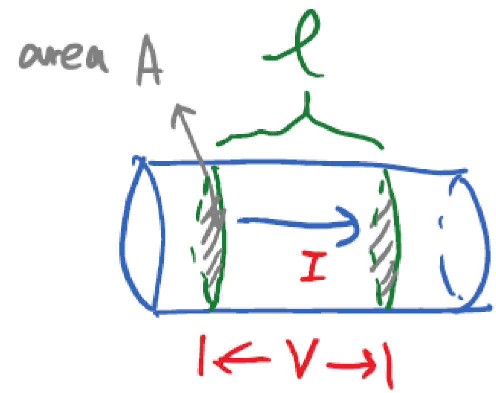
potential difference — current — resistence



$$R = \rho \frac{\ell}{A}$$

length — cross-sectional area — resistivity

$$J = \frac{I}{A}$$

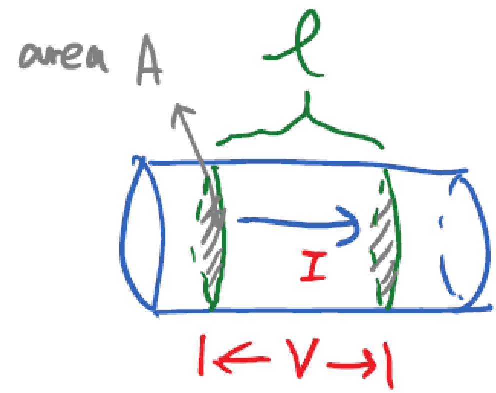
current density

$$V = E \ell$$

electric field

§ 4.1 Drude theory

- Direct-current (DC) conductivity:



Ohm's law:

$$V = I R$$

potential difference

current

resistence

$$\Rightarrow \quad E\ell = JA \cdot \rho \frac{\ell}{A}$$

$$E = \rho J \longrightarrow \vec{E} = \rho \vec{J}$$

resistivity (assumed to be isotropic)

$$\vec{J} = \sigma \vec{E}$$

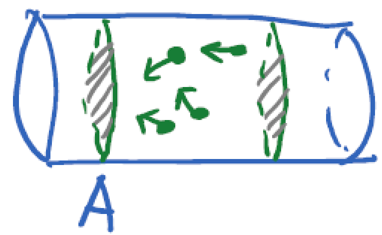
conductivity $\sigma = \frac{1}{\rho}$

§ 4.1 Drude theory

electron density (more precisely, "carrier" density)

$$n = \frac{N}{AL}$$

(N: # of electrons within volume AL)



velocity v $\xrightarrow{\text{after time } dt}$ distance $v\,dt$

$\Rightarrow$ $n \times (v\,dt) \times A$ electrons cross the area A

↳ charge $-e$ $(e>0)$

§ 4.1 Drude theory

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↳ charge $-e$ $(e>0)$

current $I = \dfrac{dQ}{dt} = \dfrac{-en(v\,dt)A}{dt} = -envA$

current density $J = \dfrac{I}{A} = -env$

$$\vec{J} = -en\vec{v}$$

§ 4.1 Drude theory

No electric field, $\vec{v}$ averaged to zero $\Rightarrow$ no charge current

With electric field,

$$\vec{v}_0 \xrightarrow{\text{time } \tau} \vec{v}_0 + \frac{(-e)\vec{E}}{m}\tau$$

relaxation time

(collision / scattering)

$$\Rightarrow \vec{V}_{avg} \simeq -\frac{e\vec{E}}{m}\tau$$

$$\vec{J} = -en\vec{V}_{avg} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma}\vec{E}$$

σ : conductivity

$$\tau = \frac{\sigma m}{ne^2} \simeq 10^{-14}s$$

(room temperature)

§ 4.1 Drude theory

- Equation of motion (E.O.M.):

$$\vec{V}(t) = \frac{\vec{p}(t)}{m}$$

momentum per electron

$$\vec{p}(t+dt)$$

- collision probability $\frac{dt}{\tau}$:

$$\vec{p}(t+dt) = \underbrace{0}_{\text{random average}} + \vec{f}(t)\,dt$$

- no collision : probability $1 - \frac{dt}{\tau}$

$$\vec{p}(t+dt) = \vec{p}(t) + \underbrace{\vec{f}(t)}_{\text{external force}}\,dt$$

