Unconventional criticality in fermionic systems

Lukas Janssen

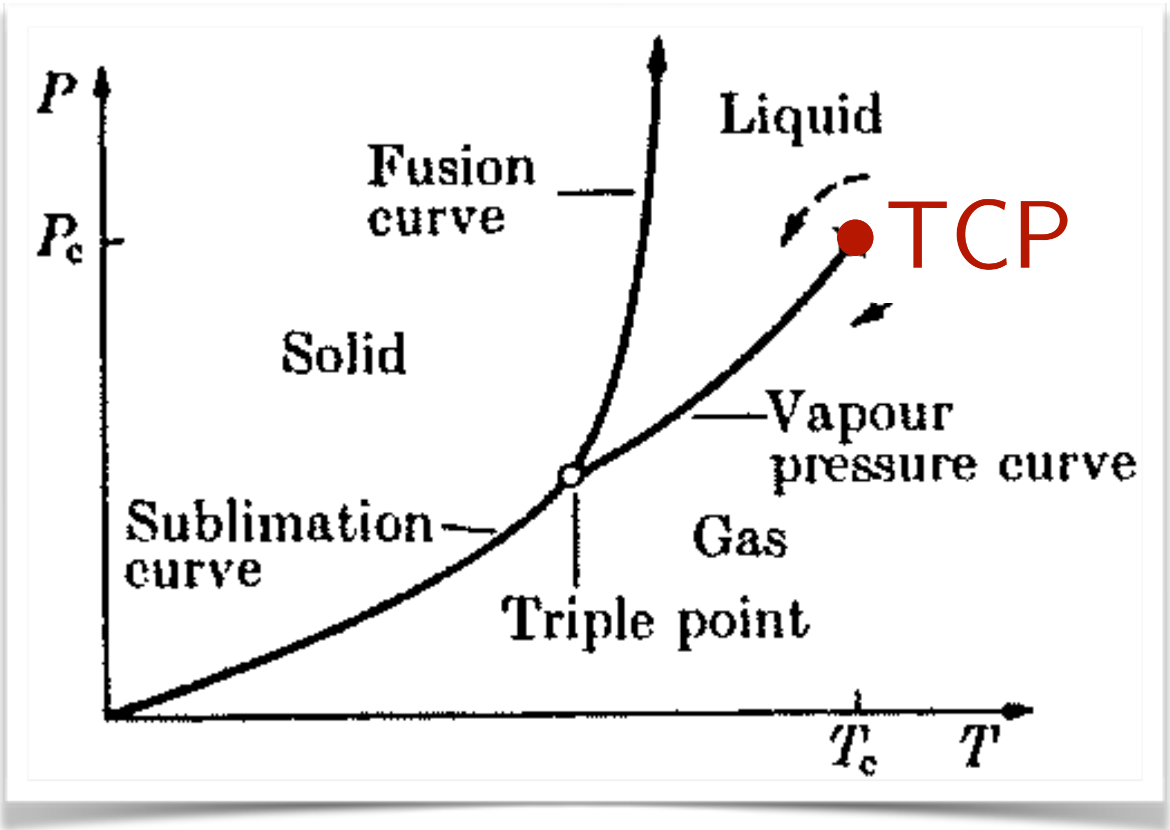
L.J. and Y.-C. He, Phys. Rev. B **96**, 205113 (2017)
B. Ihrig, L.J., L. Mihaila, and M. Scherer, arXiv:1807.04958 (PRB accepted)

TECHNISCHE UNIVERSITÄT DRESDEN

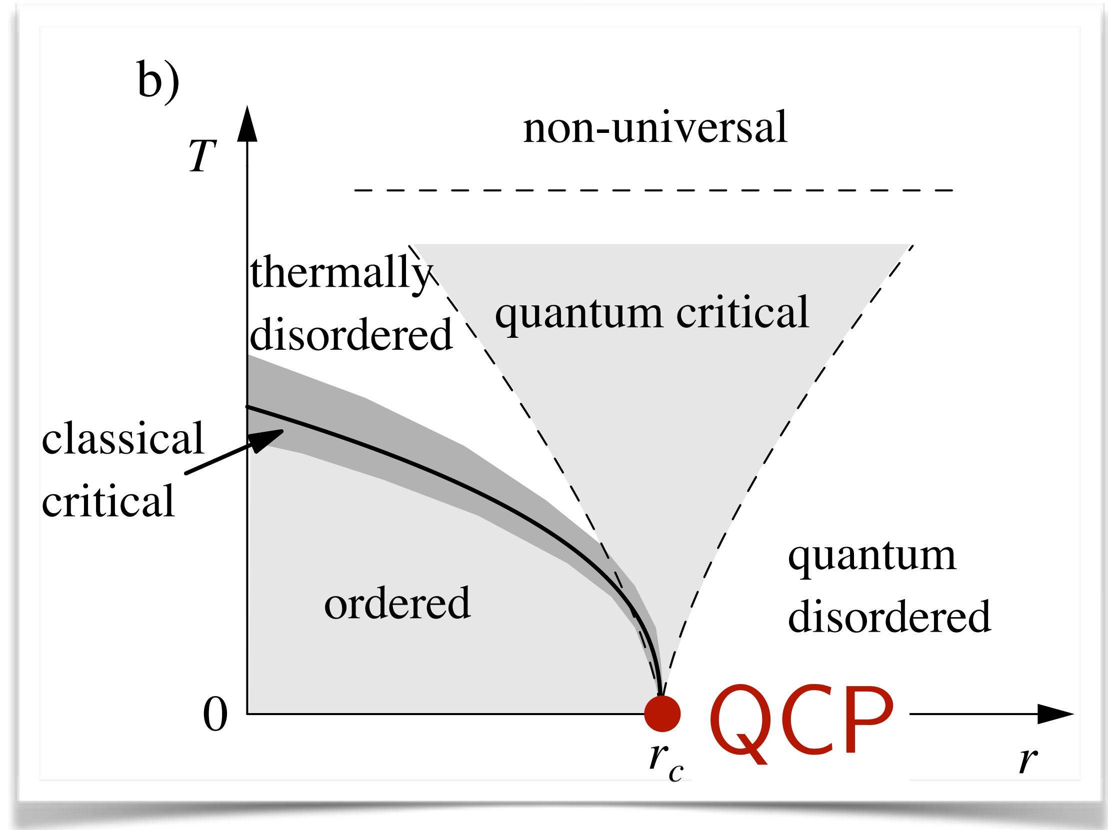
SFB 1143

Thermal critical point (TCP) vs. quantum critical point (QCP)

Thermal:



Quantum:



[M Vojta, Rep. Progr. Phys. '03]

... driven by thermal fluctuations

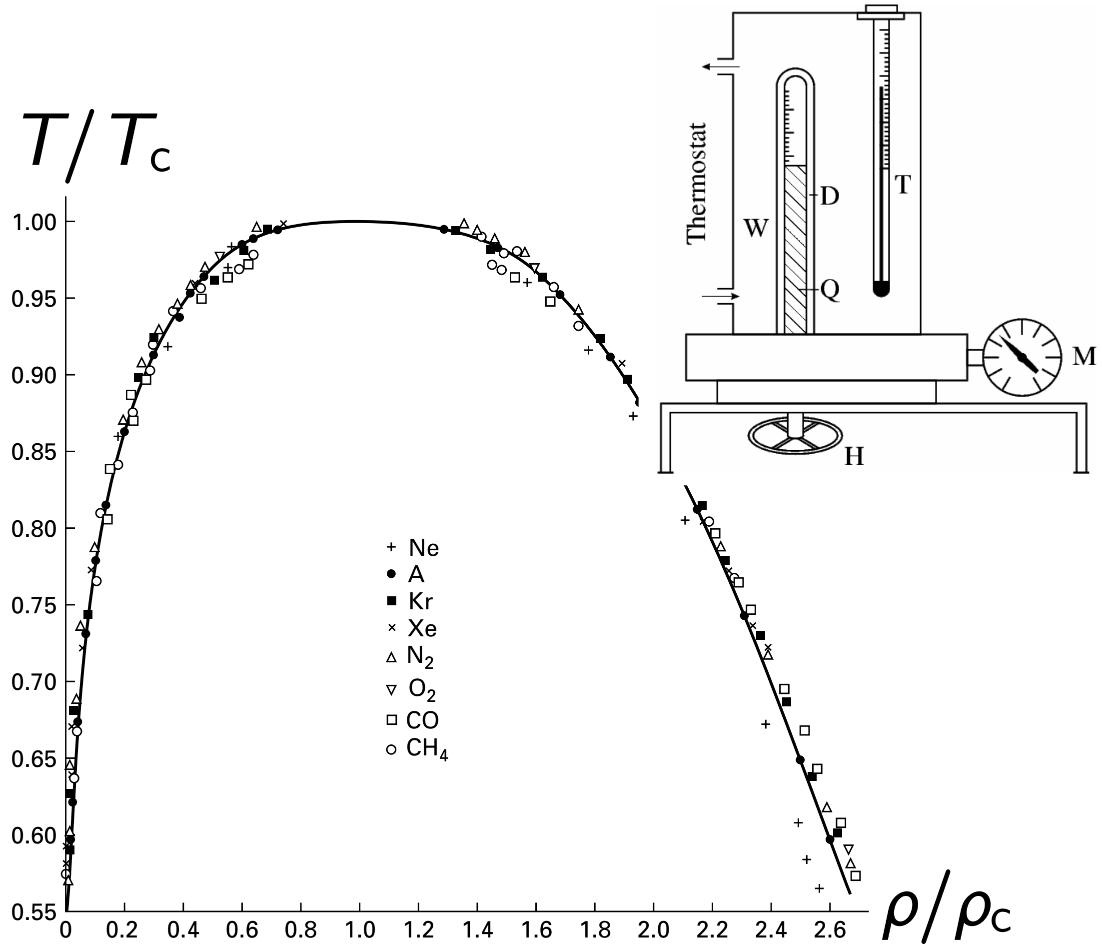
... tuned by temperature

... driven by quantum fluctuations

... tuned by pressure, field, ...

