TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

Tensor Networks (SS2021)

Lecture 12: time-evolving block decimation (TEBD)

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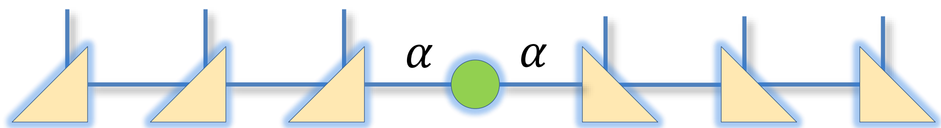
June 14th, 2021

§ 3.3 MPS in Γ-Λ form

- Apart from several canonical forms, the Γ-Λ form introduced by Vidal is also rather useful.

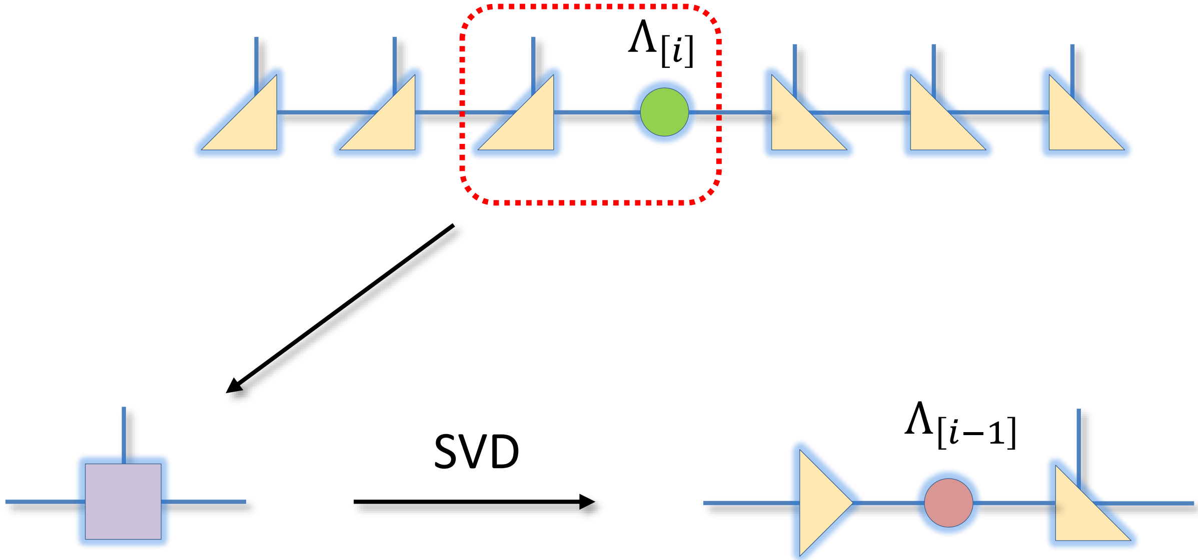
Start from the bond canonical form:



$$|\psi\rangle = \sum_{\alpha=1}^{D} \Lambda_{[i],\alpha} |\phi_\alpha^L\rangle \otimes |\phi_\alpha^R\rangle \qquad \Lambda_{[i],\alpha} > 0 \qquad \sum_{\alpha=1}^{D} \Lambda_{[i],\alpha}^2 = 1$$

G. Vidal, Phys. Rev. Lett. 91, 147902 (2003).

§ 3.3 MPS in Γ-Λ form



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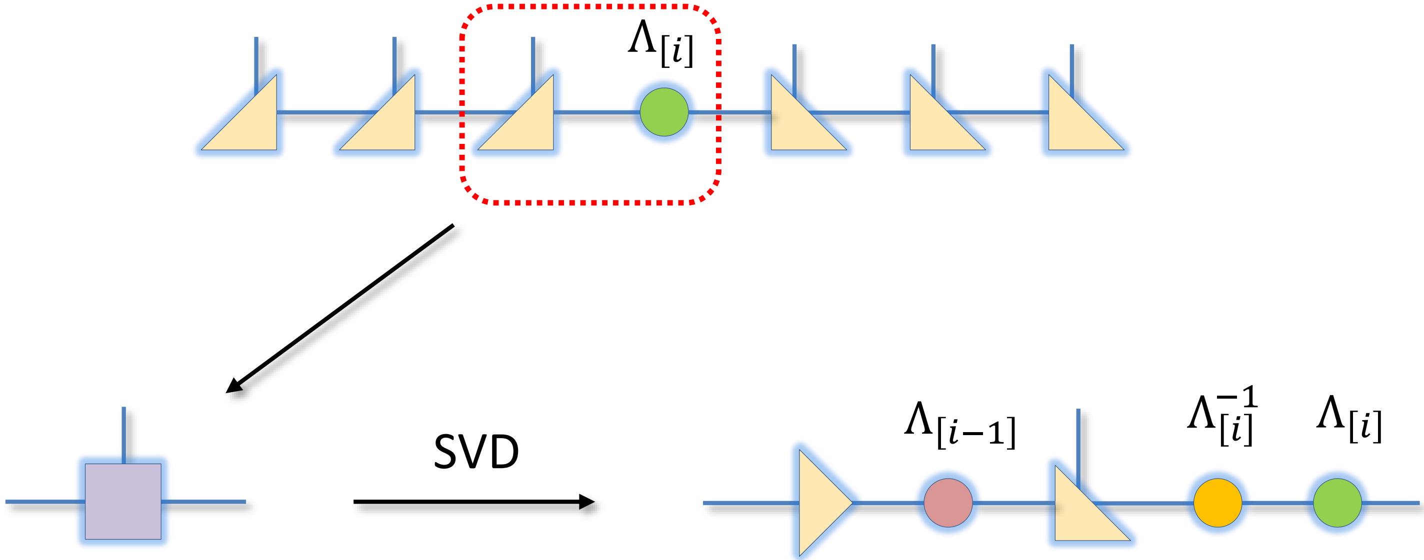

