TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

SFB 1143

# Tensor Networks (SS2021)

## Lecture 20: Tensor network renormalization


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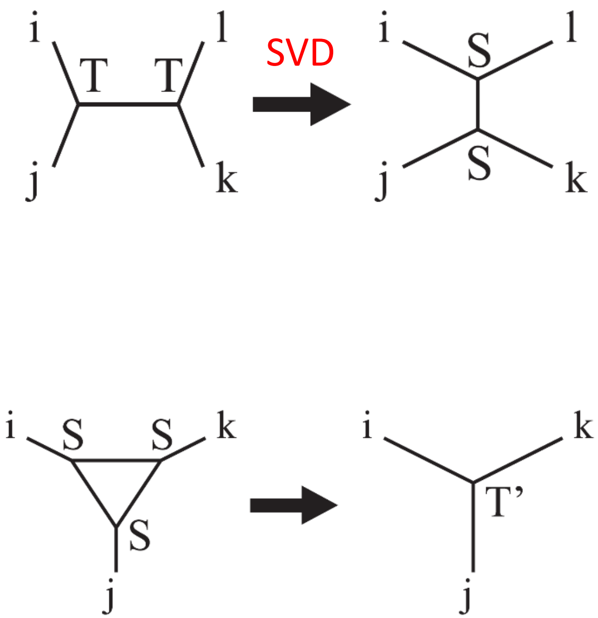
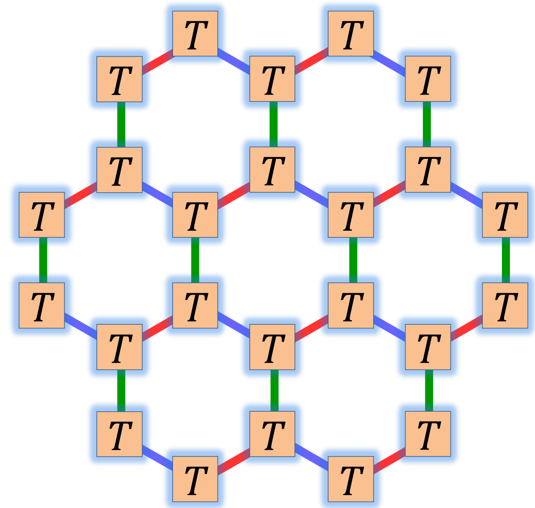
# § 4.3 Tensor renormalization group

Keep steps of the TRG:



# § 4.3 Tensor renormalization group

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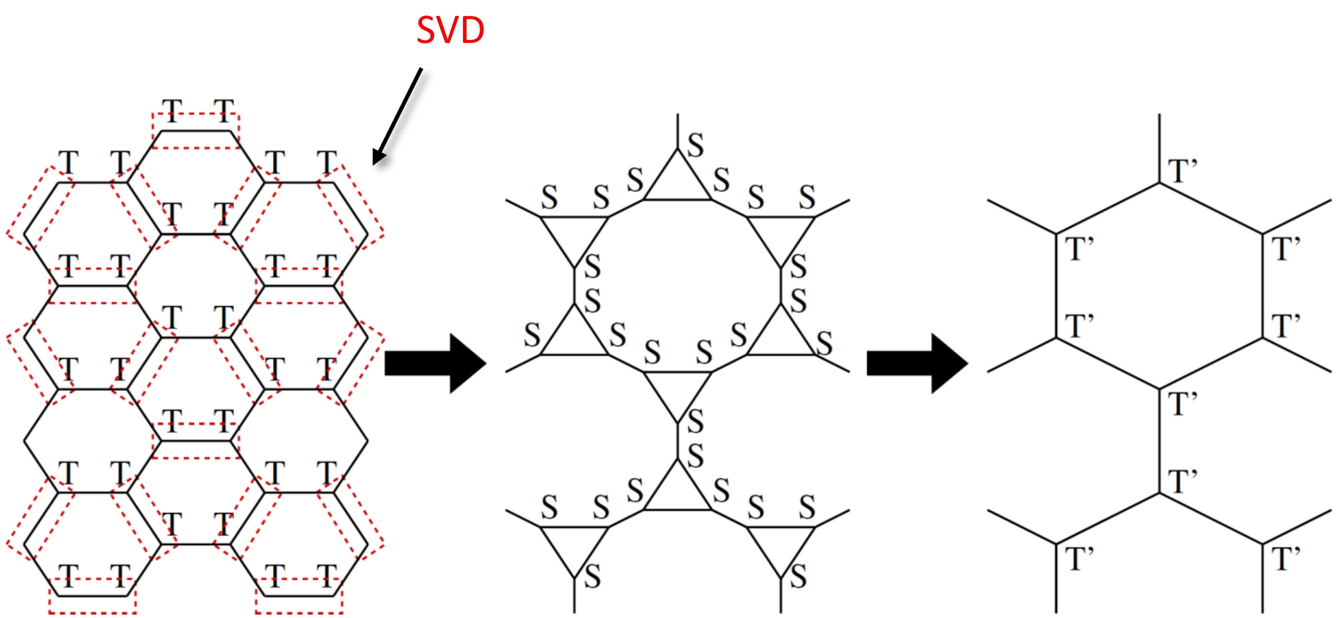
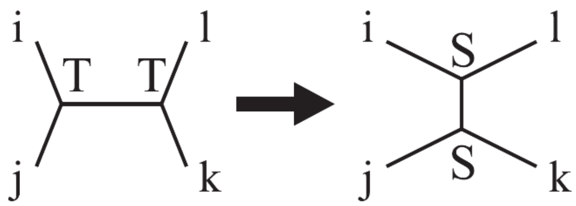
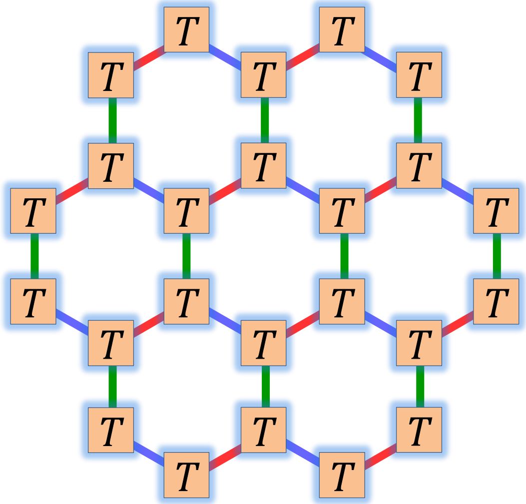



Repeat the RG steps to reduce the # of sites

# § 4.3 Tensor renormalization group

When doing the local SVD truncation, environment is ignored!



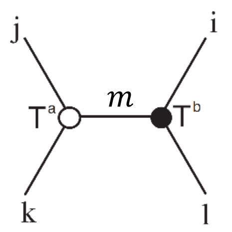



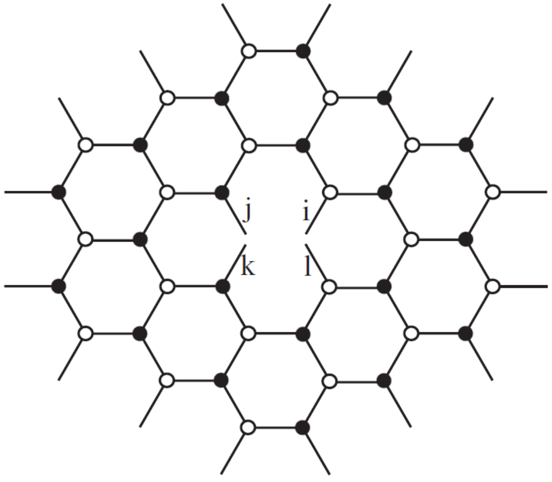

How can we perform an optimal truncation here?

# § 4.4 Tensor network renormalization

Consider the partition function:

$$Z = \sum_{\{\ldots i,j,m,k,l\ldots\}} \left(\cdots T^a_{jkm} T^b_{mil} \cdots\right) = \sum_{ij,kl} M_{ij,kl} M^e_{kl,ij}$$

