SFB 1143

Tensor Networks (SS2021)

Lecture 16: Classification of 1D gapped phases

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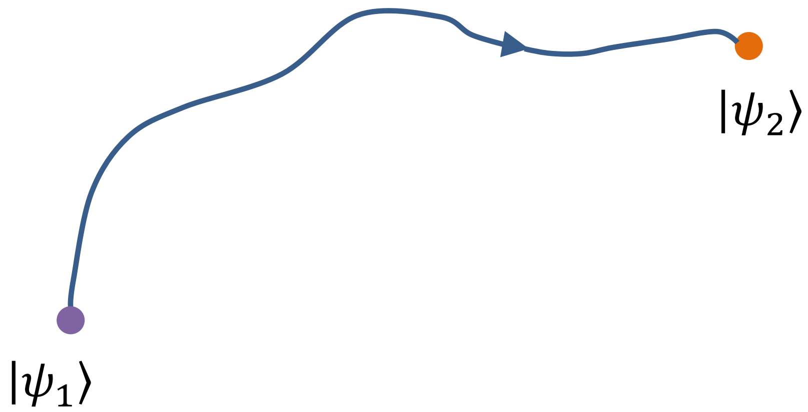
§ 1.10 Classification of 1D gapped phases

- Classification of phases: find equivalence classes of states



Equivalence under local unitary: $|\psi_1\rangle = U_1 \otimes \cdots \otimes U_m |\psi_2\rangle$

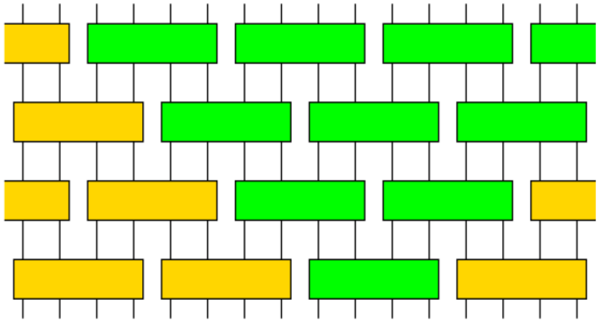
§ 1.10 Classification of 1D gapped phases

- Equivalence under finite-depth local unitary circuits:

$$|\psi_1\rangle = U|\psi_2\rangle$$

⇓

Same phase

$U =$ 

X. Chen, Z.-C. Gu & X.-G. Wen, Phys. Rev. B 82, 155138 (2010).

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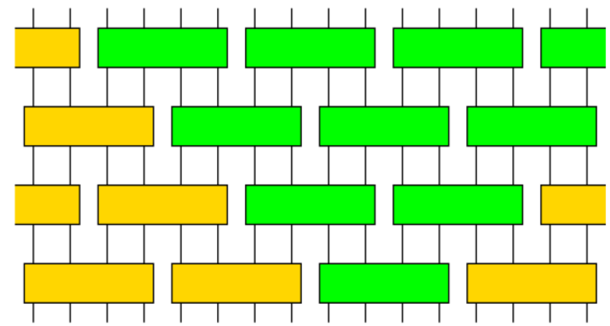

- Distinguishing phases via entanglement:

  long-range entanglement vs. short-range entanglement

  - Short-ranged entangled if transforming into a product state is possible (Note: Two different conventions in the literature!).

