Deconfined criticality from the QED$_3$-Gross-Neveu model

Lukas Janssen

L.J. and Y.-C. He, Phys. Rev. B **96**, 205113 (2017)

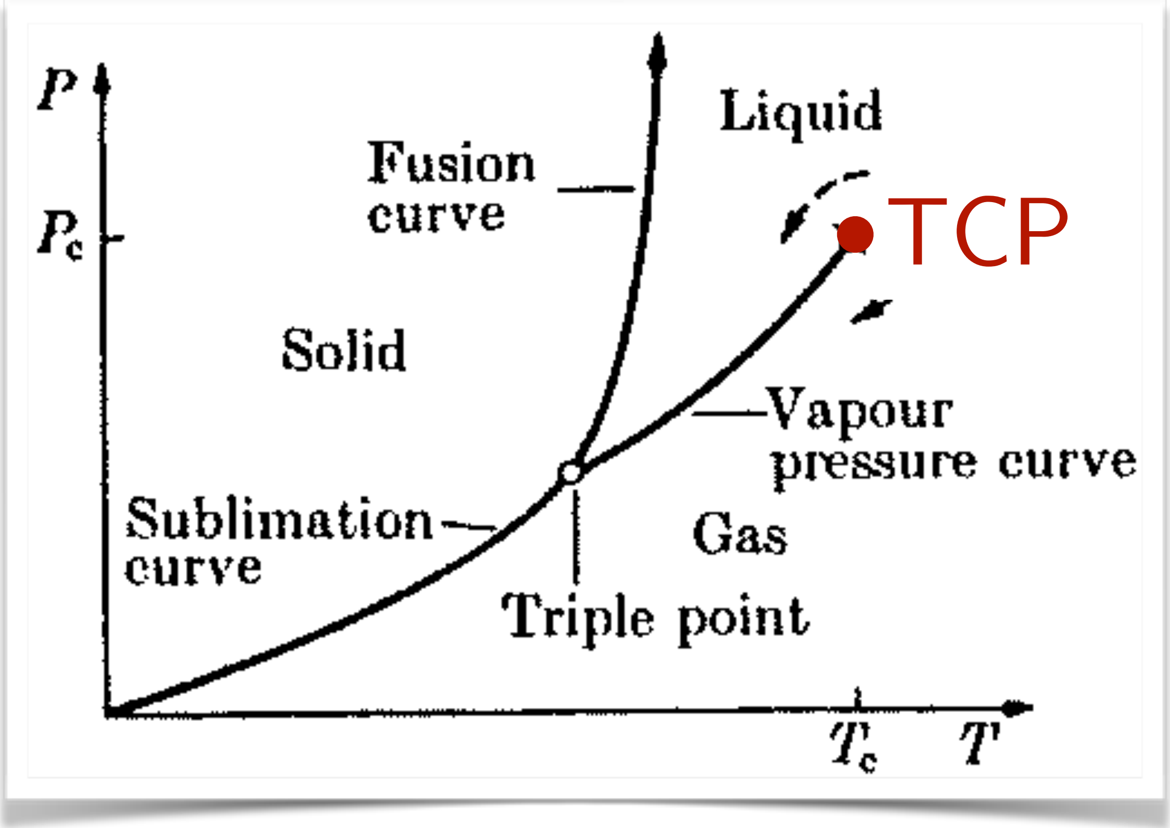
B. Ihrig, L.J., L. Mihaila, and M. Scherer, arXiv:1807.XXXX

TECHNISCHE UNIVERSITÄT DRESDEN

SFB 1143

Thermal critical point (TCP) vs. quantum critical point (QCP)

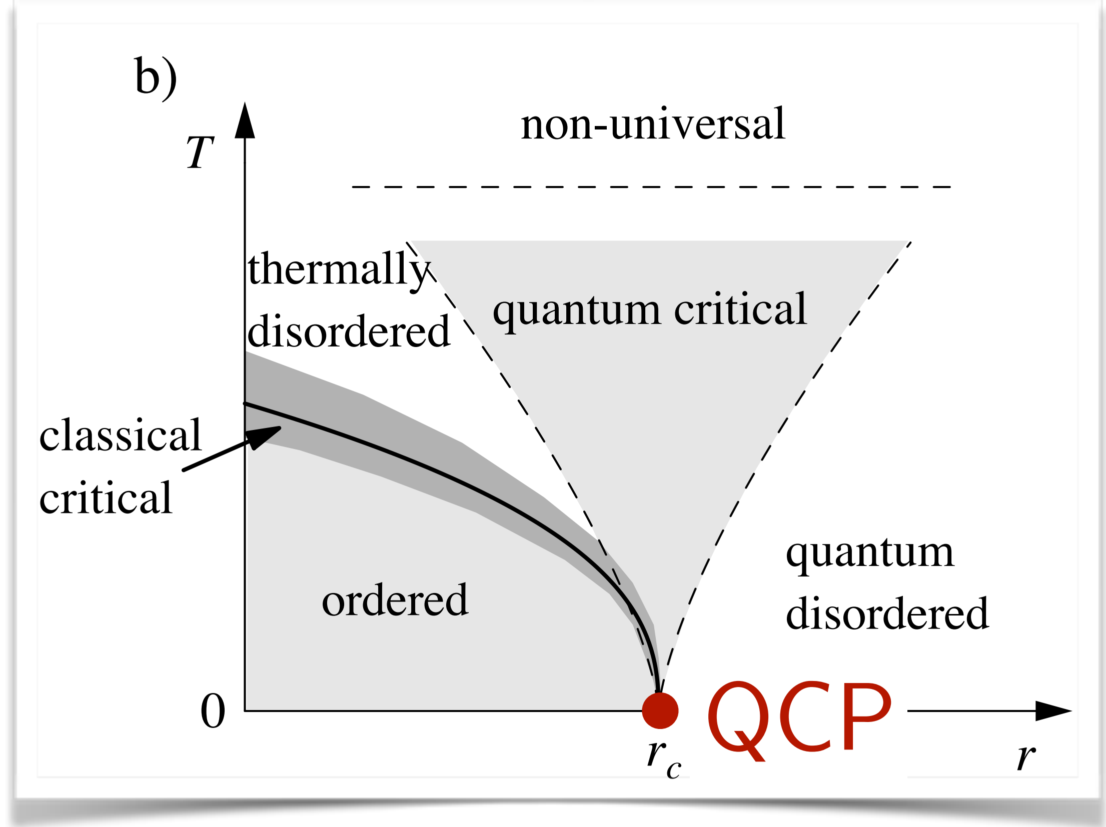
Thermal:



... driven by thermal fluctuations

... tuned by temperature

Quantum:



[M Vojta, Rep. Progr. Phys. '03]

... driven by quantum fluctuations

... tuned by pressure, field, ...

Thermal critical point (TCP) vs. quantum critical point (QCP)

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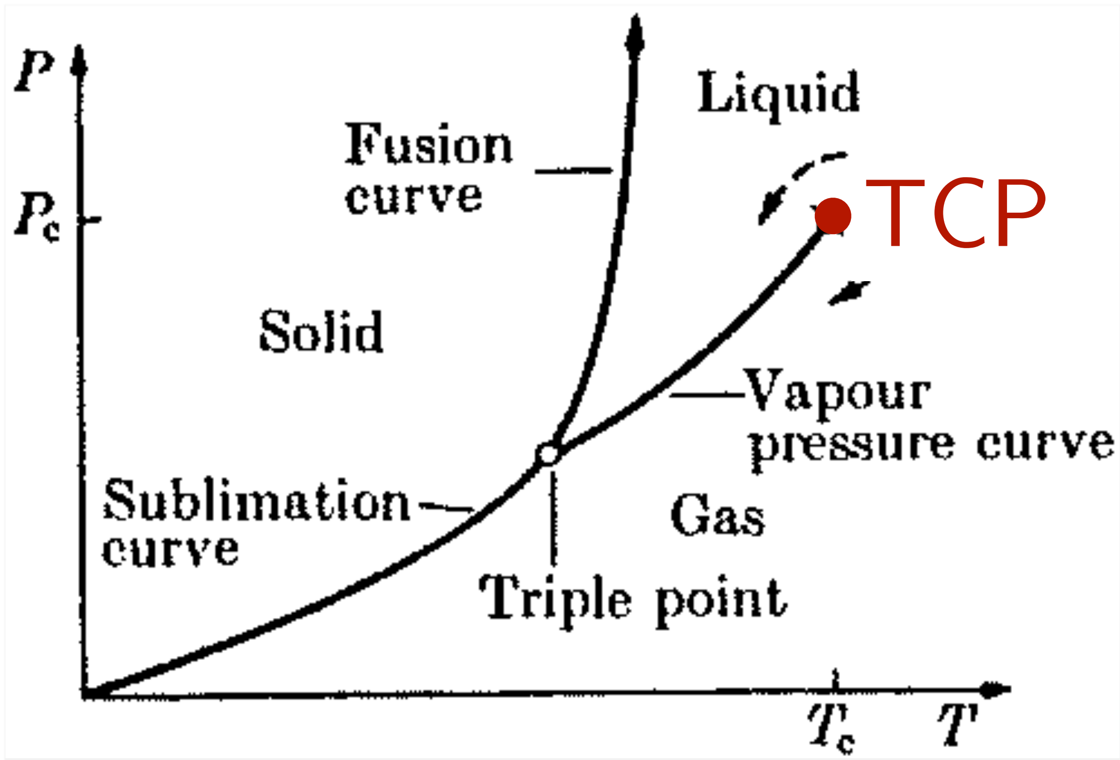
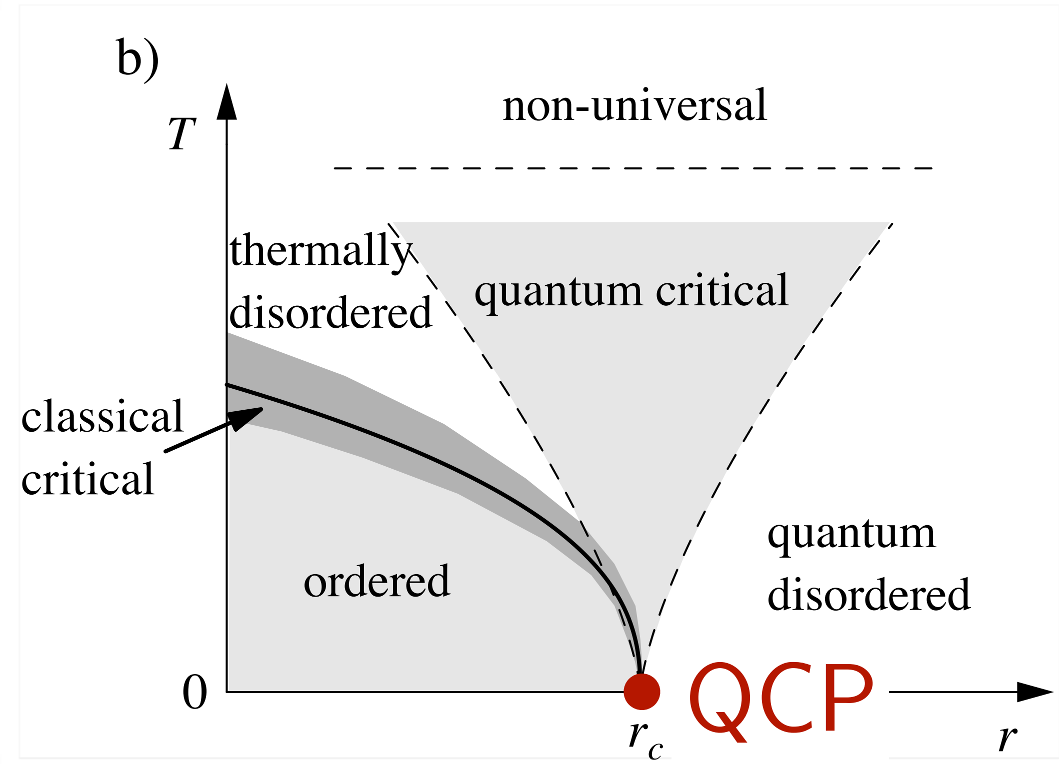


Quantum:



[M Vojta, Rep. Progr. Phys. '03]

... driven by thermal fluctuations

... tuned by temperature

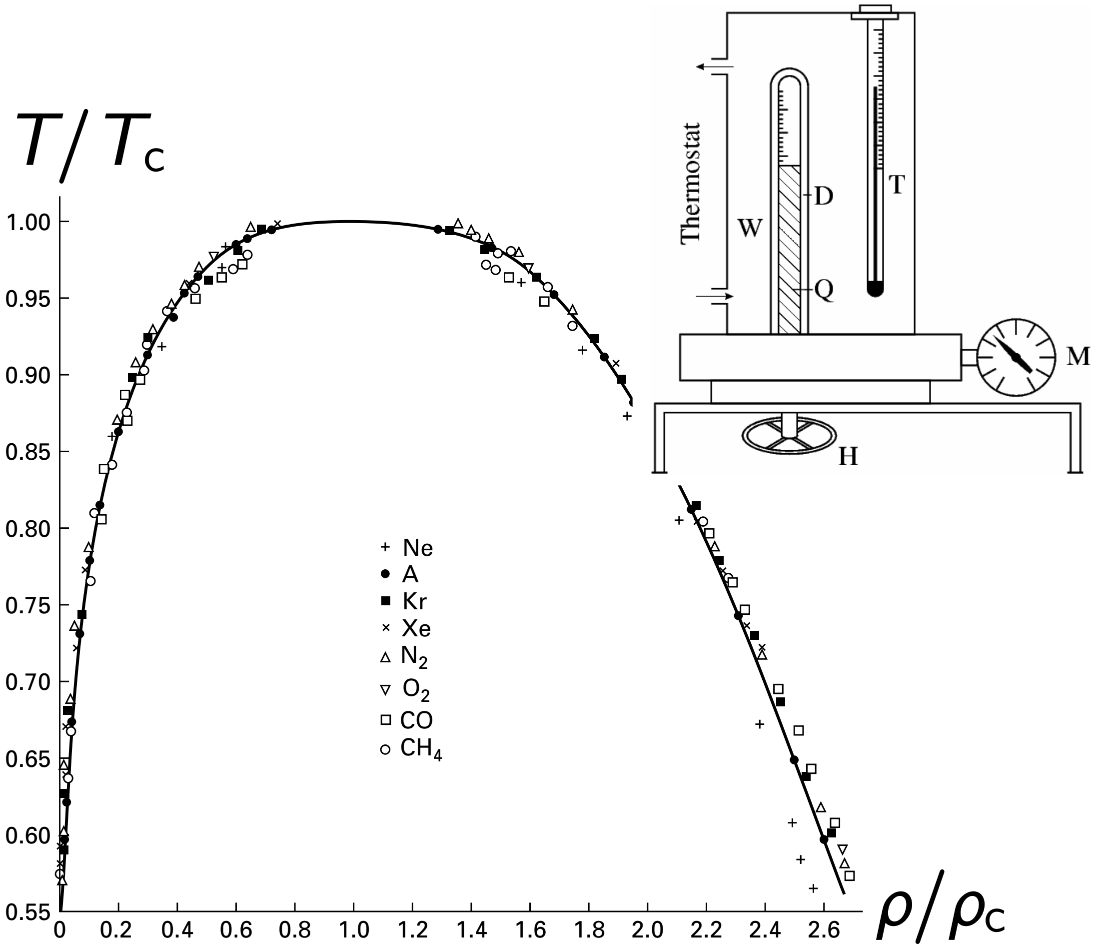
... driven by quantum fluctuations

... tuned by pressure, field, ...

