Spontaneous breaking of Lorentz symmetry in QED_3

Lukas Janssen

Technische Universität Dresden

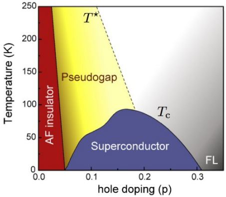
based on: LJ, arXiv:1604.06354 [hep-th]

SFB 1143

QED_3 as ...

...effective description of high-T_c's:

[Franz, Tesanovic, Vafek '02; Herbut '02; ...]



...with $N = 2$ four-component Dirac fermions

...emergent theory of fractionalized excitations in spin systems:

[Hermele *et al.* '04; Ran *et al.* '07; Xu '08; He *et al.* '15; Wang & Senthil '16; ...]

- U(1) Dirac spin liquids?

...with $N = 1, 2, 4, \ldots$

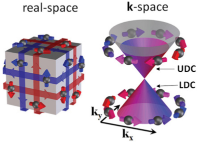
...field theory of the half-filled Landau level?

[Son '15]

...with $N = 1/2$

...dual description of topological insulators?

[Wang & Senthil '15; Metlitski & Vishwanath '15; Mross *et al.* '15]

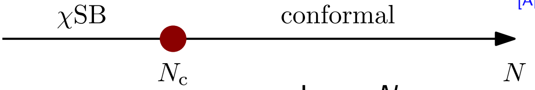


...with $N = 1/2, 3/2, 5/2, \ldots$

QED_3: phase diagram

(Expected) phase diagram of QED_3 with N massless Dirac fermions:

[Appelquist *et al.* '88; ...]



Low N:

- chiral symmetry breaking (χSB)
- dynamical mass generation
- Mott insulator

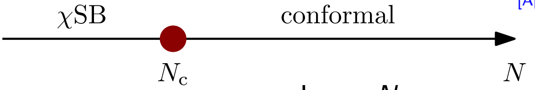
Large N:

- conformal symmetry
- interacting gapless fermions
- non-Fermi liquid

QED$_3$: phase diagram

(Expected) phase diagram of QED$_3$ with N massless Dirac fermions:

[Appelquist *et al.* '88; . . .]



Low N:

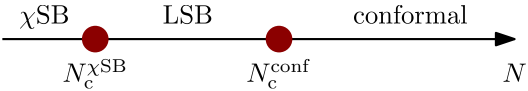
- chiral symmetry breaking (χSB)
- dynamical mass generation
- Mott insulator

Large N:

- conformal symmetry
- interacting gapless fermions
- non-Fermi liquid

We show:

[LJ, arXiv:1604.06354]



Intermediate N:

- spontaneous Lorentz symmetry breaking (LSB)
- gapless fermions, anisotropic propagation

Outline

(1) $2+\epsilon$ expansion: $N_{\rm c}^{\rm conf} \nearrow \infty$ when $d \searrow 2$

(2) (generalized) F theorem: $N_{\rm c}^{\chi{\rm SB}} \leq 1 + \mathcal{O}(d-2) < N_{\rm c}^{\rm conf}$

(3) Mean-field theory, F theorem, susceptibility analysis:

$$\langle v_\mu \rangle \neq 0 \quad \text{for} \quad N_{\rm c}^{\chi{\rm SB}} < N < N_{\rm c}^{\rm conf}$$

Model

Action:

$$S_{\text{QED}} = \int d^d x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}_i \gamma_\mu D_\mu \psi_i \right) \qquad \text{in } 2 < d < 4,$$

where $i = 1, \ldots, N$ and $\mu, \nu = 0, \ldots, d-1$ and

- Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}\mathbb{1}_4$ 4-dimensional
- Covariant derivative: $D_\mu = \partial_\mu + ieA_\mu$
- Charge: $[e^2] = 4 - d$ RG relevant in $d < 4$

  ... QED_3 (super-)renormalizable & asymptotically free
- Symmetries: "chiral" $\mathrm{SU}(2N)$, parity, Lorentz, local $\mathrm{U}(1)$

  ... "chirality" consequence of the reducible rep. [LJ & Gies '12; ...]
- Gauge fixing:

$$S_{\text{gf}} = -\frac{1}{2\xi} \int d^d x \, (\partial_\mu A_\mu)^2 \,, \qquad \xi \in \mathbb{R}$$

... allows to check gauge invariance via ξ

Renormalization group

Loop corrections can induce new operators: ...which are not present in S_{QED}

Most of them irrelevant, but

local 4-fermion terms are marginal in $d = 2$!