$M_{ij,kl} =$ 

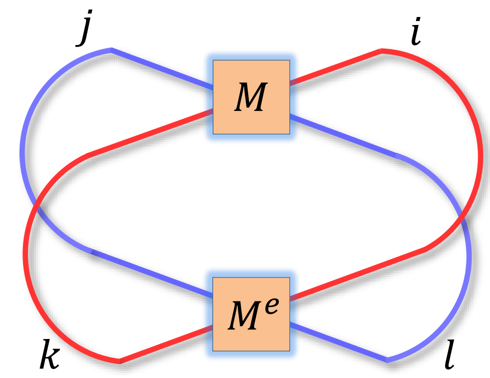

$M^e_{kl,ij} =$ 


Z. Y. Xie, H. C. Jiang, Q. N. Chen, Z. Y. Weng & T. Xiang, Phys. Rev. Lett. 103, 160601 (2009).

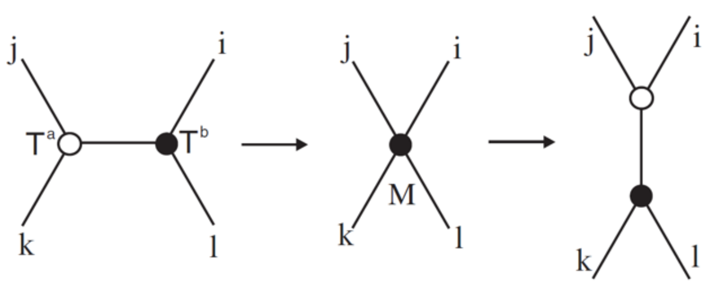
# § 4.4 Tensor network renormalization

Partition function:

$$Z = \sum_{ij,kl} M_{ij,kl} M^e_{kl,ij}$$



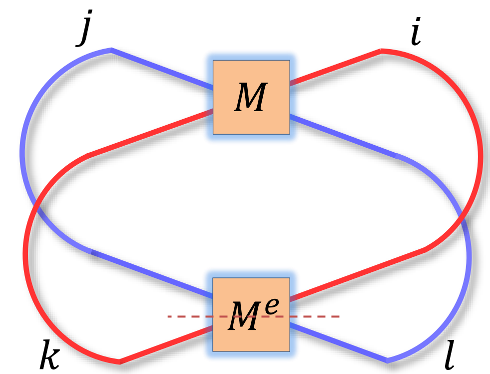

Optimal truncation for $M$ should minimize the error in $Z$:



# § 4.4 Tensor network renormalization

Partition function:

$$Z = \sum_{ij,kl} M_{ij,kl} M^e_{kl,ij}$$




$$M^e_{kl,ij} = \sum_{m=1}^{D^2} (U_e)_{kl,m} (\Lambda_e)_m (V_e^\dagger)_{m,ij}$$

$$\Rightarrow \quad Z = \mathrm{Tr}(\widetilde{M})$$

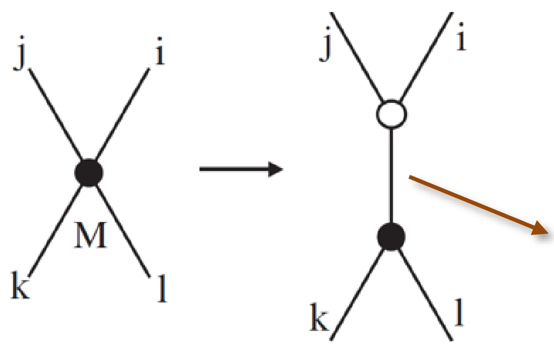
$$\widetilde{M} = \Lambda_e^{1/2} V_e^\dagger M U_e \Lambda_e^{1/2}$$

# § 4.4 Tensor network renormalization

Truncation of $M$ should minimize the error of $\widetilde{M}$:

$$Z = \mathrm{Tr}(\widetilde{M})$$

$$\widetilde{M} = \Lambda_e^{1/2} V_e^{\dagger} M U_e \Lambda_e^{1/2}$$




bond dimension $D_{\mathrm{cut}} < D^2$

# § 4.4 Tensor network renormalization

Formally, this is done by finding a low-rank approximation of $\widetilde{M}$:

$$Z = \mathrm{Tr}(\widetilde{M})$$

$$\widetilde{M} = \Lambda_e^{1/2} V_e^{\dagger} M U_e \Lambda_e^{1/2}$$

$$\widetilde{M} \approx \widetilde{U}\widetilde{\Lambda}\widetilde{V}^{\dagger}$$

Keep $D_{\mathrm{cut}}$ singular values

# § 4.4 Tensor network renormalization

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Keep $D_{\mathrm{cut}}$ singular values

$$M = V_e \Lambda_e^{-1/2} \widetilde{M} \Lambda_e^{-1/2} U_e^{\dagger} \approx V_e \Lambda_e^{-1/2} \widetilde{U}\widetilde{\Lambda}\widetilde{V}^{\dagger} \Lambda_e^{-1/2} U_e^{\dagger}$$