Any significant differences?

TCP vs. QCP: Example

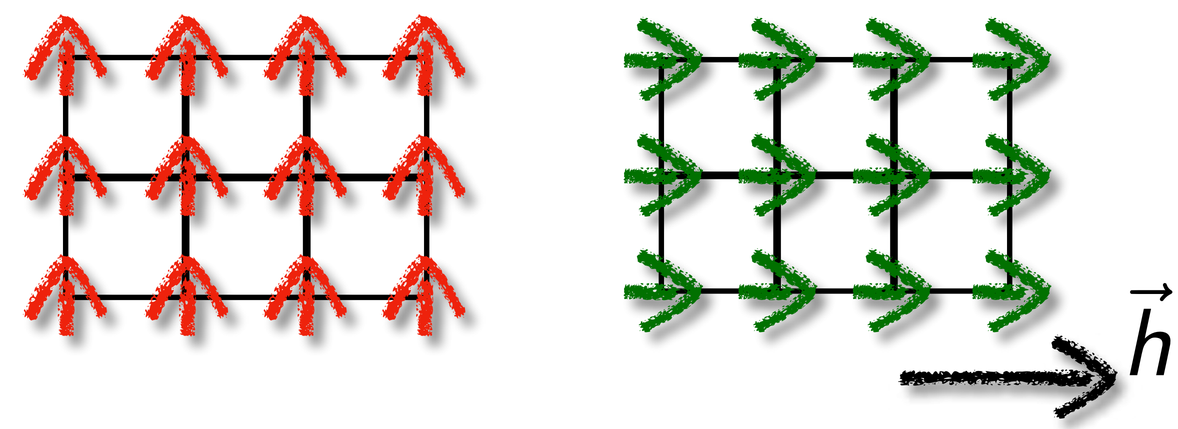
Liquid-gas transition: ... in 3D



[Guggenheim, J. Chem. Phys. '45]

Transverse-field Ising model: ... in 2D

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i^z S_j^z - \vec{h} \cdot \sum_i \vec{S}_i$$



$h/J < (h/J)_c$ $\qquad$ $h/J > (h/J)_c$

Order parameter:

$$|\rho_L - \rho_G| \propto |T - T_c|^\beta, \quad \beta \approx 0.33$$

Order parameter:

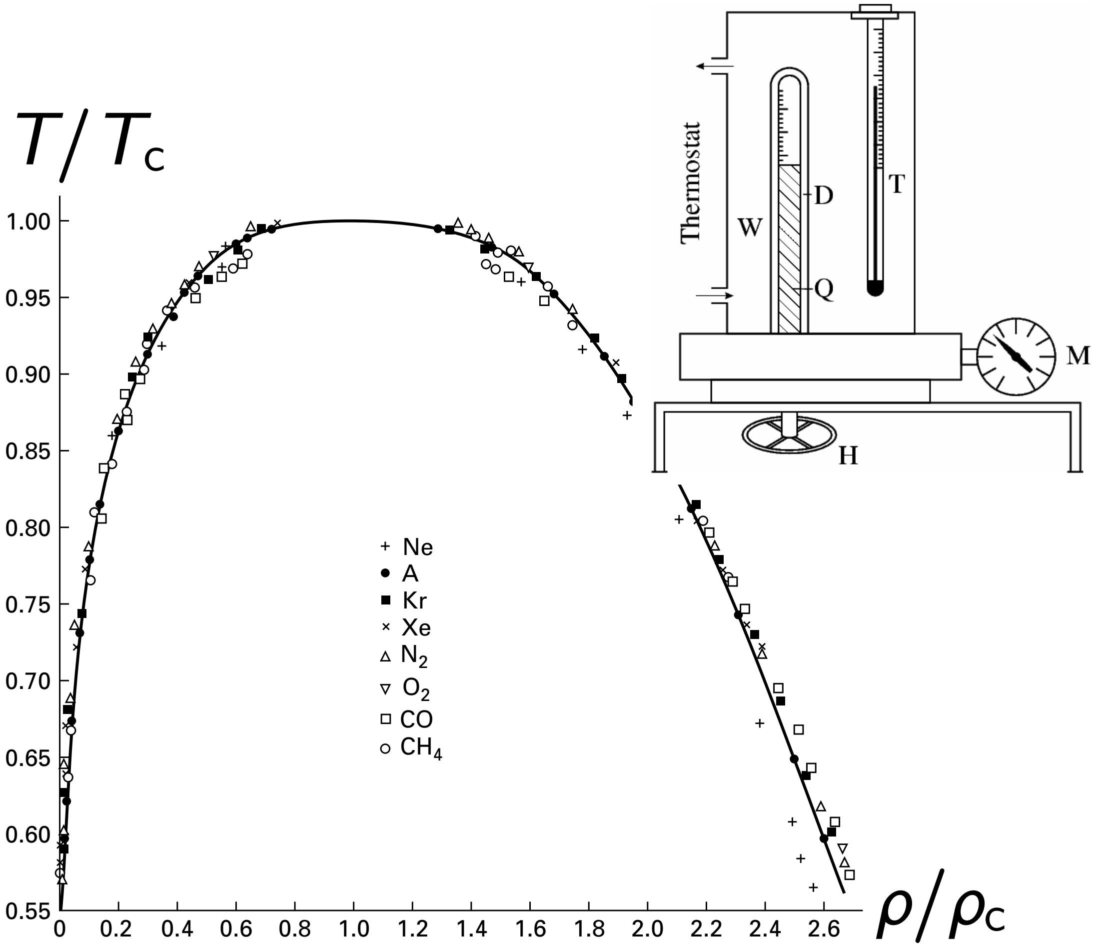
$$|m_z| \propto |J - J_c|^\beta, \quad \beta \approx 0.33$$

... and other exponents also agree

[Elliot *et al.*, PRL '70]

TCP vs. QCP: Example

Liquid-gas transition: ... in 3D



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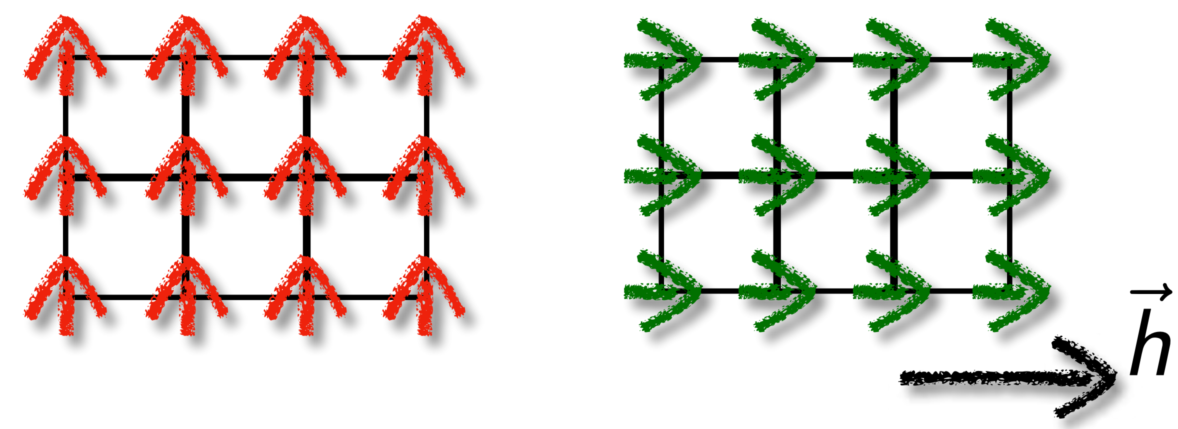


$h/J < (h/J)_c$ $h/J > (h/J)_c$

Quantum-to-classical mapping:

$$\mathrm{TCP}(d+z) \iff \mathrm{QCP}(d)$$

z ... dynamical critical exponent

Landau-Ginzburg-Wilson theory

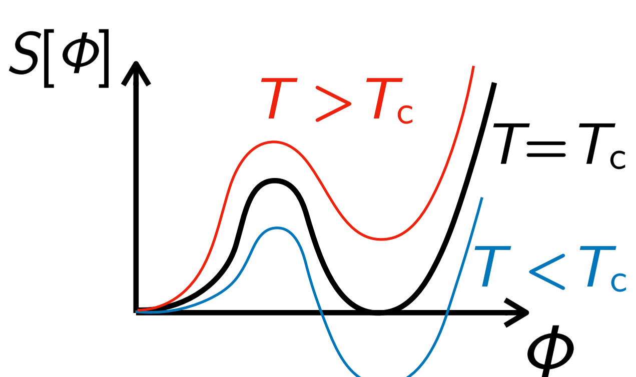
Assumption:

Transition uniquely characterized by order-parameter fluctuations

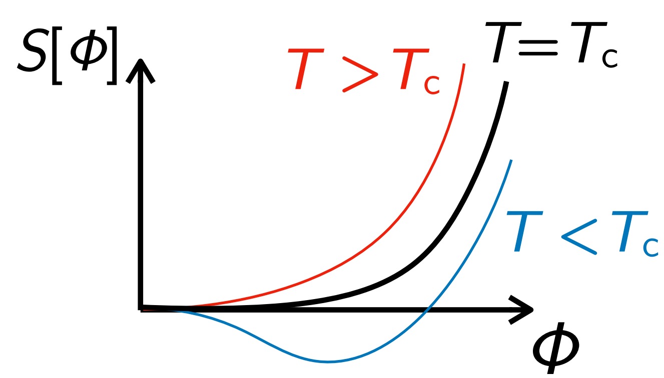
Continuum field theory: $S[\phi] = \int d^d\vec{r}\left[\frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \lambda\phi^4 + \ldots\right]$

ϕ ... order-parameter field

Mean-field theory (Landau):



$S[\phi] \propto \frac{r}{2}\phi^2 + \mu\phi^3 + \lambda\phi^4$



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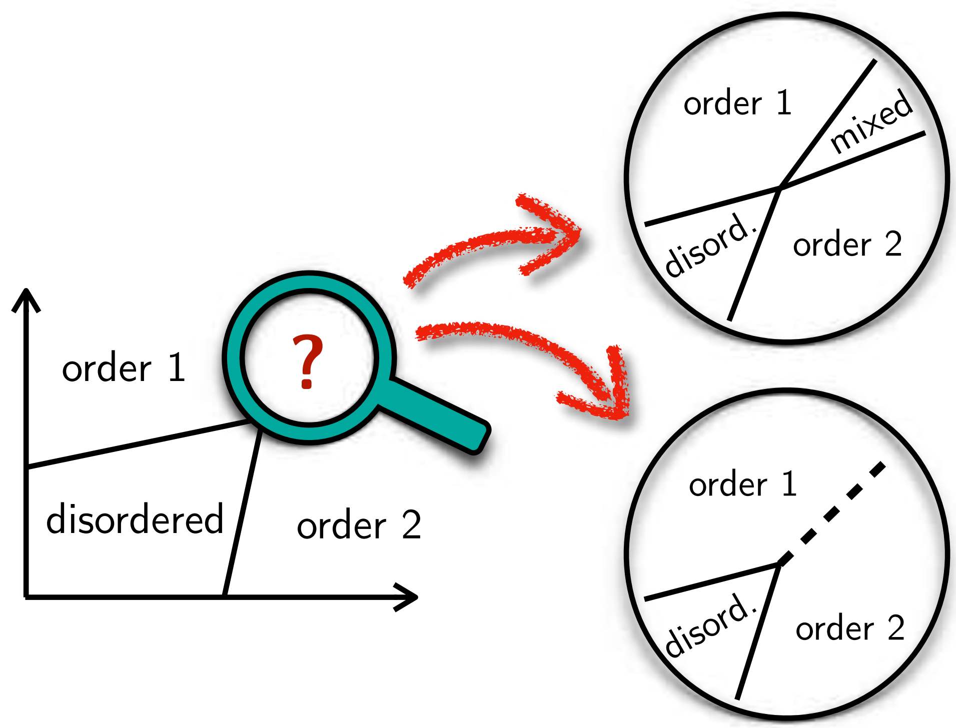
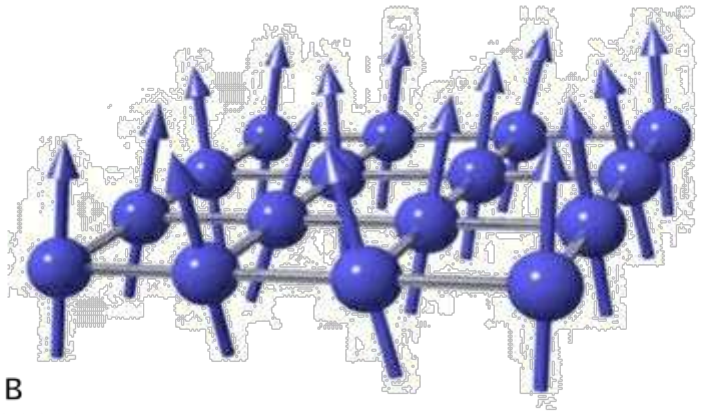


Renormalization group (Wilson):

Universality $\Longleftrightarrow$ Existence of stable RG fixed point

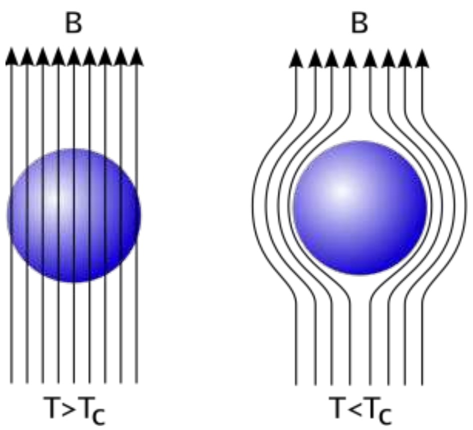
Landau-Ginzburg-Wilson theory: Successes

Ansatz works remarkably well ...



