TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

Tensor Networks (SS2021)

Lecture 17: Projected Entangled Pair State

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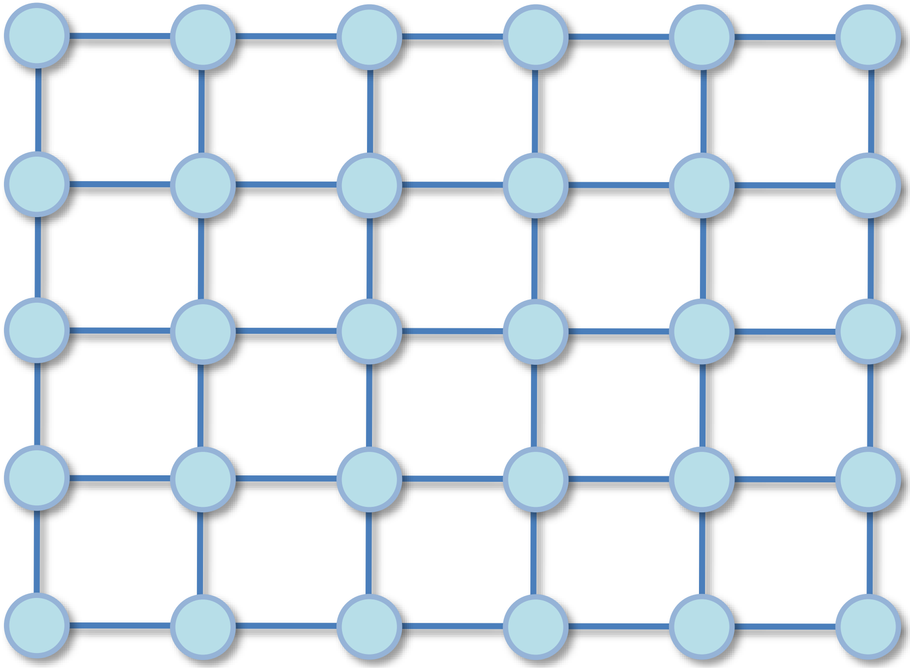
§ 4.0 From 1D to 2D: limitation of MPS

$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \cdots$$

Q: Can we approximate the ground state with an MPS (e.g. using DMRG)?



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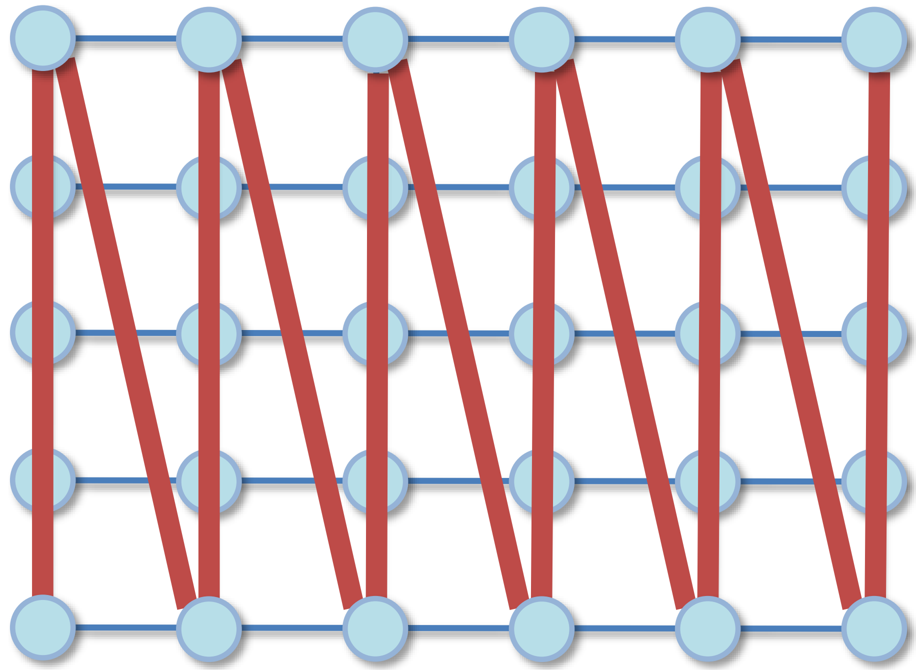

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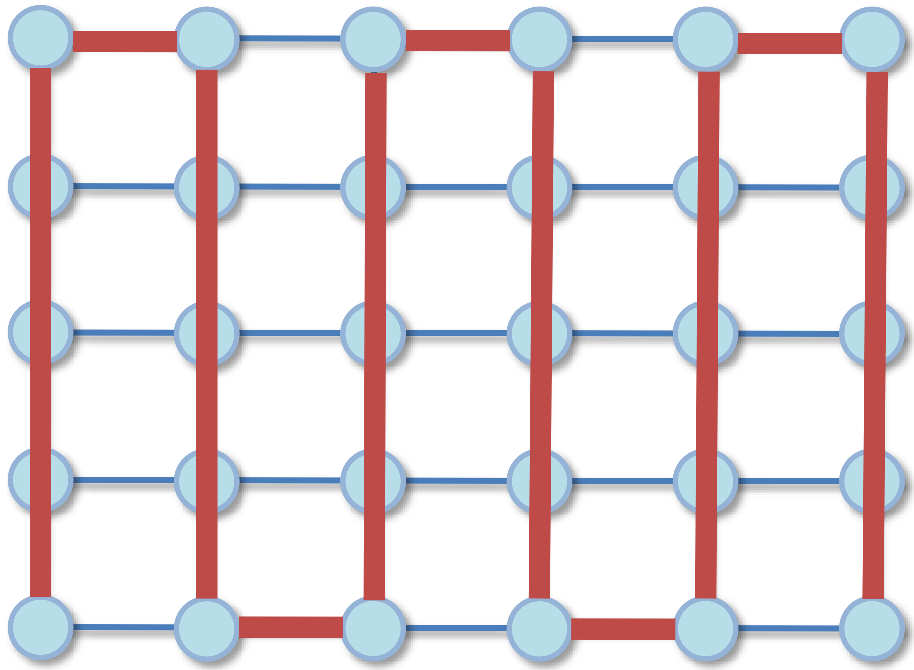

