

4 Renormalization group (RG)

4.1 Concept of RG

Assumption: The relevant physics describing phases and phase transition is governed by the behavior at large length scales ($\hat{=}$ low energy) $L \gg a$

\uparrow typical length scale
 \leftarrow microscopic length scale

Example (magnet): $\langle S_i^z S_{i+1}^z \rangle \neq 0$ for all T

$$\lim_{|i-j| \rightarrow \infty} \langle S_i^z S_j^z \rangle \begin{cases} = 0 & \text{for } T > T_c \\ \neq 0 & \text{for } T < T_c \end{cases}$$

Idea: Successively integrate out short-distance ($\hat{=}$ high-energy) modes to obtain an effective theory at large length scales ($\hat{=}$ low energy)

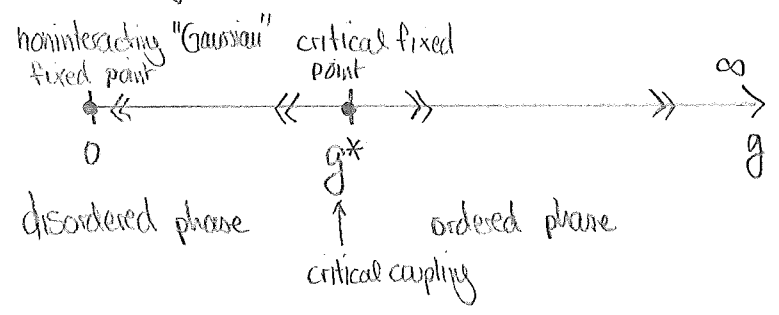
E.g.: $S(g_1, g_2, \dots) \mapsto S(g'_1, g'_2, \dots)$

\uparrow action
 \uparrow couplings
 \uparrow if S is sufficiently general

\Rightarrow couplings become scale-dependent $g_i \rightarrow g_i(L)$

RG flow: Change of couplings under the successive integration of modes

RG flow diagram (example):

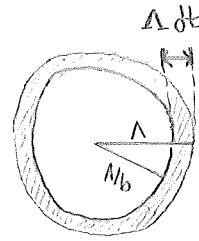


4.2 Scaling transformation and scaling dimension

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Integrate out high-energy modes with momenta:

$$\frac{\Lambda}{b} \leq |\vec{k}| < \Lambda$$



"momentum shell"

Infinitesimal RG step:

$$b = e^{dt} \quad \text{with } dt \ll 1, \quad t = \int_0^t dt' \quad \text{"RG time"}$$

RG flow:

$$\frac{dg_i}{dt} = \beta_i(g_i)$$

↑
beta functions

Fixed point:

$$\left. \frac{dg_i}{dt} \right|_{g_i^*} = 0$$

Linearized RG flow (around Gaussian fixed point $g^* = 0$ in theory with one coupling g):

$$\beta(g) = \theta g + O(g^2) \quad \text{with } \theta \equiv \text{dim}[g] = \text{const.} \quad \text{"scaling dimension of } g\text{"}$$

Integrated flow: constant term vanishes since $g=0$ is a fixed point

$$\frac{dg}{dt} = \theta g \quad \Rightarrow \quad g(t) = g(0) e^{\theta t}$$

Classification of couplings:

$$\text{dim}[g] > 0$$

"relevant coupling"

g increases in RG time

$$\text{dim}[g] < 0$$

"irrelevant coupling"

g decreases in RG time

$$\text{dim}[g] = 0$$

"marginal coupling"

higher-order terms decide its fate (marginally relevant, marginally irrelevant, or exactly marginal)

Classification of fixed points:

- "stable fixed point": all couplings irrelevant near fixed point
- "critical fixed point": exactly one relevant direction
- "multicritical fixed point": number of relevant directions $2 \leq n \leq n_0$ where n_0 is the number of tuning parameters
- "unstable fixed point": number of relevant directions $n > n_0$

4.3 Momentum-shell RG for the O(N) model

Action (O(N) model):

$$S = \int d^d \vec{x} \left[\frac{1}{2} (\vec{\nabla} \phi^a(\vec{x}))^2 + \frac{u}{2} (\phi^a(\vec{x}))^2 + \frac{u}{4!} (\phi^a(\vec{x})^2)^2 \right], \quad a=1, \dots, N$$