TCP vs. QCP: Example

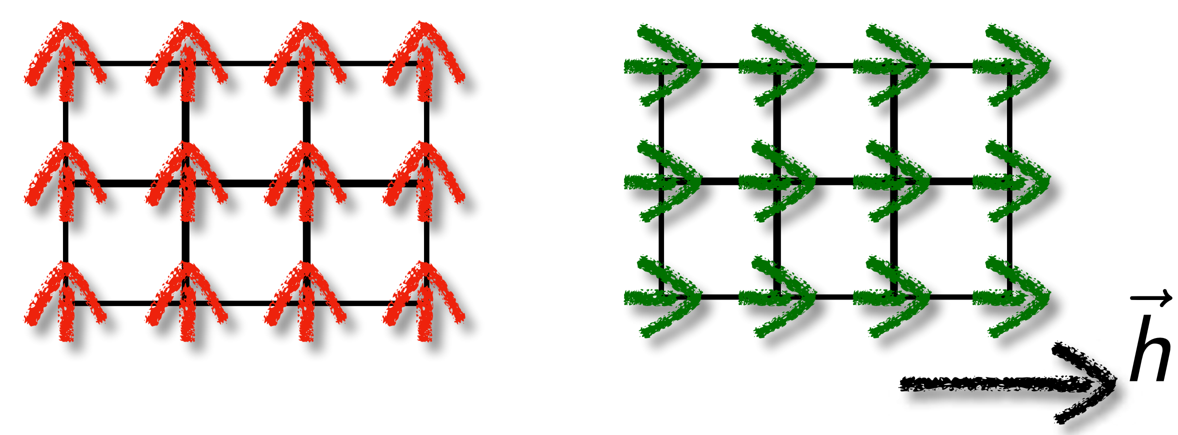
Liquid-gas transition: ... in 3D



[Guggenheim, J. Chem. Phys. '45]

Transverse-field Ising model: ... in 2D

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i^z S_j^z - h \sum_i S_i^x$$



$h/J < (h/J)_c$ $\qquad$ $h/J > (h/J)_c$

Order parameter:

$$|\rho_L - \rho_G| \propto |T - T_c|^\beta, \quad \beta \approx 0.33$$

Order parameter:

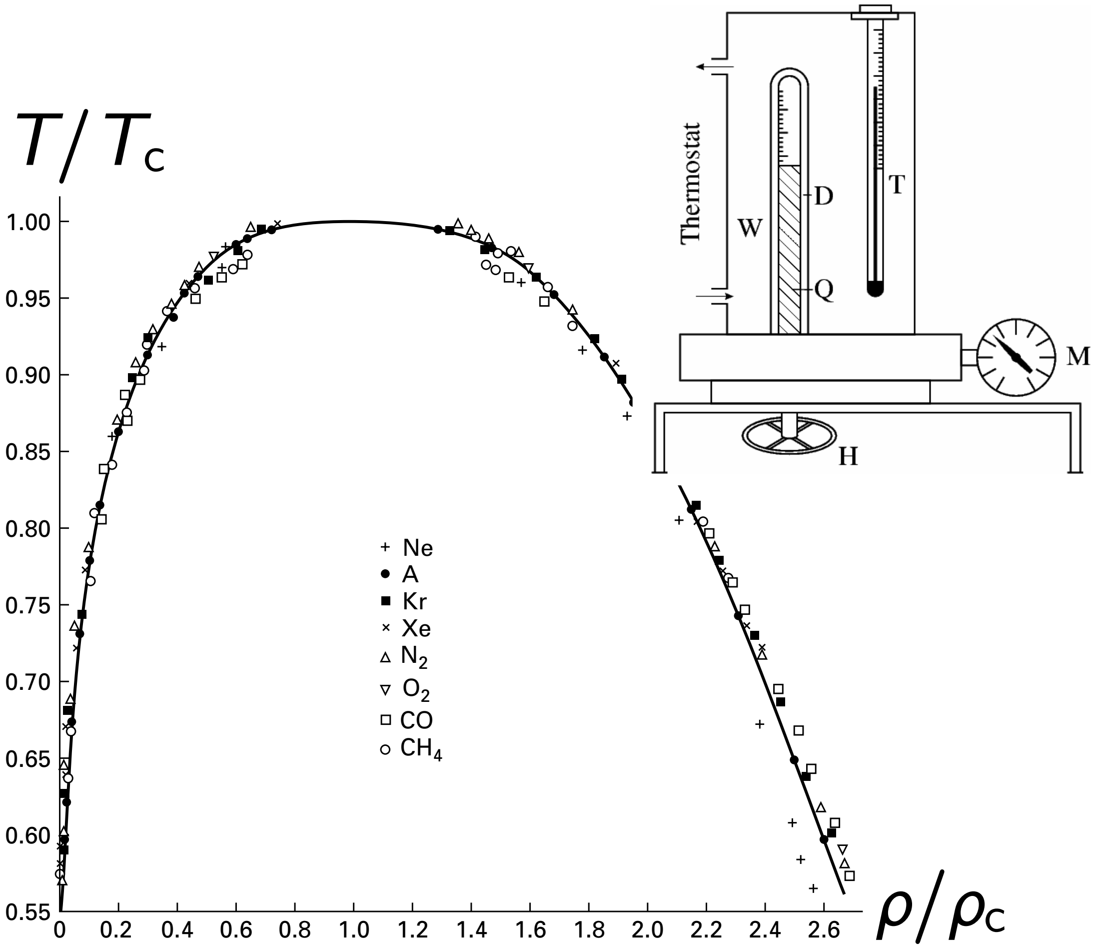
$$|m_z| \propto |J - J_c|^\beta, \quad \beta \approx 0.33$$

... and other exponents also agree

[Elliot *et al.*, PRL '70]

TCP vs. QCP: Example

Liquid-gas transition: ... in 3D



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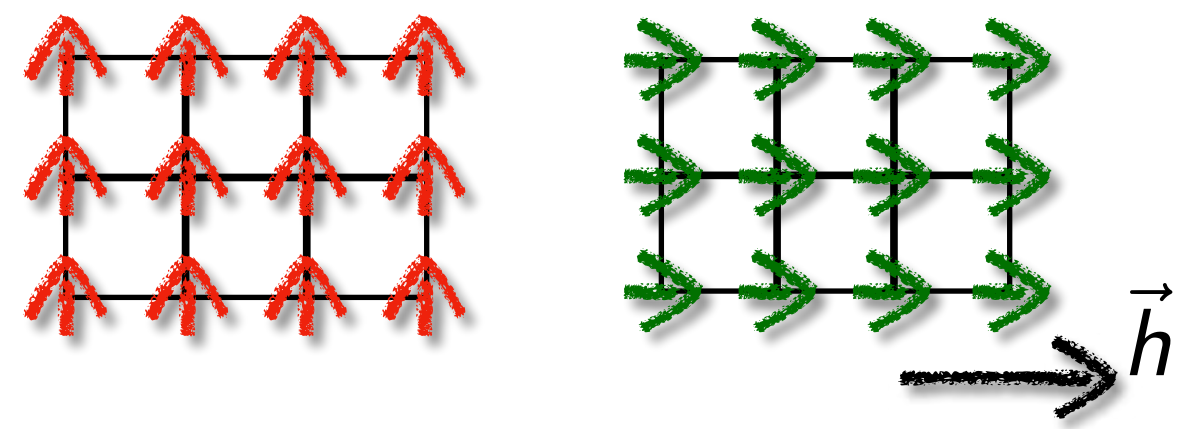


$h/J < (h/J)_c$ $h/J > (h/J)_c$

Quantum-to-classical mapping:

$$\mathrm{TCP}(d+z) \iff \mathrm{QCP}(d)$$

z ... dynamical critical exponent

Landau-Ginzburg-Wilson theory

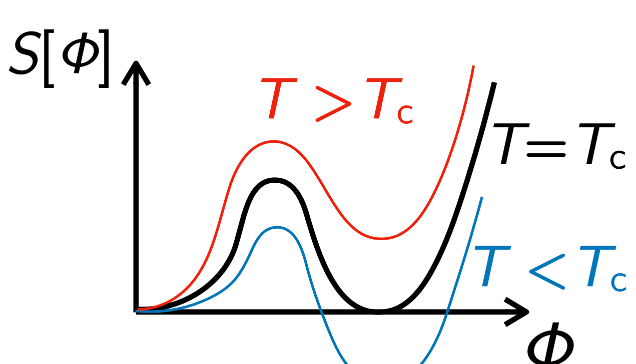
Assumption:

Transition uniquely characterized by order-parameter fluctuations

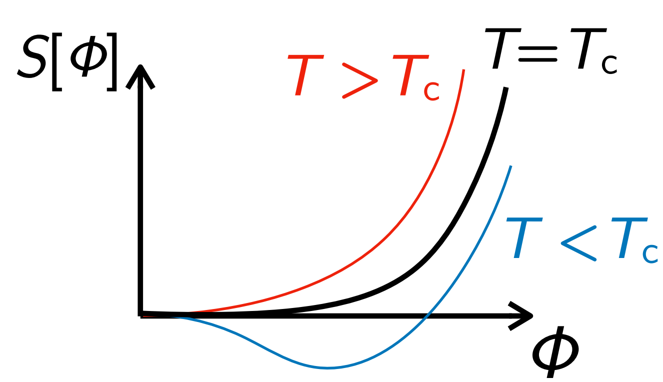
Continuum field theory: $S[\phi] = \int d^d\vec{r}\,\left[\frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \lambda\phi^4 + \ldots\right]$

Φ ... order-parameter field

Mean-field theory (Landau):



$S[\phi] \propto \frac{r}{2}\phi^2 + \mu\phi^3 + \lambda\phi^4$



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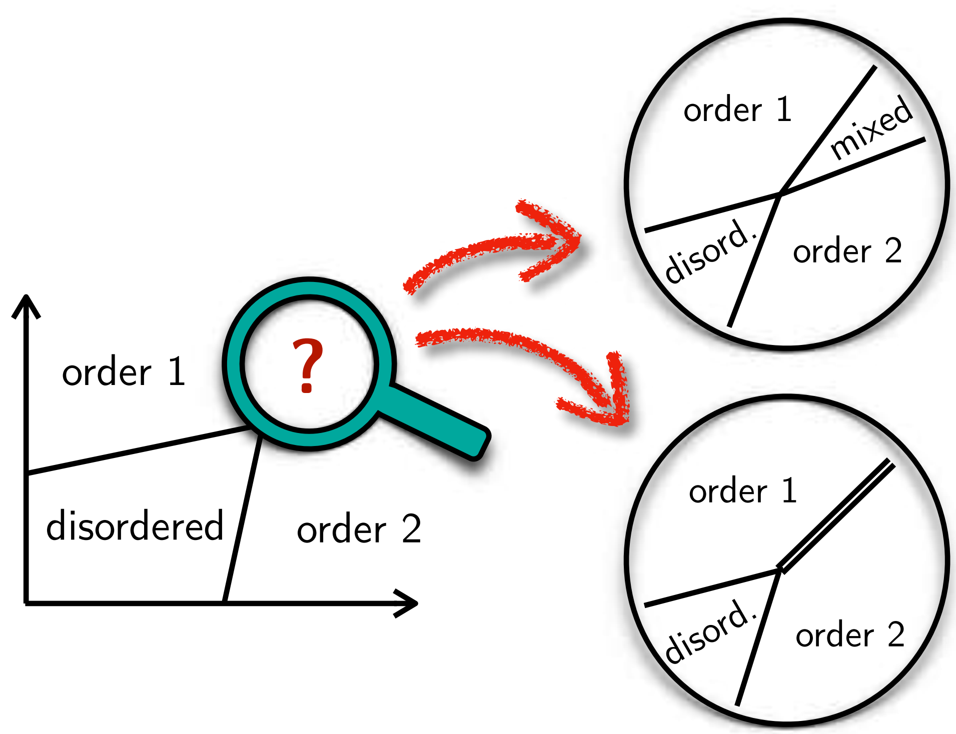
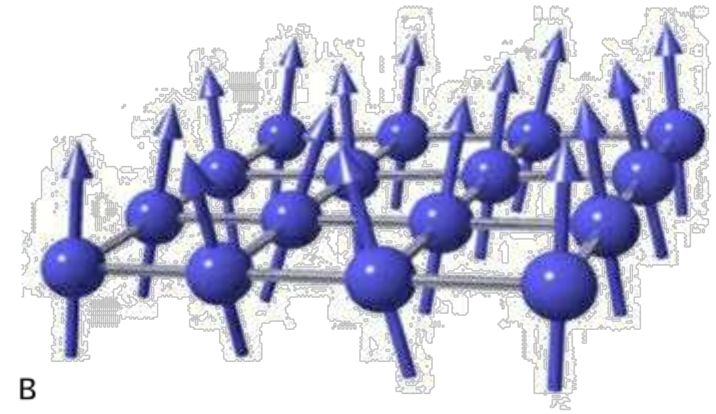