...and can thus become relevant at interacting fixed points in $d = 2 + \epsilon$

Full basis: [Gies & LJ '10; ...]

$$S_{\text{4-fermi}} = \int d^d x \left[g_1 (\bar{\psi}_i \gamma_{35} \psi_i)^2 + g_2 (\bar{\psi}_i \gamma_\mu \psi_i)^2 \right]$$

with $\gamma_{35} \equiv i\gamma_3\gamma_5$ ...and where γ_3 and γ_5 the two "left-over" gamma matrices not present in $\not{D}$

RG flow for $S = S_{\mathrm{QED}} + S_{\mathrm{gf}} + S_{\text{4-fermi}}$

Flow of charge:

$$\beta_{e^2} = (d - 4 + \eta_A)e^2 \leq 0$$

with anomalous dimension $\eta_A = \frac{4}{3}Ne^2 + \mathcal{O}(e^4)$.

- e^2 flows to strong coupling
- charged fixed point:

$$\eta_A = 4 - d$$

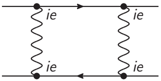
...as in Abelian Higgs model: [Herbut & Tesanovic '96]

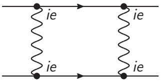
- photon propagator:

$$D_{\mu\nu}(q) \propto |q|^{\eta_A - 2} = |q|^{2-d} = \begin{cases} \text{const.}, & d = 2 \\ \frac{1}{|q|}, & d = 3 \end{cases}$$

[Schwinger '62]
[Pisarski '84]

- strong e^2 generates g_1, g_2:

$\beta_{g_i} = (d-2)g_i +$ 

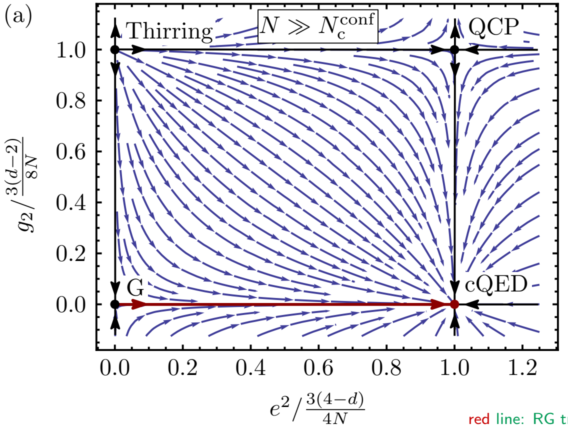
$+$ 

$+ \ldots$

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

. . . with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

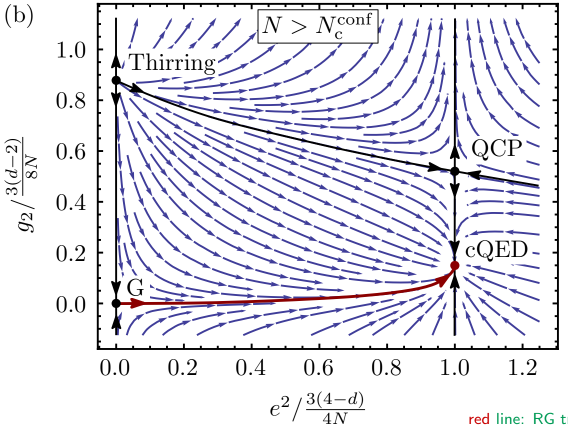
3 interacting fixed points:

Thirring [Gies & LJ,. . . '10, '12, '15], conformal (cQED), quantum critical (QCP)