↓ tuning parameter ("mass")
↑ self-interaction coupling

$$= \int_0^\Lambda \frac{d^d \vec{k}}{(2\pi)^d} \frac{1}{2} \phi^a(\vec{k}) (k^2 + \dots) \phi^a(\vec{k}) + \frac{u}{4!} \int_0^\Lambda \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{3d}} \phi^a(\vec{k}_1) \phi^a(\vec{k}_2) \phi^b(\vec{k}_3) \phi^b(-\vec{k}_1 - \vec{k}_2 - \vec{k}_3)$$

where we have rescaled $\sum_0^2 (\phi^{a2}) \mapsto \phi^a$ and introduced an ultraviolet momentum cutoff Λ , $0 \leq |\vec{k}| \leq \Lambda$, with, e.g., $\Lambda \sim \frac{\pi}{a}$ (a : lattice constant).

Three stages of RG transformation:

1. Eliminate "fast" modes $\phi_{>}$ with momenta $\frac{\Lambda}{b} \leq |\vec{k}| < \Lambda$ ("momentum shell").

$$\phi(\vec{k}) \equiv \theta\left(\frac{\Lambda}{b} - |\vec{k}|\right) \phi_{<}(\vec{k}) + \theta\left(|\vec{k}| - \frac{\Lambda}{b}\right) \phi_{>}(\vec{k})$$

\uparrow slow modes $0 \leq |\vec{k}| < \frac{\Lambda}{b}$ \uparrow fast modes $\frac{\Lambda}{b} \leq |\vec{k}| < \Lambda$

2. Rescale momenta $\vec{k} \mapsto \vec{k}' = b\vec{k}$ with $0 \leq |\vec{k}'| < \Lambda$ for slow modes.

3. Introduce "renormalized" fields $\phi'(\vec{k}') = b^y \phi_{<}(\vec{k}'/b)$ with y chosen such that the new action in terms of ϕ' has the same coefficient for a certain quadratic term.

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RG for Gaussian model ($u=0$):

1. Mode elimination:

$$Z = \int \mathcal{D}\phi_{<} \int \mathcal{D}\phi_{>} e^{-S_0[\phi_{<}, \phi_{>}]}$$

with

$$S_0[\phi_{<}, \phi_{>}] = \int_0^{\Lambda/b} \frac{d^d k}{(2\pi)^d} \frac{1}{2} \phi_{<}(-\vec{k}) (k^2 + r) \phi_{<}(\vec{k}) + \int_{\Lambda/b}^{\Lambda} \frac{d^d k}{(2\pi)^d} \frac{1}{2} \phi_{>}(-\vec{k}) (k^2 + r) \phi_{>}(\vec{k})$$

Thus:

$$Z = \int \mathcal{D}\phi_{<} e^{-\underbrace{\int_0^{\Lambda/b} \phi_{<} (k^2 + r) \phi_{<}}_{\equiv S_{\text{eff}}} \cdot \text{const.}}$$

\uparrow independent of $\phi_{<}$

$$\left[Z_{0>} = \int \mathcal{D}\phi_{>} e^{-S_{0>}} = [\det(k^2 + r)]^{-1/2} \right]_{\frac{\Lambda}{b} \leq k < \Lambda}$$

2. Momentum rescaling:

$$S_{\text{eff}} = \int_0^{1/b} \frac{d^d \vec{k}}{(2\pi)^d} \frac{1}{2} \phi_{\vec{k}}(-\vec{k}) (k^2 + \tau) \phi_{\vec{k}}(\vec{k}) \begin{cases} \vec{k}' = b\vec{k} \\ d^d \vec{k}' = b^d d^d \vec{k} \end{cases}$$

$$= \int_0^1 \frac{d^d \vec{k}'}{(2\pi)^d} b^{-d} \frac{1}{2} \phi_{\vec{k}'}(-\vec{k}'/b) (b^{-2} k'^2 + \tau) \phi_{\vec{k}'}(\vec{k}'/b)$$

3. Field renormalization:

with $\phi'(\vec{k}') = b^y \phi_{\vec{k}'}(\vec{k}'/b)$:

$$S_{\text{eff}} = \int_0^1 \frac{d^d \vec{k}'}{(2\pi)^d} \frac{1}{2} \phi'(-\vec{k}') (b^{-d-2-2y} k'^2 + b^{-d-2y} \tau) \phi'(\vec{k}')$$

has the same form as original S for $y = -\frac{d+2}{2}$.