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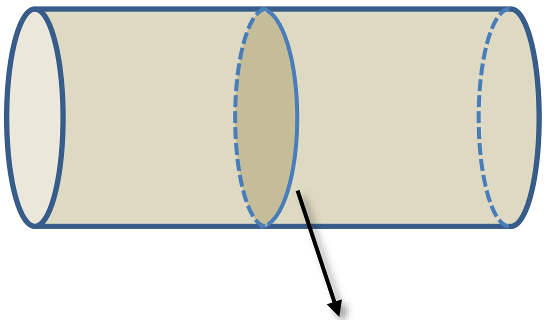
§ 4.0 From 1D to 2D: limitation of MPS

- Entanglement area law for 2D ground states:

$$S(L_y) \sim \alpha L_y + \cdots$$

  - ➢ conjectured to be true for all gapped ground states of local Hamiltonians
  - ➢ fulfilled by some gapless states (e.g. Dirac Fermi sea)



circumference: L_y

Exception: 2D fermion with a Fermi surface

$$S(L_y) \sim L_y \ln L_y + \cdots$$

Review: J. Eisert, M. Cramer & M. B. Plenio, Rev. Mod. Phys. 82, 277 (2010).

§ 4.0 From 1D to 2D: limitation of MPS

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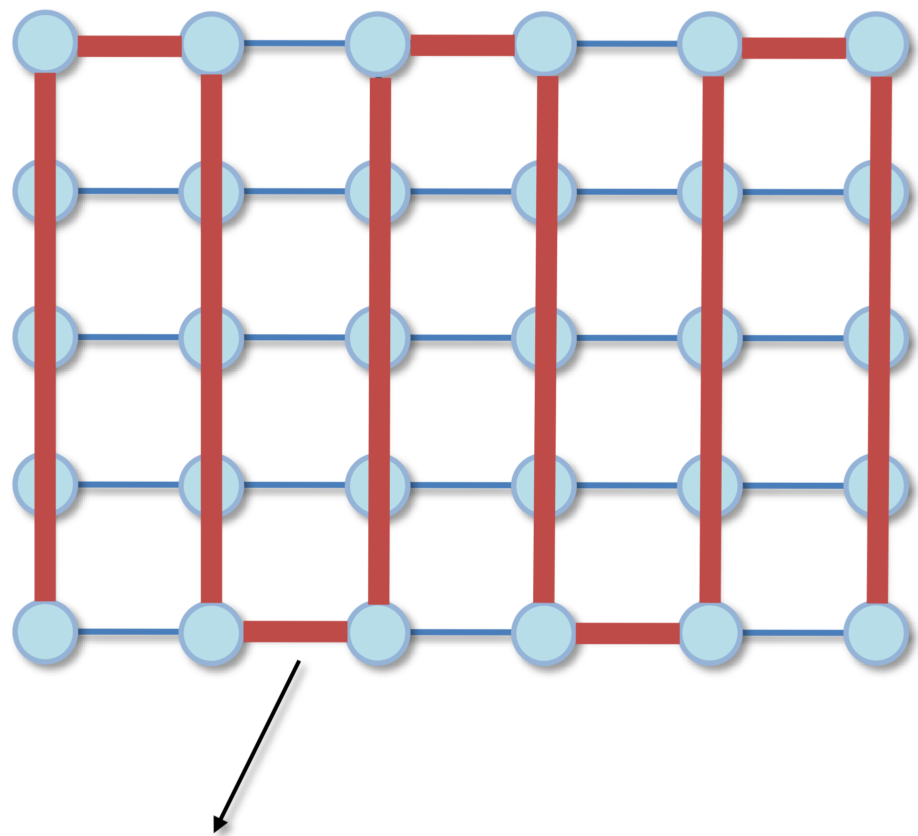