§ 4.1 Drude theory

Averaging over two possibilities:

$$\vec{p}(t+dt) = \frac{dt}{\tau} \cdot \vec{f}(t)dt + \left(1 - \frac{dt}{\tau}\right)\left(\vec{p}(t) + \vec{f}(t)dt\right)$$

$$= \vec{p}(t) - \frac{p(t)}{\tau}dt + f(t)\,dt + O(dt^2)$$

$$\vec{p}(t+dt) - \vec{p}(t) = \frac{d\vec{p}}{dt}$$

§ 4.1 Drude theory

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$$\vec{p}(t+dt) - \vec{p}(t) = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} + f(t)$$

classical EOM under the relaxation time assumption

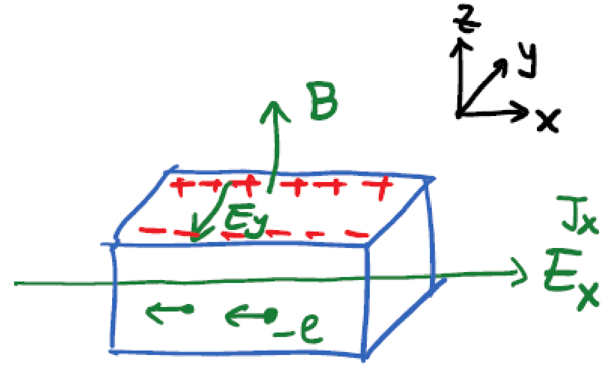
$-\frac{\vec{p}}{\tau}$: damping term

If $f(t) = 0, \vec{p}(t) = \vec{p}_0 e^{-t/\tau}$.

§ 4.1 Drude theory

- Hall effect:

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\left(\vec{E} + \underbrace{\frac{\vec{p}}{m} \times \vec{B}}_{\text{Lorentz force}}\right)$$



§ 4.1 Drude theory

- Hall effect:

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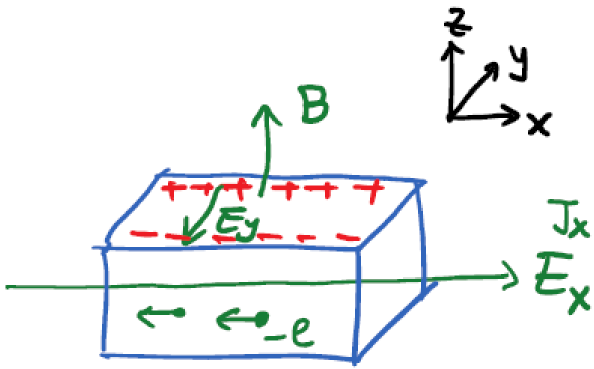
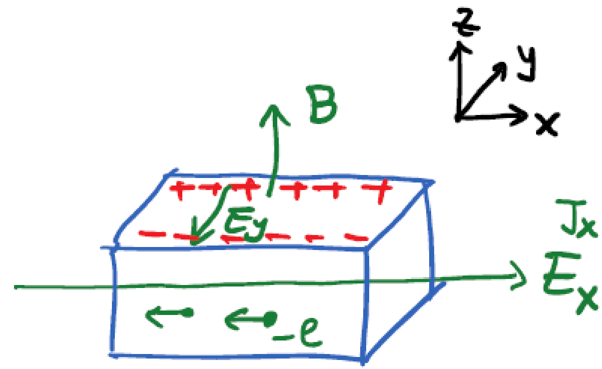


Steady state: $\frac{dp_x}{dt} = \frac{dp_y}{dt} = 0$

$$\begin{cases} 0 = -\frac{p_x}{\tau} - eE_x - e\frac{p_y}{m}B & (1) \\ 0 = -\frac{p_y}{\tau} - eE_y + e\frac{p_x}{m}B & (2) \end{cases}$$

§ 4.1 Drude theory

$$\begin{cases} 0 = -\dfrac{p_x}{\tau} - eE_x - e\dfrac{p_y}{m}B & (1) \\[2ex] 0 = -\dfrac{p_y}{\tau} - eE_y + e\dfrac{p_x}{m}B & (2) \end{cases}$$



$p_y = 0$: balance of E_y and B

(1) $\Rightarrow$

$$\left.\begin{aligned} E_x &= -\frac{p_x}{e\tau} \\ J_x &= -nev_x = -ne\frac{p_x}{m} \end{aligned}\right\}$$

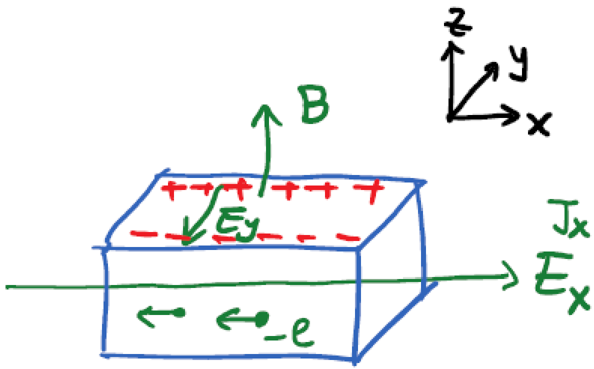
$$\Rightarrow \rho_x = \frac{E_x}{J_x} = \frac{m}{ne^2\tau}$$

magnetoresistance

(no B dependence)