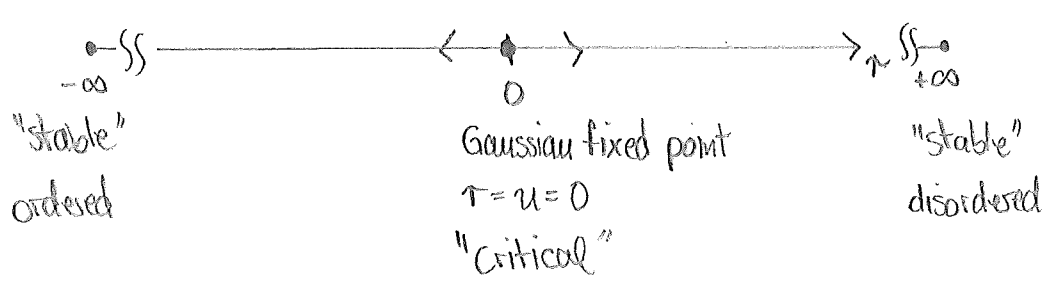
Then:

$$Z = \int \mathcal{D}\phi' e^{-\int_0^1 \frac{1}{2} \phi' (k'^2 + \tau') \phi'} \quad \text{with } \underline{\underline{\tau' = b^2 \tau}}$$

Beta function ($0 < \ln b \ll 1$):

$$\beta_{\tau} = \frac{d\tau}{d \ln b} = 2\tau \quad (\text{for } u=0)$$

RG flow diagram:



RG for ϕ^4 model ($u > 0$ with $N=1$):

1. Mode elimination:

$$Z = \int \mathcal{D}\phi_{<} \int \mathcal{D}\phi_{>} e^{-S_{0<} - S_{0>} - S_{int}[\phi_{<}, \phi_{>}]}$$

$$= \int \mathcal{D}\phi_{<} e^{-S_{0<}} \int \mathcal{D}\phi_{>} e^{-S_{0>}} \left(1 - S_{int}[\phi_{<}, \phi_{>}] + \mathcal{O}(u^2) \right)$$



with

$$S_{int}[\phi_{<}, \phi_{>}] = \frac{u}{4!} \left[\int_0^{\Lambda/b} \int_{\vec{k}_1, \vec{k}_2, \vec{k}_3} \phi_{<} \phi_{<} \phi_{<} \phi_{<} + \int_0^{\Lambda} \int_{\vec{k}_1, \vec{k}_2, \vec{k}_3} \phi_{>} \phi_{>} \phi_{>} \phi_{>} + \binom{4}{2} \int_0^{\Lambda} \int_{\vec{k}_1, \vec{k}_2, \vec{k}_3} \phi_{<} \phi_{<} \phi_{>} \phi_{>} \right]$$

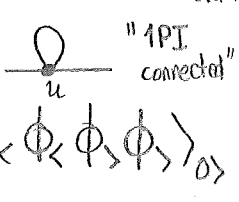
$\left[\int_0^{\Lambda} \int_{\vec{k}} \equiv \int_0^{\Lambda} \frac{d^d k}{(2\pi)^d} \right]$

Thus:

$$Z = Z_{0>} \int \mathcal{D}\phi_{<} e^{-S_{0<}} \left(1 - \frac{u}{4!} \left[\int_0^{\Lambda/b} \langle \phi_{<} \phi_{<} \phi_{<} \phi_{<} \rangle_{0>} + \int_0^{\Lambda} \langle \phi_{>} \phi_{>} \phi_{>} \phi_{>} \rangle_{0>} + \binom{4}{2} \int_0^{\Lambda} \frac{0}{u} \langle \phi_{<} \phi_{<} \phi_{>} \phi_{>} \rangle_{0>} \right] + \mathcal{O}(u^2)$$

 "tree-level"  "vacuum"

average w.r.t. $S_{0>}$
 $\langle \dots \rangle_{0>} \equiv \int \mathcal{D}\phi_{>} e^{-S_{0>}} (\dots) / Z_{0>}$

 "1PI correct"

Wick's theorem:

$$\langle \phi_{<} \phi_{<} \phi_{>} \phi_{>} \rangle_{0>} = \langle \phi_{<} \phi_{<} \rangle_{0>} \langle \phi_{>} \phi_{>} \rangle_{0>} \quad [\text{exercise sheet 3, problem 1c}]$$