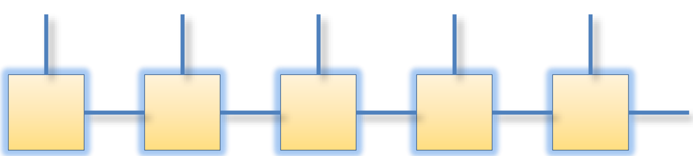
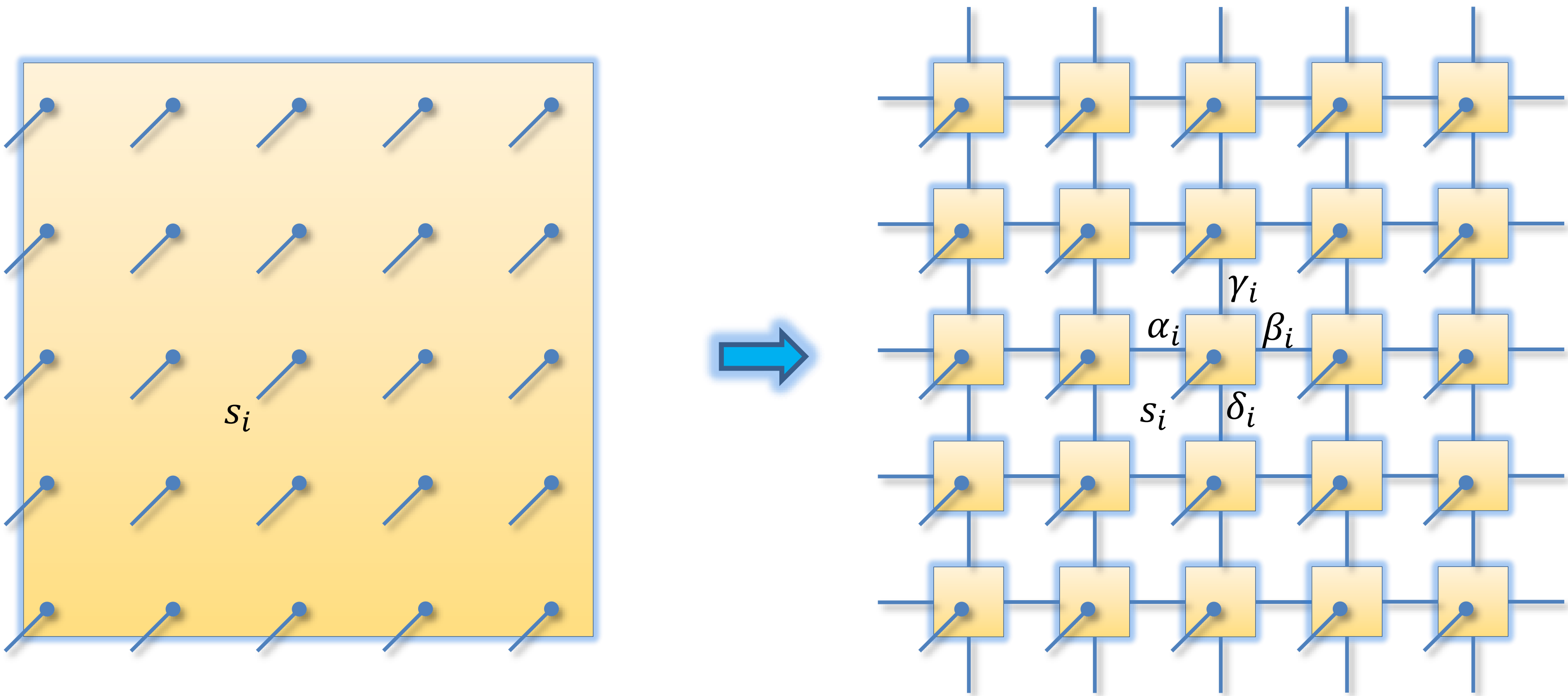




$D \sim e^{S(L_y)} \sim e^{\alpha L_y}$ (area law case)

§ 4.1 PEPS: Motivation and examples



$$|\psi\rangle = \sum_{\{s\}=1}^{d} \psi(\ldots s_i \ldots)\,|\ldots s_i \ldots\rangle$$

$$|\psi\rangle = \sum_{\{s\}=1}^{d} \left(\sum_{\{\alpha\beta\gamma\delta\}=1}^{D} \cdots A^{s_i}_{\alpha_i\beta_i\gamma_i\delta_i} \cdots \right) |\ldots s_i \ldots\rangle$$

rank-dD^4 tensor

F. Verstraete & J. I. Cirac, cond-mat/0407066.

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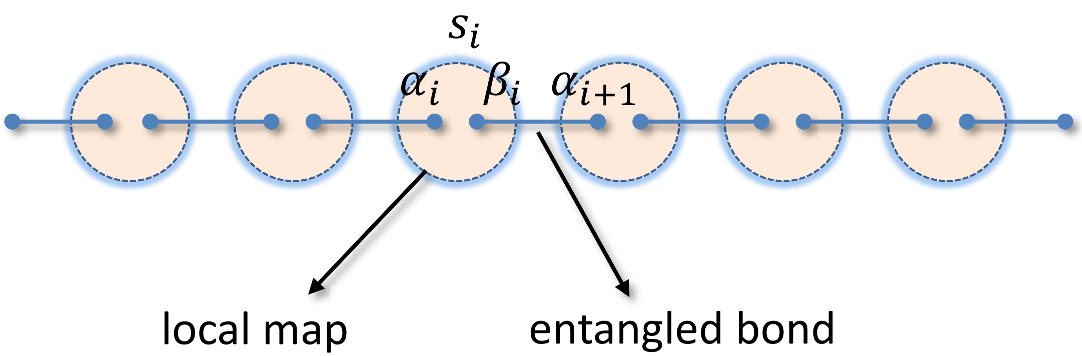


