SS2021: Tensor Networks
Institut für Theoretische Physik, TU Dresden
Lecturer: Hong-Hao Tu
Tutorial: 10.06.2021

Problem Set 4

Exercise 1: Entanglement entropy in 1D free fermionic state

- Consider the 1D spinless fermion chain with periodic boundary condition:

$$H = -\sum_{j=1}^{N} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

- Calculate the correlation matrix $G_{lj} = \langle\psi|c_j^\dagger c_l|\psi\rangle$ for the ground state $|\psi\rangle$ at half-filling.

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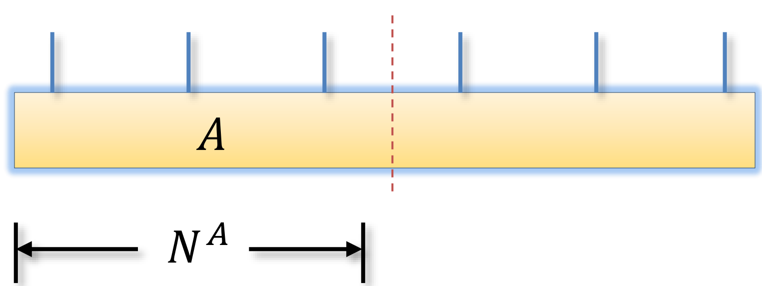
- Consider a bipartition of the system into two subsystems: A (with N_A sites) and B (with $N - N_A$ sites). Show that the reduced density matrix of the subsystem A can be written as

$$\rho^A = \prod_{p=1}^{N_A} [\Lambda_p f_p^\dagger f_p + (1 - \Lambda_p) f_p f_p^\dagger]$$

by diagonalizing the submatrix of G_{lj} with $j, l \in A$.



- Calculate the entanglement entropy $S^A = -\mathrm{tr}_A(\rho^A \ln\rho^A)$ and plot S^A as a function of N_A. How does S^A increase with the subsystem size N_A?

- Determine the central charge c by fitting S^A with the formula

$$S^A(N_A) = \frac{c}{3}\ln\left(\frac{N}{\pi}\sin\frac{\pi N_A}{N}\right) + c_1'$$

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Exercise 2: MPO-MPS evolution for preparing free fermionic state

- Consider the 1D spinless fermion chain with open boundary condition:

$$H = -\sum_{j=1}^{N-1} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

- Determine the filled single-particle orbitals $d_m^\dagger$ for the ground state $|\psi\rangle = \prod_{m=1}^{M} d_m^\dagger |0\rangle$ at half-filling.

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- Construct Wannier orbitals $f_r^\dagger$ from $d_m^\dagger$ by diagonalizing the position operator $X = \sum_{l=1}^{N} l c_l^\dagger c_l$ in the subspace of filled single-particle wave functions.

- Use the MPO-MPS evolution to prepare the ground state $|\psi\rangle$.