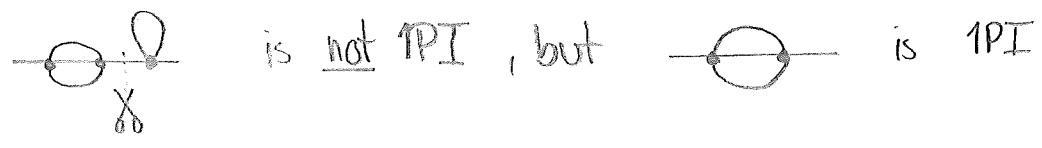
Feynman rules (momentum-shell RG):

- vertex $\times \hat{=} \frac{u}{4!} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)$
- internal line $\longleftrightarrow \hat{=} \langle \phi, \phi \rangle_0$
- external line $\longrightarrow \hat{=} \phi_k$

Remark:

Only one-particle irreducible (1PI) connected diagrams (that remain connected after cutting one internal line) contribute to the RG flow.

Example:



Reexponentiation:

$$Z = Z_0 \int D\phi_k e^{-S_{0k} - \frac{u}{4!} \left[\int_0^{\Lambda b} \phi_k \phi_k \phi_k \phi_k + \binom{4}{2} \left(\int_0^{\Lambda b} \phi_k \phi_k \right) \left(\int_{\Lambda b}^{\Lambda} \frac{1}{k^2 + r} \right) \right]} + O(u^2)$$

$= -S_{\text{eff}}$

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with $\int_{\Lambda b}^{\Lambda} \frac{1}{k^2 + r} = \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{\Lambda^2 + r} \ln b + O(\ln^2 b)$

2. Momentum rescaling: $\vec{k}' = b\vec{k}$

$$S_{\text{eff}} = S_{0k} + \frac{u}{4!} \left[\int_0^{\Lambda} b^{-3d} \phi_k \phi_k \phi_k \phi_k + \binom{4}{2} \int_0^{\Lambda} b^{-d} \phi_k \phi_k \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{\Lambda^2 + r} \ln b \right] + O(u^2, \ln^2 b)$$

3. Field renormalization: $\phi'(\vec{k}') = b^{-\frac{d+2}{2}} \phi_{<}(\vec{k}'/b)$

$$S_{\text{eff}} = \int_0^\Lambda \frac{1}{2} \phi'(\vec{k}'^2 + \underbrace{b^2 \tau + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{\Lambda^{2+\tau}} \ln b}_{\equiv \tau'}) \phi' + \int_0^\Lambda \underbrace{\frac{u}{4!} b^{4-d}}_{\frac{u'}{4!}} \phi' \phi' \phi' \phi' + \mathcal{O}(u^2, \ln^2 b) \quad [b^2 = 1 + \mathcal{O}(\ln b)]$$

Then:

$$\tau' = b^2 \tau + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{\Lambda^{2+\tau}} \ln b + \mathcal{O}(u^2, \ln^2 b)$$

$$u' = b^{4-d} u + \mathcal{O}(u^2, \ln^2 b)$$

Introduce dimensionless variables (for convenience):

$$\tau \mapsto t \equiv \frac{\tau}{\Lambda^2}$$

$$u \mapsto g \equiv \frac{S_d}{(2\pi)^d} \frac{u}{\Lambda^{4-d}}$$

Beta functions:

$$\beta_t = \frac{dt}{d \ln b} = 2t + \frac{g}{2} \frac{1}{1+t} + \mathcal{O}(g^2)$$

$$\beta_g = \frac{dg}{d \ln b} = (4-d)g + \mathcal{O}(g^2)$$

Remarks:

• Scaling dimensions $\text{dim}[\tau] = 2$ and $\text{dim}[u] = 4-d$ agree with

power-counting dimensions:

$$0 = [S] = \underbrace{[\tau^2]}_{\substack{\text{inverse-length} \\ \text{dimensional} \\ 2}} + [\phi^2] + \underbrace{[d^d]}_{-d}$$

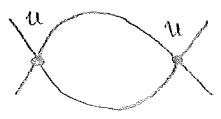
$$\Rightarrow [\phi] = \frac{d-2}{2}, \quad [\tau] = 2 = \text{dim}[\tau] \quad \checkmark$$

$$0 = [S] = [u] + \underbrace{[\phi^4]}_{2(d-2)} + \underbrace{[d^d]}_{-d}$$

$$\Rightarrow [u] = 4-d = \text{dim}[g] \quad \checkmark$$

• To compute the leading interaction correction to β_g we need to compute the g^2 contribution.