Landau-Ginzburg-Wilson theory: Successes

Ansatz works remarkably well …

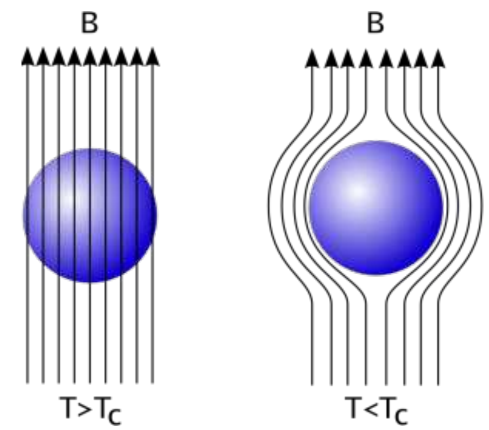


… magnets ($\vec{\varphi}$)

[Wilson & Fisher, PRL ’72]

… superconductors (ϕ, ϕ^*, a_μ)

[Halperin, Lubensky, Ma, PRL ’74]



… Mott transition in Fermi-point systems $(\vec{\varphi}, \psi^\dagger, \psi)$

2D Dirac:

[Herbut, PRL ’06]

[Raghu, Qi, Honerkamp, Zhang, PRL ’08]

[Assaad & Herbut, PRX ’13]

[LJ & Herbut, PRB ’14]

…

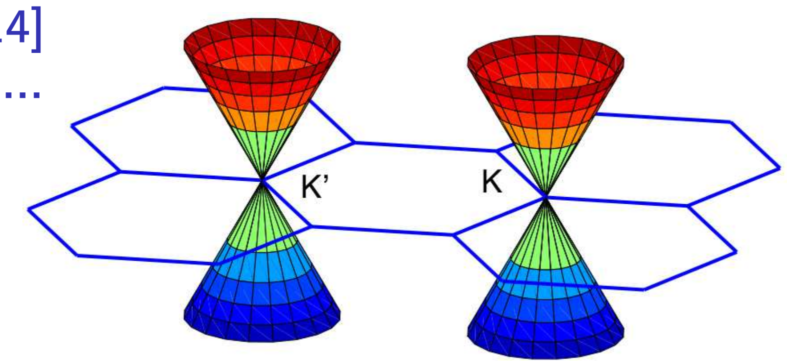
2D QBT:

[Sun *et al.*, PRL ’09]

[Scherer, Uebelacker, Honerkamp, PRB ’12]

[Pujari, Lang, Murthy, Kaul, PRL ’16]

…

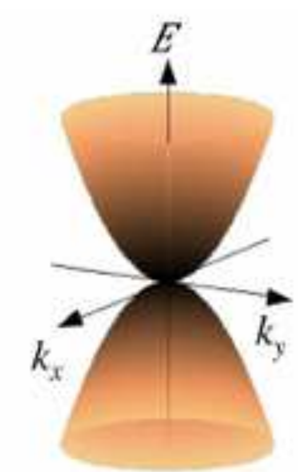


3D QBT:

[Herbut & LJ, PRL ’14]

[LJ & Herbut, PRB ’15; ’17]

…



… and more

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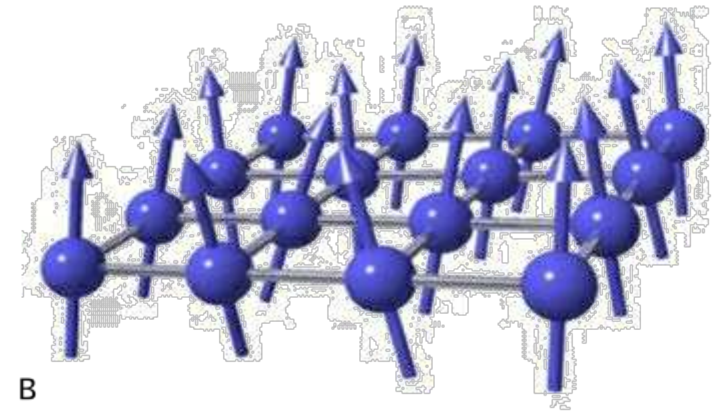

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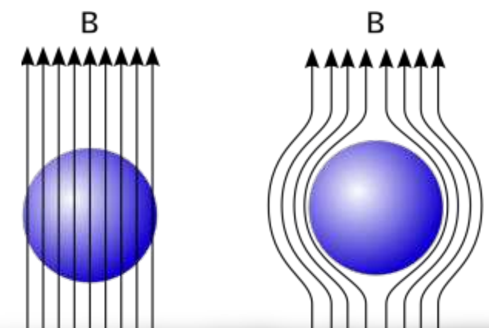

... superconductors (ϕ, ϕ^*, a_μ)

[Halperin, Lubensky, Ma, PRL '74]

Exceptions?

... M

2D Dir

PRL '14]

[Raghu

'15; '17]

...

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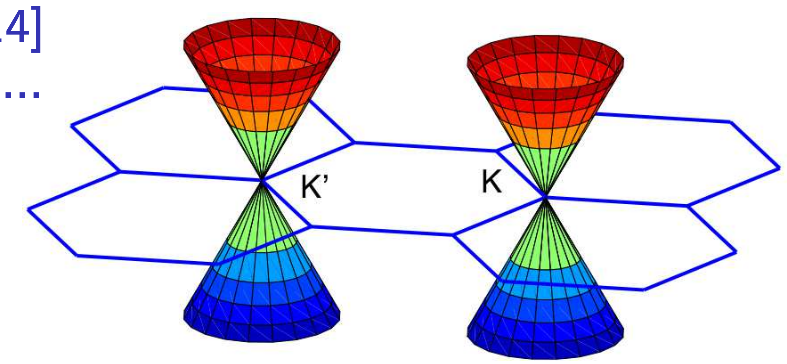
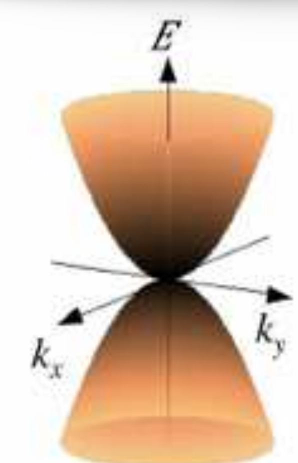




... and more

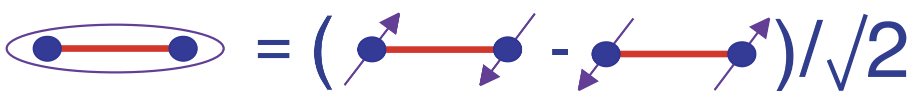
Deconfined quantum criticality

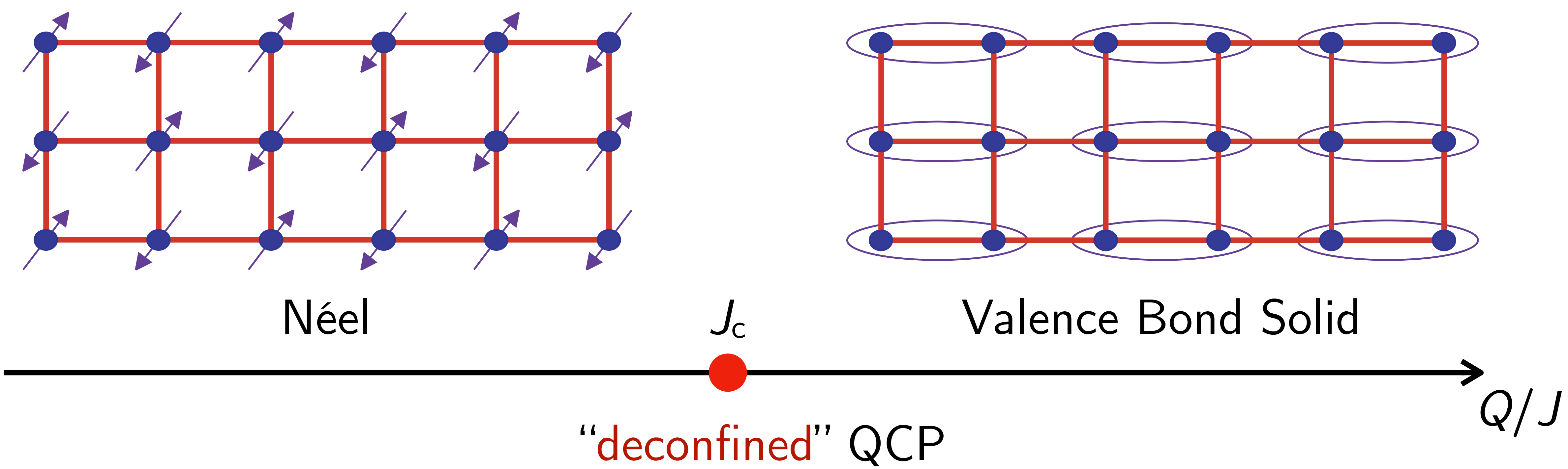
[Senthil *et al.*, Science '04; PRB '04]

Toy model (spin-1/2 on square lattice):

[Sandvik, PRL '07; PRL '10]
[Melko & Kaul, PRL '08]
[Nahum *et al.*, PRX '15; PRL '15]
[Shao *et al.*, Science '16]
...