... magnets ($\vec{\varphi}$)

[Wilson & Fisher, PRL '72]



... superconductors (ϕ, ϕ^*, a_μ)

[Halperin, Lubensky, Ma, PRL '74]

... Mott transition in Fermi-point systems $(\vec{\varphi}, \psi^\dagger, \psi)$

2D Dirac:

[Herbut, PRL '06]

[Raghu, Qi, Honerkamp, Zhang, PRL '08]

[Assaad & Herbut, PRX '13]

[LJ & Herbut, PRB '14]

[He, Xu, Sun, Assaad, Meng, Lu, PRB '18]

...

2D QBT:

[Sun *et al.*, PRL '09]

[Scherer, Uebelacker, Honerkamp, PRB '12]

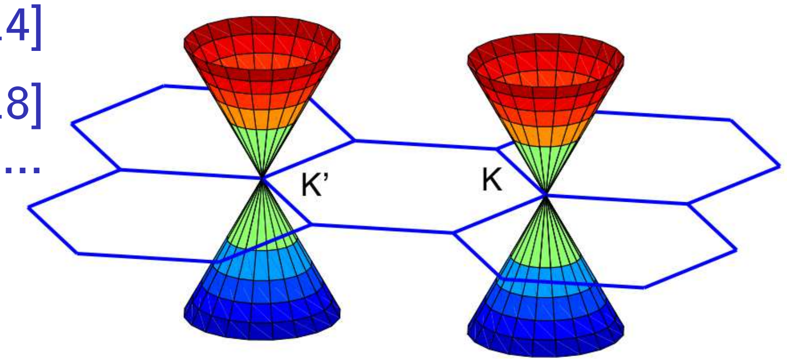
[Pujari, Lang, Murthy, Kaul, PRL '16]

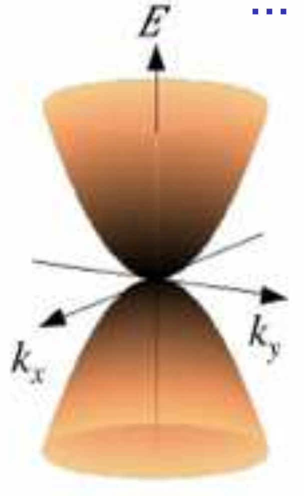
...

3D QBT:

[Herbut & LJ, PRL '14]

...





... and more

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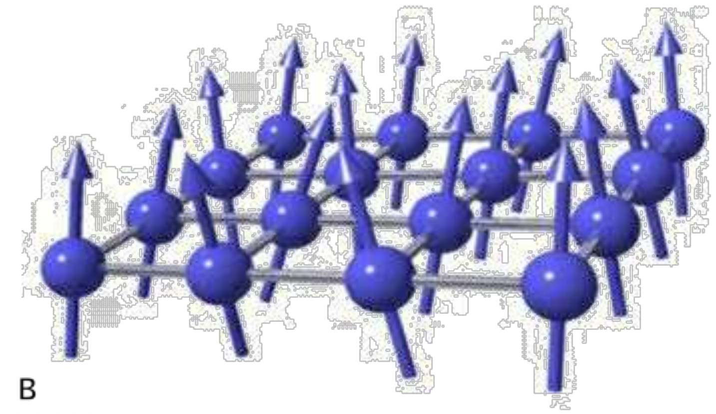

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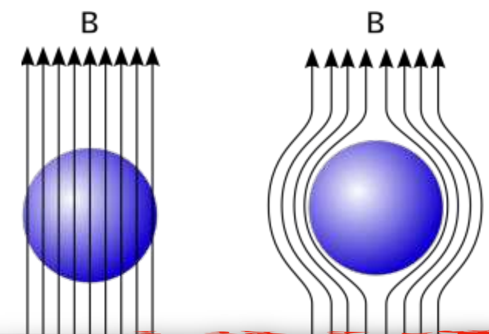

... M

2D Dir

New paradigms?

PRL '14]

[Raghu

...

[Assaad & Herbut, PRX '13]

[Pujari, Lang, Murthy, Kaul, PRL '16]

[LJ & Herbut, PRB '14]

...

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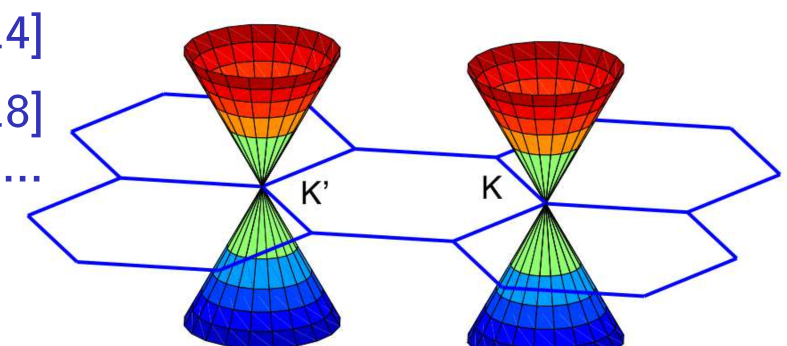
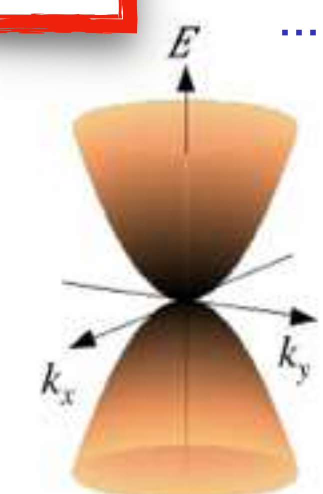




... and more

Challenging Landau's paradigm

(A) "Fluctuation-induced" quantum criticality

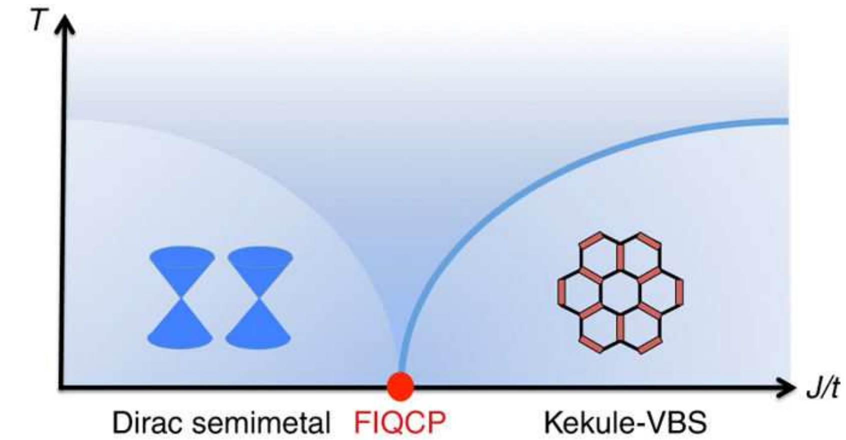
... i.e., mean-field theory becomes invalid

... Kekulé QCP

... despite the presence of cubic term in S[Φ]

[Li, Jiang, Jian, Yao, Nat. Comm. '17]

[Classen, Herbut, Scherer, PRB '17]

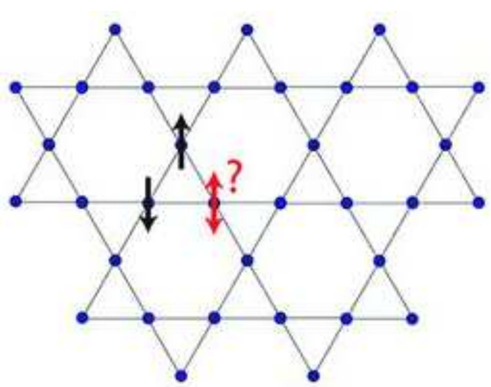


(B) "Topological" quantum criticality

... adjacent phase characterized by topological order

... i.e., no local order parameter

... Kagome spin liquid



[Hastings, PRB '00]

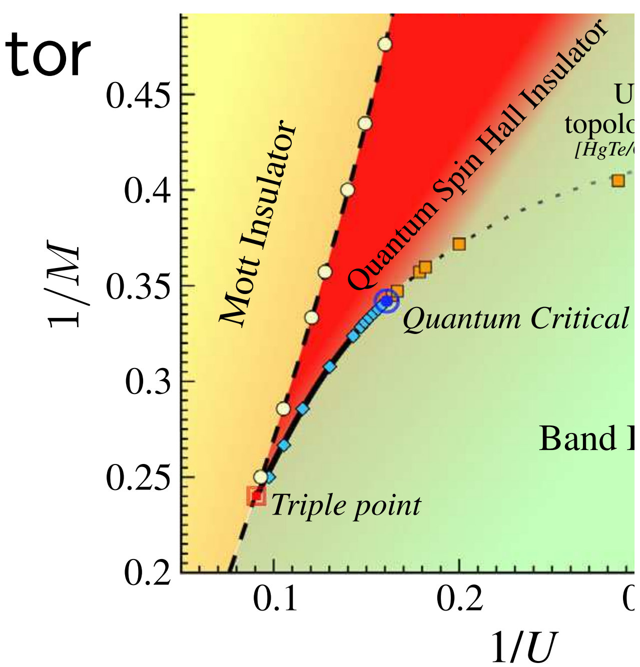
[He & Chen, PRL '15]

[He *et al.*, PRX '17]

...

... topological insulator

[Amaricci *et al.*, PRL '15]



(C) "Deconfined" quantum criticality

[Senthil, Vishwanath, Balents, Sachdev, Fisher, Science '04]

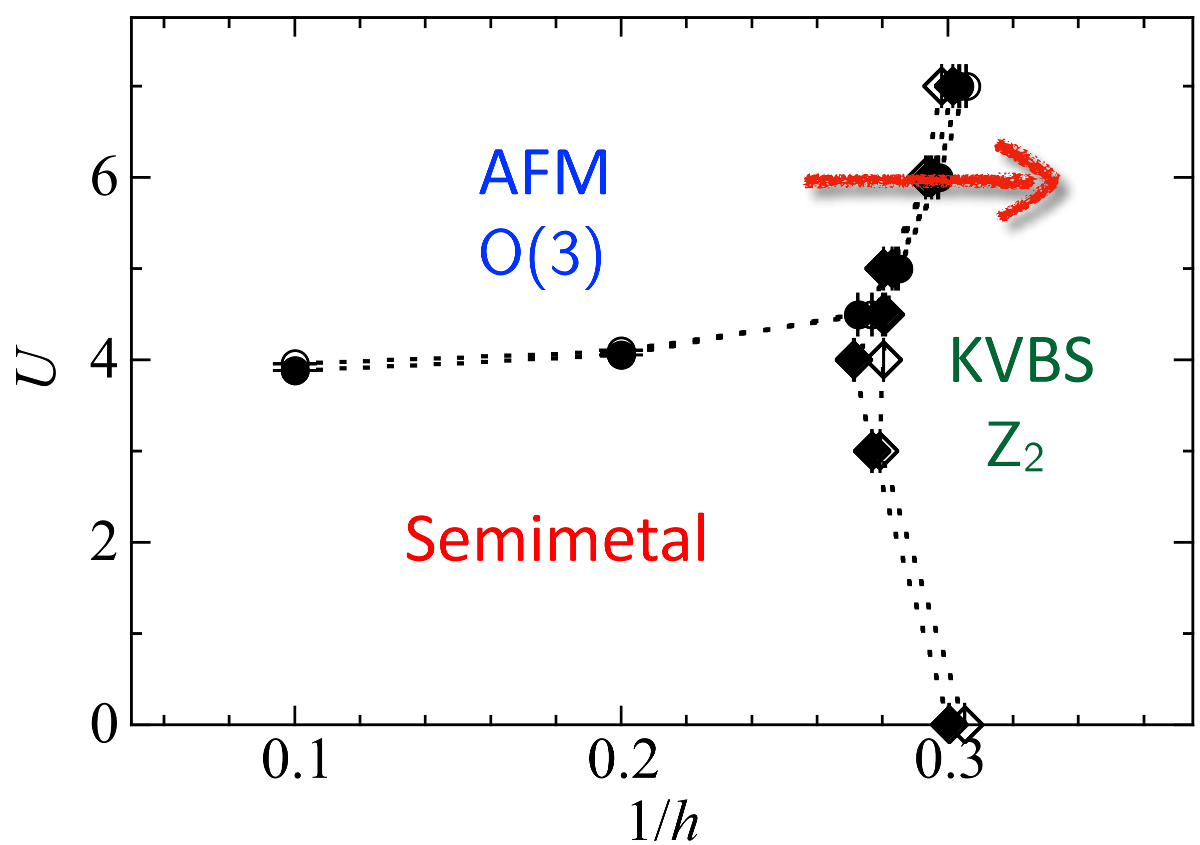
... continuous order-to-order transition

... characterized by fractionalized excitations

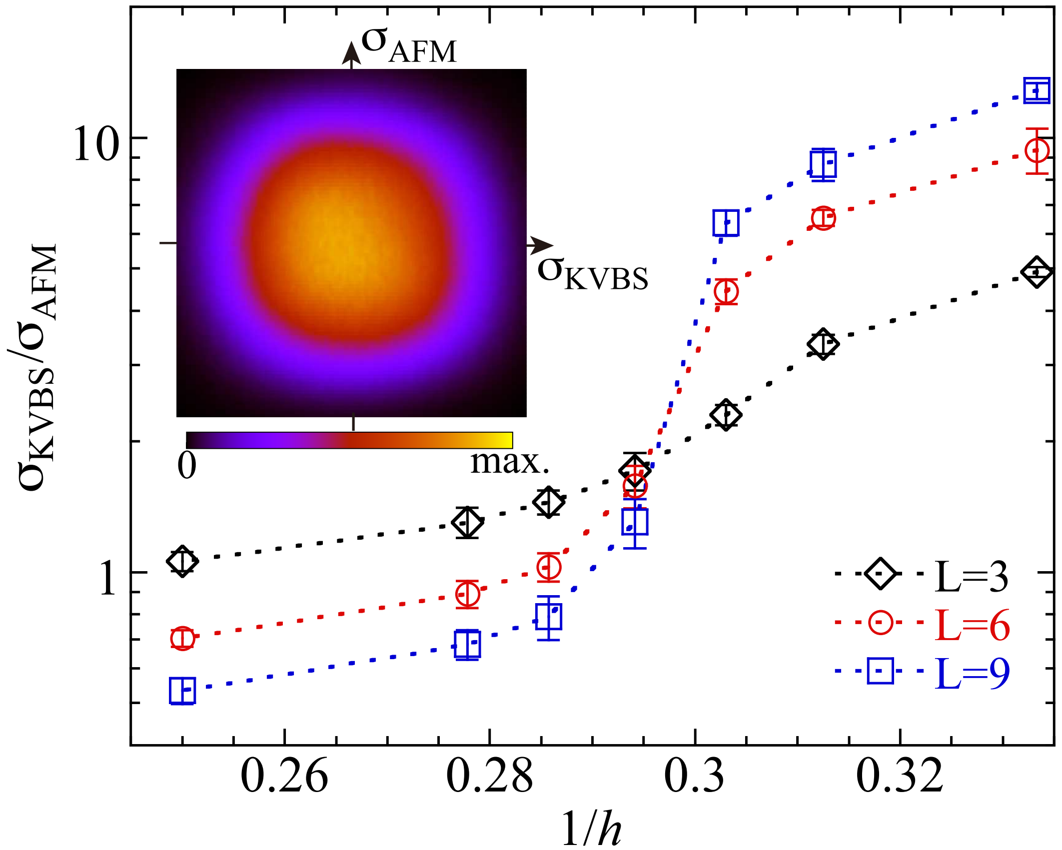
Deconfined quantum criticality

(1) Néel-to-Kekulé transition on honeycomb lattice

... anticommuting masses



standard deviation



[Sato, Hohenadler, Assaad, PRL '17]

... direct continuous transition?

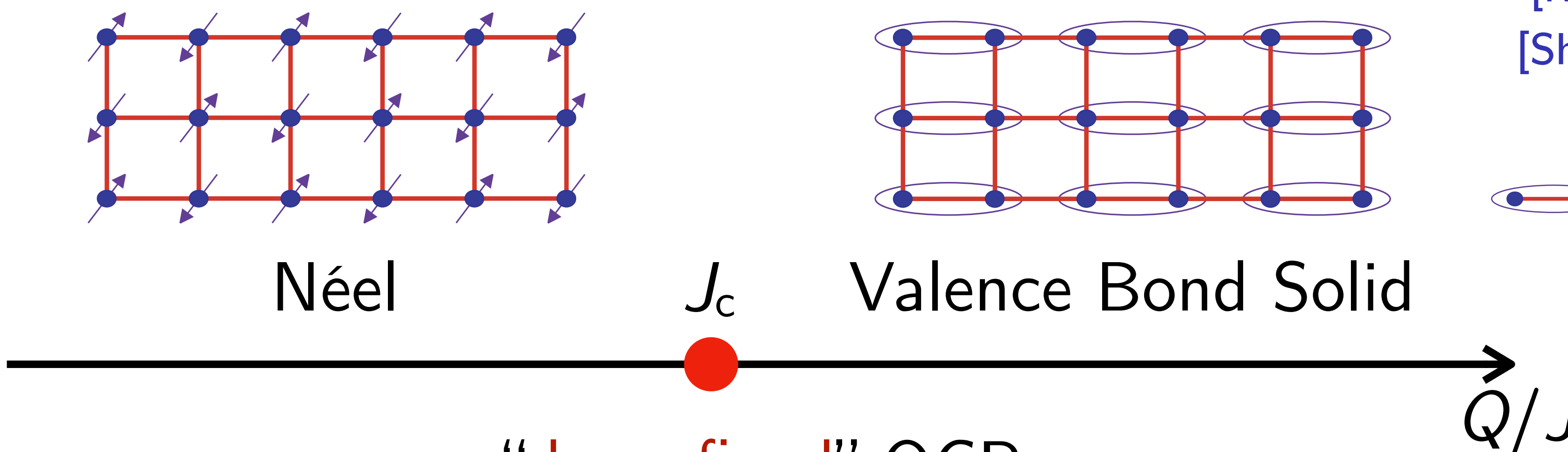
... emergent SO(4)?

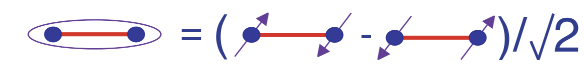
(2) Strong-coupling limit $U \to \infty$:

[Senthil *et al.*, Science ’04]

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle ijkl \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \frac{1}{4} \right)$$

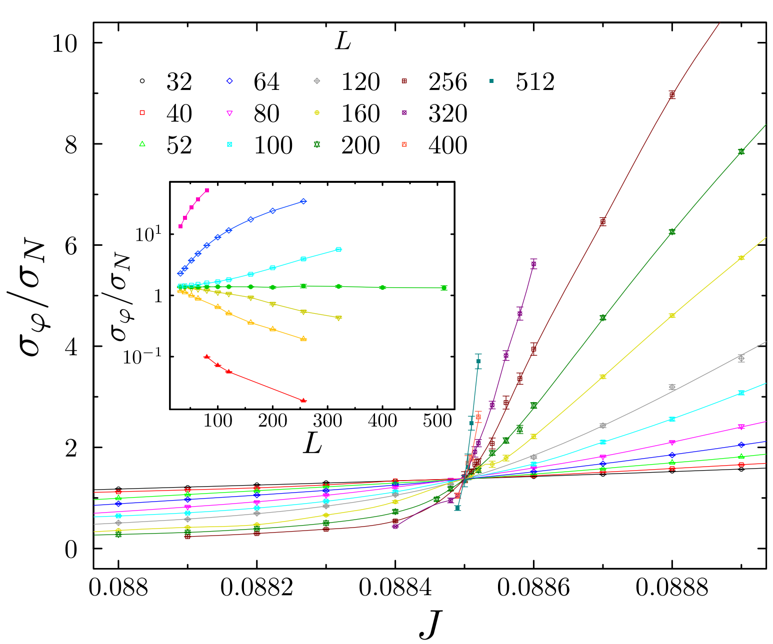
[Sandvik, PRL ’07; PRL ’10]
[Melko & Kaul, PRL ’08]
[Nahum *et al.*, PRX ‘15]
[Shao *et al.*, Science ’16]
...



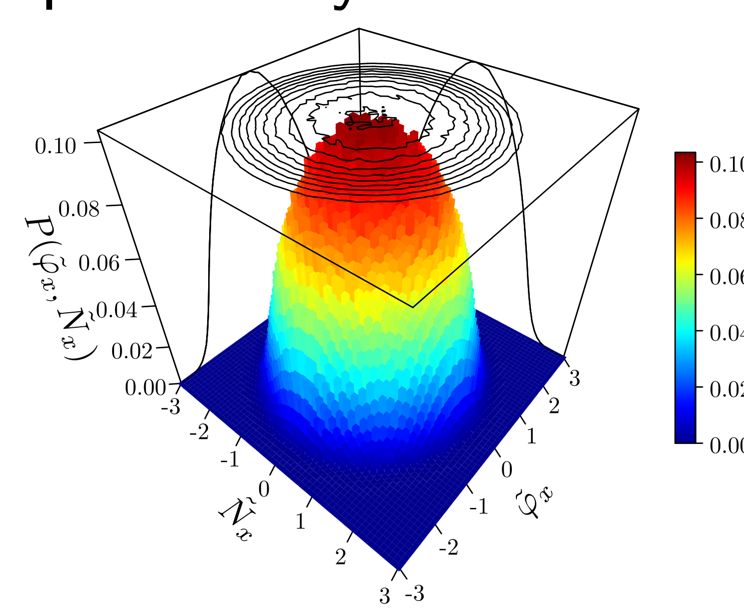


“deconfined” QCP

standard deviation



probability distribution



... isotropic!

[Nahum *et al.*, PRL ‘15]

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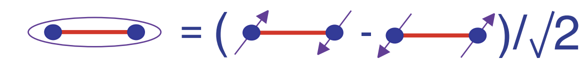


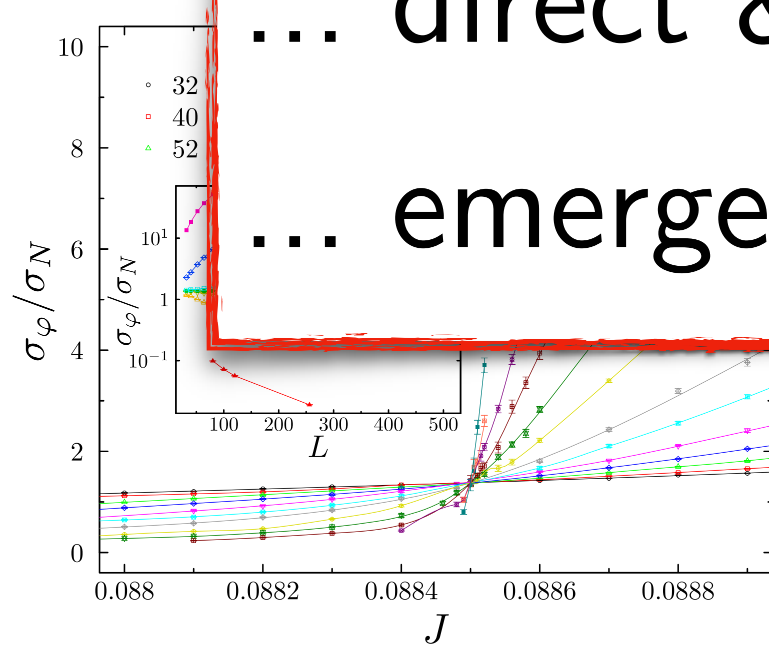
Néel J_c Valence Bond Solid

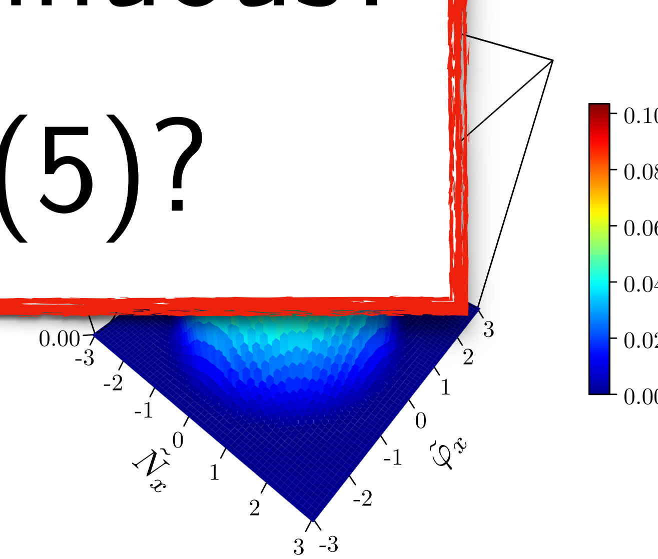
"deconfined" QCP

st... bution

... direct & continuous?

... emergent SO(5)?





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[Nahum *et al.*, PRL '15]

Breakdown of Landau-Ginzburg-Wilson

Continuum field theory for Néel phase: [Senthil *et al.*, Science '04; PRB '04]

$$S_{\vec{n}} = \frac{1}{2g}\int d^2r\, d\tau (\partial_\mu \vec{n})^2 + S_{\rm B}$$

... O(3) nonlinear σ model
... with "Berry-phase" term $S_{\rm B}$

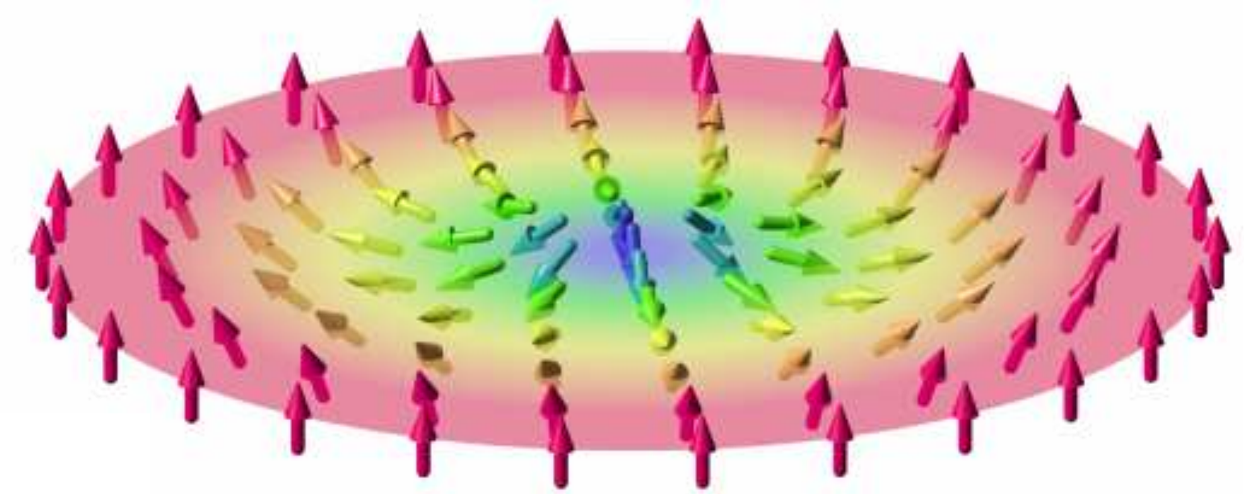
Néel order parameter: $\vec{n} \propto (-1)^r \vec{S}_r$

r ... lattice site

Spin Berry phase: $S_{\rm B} = iS \sum_r (-1)^r A_r$

A_r ... area enclosed by $\vec{n}_r(\tau)$

... nonvanishing for monopole events:



... e.g., creation of skyrmion with $Q = \frac{1}{4\pi}\int d^2r\, \vec{n}\cdot(\partial_x \vec{n} \times \partial_y \vec{n})$

Order parameters: Néel: (n_1, n_2, n_3) VBS: $(\mathrm{Re}\,\mathcal{M}, \mathrm{Im}\,\mathcal{M})$

$\mathcal{M}$ ... monopole operator

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... nonvanishing for monopole events:

Berry phase crucial for transition!

... e.g., creation of skyrmion with $Q = \frac{1}{4\pi} \int d^2 r \, \vec{n} \cdot (\partial_x \vec{n} \times \partial_y \vec{n})$

Order parameters: Néel: (n_1, n_2, n_3) VBS: $(\mathrm{Re}\,\mathcal{M}, \mathrm{Im}\,\mathcal{M})$

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Field theory for deconfined criticality

Reformulation:

... CP¹ parametrization

$$\vec{n} = z^\dagger \vec{\sigma} z$$

$z = (z_1, z_2)$ ... complex "spinon"

CP¹ model:

$$S_z = \int d^2\vec{r} d\tau \left[\sum_{\alpha=1,2} |(\partial_\mu - i b_\mu) z_\alpha|^2 - \left(|z_1|^2 + |z_2|^2\right)^2 \right]$$

b_μ ... "photon"

... monopoles = instantons in b_μ

Senthil *et al.*: **Monopoles irrelevant at critical point!**

[Senthil *et al.*, Science '04; PRB '04]

Natural field theory: noncompact CP¹ model (NCCP¹)

Field theory for deconfined criticality

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... CP^1 parametrization

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CP^1 model:

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Natural field theory: noncompact CP^1 model (NCCP^1)

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Field theory for deconfined criticality

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CP^1 mod

S

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Senthil *et*

...e '04; PRB '04]

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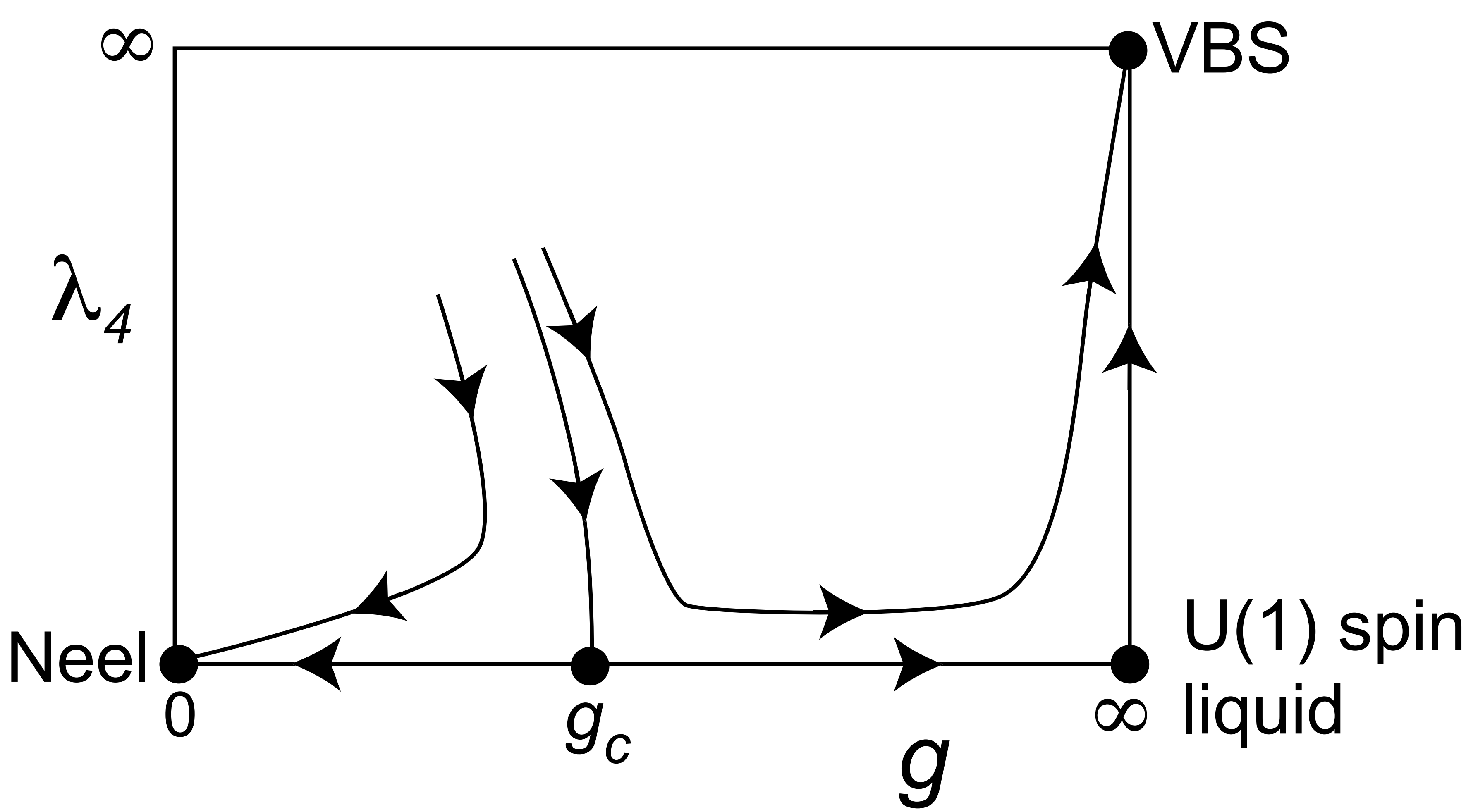


[Senthil *et al.*, JPSJ '05]

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Alternative formulations of deconfined QCP

Duality conjecture:

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

noncompact CP^1 model $\Longleftrightarrow$ QED_3-Gross-Neveu model

$$(z_1, z_2, z_1^\dagger, z_2^\dagger, b_\mu) \quad \Leftrightarrow \quad (\psi_1, \psi_2, \bar\psi_1, \bar\psi_2, a_\mu)$$

$$\sum_{\alpha=1,2} |D_b z_\alpha| - \left(|z_1|^2 + |z_2|^2\right)^2 \quad \Leftrightarrow \quad \sum_{i=1,2} \left(\bar\psi_i \not{D}_a \psi_i + \phi \bar\psi_i \psi_i\right) + V(\phi)$$

... with $V(\Phi)$ tuned to criticality

Explicitly:

$$(n_1, n_2, n_3, n_4, n_5) \sim (\overbrace{2\,\mathrm{Re}\,\mathcal{M}_b, 2\,\mathrm{Im}\,\mathcal{M}_b}^{\mathrm{U}(1)}, \overbrace{z^\dagger\sigma_x z, z^\dagger\sigma_y z, z^\dagger\sigma_z z}^{\mathrm{O}(3)})$$

$$\sim \Big[\underbrace{\mathrm{Re}\,(\psi_1^\dagger\mathcal{M}_a), -\mathrm{Im}\,(\psi_1^\dagger\mathcal{M}_a), \mathrm{Re}\,(\psi_2^\dagger\mathcal{M}_a), \mathrm{Im}\,(\psi_2^\dagger\mathcal{M}_a)}_{\mathrm{U}(2)}, \phi\Big]$$

... naturally explains emergent SO(5)!

... part of "duality web" in 2+1D:

[Seiberg, Senthil, Wang, Witten, Ann. Phys. '16]

[Karch & Tong, PRX '16]

[Thomson & Sachdev, arXiv '17]

...

Consequences of $NCCP^1 \Longleftrightarrow QED_3$-Gross-Neveu

Predictions for critical behavior:

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

(1) $[z^\dagger \sigma^z z] = [\phi] \quad \Rightarrow \quad \eta_{\text{QED}_3\text{-GN}} = \eta_{\text{Néel}} = \eta_{\text{VBS}}$ ... from $\phi \sim z^\dagger \sigma^z z$

... $\eta_{\text{Néel}} = \eta_{\text{VBS}}$ consistent with QMC [Sandvik, PRL '07]

(2) $[z^\dagger z] = [\phi^2] \quad \Rightarrow \quad \nu_{\text{QED}_3\text{-GN}} = \nu_{\text{Néel-VBS}}$ ... from $(\phi^2, \dots) \sim (z^\dagger \sigma^z z, z^\dagger z, \dots)$

(3) $[\bar\psi \sigma^z \psi] = [\phi^2] \quad \Rightarrow \quad [\bar\psi \sigma^z \psi] = 3 - 1/\nu_{\text{QED}_3\text{-GN}}$ ... from $\bar\psi \sigma^z \psi \sim z^\dagger z$

... nontrivial prediction fully within QED_3-GN

... allows quantitative test of duality conjecture

Here: (a) Existence of QCP in QED_3-GN model? ... prerequisite for duality

(b) Critical behavior? ... & comparison with duality prediction

QED$_3$-Gross-Neveu model: GN limit

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - \cancel{iea_\mu})\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

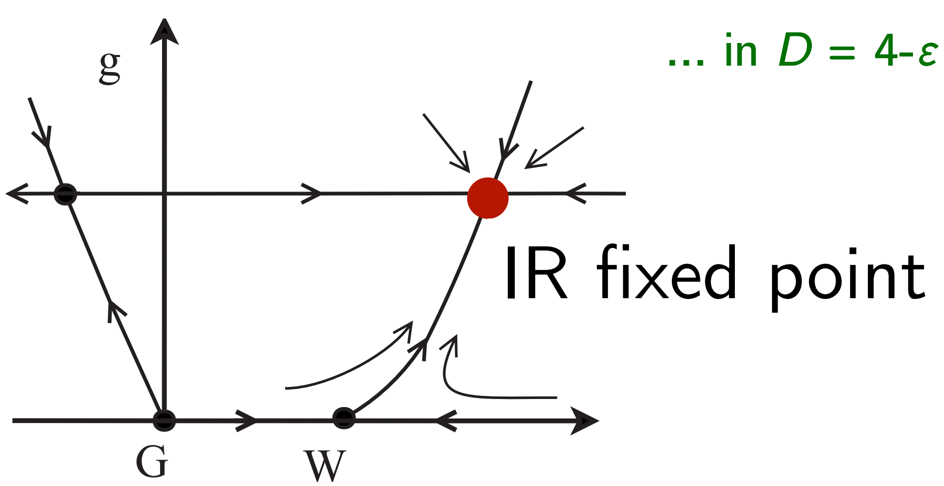
... in D = 2+1

... i = 1,...,2N

Gross-Neveu limit $(e^2 \to 0)$:

Gross-Neveu-Yukawa theory

... in D = 4-ε



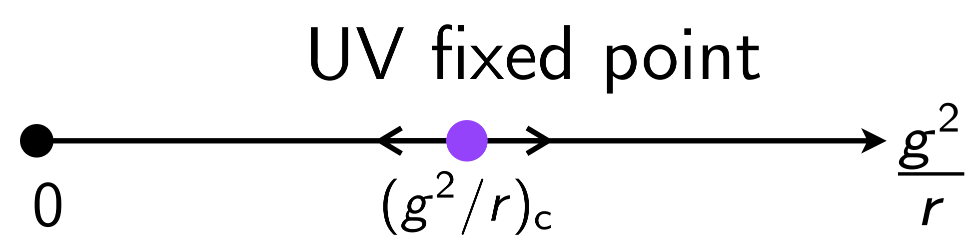
[Herbut, Juricic, Vafek, PRB '09]

$\Leftrightarrow$

Gross-Neveu theory

... in D = 2+ε

$$g\phi\bar{\psi}\psi + \frac{r}{2}\phi^2 \sim -\frac{g^2}{r}(\bar{\psi}\psi)^2$$



GN-QCP exists for all $2 < D < 4$ and can be understood as either ...

... **IR** fixed point of GNY or ... **UV** fixed point of GN

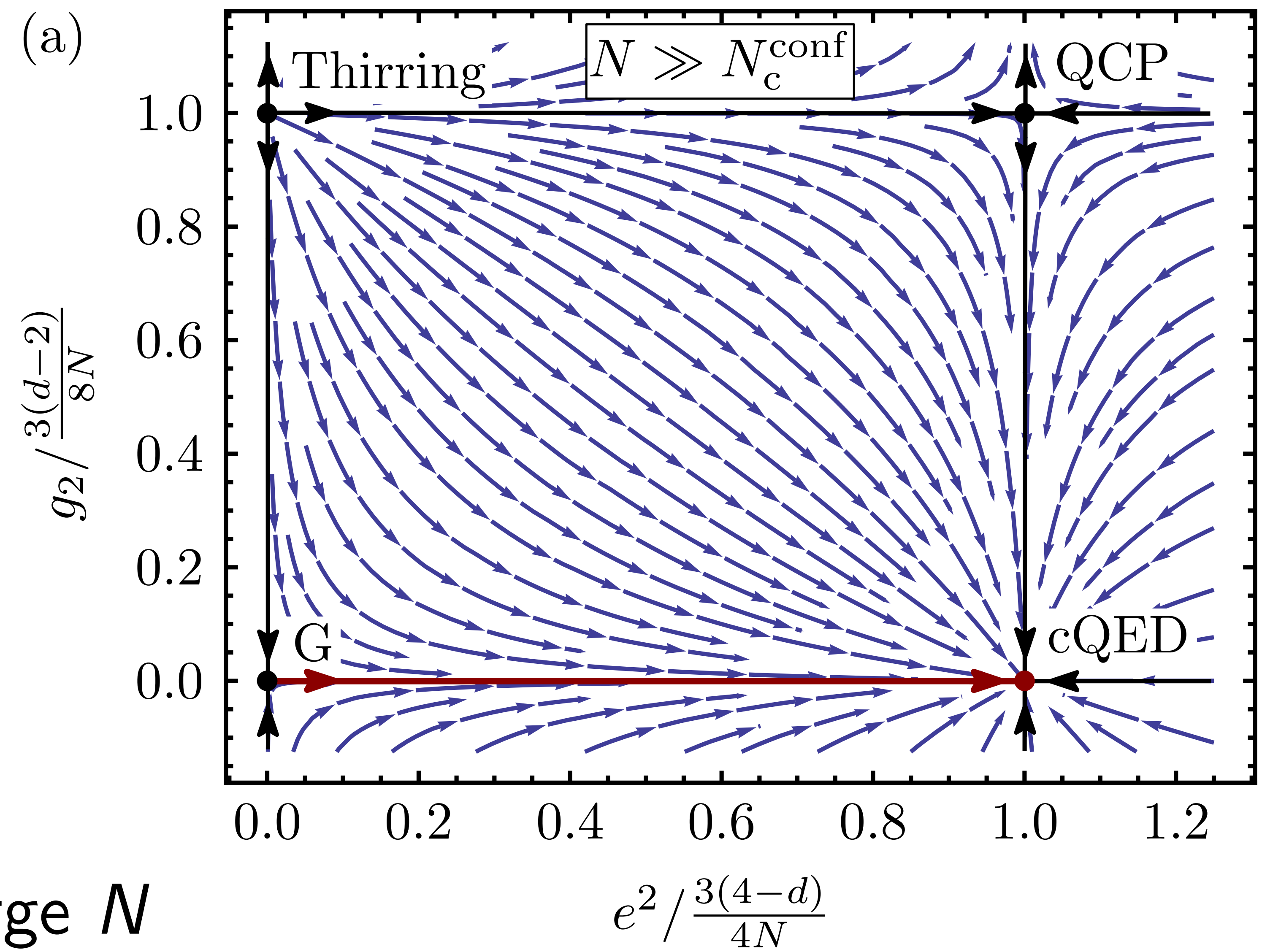
[Zinn-Justin, NPB '91]

QED$_3$-Gross-Neveu model: QED$_3$ limit

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - iea_\mu)\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

QED$_3$ limit $(g \to 0)$:



... conformal phase at large N

[LJ, PRD '16]

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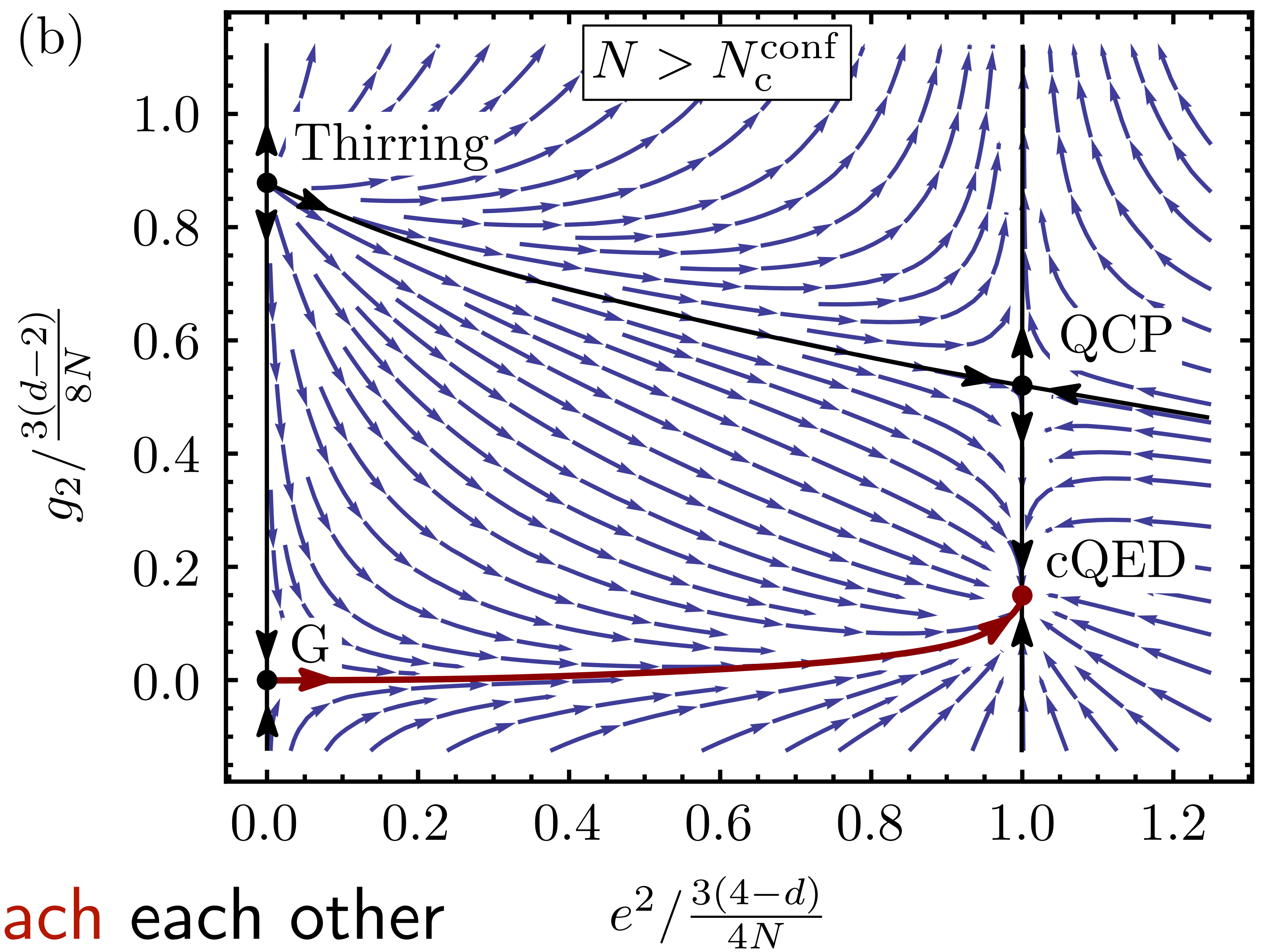

... QCP and cQED approach each other

[LJ, PRD '16]

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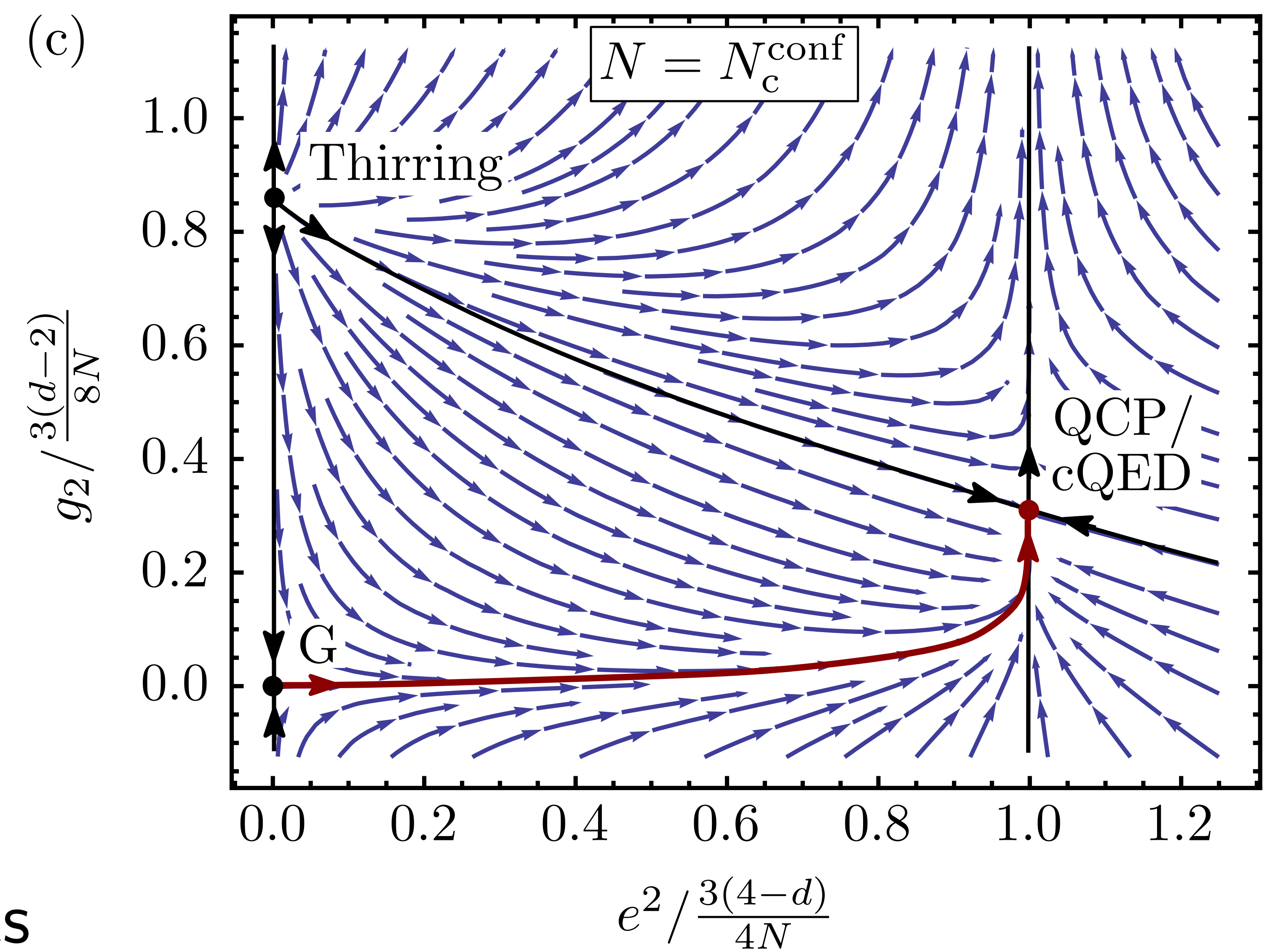

... collision of fixed points

[LJ, PRD '16]

QED$_3$-Gross-Neveu model: QED$_3$ limit

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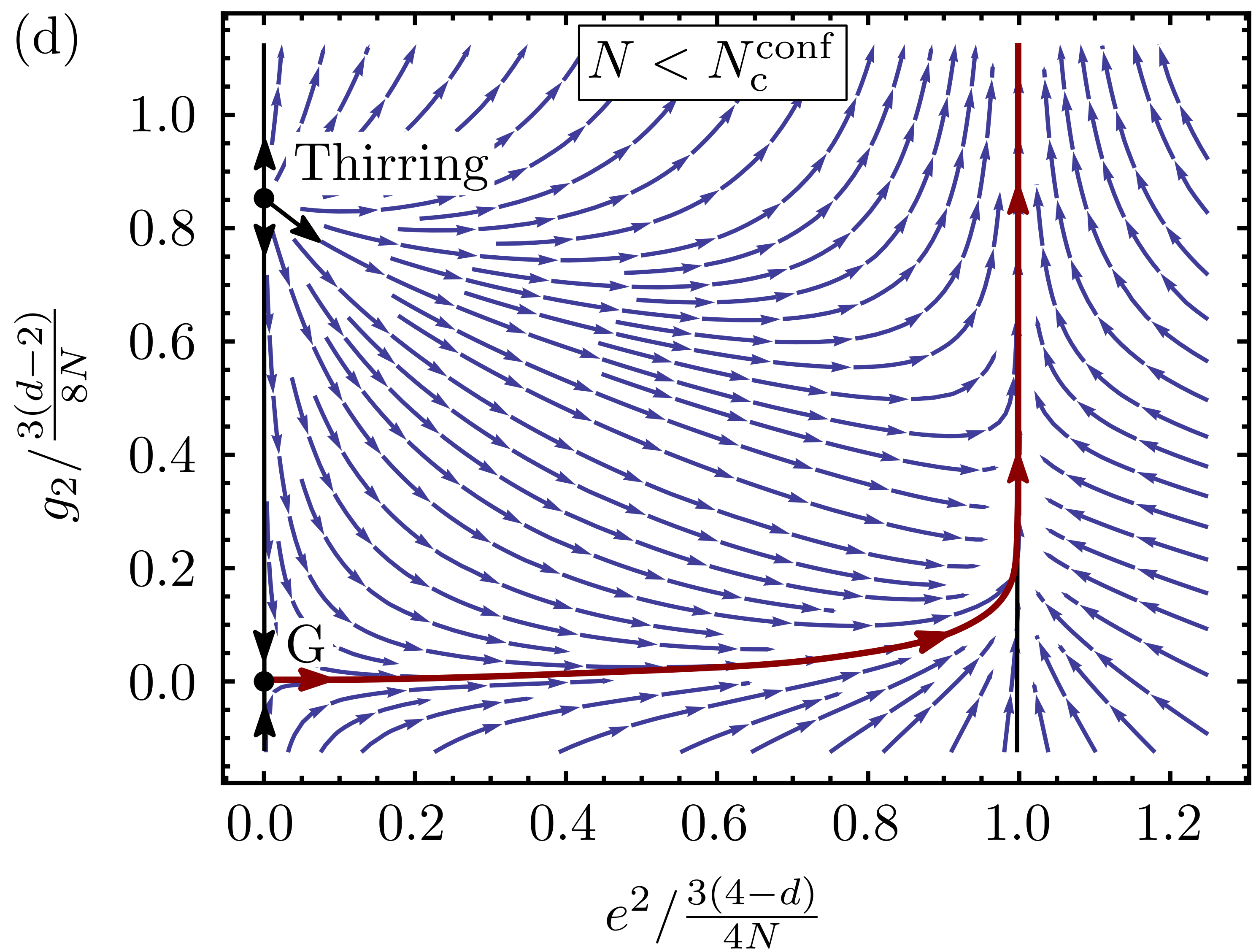

... runaway flow!

[LJ, PRD '16]

QED$_3$-Gross-Neveu model: QED$_3$ limit

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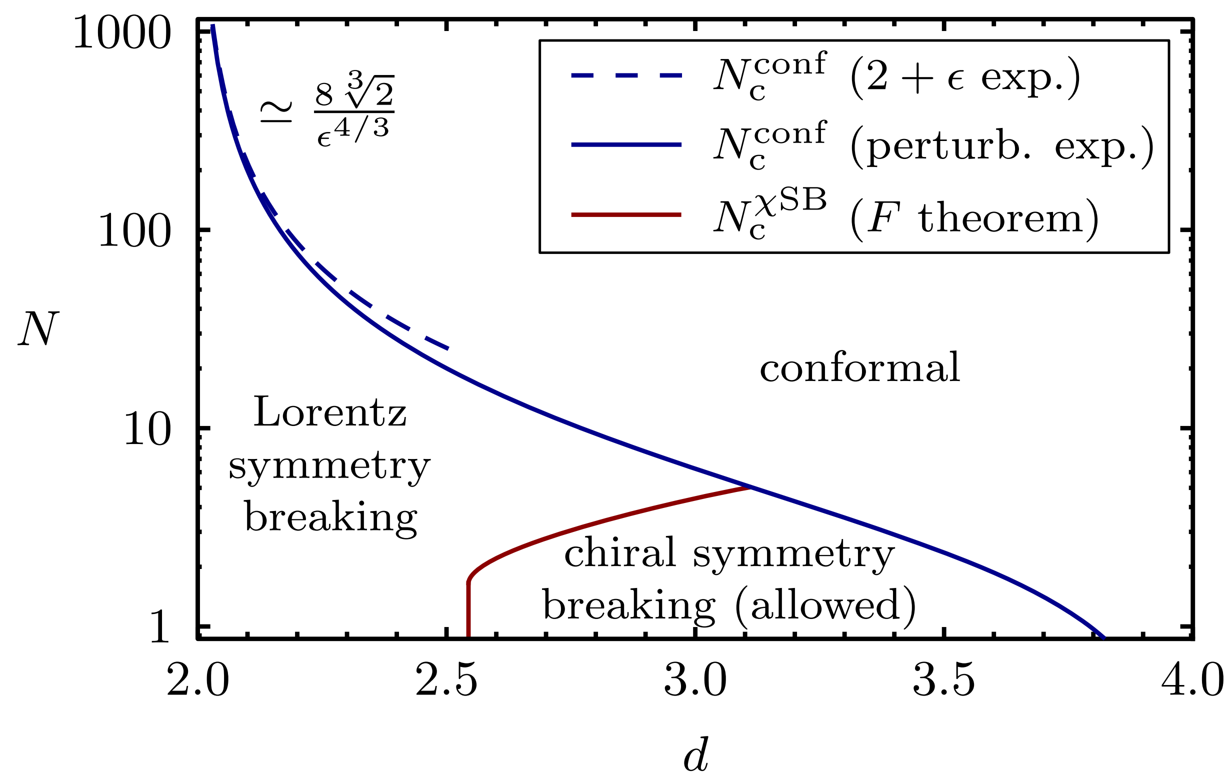
QED$_3$ limit $(g \to 0)$:

[LJ, PRD '16]



... QED$_3$ (potentially) unstable at low N!