Leading interaction correction to β_g (diagrammatically):



$$= (-1) \frac{1}{2!} \left(\frac{-u}{4!} \right)^2 (\underbrace{\phi\phi\phi\phi}_{\substack{\text{re-exponentiation} \\ \text{of exp(-)}}} \phi\phi\phi\phi) \times \left(\frac{4}{2} \right)^2 \cdot 2$$

where $\underbrace{\phi\phi}_{\phi^2} \equiv \langle \phi, \phi \rangle_0$

$$= - \frac{1}{4!} \frac{3}{2} u^2 \phi\phi \int \frac{1}{(k^2 + \uparrow)^2}$$

Thus:

$$\beta_g = (4-d)g - \frac{3}{2} g^2 \frac{1}{(1+t)^2} + \mathcal{O}(g^3)$$

Generalization to $O(N)$ model:

$$\beta_t = \frac{dt}{d\ln b} = 2t + \frac{N+2}{6} \frac{g}{1+t} + \mathcal{O}(g^2)$$

$$\beta_g = \frac{dg}{d\ln b} = (4-d)g - \frac{N+8}{6} \frac{g^2}{(1+t)^2} + \mathcal{O}(g^3)$$

Fixed points:

(a) Gaussian fixed point: $t^* = g^* = 0$

Near this fixed point:

$\dim[u] = 4-d$ irrelevant for $d > 4$

$\dim[u_6] = 6-2d$ irrelevant for $d > 3$

\uparrow
 ϕ^6 coupling

(b) Wilson-Fisher fixed point: Assume $\epsilon = 4-d \ll 1$

"fractional dimension"

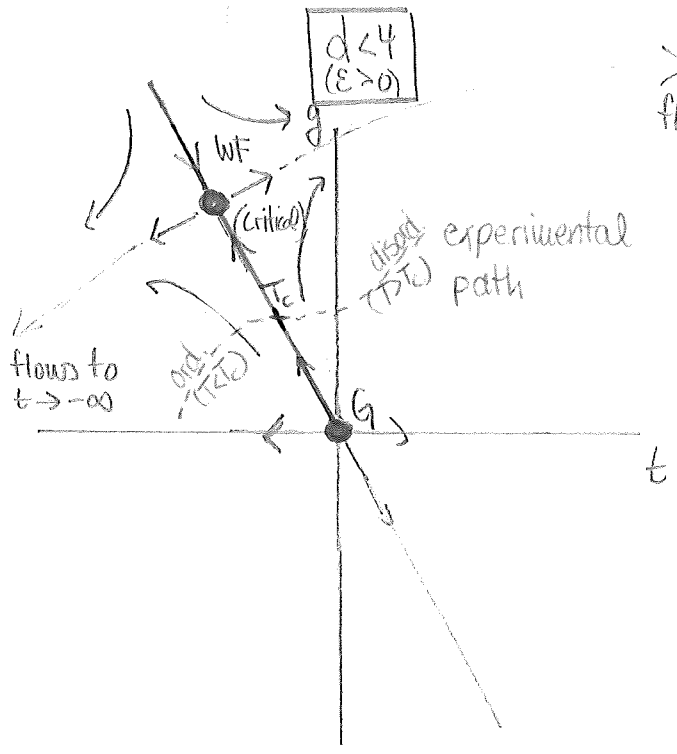
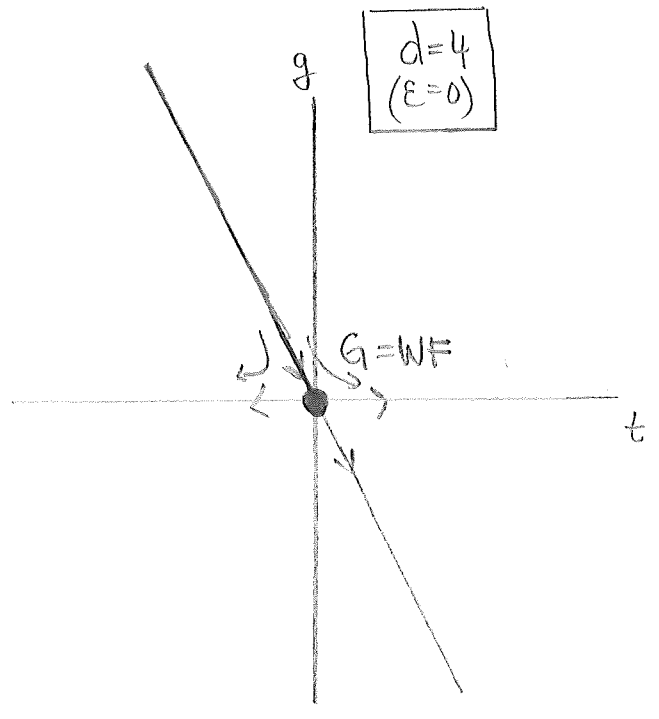
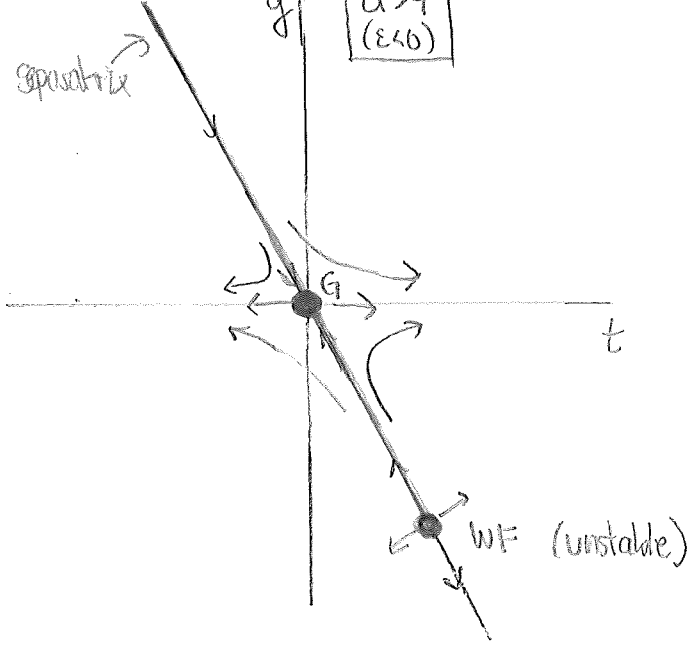
$$g^* = \frac{6}{N+8} \epsilon + \mathcal{O}(\epsilon^2)$$