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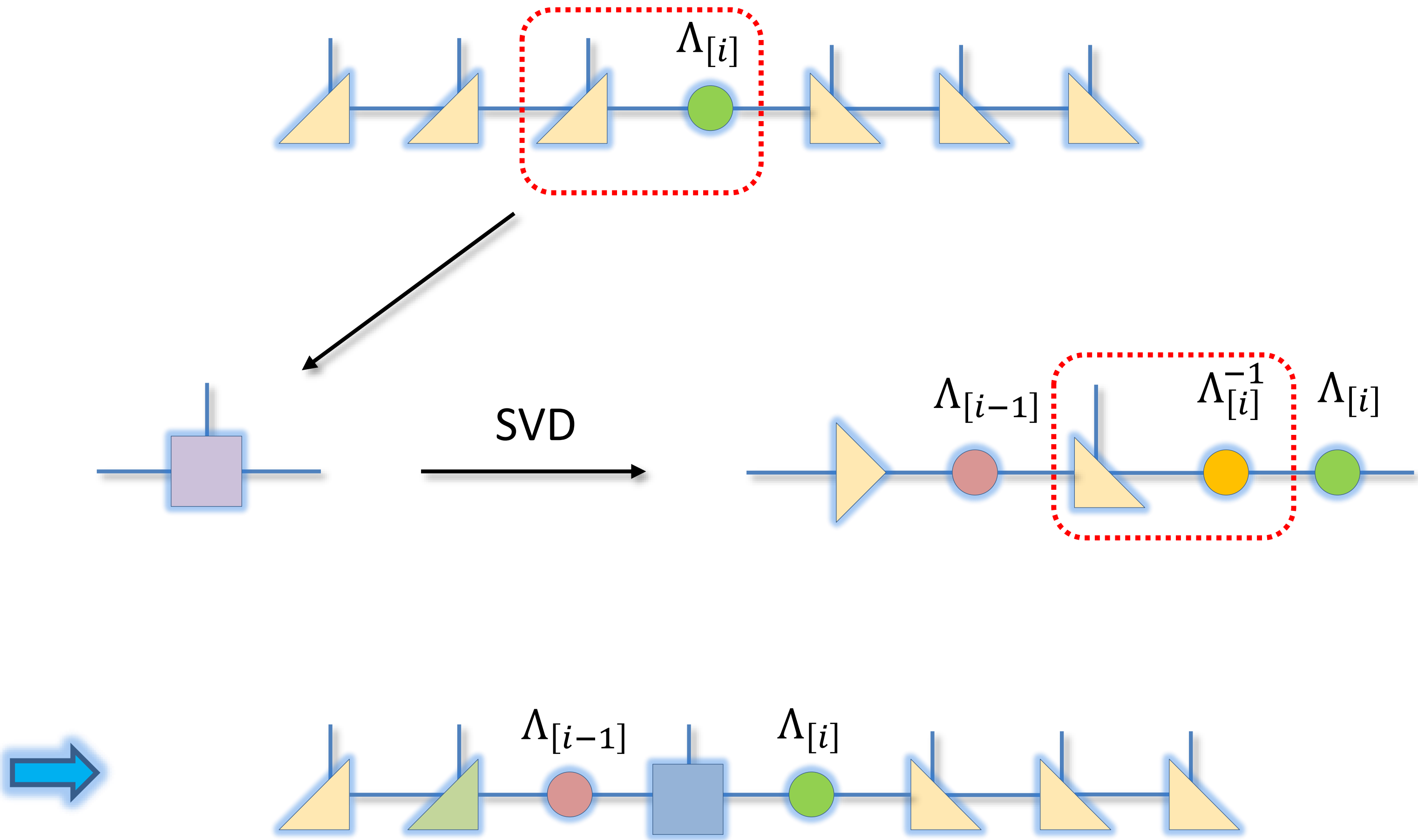
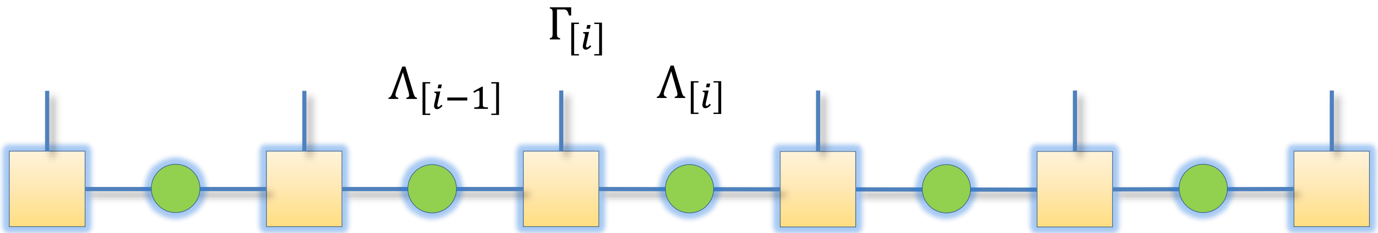


§ 3.3 MPS in Γ-Λ form

- Repeating the above procedure generates an MPS in Γ-Λ form. This gives Schmidt decompositions for all bipartitions.



