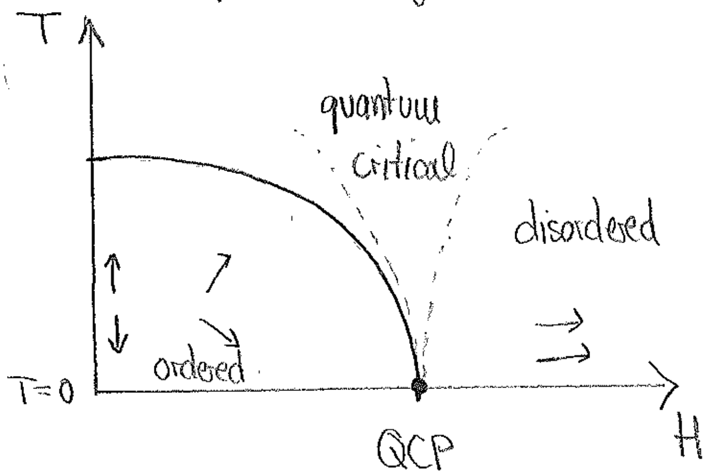
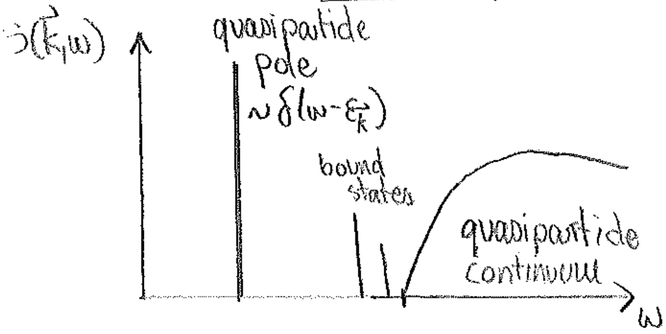
Emergent symmetries at quantum critical points

Generic phase diagram near a QCP (AFM in field):



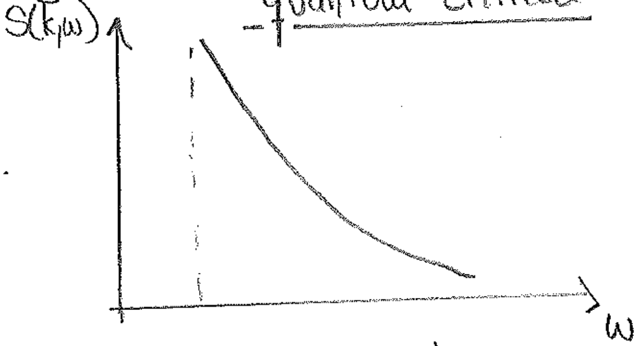
Spectrum of excitations:

<u>stable phases</u>



vs.

<u>quantum critical</u>



e.g., Fermi liquids : $C_V \sim T$
ordered magnets : $C_V \sim \frac{1}{T} e^{-\frac{\Delta}{k_B T}}$ ← gap

e.g., $C_V \sim T^{d/z}$ ← dynamical critical exponent

"novel state of matter"

Scale invariance (classical critical points):

Correlation length: $\xi \sim \left| \frac{T_c - T}{T_c} \right|^{-\nu}$ diverges!

Specific heat: $C_V \sim \left| \frac{T_c - T}{T} \right|^{\alpha}$

Order parameter: $m \sim \left(\frac{T_c - T}{T_c} \right)^{\beta}$ $(T < T_c)$

power laws $\Longleftrightarrow$ scale invariance

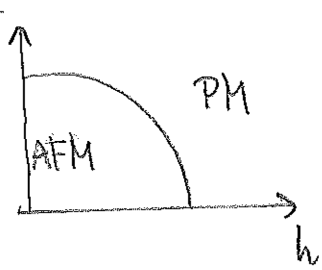
Universality:

Power laws determined by <u>dimension</u>, <u>symmetry</u>, (range of interactions)

Example (3D H'berg AFM, $T>0$)

Microscopic Hamiltonian:

$$\mathcal{H} = J \left[\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i \right]$$

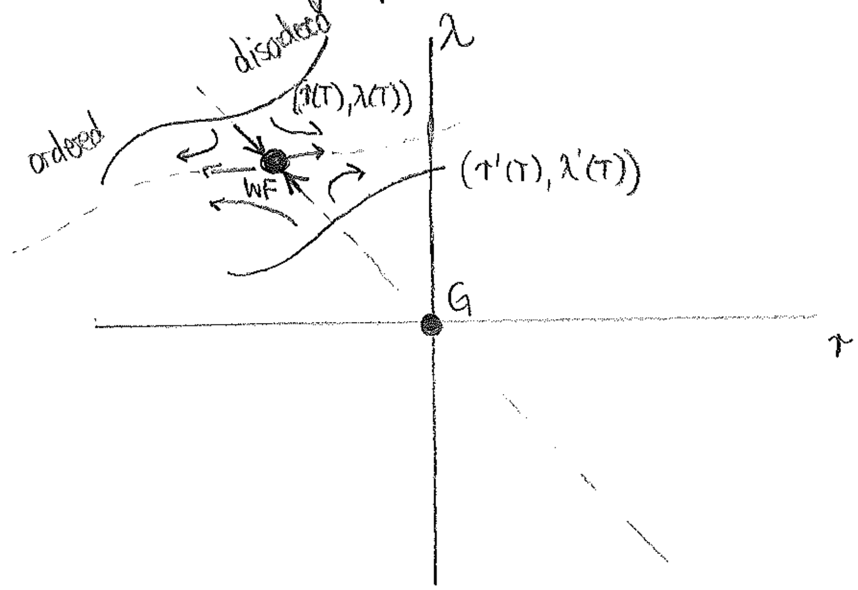


Continuum limit: $\frac{a}{\xi} \to 0$

$$S = \int d^d r \left[\frac{1}{2} (\vec{\nabla} \phi_i)^2 + \frac{r}{2} \phi_i^2 + \lambda (\phi_i^2)^2 + \dots \right], \quad i = x, y$$

with $r = r(T, h)$, $\lambda = \lambda(T, h)$

Renormalization group flow (Wilson):



⇒ different theories exhibit exactly the <u>same</u> critical behavior!

Symmetry-breaking perturbations:

E.g.: $\mathcal{H}_{pert} = J' \sum_{\langle ij \rangle} \left[(S_i^x S_j^x)^2 + (S_i^y S_j^y)^2 + (S_i^z S_j^z)^2 \right]$ cubic anisotropy

$$S_{pert} = \int d^d \vec{r} \; \lambda' (\phi_x^4 + \phi_y^4 + \phi_z^4) + \dots$$

RG analysis: λ' is relevant at WF!

Critical behavior:

H'berg ($\lambda'=0$)	Cubic ($\lambda' \neq 0$)
$\eta = 0.038$	$\eta = 0.033$
$\nu = 0.711$	$\nu = 0.706$

General analysis: Emergent SO(N) symmetry only for N<3

[Calabrese, PRB'03]

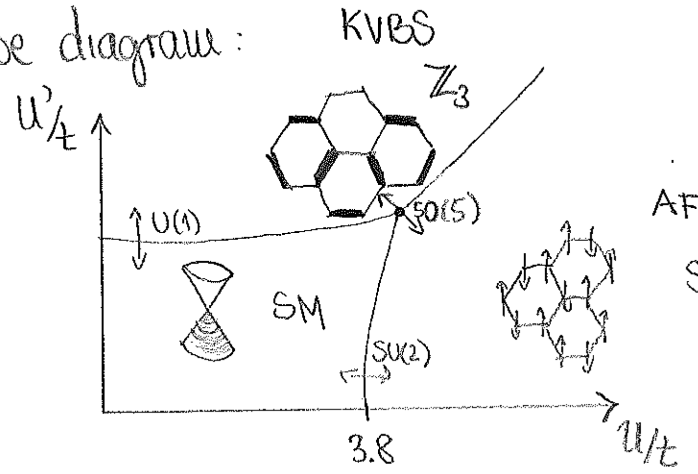
Quantum critical Dirac systems ($T=0$):

(Extended) Hubbard model on the honeycomb lattice:

$$\mathcal{H} = -t \sum_{\langle ij\rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \ldots$$

Phase diagram:

[keep!!!]