$$t^* = -\frac{N+2}{2(N+8)} \epsilon + \mathcal{O}(\epsilon^2)$$

Remark:

Systematic loop expansion: contributions at $\mathcal{O}(\epsilon^n)$ arise from n -loop Feynman diagrams

RG flow diagrams:



flows to $t \rightarrow +\infty$

Remarks:

- The Gaussian (Wilson-Fisher) fixed governs the critical behavior for $d > 4$ ($d < 4$).
- $d = d_c^+ = 4$ is the upper critical dimension.

- For $d > d_c^+$ Landau theory becomes (asymptotically) exact because the theory is effectively Gaussian at criticality.
- An experimental system at T_c flows to the respective critical fixed point and the system becomes scale invariant.
- The critical behavior is governed by the flow in the vicinity of the critical fixed point.

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Perturbations to the Wilson-Fisher fixed point

Consider RG flow near the WF fixed point:

$$t = t^* + \delta t$$

$$g = g^* + \delta g$$

with $\delta t \ll t^*$ and $\delta g \ll g^*$.

Linearized flow equations ($O(N)$ model):

$$\frac{d}{d \ln b} \begin{pmatrix} \delta t \\ \delta g \end{pmatrix} = \underbrace{\begin{pmatrix} 2 - \epsilon \frac{N+2}{N+8} & \frac{N+2}{6} \left(1 + \epsilon \frac{N+2}{N+8}\right) \\ 0 & -\epsilon \end{pmatrix}}_{=: (B_{ij}) \text{ "stability matrix" }} \begin{pmatrix} \delta t \\ \delta g \end{pmatrix} + O(\delta^2)$$

Diagonalization of stability matrix:

$$\sum_{j=1}^2 B_{ij} v_j^I = \theta^I v_i^I \quad I=1,2 \text{ (no sum!)}$$

\uparrow eigenvectors \uparrow eigenvalues

Remarks:

- $\theta^I = \text{dim}[v^I]$ is the scaling dimension of the coupling v^I at the WF fixed point
- Any critical fixed point has exactly one $\theta^I > 0$ (w.l.o.g. for $I=1$)