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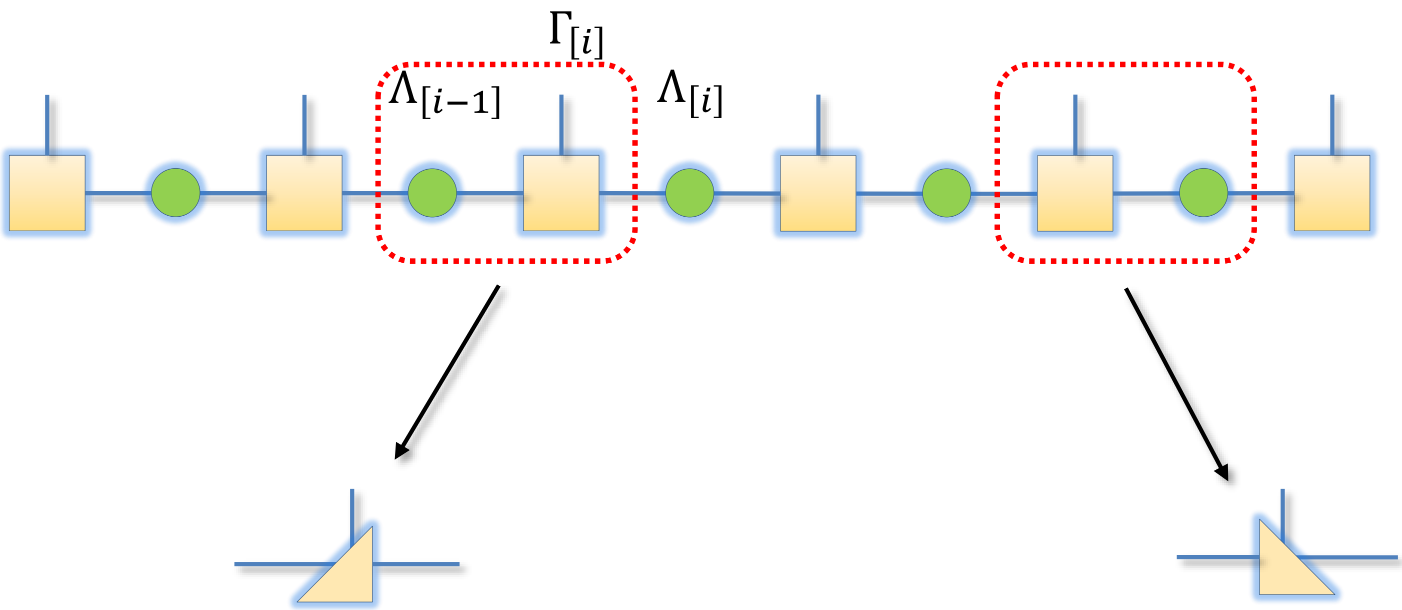

§ 3.3 MPS in Γ-Λ form

- The Γ-Λ form of MPS is a convenient basis for classically simulating quantum circuits (including single-qubit and two-qubit gates).

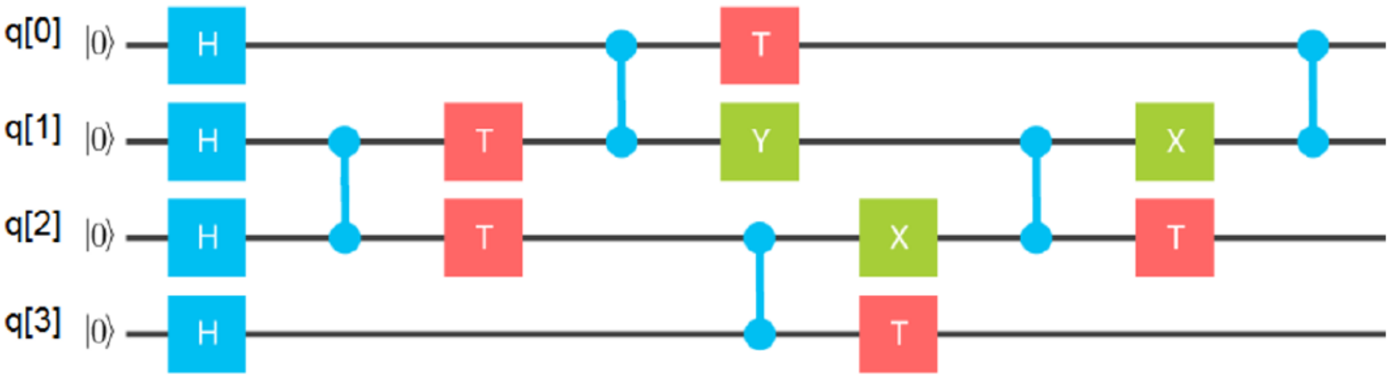

Figure from arXiv:1710.05867

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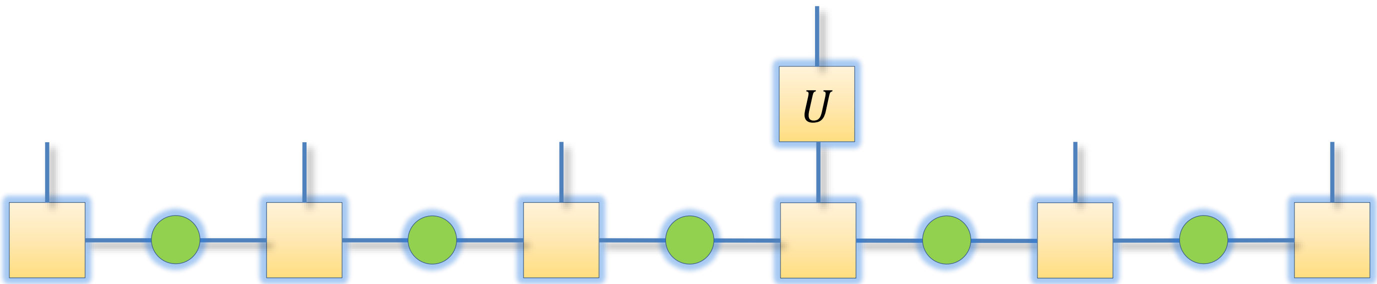
Single-qubit gate:



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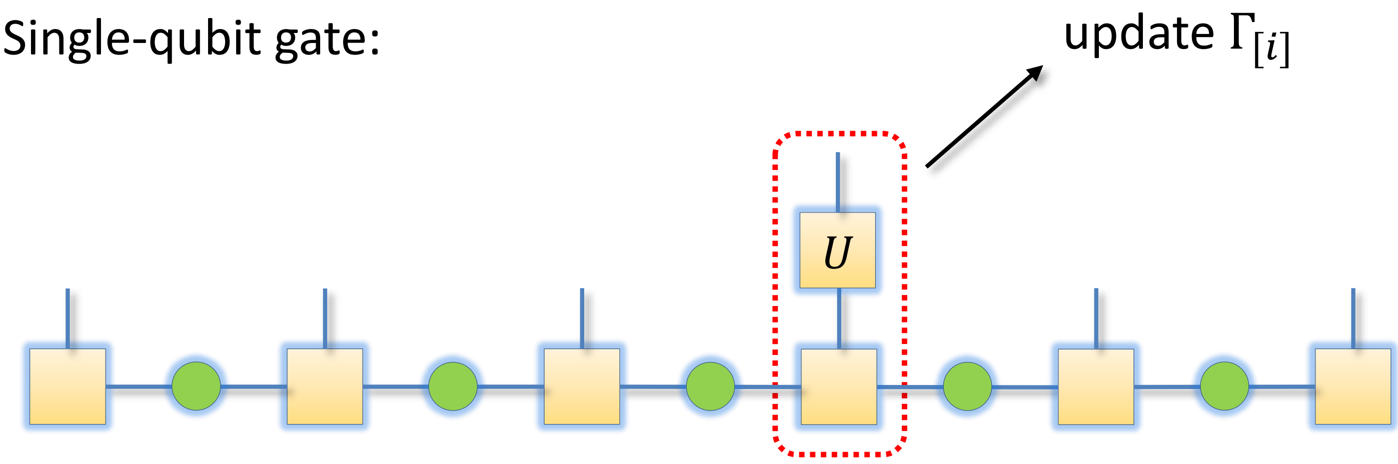