Continuum limit (Kekulé transition):

$$S = \int d^2\vec{r} \int d\tau \left[\phi^* \left(-\partial_\mu^2 + r\right)\phi + \overbrace{g\left(\phi^3 + \phi^{*3}\right)}^{\mathbb{Z}_3} + \lambda |\phi|^4 \right.$$
$$\left. + \bar{\psi}\gamma_\mu \partial_\mu \psi + y\left((\mathrm{Re}\,\phi)\, \bar{\psi} i \gamma_3 \psi + (\mathrm{Im}\,\phi)\, \bar{\psi} i \gamma_5 \psi \right) \right]$$

RG analysis: $g \to 0$ at QCP $\Rightarrow$ <u>emergent</u> U(1) symmetry!

Continuum limit (multicritical point):

$$S = \int d^2\vec{r} \int d\tau \Big[\tfrac{1}{2}\phi_a(-\partial_\mu^2 + r_1)\phi_a + \tfrac{1}{2}\chi_b(-\partial_\mu^2 + r_2)\chi_b$$

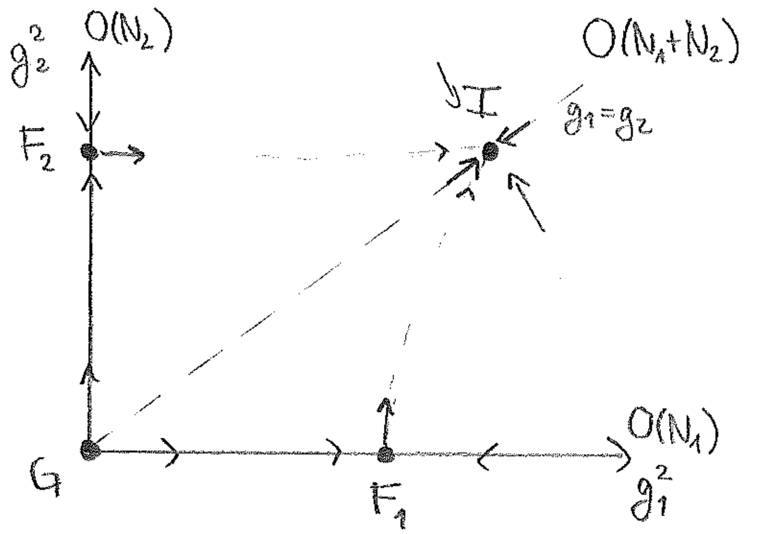
$$+ \lambda_1 (\phi_a^2)^2 + \lambda_2 (\chi_b^2)^2 + 2\lambda_3 \phi_a^2 \chi_b^2$$

$$+ \bar{\psi}\gamma_\mu \partial_\mu \psi + g_1 \phi_a \bar{\psi}\gamma_0 M_a^\phi \psi + g_2 \chi_b \bar{\psi}\gamma_0 M_b^\chi \psi \Big]$$

with $a = 1, \dots, N_1$ and $b = 1, \dots, N_2$ with $\{M_a^\phi, M_b^\chi\} = 0$

Symmetry: $\begin{cases} O(N_1+N_2) & \text{for } \lambda_1 = \lambda_2 = \lambda_3 \text{ and } g_1 = g_2 \\ O(N_1)\times O(N_2) & \text{otherwise} \end{cases}$

RG flow:



<u>Emergent</u> $O(N_1+N_2)$ for all N_1, N_2!

Conclusion:

- Classical criticality ($d=3$): Emergent $O(N)$ only for $N<3$
- Quantum criticality with gapless Dirac fermions ($d+z=3$): Emergent $O(N)$ for all N

Outlook:

- Kitaev-Heisenberg in field (α-$RuCl_3$, iridates, ...)
- Transition into and out of spin liquid phases