Heisenberg-Kitaev physics in magnetic fields

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&

Eric Andrade, Matthias Vojta

L.J., E. Andrade, and M. Vojta, Phys. Rev. Lett. **117**, 277202 (2016)

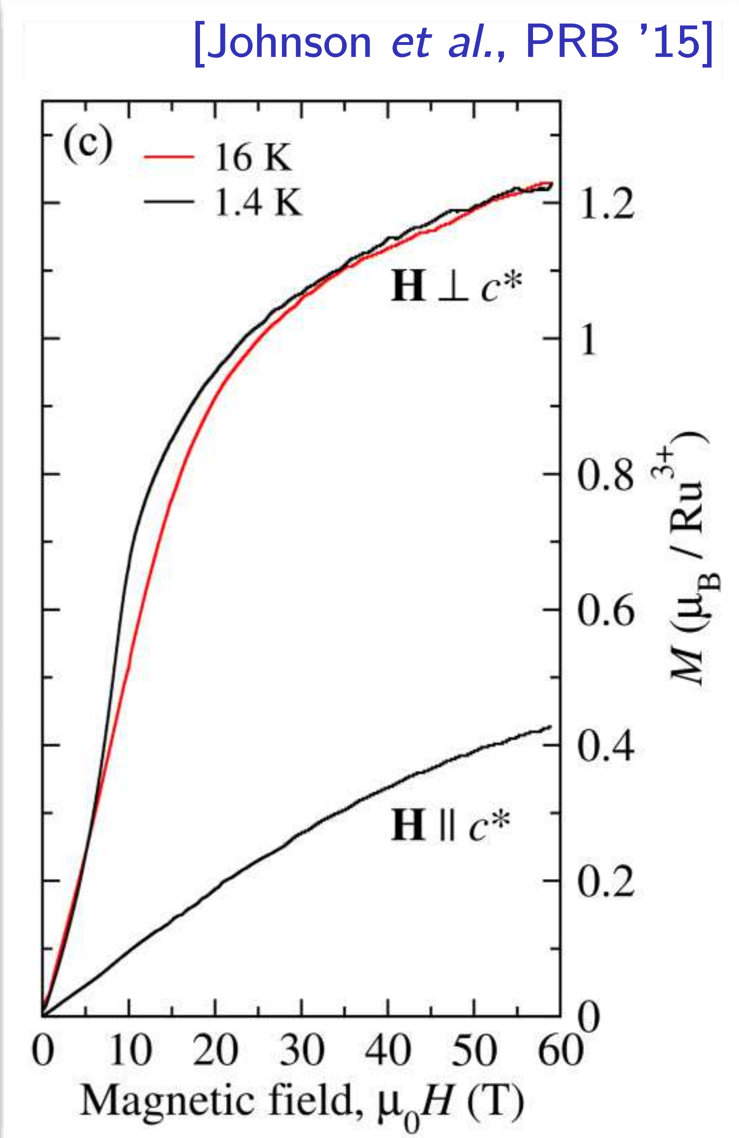
L.J., E. Andrade, and M. Vojta, Phys. Rev. B **96**, 064430 (2017)

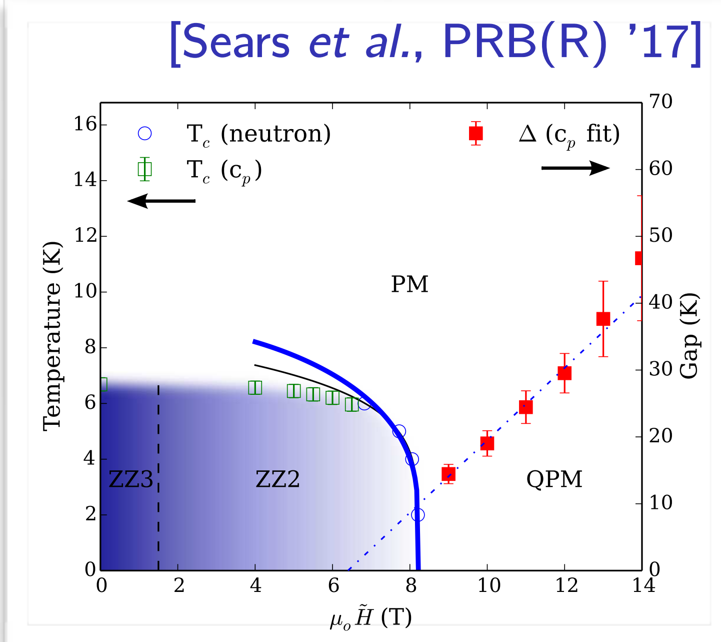
A. Wolter, L. Corredor, **L.J.**, *et al.*, Phys. Rev. B **96**, 041405(R) (2017)

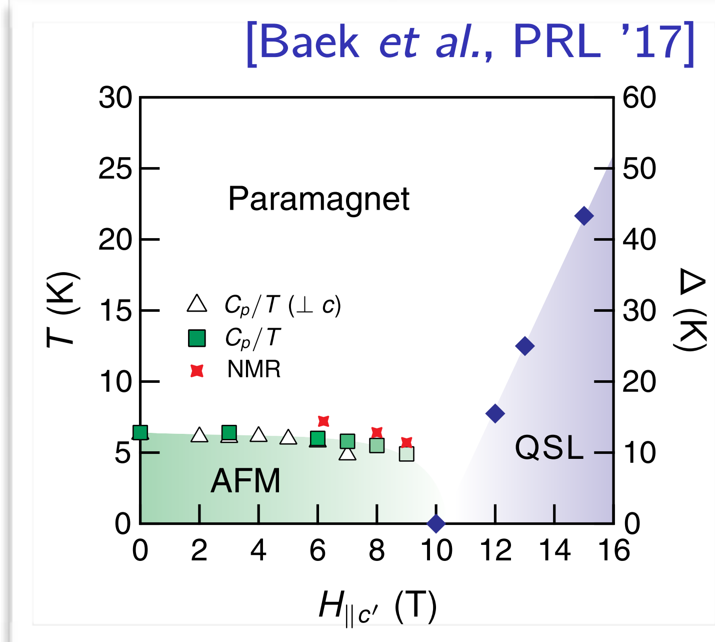
TECHNISCHE UNIVERSITÄT DRESDEN

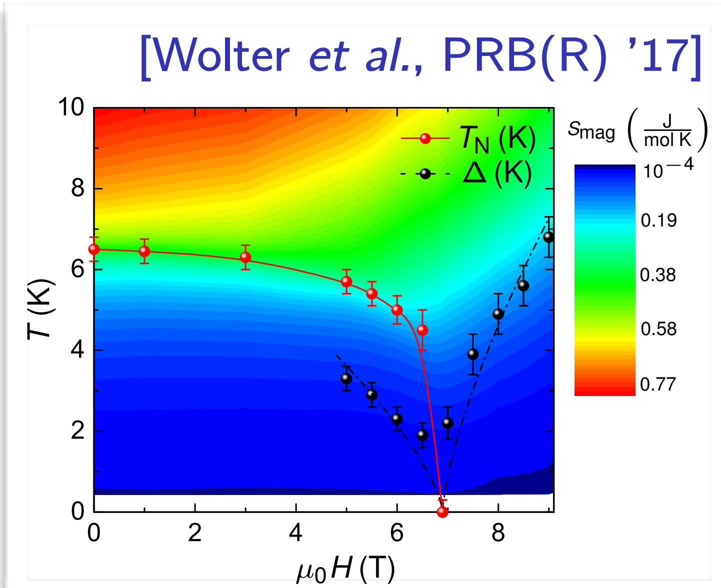
SFB 1143

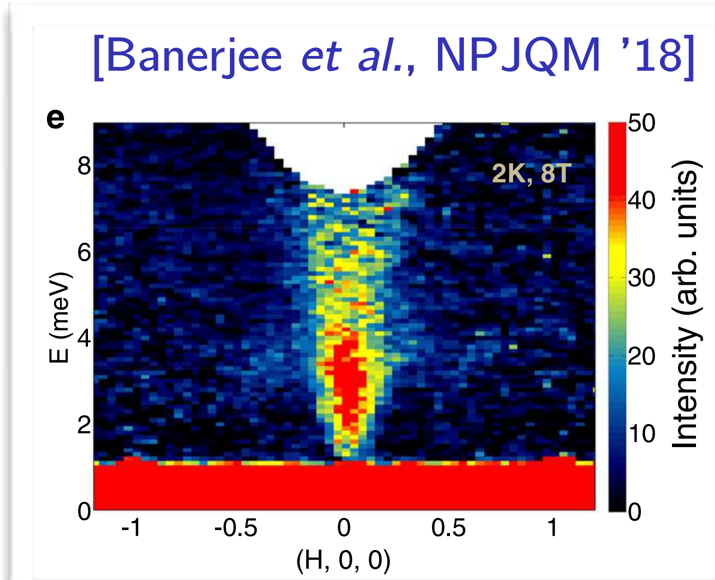
α-$RuCl_3$ in field:











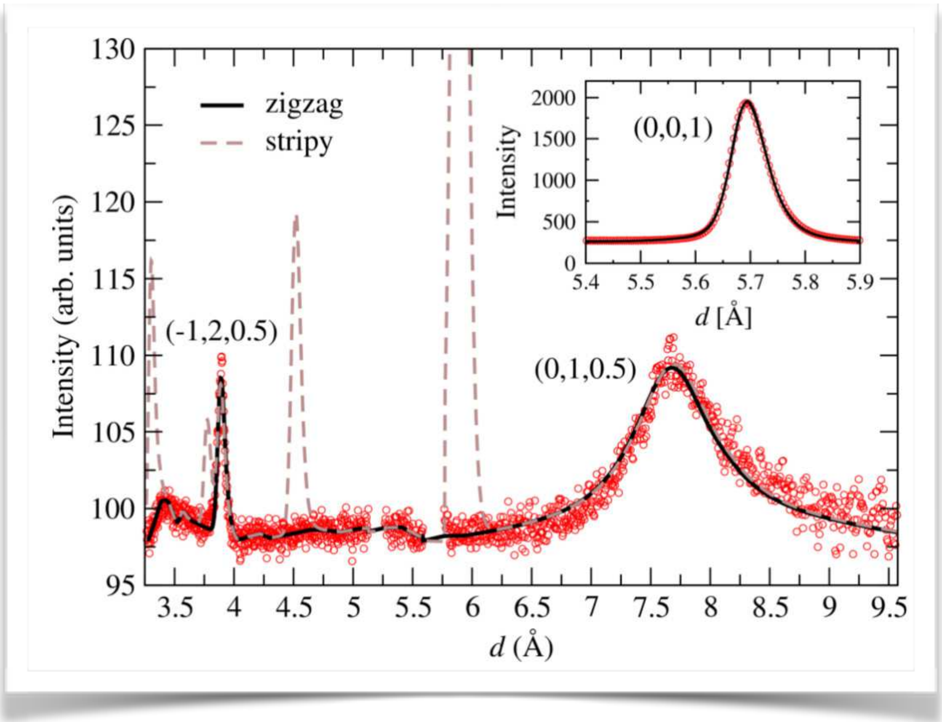
Questions:

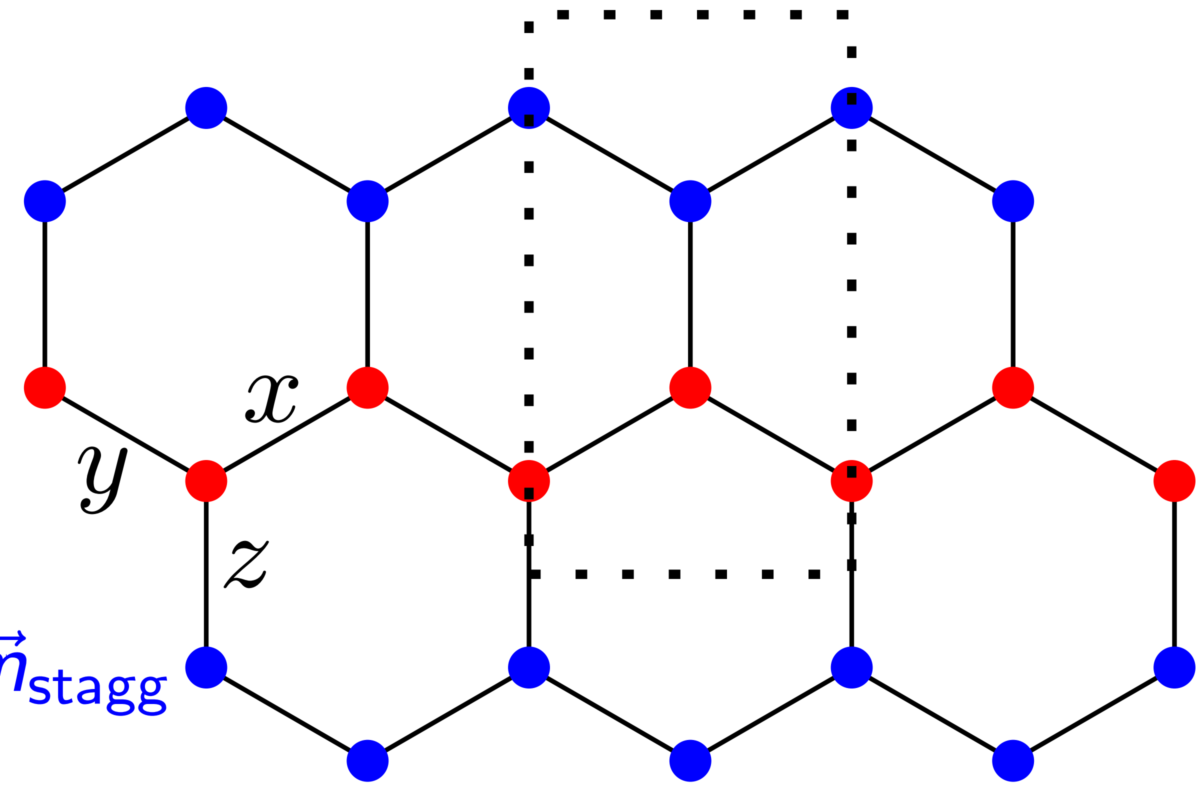
- Reason for anisotropy? ... g factors or intrinsic?
- Nature of field-induced phases? ... quantum paramagnet/topological spin liquid?
- Appropriate spin model? ... sign of K_1, role of J_3, Γ_1?

α-RuCl₃ in zero field: Zigzag antiferromagnet

Neutron diffraction:

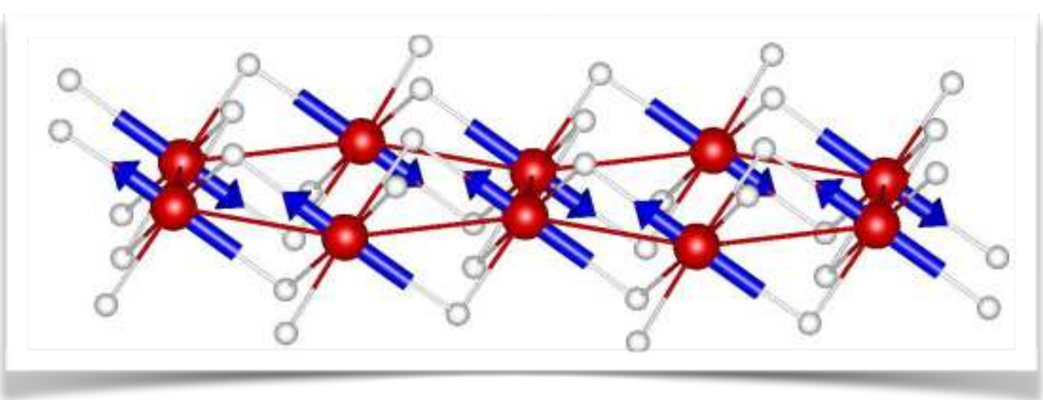
[Johnson *et al.*, PRB '15]



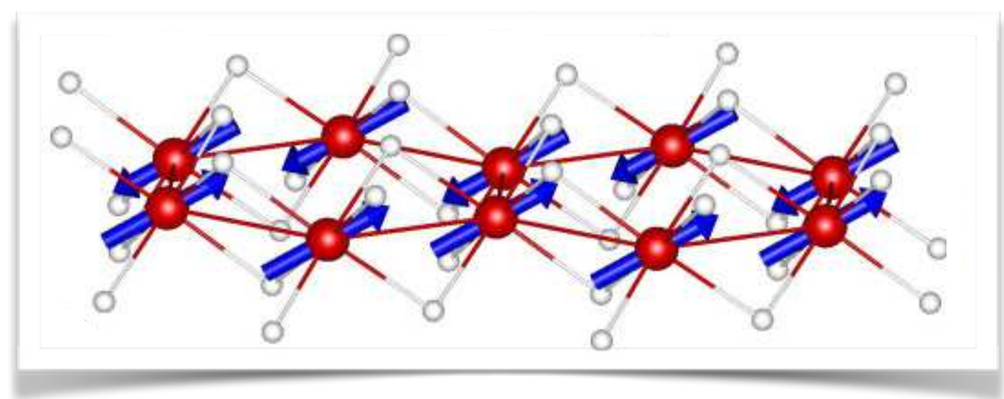


Moment direction:

[Cao *et al.*, PRB '16]



or



$\vec{S} \parallel \pm[001] \in ac^*, \quad \measuredangle(\vec{S}, a) \simeq -35^\circ$

$\vec{S} \parallel \pm[xxz] \in ac^*, \quad \measuredangle(\vec{S}, a) \simeq +35^\circ$

Mechanisms to stabilize zigzag

Extended Heisenberg-Kitaev models:

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right] + \sum_{\langle\langle ij \rangle\rangle} \left(J_2 \vec{S}_i \cdot \vec{S}_j + K_2 S_i^\gamma S_j^\gamma \right) + \sum_{\langle\langle\langle ij \rangle\rangle\rangle} J_3 \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

... neglect trigonal distortion

Set	Material	J_1 (meV)	K_1 (meV)	Γ_1 (meV)	J_2 (meV)	K_2 (meV)	J_3 (meV)	Method	Ref.	Year
1	α-$RuCl_3$	−4.6	+7.0	–	–	–	–	Fit to neutron scattering	[35,36]	2016
1′	Na_2IrO_3	−4.0	+10.5	–	–	–	–	Fit to susceptibility and neutron scattering	[30]	2013
$1+\Gamma$	α-$RuCl_3$	−12	+17	+12	–	–	–	DFT + t/U expansion	[44]	2015
2	Na_2IrO_3	0	−17	0	0	–	+6.8	DFT + exact diagonalization	[32]	2016
$2+\Gamma$	Na_2IrO_3	+3	−17	+1	−3	+6	+1	DFT + t/U expansion, direction of moments	[40,45]	2016
$(2+\Gamma)'$	Na_2IrO_3	+3	−17.5	+1	+5	–	+5	MRCI, fit to θ_{CW}	[47]	2014
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$3+J_3$	α-$RuCl_3$	−0.5	−5.0	+2.5	–	–	+0.5	Fit to neutron scattering	[39]	2017

[LJ, Andrade, Vojta, PRB '17]

Mechanisms to stabilize zigzag

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Three scenarios:

(1) Antiferromagnetic K_1, ferromagnetic J_1 [Chaloupka *et al.*, PRL '13]

Mechanisms to stabilize zigzag

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(3) Ferromagnetic K_1, positive Γ_1 [Rau *et al.*, PRL '14; Ran *et al.*, PRL '17]

... we will show that finite $J_1 < 0$ is also needed

Mechanisms to stabilize zigzag

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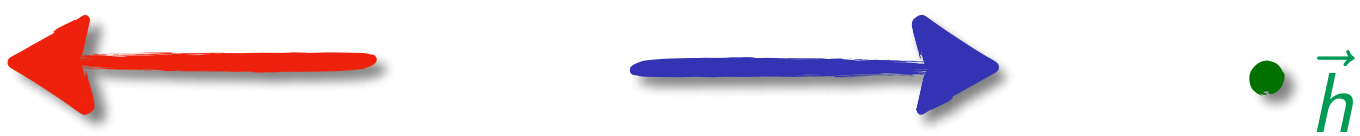
... we will show that finite $J_1 < 0$ is also needed

This talk: Magnetic-field behavior allows to distinguish scenarios!