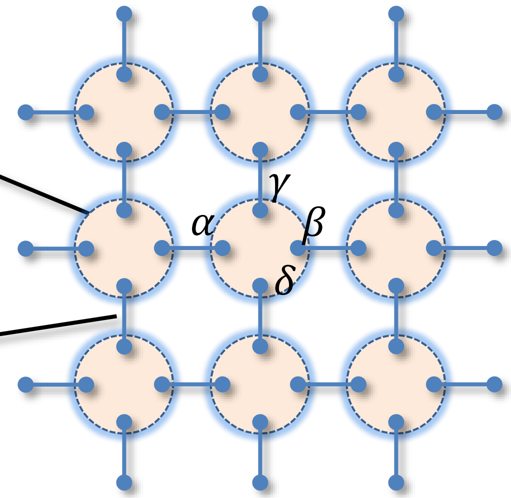
$$P_i = \sum_{s_i=1}^{d} \sum_{\alpha_i,\beta_i=1}^{D} B^{s_i}_{\alpha_i\beta_i} |s_i\rangle\langle\alpha_i,\beta_i| \qquad |I\rangle_{i,i+1} = \sum_{\beta_i,\alpha_{i+1}=1}^{D} R_{\beta_i,\alpha_{i+1}} |\beta_i,\alpha_{i+1}\rangle$$

$$|\psi\rangle = \left(\bigotimes_{i=1}^{N} P_i\right)\left(\bigotimes_{j=1}^{N} |I\rangle_{j,j+1}\right) = \sum_{s_1,s_2,\dots,s_N=1}^{d} \mathrm{Tr}(A^{s_1}A^{s_2}\cdots A^{s_N})\,|s_1,s_2,\dots,s_N\rangle$$

§ 4.1 PEPS: Motivation and examples

$$P_i = \sum_{s_i=1}^{d} \sum_{\alpha_i,\beta_i,\gamma_i,\delta_i=1}^{D} B^{s_i}_{\alpha_i,\beta_i\gamma_i\delta_i} |s_i\rangle\langle\alpha_i,\beta_i,\gamma_i,\delta_i|$$

$$|I\rangle_{ij} = \sum_{\delta_i,\gamma_j=1}^{D} R_{\delta_i,\gamma_j} |\delta_i,\gamma_j\rangle$$



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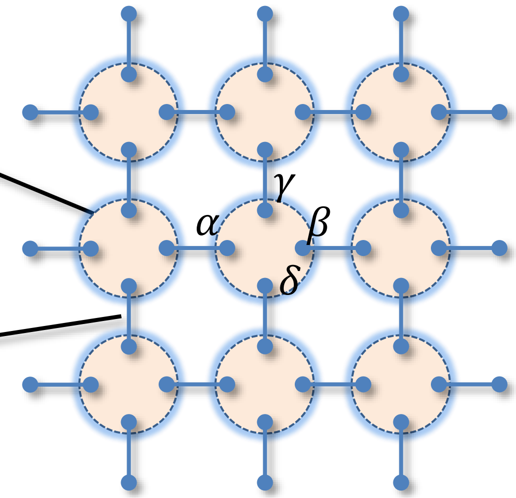
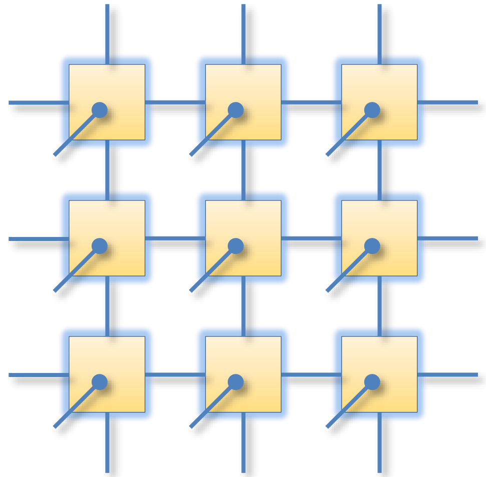
$$|I\rangle_{ij} = \sum_{\delta_i,\gamma_j=1}^{D} R_{\delta_i,\gamma_j} |\delta_i, \gamma_j\rangle$$



$$|\psi\rangle = \left(\bigotimes_{i=1}^{N} P_i\right)\left(\bigotimes_{\langle ij\rangle} |I\rangle_{ij}\right)$$

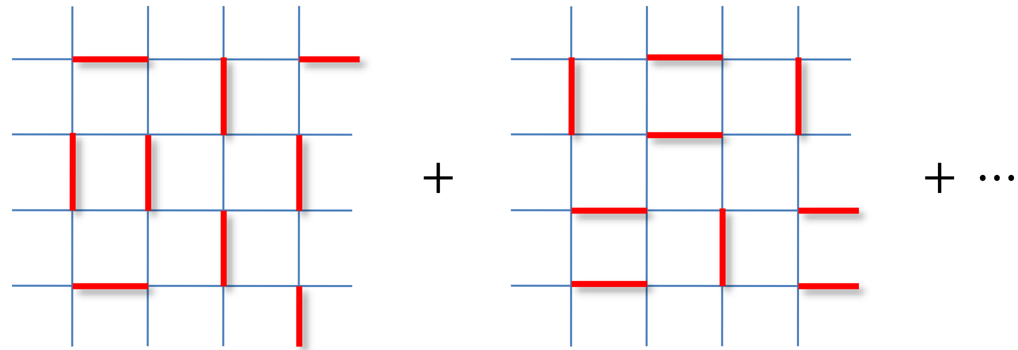
$$= \sum_{\{s\}} \left(\sum_{\{\alpha\beta\gamma\delta\}} \cdots B^{s_i}_{\alpha_i\beta_i\gamma_i\delta_i} R_{\delta_i\gamma_j} \cdots \right) |\ldots s_i \ldots\rangle$$



