Instability of the conformal phase in QED_3

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LJ, Phys. Rev. D **94**, 094013 (2016);

J. Braun, H. Gies, **LJ**, D. Roscher, Phys. Rev. D **90**, 036002 (2014)

SFB 1143

QED$_3$ historically . . .

[Pisarski ’84, Appelquist *et al.* ’88]

. . . toy model for **QCD**$_4$:

- asymptotic freedom

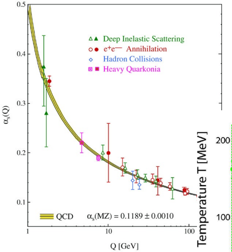
  . . . due to dimensionful fine structure constant

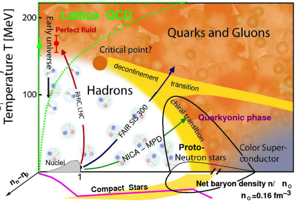
- chiral symmetry breaking

  . . . or conformal phase

- confinement

  . . . in its compact version

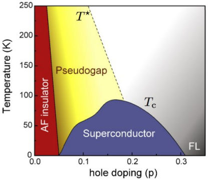




QED$_3$ today . . .

. . . effective description of high-T_c's:

[Franz, Tesanovic, Vafek '02; Herbut '02; Hermele, Senthil, Fisher '05; . . .]



. . . with $N = 2$ four-component Dirac fermions

. . . emergent theory of fractionalized excitations in spin systems:

[Hermele *et al.* '04; Ran *et al.* '07; Xu '08; He *et al.* '15; Wang & Senthil '16; . . .]

. . . with $N = 1, 2, 4, \ldots$

. . . field theory of the half-filled Landau level?

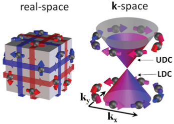
[Son '15]

. . . with $N = 1/2$

→ Talk by D. T. Son, Wed 2:45pm

. . . dual description of topological insulators?

[Wang & Senthil '15; Metlitski & Vishwanath '15; Mross *et al.* '15]

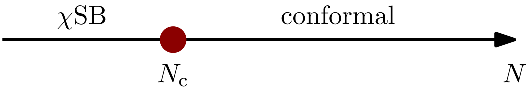


. . . with $N = 1/2, 3/2, 5/2, \ldots$

→ Talk by T. Senthil, Wed 11:45pm

QED$_3$: phase diagram

Possible phase diagrams of QED$_3$ with N massless Dirac fermions:



[Appelquist *et al.* '88]
[Hands *et al.* '04]
[Raviv *et al.* '14]
[Pietro *et al.* '16]
[Herbut '16]
...

Low N:

- chiral symmetry breaking (χSB)
- dynamical mass generation
- Mott insulator

Large N:

- conformal symmetry
- interacting gapless fermions
- non-Fermi liquid

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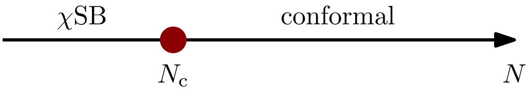

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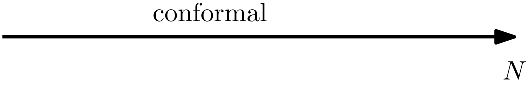

[Karthik & Narayanan '16]
[Chester & Pufu '16]

$\rightarrow$ Talk by R. Narayanan, Mo 4:15pm

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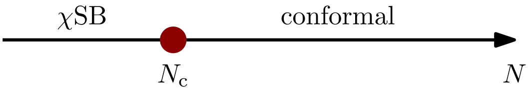

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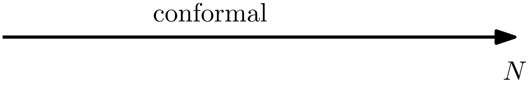
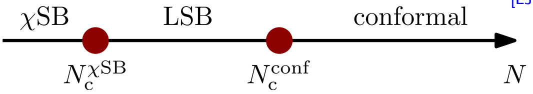