Warm-up: SU(2) Heisenberg antiferromagnet in field

Minimal model:

$$\mathcal{H} = \sum_{\langle ij \rangle} J_1 \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$



$$|\vec{h}| = 0$$

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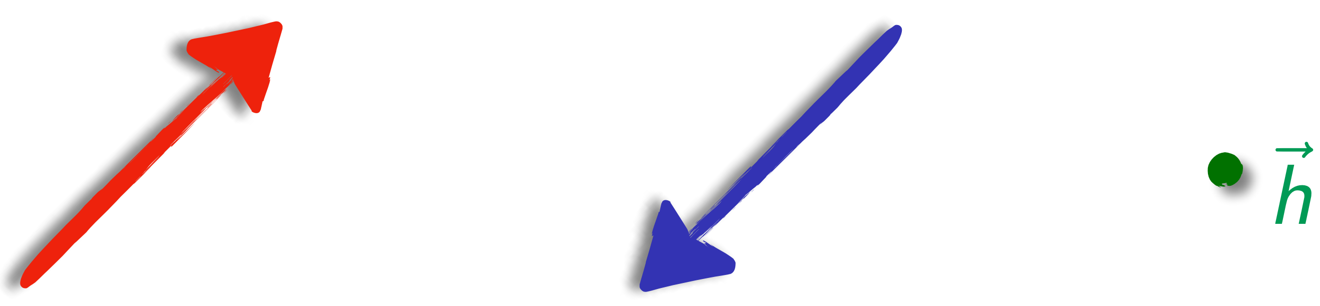

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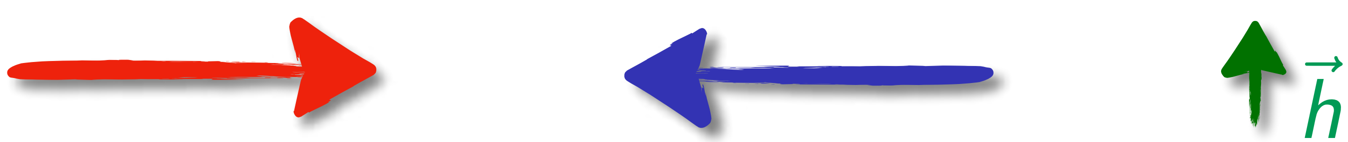

$$0 < |\vec{h}| \ll J_1 S$$

$\vec{S}_i \perp \vec{h}$ (largest susceptibility)

Warm-up: SU(2) Heisenberg antiferromagnet in field

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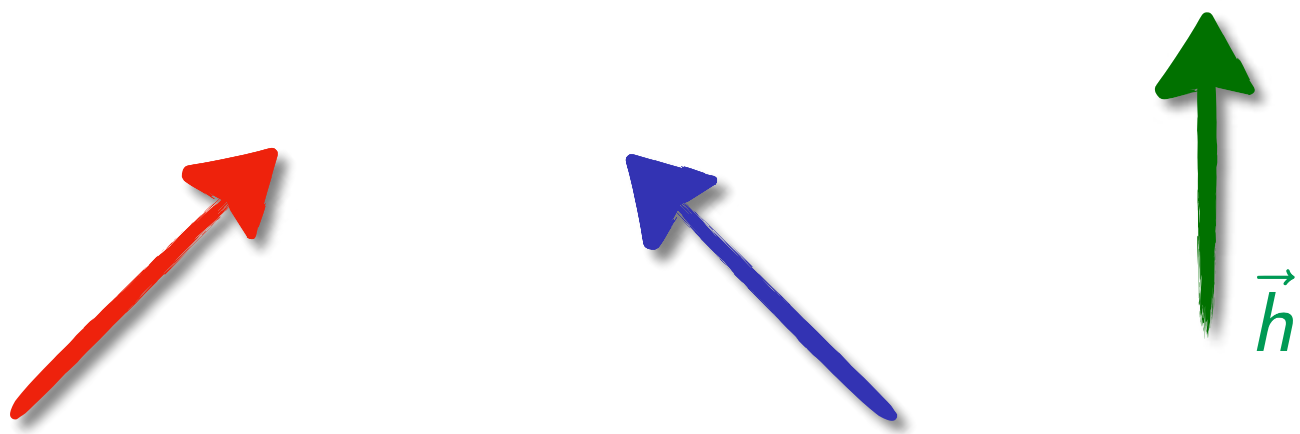
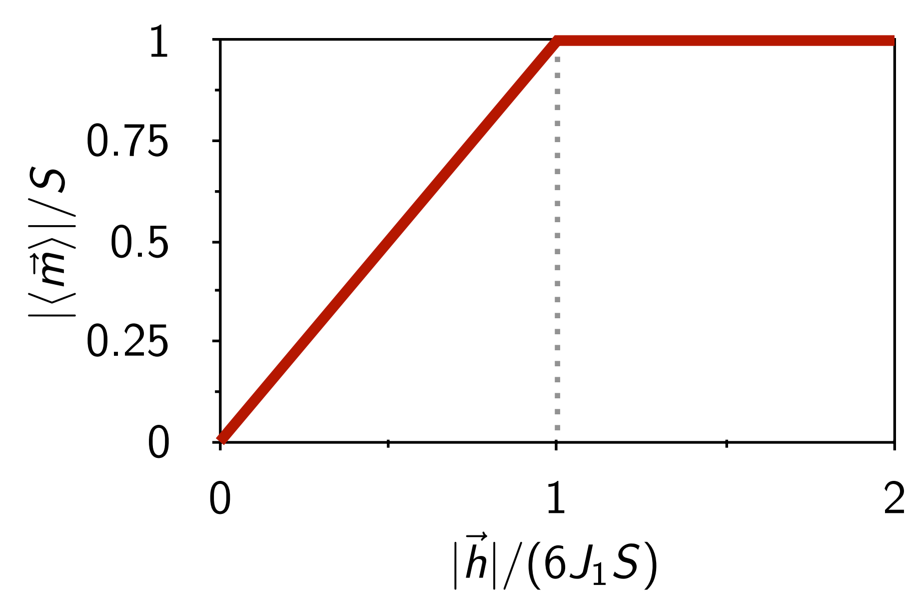


$0 < |\vec{h}| < 6 J_1 S$

"homogenous" canting



Warm-up: SU(2) Heisenberg antiferromagnet in field

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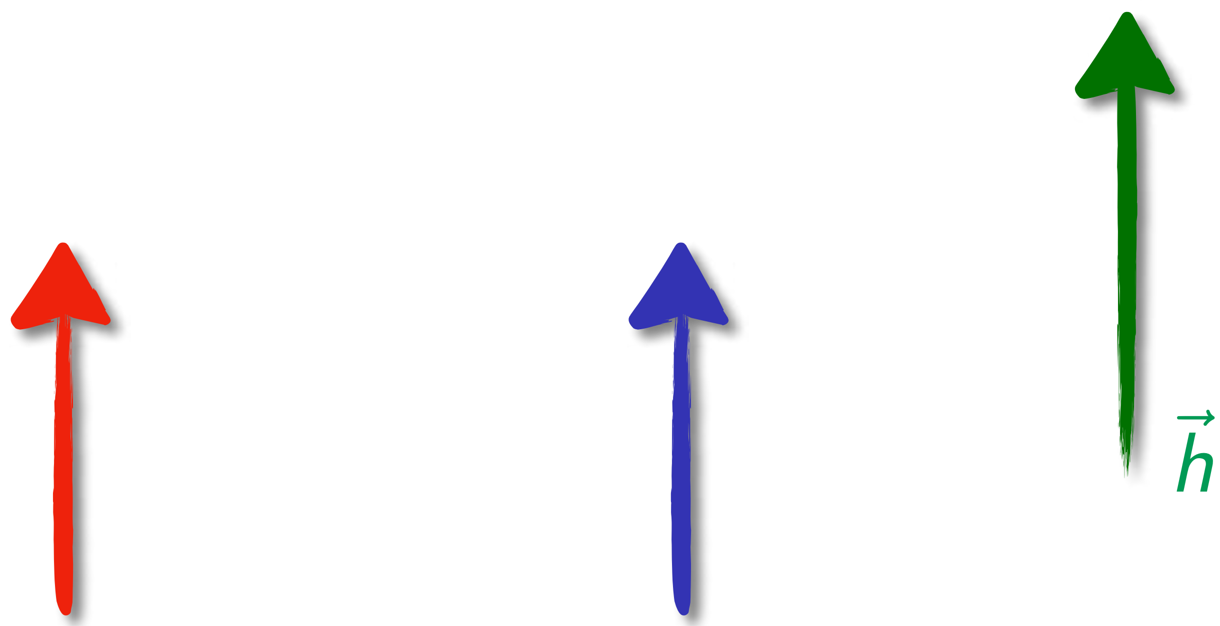
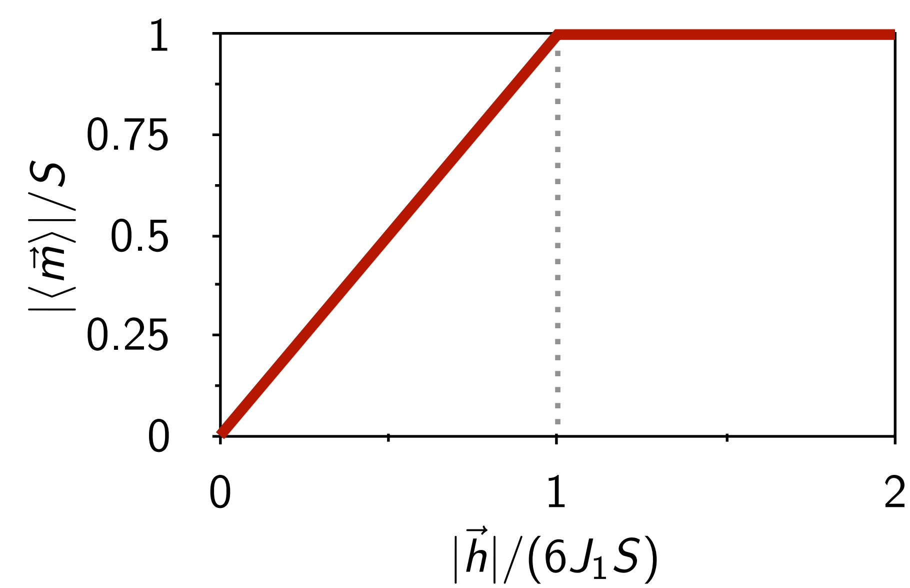


$6 J_1 S \leq |\vec{h}|$

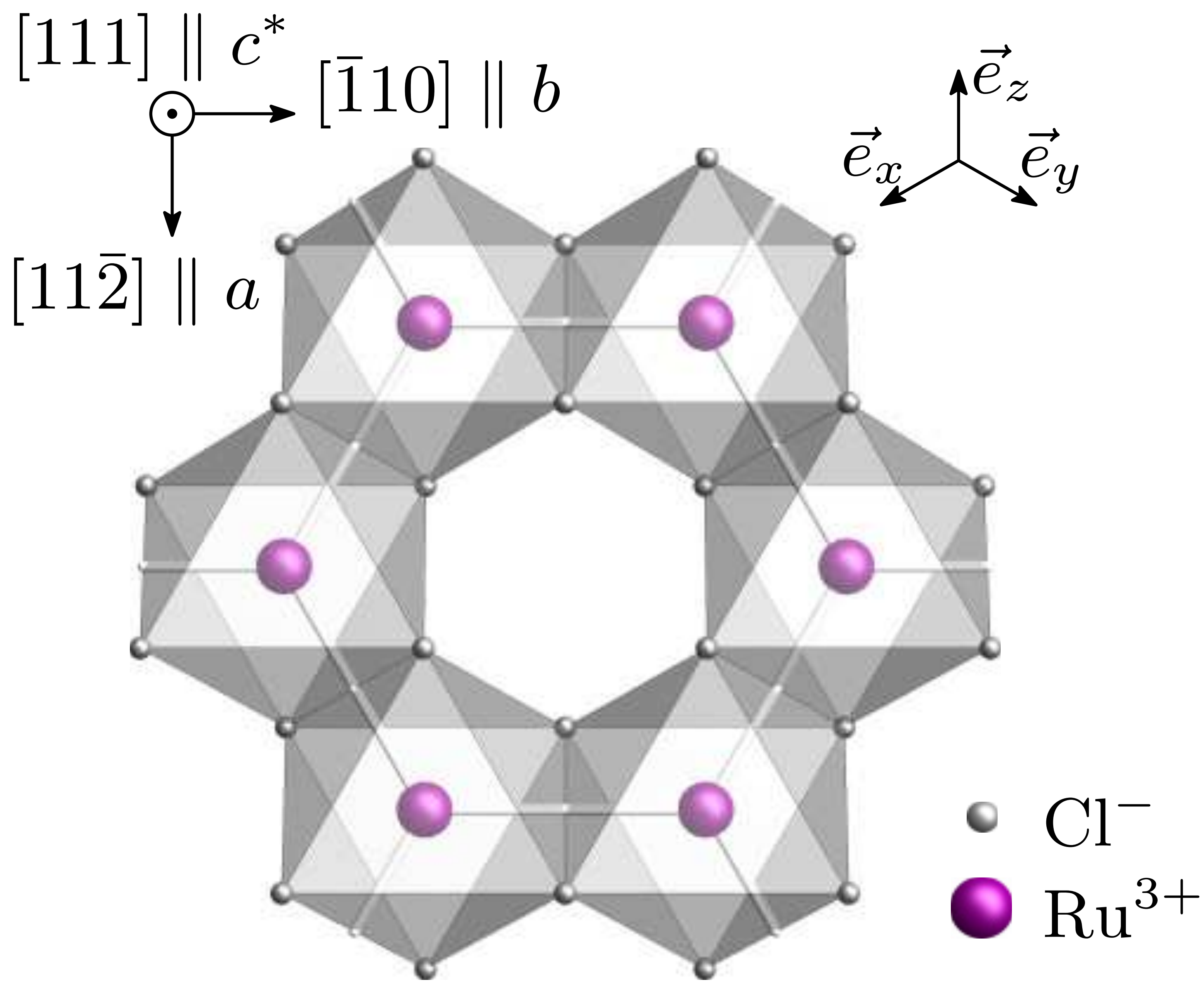
fully polarized

$|\langle \vec{m} \rangle| / S = 1$

... since one-magnon state eigenstate of $\mathcal{H}$



α-$RuCl_3$: Field directions

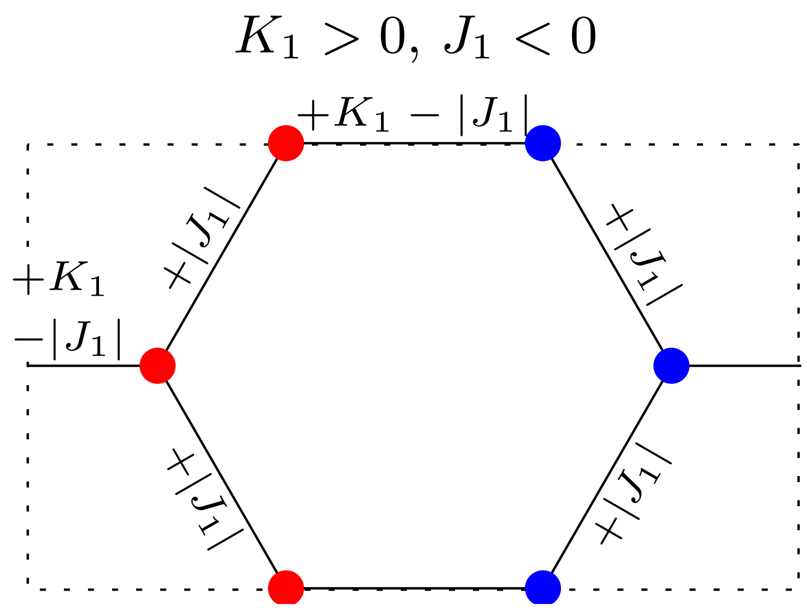


Scenario 1: Antiferromagnetic K_1, ferromagnetic J_1

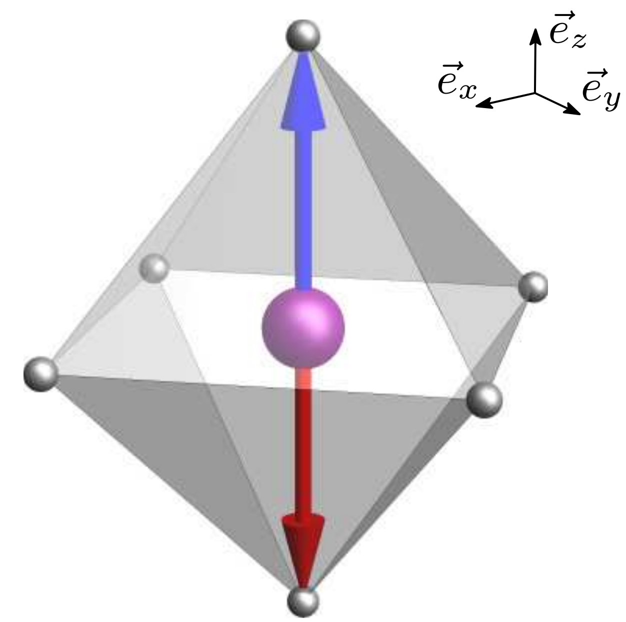
Minimal model: [Chaloupka *et al.*, PRL '10; PRL '13]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma \right]$$

Zero field:

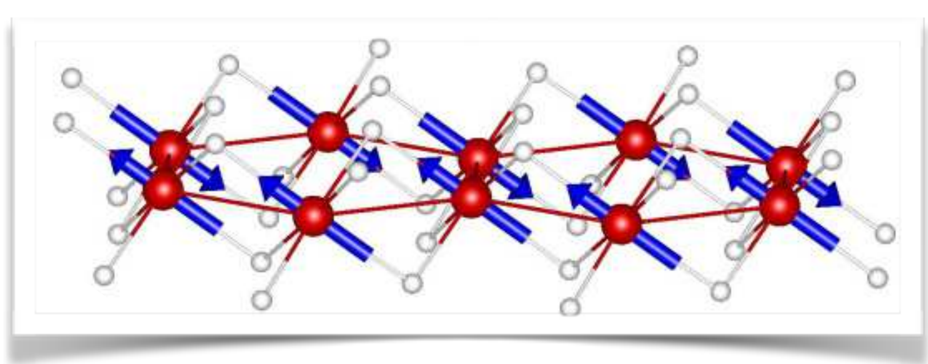


"cubic-axes zigzag"



$\vec{S} \parallel \pm\vec{e}_z$ ... along Ru-Cl bonds

... consistent with one of the two options in α-$RuCl_3$:

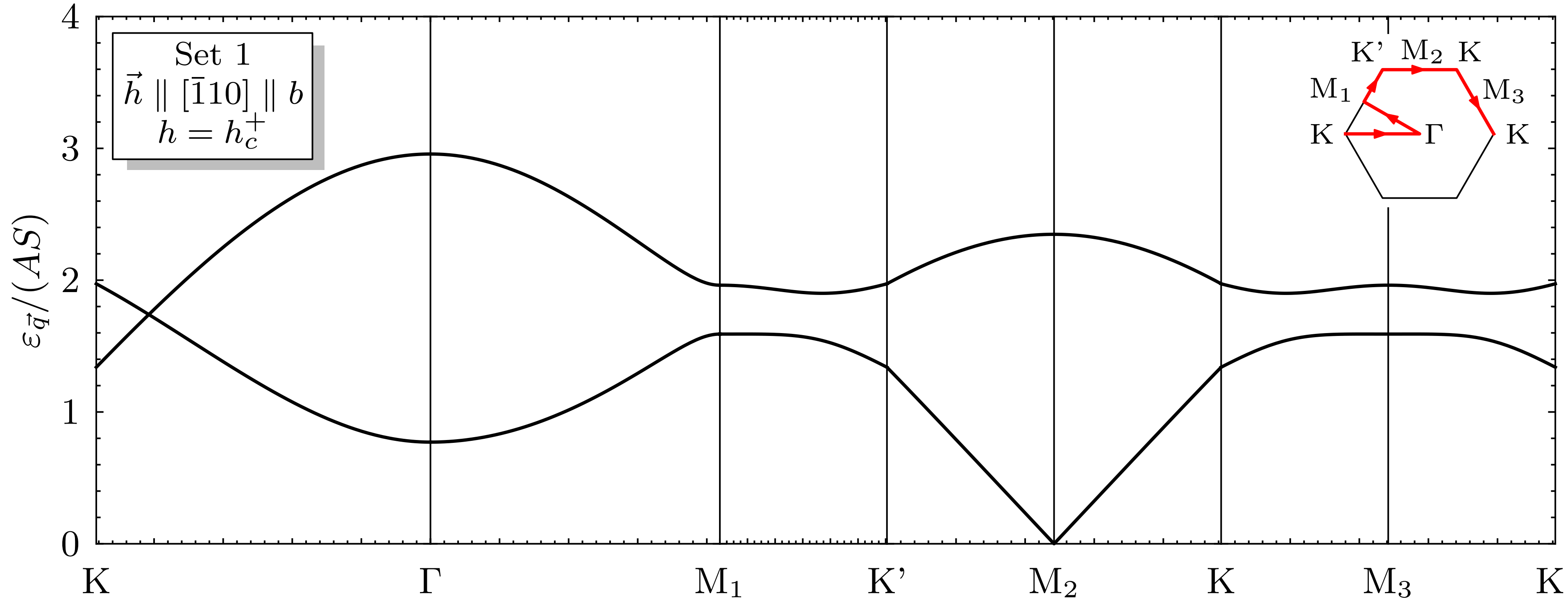


[Cao *et al.*, PRB '16]

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [\bar{1}10] \parallel b$

... along Ru-Ru bonds

Spin-wave spectrum in high-field phase:



(J_1, K_1) = (-4.6, +7.0) meV

[Banerjee *et al.*, Nat. Mat. '16]

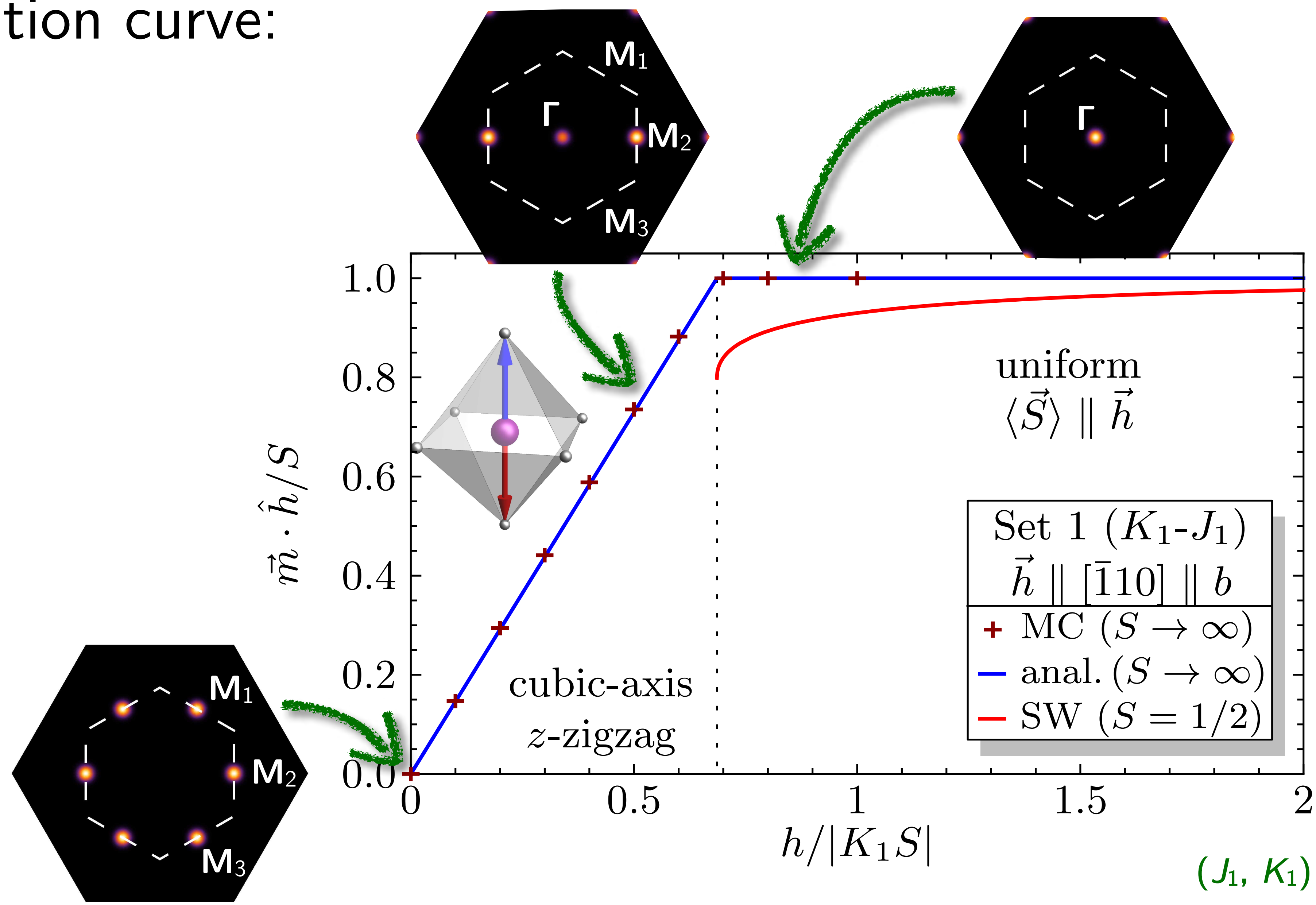
... gap closes precisely at z zigzag ordering wavevector $\vec{Q} = \mathbf{M}_2$