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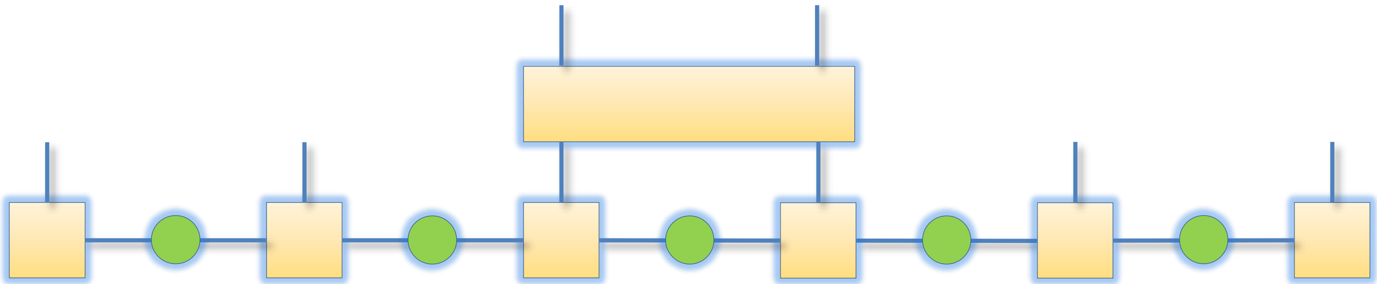
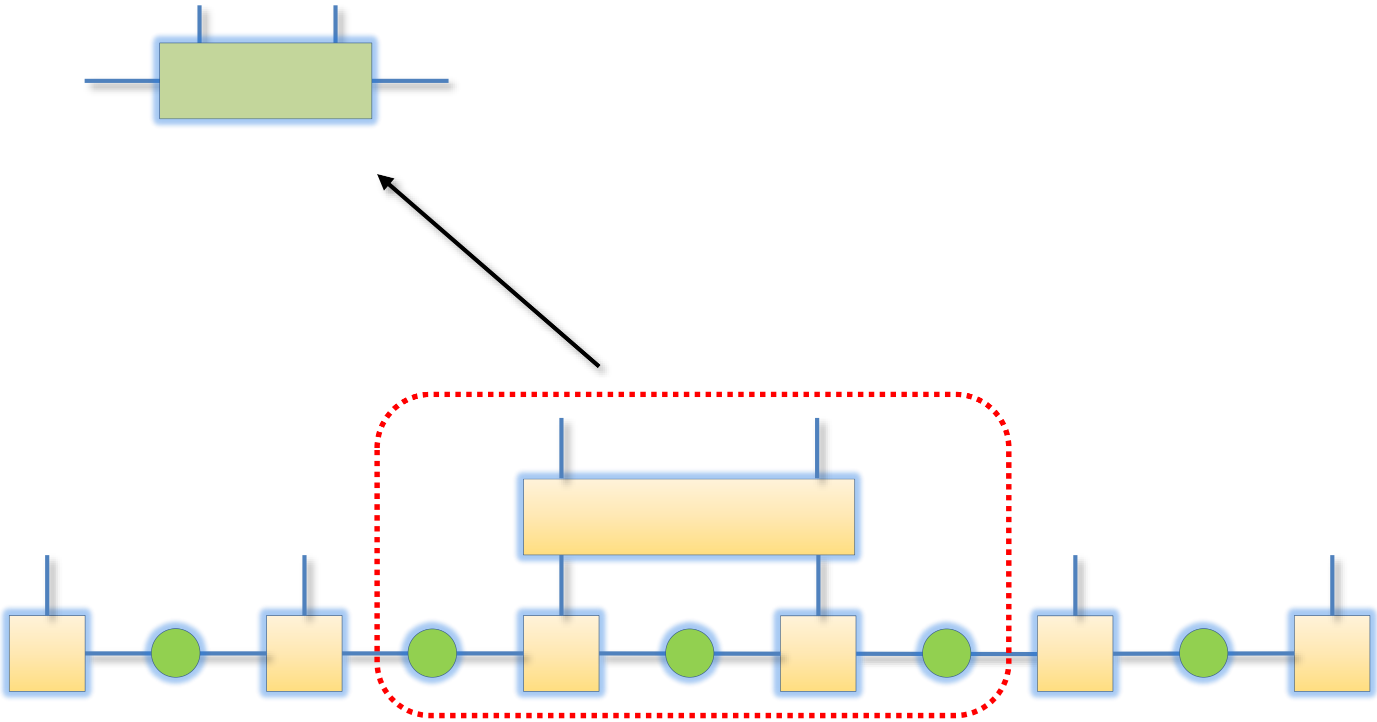
Two-qubit gate:



§ 3.3 MPS in Γ-Λ form

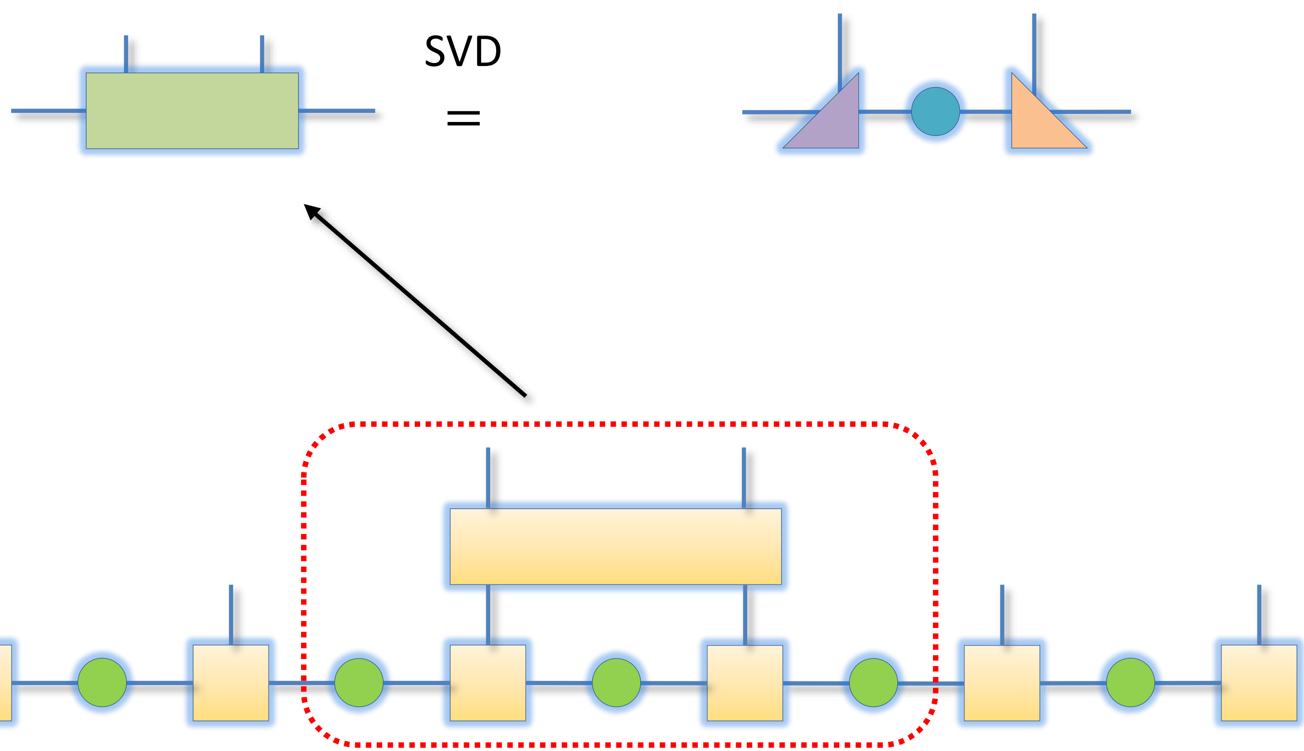
Two-qubit gate:



§ 3.3 MPS in Γ-Λ form

Two-qubit gate:

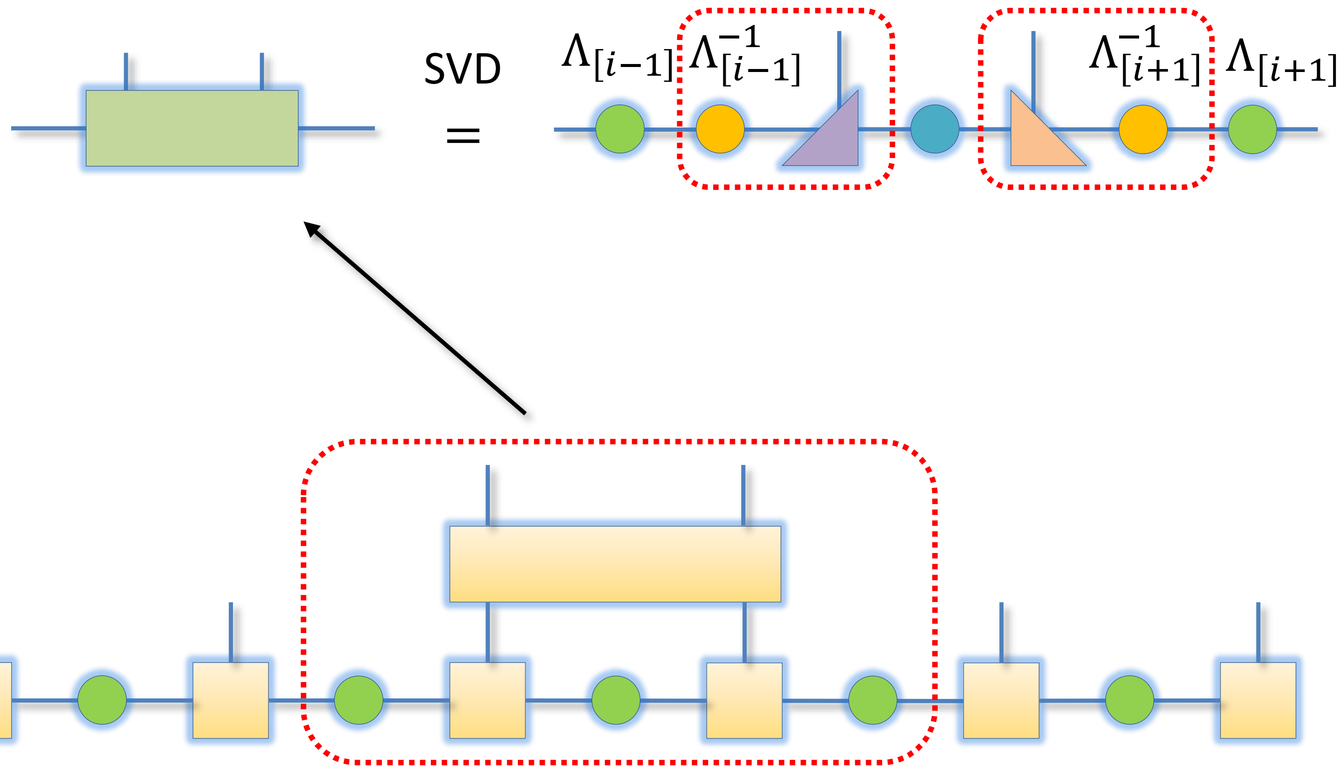
If truncation is needed, remove smallest singular values (same as “two-site DMRG”).