[Appelquist, Nash, Wijewardhana, PRL '88]
[Braun, Gies, LJ, Roscher, PRD '14]
[Di Pietro *et al.*, PRL '16]
[Herbut, PRD '16]
...

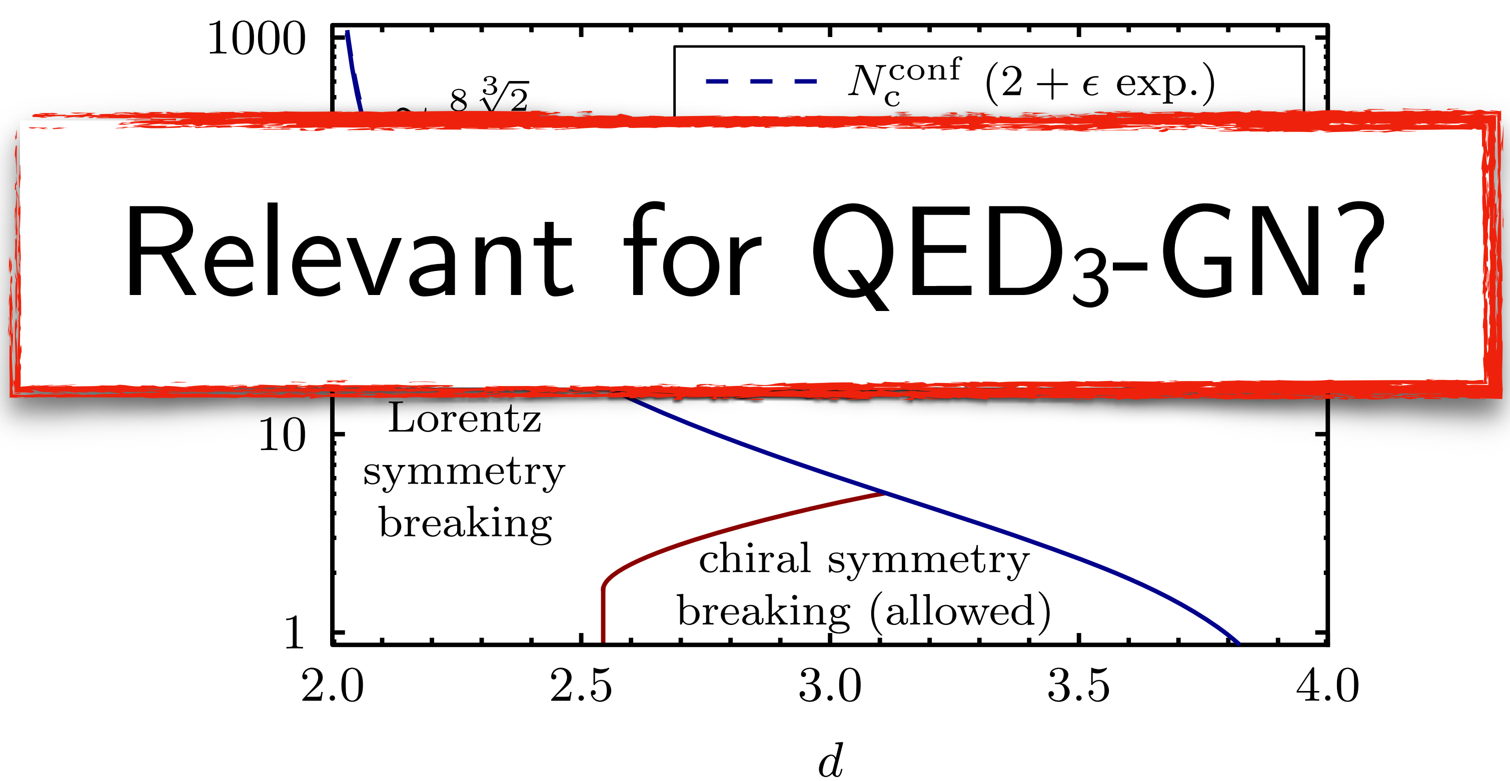
QED3-Gross-Neveu model: QED3 limit

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - iea_\mu)\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

QED3 limit $(g \to 0)$:

[LJ, PRD '16]



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[Braun, Gies, LJ, Roscher, PRD '14]
[Di Pietro *et al.*, PRL '16]
[Herbut, PRD '16]
...

QED$_3$-GN model: Fermionic RG

Integrate out ϕ:

$$g\phi\bar{\psi}_i\psi_i + \frac{r}{2}\phi^2 \mapsto u(\bar{\psi}_i\psi_i)^2$$

... u will also generate other four-fermion terms

General four-fermion theory compatible with U($2N$):

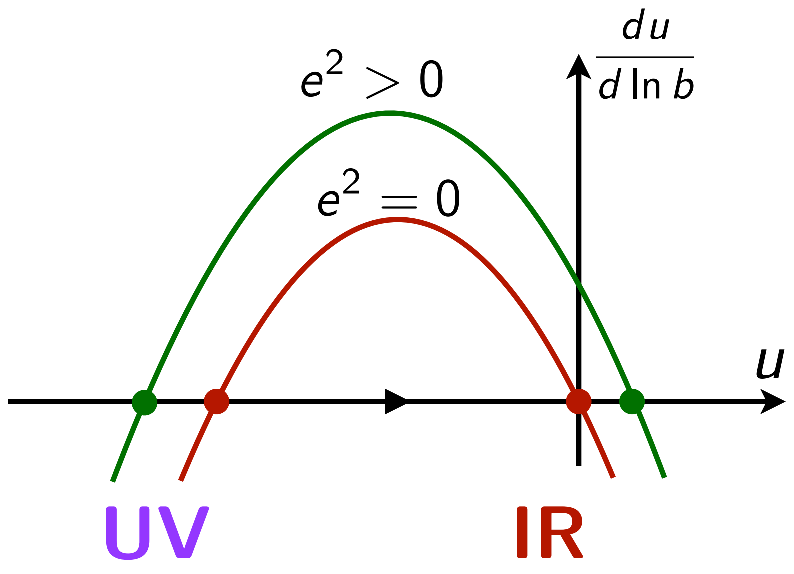
[Gies & LJ, PRD '10]

$$\mathcal{L}_\psi = \bar{\psi}_i\gamma_\mu(\partial_\mu - iea_\mu)\psi + u(\bar{\psi}_i\psi_i)^2 + v(\bar{\psi}_i\gamma_\mu\psi_i)^2$$

One-loop RG:

... at large N

$$\frac{du}{d\ln b} = -u - 8u^2 + 2e^4$$



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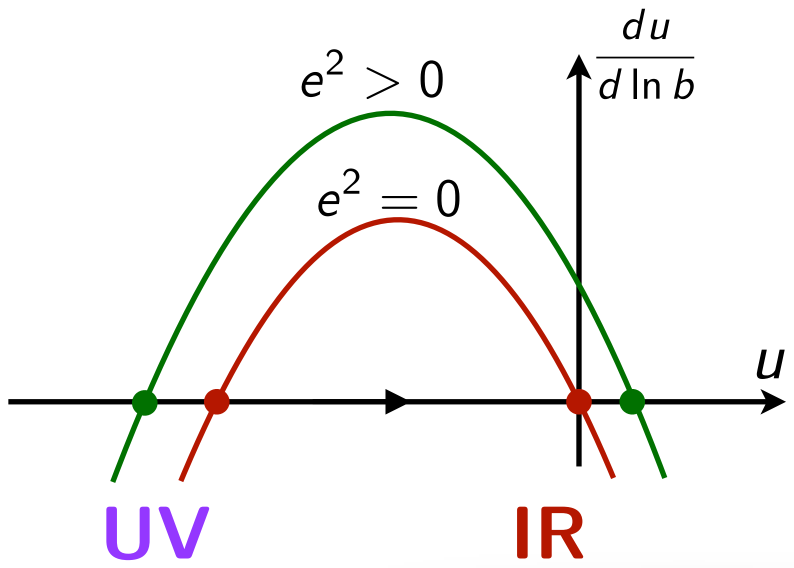


... gauge fluctuations "stabilize" QED$_3$-GN fixed point!

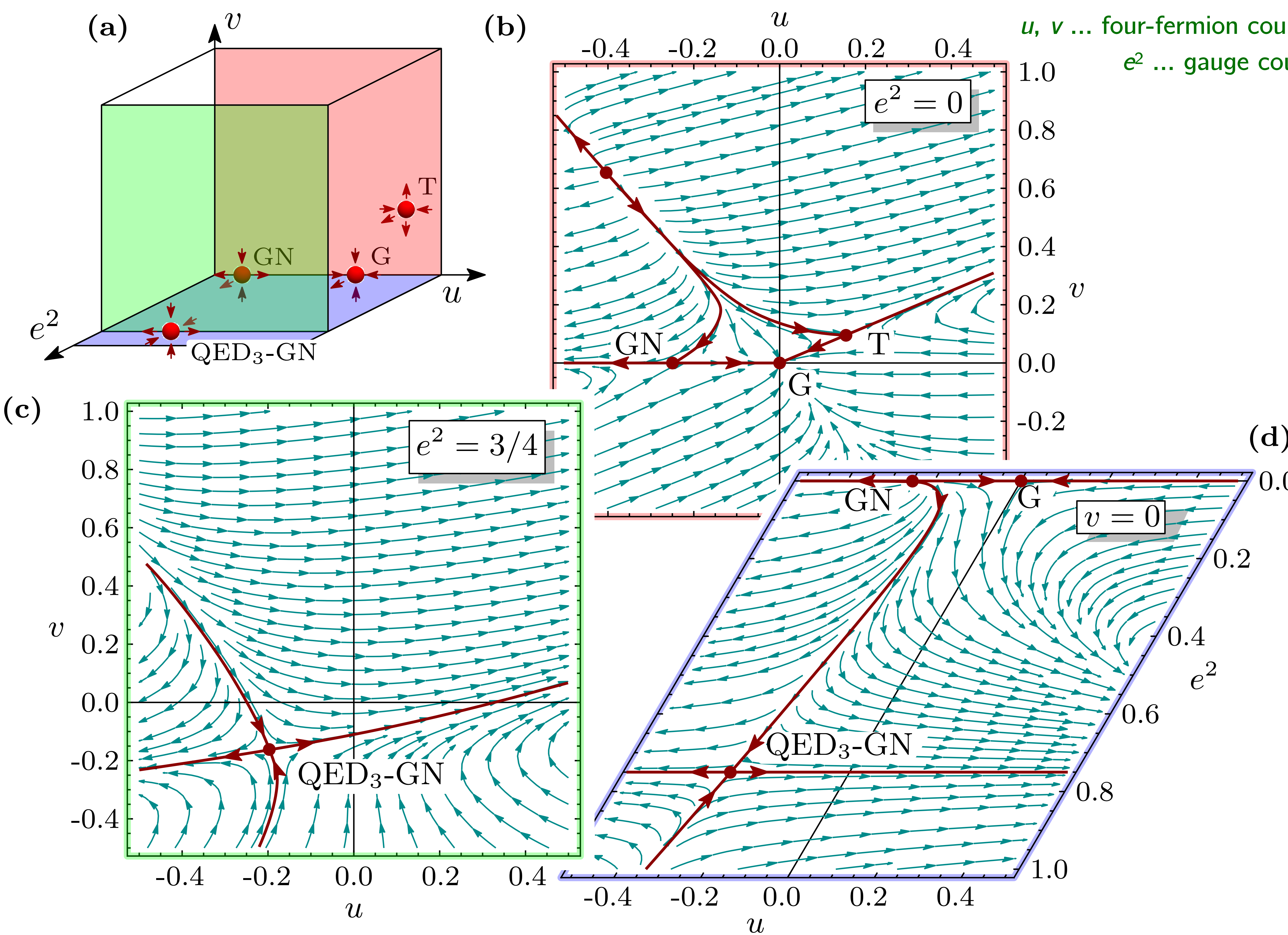
... in contrast to QED$_3$-Thirring fixed point

Fermionic RG: Flow diagram

[LJ & Y-C He, PRB '17]

