

## Exercises for “Quantum Phase Transitions”

Summer 26

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## Exercise 4 (12.06.26)

**1. Anisotropic perturbation to the  $O(2)$  Wilson-Fisher fixed point** (10 points)

Consider the  $O(2)$  model for the two-component boson field  $\phi = (\phi_1, \phi_2)$  (with  $\phi_1, \phi_2$  being real scalars) in the presence of an anisotropic perturbation:

$$S = \int d^d x \left[ \frac{1}{2} (\nabla \phi_1)^2 + \frac{1}{2} (\nabla \phi_2)^2 + \frac{r}{2} (\phi_1^2 + \phi_2^2) + \frac{u}{4!} (\phi_1^4 + \phi_2^4) + \frac{2v}{4!} \phi_1^2 \phi_2^2 \right]. \quad (1)$$

For  $v \neq u$  the continuous  $O(2)$  rotational symmetry is explicitly broken, but a residual  $\mathbb{Z}_4$  symmetry (fourfold rotations by integer multiples of  $\pi/2$ ) remains intact.

- Classify all possible symmetry-allowed operators with respect to their scaling dimension. Are there any relevant or marginal operators near  $d = 4$  dimensions that have been omitted in Eq. (1)?
- Show that the one-loop RG flow of the suitably rescaled couplings in  $d = 4 - \epsilon$  can be written as:

$$\frac{dr}{d\ell} = 2r + \frac{1}{2}u + \frac{1}{6}v \quad (2)$$

$$\frac{du}{d\ell} = \epsilon u - \frac{3}{2}u^2 - \frac{1}{6}v^2 \quad (3)$$

$$\frac{dv}{d\ell} = \epsilon v - \frac{2}{3}v^2 - uv \quad (4)$$

- Show that these equations reduce to the expected flow equations of the  $O(2)$  model in the limit  $u = v$ .
- Determine the linearized RG flow in the vicinity of the Wilson-Fisher fixed point at  $r = r^*$  and  $u = v = u^*$ :

$$\frac{d\delta g_i}{d\ell} = \sum_{j=1}^3 B_{ij} \delta g_j + \mathcal{O}(\delta g^2), \quad \delta g_i \equiv g_i - g_i^*,$$

with the “stability matrix”  $B_{ij} = \left. \frac{\partial (dg_i/d\ell)}{\partial g_j} \right|_{g=g^*}$  and  $(g_i) \equiv (r, u, v)$ . Is the  $\mathbb{Z}_4$  anisotropy  $\propto u - v$  relevant or irrelevant at the Wilson-Fisher fixed point? What is the corresponding eigenvalue of the stability matrix?

**2. Teaching evaluation**

Fill out the evaluation survey (lecture only<sup>1</sup>):

<https://befragung.zqa.tu-dresden.de/uz/de/sl/7cLw15omM47B>



<sup>1</sup>Note that a survey to evaluate the exercise class is not available.