Integration of the relevant direction:

$$v^1(b) = v^1(0) b^{\theta^1} \quad \text{with } \theta^1 > 0$$

Scaling transformation of reduced temperature t_{red} (or other tuning parameter):

$$t_{\text{red}} \sim \delta t \sim v^2 \Rightarrow t_{\text{red}} \mapsto b^{\theta_1} t_{\text{red}} \Rightarrow \theta_1 \equiv y_t$$

Correlation-length exponent:

$$\boxed{\nu = \frac{1}{\theta^1}}$$

Wilson-Fisher fixed point:

$$\nu = \frac{1}{2} + \frac{N+2}{4(N+8)} \epsilon + \mathcal{O}(\epsilon^2)$$

Gaussian fixed point:

$$\nu = \frac{1}{2}$$

Remarks:

• For $N=1$ and $\epsilon=1$ we get

$$v = \frac{1}{2} + \frac{1}{12} \pm \dots \approx 0.58$$

• Higher-order calculations ($D=3$):

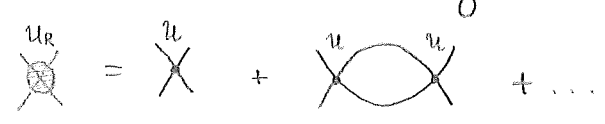
- $v = 0.629(3)$ (six-loop ϵ expansion + Borel summation)
- $v = 0.631(4)$ (high-temperature expansion)
- $v = 0.6300(1)$ (MC simulation)
- $v = 0.64(1)$ (neutron scattering of FeF_2)

[Guida & Zinn-Justin, J.Phys A 31, 8103 (1998)]

4.4 Field-theoretical perspective and anomalous dimension

Idea: Perturbation theory in "renormalized" coupling u_R :

$$u_R = u - \frac{N+8}{6} u^2 \int_0^\Lambda \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + \tau)^2} + O(u^3)$$



Remarks:

• u_R is the effective coupling after integrating out all modes:

• Dimensionless coupling:

$$u \mapsto g \equiv \frac{S_d}{(2\pi)^d} \frac{u}{|\tau/\epsilon|^{(d-1)/2}} \quad \text{diverges for } \tau \rightarrow 0 \text{ when } d < d_c^+ = 4$$

\Rightarrow standard perturbation theory (in u) breaks down at criticality

• "Renormalized" perturbation theory (in u_R) can be set up to yield finite result.

Example (anomalous dimension, sketch):

Expected critical correlator:

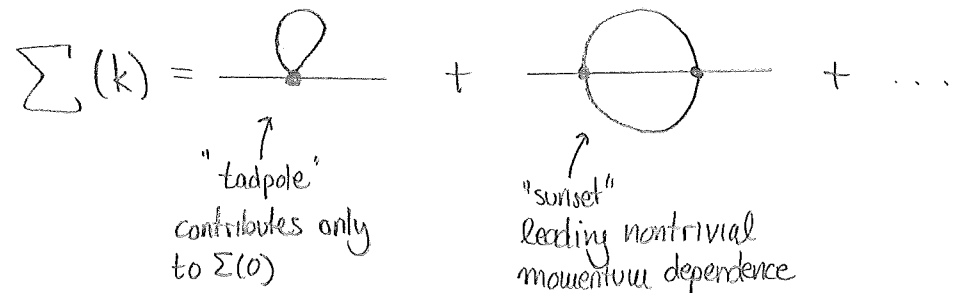
$$\langle \phi(-k)\phi(k) \rangle \propto \frac{1}{k^{2-\eta}}$$

anomalous dimension

Standard perturbation theory:

$$\langle \phi(-k)\phi(k) \rangle \propto \frac{1}{k^2 + r - \Sigma(k)}$$

with the "self-energy"



Critical point: $r_R = r - \Sigma(0) = 0$

Sunset diagram yields in $D=4-\epsilon$:

$$\Sigma(k) - \Sigma(0) = u^2 \left[c_1 k^2 \ln\left(\frac{\Lambda}{k}\right) + \mathcal{O}(k^4, \epsilon) \right] + \dots$$

↑ constant

To the leading order $u_R = u + \mathcal{O}(u^2)$ and thus

$$\langle \phi(-k)\phi(k) \rangle \propto \frac{1}{k^2 \left[1 + c_2 g_R^2 \ln\left(\frac{\Lambda}{k}\right) \right]} + \mathcal{O}(g_R^3)$$

$$= \frac{1}{k^2} \left(\frac{\Lambda}{k}\right)^{-c_2 g_R^2} + \mathcal{O}(g_R^3) \quad [k^x = 1 + x \ln k + \mathcal{O}(\epsilon)]$$

with $g_R = g^*$ at the critical point.