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle ijkl \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \tfrac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \tfrac{1}{4} \right)$$

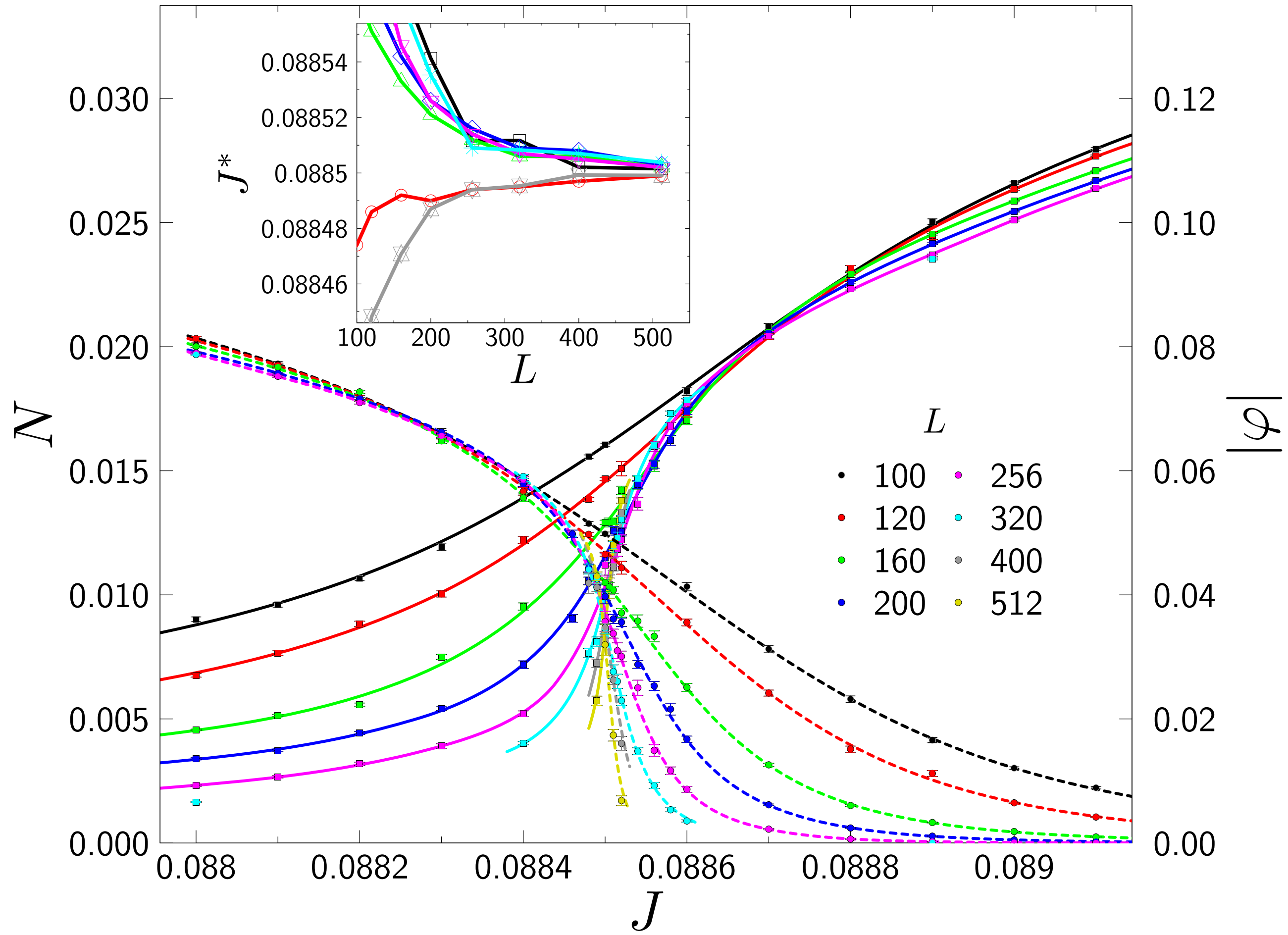




Monte Carlo result (J-Q model)

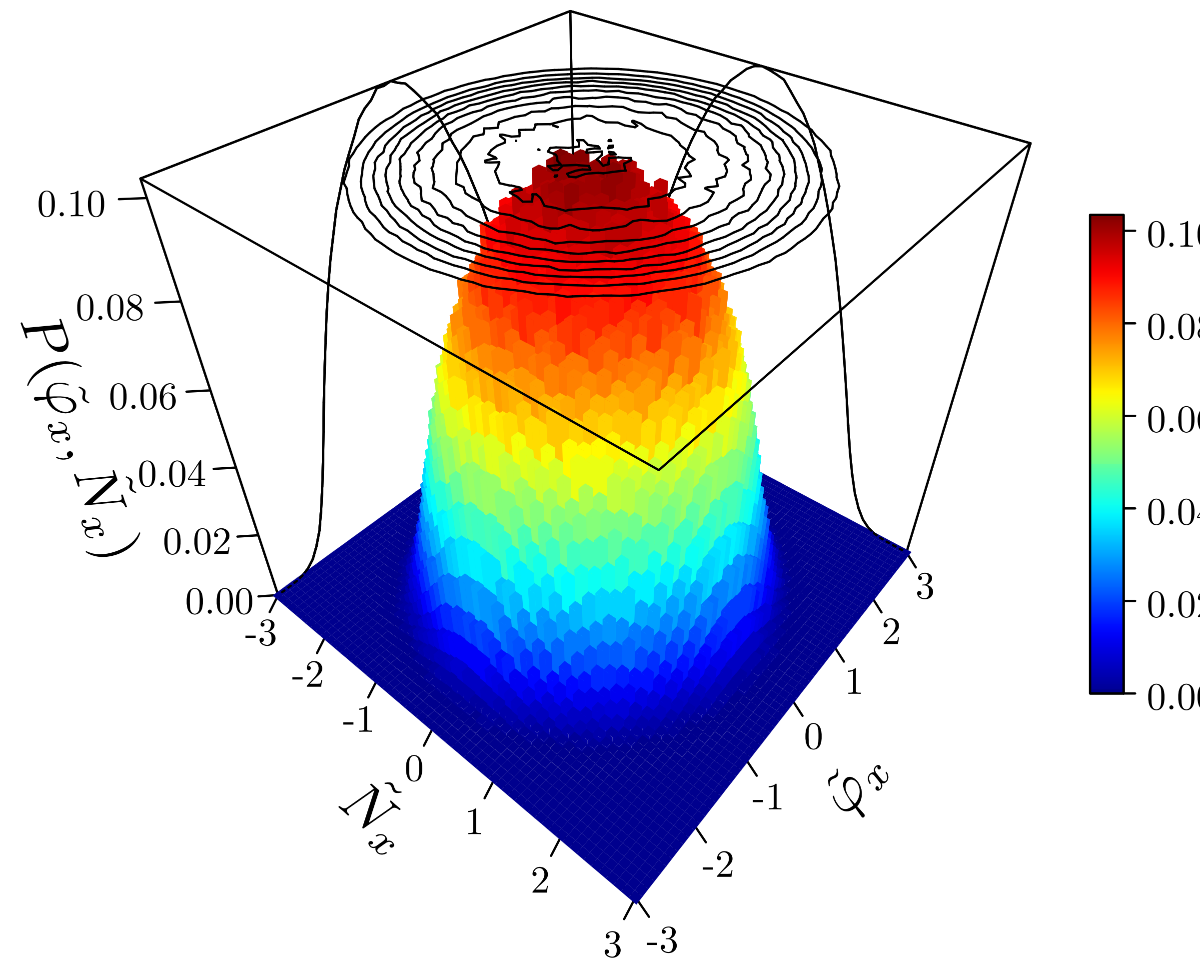
[Nahum *et al.*, PRX '15; PRL '15]

order parameters



... single crossing point

probability distribution



... isotropic!

... direct & continuous?

... emergent SO(5)?

Field theory for deconfined criticality

Fractionalization:

... CP^1 parametrization

$$\vec{n} = z^\dagger \vec{\sigma} z$$

$z = (z_1, z_2)$ ... complex "spinon"

Continuum field theory:

$$S_z = \int d^2\vec{r} d\tau \left[\sum_{\alpha=1,2} |(\partial_\mu - i b_\mu) z_\alpha|^2 - \left(|z_1|^2 + |z_2|^2\right)^2 \right]$$

b_μ ... "photon"

Monopoles irrelevant at critical point!

[Senthil *et al.*, Science '04; PRB '04]

→ "noncompact CP^1 model" ($NCCP^1$)

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→ "noncompact CP^1 model" ($NCCP^1$)

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Field theory for deconfined criticality

Fractionalization:

... CP¹ parametrization

complex "spinon"

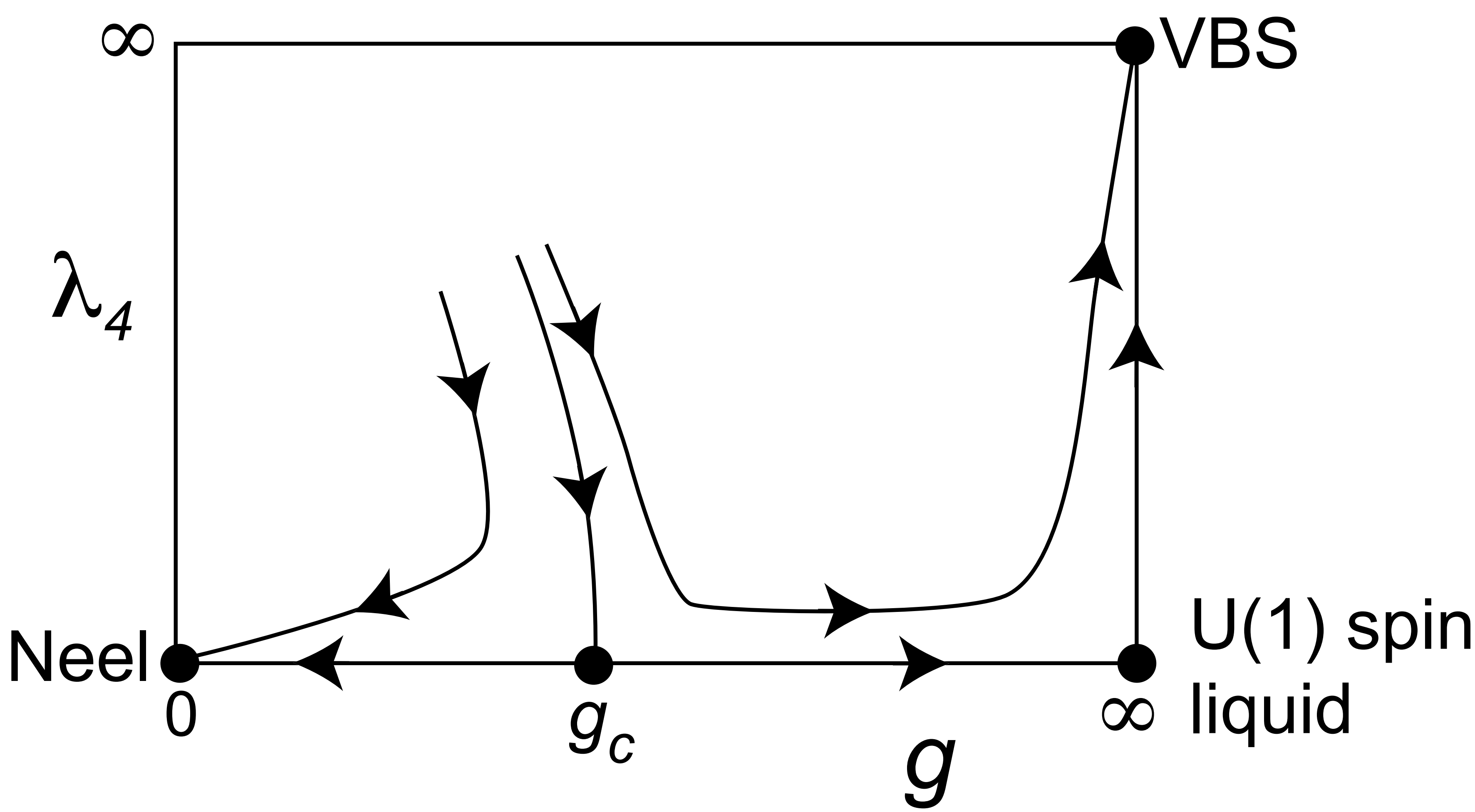
Continuu

S

b_μ ... "photon"

e '04; PRB '04]

→ "nonco



[Senthil *et al.*, JPSJ '05]

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Critical exponents

$1/N$ expansion [NCCP¹]:

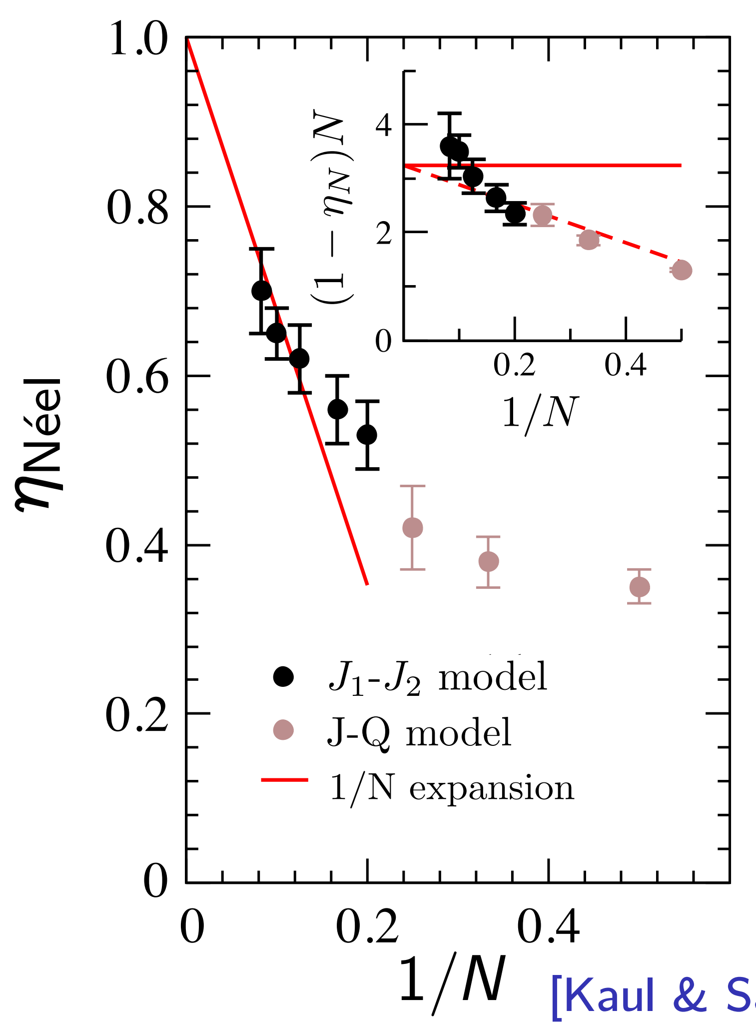
$$\eta_{\text{Néel}} = 1 - \frac{32}{\pi^2 N} + \mathcal{O}(1/N^2)$$

[Kaul & Sachdev, PRB '08]

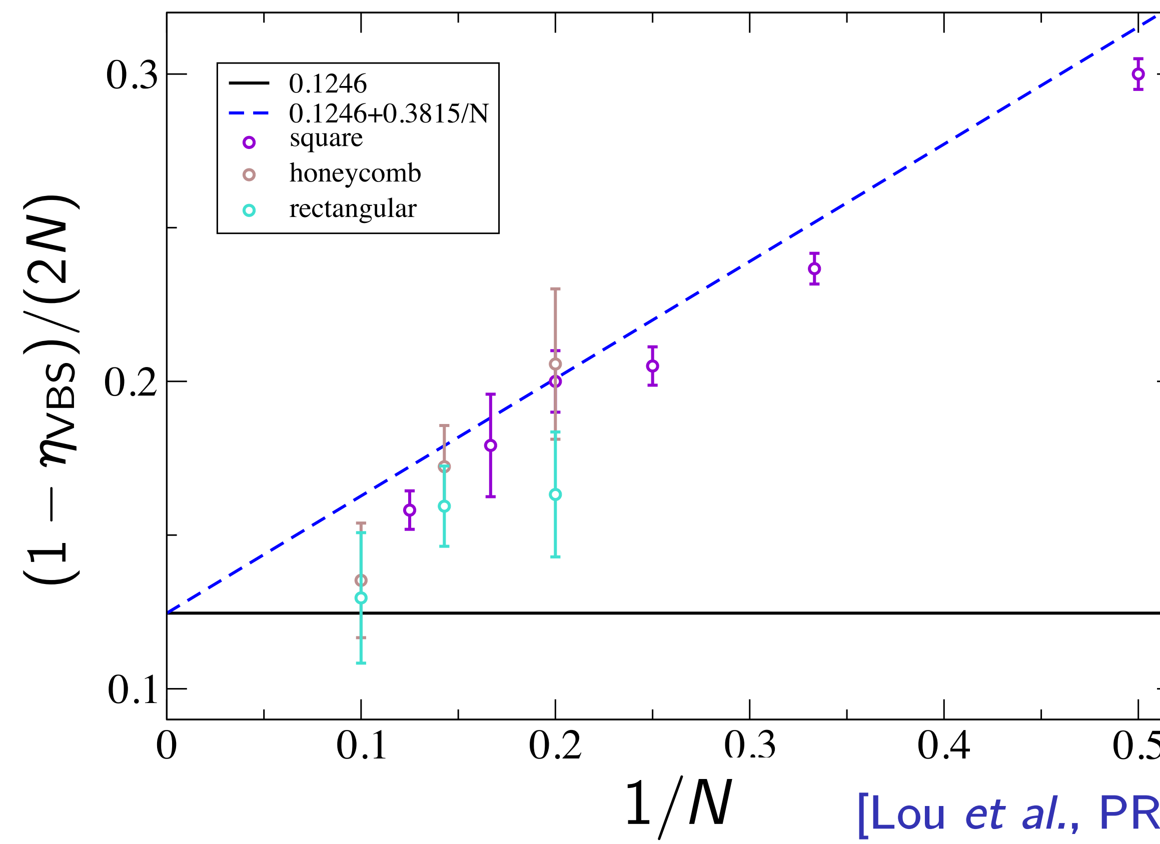
$$\eta_{\text{VBS}} = 0.249N - 0.237 + \mathcal{O}(1/N)$$

[Dyer *et al.*, JHEP '16]

MC [SU(N) J_1-J_2 Heisenberg model]:



[Kaul & Sandvik, PRL '12]



[Lou *et al.*, PRB '09]

[Kaul & Sandvik, PRL '12]

[Block *et al.*, PRL '13]

... excellent agreement

Critical exponents

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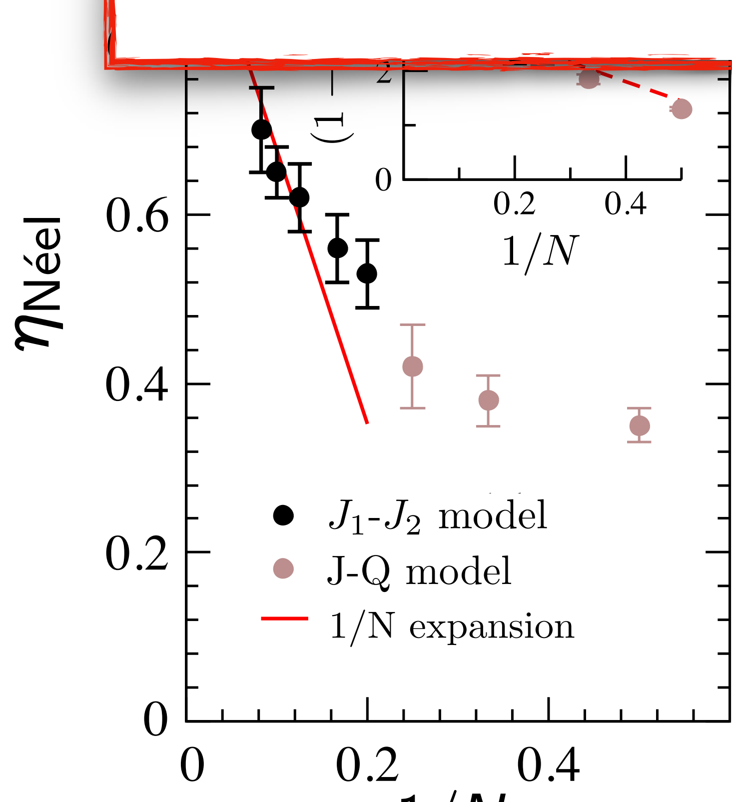
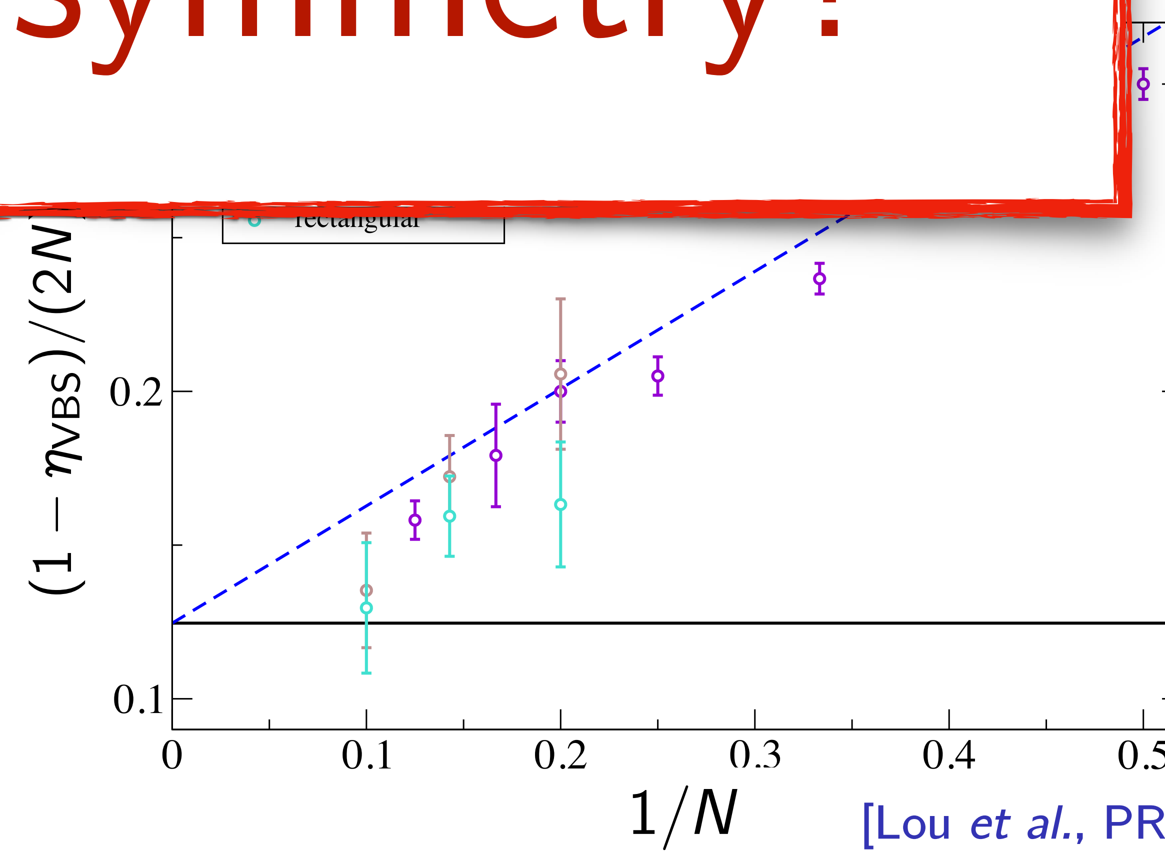
[Kaul & Sachdev, PRB '08] [Dyer *et al.*, JHEP '16]

MC [S

Emergent symmetry?