§ 4.1 Drude theory

$$\begin{cases} 0 = -\dfrac{p_x}{\tau} - eE_x - e\dfrac{p_y}{m}B & (1) \\ 0 = -\dfrac{p_y}{\tau} - eE_y + e\dfrac{p_x}{m}B & (2) \end{cases}$$



$p_y = 0$: balance of E_y and B

$$(2) \Rightarrow \left.\begin{aligned} E_y &= \frac{B}{m}p_x \\ J_x &= -nev_x = -ne\frac{p_x}{m} \end{aligned}\right\}$$

$$\Rightarrow \frac{E_y}{J_x} = -\frac{B}{ne}$$

Hall coefficient: $R_H = \dfrac{E_y}{BJ_x} = -\dfrac{1}{ne}$

§ 4.1 Drude theory

Hall coefficient $\quad -\frac{1}{R_H ne}$

Li	0.8
Na	1.2
K	1.1
Cu	1.5
Ag	1.3

1 valence electron as "carrier"

§ 4.1 Drude theory

Hall coefficient $-\frac{1}{R_H n e}$

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1 valence electron as "carrier"

Be	−0.2
Mg	−0.4
Al	−0.3

2 valence electrons

3 valence electrons

?!

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Hall coefficient $-\frac{1}{R_H ne}$

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1 valence electron as "carrier"

Positively charged "holes" as carriers (quantum theory needed)

2 valence electrons

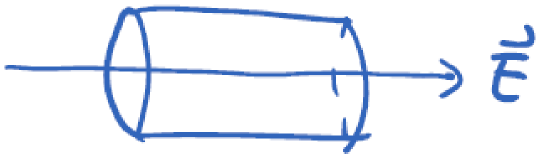
3 valence electrons

?!

§ 4.1 Drude theory

- Alternating-current (AC) conductivity:

$$\vec{E}(t) = \mathrm{Re}\left(\vec{E}(\omega) e^{-i\omega t}\right)$$



$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}(t)$$

$$= -\frac{\vec{p}}{\tau} - e\,\mathrm{Re}\left(\vec{E}(\omega) e^{-i\omega t}\right)$$

§ 4.1 Drude theory

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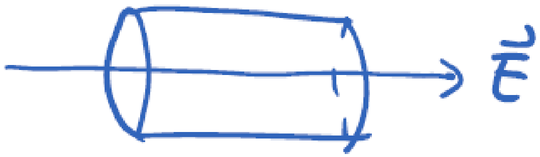

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$$= -\frac{\vec{p}}{\tau} - e\,\mathrm{Re}\left(\vec{E}(\omega) e^{-i\omega t}\right)$$

steady state: $\vec{p}(t) = \mathrm{Re}\left(\vec{p}(\omega) e^{-i\omega t}\right)$

$$\begin{cases} \vec{E}(\omega) = \mathrm{Re}\, E(\omega) + i\, \mathrm{Im}\, E(\omega) \\ \vec{p}(\omega) = \mathrm{Re}\, \vec{p}(\omega) + i\, \mathrm{Im}\, P(\omega) \end{cases}$$

§ 4.1 Drude theory

E.O.M.: $\Rightarrow$ $$-i\omega \vec{p}(\omega) = -\frac{\vec{p}(\omega)}{\tau} - e\vec{E}(\omega)$$

$$\left(i\omega - \frac{1}{\tau}\right)\vec{p}(\omega) = e\vec{E}(\omega)$$

Alternating current:

$$\vec{J}(t) = -ne\frac{\vec{p}(t)}{m} = -\frac{ne}{m} \mathrm{Re}\left[\vec{p}(\omega)e^{-i\omega t}\right]$$

$$= \mathrm{Re}\left[\vec{J}(\omega)e^{-i\omega t}\right]$$

§ 4.1 Drude theory

E.O.M.: $\Rightarrow$ $$-i\omega \vec{P}(\omega) = -\frac{\vec{P}(\omega)}{\tau} - e\vec{E}(\omega)$$

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Alternating current:

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$$= \mathrm{Re}\left[\vec{J}(\omega)e^{-i\omega t}\right]$$

$$\Rightarrow \vec{J}(\omega) = -\frac{ne}{m}\vec{P}(\omega)$$

$$= \underbrace{-\frac{ne}{m}\frac{e}{i\omega - \frac{1}{\tau}}}_{=\,\sigma(\omega)}\vec{E}(\omega)$$

$\sigma(\omega)$: AC conductivity