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

...with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



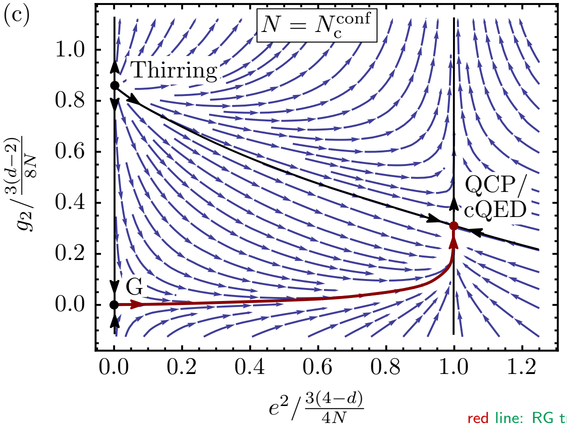
red line: RG trajectory of pure QED

cQED and QCP approach each other when lowering N

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

. . . with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



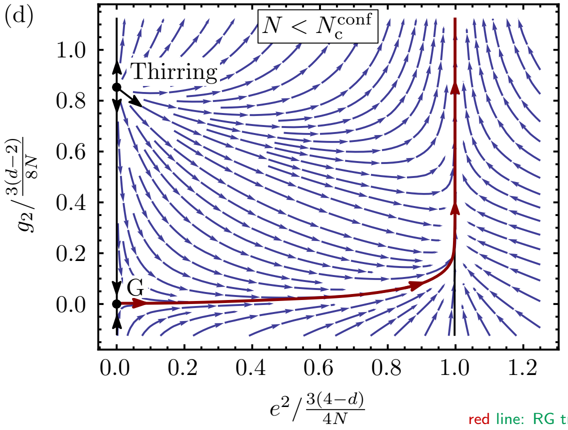
red line: RG trajectory of pure QED

cQED and QCP merge when $N = N_c^{\rm conf}$

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

...with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

cQED and QCP disappear for $N < N_c^{\text{conf}}$ ...

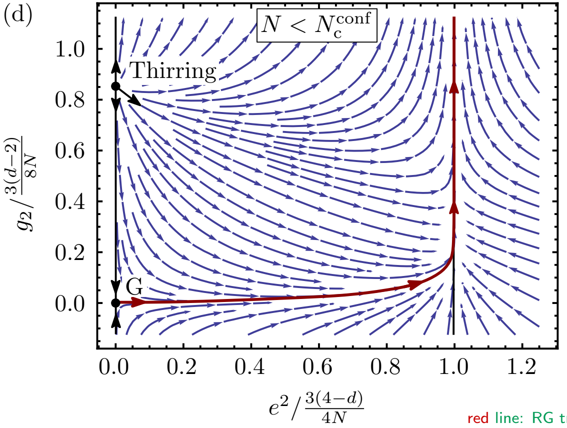
...leaving behind the runaway flow!

...i.e., conformal phase is unstable below N_c^{conf}

$2 + \epsilon$ expansion

RG flow in e^2–g_2 plane:

... with $g_1 \equiv g_1^*(e^2, g_2)$ chosen such that $\beta_{g_1} = 0$



red line: RG trajectory of pure QED

$$N_c^{\text{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}} \nearrow \infty \quad \text{for} \quad \epsilon \searrow 0$$

... agrees with [Di Pietro *et al.* '16]

... perturbative expansion under control

RG monotonicity

Observation:

"effective" number of degrees of freedom decreases under RG

. . . e.g., N critical modes at Wilson-Fisher $\mathrm{O}(N)$ fixed point $\rightarrow$ $(N-1)$ massless modes in the SSB phase
or 0 massless modes in the symmetric phase

Quantification?

1+1D: *c* theorem

[Zamolodchikov '86]

$$\boxed{c_{\mathrm{UV}} > c_{\mathrm{IR}}}$$

c central charge of conformal algebra

. . . RG flows go "downhill"

3+1D: *a* theorem

[Cardy '88; Komargodski & Schwimmer '11]

$$\boxed{a_{\mathrm{UV}} > a_{\mathrm{IR}}}$$

anomaly coefficient $a \sim \int_{S^4} \langle T^\mu_\mu \rangle$

2+1D: *F* theorem

[Jafferis '10; Jafferis *et al.* '11; Casini & Huerta '12; . . .]

$$\boxed{F_{\mathrm{UV}} > F_{\mathrm{IR}}}$$

"sphere free energy" $F = -\log Z_{S^3}$

= universal part of entanglement-entropy scaling $S_A = \alpha L - \gamma + \mathcal{O}(1/L) \;\Rightarrow\; \gamma = F$

[Kitaev & Preskill '06; Casini, Huerta, Myers '11]

RG monotonicity: continuous dimension?