Reinstating the constants, we read off

$$\eta = c_2 (g^*)^2 = \frac{N+2}{2(N+8)^2} \epsilon^2 + O(\epsilon^3)$$

Remarks:

• The last step $1 + c_2 g_R^2 \ln(\frac{\Lambda}{k}) = (\frac{\Lambda}{k})^{c_2 g_R^2} + O(g_R^4)$ effectively resums an infinite number of diagrams

• For $N=1$ and $\epsilon=1$ (3D Ising):

$$\eta = \frac{1}{54} + \dots \approx 0.02$$

to be compared with (almost exact) value from MC

$$\eta_{MC} = 0.0363(1)$$

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4.5 Phase transitions and critical dimensions

Universality: different microscopic models flow to the same RG fixed point at criticality

Critical dimensions:

• Upper critical dimension d_c^+ : Mean-field theory asymptotically exact for $d \geq d_c^+$

• Lower critical dimension d_c^- : Fluctuations destroy ordered phase at any temperature for $d \leq d_c^-$

• Critical exponents typically depend on d for $d_c^- < d < d_c^+$ and become d -independent for $d > d_c^+$ [exception: system with sufficiently long-ranged interactions].

- Classical magnets with short-range interactions [$O(N)$ models]: (48)

$$d_c^+ = 4 \quad \text{and} \quad d_c^- = \begin{cases} 2 & \text{for } N > 2 \\ 1 & \text{for } N = 1 \end{cases}$$

(The case $N=2$ and $d=2$ is special.)

Physics near upper critical dimension [$O(N)$ models]:

- For $d < d_c^+ = 4$: critical fixed point = Wilson-Fisher fixed point, observables computable in renormalized perturbation theory in $u^* = u^*(d)$, hyperscaling valid.

- For $d > d_c^+ = 4$: critical fixed point = Gaussian fixed point, observables computable in standard perturbation theory in u , exponents take mean-field values, e.g. $\alpha=0$, $\eta=0$, $\nu=\frac{1}{2}$, etc. hyperscaling violated: e.g.

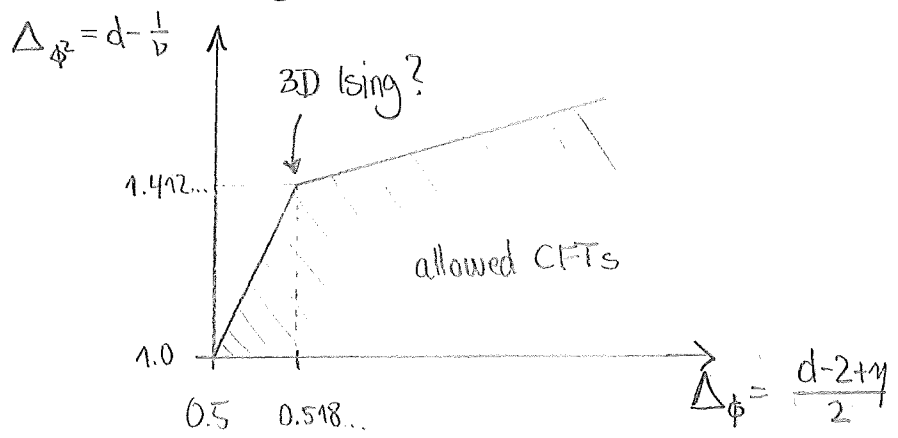
$$2 - \alpha \neq d\nu \quad (\text{Josephson})$$

can be traced back to presence of dangerously irrelevant coupling u (free energy nonanalytic at $u = u^* = 0$.)

- For $d = d_c^+ = 4$: logarithmic corrections to mean-field behavior

Analytical alternatives to $\epsilon = 4-D$ expansion:

- $\frac{1}{N}$ expansion (exercise sheet 3)
- $2+\epsilon$ expansion: expansion in $T_c(\epsilon) \propto O(\epsilon)$
- Conformal bootstrap: use symmetry and unitarity arguments to constrain scaling dimensions of operators assuming conformal invariance



world record in precision, e.g.:

$\nu = 0.629971(4)$

(3D Ising, Kos et al., 2016)
[tinyurl.com/ising-bs]

Summary:

