SS2021: Tensor Networks
Institut für Theoretische Physik, TU Dresden
Lecturer: Hong-Hao Tu
Tutorial: 06.05.2021

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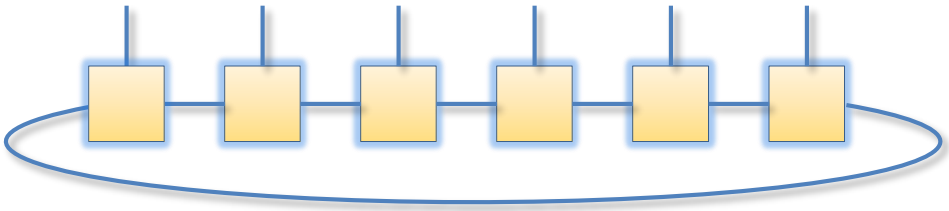
# Problem Set 2

## Exercise 1: AKLT model

- The ground state of the spin-1 AKLT model has the following MPS form:

$$A^{s=1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad A^{s=0} = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad A^{s=-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$$



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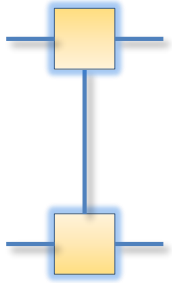
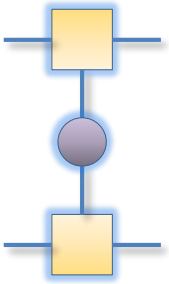
- Calculate the transfer matrix $T$ and determine its eigenvalues and eigenvectors:



$$T = \sum_s \bar{A}^s \otimes A^s$$

- Calculate the transfer matrices for $S^z$ and $e^{i\pi S^z}$:



$$T^O = \sum_{s,s'} (\bar{A}^{s'} \otimes A^s) \langle s'|O|s\rangle$$

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- Determine the asymptotic behaviors of the spin-spin and string correlation functions for the AKLT state:

$$C_{ij}^{zz} = \langle S_i^z S_j^z \rangle$$

$$C_{ij}^{\text{string}} = \langle S_i^z \prod_{k=i+1}^{j-1} e^{i\pi S_k^z} S_j^z \rangle$$

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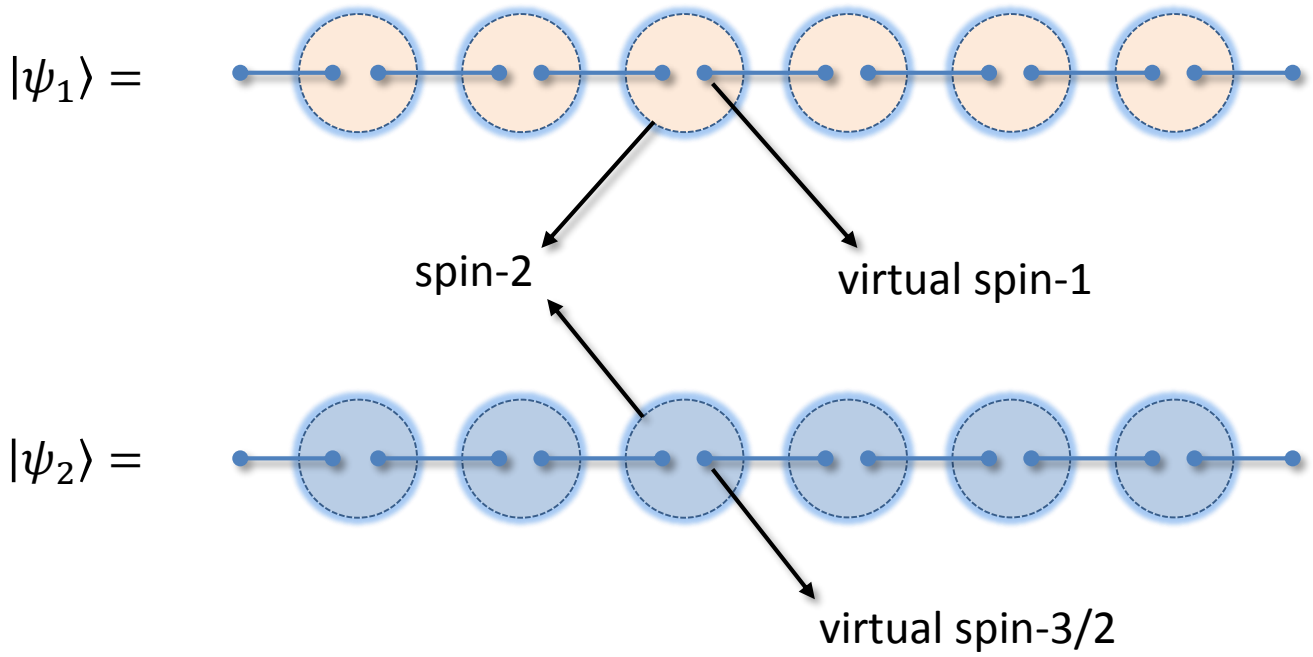
## Exercise 2: Projective construction of MPS and parent Hamiltonian

- Consider two SU(2)-symmetric MPS for spin-2 chains:



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- Calculate their corresponding local MPS tensors.

$A^s_{\alpha\beta}$

- Derive the parent Hamiltonians for these two MPS.

  Hint: It's sufficient to consider nearest-neighbor interactions written in terms of projectors.