Generalized F for $d \in \mathbb{R}$: [Giombi & Klebanov '14]

$$\tilde{F} = \sin\left(\frac{\pi d}{2}\right) \log Z_{S^d}$$

with:

$\tilde{F}\big\vert_{d=2} = \frac{\pi}{6} c$	c theorem	✓
$\tilde{F}\big\vert_{d=3} = F$	F theorem	✓
$\tilde{F}\big\vert_{d=4} = \frac{\pi}{2} a$	a theorem	✓

Thus:

$$\boxed{\tilde{F}_{\text{UV}} > \tilde{F}_{\text{IR}}} \qquad \text{for all integer } d$$

. . . and various evidence for continuous d:

$$d = 2 + \epsilon, \;\; d = 3 - \epsilon, \;\; d = 4 - \epsilon, \;\; d = 6 - \epsilon, \; \ldots$$

[Giombi, Klebanov, Tarnopolsky, Fei, . . . '14, '15, '16]

. . . though no rigorous proof yet

QED$_3$: breaking of $\mathrm{U}(2N) \to \mathrm{U}(N) \times \mathrm{U}(N)$?

Compute $\tilde{F}$ in $d = 2 + \epsilon$: [Giombi, Klebanov, Tarnopolsky '16]

UV: $$\tilde{F}_{\mathrm{UV}} = \underbrace{N\tilde{F}_{\mathrm{f}}}_{\text{fermions}} + \underbrace{(d-2)\tilde{F}_{\mathrm{b}}}_{\text{photon}} = N\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

$\tilde{F}_{\mathrm{f}}$: free fermion
$\tilde{F}_{\mathrm{b}}$: free boson

cQED: $$\tilde{F}_{\mathrm{conf}} = \left(N - \tfrac{1}{2}\right)\tilde{F}_{\mathrm{f}} = \underbrace{(2N-1)}_{c_{\text{Schwinger}}\ \checkmark}\frac{\pi}{6} + \mathcal{O}(\epsilon)$$

χSB: $$\tilde{F}_{\chi\mathrm{SB}} = \left(2N^2 + (d-2)\right)\tilde{F}_{\mathrm{b}} = N^2\frac{\pi}{3} + \mathcal{O}(\epsilon)$$

$$\Rightarrow \quad \tilde{F}_{\chi\mathrm{SB}} > \tilde{F}_{\mathrm{UV}} > \tilde{F}_{\mathrm{conf}} \quad \text{for all } N > 1$$

Hence:

$$\boxed{N_{\mathrm{c}}^{\chi\mathrm{SB}} \leq 1 + \mathcal{O}(d-2) < N_{\mathrm{c}}^{\mathrm{conf}}}$$

…the phase below $N_{\mathrm{c}}^{\mathrm{conf}}$ cannot exhibit χSB!

[LJ, arXiv:1604.06354]

Intermediate phase

Vafa-Witten theorem: QED_3 should have [Vafa & Witten '84]

(a) unbroken $\mathrm{U}(N) \times \mathrm{U}(N)$ symmetry, and . . . at least

. . . rules out other chiral breaking patterns

(b) gapless spectrum

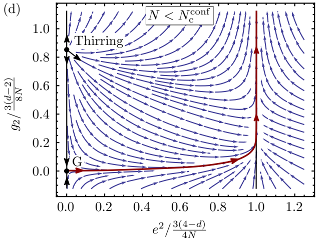
. . . rules out plain parity breaking

What else can it be?

Mean-field theory

Recall flow . . .

. . . towards divergent g_2 (and finite e^2, g_1)!

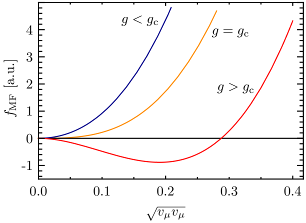


Effective description: Thirring model!

$$S_{\text{Thirring}} = \int d^d x \left[\bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + g_2 \left(\bar{\psi}_i \gamma_\mu \psi_i \right)^2 \right]$$

Large-N (saddle-point) solution:

$$f_{\text{MF}}(v_\mu) = \frac{1}{2} \left(\frac{1}{g_2} - \frac{40N}{3} \right) v_\mu^2 + \sqrt{6}\pi N \left(v_\mu^2 \right)^{3/2} + \mathcal{O}(v^4)$$



for vector order parameter $v_\mu \propto \langle \bar{\psi}_i \gamma_\mu \psi_i \rangle$

