TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

Solid State Theory (SS2020)

Lecture 17: Landau levels

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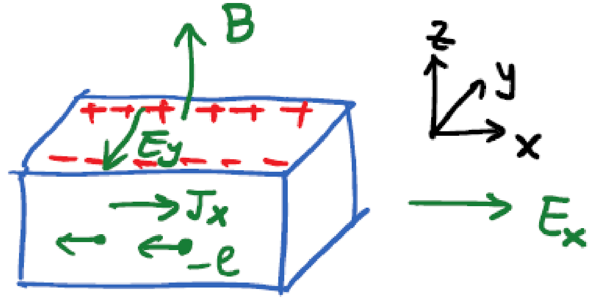
§ 5.1 Landau levels

- Classical Hall effect:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}, \quad J_y = 0$$



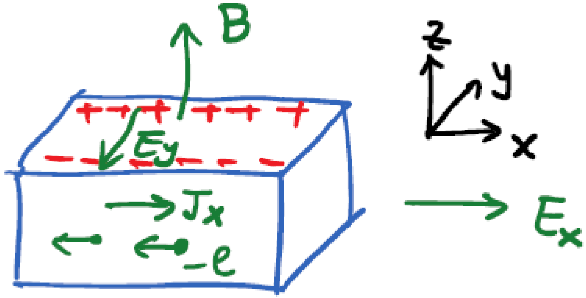
$$\Rightarrow \begin{cases} E_x = \rho_{xx} J_x \\ E_y = \rho_{yx} J_x \end{cases}, \quad \rho_{yx} = -\rho_{xy}$$

§ 5.1 Landau levels

- Classical Hall effect:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

$J_y = 0$



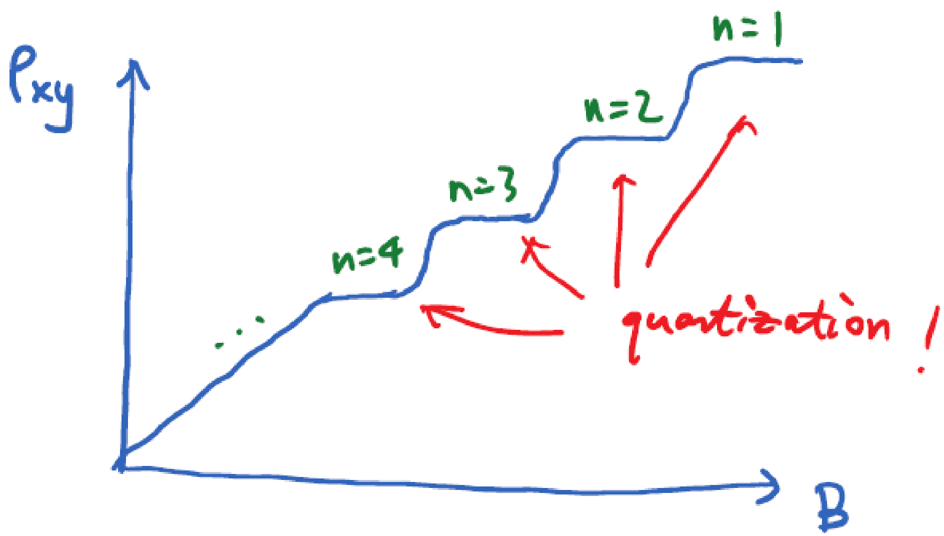
Magnetoresistance: $\rho_{xx} = \frac{E_x}{J_x} = \frac{m}{ne^2\tau}$

Hall coefficient:

$$R_H = \frac{1}{B}\rho_{yx} = \frac{1}{B}\frac{E_y}{J_x} = -\frac{1}{ne} \Rightarrow \rho_{xy} \propto B$$

§ 5.1 Landau levels

- Integer quantum Hall (IQH) effect (von Klitzing *et al.*, 1980):



Nobel Prize in 1985

$$\frac{h}{e^2} = 25812.807\,\Omega$$

(resistence standard)