... consistent with a direct continuous transition to canted zigzag

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [\bar{1}10] \parallel b$

... along Ru-Ru bonds

Magnetization curve:

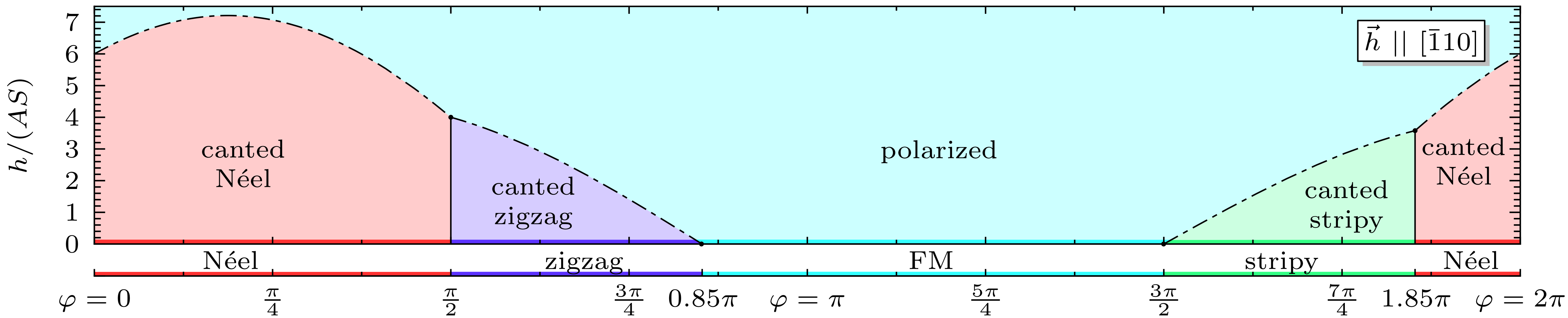


(J_1, K_1) = (-4.6, +7.0) meV

... but: high-field state not fully polarized: $|\langle \vec{m} \rangle| / S < 1$

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [\bar{1}10] \parallel b$

Classical phase diagram in $[\bar{1}10]$ field:



$$J_1 = A \cos \varphi$$

$$K_1 = 2A \sin \varphi$$

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

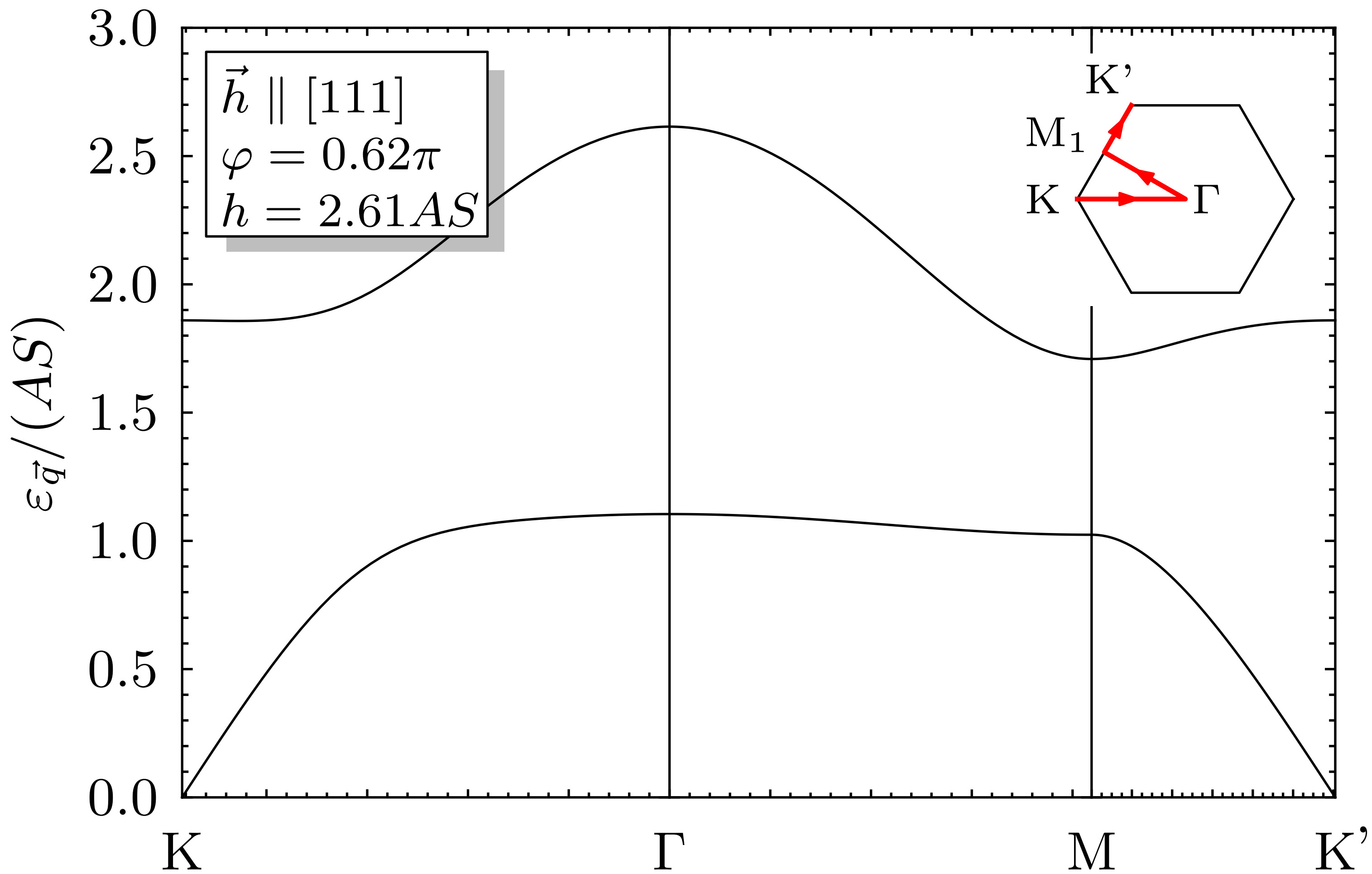
Zero field: $\vec{S}_i$ along cubic axes

... "cubic-axes zigzag"

$\Rightarrow$ simple canting impossible!

... canted zigzag will compete with other states

Spin-wave spectrum in high-field phase:



... magnon gap closes at **K** points!

... different from zigzag ordering wavevector

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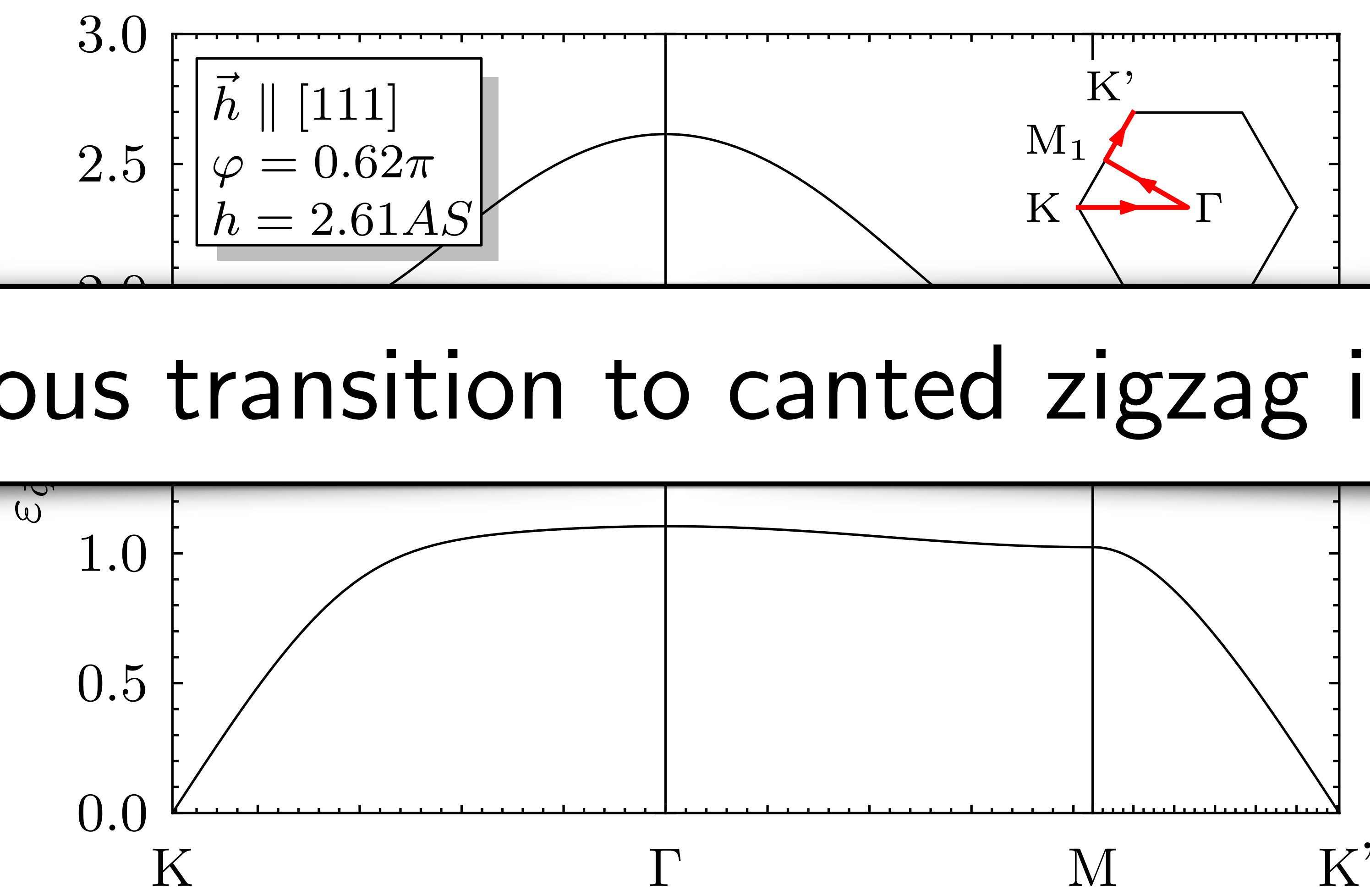


✘ Continuous transition to canted zigzag impossible!

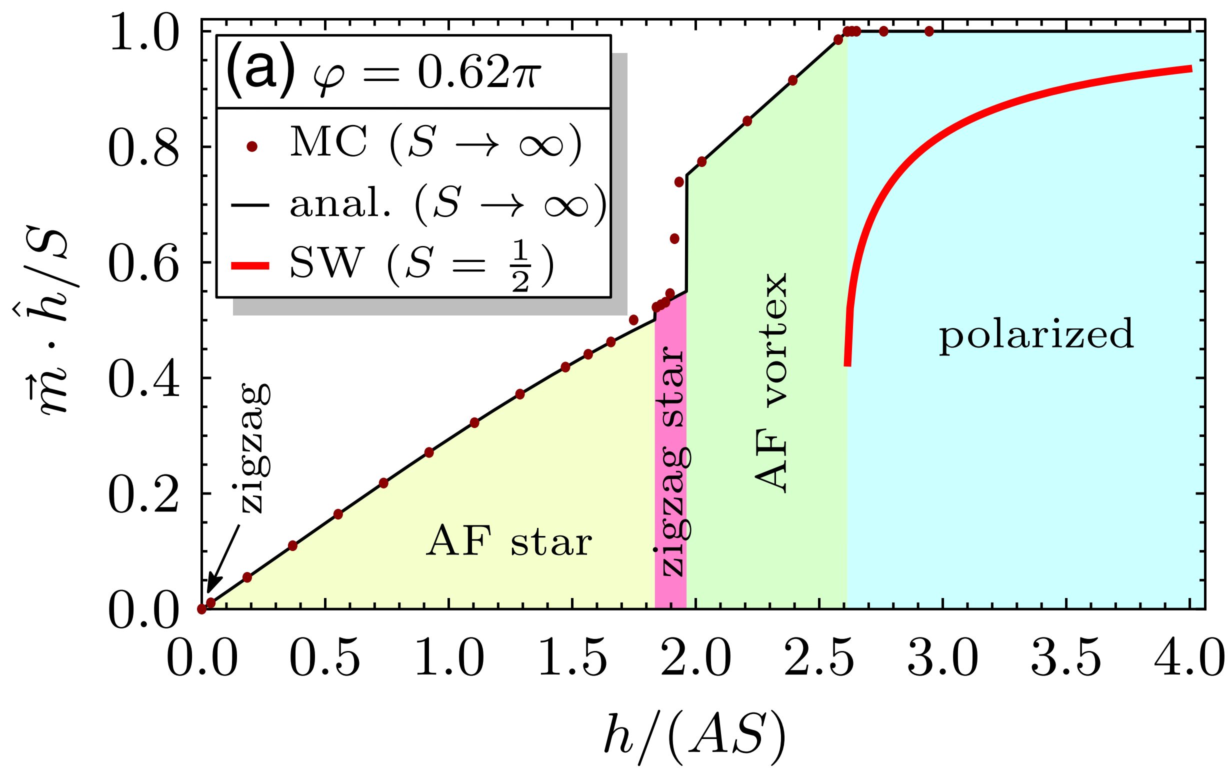
... magnon gap closes at **K** points!

... different from zigzag ordering wavevector

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

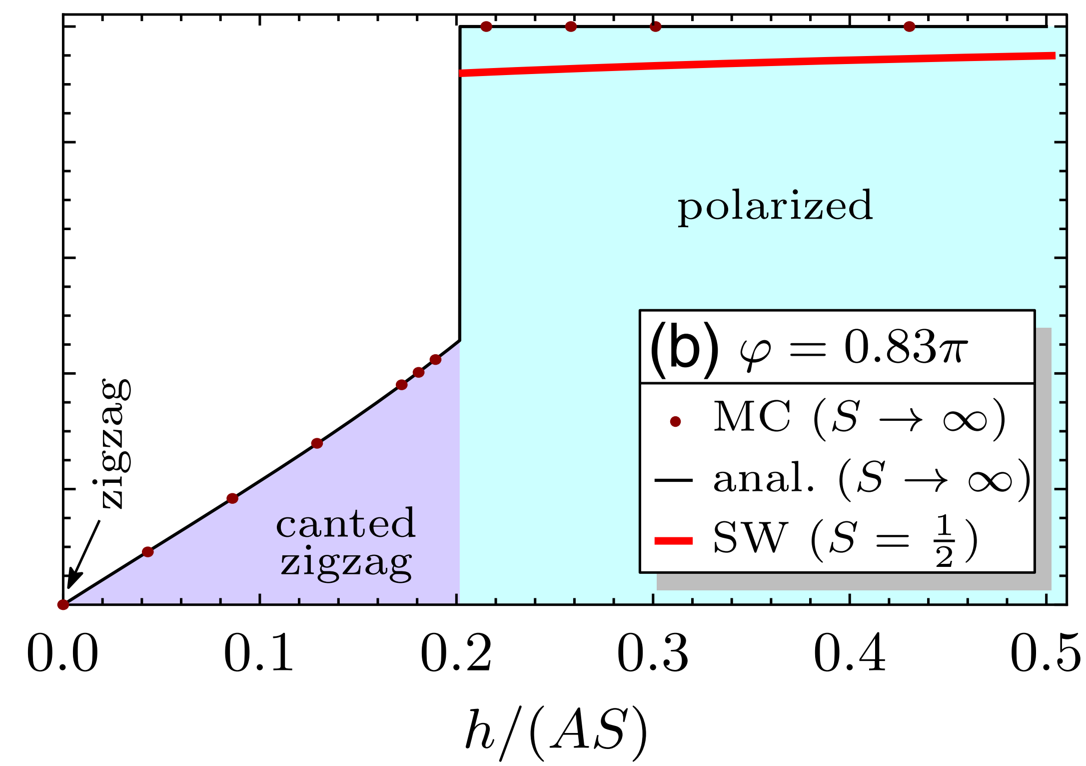
$K_1 > -2J_1$:

intermediate phases



$-2J_1 > K_1 > -J_1$:

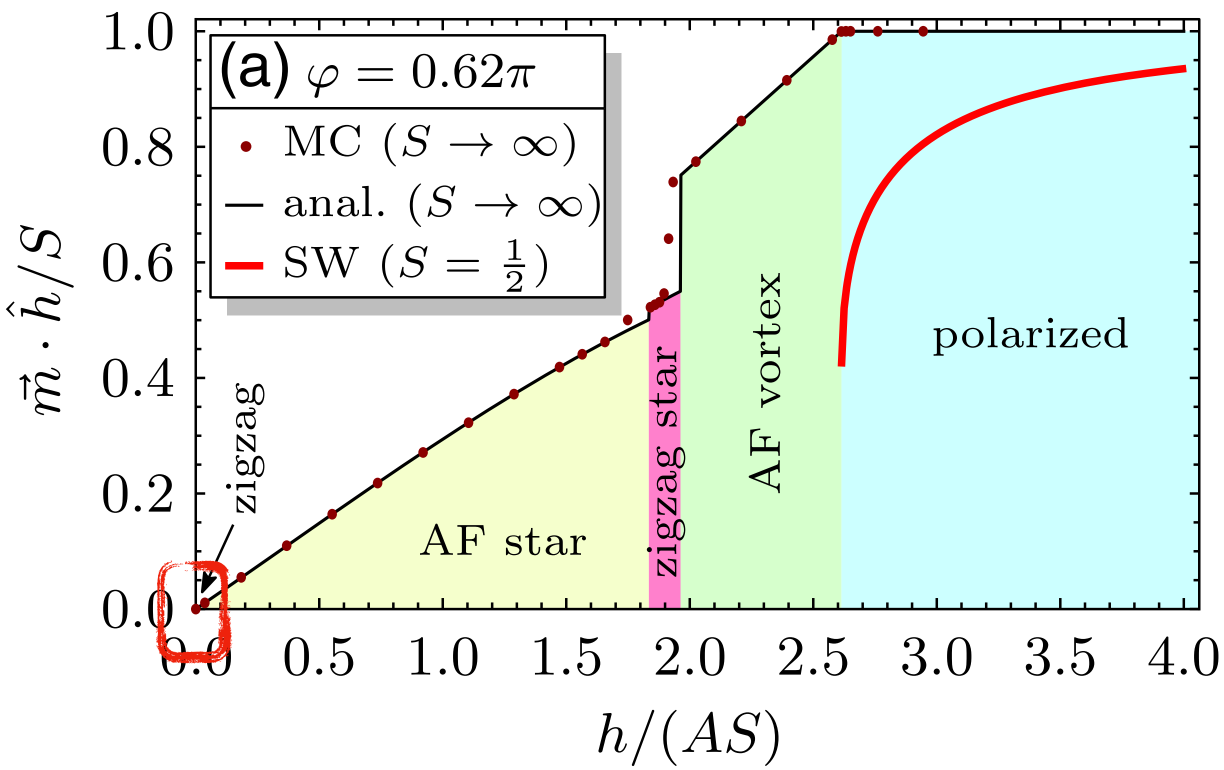
first-order transition



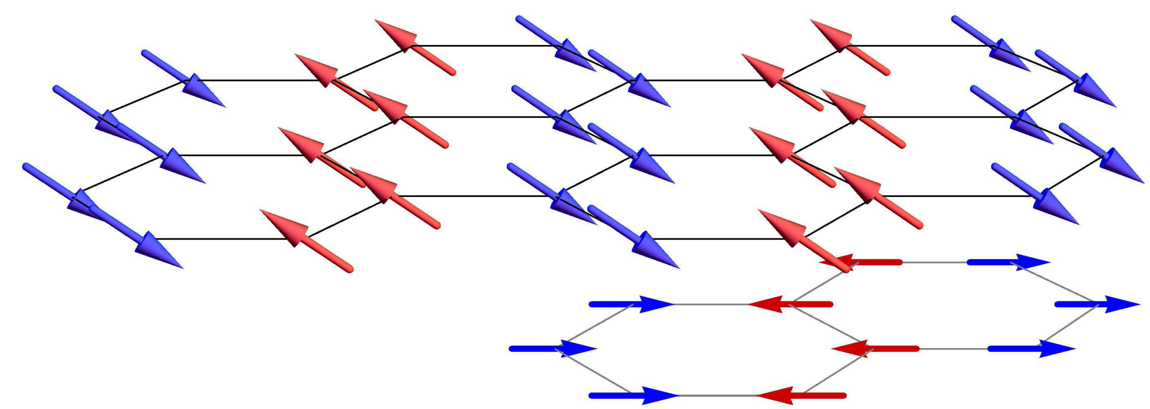
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

$K_1 > -2J_1$:

intermediate phases



$h = 0$:

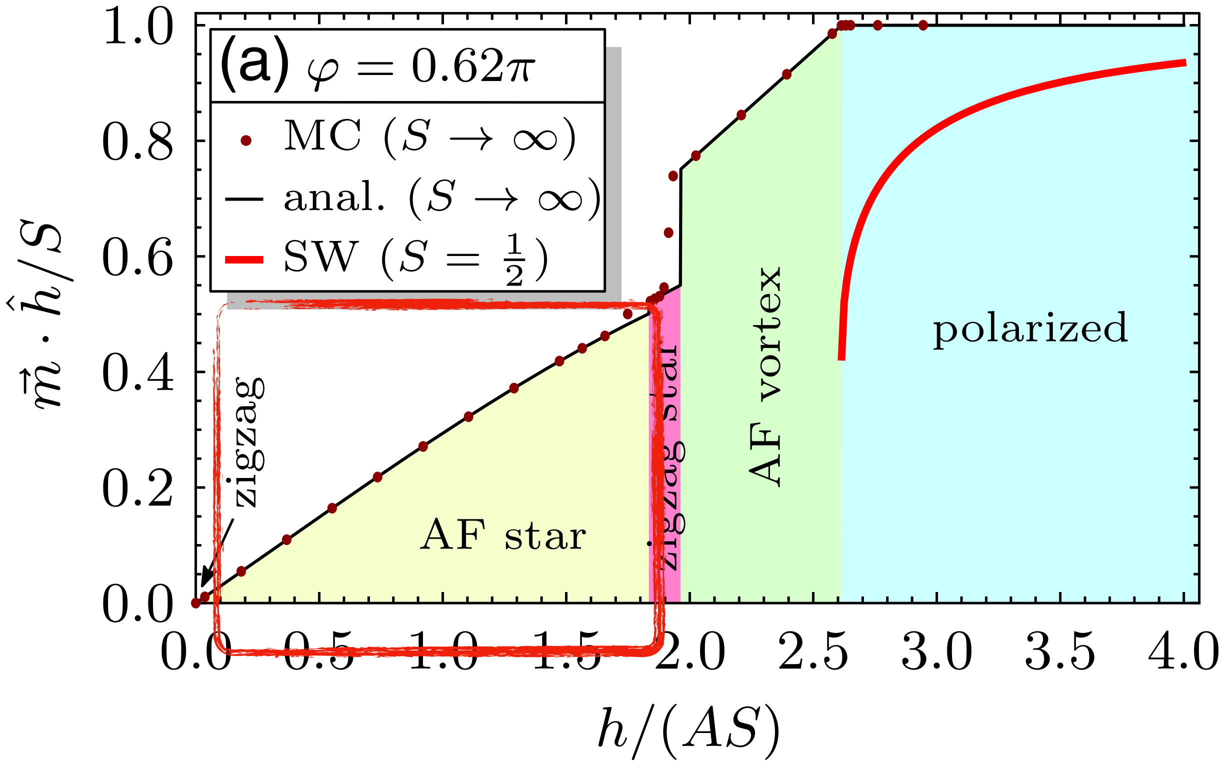


“zigzag”

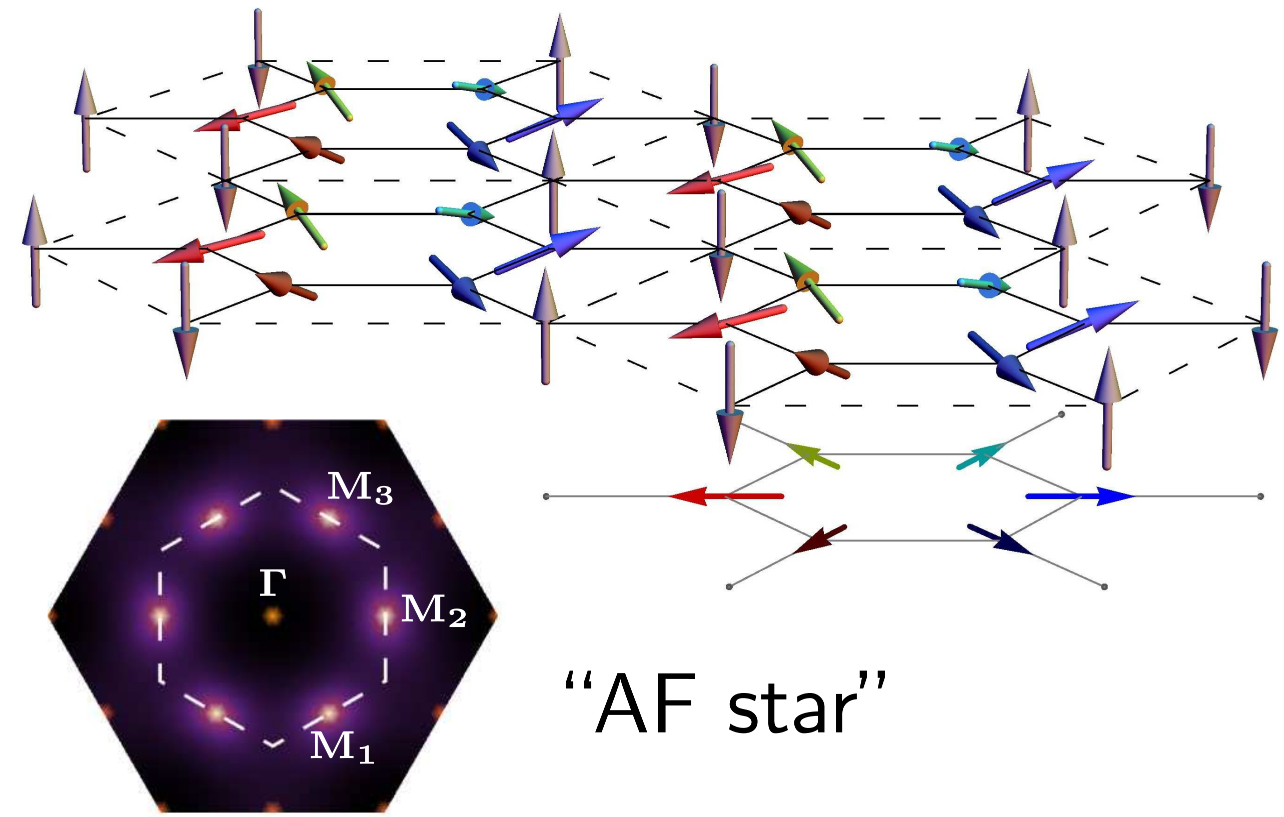
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

$K_1 > -2J_1$:

intermediate phases



$0 < h/(AS) < 1.8$:

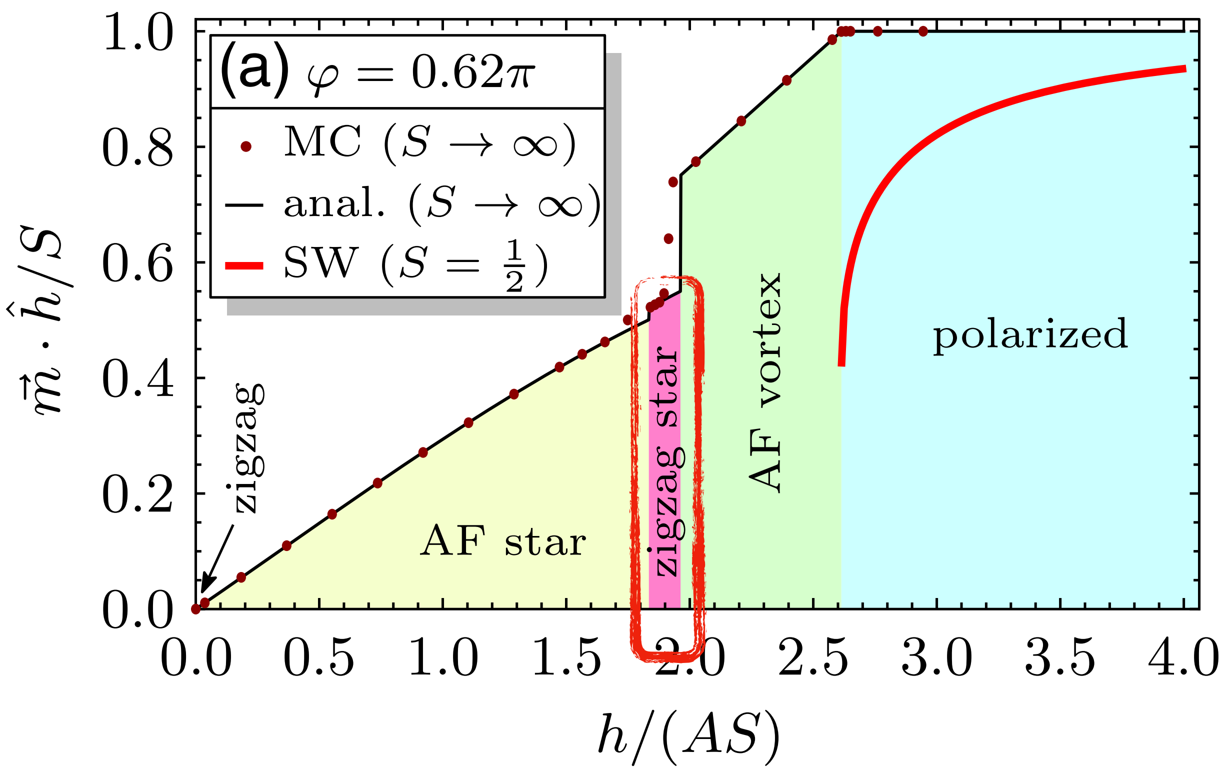


"AF star"

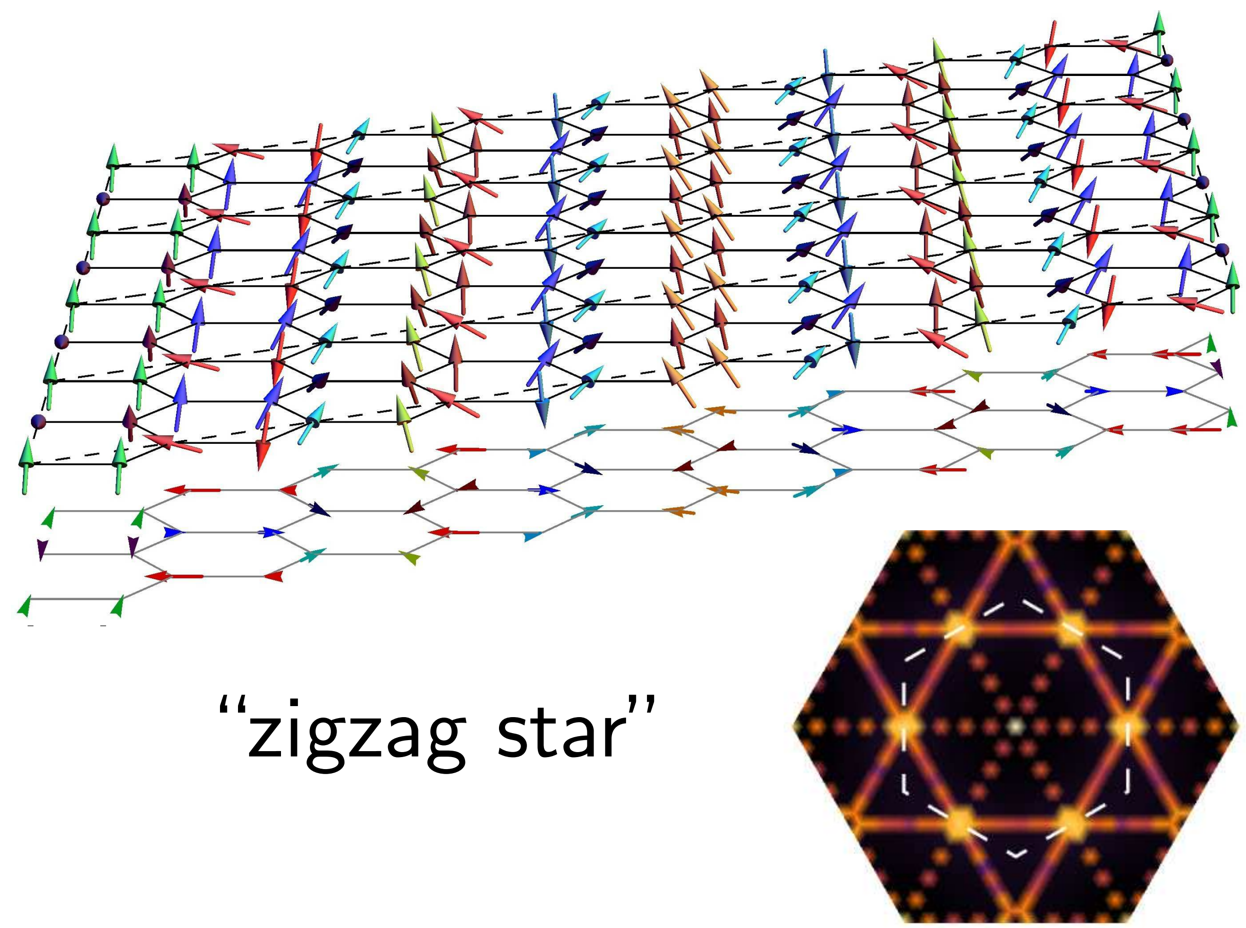
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

$K_1 > -2J_1$:

intermediate phases



$1.8 < h/(AS) < 2.0$:



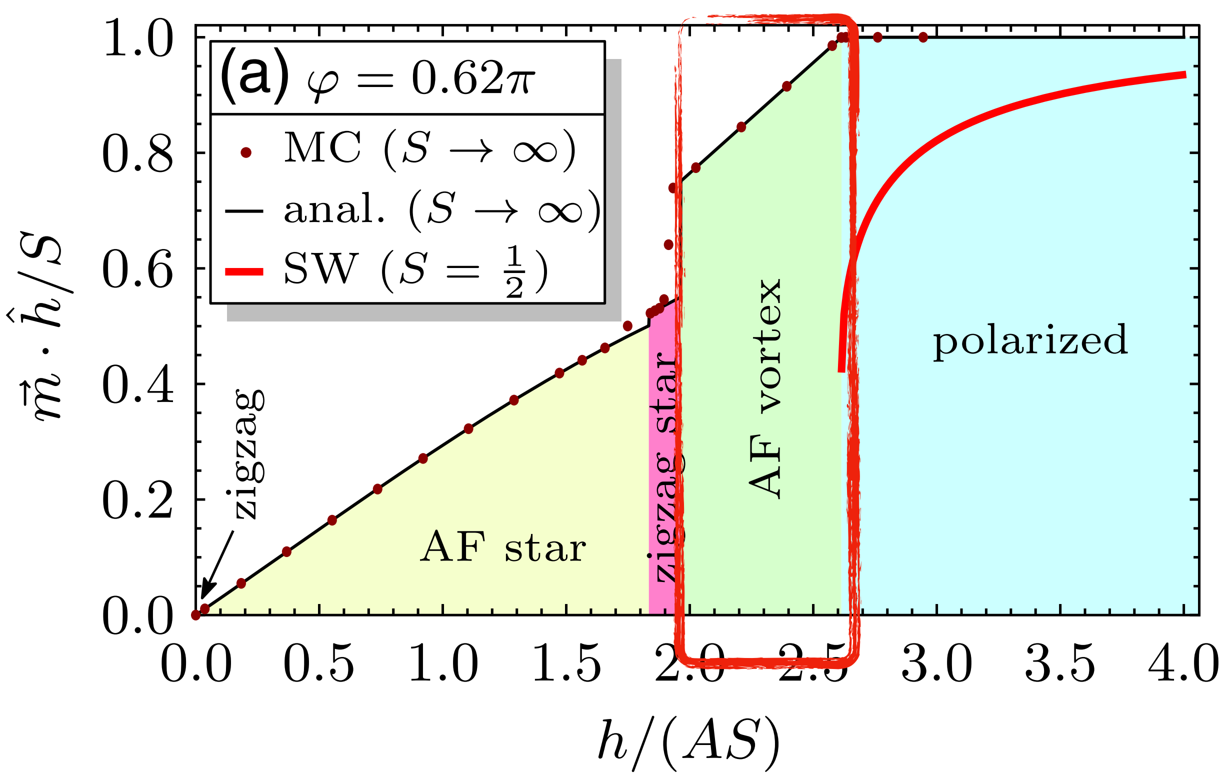
"zigzag star"

... cf. [Chern *et al.*, PRB '17]

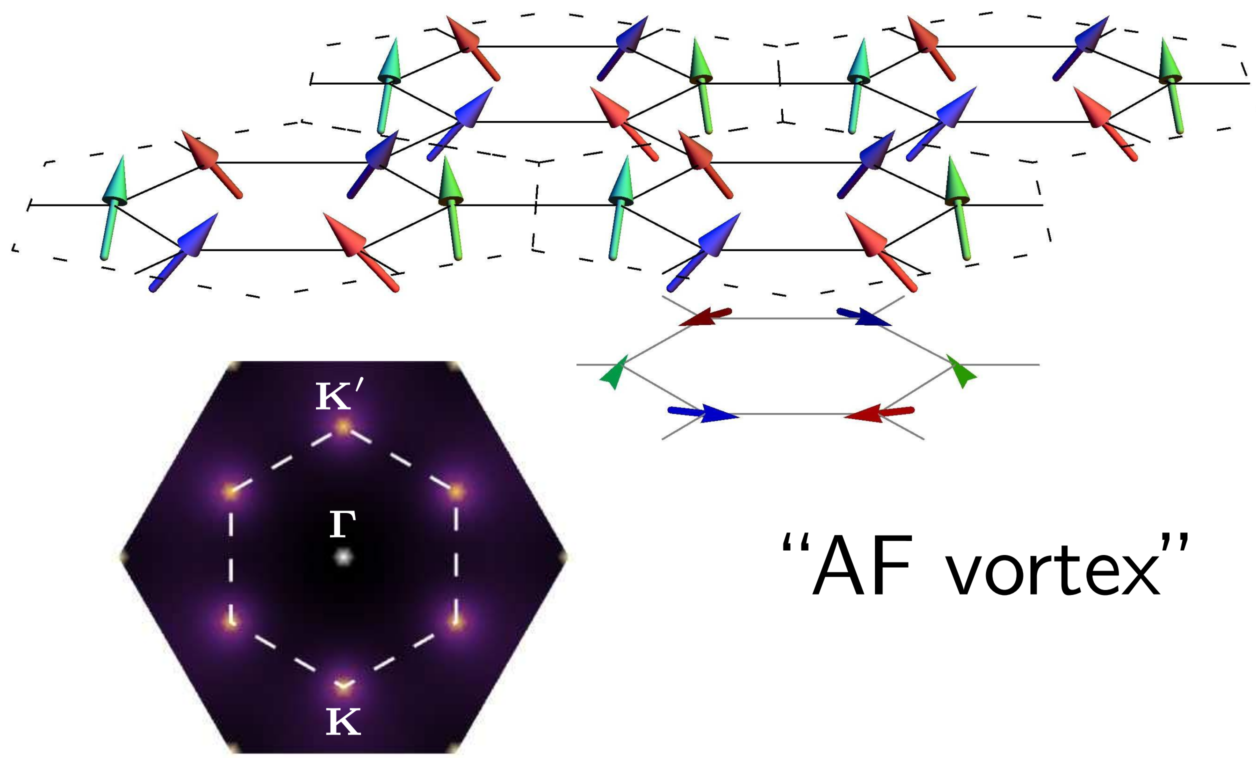
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

$K_1 > -2J_1$:

intermediate phases



$2.0 < h/(AS) < 2.6$:



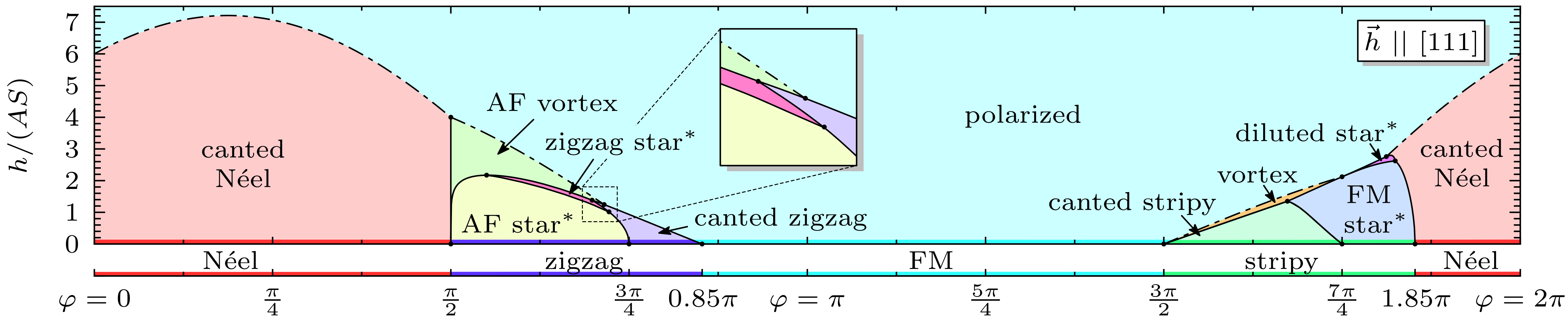
"AF vortex"

... consistent with spin-wave analysis

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

Classical phase diagram in [111] field:

[LJ, Andrade, Vojta, PRL '16]



$$J_1 = A\cos\varphi$$

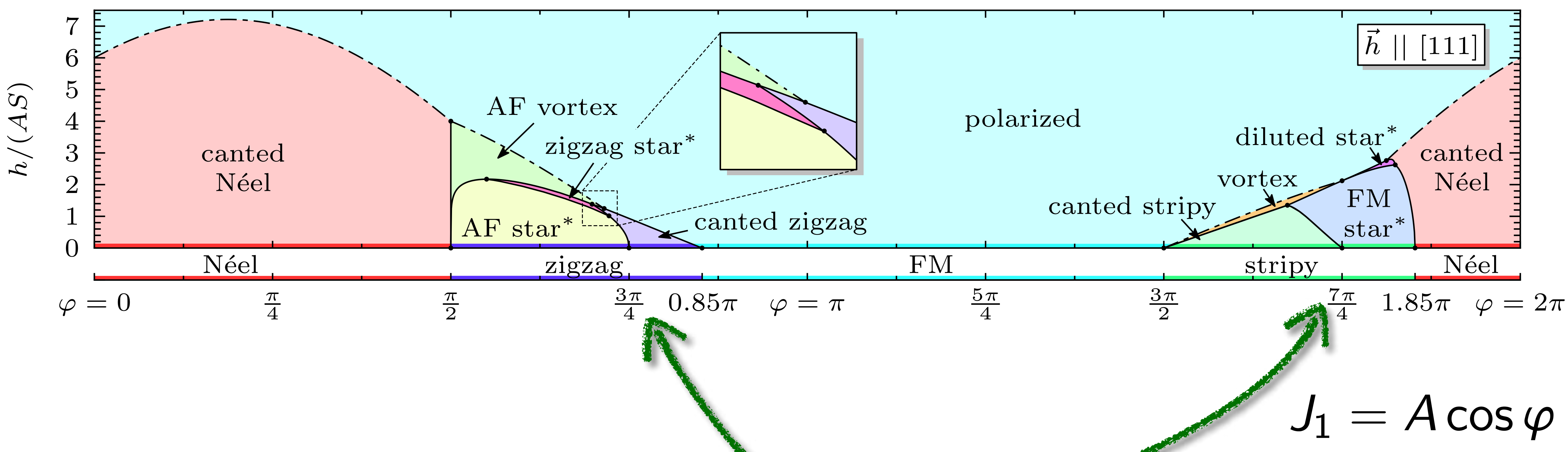
$$K_1 = 2A\sin\varphi$$

... similarly complicated phase diagram for [11$\bar{2}$] direction (a axis)

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

Classical phase diagram in [111] field:

[LJ, Andrade, Vojta, PRL '16]



$J_1 = A \cos \varphi$

$K_1 = 2A \sin \varphi$

zigzag/stripy & "star"

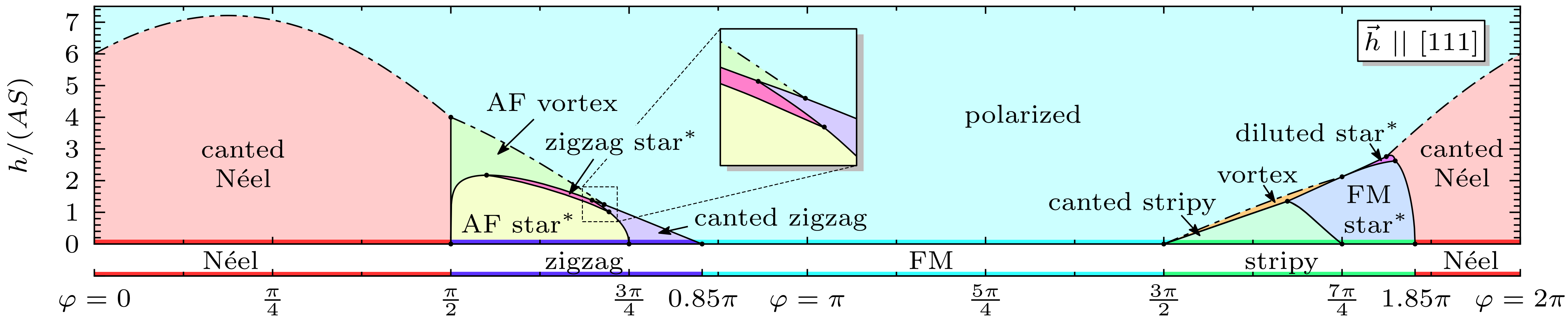
degenerate @ "Klein" points

$\Rightarrow$ intermediate phases also for $S = 1/2$

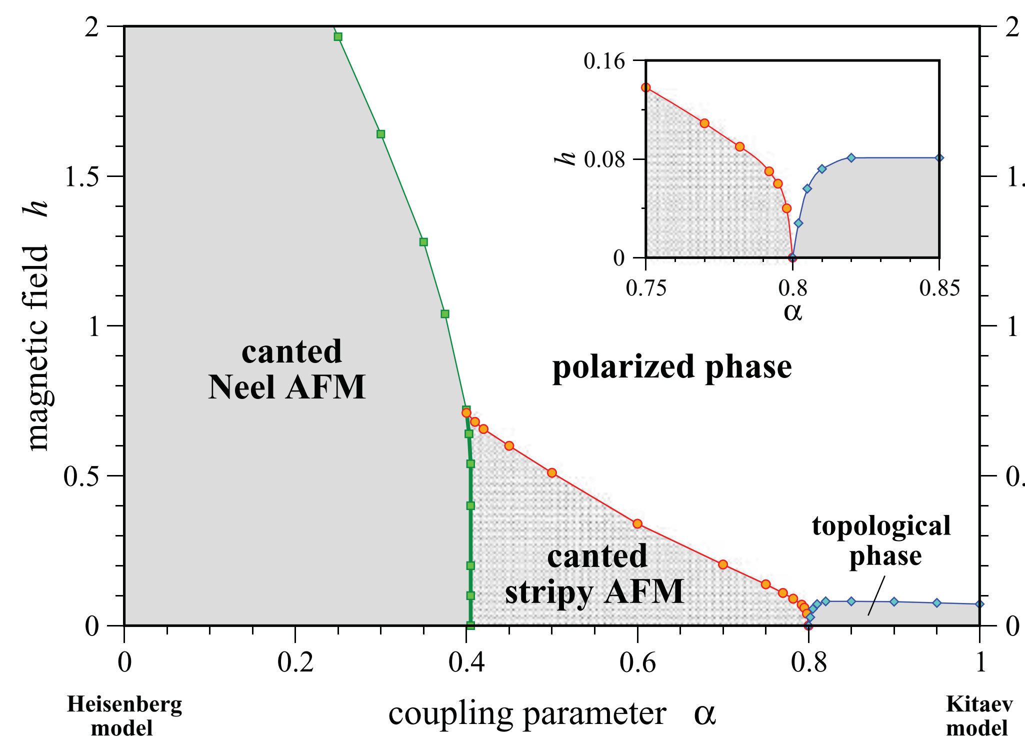
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

Classical phase diagram in [111] field:

[LJ, Andrade, Vojta, PRL '16]



DMRG:



$J_1 = 1 - \alpha$

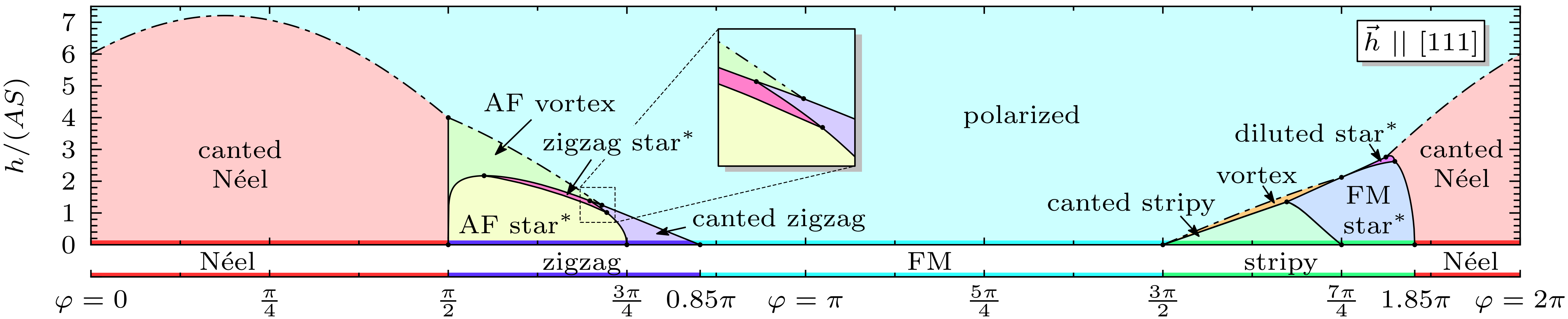
$K_1 = -2\alpha$

[Jiang, Gu, Qi, Trebst, PRB '11]

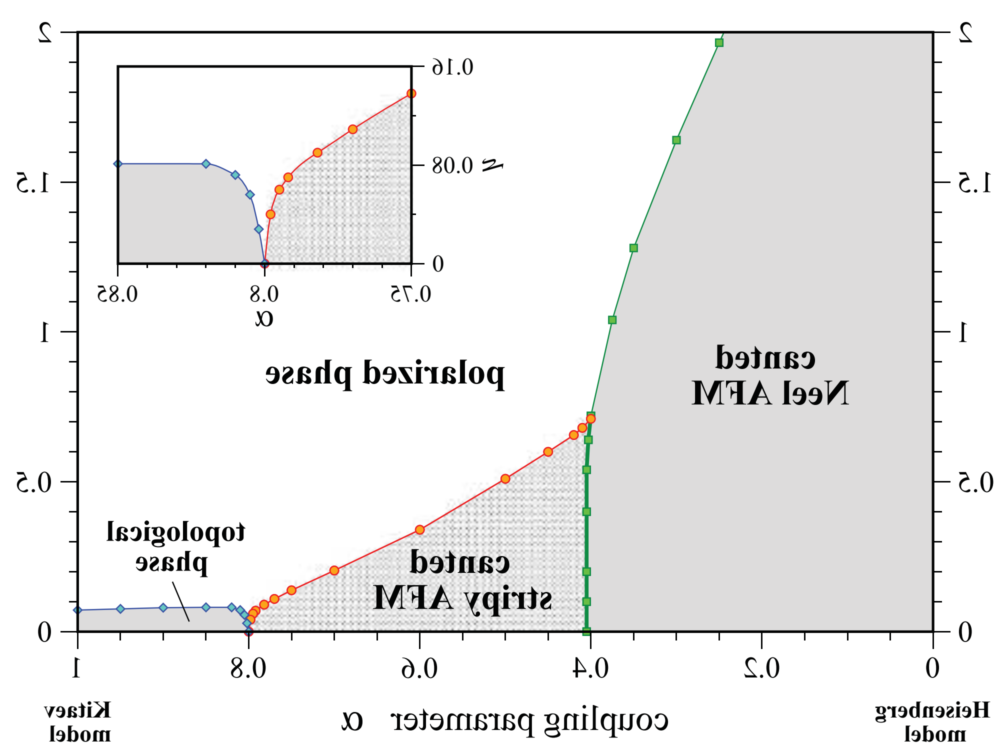
Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

Classical phase diagram in [111] field:

[LJ, Andrade, Vojta, PRL '16]



DMRG:



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[Jiang, Gu, Qi, Trebst, PRB '11]

Scenario 1 ($K_1 > 0$, $J_1 < 0$) in field: $\vec{h} \parallel [111] \parallel c^*$

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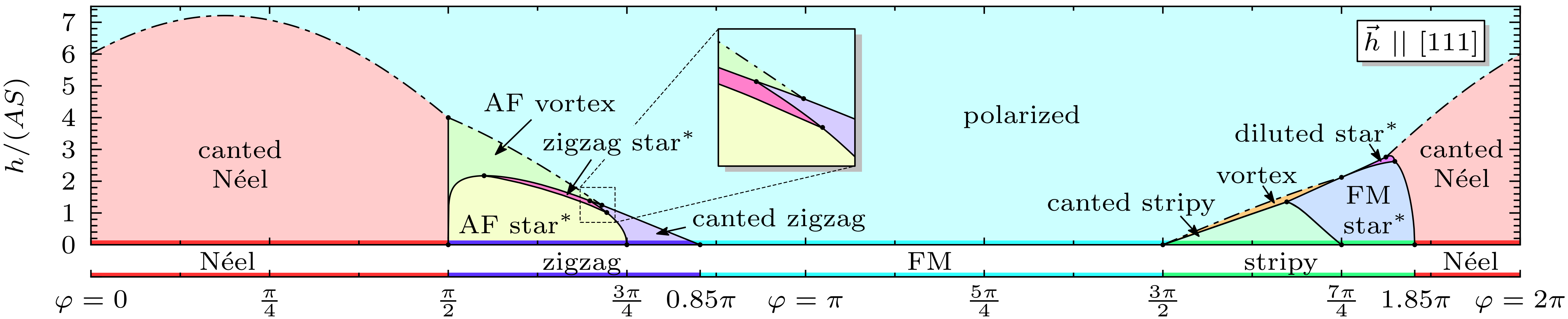
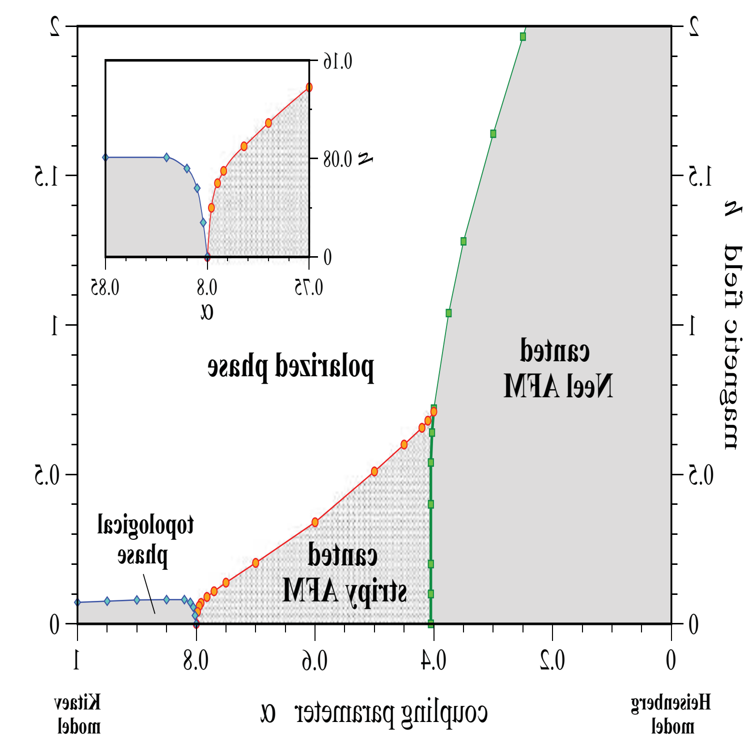


DMRG:



[Jiang, Gu, Qi, Trebst, PRB '11]

Summary: Scenario 1 ($K_1 > 0$, $J_1 < 0$)

Magnetization process ...

... strongly anisotropic

... but only weak angle dependence of susceptibility and h_c

... does not explain anisotropy in α-$RuCl_3$

... typically no simple canting

... unless field along high-symmetry direction

... exotic intermediate phases

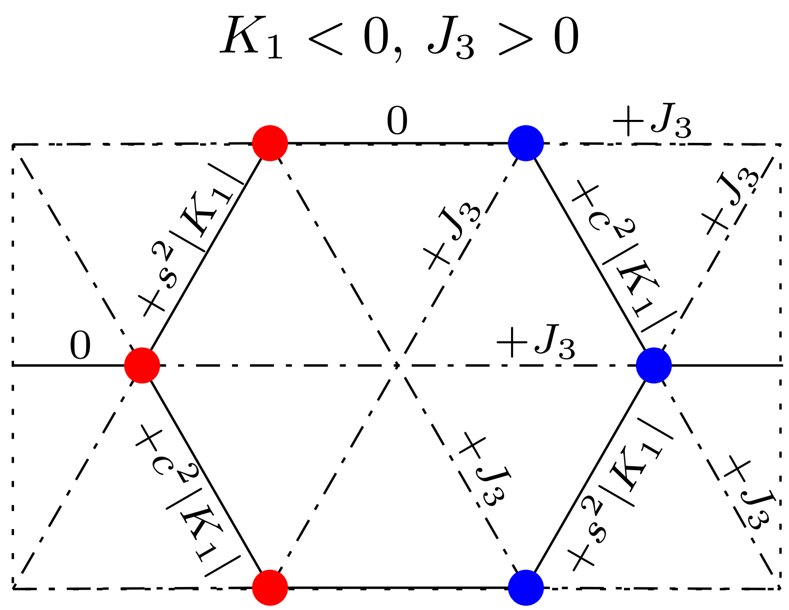
... large unit cell, multi-$\vec{Q}$, vortex

Scenario 2: Ferromagnetic K_1, antiferromagnetic J_3

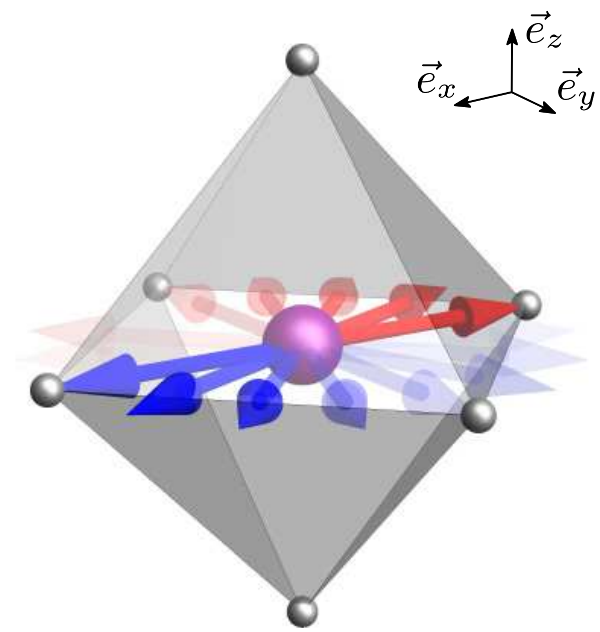
Minimal model: [Winter *et al.*, PRB '16]

$$\mathcal{H} = \sum_{\langle ij \rangle} K_1 S_i^\gamma S_j^\gamma + \sum_{\langle\langle\langle ij \rangle\rangle\rangle} J_3 \vec{S}_i \cdot \vec{S}_j$$

Zero field:



"cubic-planes zigzag"



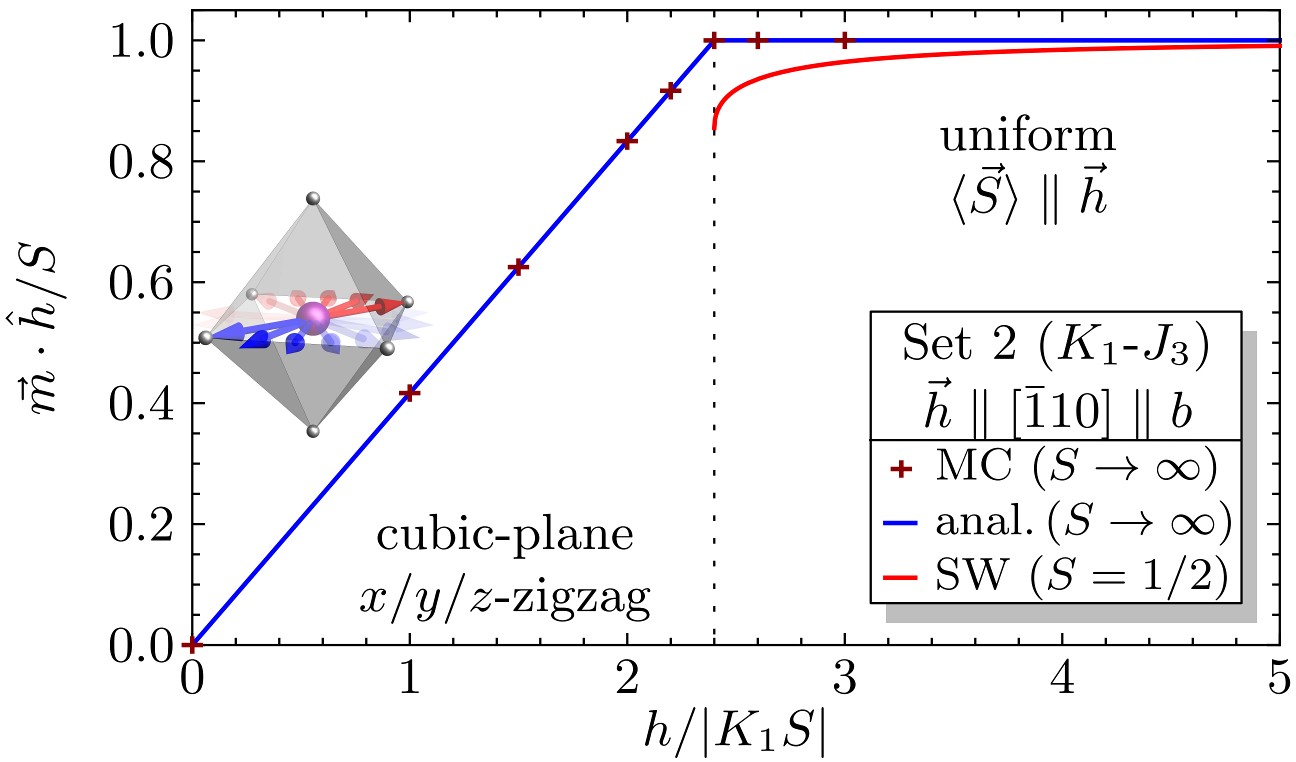
$\vec{S} \parallel \pm(c\vec{e}_x + s\vec{e}_y), \quad c^2 + s^2 = 1$

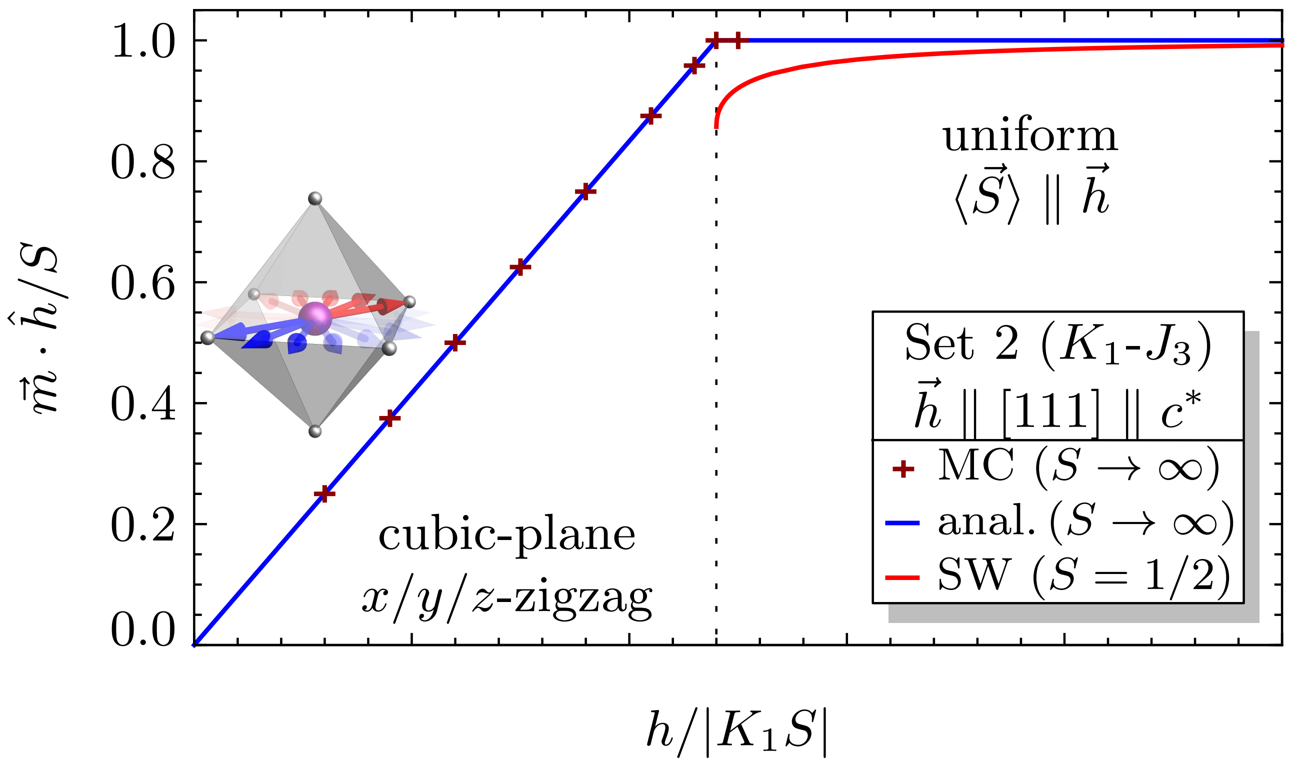
... i.e., within Ru_2Cl_2 planes

... degeneracy will be lifted in real materials

Scenario 2 ($K_1 < 0$, $J_3 > 0$) in field

For $\Gamma_1 = 0$:





(K_1, J_3) = (-17, +6.8) meV

[Winter *et al.*, PRB '16]

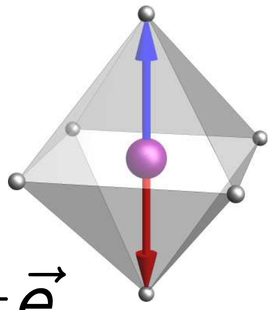
... classical magnetization curve independent of field direction!