§ 4.1 PEPS: Motivation and examples

$$|\psi_{\mathrm{RVB}}\rangle = \sum |\mathrm{dimers}\rangle$$

 $= |01\rangle - |10\rangle$



P. W. Anderson, Science 235, 1196 (1987);

D. S. Rokhsar & S. A. Kivelson, Phys. Rev. Lett. 61, 2376 (1988).

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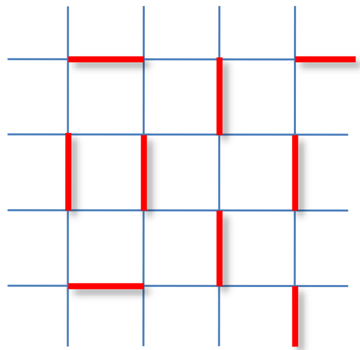
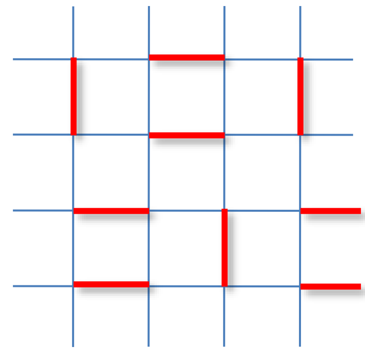


$+$

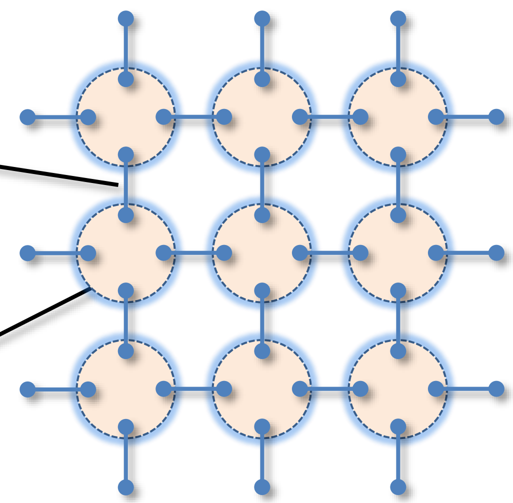


$+ \cdots$

$$— = |01\rangle - |10\rangle$$

$$|I\rangle = |01\rangle - |10\rangle + |22\rangle$$

$$P = |0\rangle(\langle 0222| + \langle 2022| + \langle 2202| + \langle 2220|)$$
$$+|1\rangle(\langle 1222| + \langle 2122| + \langle 2212| + \langle 2221|)$$



F. Verstraete, M. M. Wolf, D. Perez-Garcia & J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006).

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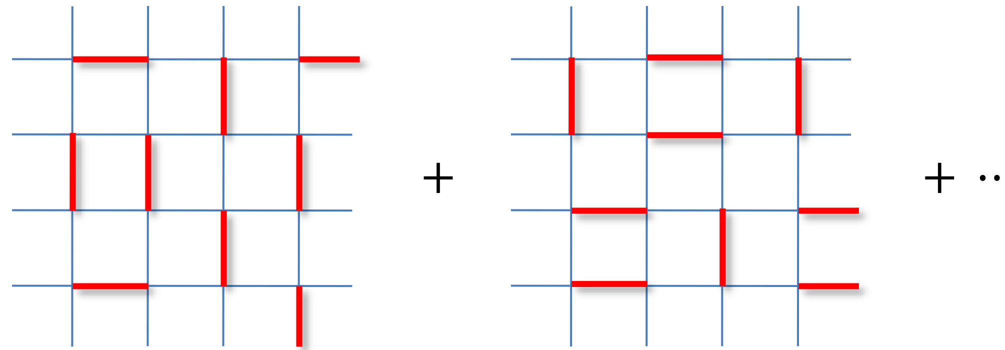
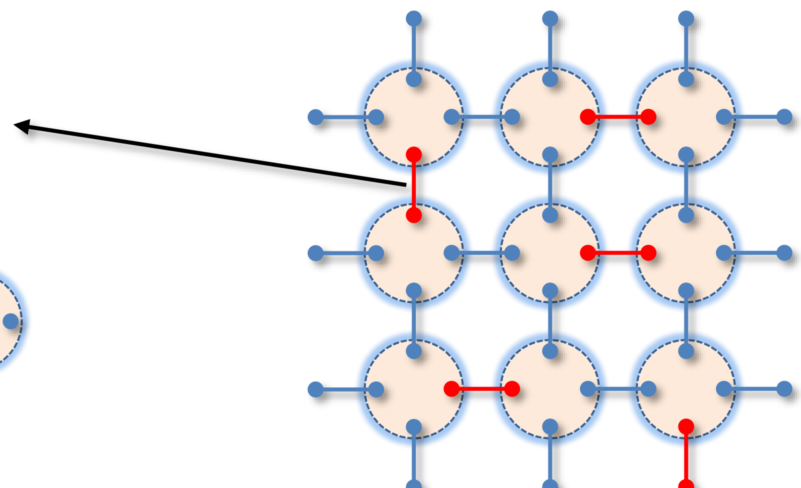
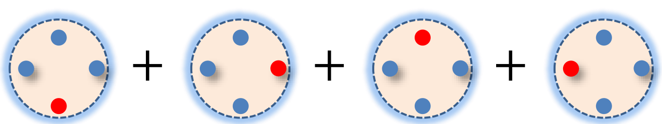


(red line) $= |01\rangle - |10\rangle$

$|I\rangle = \boxed{|01\rangle - |10\rangle} + |22\rangle$



$P =$



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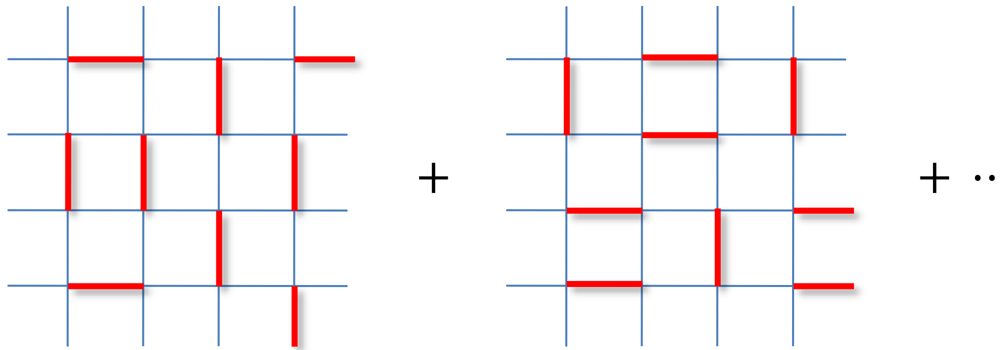

— $= |01\rangle - |10\rangle$

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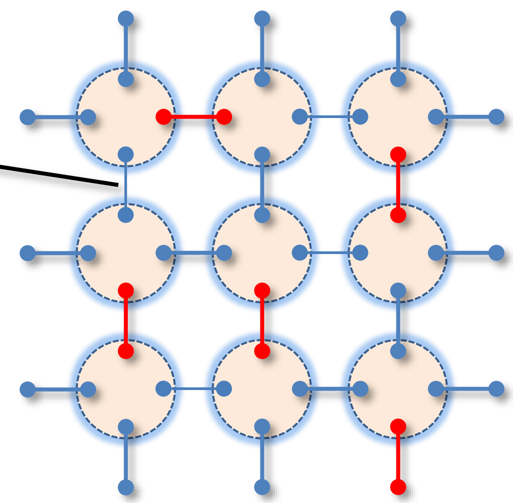
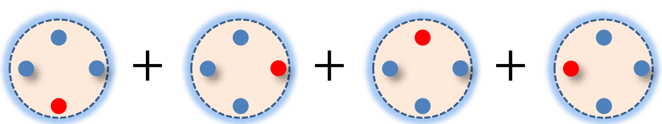


$P =$



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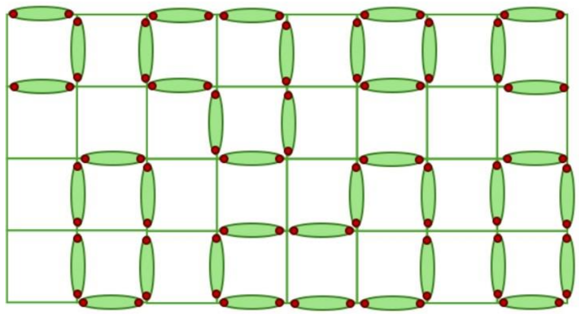
$$|\psi_{\mathrm{RAL}}\rangle = \sum |\mathrm{AKLT\ loops}\rangle$$

H. Yao, L. Fu & X.-L. Qi, arXiv:1012.4470



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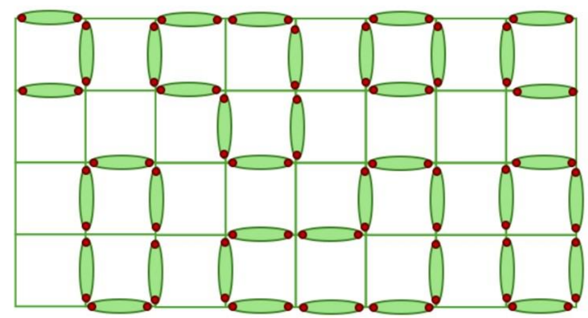
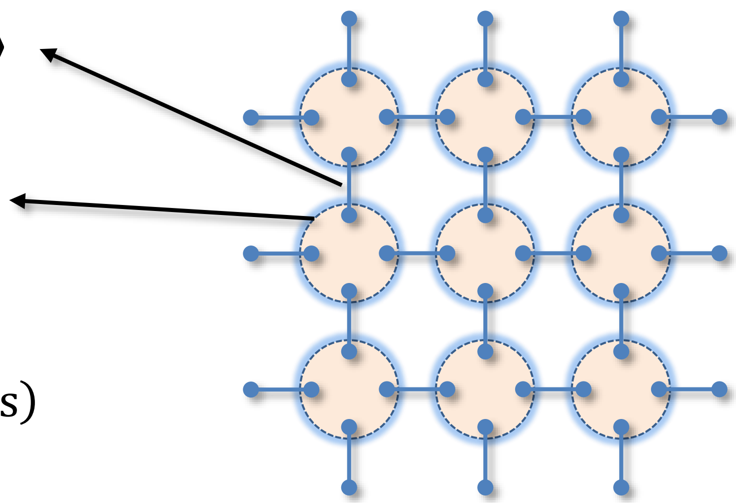