[Kaul & Sandvik, PRL '12]



[Lou *et al.*, PRB '09]

[Kaul & Sandvik, PRL '12]

[Block *et al.*, PRL '13]

... excellent agreement

Alternative formulations

Duality conjecture:

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

noncompact CP^1 model $\Longleftrightarrow$ QED_3-Gross-Neveu model

$$\sum_{\alpha=1,2} |D_b z_\alpha| - \left(|z_1|^2 + |z_2|^2\right)^2 \quad \Leftrightarrow \quad \sum_{i=1,2} \left(\bar{\psi}_i \not{D}_a \psi_i + \phi \bar{\psi}_i \psi_i\right) + V(\phi)$$

... with $V(\Phi)$ tuned to criticality

Explicitly:

$$(n_1, n_2, n_3, n_4, n_5) \sim (\overbrace{2\,\mathrm{Re}\,\mathcal{M}_b, 2\,\mathrm{Im}\,\mathcal{M}_b}^{\mathrm{U}(1)}, \overbrace{z^\dagger \sigma_x z, z^\dagger \sigma_y z, z^\dagger \sigma_z z}^{\mathrm{O}(3)})$$

$$\sim \Big[\underbrace{\mathrm{Re}\,(\psi_1^\dagger \mathcal{M}_a), -\mathrm{Im}\,(\psi_1^\dagger \mathcal{M}_a), \mathrm{Re}\,(\psi_2^\dagger \mathcal{M}_a), \mathrm{Im}\,(\psi_2^\dagger \mathcal{M}_a)}_{\mathrm{U}(2)}, \phi\Big]$$

... naturally explains emergent SO(5)!

... part of "duality web" in 2+1D:

[Seiberg, Senthil, Wang, Witten, Ann. Phys. '16]

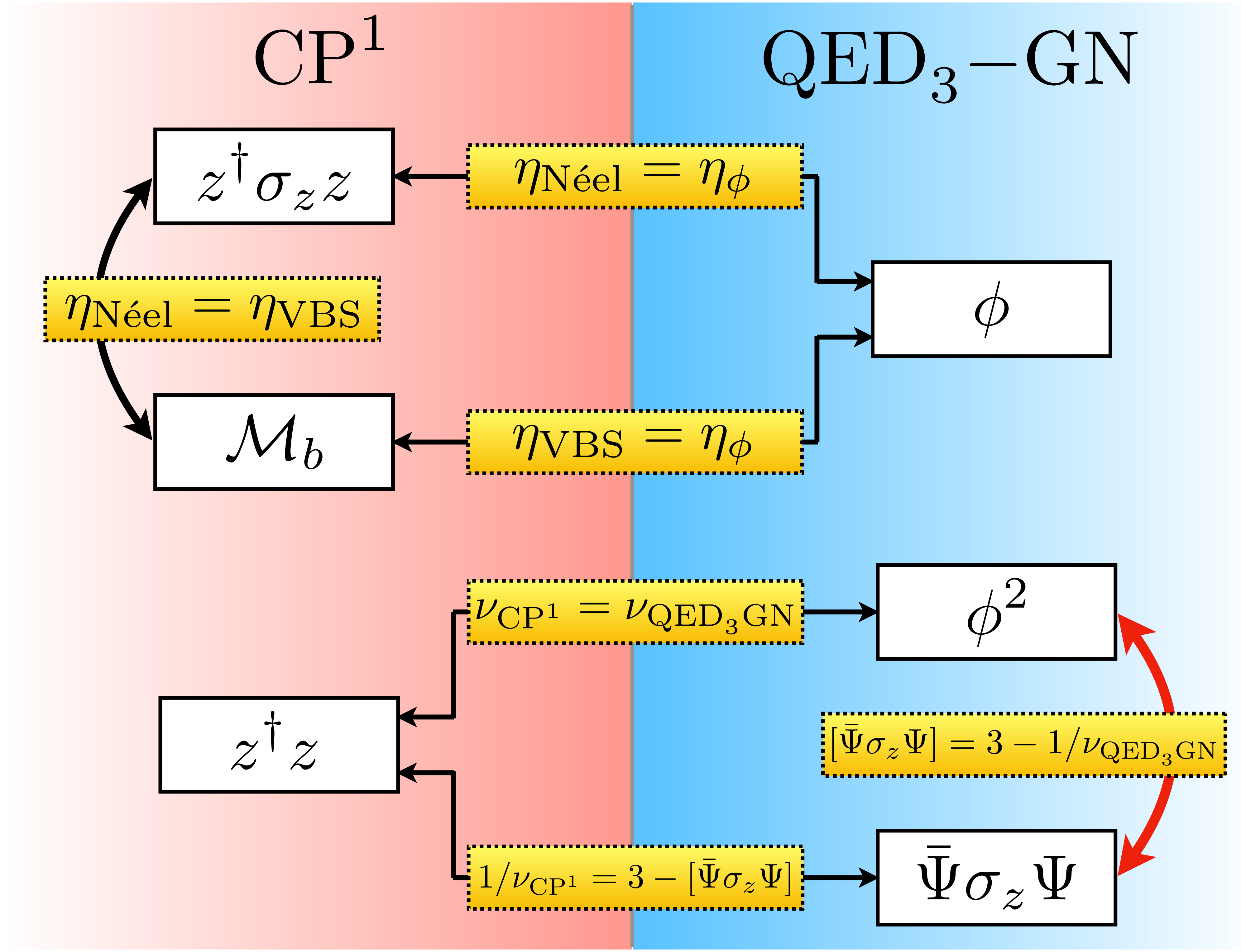
[Karch & Tong, PRX '16]

[Thomson & Sachdev, PRX '17]

...

Consequences of NCCP^1 $\Longleftrightarrow$ QED$_3$-Gross-Neveu

Predictions for critical behavior:



[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

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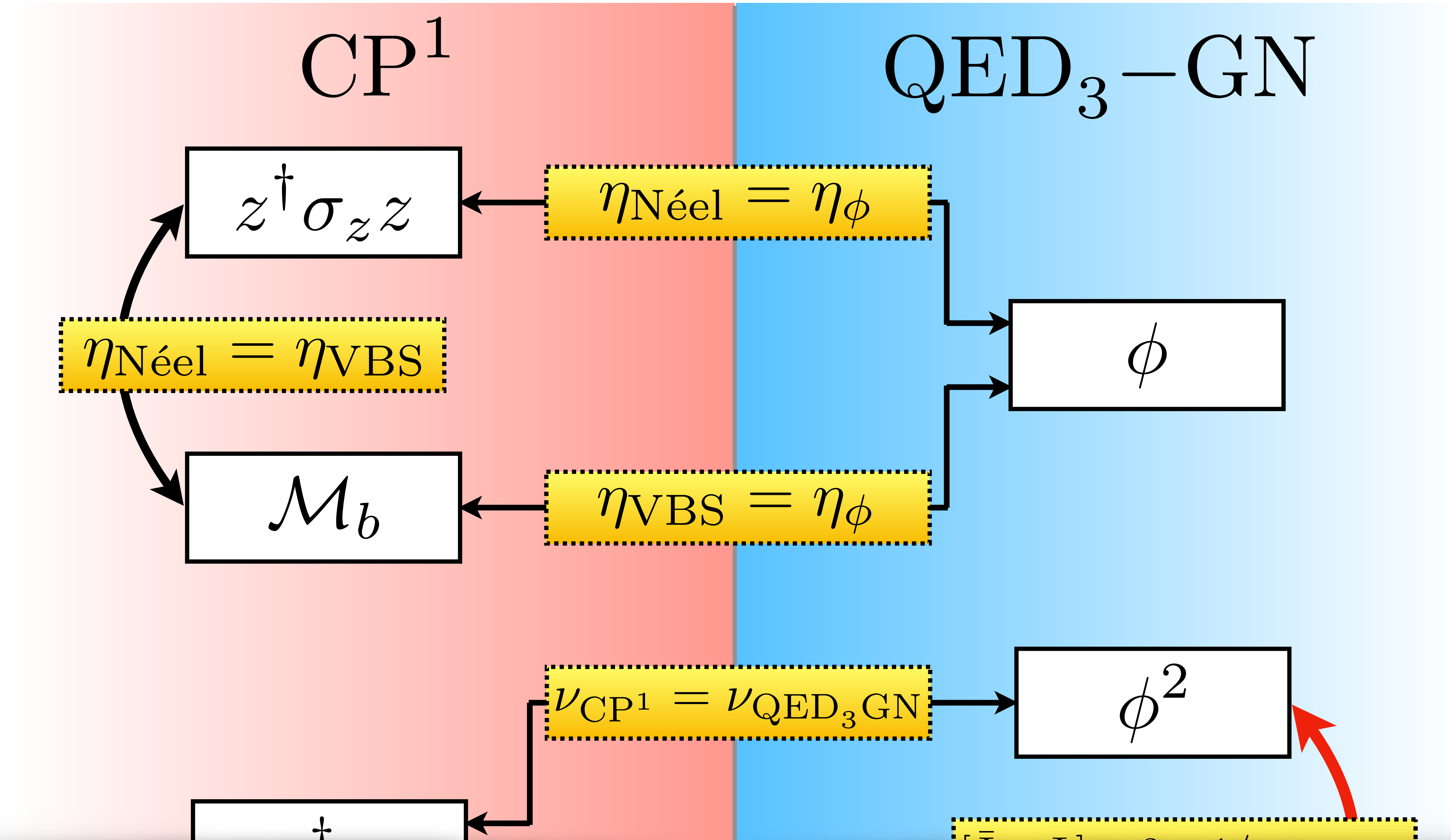


Here: (a) Existence of QCP in QED_3-GN model? ... prerequisite for duality

(b) Critical behavior? ... & comparison with duality prediction

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

QED3-Gross-Neveu model: GN limit

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - \cancel{iea_\mu})\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

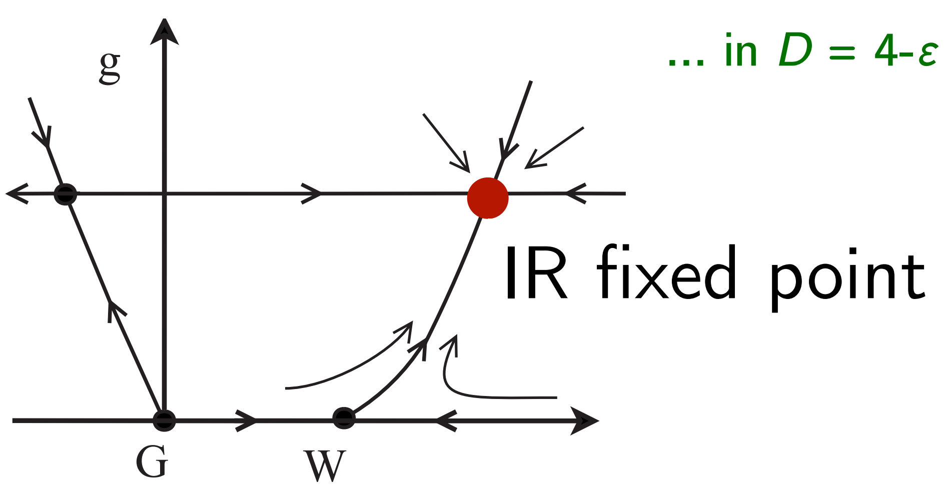
... in D = 2+1

... i = 1,...,2N

Gross-Neveu limit ($e^2 \to 0$) :

Gross-Neveu-Yukawa theory

... in D = 4-ε



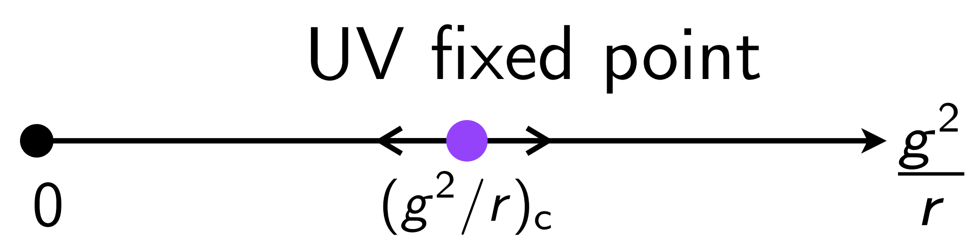
[Herbut, Juricic, Vafek, PRB '09]

$\Leftrightarrow$

Gross-Neveu theory

... in D = 2+ε

$$g\phi\bar{\psi}\psi + \frac{r}{2}\phi^2 \sim -\frac{g^2}{r}(\bar{\psi}\psi)^2$$



GN-QCP exists for all $2 < D < 4$ and can be understood as either ...