X. Chen, Z.-C. Gu & X.-G. Wen, Phys. Rev. B 82, 155138 (2010).

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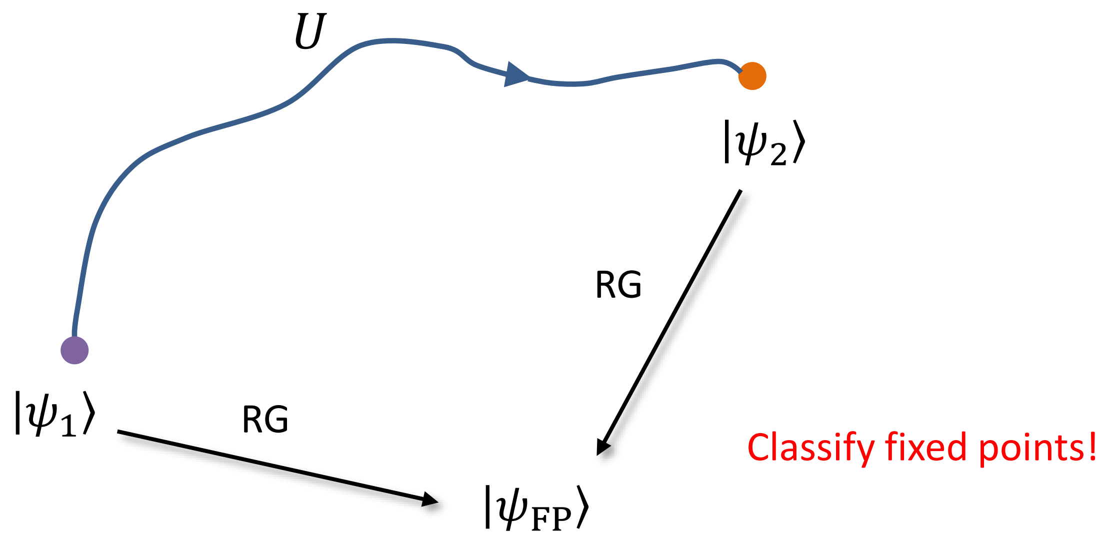
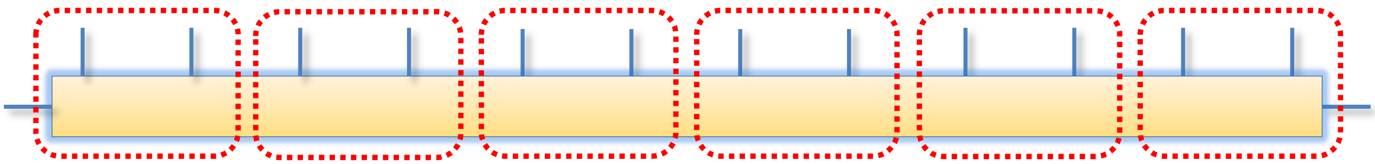
- Renormalization group (RG) and fixed-point wave functions:

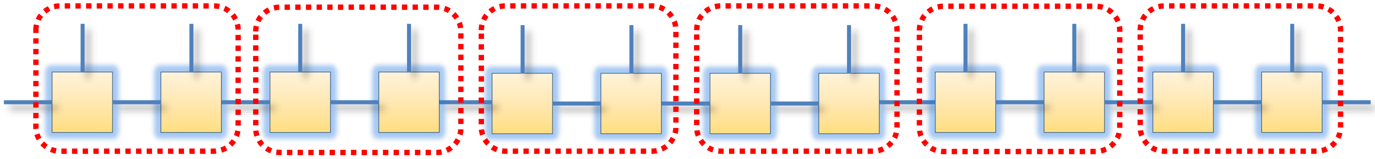




F. Verstraete, J. I. Cirac, J. I. Latorre, E. Rico & M. M. Wolf, Phys. Rev. Lett. 94, 140601 (2005).

§ 1.10 Classification of 1D gapped phases

- The classification of phases can be explicitly worked out for 1D gapped systems.
  - ➢ The gapped ground states are well described by MPS.
  - ➢ The wave-function RG (keeping the most relevant degrees of freedom) can be carried out exactly for MPS.



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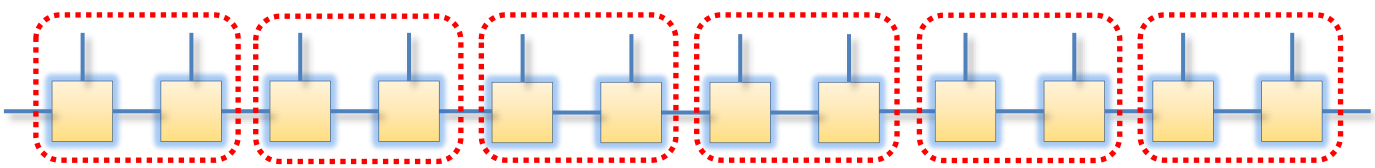
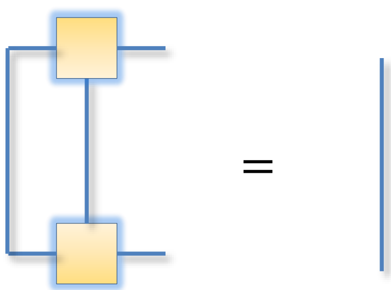
Unique gapped ground state ⇒ Injective MPS