[Karthik & Narayanan '16]
[Chester & Pufu '16]

We propose:

[LJ, PRD **94**, 094013 (2016)]



Intermediate N:

- spontaneous Lorentz symmetry breaking (LSB)
- gapless fermions, anisotropic propagation
- generation of charge $\langle \psi^\dagger \psi \rangle$ or current $\langle \bar{\psi} \vec{\gamma} \psi \rangle$

Outline

(1) $2 + \epsilon$ expansion: $N_c^{\text{conf}} \nearrow \infty$ when $d \searrow 2$

(2) (generalized) F theorem: $N_c^{\chi\text{SB}} \le 1 + \mathcal{O}(d-2) < N_c^{\text{conf}}$

(3) Mean-field theory & susceptibility analysis:

$$\langle v_\mu \rangle \neq 0 \quad \text{for} \quad N_c^{\chi\text{SB}} < N < N_c^{\text{conf}}$$

Model

Action:

$$S_{\mathrm{QED}} = \int d^d x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}_i \gamma_\mu D_\mu \psi_i \right) \qquad \text{in } 2 < d < 4,$$

where $i = 1, \ldots, N$ and $\mu, \nu = 0, \ldots, d-1$ and

- Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}\mathbb{1}_4$ 4-dimensional
- Covariant derivative: $D_\mu = \partial_\mu + ieA_\mu$
- Charge: $[e^2] = 4 - d$ RG relevant in $d < 4$

  ... QED_3 (super-)renormalizable & asymptotically free
- Symmetries: "chiral" SU($2N$), parity, Lorentz, local U(1)

  ... "chirality" consequence of the reducible rep. [LJ & Gies '12; ...]
- Gauge fixing:

$$S_{\mathrm{gf}} = -\frac{1}{2\xi} \int d^d x \, (\partial_\mu A_\mu)^2 \,, \qquad \xi \in \mathbb{R}$$

... allows to check gauge invariance via ξ

Renormalization group

Loop corrections can induce new operators: ...which are not present in S_{QED}

Most of them irrelevant, but

local 4-fermion terms are marginal in $d = 2$!

...and can thus become relevant at interacting fixed points in $d = 2 + \epsilon$

Full basis: [Gies & LJ '10; ...]

$$S_{\text{4-fermi}} = \int d^d x \left[g_1 (\bar{\psi}_i \gamma_{35} \psi_i)^2 + g_2 (\bar{\psi}_i \gamma_\mu \psi_i)^2 \right]$$

with $\gamma_{35} \equiv i\gamma_3\gamma_5$ ...and where γ_3 and γ_5 the two "left-over" gamma matrices not present in $\not{D}$

RG flow for $S = S_{\text{QED}} + S_{\text{gf}} + S_{\text{4-fermi}}$

Flow of charge:

RG "time" $\ell \in [0, \infty)$

$$\frac{de^2}{d\ell} = (4 - d - \eta_A)e^2 \leq 0$$

. . . no vertex corrections $\Leftrightarrow$ Ward identity

with anomalous dimension $\eta_A = \frac{4}{3}Ne^2 + \mathcal{O}(e^4)$.

- e^2 flows to strong coupling
- charged fixed point:

$$\eta_A = 4 - d$$

. . . exactly

. . . as in Abelian Higgs model: [Herbut & Tesanovic '96]

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- photon propagator:

$$D_{\mu\nu}(q) \propto |q|^{\eta_A - 2} = |q|^{2-d} = \begin{cases} \text{const.}, & d = 2 \\ \frac{1}{|q|}, & d = 3 \end{cases}$$

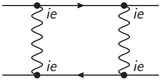
[Schwinger '62]

[Pisarski '84]

[Gusynin, Hams, Reenders '01]

- strong e^2 generates g_1, g_2:

$$\frac{dg_i}{d\ell} = (2 - d)g_i +$$



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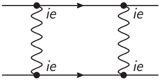