... **IR** fixed point of GNY or ... **UV** fixed point of GN

[Zinn-Justin, NPB '91]

QED3-GN model: Fermionic RG

[LJ & He, PRB ’17]

Integrate out ϕ:

$$g\phi\bar{\psi}_i\psi_i + \frac{r}{2}\phi^2 \mapsto u(\bar{\psi}_i\psi_i)^2$$

... u will also generate other four-fermion terms

General four-fermion theory compatible with U(2N):

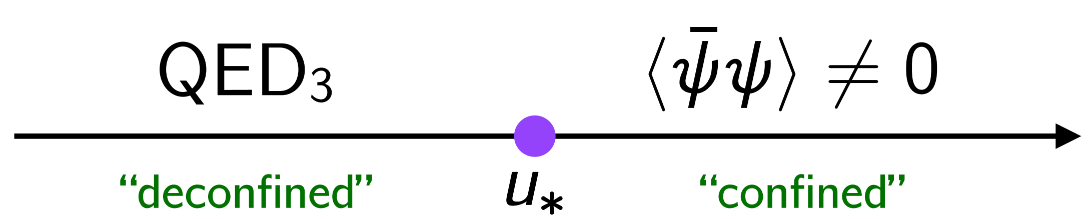
$$\mathcal{L}_\psi = \bar{\psi}_i\gamma_\mu(\partial_\mu - iea_\mu)\psi + u(\bar{\psi}_i\psi_i)^2 + v(\bar{\psi}_i\gamma_\mu\psi_i)^2$$

UV fixed point:

$$(e^2, u, v)_* = \left(\frac{3}{4N}, -\frac{1}{8N}, 0\right) + \mathcal{O}(1/N^2)$$

... with precisely one RG relevant direction

... “critical” fixed point

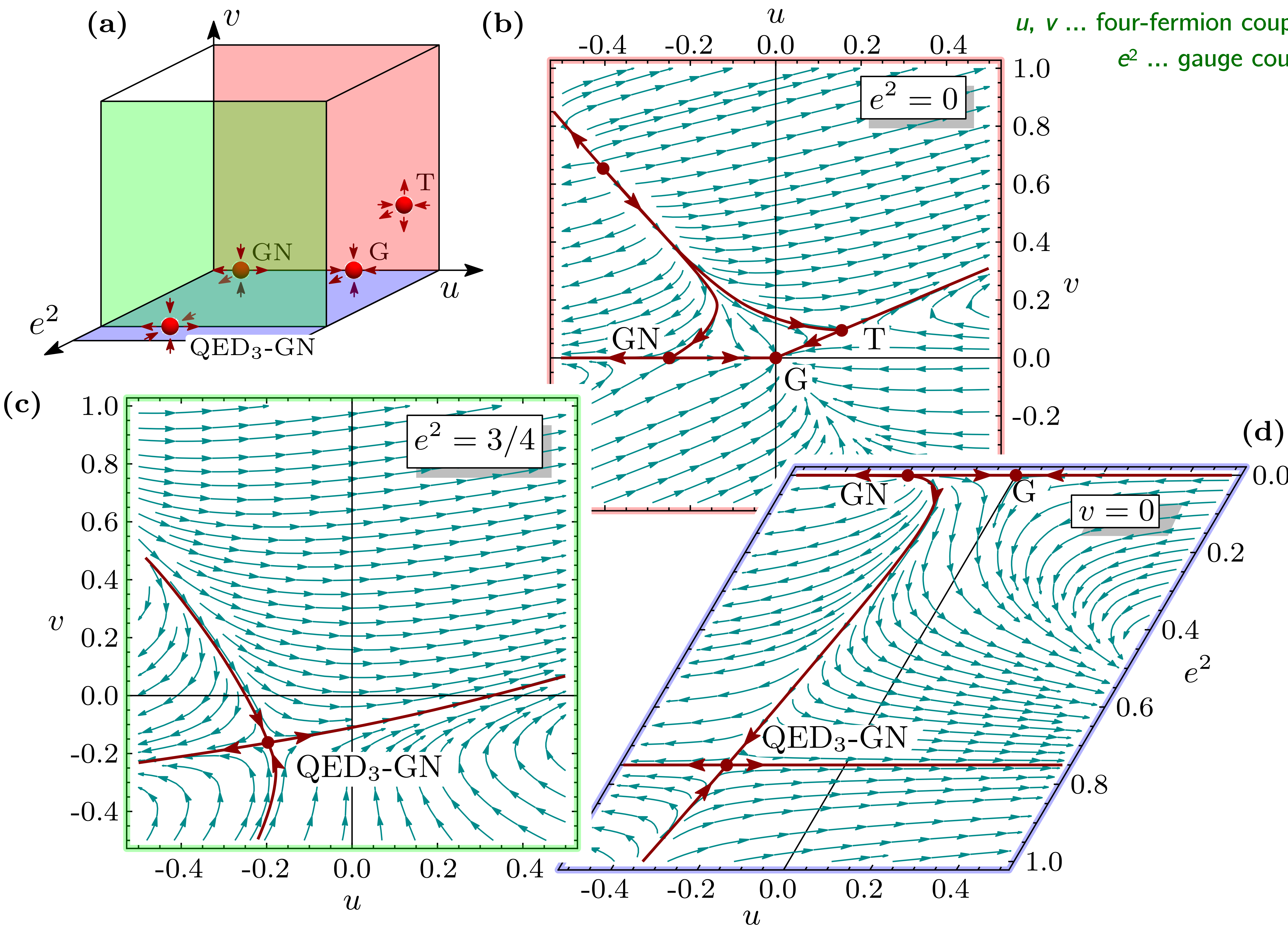


Fermionic RG: Flow diagram

[LJ & He, PRB '17]

u, *v* ... four-fermion couplings

e^2 ... gauge coupling



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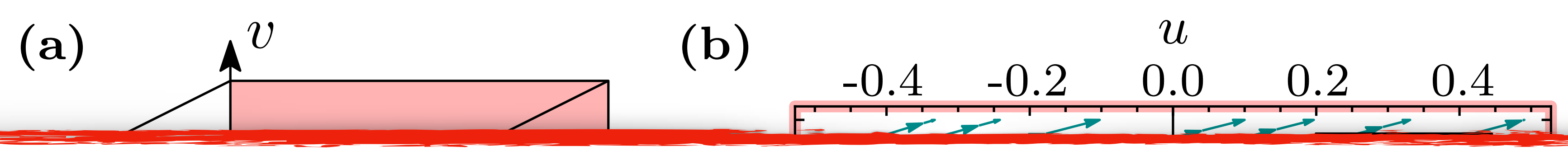


u, v ... four-fermion couplings

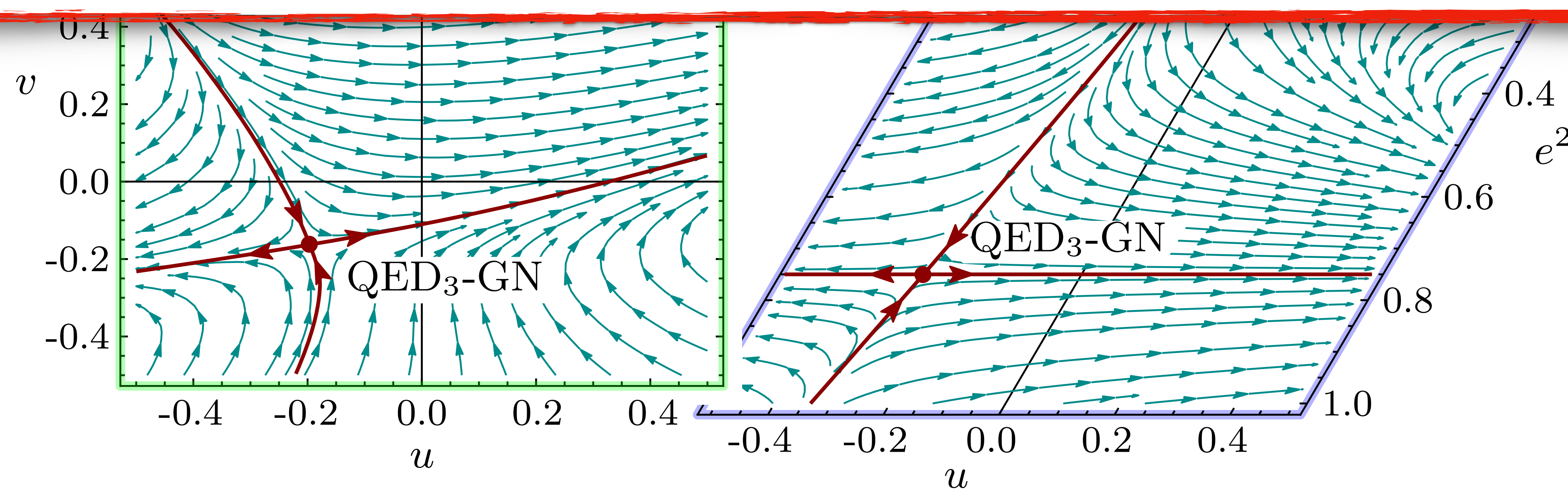
e^2 ... gauge coupling

Critical exponents:

$$1/\nu = 1 + \mathcal{O}(1/N)$$

$$[\bar\psi\psi] = 1 + \mathcal{O}(1/N) \quad \Rightarrow \quad \eta_\phi = 1 + \mathcal{O}(1/N)$$ ... large anom. dimension!

$$[\bar\psi\sigma^z\psi] = 2 + \mathcal{O}(1/N) \quad \Rightarrow \quad \eta_{\bar\psi\sigma^z\psi} = \mathcal{O}(1/N)$$ ... trivial