... and angle dependence of quantum corrections very weak

Scenario 2 ($K_1 < 0$, $J_3 > 0$) in field: Perturbations

$\mathbb{Z}_3 \times O(2)$ degeneracy lifted by:

(i) quantum fluctuations:



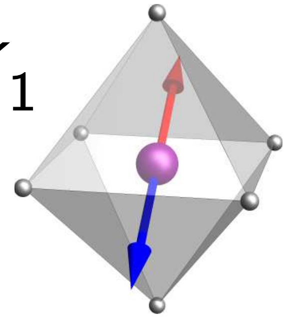
[Sizyuk *et al.*, PRB '16]
[Chaloupka *et al.*, PRB '16]

$$\vec{S}_i \parallel \pm\vec{e}_x, \pm\vec{e}_y, \pm\vec{e}_z$$

"cubic-axes zigzag"

(ii) off-diagonal $\Gamma_1 > 0$:

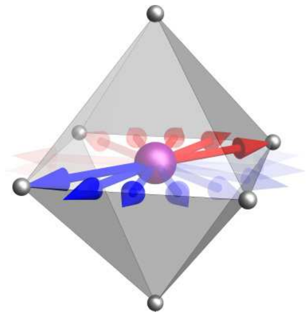
$$\vec{S}_i \parallel \begin{cases} \pm[110] \text{ or symmetry-equivalent,} & \text{if } 0 < \Gamma_1 \ll K_1 \\ \pm[11\bar{1}] \text{ or symmetry-equivalent,} & \text{if } \Gamma_1 \gg K_1 \end{cases}$$



"face-center zigzag"

(iii) magnetic field:

$$\vec{S}_i \perp \vec{h}$$



"cubic-plane zigzag"

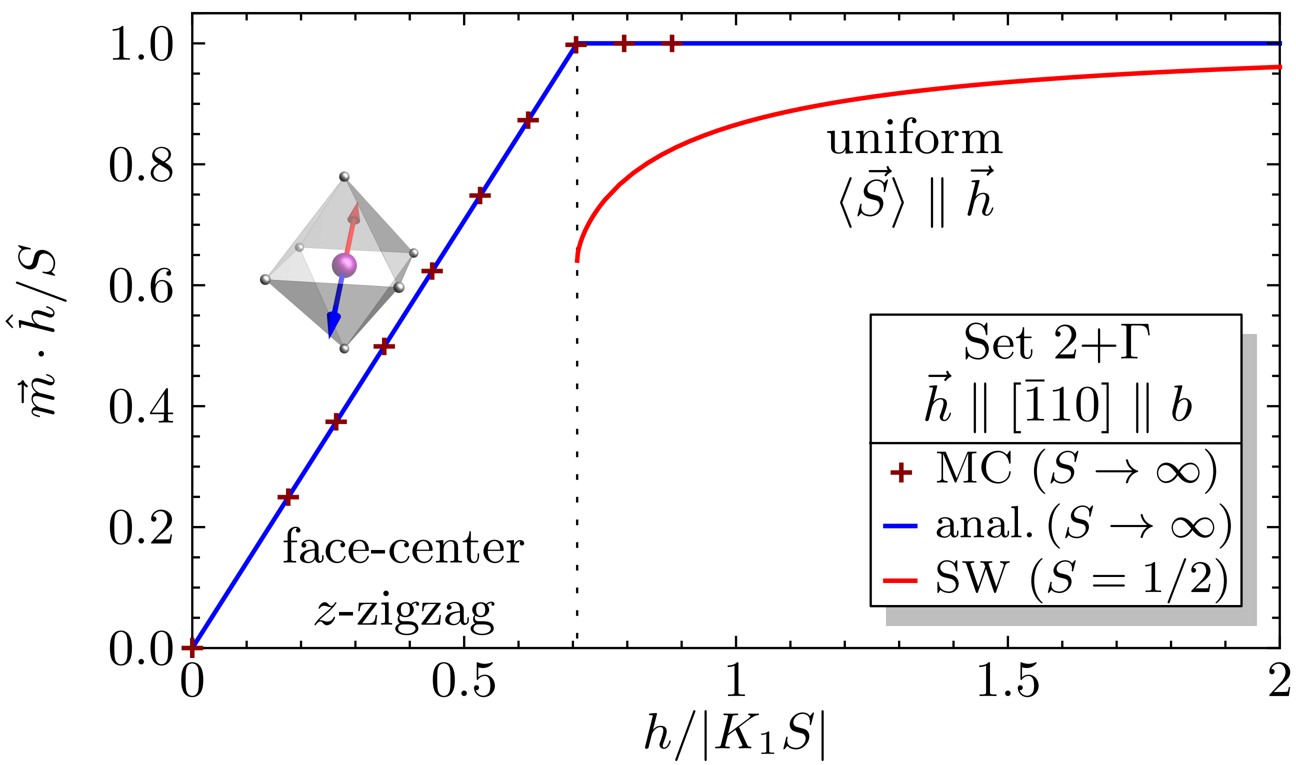
$\Rightarrow$ competition between different effects

... can be tuned by h

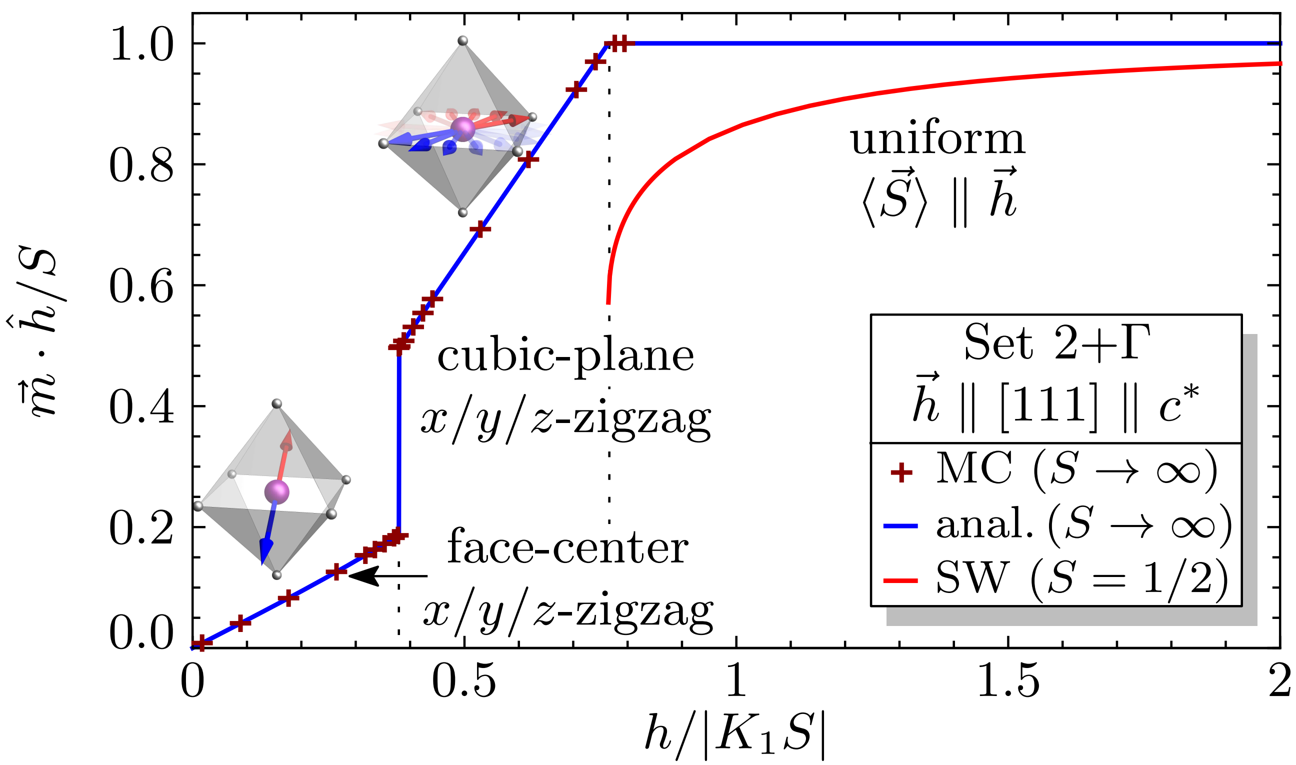
Scenario 2 ($K_1 < 0$, $J_3 > 0$) in field: Perturbations

Metamagnetic transitions between canted zigzag states:

$\vec{h} \parallel [\bar{1}10] \parallel b$



$\vec{h} \parallel [111] \parallel c^*$



(J_1, K_1, Γ_1, J_2, K_2, J_3) = (+3, -17, +1, -3, +6, +1) meV

[Sizyuk *et al.*, PRB '16]

Summary: Scenario 2 ($K_1 < 0$, $J_3 > 0$)

Magnetization process ...

... mostly isotropic

... very weak angle dependence of susceptibility and h_c

... does not explain anisotropy in α-$RuCl_3$

... transitions between different canted zigzag states possible

... for some field directions

... no exotic intermediate phases

... high-field transition continuous

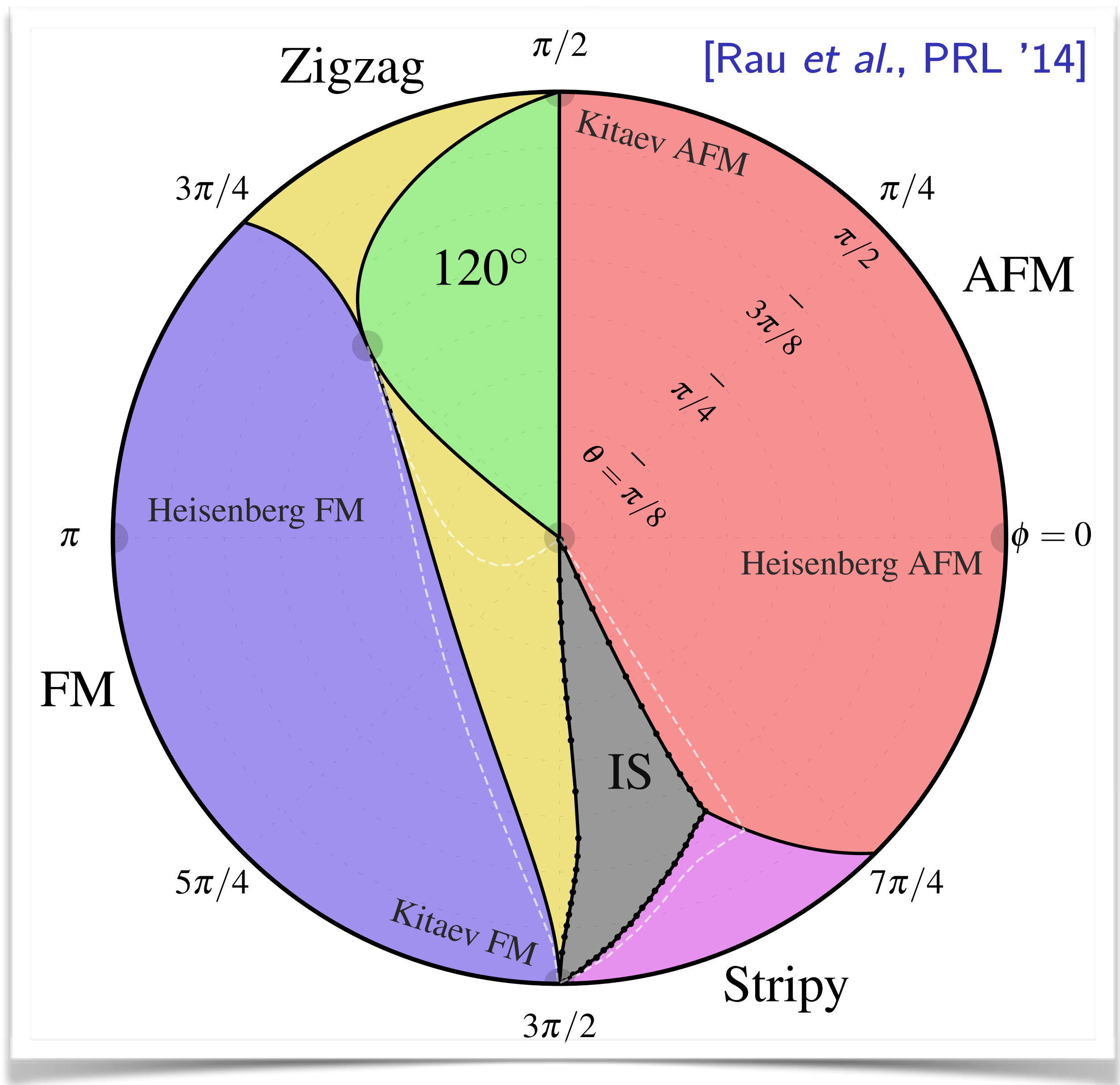
Scenario 3: Ferromagnetic K_1, positive Γ_1

Minimal model:

[Ran *et al.*, PRL '17]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

Zero-field phase diagram?



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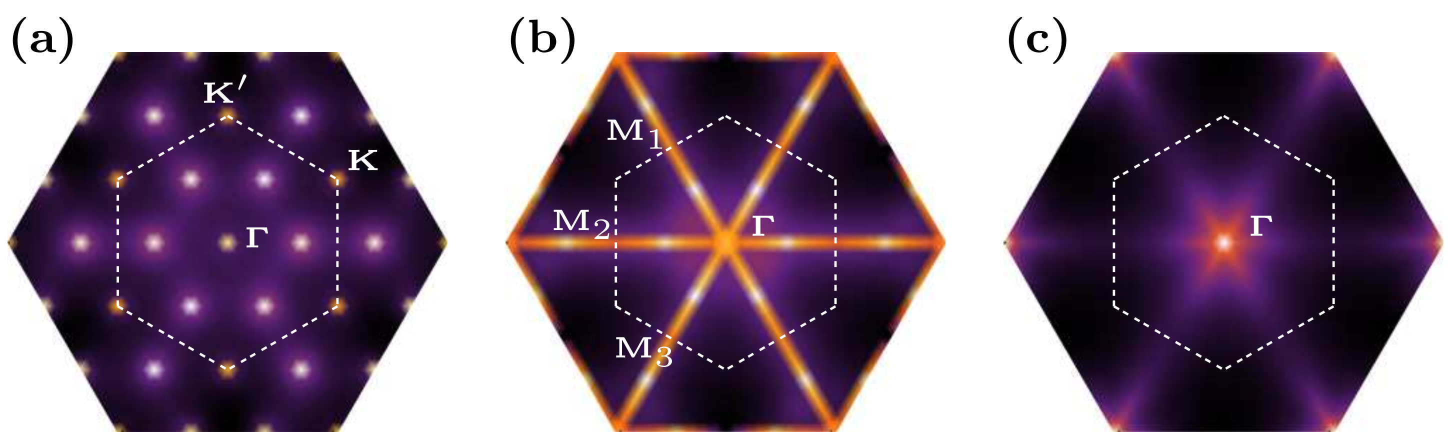
Zero-field phase diagram?

(K_1, Γ_1) = (-6.8, +9.5) meV

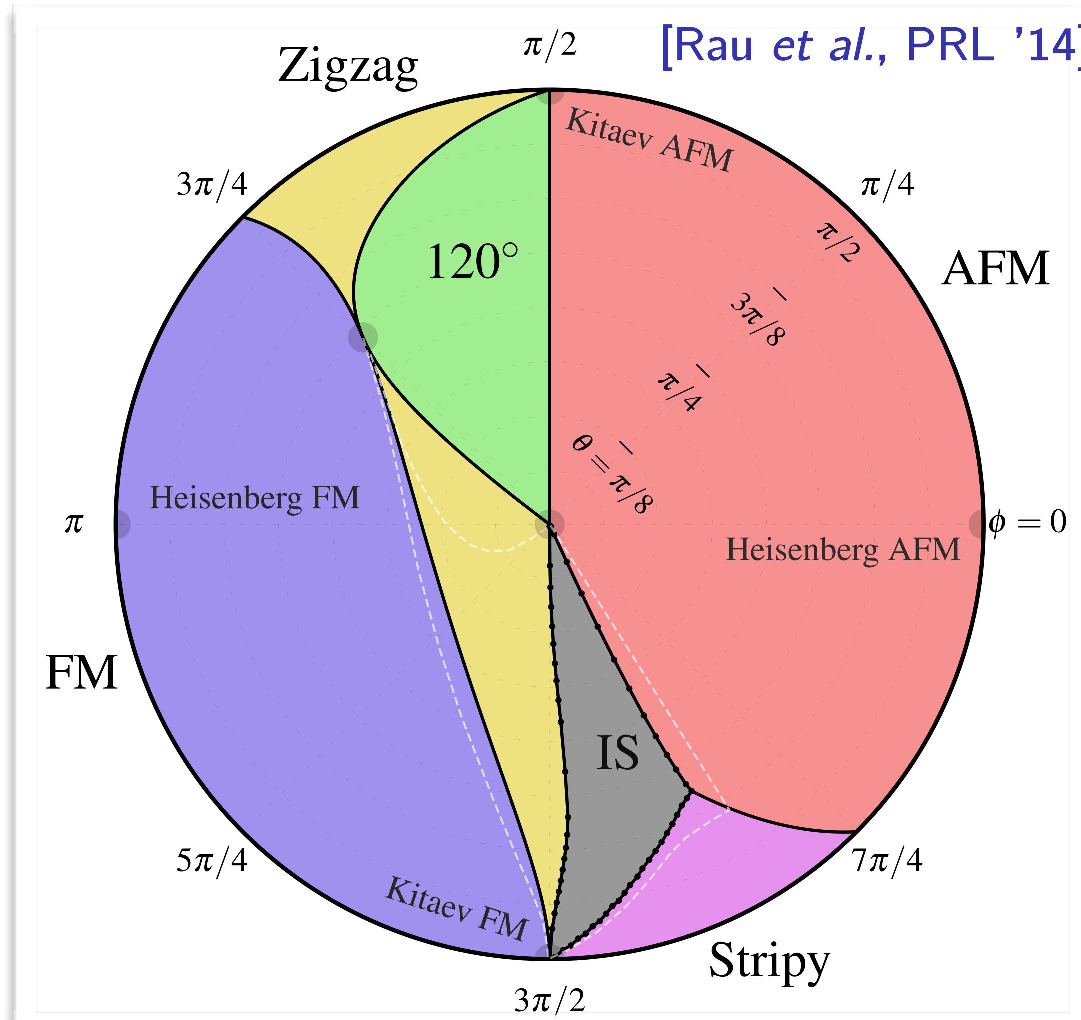
[Ran *et al.*, PRL '17]



$0 \leq \frac{-J_1}{|K_1|} \lesssim 0.25$ — multi-$\vec{Q}$

$0.25 \lesssim \frac{-J_1}{|K_1|} \lesssim 0.55$ — incommensurate

$0.55 \lesssim \frac{-J_1}{|K_1|}$ — ferromagnet



[Rau *et al.*, PRL '14]

... failure of Luttinger-Tisza

Scenario 3: Ferromagnetic K_1, positive Γ_1

Minimal model:

[Ran *et al.*, PRL '17]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

Zero-field phase diagram?