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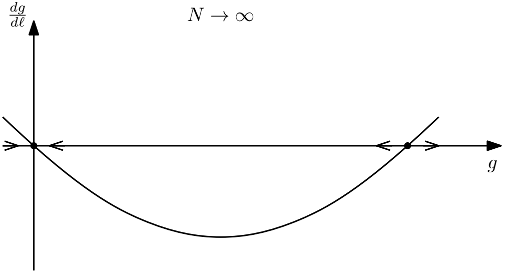

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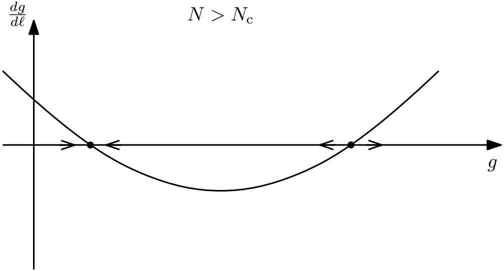

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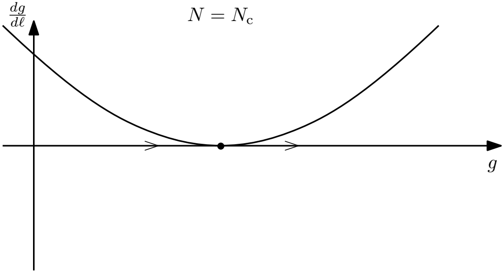

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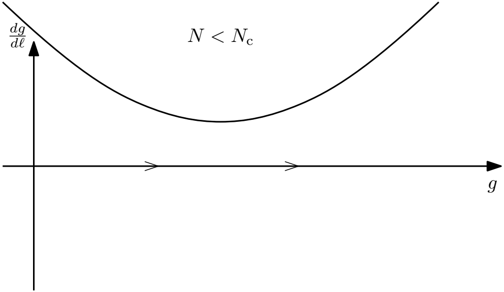
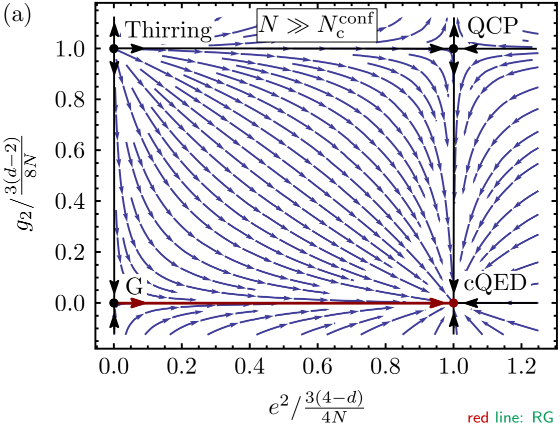


$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

...with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

3 interacting fixed points:

Thirring [Gies & LJ,...'10, '12, '15], conformal (cQED), quantum critical (QCP)

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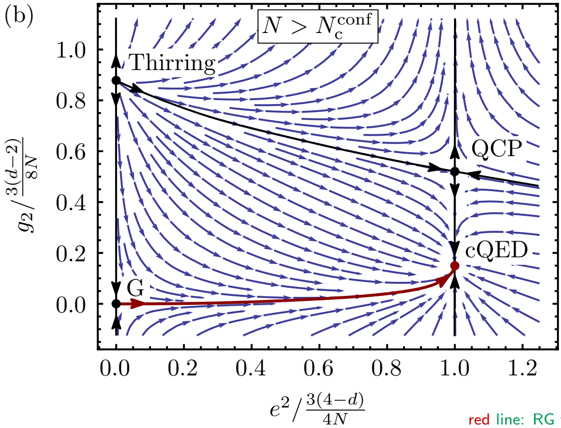