§ 1.10 Classification of 1D gapped phases

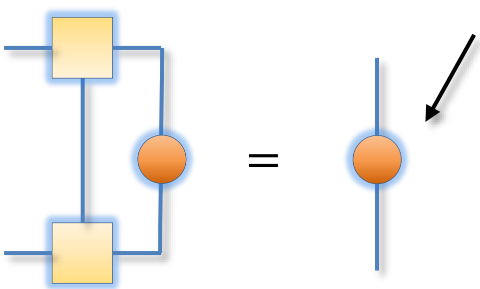
- This problem is reduced to finding equivalence classes of injective MPS.

- The transfer matrix of injective MPS has a unique leading eigenvector with eigenvalue 1.



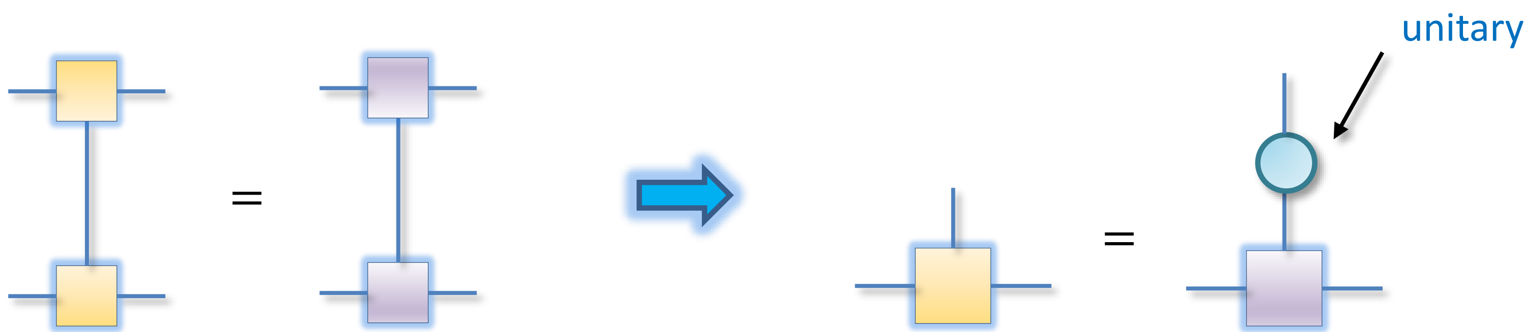
$\Lambda > 0$

$\mathrm{tr}(\Lambda) = 1$



§ 1.10 Classification of 1D gapped phases

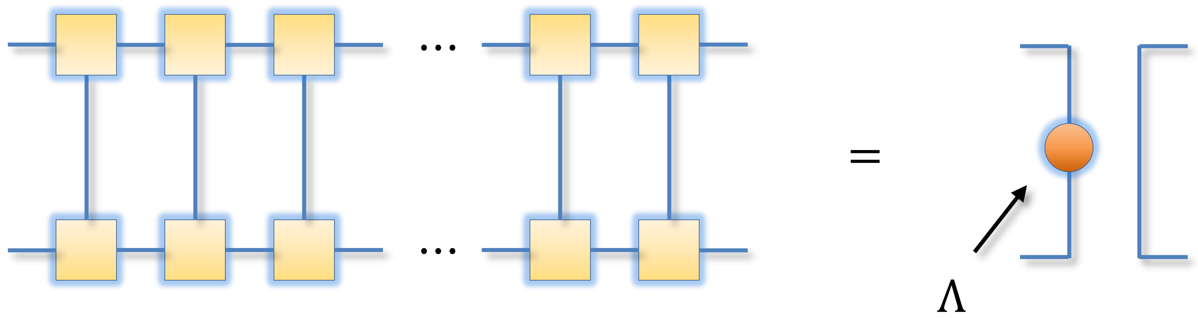
- Transfer matrix uniquely determines an MPS up to a unitary basis rotation:



See, e.g., Appendix A in X. Chen, Z.-C. Gu & X.-G. Wen, Phys. Rev. B 83, 035107 (2011).