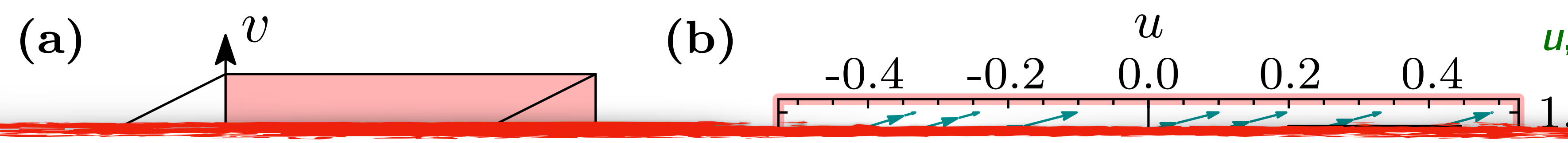
u, *v* ... four-fermion couplings

e^2 ... gauge coupling



Fermionic RG: Flow diagram

[LJ & Y-C He, PRB ’17]



u, v ... four-fermion couplings

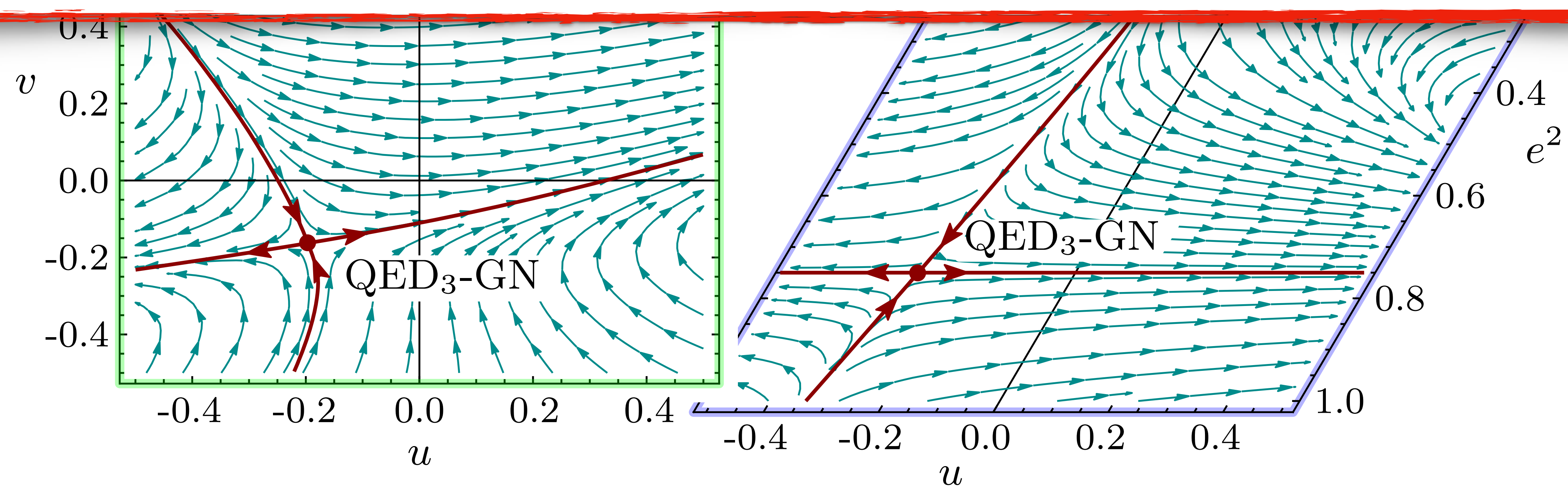
e^2 ... gauge coupling

Critical exponents:

$$1/\nu = 1 + \mathcal{O}(1/N)$$

$$[\bar\psi\psi] = 1 + \mathcal{O}(1/N) \quad \Rightarrow \quad \eta_\phi = 1 + \mathcal{O}(1/N)$$ ... large anom. dimension!

$$[\bar\psi\sigma^z\psi] = 2 + \mathcal{O}(1/N) \quad \Rightarrow \quad \eta_{\bar\psi\sigma^z\psi} = \mathcal{O}(1/N)$$ ... trivial



Gauged four-fermion model: Large-N expansion

Lagrangian:

$$\mathcal{L}_\psi = \bar{\psi}_i(\partial_\mu - iea_\mu)\psi_i + u\,(\bar{\psi}_i\psi_i)^2$$

. . . without $\partial^2\phi^2$ and ϕ^4 terms

Critical exponents in $2 < D < 4$: [Gracey, Ann. Phys. '93]

$$\eta_\phi = 4 - D + \frac{(D-1)\Gamma(D-1)}{[\Gamma(D/2)]^3\Gamma\left(\frac{4-D}{2}\right)}\frac{1}{N} + \mathcal{O}(1/N^2)$$
$$= 1 + \frac{16}{\pi^2 N} + \mathcal{O}(1/N^2)$$

... in D = 2+1

$$\nu^{-1} = D - 2 - \frac{\Gamma(D+1)}{2[\Gamma(D/2)]^3\Gamma\left(\frac{4-D}{2}\right)}\frac{1}{N} + \mathcal{O}(1/N^2)$$
$$= 1 - \frac{24}{\pi^2 N} + \mathcal{O}(1/N^2)$$

... in D = 2+1

QED$_3$-GN model: 4-ε expansion

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - iea_\mu)\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

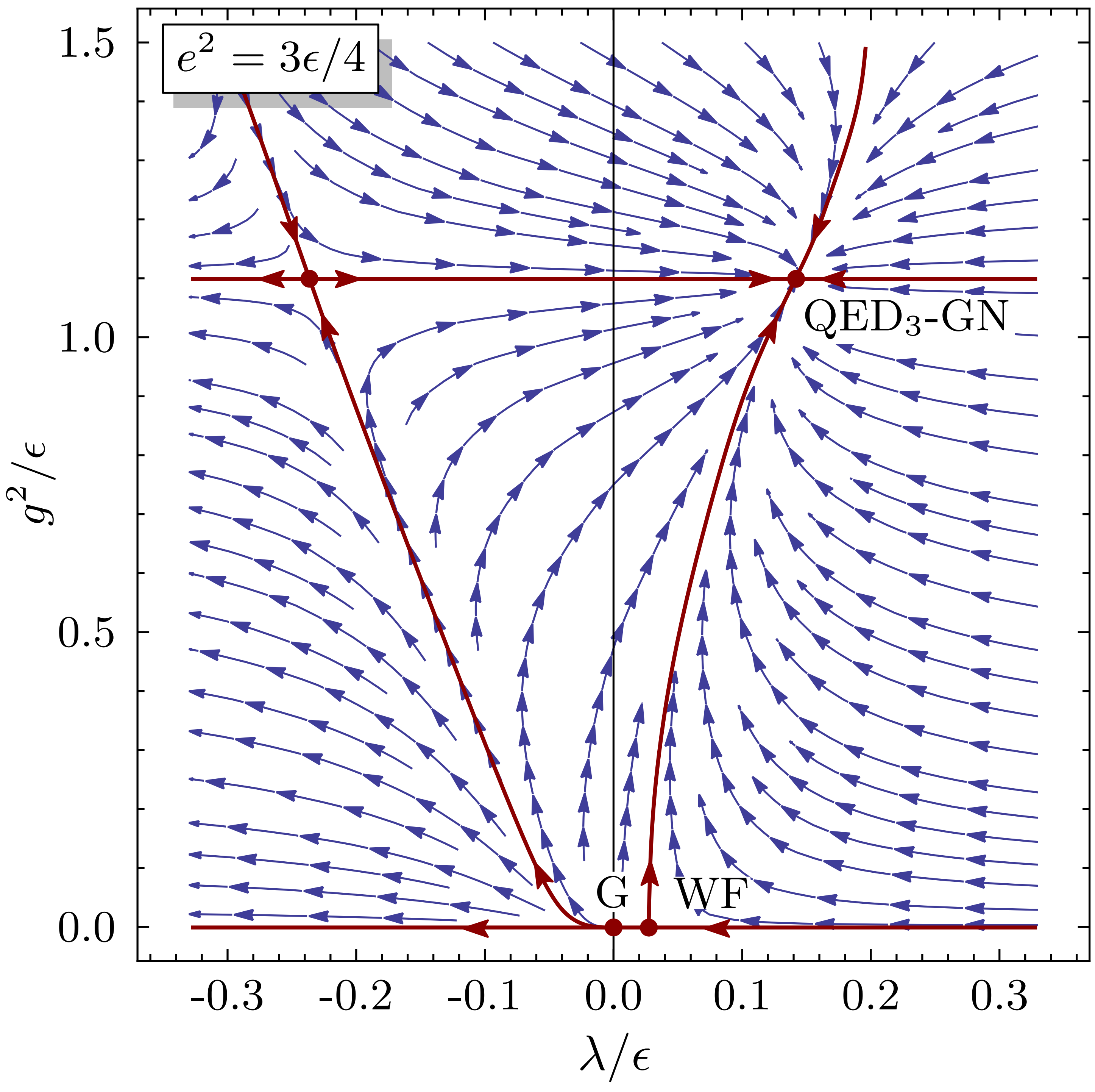
Engineering dimensions:

$$[e^2] = 4 - D, \qquad [g] = \frac{4-D}{2}, \qquad [\lambda] = 4 - D$$

... become simultaneously marginal near $D = 3+1$!

ε expansion in $D = 4$-ε possible!

QED$_3$-GN model: Flow diagram in $D = 4$-ε



... for $N = 1$

... fully IR stable fixed point

[LJ & Y-C He, PRB '17]

QED$_3$-GN model: Critical exponents at $O(\varepsilon)$

[LJ & Y-C He, PRB '17]

Gauge-field anomalous dimension:

$$\eta_a = 4 - D$$

... consequence of Ward identity

Gauge propagator:

$$G_a(p) \propto \frac{1}{|p|^{2-\eta_a}} = \frac{1}{|p|^{D-2}}$$

... exactly

... as in pure QED$_3$

Critical exponents:

$$\eta_\phi = \frac{2N+9}{2N+3}\epsilon + \mathcal{O}(\epsilon^2)$$

$$\nu = \frac{1}{2} + \frac{10N^2 + 39N + f(N)}{24N(2N+3)}\epsilon + \mathcal{O}(\epsilon^2)$$

... with $f(N) \equiv \sqrt{4N^4 + 204N^3 + 1521N^2 + 2916N}$

$$[\bar{\psi}\sigma^z\psi] = 3 - \frac{2N+6}{2N+3}\epsilon + \mathcal{O}(\epsilon^2)$$

... large $\mathcal{O}(\epsilon)$ corrections

QED$_3$-GN model: Critical exponents at $O(\varepsilon^3)$

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

Gauge-field anomalous dimension:

$$\eta_a = \epsilon + \mathcal{O}(\epsilon^4)$$

... consistent with Ward identity

Critical exponents (Large N):

$$\eta_\phi = \left(1 + \frac{3}{N}\right)\epsilon - \frac{\epsilon^2}{N} - \frac{3\epsilon^3}{4N} + \mathcal{O}(1/N^2, \epsilon^4)$$

$$\nu^{-1} = 2 - \left(1 + \frac{6}{N}\right)\epsilon + \frac{7\epsilon^2}{2N} + \frac{\epsilon^3}{N} + \mathcal{O}(1/N^2, \epsilon^4)$$

$$[\bar{\psi}\sigma^z\psi] = 3 - \left(1 + \frac{3}{2N}\right)\epsilon + \frac{\epsilon^2}{2N} + \frac{3}{8N}\epsilon^3 + \mathcal{O}(1/N^2, \epsilon^4)$$

... coincide with $1/N$ expansion of four-fermion model!

QED3-GN model: Critical exponents at $O(\varepsilon^3)$

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

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QED3-GN (**IR** FP) = gauged four-fermion (**UV** FP)

... coincide with $1/N$ expansion of four-fermion model!

QED$_3$-GN model: Critical exponents at $O(\varepsilon^3)$ for $N = 1$

Critical exponents ($N = 1$):

$$\eta_\phi = 2.2\epsilon - 0.222725\epsilon^2 + 16.8838\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\nu^{-1} = 2 - 3.90514\epsilon + 7.47146\epsilon^2 - 90.5962\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$[\bar{\psi}\sigma^z\psi] = 3 - 1.6\epsilon + 1.987\epsilon^2 - 17.46\epsilon^3 + \mathcal{O}(\epsilon^4)$$

... large $\mathcal{O}(\epsilon^3)$ corrections

Padé approximant:

$$[m/n] = \frac{a_0 + a_1\epsilon + \ldots a_m\epsilon^m}{1 + b_1\epsilon + \cdots + b_n\epsilon^n}$$

QED$_3$-GN model: 2+1D estimates ($N = 1$)

Padé estimates for $N = 1$:

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

Order	$[m/n]$	$1/\nu$	η_ϕ	$[\bar{\Psi}\sigma_z\Psi]$
ϵ^2	$[0/2]$	0.6602	–	2.5964
	$[1/1]$	0.6595	1.9978	2.2863
ϵ^3	$[1/2]$	0.6774	–	1.9894
	$[2/1]$	–	2.1971	1.6030

Mean values:

$$1/\nu = 0.67(1)$$

$$[\bar{\psi}\sigma^z\psi] \approx 2.12(50)$$

QED$_3$-GN vs. NCCP^1 duality: SO(5) scaling relation

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

Scaling relation from SO(5) symmetry:

$$[\bar{\psi}\sigma^z\psi] = 3 - 1/\nu$$

Our estimates:

$$[\bar{\psi}\sigma^z\psi] \approx 2.12(50) \qquad\qquad 3 - 1/\nu \approx 2.33(1)$$

... consistent with duality prediction!

QED$_3$-GN vs. NCCP^1 duality: AFM-VBS numerics

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

Duality prediction:

$$\eta_{\text{QED}_3\text{-GN}} = \eta_{\text{Néel}} = \eta_{\text{VBS}}$$

We find:

$$\eta_{\text{QED}_3\text{-GN}} > 1$$

... in agreement with $1/N$ expansion

[Gracey, Ann. Phys. '93]

AFM-VBS transition (MC):

[Sandvik, PRL '07; PRL '10]

[Nahum *et al.*, PRX '15]

[Shao *et al.*, Science '16]

...

$$\eta_{\text{Néel}} \approx \eta_{\text{VBS}} < 1$$

... inconsistent with duality prediction!

... similar inconsistency for ν

QED_3-GN vs. $NCCP^1$ duality: Possible scenarios

QED_3-GN critical behavior is ...

... consistent with SO(5) duality relation

... inconsistent with numerics for AFM-VBS transition

Three potential scenarios:

(A) Only weak duality holds

... i.e., not the same IR fixed points

(B) Perturbative approach fails

... i.e., emergence of SO(5) correctly predicted,
but absolute values incorrect

(C) No unitary fixed point

... i.e., annihilation & complexification of fixed point
[Nahum *et al.*, PRX '15]

Conclusions

QED$_3$-Gross-Neveu model ... [LJ & Y-C He, PRB '17]

... interesting due to possible duality with NCCP^1

... i.e., theory of Néel-VBS deconfined critical point

... has a stable fixed point

... even when cQED$_3$ collides with another QCP

... prerequisite for duality to hold

... critical behavior computable within 4-ε expansion

... all couplings simultaneously marginal

... three-loop exponents: [Ihrig, LJ, Mihaila, Scherer, arXiv:1807.XXXX]

- consistent with SO(5) scaling relation
- inconsistent with AFM-VBS numerics ... large anomalous dimension η_ϕ

... however: large η_ϕ necessary for emergent SO(5)

[Nakayama & Ohtsuki, PRL '16]