$$\rho_{xy} = \frac{1}{n}\frac{h}{e^2} \quad (n = 1, 2, 3, \ldots)$$

ρ_{xx} vanishingly small when ρ_{xy} quantized

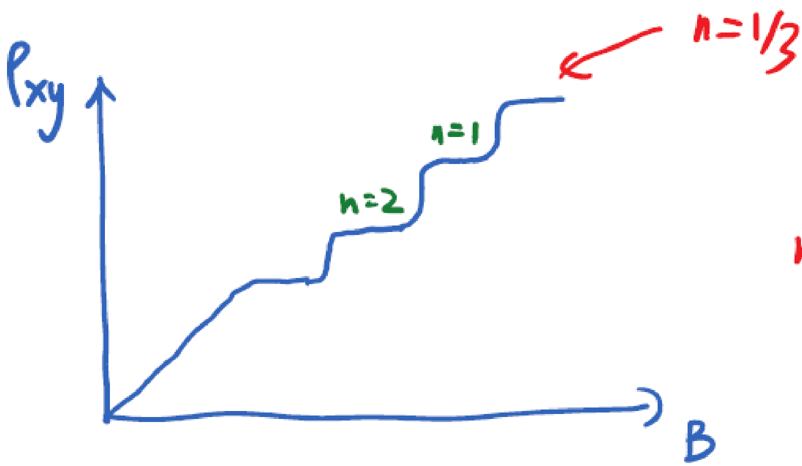
See Wikipedia for IQH

§ 5.1 Landau levels

- Fractional quantum Hall (FQH) effect:

  Experiment: Tsui, Störmer & Gossard, 1982
  Theory: Laughlin, 1983

Nobel Prize in 1998



$n = 1/3,\ 1/5,\ 2/3,\ \ldots$

See Wikipedia for FQH

§ 5.1 Landau levels

- Fractional quantum Hall (FQH) effect:

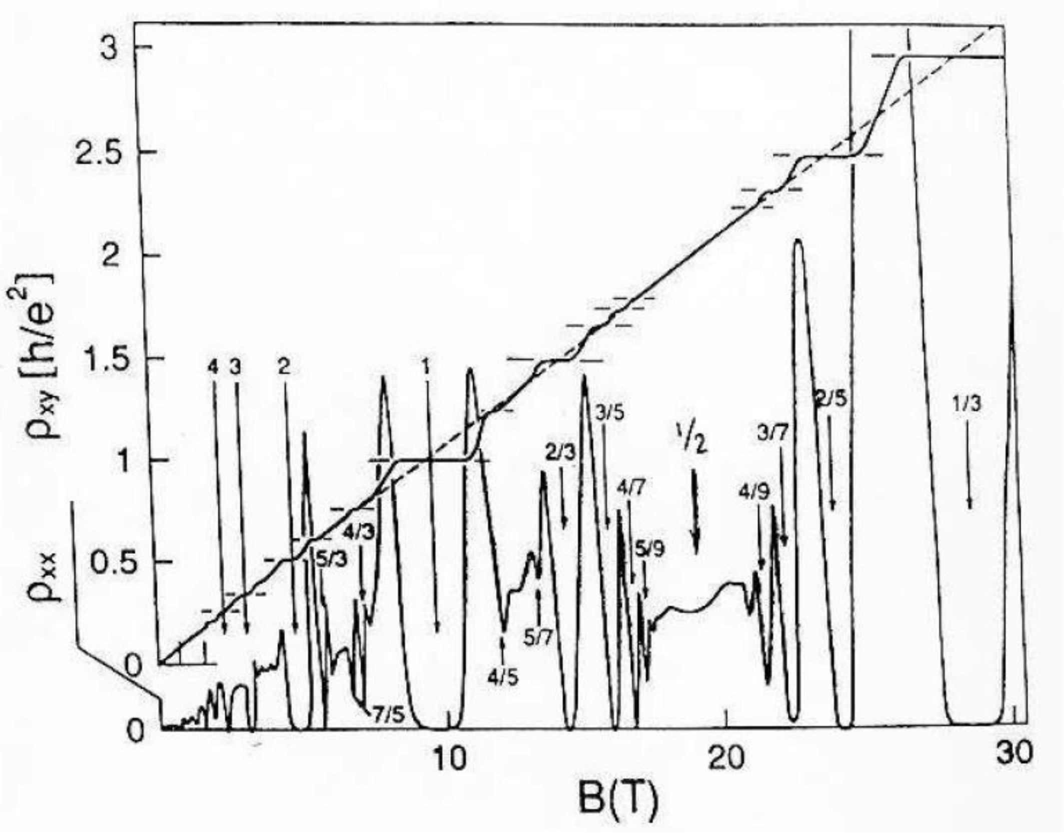


Figure from Störmer, Physica B 177, 401 (1992)

§ 5.1 Landau levels

- Single-particle states and Landau levels:

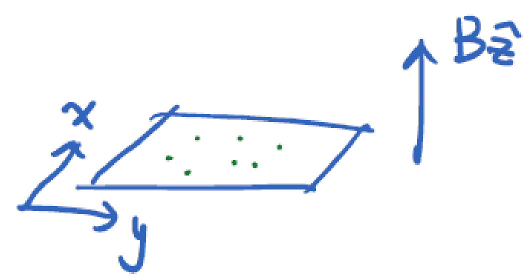
2D spin-polarized electron gas in strong B-field:

$$H = \frac{1}{2m}(P_x + eA_x)^2 + \frac{1}{2m}(P_y + eA_y)^2$$

m: effective mass

A: vector potential

$$\vec{\nabla} \times \vec{A} = \vec{B} = B\hat{z}$$



§ 5.1 Landau levels

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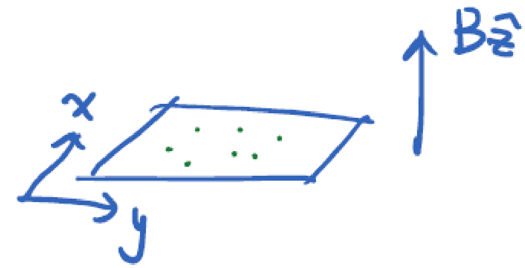

$$\vec{\nabla} \times \vec{A} = \vec{B} = B\hat{z}$$

Landau gauge: $\vec{A}(\vec{r}) = (A_x, A_y, A_z) = (-By, 0, 0)$

$$B_z = \partial_x A_y - \partial_y A_x = -\partial_y(-By) = B$$

§ 5.1 Landau levels

- Single-particle states and Landau levels:

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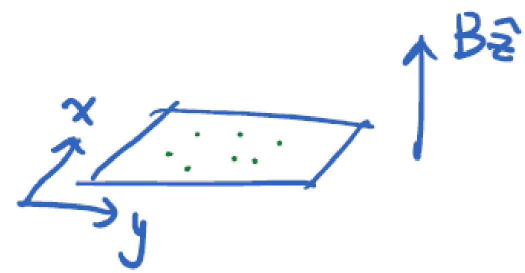


Other gauge choices are possible, e.g.

symmetric gauge: $\vec{A} = \left(-\frac{B}{2}y, \frac{B}{2}x, 0\right)$

§ 5.1 Landau levels

Below we use the Landau gauge $\vec{A} = (-By, 0, 0)$:

$$H = \frac{1}{2m}(\hat{P}_x - eB\hat{y})^2 + \frac{1}{2m}\hat{P}_y^2$$

$[H, \hat{P}_x] = 0$ → conserved quantity

$[\hat{x}, \hat{P}_x] = [\hat{y}, \hat{P}_y] = i\hbar$

$[H, \hat{P}_y] \neq 0$

§ 5.1 Landau levels

$$\Rightarrow \quad H = \frac{1}{2m}(\hbar k_x - eB\hat{y})^2 + \frac{1}{2m}\hat{p}_y^2$$

$\hbar k_x$ ↓ number (instead of operator): $k_x = 0, \pm\frac{2\pi}{L_x}, \pm\frac{4\pi}{L_x} \cdots$

(PBC with length L_x in x-direction)

$$= \frac{1}{2m}\hat{p}_y^2 + \frac{1}{2}m\omega_c^2(\hat{y} - y_0)^2$$

$$\omega_c = \frac{eB}{m}$$

(cyclotron frequency)

$$y_0 = \frac{\hbar}{eB}k_x$$

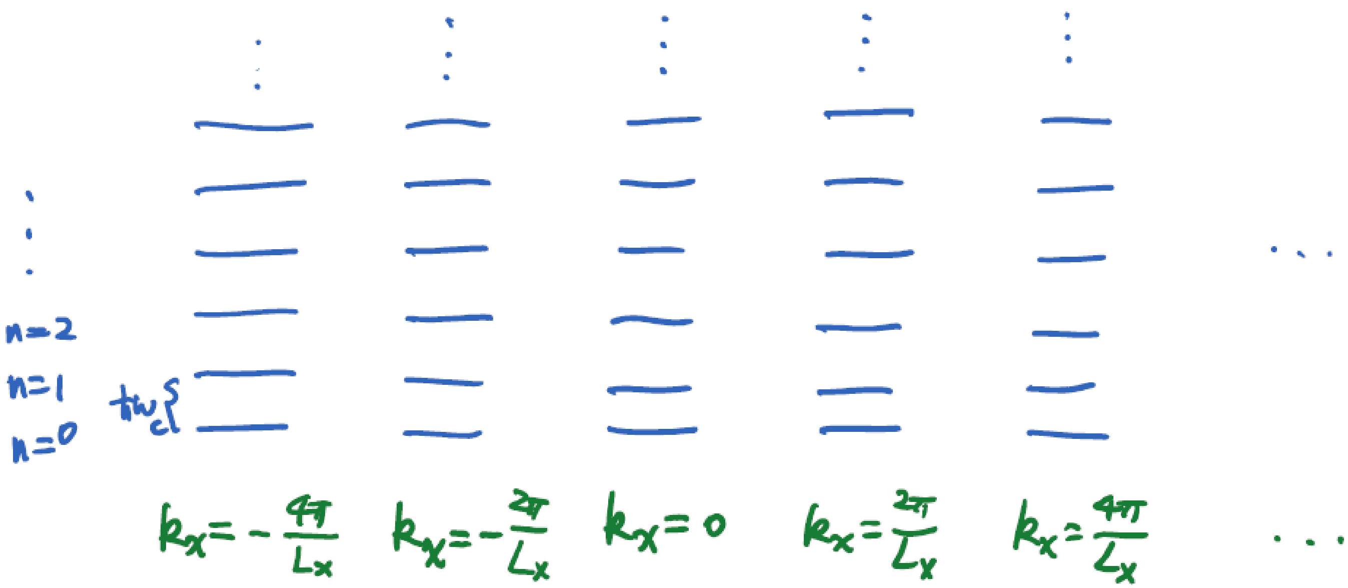
Harmonic oscillator for each $y_0 = \frac{\hbar}{eB}k_x$!