# § 4.4 Tensor network renormalization

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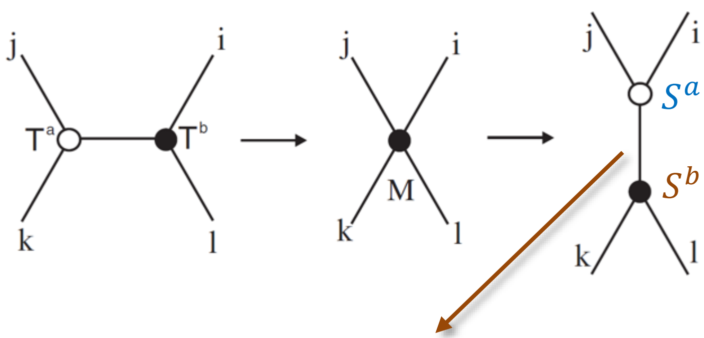


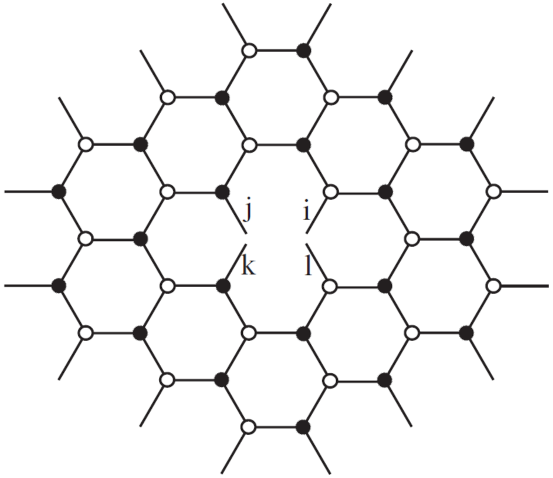

bond dimension $D_{\mathrm{cut}}$

$$M = V_e \Lambda_e^{-1/2} \widetilde{M} \Lambda_e^{-1/2} U_e^{\dagger} \approx V_e \Lambda_e^{-1/2} \widetilde{U} \widetilde{\Lambda}^{1/2} \widetilde{\Lambda}^{1/2} \widetilde{V}^{\dagger} \Lambda_e^{-1/2} U_e^{\dagger} = S^a S^b$$

# § 4.3 Tensor renormalization group

- The computation of the environment tensor $M^e_{kl,ij}$ is not easy. One still needs some approximation schemes.

$M^e_{kl,ij} =$



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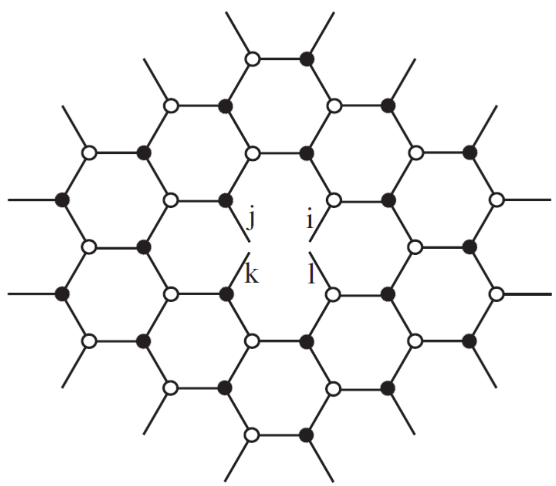
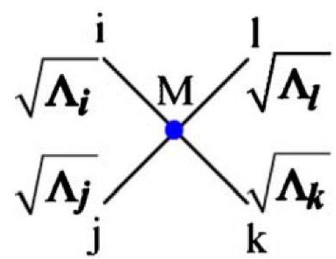



An approximate ansatz (c.f. iTEBD and simple update):