red line: RG trajectory of pure QED

cQED and QCP approach each other when lowering N

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

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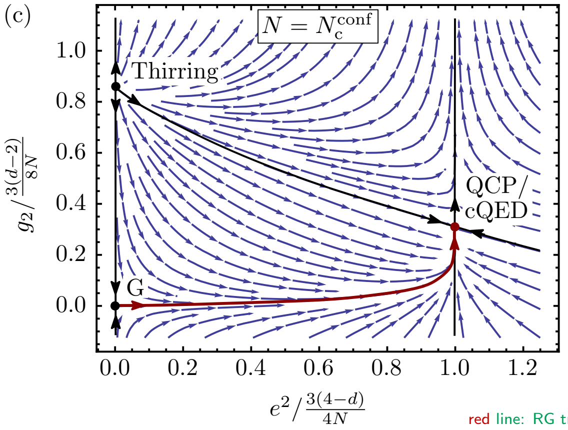


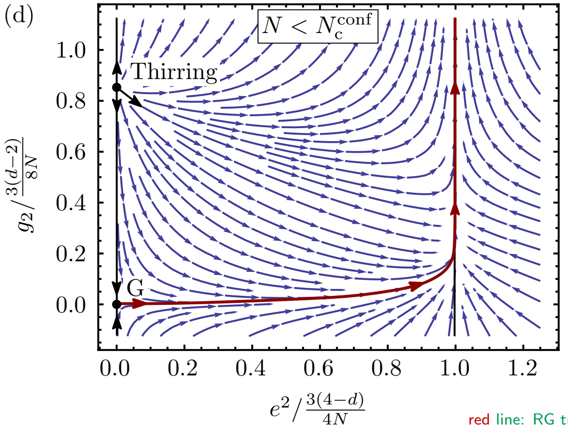
red line: RG trajectory of pure QED

cQED and QCP merge when $N = N_c^{\text{conf}}$

$2+\epsilon$ expansion

RG flow in e^2–g_2 plane:

...with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

cQED and QCP disappear for $N < N_c^{\rm conf}$ ...

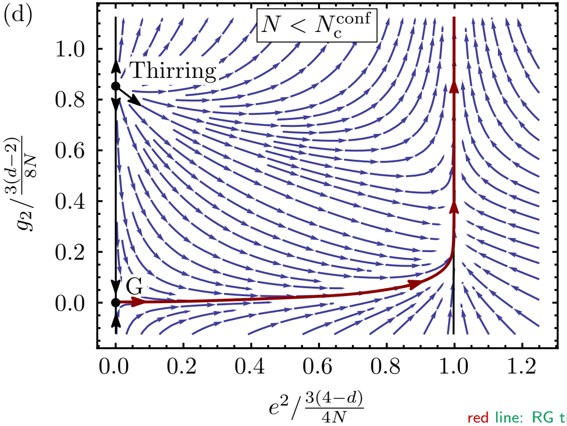
...leaving behind the runaway flow!

...i.e., conformal phase is unstable below $N_c^{\rm conf}$

$2+\epsilon$ expansion

RG flow in e^2–g_2 plane:

...with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

$$\boxed{N_c^{\mathrm{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}} \nearrow \infty \quad \text{for} \quad \epsilon \searrow 0}$$

...agrees with [Di Pietro *et al.* '16]

...perturbative expansion under control

Instability of the conformal state

Common mechanism to destabilize conformal state: [Kaplan, Lee, Son, Stephanov '09]

- Abelian Higgs model [Halperin, Lubensky, Ma '74]
- Many-flavor QCD_4 [Gies & Jaeckel '06]
- Quadratic Fermi node systems in 3D [Herbut & LJ '14, '15, '16, '17]

  $\rightarrow$ Talk by I. Herbut, Wed 2pm
- ...