§ 1.10 Classification of 1D gapped phases

- Wave-function RG can be conveniently performed with transfer matrices:



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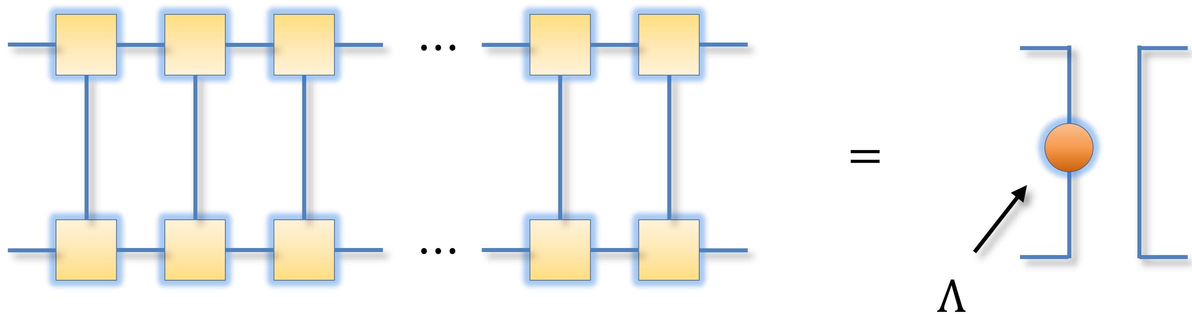
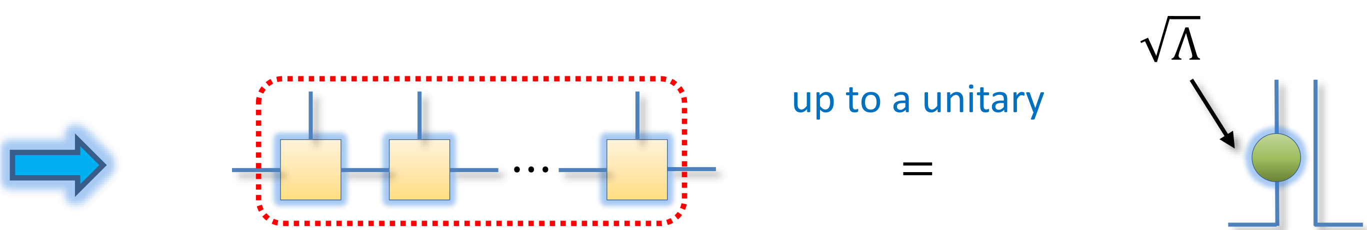
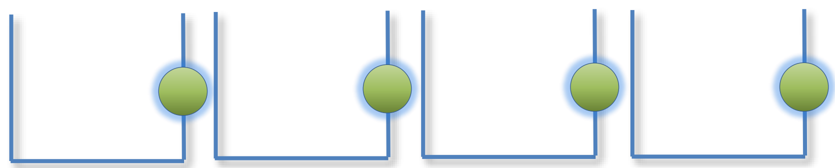




§ 1.10 Classification of 1D gapped phases

- Fixed-point MPS obtained from wave-function RG:

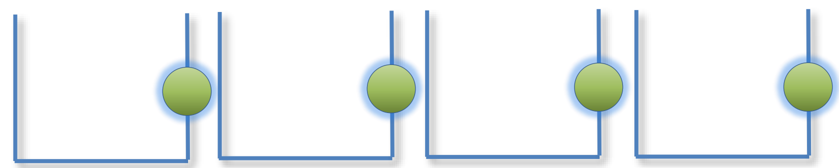
$$|\psi_{\mathrm{FP}}\rangle \quad = $$



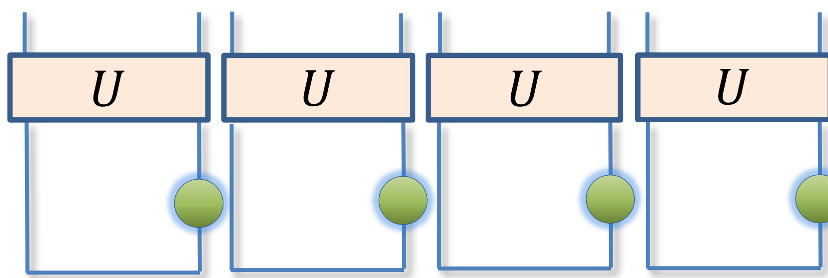
§ 1.10 Classification of 1D gapped phases

- There is a single trivial gapped phase in 1D gapped spin systems (i.e., no topological order!).

$|\psi_{\rm FP}\rangle \quad =$



⇒



⇒ Product state!

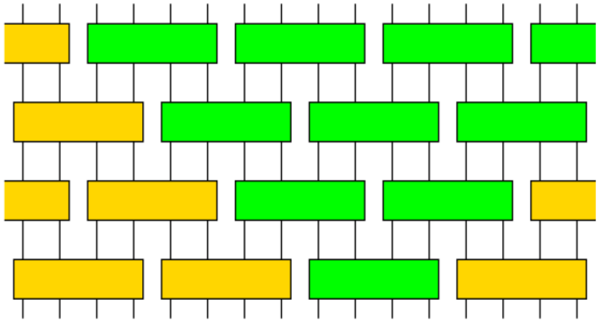
§ 1.10 Classification of 1D gapped phases

- What happens if symmetry is present?

$$|\psi_1\rangle = U|\psi_2\rangle$$

Same phase?

$U =$

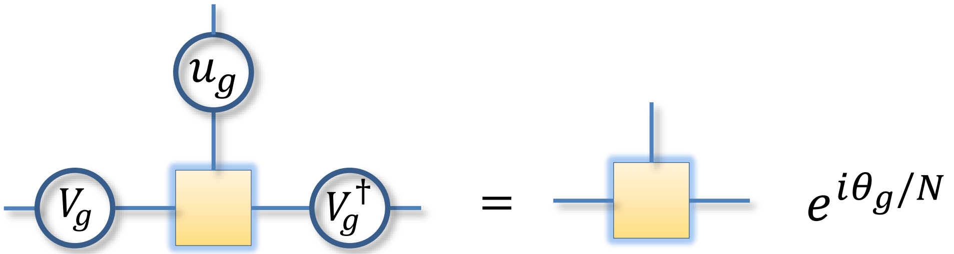


➢ U is a symmetry-preserving finite-depth local unitary circuit.

➢ Same phase if such symmetry-preserving circuit exists (this defines equivalence classes under a certain symmetry).