§ 3.3 MPS in Γ-Λ form

Two-qubit gate:

Update $\Gamma_{[i]}, \Gamma_{[i+1]}, \Lambda_{[i]}$.



§ 3.3 MPS in Γ-Λ form

- The Γ-Λ form of MPS is a convenient basis for classically simulating quantum circuits (including single-qubit and two-qubit gates).

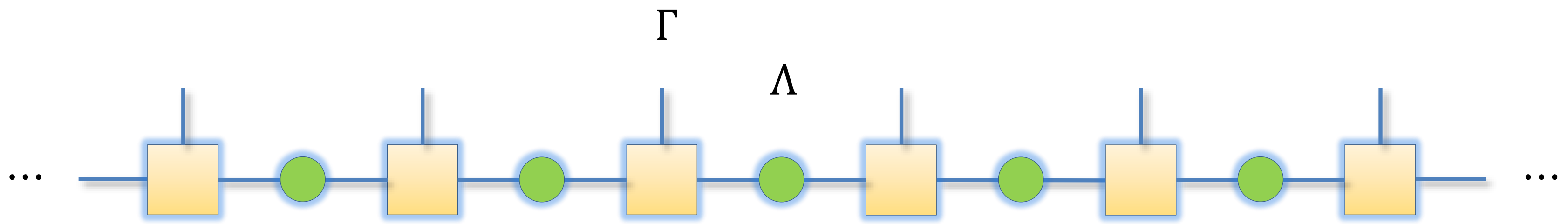
  - The fidelity of the MPS-based simulation is controlled by the truncation errors (this depends on the entangling power of the two-qubit gates).

  - If the quantum computation is performed within a class of states with low entanglement, MPS provides a high-fidelity classical simulation in polynomial time.

See Stoudenmire's recent talk on this topic: https://youtu.be/HwrBssNJfFA

§ 3.4 infinite time-evolving block decimation

- The Γ-Λ form provides a natural way to define translationally invariant MPS in the thermodynamic limit ("iMPS").



- The canonical form ("orthogonality") ensures normalization and gives the Schmidt decomposition in the thermodynamic limit.

§ 3.4 infinite time-evolving block decimation

- iTEBD: a power method for obtaining iMPS approximation to the ground state (by "cooling")

$$\lim_{\beta\to\infty} e^{-\beta H}\,|\psi(\Gamma,\lambda)\rangle = \lim_{M\to\infty}\left[e^{-\delta\tau(H_{\text{even}}+H_{\text{odd}})}\right]^M |\psi(\Gamma,\lambda)\rangle$$

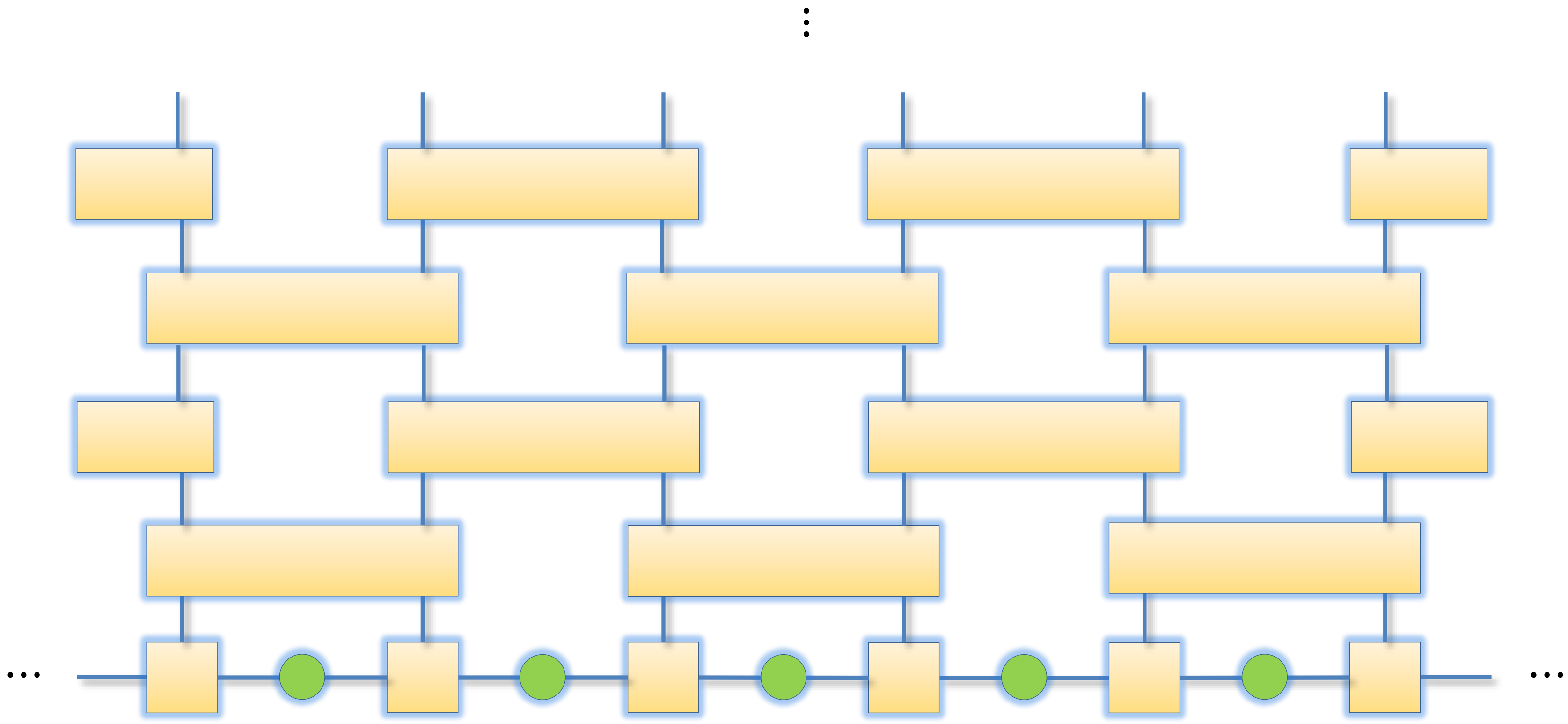
"Trotter-Suzuki"

$$= \lim_{M\to\infty}\left[e^{-\delta\tau H_{\text{even}}}e^{-\delta\tau H_{\text{odd}}} + O(\delta\tau^2)\right]^M |\psi(\Gamma,\lambda)\rangle$$

➢ Higher-order Trotter-Suzuki decomposition can be used to make the Trotter error smaller.