$$|I\rangle = |01\rangle - |10\rangle + |22\rangle$$

$$P = |1\rangle(\langle 0022| + \mathrm{permutations})$$
$$+|-1\rangle(\langle 1122| + \mathrm{permutations})$$
$$+\frac{1}{\sqrt{2}}|0\rangle(\langle 0122| + \mathrm{permutations})$$



W. Li, S. Yang, M. Cheng, Z.-X. Liu & HHT, Phys. Rev. B 89, 174411 (2014).

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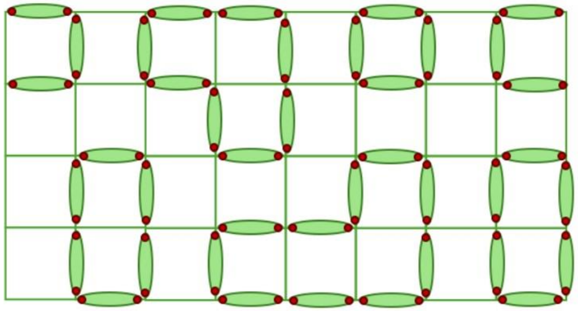

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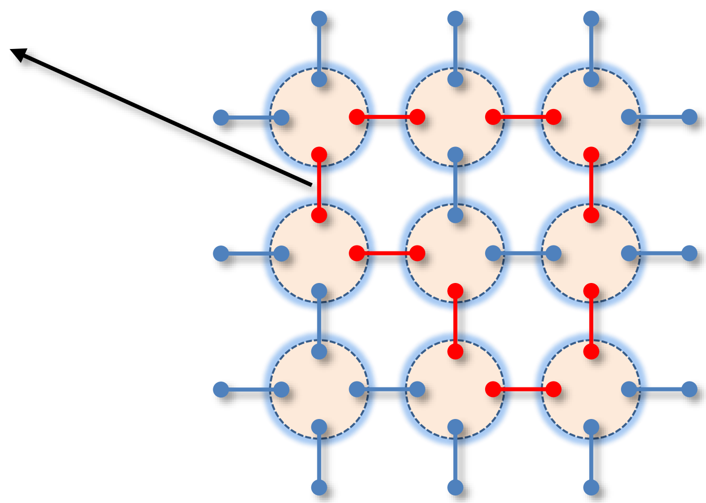
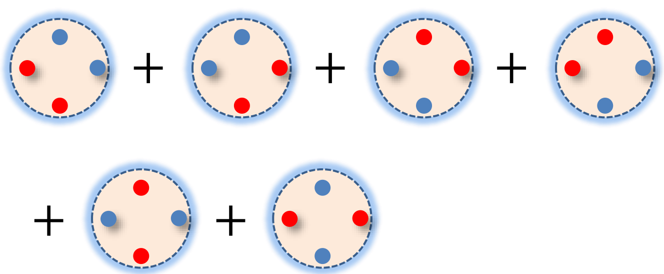


$P =$

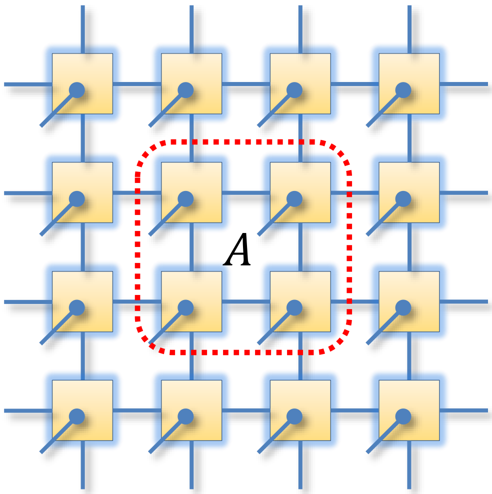


§ 4.1 PEPS: Motivation and examples

- # of parameters in PEPS scales polynomially

- PEPS satisfy the entanglement area law

$$S(A) \sim N_{\partial A}$$



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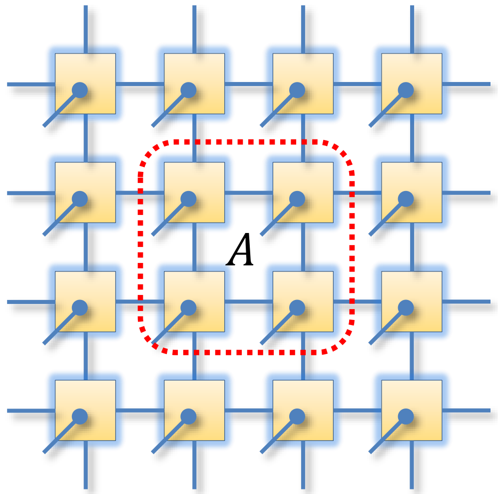

Theoretical foundation:

- Exact representation for many 2D (nonchiral) topologically ordered and symmetry-protected topological states.