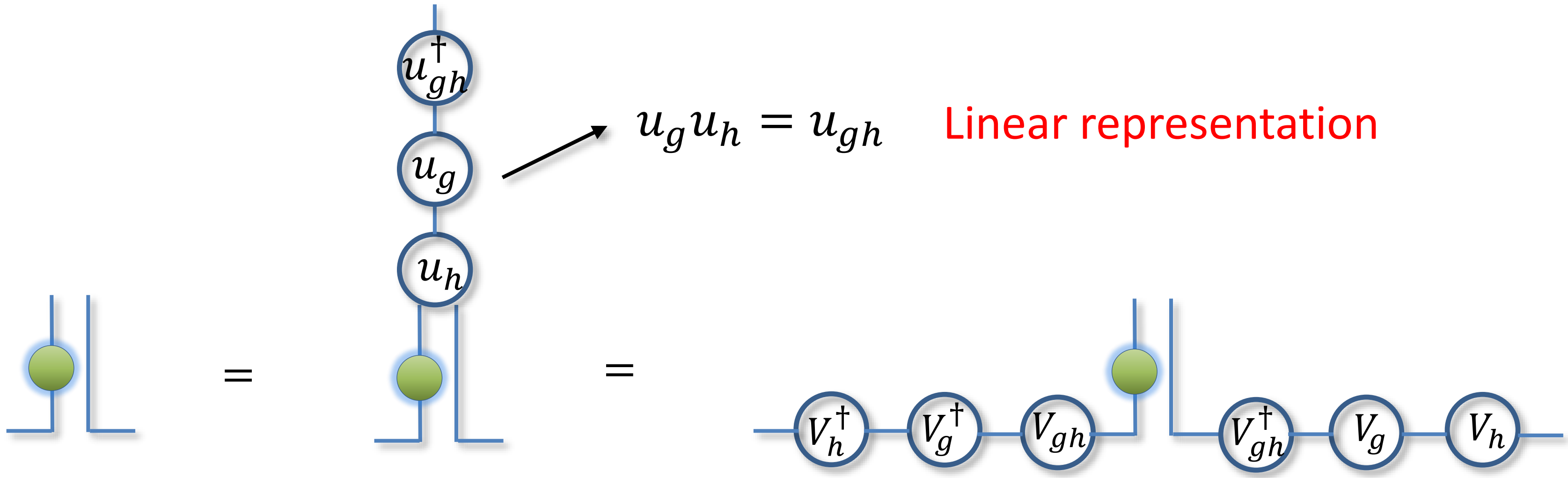
§ 1.10 Classification of 1D gapped phases

- Consider on-site unitary symmetry G:



§ 1.10 Classification of 1D gapped phases

- Onsite unitary symmetry for fixed-point MPS (no translational symmetry assumed!) :



$$u_g u_h = u_{gh}$$ Linear representation

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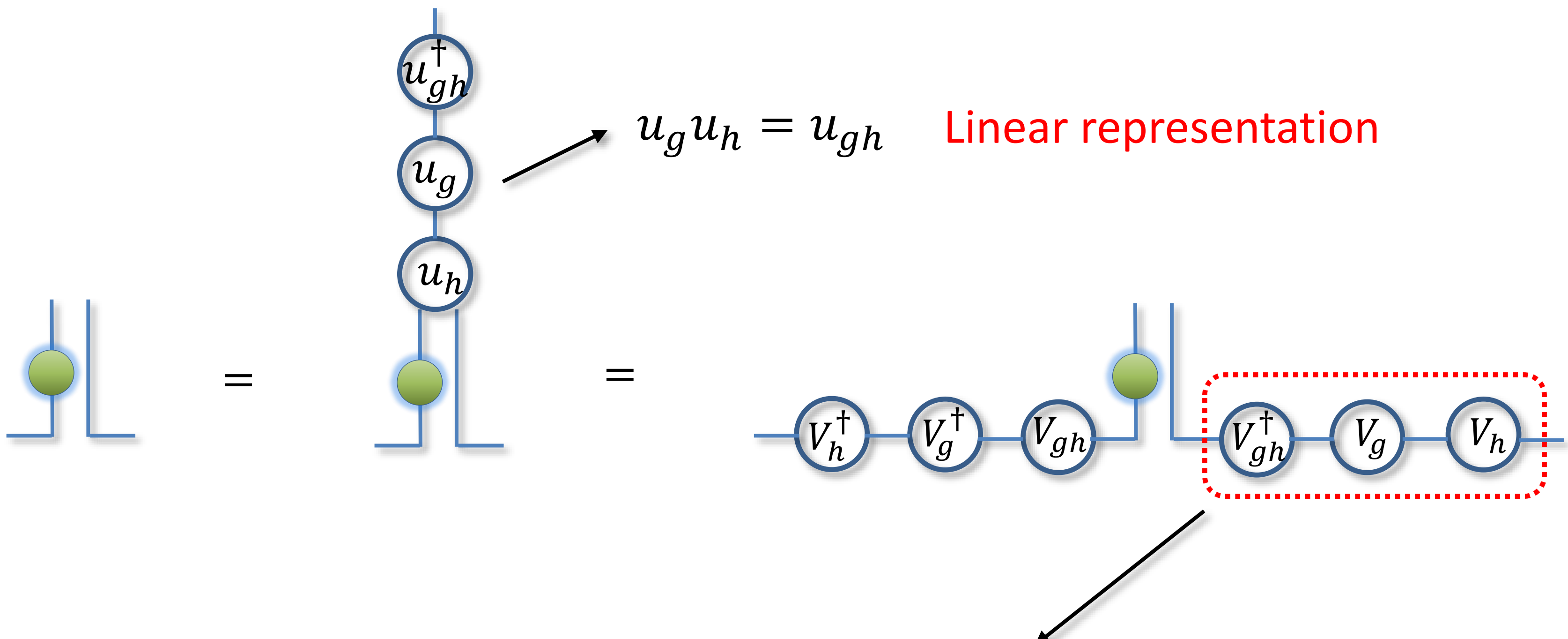
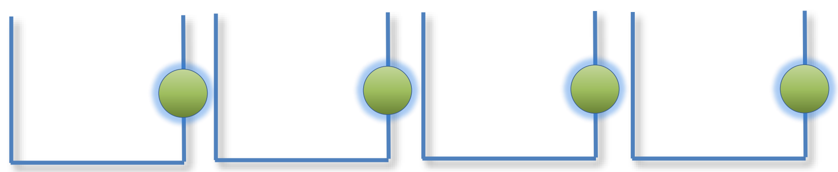


Classified by inequivalent projective representations!

$$V_g V_h = e^{i\phi(g,h)} V_{gh}$$

§ 1.10 Classification of 1D gapped phases

Example: spin chain with SO(3) symmetry

$|\psi_{\mathrm{FP}}\rangle$ = 

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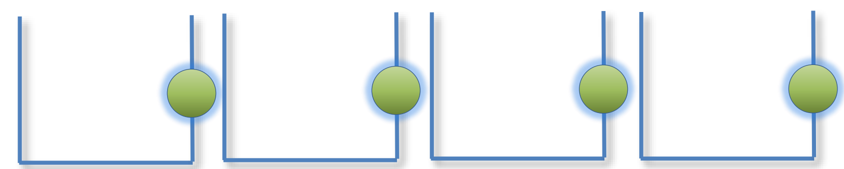
$|\psi_{\mathrm{FP}}\rangle \quad =$ 

- Nontrivial projective representation => SU(2) half-integer representation at the boundary (e.g. spin-1 AKLT with spin-1/2 edge state)

- Trivial projective (i.e. linear) representation phase => SU(2) integer representation at the boundary (e.g. spin-2 AKLT with spin-1 edge state)

X. Chen, Z.-C. Gu & X.-G. Wen, Phys. Rev. B 83, 035107 (2011); Phys. Rev. B 84, 235128 (2011);
Norbert Schuch, David Pérez-García, and Ignacio Cirac, Phys. Rev. B 84, 165139 (2011).

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- Remarks on higher-dimensional systems:
  - How to carry out the wave-function RG under control?
  - The current classification results (e.g. SPT phases in higher dimensions, string-net models, etc) are often based on fixed-point wave functions (the free-fermion classification is an exception).

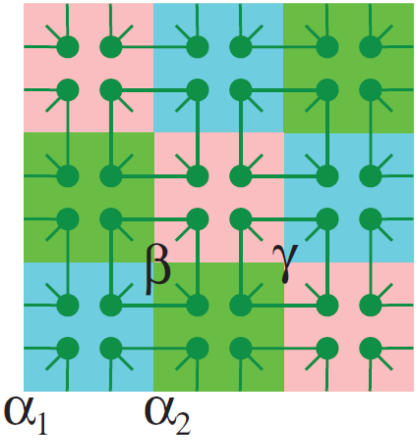

Figure from PRB 87, 155114 (2013)