§ 3.4 infinite time-evolving block decimation

$$\lim_{\beta \to \infty} e^{-\beta H} |\psi(\Gamma, \lambda)\rangle =$$



§ 3.4 infinite time-evolving block decimation

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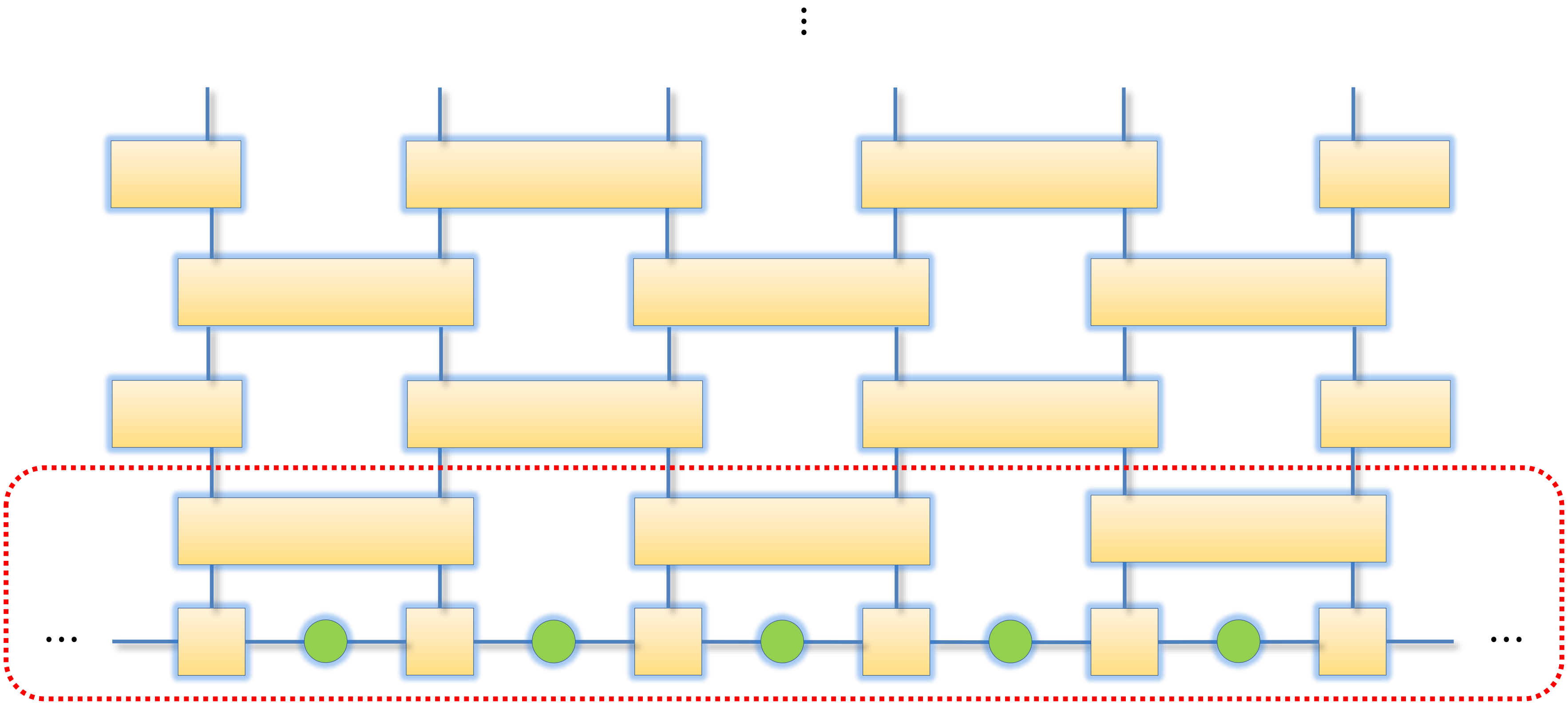
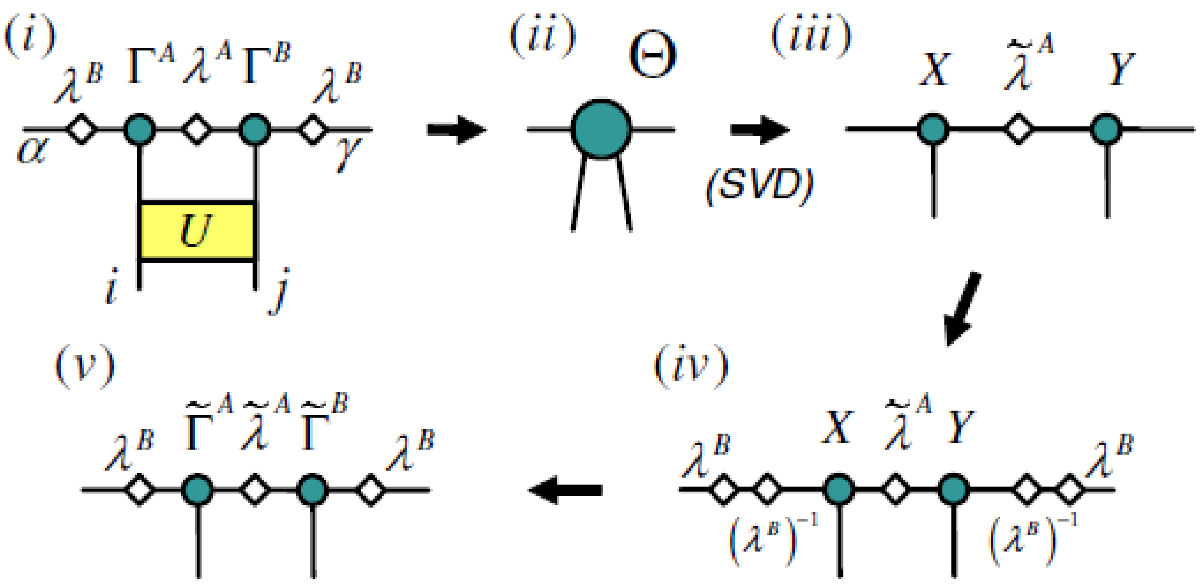


§ 3.4 infinite time-evolving block decimation

Update $|\psi(\Gamma, \lambda)\rangle$ after evolving a small (imaginary) time interval:



- ➢ Apply imaginary-time evolutions many times until convergence is achieved.

