TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

Solid State Theory (SS2020)

Lecture 20: Fractional quantum Hall effect

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§ 5.4 Fractional quantum Hall effect

- Fractional quantum Hall (FQH) effect:

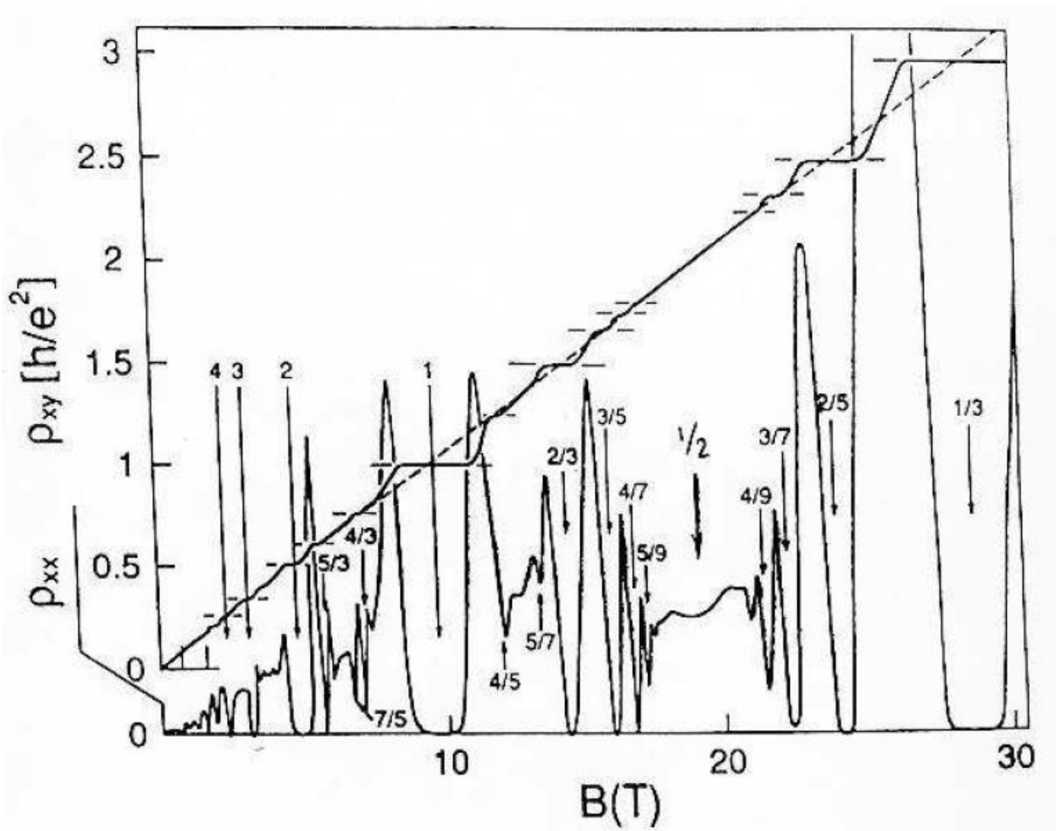
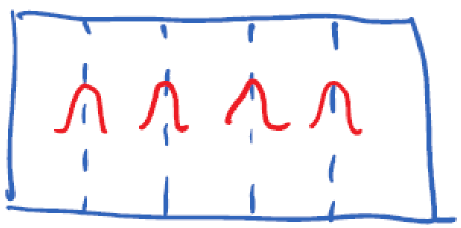


Figure from Störmer, Physica B 177, 401 (1992)

§ 5.4 Fractional quantum Hall effect

- Fractionally filled Landau level:



$$\frac{\hbar}{eB}\frac{2\pi}{L_x} = \frac{2\pi \ell^2}{L_x}$$

magnetic length:

$$\ell = \sqrt{\frac{\hbar}{eB}}$$

IQH: $\nu =$ integers (fully filled Landau levels)

Q: What happens if Landau levels are partially filled?

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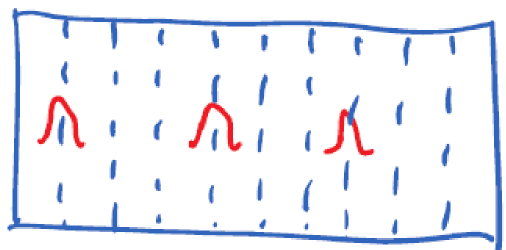
Macroscopic degeneracy if interactions are not considered.

However, electron interactions will make a choice!

Naive expectation:



Wigner crystal (elecerons try to be far from each other to minimize repulsion)

But this doesn't explain FQH plateaux ...

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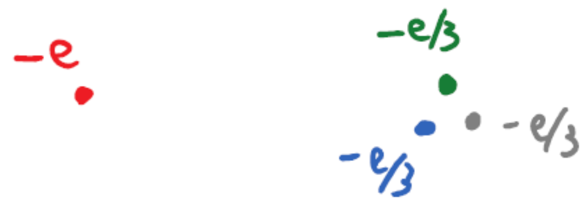
- Parton picture: $\nu = 1/3$ case



electron parton

$$c^\dagger(\vec{r}) \longleftrightarrow d_1^\dagger(\vec{r})\, d_2^\dagger(\vec{r})\, d_3^\dagger(\vec{r})$$

$d_a^\dagger$ carry electric charge $-\frac{e}{3}$!

$\downarrow$

$a = 1, 2, 3$

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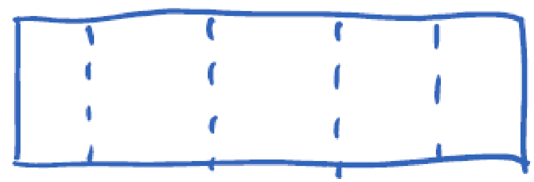
Landau levels for partons :



$$\frac{\hbar}{\frac{e}{3}B}\frac{2\pi}{L_x} = 3\,\frac{\hbar}{eB}\frac{2\pi}{L_x}$$

distance between two single-particle states three times larger !

$\nu = 1/3$ for electrons = fully filled LLL of partons

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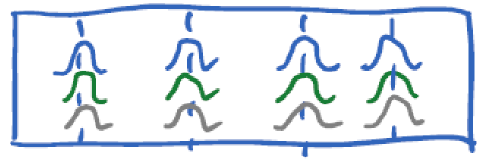
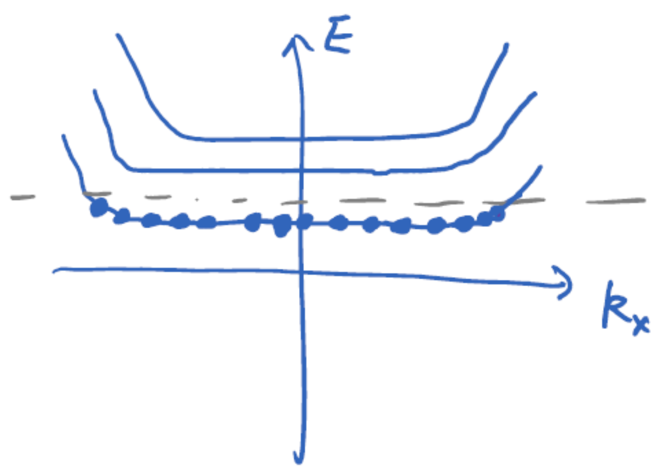
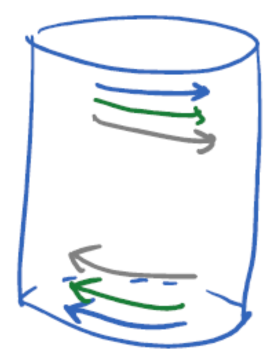


Each single-particle level can accommodate three partons with different colors $(a=1,2,3)$.





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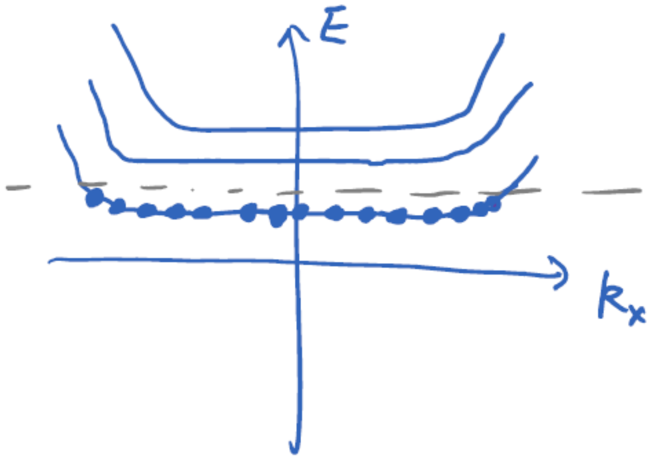