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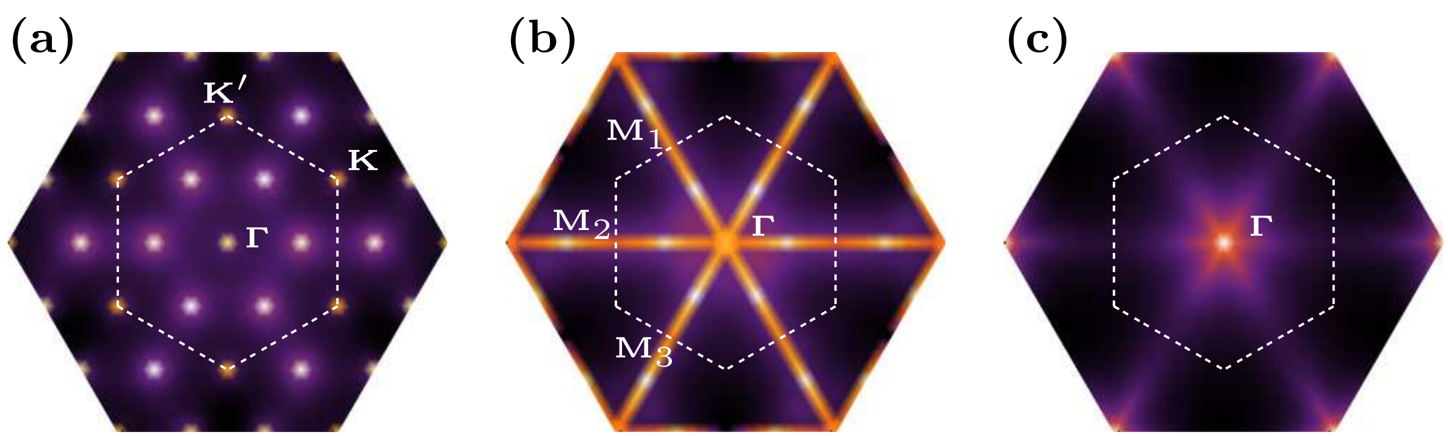
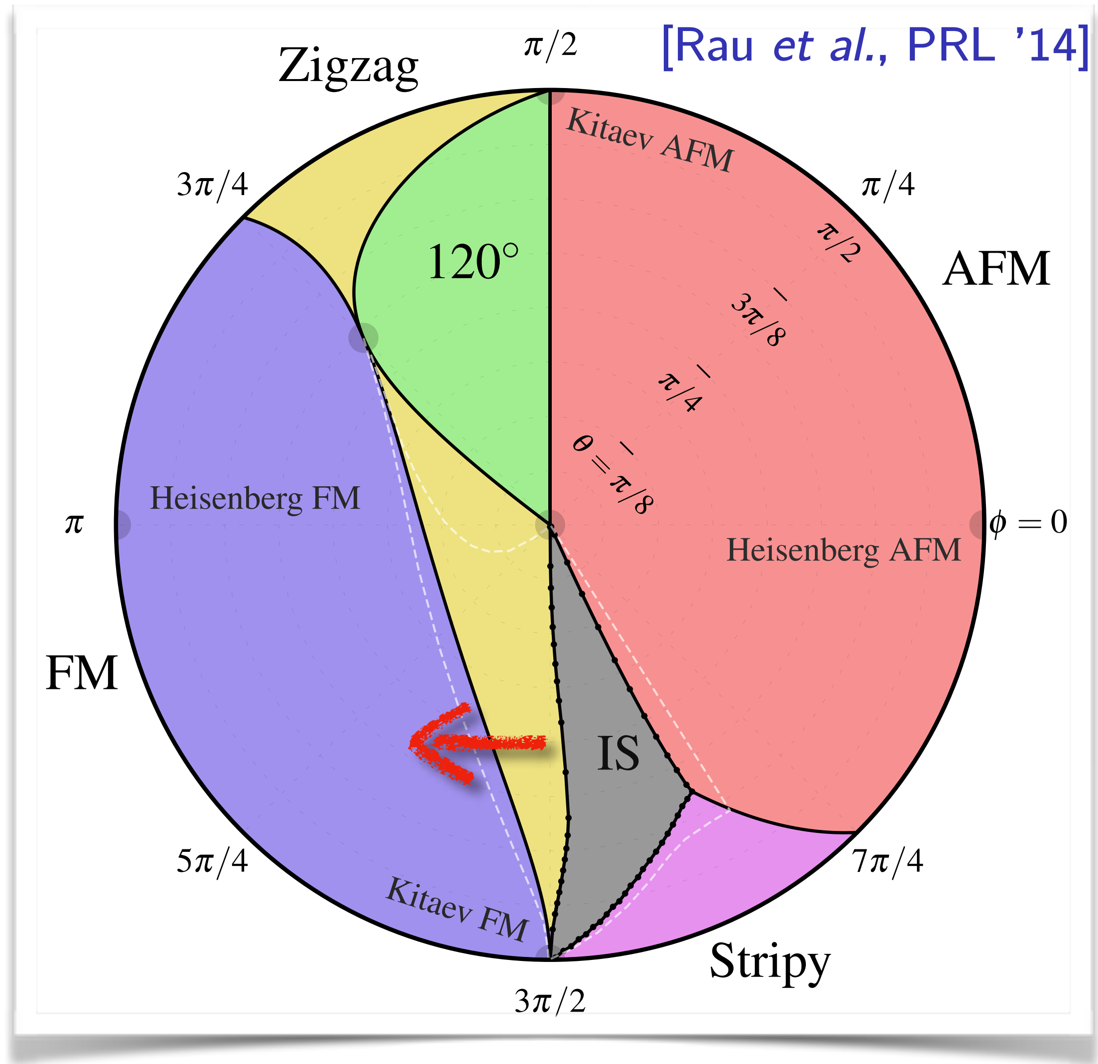


$0 \le \frac{-J_1}{|K_1|} \lesssim 0.25$ multi-$\vec{Q}$

$0.25 \lesssim \frac{-J_1}{|K_1|} \lesssim 0.55$ incommensurate

$0.55 \lesssim \frac{-J_1}{|K_1|}$ ferromagnet



... failure of Luttinger-Tisza

Scenario 3: Ferromagnetic K_1, positive Γ_1

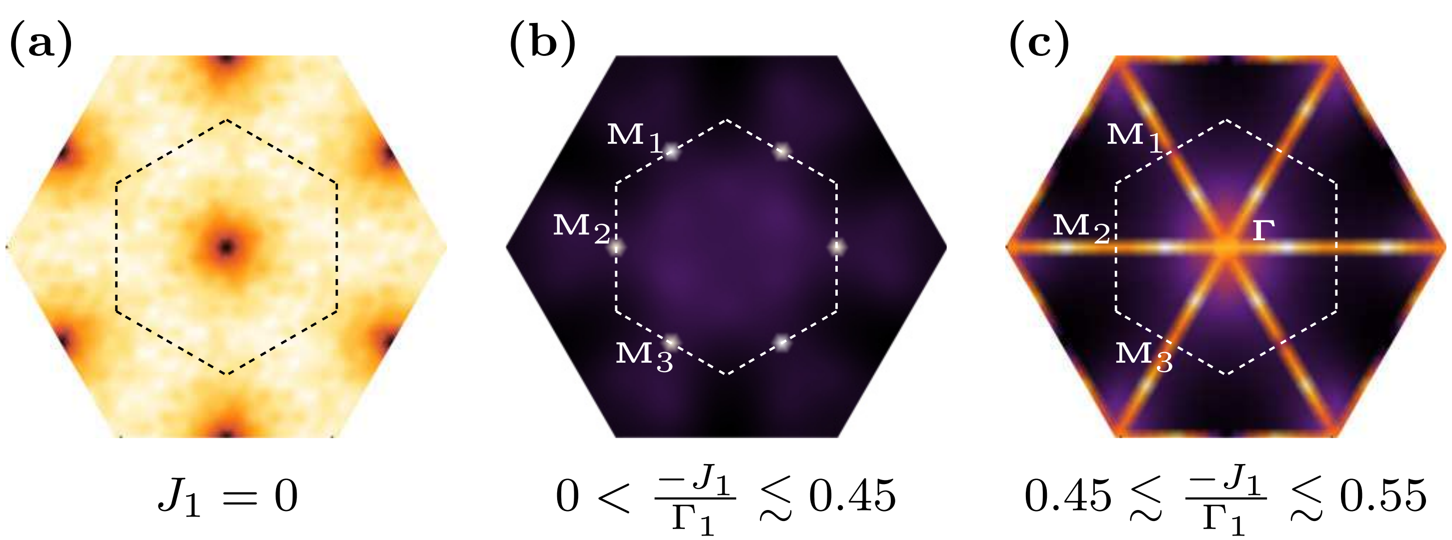
Minimal model:

[Ran *et al.*, PRL '17]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

Zero-field phase diagram?

... in J_1-Γ_1 model



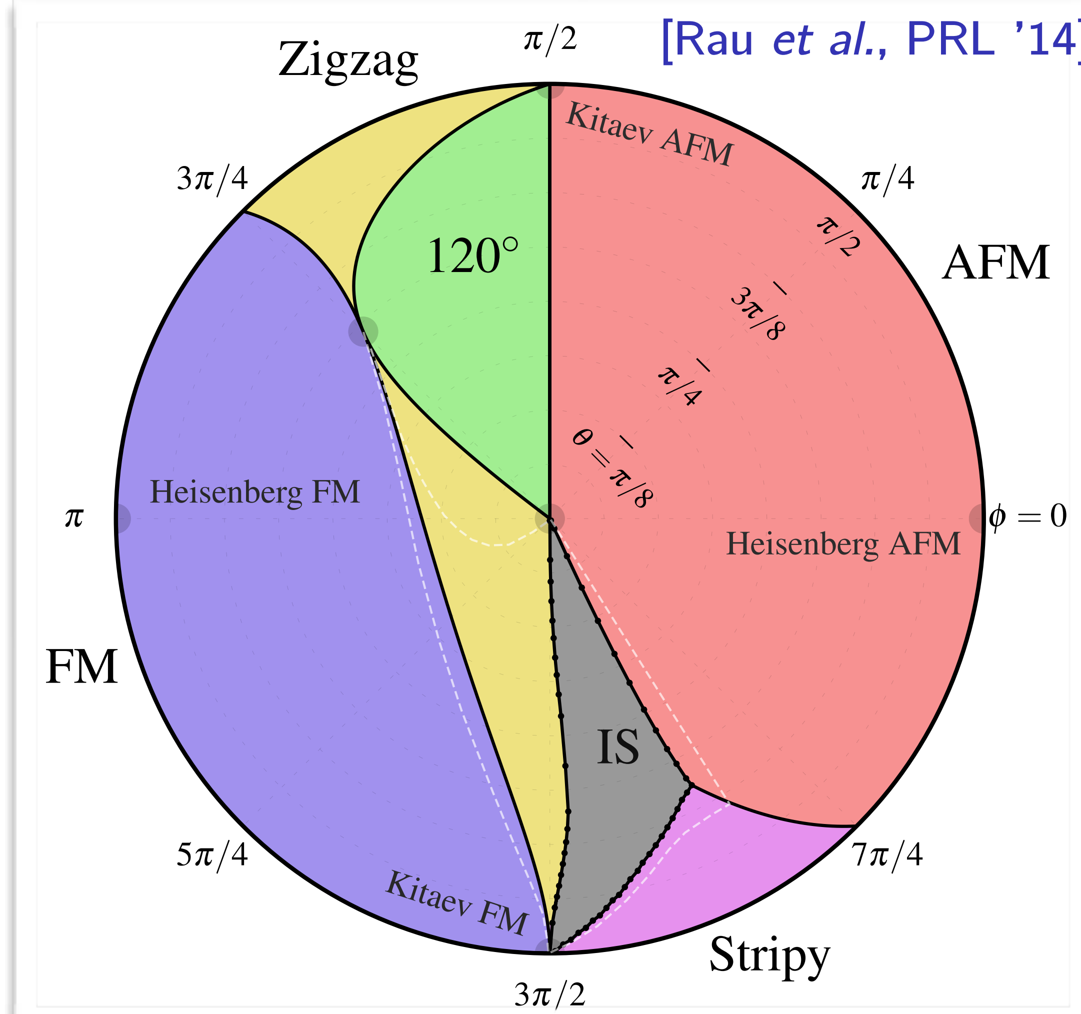
$J_1 = 0$ — classical SL

$0 < \frac{-J_1}{\Gamma_1} \lesssim 0.45$ — zigzag

$0.45 \lesssim \frac{-J_1}{\Gamma_1} \lesssim 0.55$ — incommensurate

[Rousochatzakis & Perkins, PRL '17]

[Rau *et al.*, PRL '14]



... zigzag can be recovered for $\Gamma_1 \gg |K_1|$

... failure of Luttinger-Tisza

Scenario 3: Ferromagnetic K_1, positive Γ_1

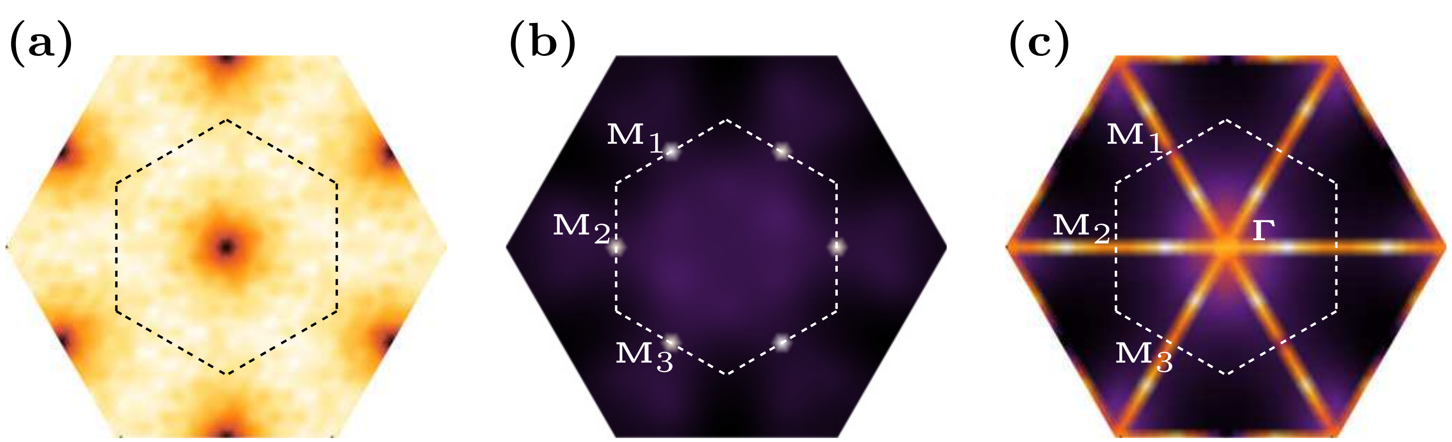
Minimal model:

[Ran *et al.*, PRL '17]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

Zero-field phase diagram?

... in J_1-Γ_1 model

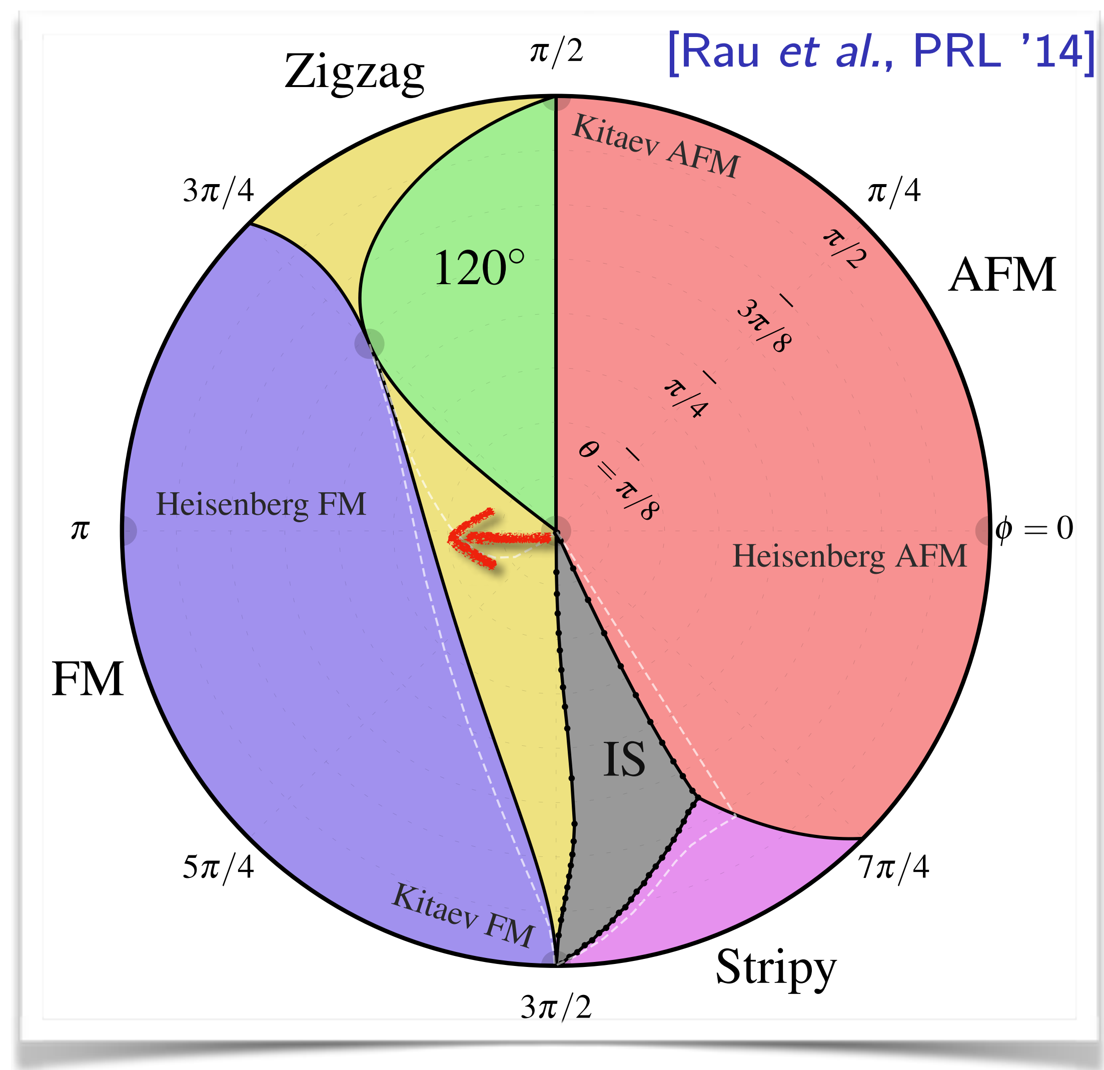


$J_1 = 0$ | $0 < \frac{-J_1}{\Gamma_1} \lesssim 0.45$ | $0.45 \lesssim \frac{-J_1}{\Gamma_1} \lesssim 0.55$

classical SL | zigzag | incommensurate

[Rousochatzakis & Perkins, PRL '17]

[Rau *et al.*, PRL '14]



... zigzag can be recovered for $\Gamma_1 \gg |K_1|$

... failure of Luttinger-Tisza

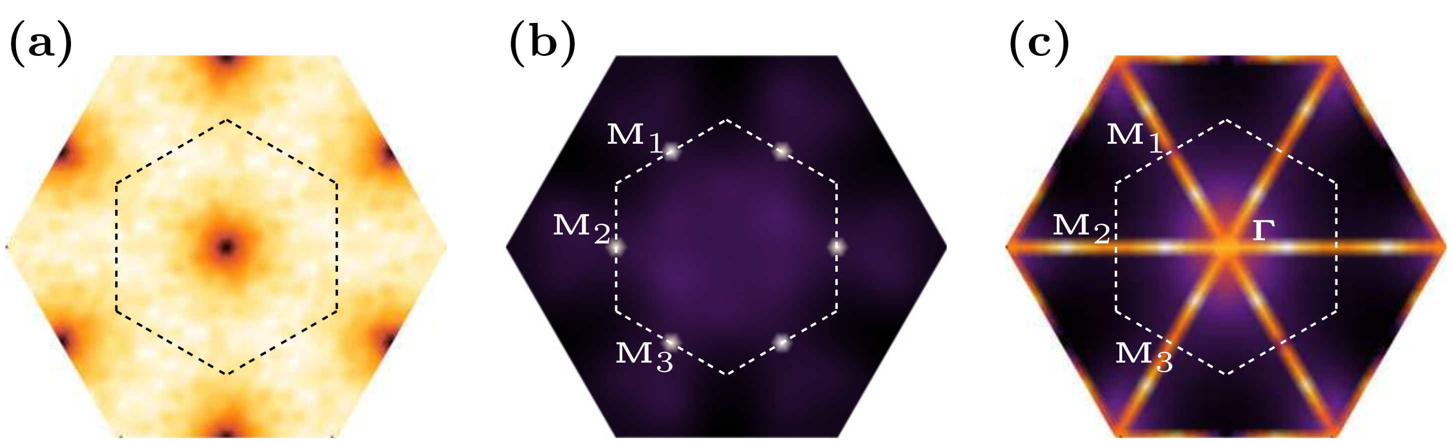
Scenario 3: Ferromagnetic K_1, positive Γ_1

Minimal model: [Ran *et al.*, PRL '17]

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + K_1 S_i^\gamma S_j^\gamma + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

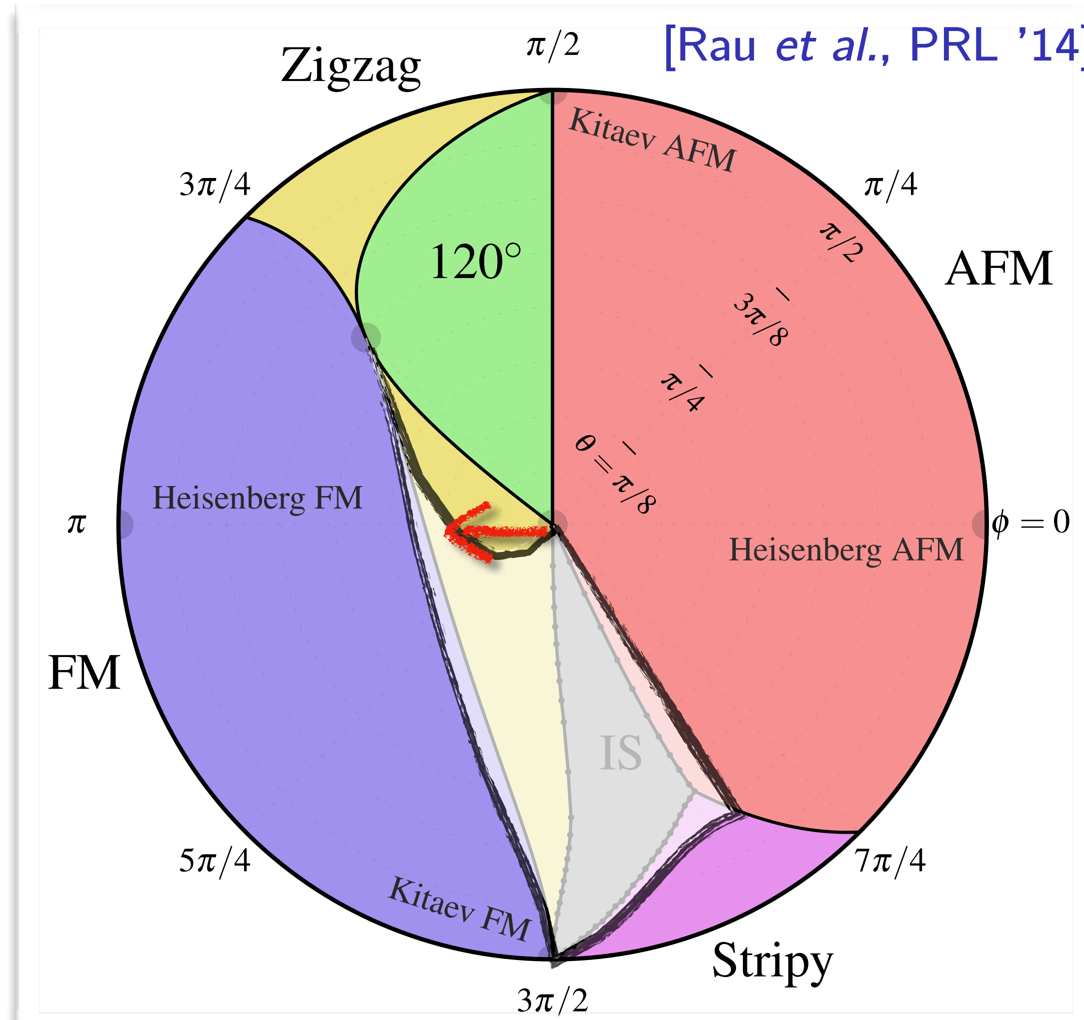
Zero-field phase diagram?