$\Rightarrow$ transition towards $v_\mu \neq 0$: **spontaneous Lorentz symmetry breaking!**

. . . spontaneous formation of electron-hole puddles

. . . consistent with Vafa-Witten and F theorem

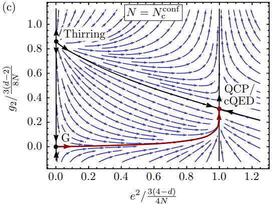
Susceptibility analysis in $d = 2 + \epsilon$

Add small symmetry-breaking "seeds": ...à la magnetic field

$$S_\Delta = \int d^d x\, \bar\psi_i \left[\Delta_{\chi\mathrm{SB}} \mathbb{1}_4 + \Delta_{\mathrm{PSB}}\gamma_{35} + i\Delta_{\mathrm{Kek}}\left(\gamma_3 \cos\varphi + \gamma_5 \sin\varphi\right) + i\Delta^\mu_{\mathrm{LSB}}\gamma_\mu\right]\psi_i$$

$N \lesssim N_{\mathrm{c}}^{\mathrm{conf}}$: Flow "hovers" over complex pair of fixed points QCP/cQED:

$\Rightarrow$ dominant order: largest susceptibility at QCP



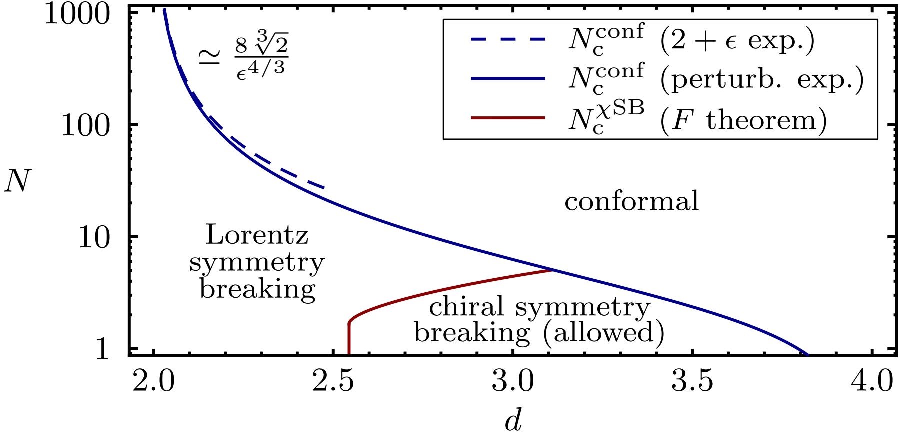
$$\gamma_{\chi\mathrm{SB}} = -\epsilon + \frac{8}{N} + \mathcal{O}(\epsilon^{3/2}) = \gamma_{\mathrm{Kek}} < 0$$

$$\gamma_{\mathrm{PSB}} = -\epsilon - \frac{12}{\sqrt{N}} + \frac{8}{N} + \mathcal{O}(\epsilon^{3/2}) < 0$$

$$\gamma_{\mathrm{LSB}} = \epsilon\sqrt{1 - \left(\frac{N_{\mathrm{c}}^{\mathrm{conf}}}{N}\right)^{3/2}} + \mathcal{O}(\epsilon^{3/2}) > 0$$

$\Rightarrow$ instability towards spontaneous Lorentz symmetry breaking!

Phase diagram of $QED_{2<d<4}$



$N_c^{\chi\text{SB}} < N_c^{\text{conf}}$ possibly even in $d = 3$!

[LJ, arXiv:1604.06354]

Conclusions

(1) QED_3 has conformal ground state at large $N > N_c^{\text{conf}} = \frac{8\sqrt[3]{2}}{\epsilon^{4/3}}$

...in $d = 2 + \epsilon$

(2) Common scenario with direct transition towards χSB inconsistent with (generalized) F theorem

...at least in $d = 2 + \epsilon$

...there are just too many Goldstones!

(3) Only possibility for $N \lesssim N_c^{\text{conf}}$: ...compatible with F theorem & Vafa-Witten

spontaneous Lorentz symmetry breaking (LSB)!

(4) Mean-field & susceptibility analyses: ...independently

confirm existence of LSB phase

(5) Extrapolation $\epsilon \to 1$ & perturbative expansion in fixed $d = 3$:

suggest finite window of LSB phase also in physical dimension