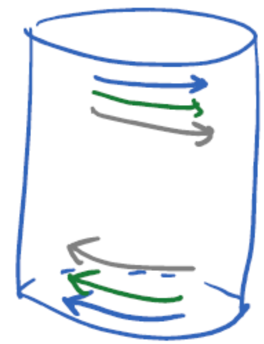

$$\Rightarrow \quad \sigma_{xy} = 3 \cdot \frac{\left(\frac{e}{3}\right)^2}{h} = \frac{1}{3}\frac{e^2}{h}$$

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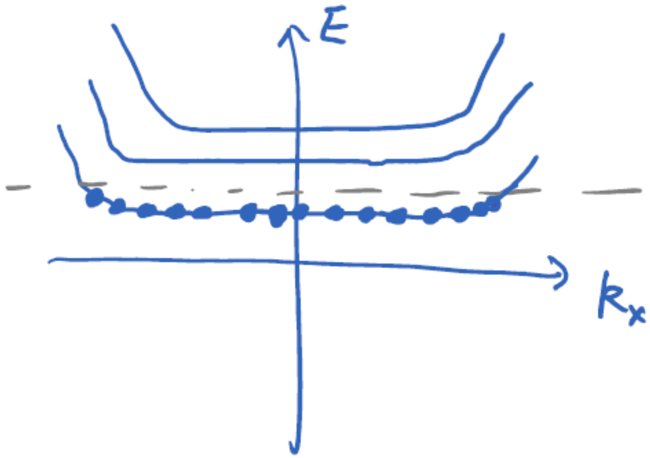

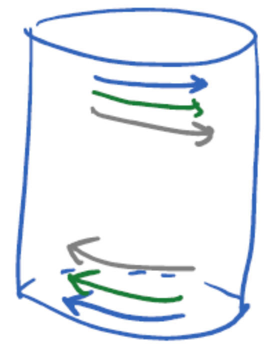

Remark: Many other FQH plateaux can be explained with a similar parton picture.

But it requires careful justification with microscopic calculations !

§ 5.4 Fractional quantum Hall effect

- Laughlin's wave function:

$$|\psi\rangle = \underset{\text{enforce } c^\dagger(\vec r) = d_1^\dagger(\vec r)\, d_2^\dagger(\vec r)\, d_3^\dagger(\vec r)}{P} \prod_{a=1,2,3} \underbrace{\prod_m \psi^\dagger_{m,a} |0\rangle}_{\nu=1 \text{ for partons}}$$

$$\psi^\dagger_{m,a} = \int d\vec r\, \underbrace{\phi_m(\vec r)}_{\text{LLL single-particle wave function}}\, d_a^\dagger(\vec r)$$

LLL single-particle wave function:

$$\phi_m(\vec r) \propto z^m e^{-|z|^2/4\ell_B^2} \quad \text{(disk geometry)}$$

§ 5.4 Fractional quantum Hall effect

$$\gamma^{+}_{m,a} = \int d\vec{r}\ \underbrace{\phi_m(\vec{r})}_{\text{LLL single-particle wave function:}}\ d^{+}_{a}(\vec{r})$$

LLL single-particle wave function:

$$\phi_m(\vec{r}) \propto z^m e^{-|z|^2/4l_p^2}$$ (disk geometry)

Note that $l_p^2 = \dfrac{\hbar}{\frac{e}{3}B} = 3\,l^2$

l_p : magnetic length for partons

l : magnetic length for electrons

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$$|\psi\rangle \propto P \prod_{a=1,2,3} \Bigg[\int d\vec{r}_{1,a} \cdots d\vec{r}_{N,a} \underbrace{\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ z_{1,a} & z_{2,a} & \cdots & z_{N,a} \\ \vdots & \vdots & \ddots & \vdots \\ z_{1,a}^{N-1} & z_{2,a}^{N-1} & \cdots & z_{N,a}^{N-1} \end{pmatrix}}_{= \prod_{i<j} (z_{i,a} - z_{j,a})}$$

$$\times e^{\sum_j \frac{|z_{j,a}|^2}{4\ell_B^2}} d_a^\dagger(\vec{r}_{1,a}) \cdots d_a^\dagger(\vec{r}_{N,a}) \Bigg] |0\rangle$$