§ 5.1 Landau levels

Single-particle energies:

Landau levels

$$E_{n,k_x} = \left(n+\frac{1}{2}\right)\hbar\omega_c$$



§ 5.1 Landau levels

1) E_{n,k_x} does NOT depend on k_x.

⇒ huge degeneracy!

2) Equally spaced energy levels (Landau levels)

$n=0$: lowest Landau level (LLL)

$n=1$: First Landau level

⋮

Many-electron states:

occupation of single-particle levels (which ones?)

§ 5.1 Landau levels

- Single-electron wave function:

$$\phi_{n,k_x}(\vec{r}) = \frac{1}{\sqrt{L_x}} e^{ik_x x} \left(\frac{m\omega_c}{\pi\hbar}\right) \frac{1}{2^{n/2}\sqrt{n!}} e^{-\frac{m\omega_c}{2\hbar}(y-y_0)^2} H_n\left[(y-y_0)\sqrt{\frac{m\omega_c}{\hbar}}\right]$$

plane wave in x-direction ($\hat{P}_x$ eigenstate)

Hermite polynomial

$H_0(t) = 1$

$H_1(t) = t$

$\vdots$

§ 5.1 Landau levels

- Single-electron wave function:

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plane wave in x-direction ($\hat{p}_x$ eigenstate)

Hermite polynomial

$H_0(t) = 1$

$H_1(t) = t$

$\vdots$

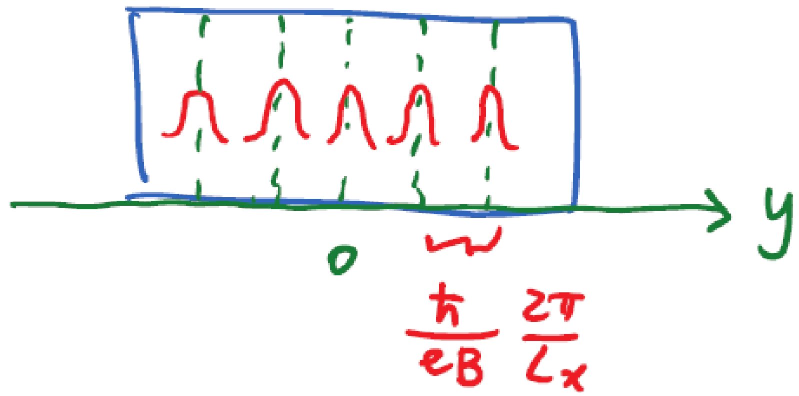
ϕ_{n,k_x} exponentially localized around

$$y = y_0 = \frac{\hbar}{eB} k_x = \frac{\hbar}{eB} \frac{2\pi}{L_x} n_x \qquad (n_x = 0, \pm 1, \pm 2, \dots)$$