Consequences:

- "Infinite-order" transition ...à la BKT transition

$$\xi \propto \exp\left[\frac{2\pi/\epsilon}{\sqrt{1-(N_\mathrm{c}/N)^{3/2}}}\right]$$

...agrees with gap-equation solution: [Appelquist *et al.* '88]

...and $4-\epsilon$ expansion: [Herbut '16]

- Condensates exponentially suppressed ...numerics need large lattices for $N \lesssim N_\mathrm{c}$

RG monotonicity

Observation:

"effective" number of degrees of freedom decreases under RG

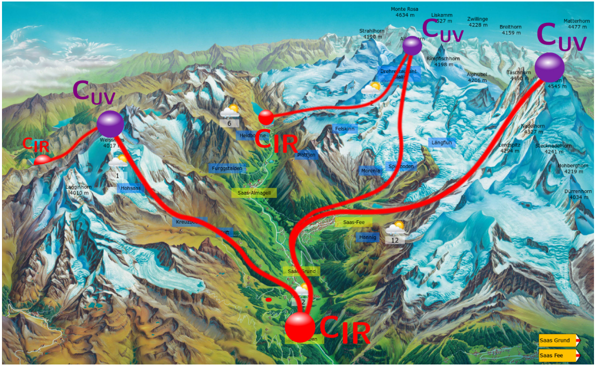
. . . e.g., N critical modes at Wilson-Fisher O(N) fixed point $\rightarrow$ $(N-1)$ massless modes in the SSB phase
or 0 massless modes in the symmetric phase

Quantification?

1+1D: *c* theorem [Zamolodchikov '86]

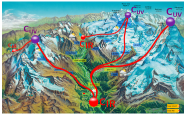
$$\boxed{c_{\mathrm{UV}} > c_{\mathrm{IR}}}$$

c central charge of conformal algebra



. . . RG flow goes "downhill"

RG monotonicity



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3+1D: *a* theorem

[Cardy '88; Komargodski & Schwimmer '11]

$$\boxed{a_{\mathrm{UV}} > a_{\mathrm{IR}}}$$

anomaly coefficient $a \sim \int_{S^4} \langle T^{\mu}_{\mu} \rangle$

2+1D: *F* theorem

[Jafferis '10; Jafferis *et al.* '11; Casini & Huerta '12; . . .]

$$\boxed{F_{\mathrm{UV}} > F_{\mathrm{IR}}}$$

"sphere free energy" $F = -\log Z_{S^3}$

RG monotonicity: continuous dimension?

Generalized F for $d \in \mathbb{R}$: [Giombi & Klebanov '14]

$$\tilde{F} = \sin\left(\frac{\pi d}{2}\right) \log Z_{S^d}$$

with:

$\tilde{F}\big|_{d=2} = \frac{\pi}{6} c$ — c theorem ✓

$\tilde{F}\big|_{d=3} = F$ — F theorem ✓

$\tilde{F}\big|_{d=4} = \frac{\pi}{2} a$ — a theorem ✓

Thus:

$$\boxed{\tilde{F}_{\text{UV}} > \tilde{F}_{\text{IR}}} \qquad \text{for all integer } d$$

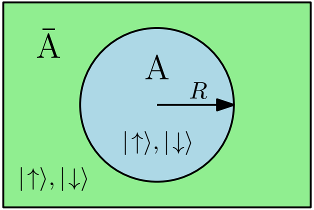
…and various evidence for continuous d:

$$d = 2 + \epsilon, \;\; d = 3 - \epsilon, \;\; d = 4 - \epsilon, \;\; d = 6 - \epsilon, \;\ldots$$

[Giombi, Klebanov, Tarnopolsky, Fei, …'14, '15, '16]

…though no rigorous proof yet

RG monotonicity: entanglement



Reduced density matrix:

$$\rho_{\mathrm{A}} = \mathrm{Tr}_{\bar{\mathrm{A}}} |\psi\rangle\langle\psi|$$

Entanglement entropy:

$$S_{\mathrm{A}} = -\,\mathrm{Tr}_{\mathrm{A}}\, \rho_{\mathrm{A}} \log \rho_{\mathrm{A}}$$

E.g., $\mathrm{A} = \bar{\mathrm{A}} = \{|\uparrow\rangle, |\downarrow\rangle\}$: $\quad S_{\mathrm{A}} = \begin{cases} 0 & \text{for} \quad |\psi\rangle = |\uparrow\uparrow\rangle \\ \log 2 & \text{for} \quad |\psi\rangle = \frac{1}{2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \end{cases}$

. . . measures entanglement between A and $\bar{\mathrm{A}}$

Scaling with subsystem size $\sim R$?