... in J_1-Γ_1 model



$J_1 = 0$	$0 < \frac{-J_1}{\Gamma_1} \lesssim 0.45$	$0.45 \lesssim \frac{-J_1}{\Gamma_1} \lesssim 0.55$
classical SL	zigzag	incommensurate

[Rousochatzakis & Perkins, PRL '17]



[Rau *et al.*, PRL '14]

... zigzag can be recovered for $\Gamma_1 \gg |K_1|$

... failure of Luttinger-Tisza

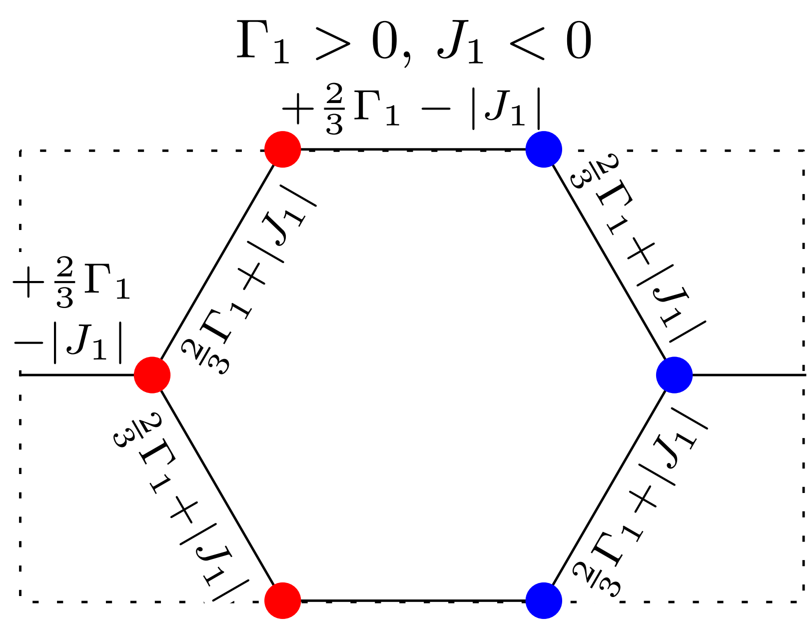
Scenario 3: Ferromagnetic K_1, J_1, positive Γ_1

Minimal model:

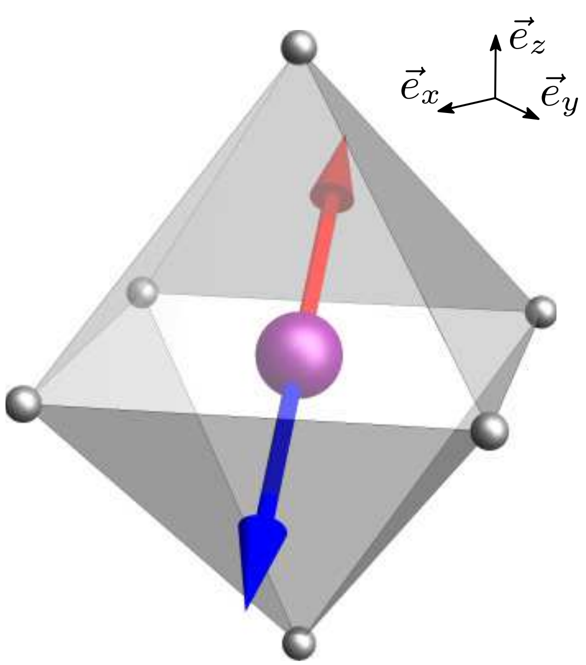
$$\mathcal{H} = \sum_{\langle ij \rangle} \left[J_1 \vec{S}_i \cdot \vec{S}_j + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right]$$

... i.e., take the limit $K_1/\Gamma_1 \to 0$

Zero field:



"face-center zigzag"



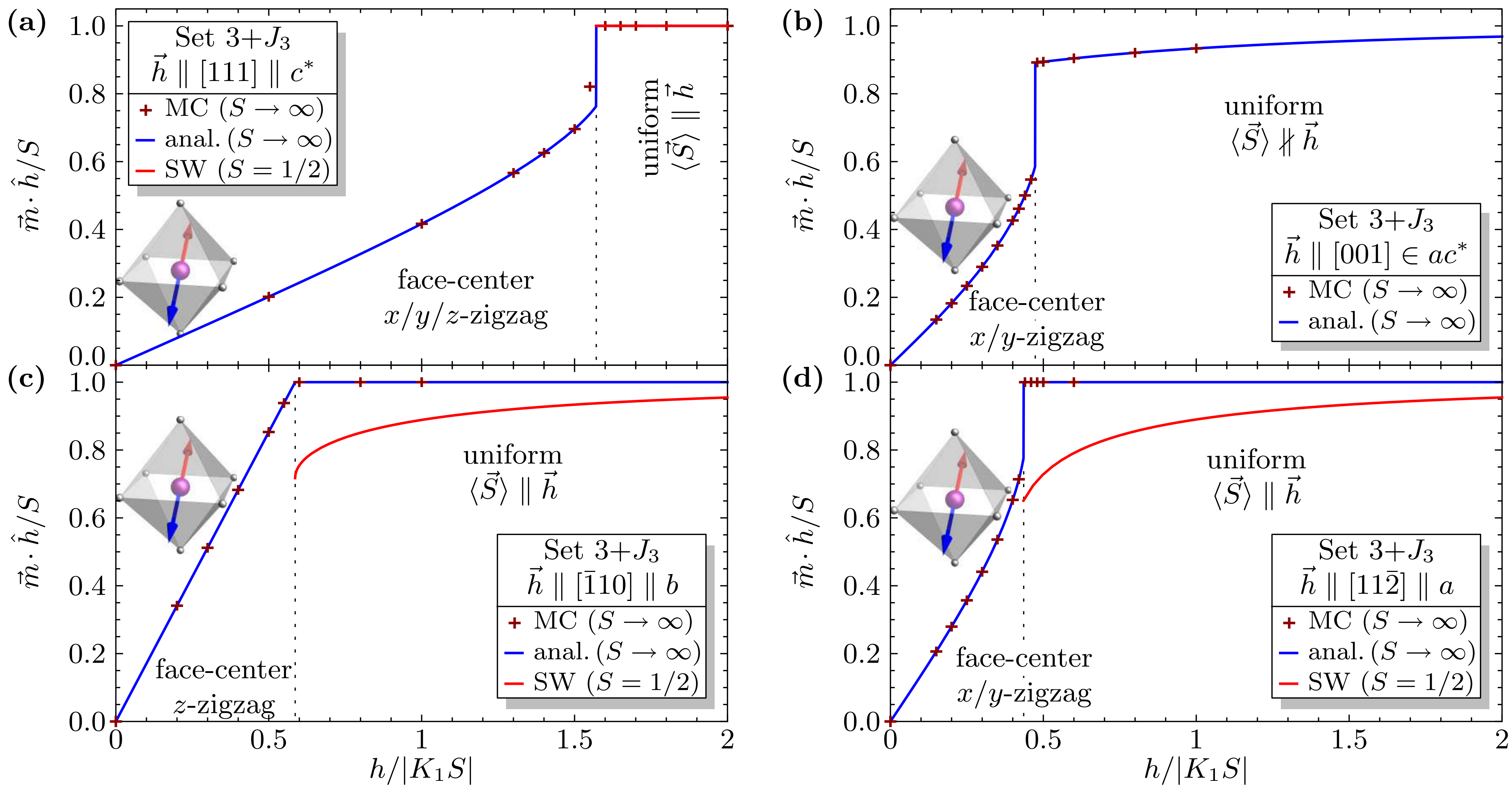
$\vec{S} \parallel \pm[11\bar{1}]$

... towards faces of $RuCl_6$ octahedra

Scenario 3: Ferromagnetic K_1, J_1, positive Γ_1

(J_1, K_1, Γ_1, J_3) = (-0.5, -5.0, +2.5, +0.5) meV

[Winter *et al.*, Nat. Comm. '17]



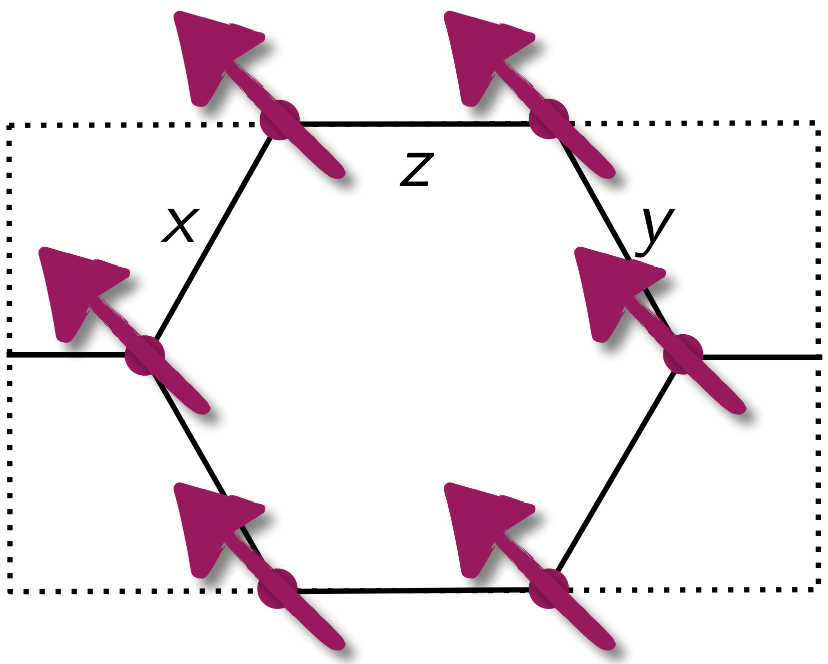
Observations:

(1) Out-of-plane h_c much larger than in-plane h_c ... as in α-$RuCl_3$

(2) $|\langle \vec{m}_{\parallel} \rangle|/S < 1$ in polarized phase already in classical limit

Scenario 3 (K_1, $J_1 < 0$, $\Gamma_1 > 0$): Consequences of strong Γ_1

Toy model:



$$\mathcal{H} = \sum_{\langle ij \rangle} \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha)$$

In polarized state $\vec{S}_i \equiv S$

$$\langle E_0/(NS^2) \rangle = \begin{cases} +\Gamma_1 & \text{if} \quad \vec{S}_i \parallel [111] \parallel c^* \\ -\Gamma_1/2 & \text{if} \quad \vec{S}_i \parallel [\bar{1}10] \parallel b \\ -\Gamma_1/2 & \text{if} \quad \vec{S}_i \parallel [11\bar{2}] \parallel a \end{cases}$$

... similarly: components of $\vec{S}_i$ in out-of-plane vs. in-plane direction

$\Rightarrow \Gamma_1 > 0$ means ...

... "antiferromagnetic" for out-of-plane spins

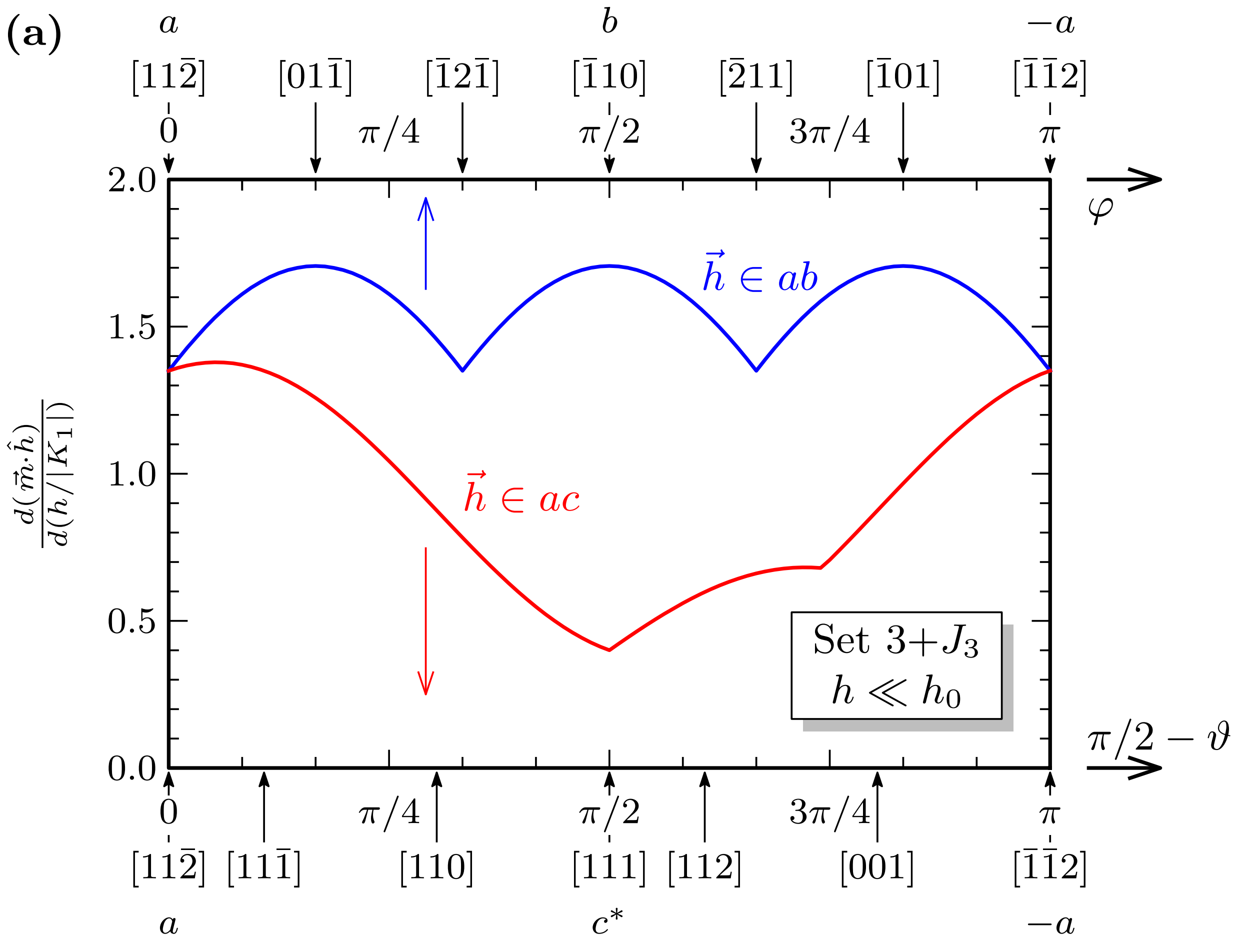
... "ferromagnetic" for in-plane spins

... naturally explains in-plane vs. out-of-plane anisotropy

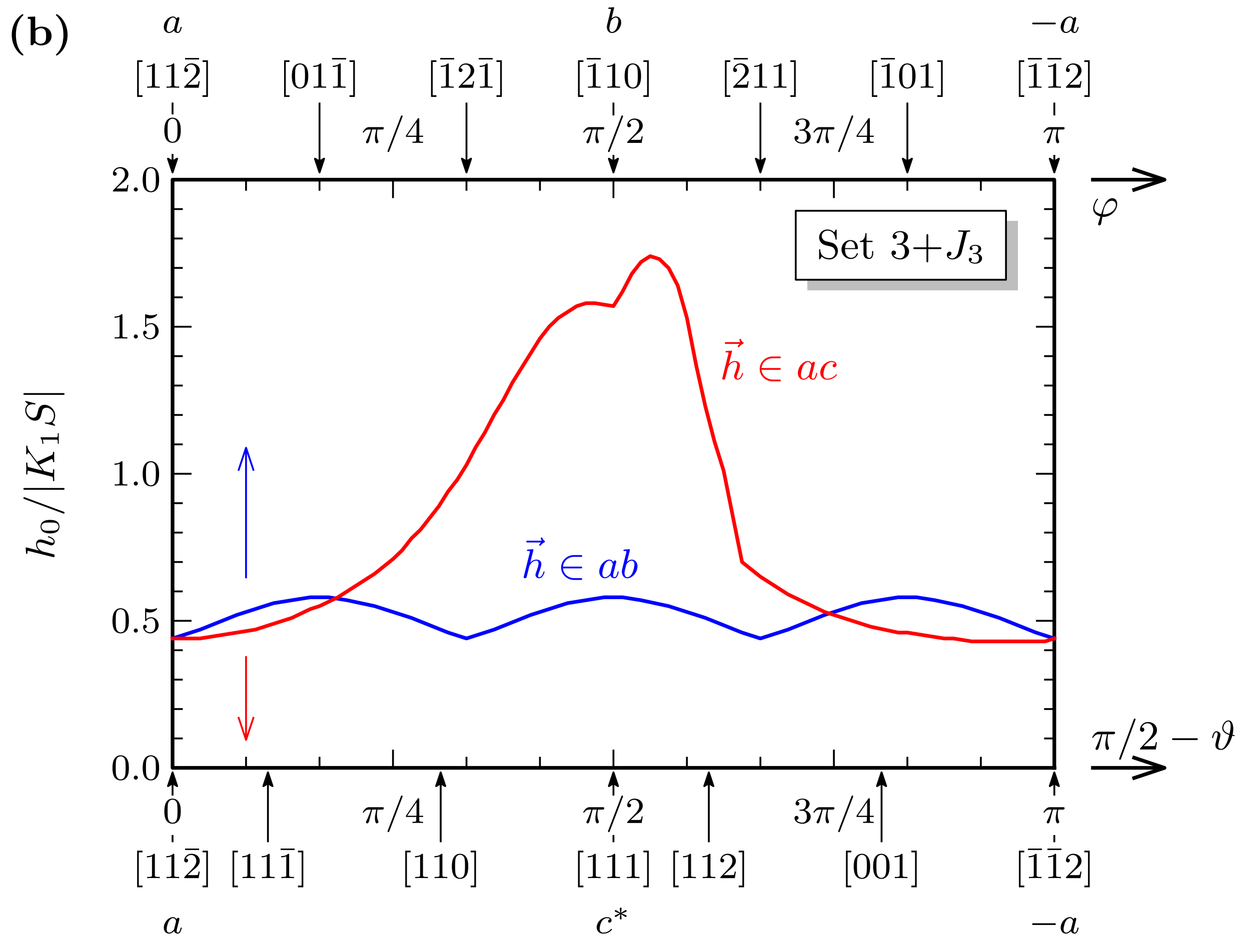
Scenario 3 (K_1, $J_1 < 0$, $\Gamma_1 > 0$): Field-angle dependence

[LJ, Andrade, Vojta, PRB ’17]

Susceptibility:



Critical field h_c:



(J_1, K_1, Γ_1, J_3) = (-0.5, -5.0, +2.5, +0.5) meV

... immediately testable in α-$RuCl_3$

Scenario 3 (K_1, $J_1 < 0$, $\Gamma_1 > 0$): High-field transversal magnetization

High fields: expect ground state w/o spontaneous symmetry breaking

... i.e., expect high-field state to be adiabatically connected to fully polarized state

If $\Gamma_1 = 0$: $\quad S_i^x \mapsto -S_i^x, \quad S_i^y \mapsto -S_i^y, \quad S_i^z \mapsto -S_i^z$

... is an accidental higher symmetry

For $\vec{h} \parallel [001]$:

(a) $\Gamma_1 = 0$: $\quad \langle \vec{S}_i \rangle \parallel [001] \quad \Rightarrow \quad |\langle \vec{m}_{||} \rangle|/S = 1$

(b) $\Gamma_1 \neq 0$: $\quad \langle \vec{S}_i \rangle \parallel [xxz]$ with $x \neq 0$ not forbidden

$\Rightarrow$ finite $\langle \vec{m}_\perp \rangle \perp \vec{h}$ possible

$\vec{m}_\perp$ is a direct measure of Γ_1 ... independent of K_1, J_1, J_3, ...

$$|\langle \vec{m}_\perp \rangle|/|\langle \vec{m}_{||} \rangle| \simeq 0.51$$

at $h = h_c^+$ for $(J_1, K_1, \Gamma_1, J_3) = (-0.5, -5.0, +2.5, +0.5)$ meV

Summary: Scenario 3 (K_1, $J_1 < 0$, $\Gamma_1 > 0$)

Magnetization process ...

... strongly anisotropic

... χ_{ab} much larger than χ_c

... even without *g*-factor anisotropy

... $\chi_{ab}/\chi_c \simeq 3 \ldots 4$ for $(J_1, K_1, \Gamma_1, J_3) = (-0.5, -5.0, +7.0, +0.5)$ meV

... consistent with small trigonal distortion [Agrestini *et al.*, PRB(R) '17]

... high-field state with transversal magnetization

... for field in intermediate direction, e.g., [001]

Conclusions:

Magnetization process crucially dependent on zigzag stabilization:

- Scenario 1 ($K_1 > 0$, $J_1 < 0$)
  - Weak anisotropy in χ and h_c
  - Various exotic intermediate phases
- Scenario 2 ($K_1 < 0$, $J_3 > 0$)
  - Nearly no anisotropy in χ and h_c
  - Metamagnetic transitions between different canted zigzag
- Scenario 3 (K_1, $J_1 < 0$, $\Gamma_1 > 0$)
  - Strong anisotropy in χ and h_c ... natural explanation for anisotropy in α-$RuCl_3$
  - Finite transversal magnetization in high-field state ... possibly measurable?
  - Intermediate phases?

α-$RuCl_3$: expect strong $\Gamma_1 > 0$, $K_1 < 0$, and small $J_3 > 0$