O. Buerschaper, M. Aguado & G. Vidal, Phys. Rev. B 79, 085119 (2009);

Z.-C. Gu, M. Levin, B. Swingle & X.-G. Wen, Phys. Rev. B 79, 085118 (2009);

D. J. Williamson, N. Bultinck, M. Mariën, M. B. Şahinoğlu, J. Haegeman & F. Verstraete, Phys. Rev. B 94, 205150 (2016).

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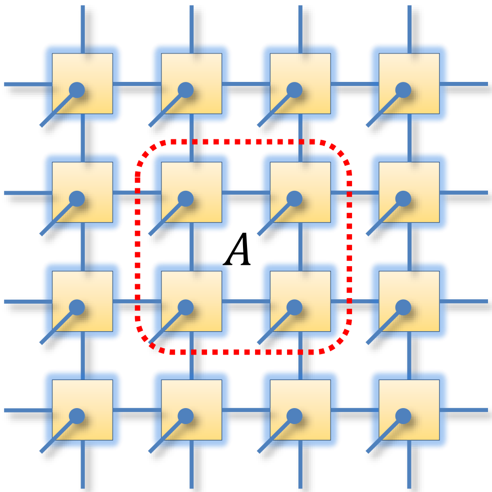

Theoretical foundation:

➢ Chiral topological states? (No-go theorem for free fermionic cases, interacting cases still not settled)

J. Dubail & N. Read, Phys. Rev. B 92, 205307 (2015);

T. B. Wahl, HHT, N. Schuch & J. I. Cirac, Phys. Rev. Lett. 111, 236805 (2013).

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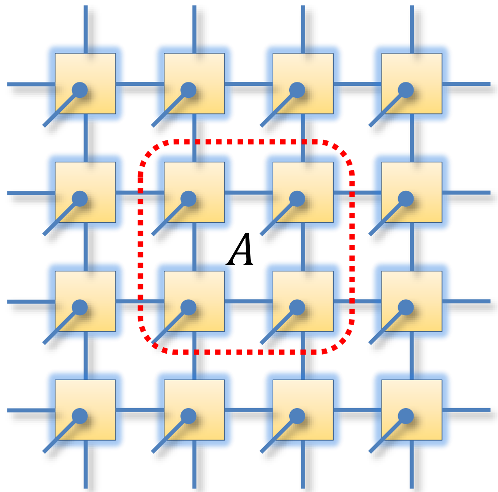
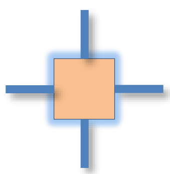


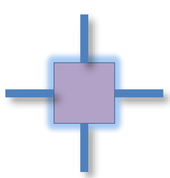
Main challenges in numerical applications:

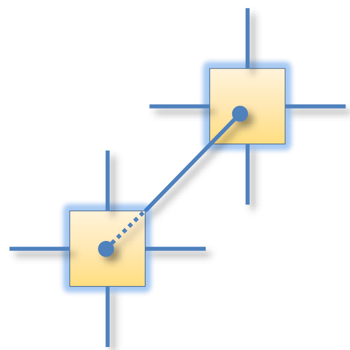
- Optimization: finding the best PEPS ansatz for a given Hamiltonian
- Contraction: Computing physical observables

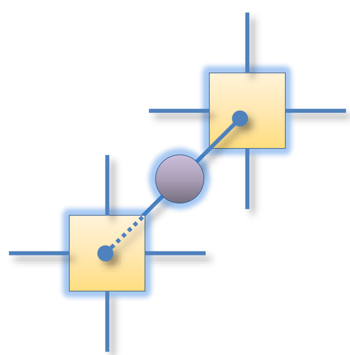
§ 4.1 PEPS: Motivation and examples

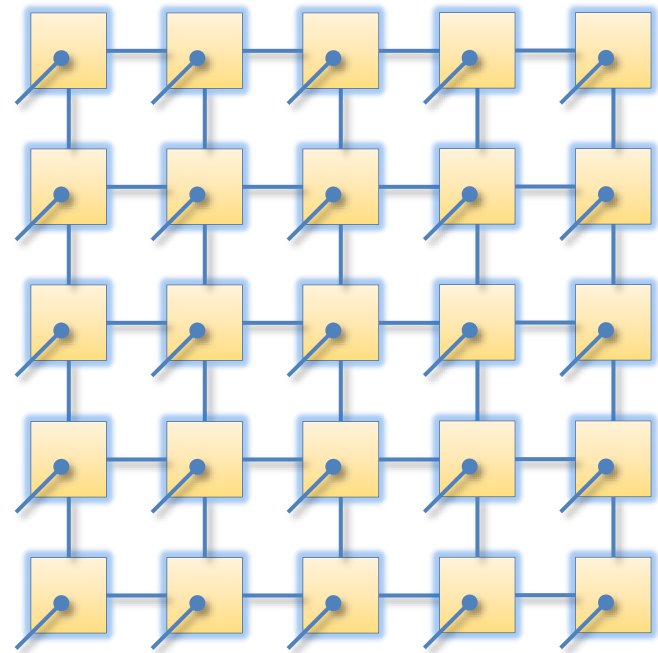
$\langle\psi|O_i O_j|\psi\rangle$











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$\langle\psi|O_i O_j|\psi\rangle =$