§ 5.1 Landau levels

LLL $(n=0)$: $\phi_{0,k_x} \sim e^{ik_x x} e^{-(y-y_0)^2/2l^2}$

Gaussian wave packet



§ 5.1 Landau levels

LLL $(n=0)$: $\phi_{0,k_x} \sim e^{ik_x x} e^{-(y-y_0)^2/2\ell^2}$

Gaussian wave packet

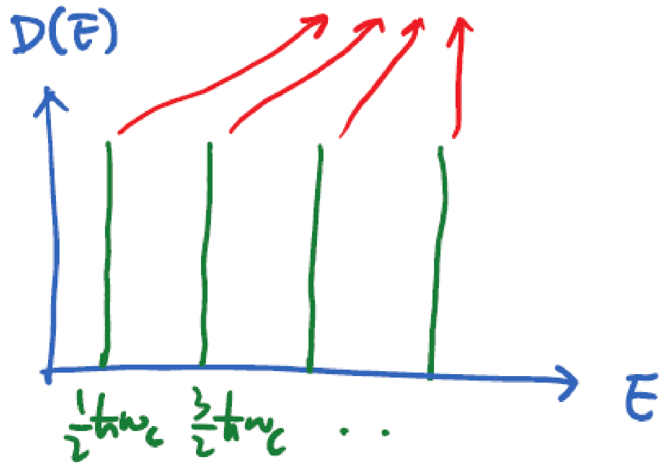
magnetic length $\ell = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar}{m}\frac{m}{eB}} = \sqrt{\frac{\hbar}{eB}}$

$\ell \sim 10^{-8}\,m$ $\gg$ lattice spacing (lattice ignored)

§ 5.1 Landau levels

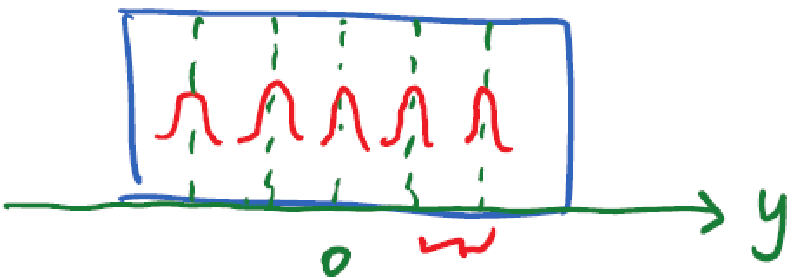
Density of states :

degeneracy of each Landau level: $N_L(B)$



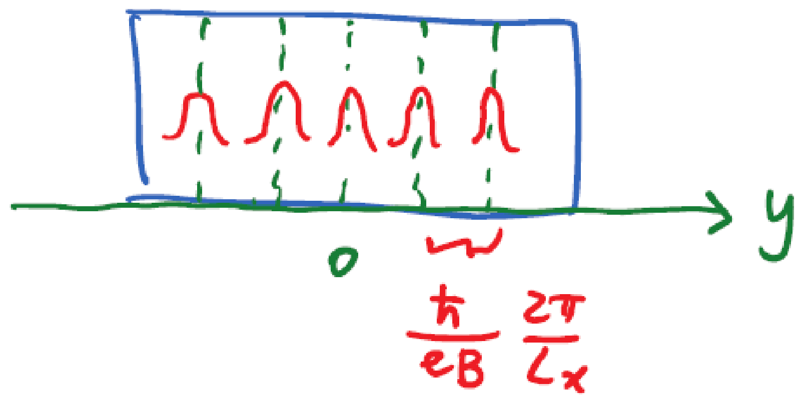
§ 5.1 Landau levels

Open boundary condition (OBC) with length L_y in y-direction $\Rightarrow$ count the degeneracy !



$$\frac{\hbar}{eB}\,\frac{2\pi}{L_x}$$

§ 5.1 Landau levels



Degeneracy:

$$N_L(B) = \frac{L_y}{\frac{\hbar}{eB}\frac{2\pi}{L_x}} = \frac{eBL_xL_y}{h} = \frac{e}{h}BS$$

L_y: length in y-direction

$\frac{\hbar}{eB}\frac{2\pi}{L_x}$: distance between wave packets

$S = L_xL_y$ (total area)

§ 5.1 Landau levels

Degeneracy:

$$N_L(B) = \frac{L_y}{\frac{\hbar}{eB}\frac{2\pi}{L_x}} = \frac{eBL_xL_y}{h} = \frac{e}{h}BS$$

L_y → length in y-direction

$\frac{\hbar}{eB}\frac{2\pi}{L_x}$ ← distance between wave packets

$S = L_xL_y$ (total area)

$$= \frac{\phi(B)}{\phi_0}$$

$\phi(B)$ → flux $\phi = BS$

ϕ_0 → unit of flux quantum

$$\phi_0 = \frac{h}{e}$$

§ 5.1 Landau levels

of electrons: N

filling fraction: $\nu = \frac{N}{\phi/\phi_0}$ (tunable by B)

e.g. $\nu = 1 \Rightarrow$ LLL fully occupied

↑ $B \sim 10\,T$

(each electron has one flux quantum)