§ 5.4 Fractional quantum Hall effect

$$|\psi\rangle \propto P \prod_{a=1,2,3} \Bigg[\int d\vec{r}_{1,a} \cdots d\vec{r}_{N,a} \, \underbrace{\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ z_{1,a} & z_{2,a} & \cdots & z_{N,a} \\ \vdots & \vdots & \ddots & \vdots \\ z_{1,a}^{N-1} & z_{2,a}^{N-1} & \cdots & z_{N,a}^{N-1} \end{pmatrix}}_{= \prod_{i<j} (z_{i,a} - z_{j,a})}$$

$$\times \, e^{\sum_j \frac{|z_{j,a}|^2}{4\ell_B^2}} \, d_a^\dagger(\vec{r}_{1,a}) \cdots d_a^\dagger(\vec{r}_{N,a}) \Bigg] |0\rangle$$

P requires that $\underbrace{\vec{r}_{j,a=1} = \vec{r}_{j,a=2} = \vec{r}_{j,a=3}}$.

partons with different colors
form an electron !

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$$|\psi\rangle \propto P \int d\vec{r}_1 \cdots d\vec{r}_N \left[\prod_{i<j} (z_i - z_j)\, e^{-\sum_j \frac{|z_j|^2}{4 l_p^2}} \right]^3$$

$$\times \underbrace{d_1^\dagger(\vec{r}_1)\, d_2^\dagger(\vec{r}_1)\, d_3^\dagger(\vec{r}_1)}_{= C^\dagger(\vec{r}_1)} \cdots \underbrace{d_1^\dagger(\vec{r}_N)\, d_2^\dagger(\vec{r}_N)\, d_3^\dagger(\vec{r}_N)}_{= C^\dagger(\vec{r}_N)} |0\rangle$$

§ 5.4 Fractional quantum Hall effect

$$|\psi\rangle \propto P \int d\vec{r}_1 \cdots d\vec{r}_N \left[\prod_{i<j} (z_i - z_j)\, e^{-\sum_j \frac{|z_j|^2}{4 l_p^2}} \right]^3$$

$$\times \underbrace{d_1^{+}(\vec{r}_1)\, d_2^{+}(\vec{r}_1)\, d_3^{+}(\vec{r}_1)}_{= C^{+}(\vec{r}_1)} \cdots \underbrace{d_1^{+}(\vec{r}_N)\, d_2^{+}(\vec{r}_N)\, d_3^{+}(\vec{r}_N)}_{= C^{+}(\vec{r}_N)} |0\rangle$$

$$= \int d\vec{r}_1 \cdots d\vec{r}_N \; \prod_{i<j} (z_i - z_j)^3 \, e^{-\sum_j \frac{|z_j|^2}{4 l^2}} \; C^{+}(\vec{r}_1) \cdots C^{+}(\vec{r}_N) |0\rangle$$

Laughlin's wave function for $\nu = 1/3$!

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$$|\Psi\rangle = \int d\vec{r}_1 \cdots d\vec{r}_N \prod_{i<j} (z_i - z_j)^3 e^{-\sum_j \frac{|z_j|^2}{4\ell^2}} C^\dagger(\vec{r}_1) \cdots C^\dagger(\vec{r}_N)|0\rangle$$

Numerics show that Laughlin's wave function is indeed a very good variational ansatz for the microscopic model ($\nu = 1/3$ with electron interactions projected to the LLL) !

§ 5.4 Fractional quantum Hall effect

- Many other important topics about quantum Hall problems:
  - Role of spin and/or disorder?
  - Anyons: exotic excitations which are neither bosons or fermions
  - Non-Abelian anyons for topological quantum computation?
  - Topological phases of matter: characterization & classification
  - …

See, e.g., J. K. Jain, *Composite fermions* (Cambridge University Press, 2007);
C. Nayak, S.H. Simon, A. Stern, M. Freedman & S. Das Sarma, RMP 80, 1083 (2008).