$$M^{e}_{kl,ij} \approx \sqrt{\Lambda_i \Lambda_j \Lambda_k \Lambda_l}$$



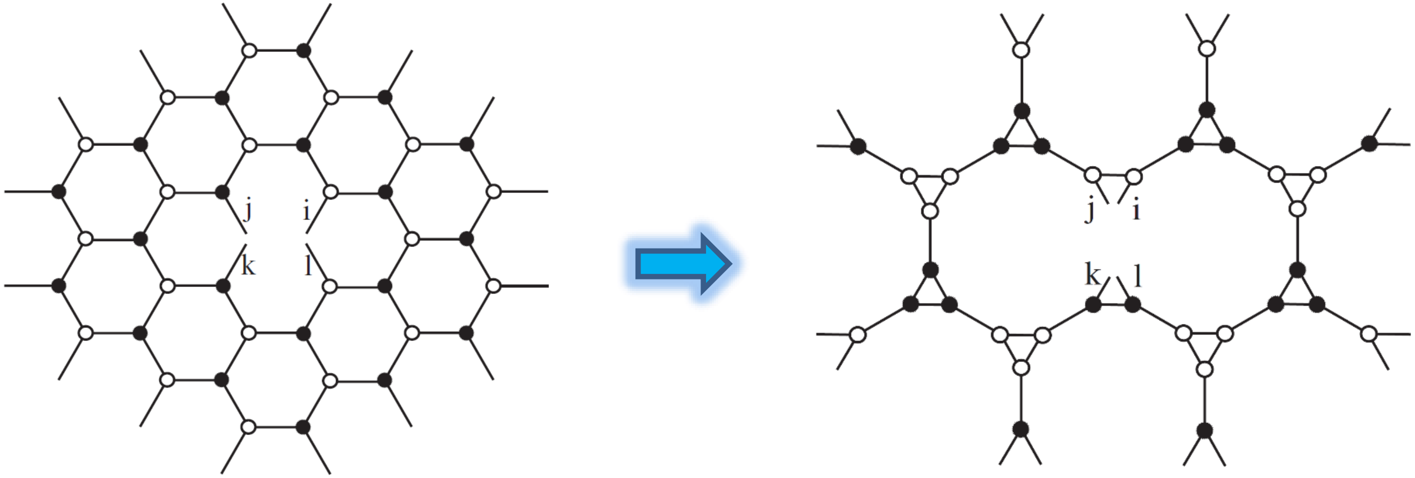

H. H. Zhao, Z. Y. Xie, Q. N. Chen, Z. C. Wei, J. W. Cai & T. Xiang, Phys. Rev. B 81, 174411 (2010).

# § 4.3 Tensor renormalization group

Second renormalization group (SRG) method:

Use several TRG iterations to obtain an approximate $M^e$:

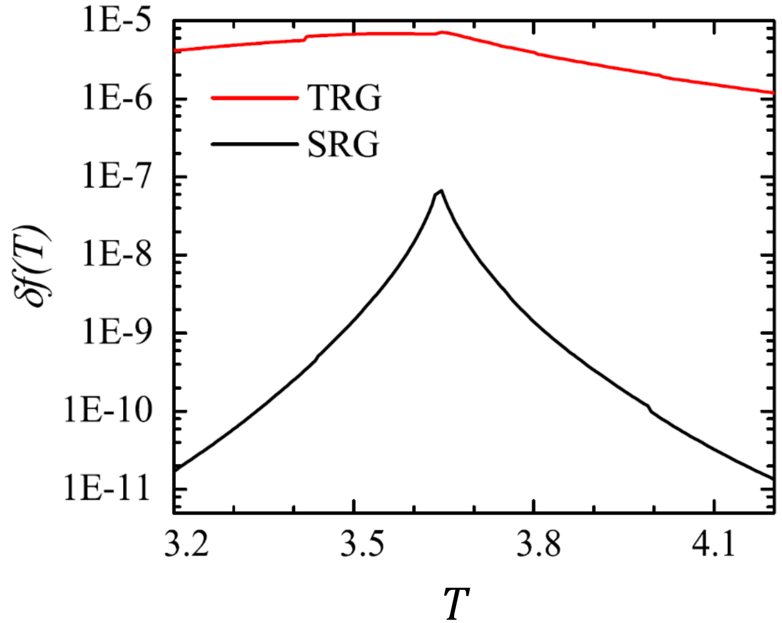
$M^e_{kl,ij} =$



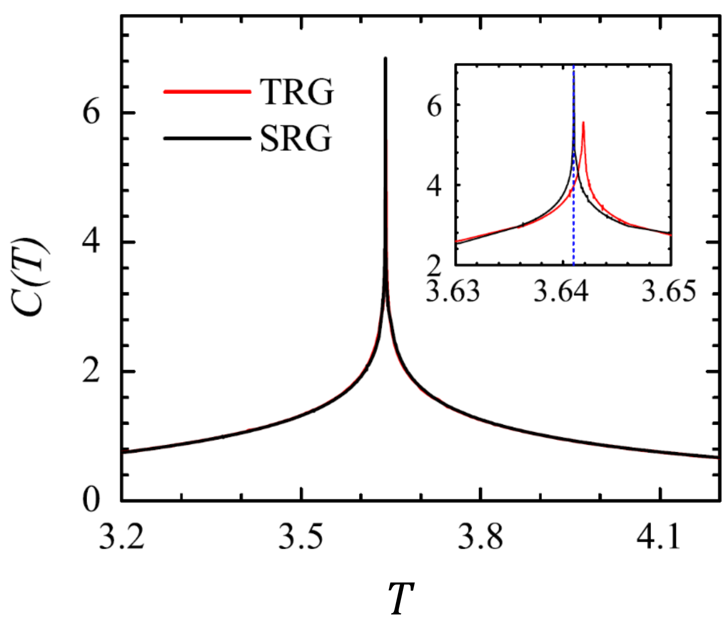
# § 4.3 Tensor renormalization group

Triangular-lattice Ising model: TRG vs. SRG

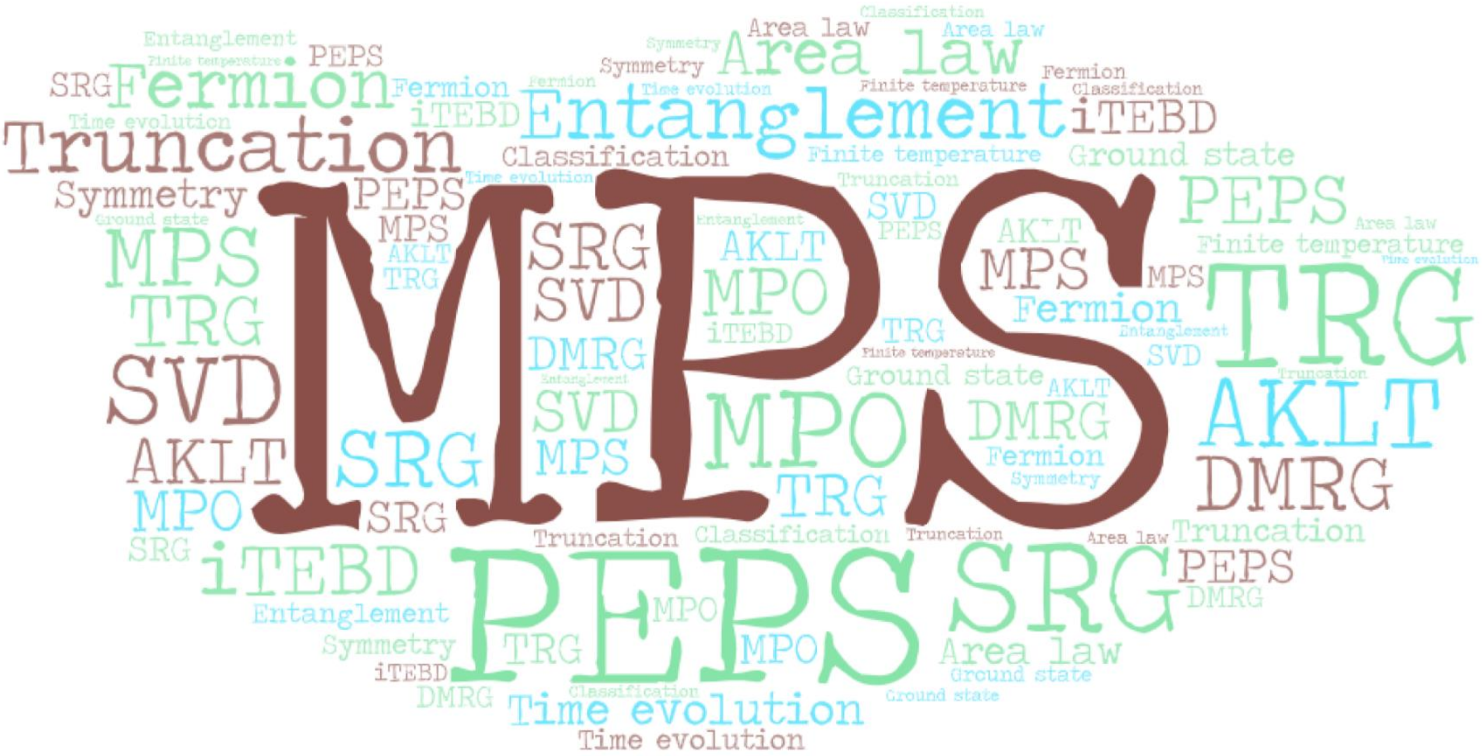
Relative error of the free energy
$\delta f(T) = |f(T) - f_{\rm ex}(T)|/f_{\rm ex}(T)$




Specific heat



# § 5.0 Summary

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