- (trivial) gapped states: $S_{\mathrm{A}} \propto R^{(d-1)-1}$ . . . "area law"
- conformal phases (or points):

$$\boxed{\begin{array}{ll} 1{+}1\text{D:} & S_{\mathrm{A}} \propto c \log R + \mathcal{O}(1/R) \\ 2{+}1\text{D:} & S_{\mathrm{A}} \propto R - \gamma + \mathcal{O}(1/R), \quad \text{with } \gamma = F \\ 3{+}1\text{D:} & S_{\mathrm{A}} \propto R^2 + a \log R + \mathcal{O}(1/R) \end{array}}$$

[Holzhey, Larsen, Wilczek '94]
[Calabrese & Cardy '04]
[Kitaev & Preskill '06]
[Casini, Huerta, Myers '11]
. . .

$\Rightarrow$ Universal coefficients of S_{A} scaling are monotonic under RG!

QED$_3$: breaking of $\mathrm{U}(2N) \to \mathrm{U}(N) \times \mathrm{U}(N)$?

Compute $\tilde{F}$ in $d = 2 + \epsilon$: [Giombi, Klebanov, Tarnopolsky '16]

UV: $$\tilde{F}_{\mathrm{UV}} = \underbrace{N\tilde{F}_{\mathrm{f}}}_{\text{fermions}} + \underbrace{(d-2)\tilde{F}_{\mathrm{b}}}_{\text{photon}} = N\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

$\tilde{F}_{\mathrm{f}}$: free fermion
$\tilde{F}_{\mathrm{b}}$: free boson

cQED: $$\tilde{F}_{\mathrm{conf}} = \left(N - \tfrac{1}{2}\right)\tilde{F}_{\mathrm{f}} = \underbrace{(2N-1)}_{c_{\mathrm{Schwinger}}\ \checkmark}\frac{\pi}{6} + \mathcal{O}(\epsilon)$$

χSB: $$\tilde{F}_{\chi\mathrm{SB}} = \left(2N^2 + (d-2)\right)\tilde{F}_{\mathrm{b}} = N^2\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

$$\Rightarrow \quad \tilde{F}_{\chi\mathrm{SB}} > \tilde{F}_{\mathrm{UV}} > \tilde{F}_{\mathrm{conf}} \quad \text{for all } N > 1$$

Hence:

$$\boxed{N_{\mathrm{c}}^{\chi\mathrm{SB}} \leq 1 + \mathcal{O}(d-2) < N_{\mathrm{c}}^{\mathrm{conf}}}$$

... the phase below $N_{\mathrm{c}}^{\mathrm{conf}}$ cannot exhibit χSB!

[LJ, PRD **94**, 094013 (2016)]

Intermediate phase

Vafa-Witten theorem: QED_3 should have [Vafa & Witten '84]

(a) unbroken $\mathrm{U}(N) \times \mathrm{U}(N)$ symmetry, and ...at least

...rules out other chiral breaking patterns

(b) gapless spectrum

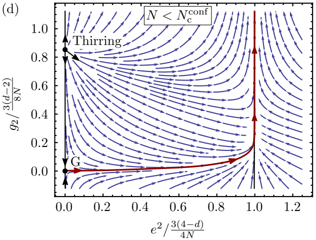
...rules out plain parity breaking

What else can it be?

Mean-field theory

Recall flow ...

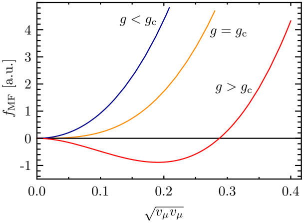
... towards divergent g_2 (and finite e^2, g_1)!



Effective description: Thirring model!

$$S_{\text{Thirring}} = \int d^d x \left[\bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + g_2 \left(\bar{\psi}_i \gamma_\mu \psi_i\right)^2\right]$$

Large-N (saddle-point) solution:



$$f_{\mathrm{MF}}(v_\mu) = \frac{1}{2}\left(\frac{1}{g_2} - \frac{40N}{3}\right) v_\mu^2 + \sqrt{6}\pi N \left(v_\mu^2\right)^{3/2} + \mathcal{O}(v^4)$$

for vector order parameter $v_\mu \propto \langle \bar{\psi}_i \gamma_\mu \psi_i \rangle$

$\Rightarrow$ transition towards $v_\mu \neq 0$: $\boxed{\text{spontaneous Lorentz symmetry breaking!}}$

... spontaneous formation of charge and/or current

... consistent with Vafa-Witten and F theorem

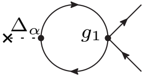
Susceptibility analysis in $d = 2 + \epsilon$: bilinear terms

Add small symmetry-breaking "seeds": ...à la magnetic field

$$S_\Delta = \int d^d x \, \bar{\psi}_i \left[\Delta_{\chi\mathrm{SB}} \mathbb{1}_4 + \Delta_{\mathrm{PSB}} \gamma_{35} + i \Delta_{\mathrm{Kek}} \left(\gamma_3 \cos\varphi + \gamma_5 \sin\varphi \right) + i \Delta^\mu_{\mathrm{LSB}} \gamma_\mu \right] \psi_i$$

Flow of bilinears:

$$\partial_\ell \Delta_\alpha = (1 - \eta_\psi) \, \Delta_\alpha +$$



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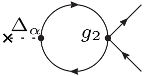

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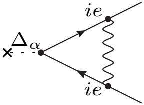


$$\equiv x_\alpha \, \Delta_\alpha + \mathcal{O}(\Delta^2)$$

...with $\Delta_\alpha \in \{\Delta_{\chi\mathrm{SB}}, \Delta_{\mathrm{PSB}}, \Delta_{\mathrm{Kek}}, \Delta^\mu_{\mathrm{LSB}}\}$

Critical exponents in $d = 2 + \epsilon$

Scaling form of dimensionless free energy:

$$f(\delta\vec{g}, \Delta_\alpha) = b^{-d} f(b^y \delta\vec{g}, b^{x_\alpha} \Delta_\alpha)$$

$$\Rightarrow \quad \boxed{\gamma_\alpha = \frac{2x_\alpha - d}{y}} \quad \& \quad \boxed{\nu = \frac{1}{y}}$$

... where $b \equiv \frac{\Lambda}{k} = e^{-\ell}$

$\partial_\ell \delta\vec{g} = y\delta\vec{g} + \mathcal{O}(\delta g^2)$

$\partial_\ell \Delta_\alpha = x_\alpha \Delta_\alpha$

$N \lesssim N_c^{\mathrm{conf}}$: Flow "hovers" over complex pair of fixed points QCP/cQED:

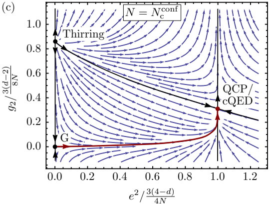
$\Rightarrow$ dominant order $\widehat{=}$ largest γ at QCP

$$1/\nu = \epsilon\sqrt{1 - (N_c^{\mathrm{conf}}/N)^{3/2}} + \mathcal{O}(\epsilon^2),$$

$$\gamma_{\chi\mathrm{SB}} < 0, \qquad \gamma_{\mathrm{PSB}} < 0, \qquad \gamma_{\mathrm{Kek}} < 0,$$

... superconducting gaps also lead to $\gamma_{\mathrm{SC}} < 0$

$$\boxed{\gamma_{\mathrm{LSB}} = 1 + \mathcal{O}(\epsilon) > 0 \ !}$$



$\Rightarrow$ $\boxed{\text{instability towards spontaneous Lorentz symmetry breaking!}}$

Towards $d = 3$

In $d = 3$ and finite N: strong-coupling problem ... difficult ...

$N_c^{\chi\mathrm{SB}}$ controlled by F theorem:

$$F_{\mathrm{conf}} = NF_{\mathrm{f}} + \frac{1}{2}\log\left(\frac{\pi N}{4}\right) + \mathcal{O}(N^{-1})$$

[Klebanov, Pufu, Sachdev, Safdi '12]

$$F_{\chi\mathrm{SB}} = (2N^2 + 1)F_{\mathrm{b}}$$

$$\Rightarrow \quad \boxed{N_c^{\chi\mathrm{SB}} \leq 4.422}$$

in $< 1‰$ agreement with bound from $4 - \epsilon$ expansion of $\tilde{F}_{\mathrm{conf}}$:
[Giombi, Klebanov, Tarnopolsky '16]

N_c^{conf} by RG flow:

naive extrapolation to $\epsilon \to 1$: $N_c^{\mathrm{conf}} \simeq 8\sqrt[3]{2} \approx 10.1$

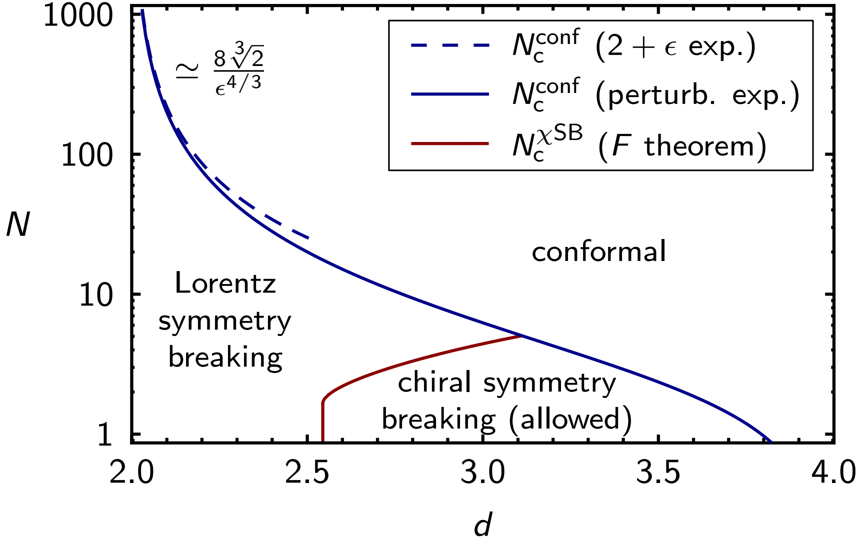
one-loop in fixed $d = 3$: $N_c^{\mathrm{conf}} \approx 6.24$

functional RG: $N_c^{\mathrm{conf}} \approx 4.1 \ldots 10.0$ [Braun, Gies, LJ, Roscher '14]

$$\boxed{N_c^{\chi\mathrm{SB}} < N_c^{\mathrm{conf}} \text{ possibly even in } d = 3!}$$

... higher-order calculations necessary

Phase diagram of $\mathrm{QED}_{2<d<4}$



[LJ, PRD **94**, 094013 (2016)]

Conclusions

(1) QED_3 has conformal ground state at large $N > N_c^{\mathrm{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}}$

...in $d = 2 + \epsilon$

(2) Common scenario with direct transition towards χSB inconsistent with (generalized) F theorem

...at least in $d = 2 + \epsilon$

...there are just too many Goldstones!

(3) Only possibility for $N \lesssim N_c^{\mathrm{conf}}$: ...compatible with F theorem & Vafa-Witten

spontaneous Lorentz symmetry breaking (LSB)!

(4) Mean-field & susceptibility analyses: ...independently

confirm existence of LSB phase

(5) Extrapolation $\epsilon \to 1$ & perturbative expansion in fixed $d = 3$:

suggest finite window of LSB phase also in physical dimension