

## Problem Set 6

### Exercise: Entanglement spectrum in Haldane chains

- The **two-fold** degenerate entanglement spectrum of the ground state is an important feature of Haldane chains. In Lecture 15, we discussed how  $\mathbb{Z}_2 \times \mathbb{Z}_2$ , as an onsite unitary symmetry, can protect this degeneracy.
- In fact, **time-reversal symmetry** or **bond-centered inversion symmetry** can also protect this two-fold degeneracy.

Ref: F. Pollmann, A. M. Turner, E. Berg & M. Oshikawa, Phys. Rev. B 81, 064439 (2010).



- Consider, e.g., the spin-1 Heisenberg chain with single-ion anisotropy:

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D \sum_j (S_j^z)^2$$

The **time-reversal symmetry** and the **bond-centered inversion symmetry** are realized with  $S_j^{x,y,z} \rightarrow -S_j^{x,y,z}$  and  $S_j^{x,y,z} \rightarrow S_{-j+1}^{x,y,z}$ , respectively.

- Can you use the MPS techniques to prove that **one of these two symmetries** is sufficient to protect the two-fold degeneracy in the entanglement spectrum?

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Institut für Theoretische Physik, TU Dresden

Lecturer: Hong-Hao Tu

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- **Bonus:** You are encouraged to check your results with the iTEBD code.