G. Vidal, Phys. Rev. Lett. 98, 070201 (2007).

§ 3.4 infinite time-evolving block decimation

- iTEBD: a power method for obtaining iMPS approximation to the ground state (by "cooling")

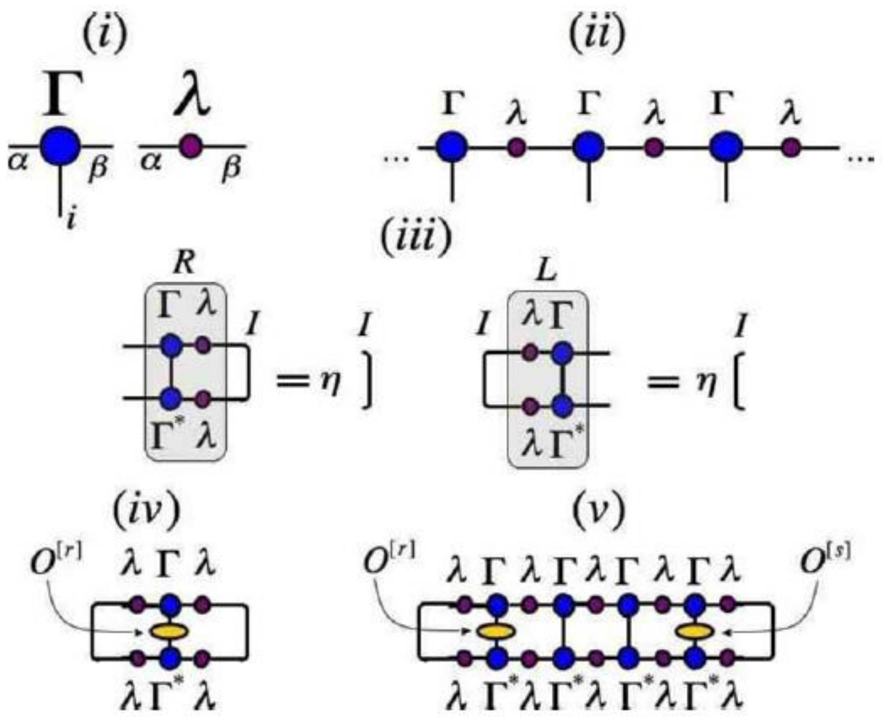
Advantage: computational cost does not depend on system size, method works directly in the thermodynamic limit.

Drawback: Orthogonality of iMPS is not preserved (intrinsic in non-unitary evolutions and due to accumulation of errors in unitary evolutions), method is limited to short-range interactions.

G. Vidal, Phys. Rev. Lett. 98, 070201 (2007).

§ 3.4 infinite time-evolving block decimation

Reorthogonalization of iMPS:



R. Orus & G. Vidal, Phys. Rev. B 78, 155117 (2008).

§ 3.4 infinite time-evolving block decimation

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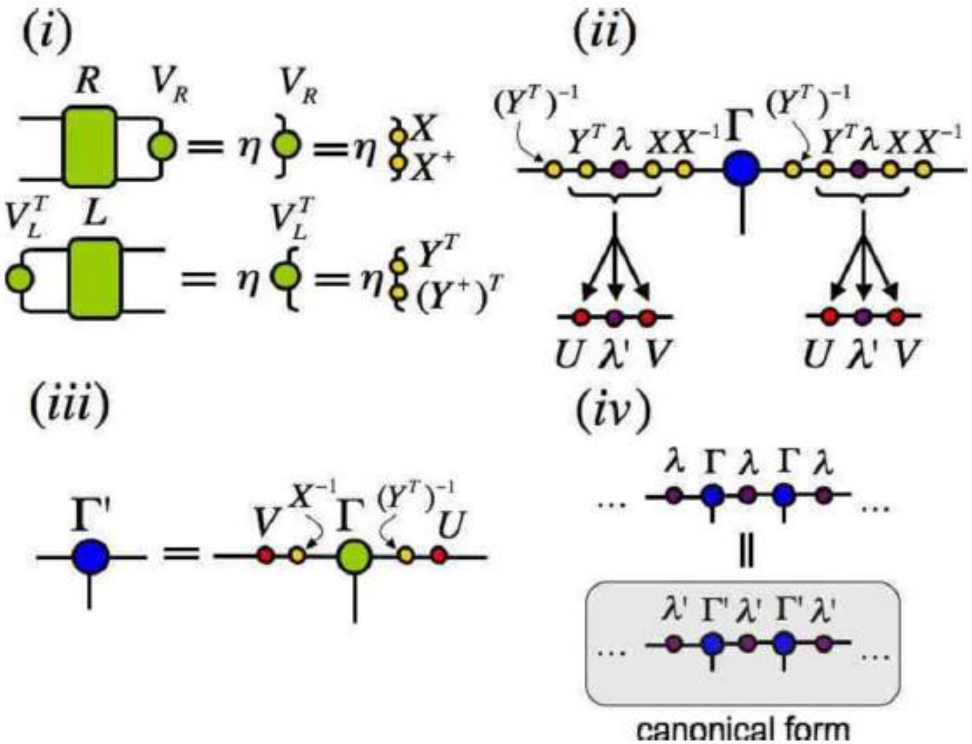

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