Gauged four-fermion model: Large-N expansion

Lagrangian:

$$\mathcal{L}_\psi = \bar{\psi}_i(\partial_\mu - iea_\mu)\psi_i + u\,(\bar{\psi}_i\psi_i)^2$$

... without $\partial^2\phi^2$ and ϕ^4 terms

Critical exponents in $2 < D < 4$:

[Gracey, Ann. Phys. '93]

[Gracey, arXiv:1808.07697]

$$\eta_\phi = 4 - D + \frac{(D-1)\Gamma(D-1)}{[\Gamma(D/2)]^3\Gamma\left(\frac{4-D}{2}\right)}\frac{1}{N} + \mathcal{O}(1/N^2)$$

$$= 1 + \frac{16}{\pi^2 N} + \mathcal{O}(1/N^2)$$

... in D = 2+1

$$\nu^{-1} = D - 2 - \frac{\Gamma(D+1)}{2[\Gamma(D/2)]^3\Gamma\left(\frac{4-D}{2}\right)}\frac{1}{N} + \mathcal{O}(1/N^2)$$

$$= 1 - \frac{24}{\pi^2 N} + \mathcal{O}(1/N^2)$$

... in D = 2+1

QED$_3$-GN model: 4-ε expansion

Lagrangian:

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - iea_\mu)\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

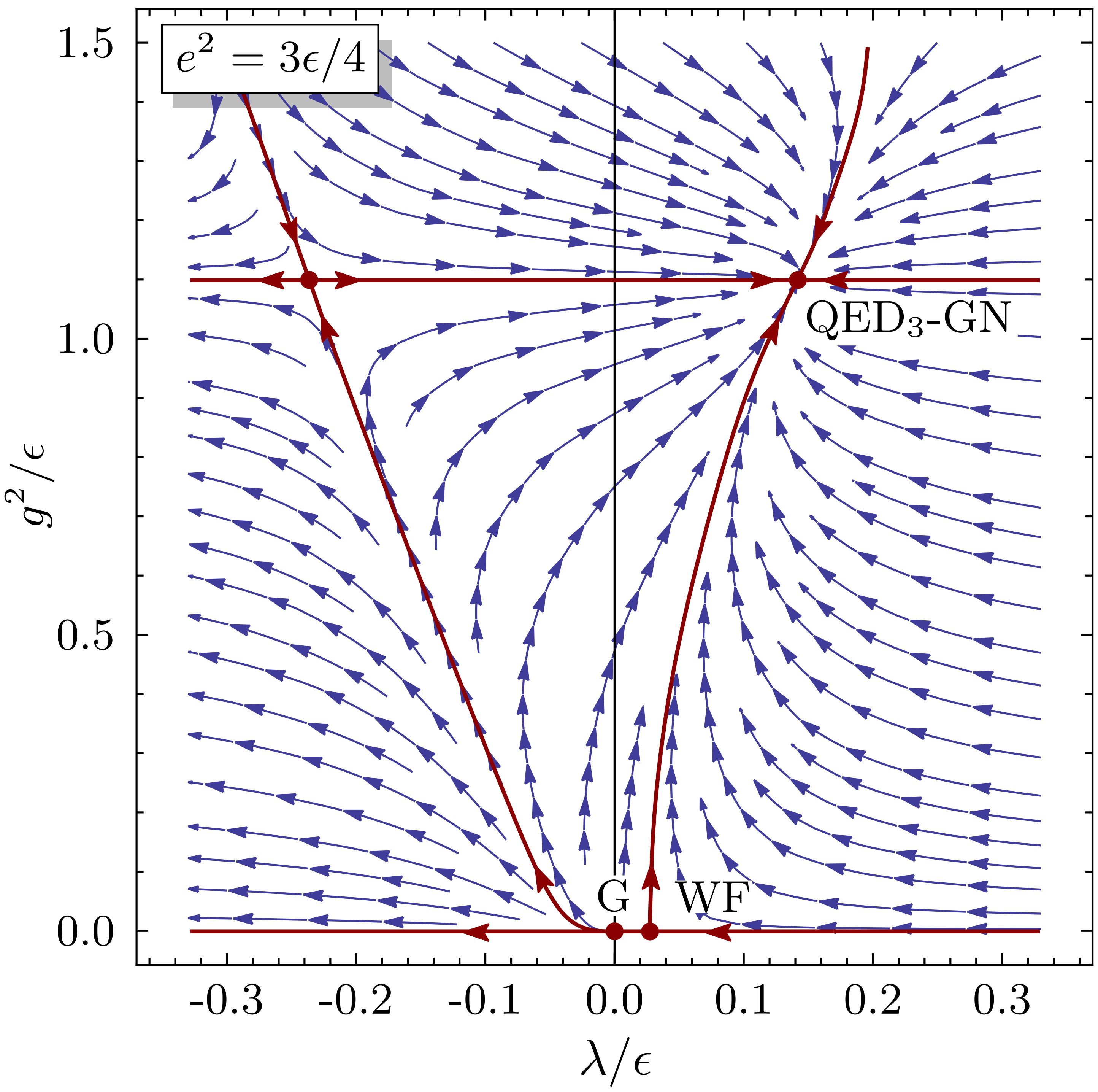
Engineering dimensions:

$$[e^2] = 4 - D, \qquad [g] = \frac{4-D}{2}, \qquad [\lambda] = 4 - D$$

... become simultaneously marginal near $D = 3+1$!

ε expansion in $D = 4$-ε possible!

QED$_3$-GN model: Flow diagram in $D = 4$-ε



... for $N = 1$

... fully IR stable fixed point

[LJ & He, PRB '17]

QED$_3$-GN model: Critical exponents at $O(\varepsilon)$

[LJ & He, PRB '17]

Gauge-field anomalous dimension:

$$\eta_a = 4 - D$$

... consequence of Ward identity

Gauge propagator:

$$G_a(p) \propto \frac{1}{|p|^{2-\eta_a}} = \frac{1}{|p|^{D-2}}$$

... exactly

... as in pure QED$_3$

Critical exponents:

$$\eta_\phi = \frac{2N+9}{2N+3}\epsilon + \mathcal{O}(\epsilon^2)$$

$$\nu = \frac{1}{2} + \frac{10N^2 + 39N + f(N)}{24N(2N+3)}\epsilon + \mathcal{O}(\epsilon^2)$$

... with $f(N) \equiv \sqrt{4N^4 + 204N^3 + 1521N^2 + 2916N}$

$$[\bar{\psi}\sigma^z\psi] = 3 - \frac{2N+6}{2N+3}\epsilon + \mathcal{O}(\epsilon^2)$$

... large $\mathcal{O}(\epsilon)$ corrections

QED$_3$-GN model: Critical exponents at $O(\varepsilon^3)$

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

Gauge-field anomalous dimension:

$$\eta_a = \epsilon + \mathcal{O}(\epsilon^4)$$

... consistent with Ward identity

Critical exponents (Large N):

$$\eta_\phi = \left(1 + \frac{3}{N}\right)\epsilon - \frac{\epsilon^2}{N} - \frac{3\epsilon^3}{4N} + \mathcal{O}(1/N^2, \epsilon^4)$$

$$\nu^{-1} = 2 - \left(1 + \frac{6}{N}\right)\epsilon + \frac{7\epsilon^2}{2N} + \frac{\epsilon^3}{N} + \mathcal{O}(1/N^2, \epsilon^4)$$

$$[\bar{\psi}\sigma^z\psi] = 3 - \left(1 + \frac{3}{2N}\right)\epsilon + \frac{\epsilon^2}{2N} + \frac{3}{8N}\epsilon^3 + \mathcal{O}(1/N^2, \epsilon^4)$$

... coincide with $1/N$ expansion of four-fermion model!

QED$_3$-GN model: Critical exponents at $O(\varepsilon^3)$

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QED$_3$-GN (**IR** FP) = gauged four-fermion (**UV** FP)

... coincide with $1/N$ expansion of four-fermion model!

QED$_3$-GN model: Critical exponents at $O(\varepsilon^3)$ for $N = 1$

Critical exponents ($N = 1$):

$$\eta_\phi = 2.2\epsilon - 0.222725\epsilon^2 + 16.8838\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\nu^{-1} = 2 - 3.90514\epsilon + 7.47146\epsilon^2 - 90.5962\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$[\bar{\psi}\sigma^z\psi] = 3 - 1.6\epsilon + 1.987\epsilon^2 - 17.46\epsilon^3 + \mathcal{O}(\epsilon^4)$$

. . . large $\mathcal{O}(\epsilon^3)$ corrections

Padé approximant:

$$[m/n] = \frac{a_0 + a_1\epsilon + \ldots a_m\epsilon^m}{1 + b_1\epsilon + \cdots + b_n\epsilon^n}$$

QED$_3$-GN model: 2+1D estimates ($N = 1$)

Padé estimates for $N = 1$:

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

Order	$[m/n]$	$1/\nu$	η_ϕ	$[\bar{\Psi}\sigma_z\Psi]$
ϵ^2	$[0/2]$	0.6602	–	2.5964
	$[1/1]$	0.6595	1.9978	2.2863
ϵ^3	$[1/2]$	0.6774	–	1.9894
	$[2/1]$	–	2.1971	1.6030

Mean values:

$$1/\nu = 0.67(1)$$

$$[\bar{\psi}\sigma^z\psi] \approx 2.12(50)$$

QED_3-GN vs. $NCCP^1$ duality: SO(5) scaling relation

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

Scaling relation from SO(5) symmetry:

$$[\bar{\psi}\sigma^z\psi] = 3 - 1/\nu$$

Our estimates:

$$[\bar{\psi}\sigma^z\psi] \approx 2.12(50) \qquad\qquad 3 - 1/\nu \approx 2.33(1)$$

... consistent with duality prediction!

QED$_3$-GN vs. NCCP^1 duality: AFM-VBS numerics

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

Duality prediction:

$$\eta_{\text{QED}_3\text{-GN}} = \eta_{\text{Néel}} = \eta_{\text{VBS}}$$

We find:

$$\eta_{\text{QED}_3\text{-GN}} > 1$$

... in agreement with $1/N$ expansion

[Gracey, Ann. Phys. '93]

AFM-VBS transition (MC):

[Sandvik, PRL '07; PRL '10]

[Nahum *et al.*, PRX '15]

[Shao *et al.*, Science '16]

...

$$\eta_{\text{Néel}} \approx \eta_{\text{VBS}} < 1$$

... inconsistent with duality prediction!

... similar inconsistency for ν

QED_3-GN vs. $NCCP^1$ duality: Possible scenarios

QED_3-GN critical behavior is ...

... consistent with SO(5) duality relation

... inconsistent with numerics for AFM-VBS transition

Three potential scenarios:

(A) Only weak duality holds

... i.e., not the same IR fixed points

(B) Perturbative approach fails

... i.e., emergence of SO(5) correctly predicted,
but absolute values incorrect

(C) No unitary fixed point

... i.e., annihilation & complexification of fixed point
[Nahum *et al.*, PRX '15]

Conclusions

QED$_3$-Gross-Neveu model ... [LJ & He, PRB '17]

... interesting due to possible duality with NCCP^1

... i.e., theory of Néel-VBS deconfined critical point

... has a stable fixed point

... prerequisite for duality to hold

... critical behavior computable within 4-ε expansion

... all couplings simultaneously marginal

... three-loop exponents: [Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]

- consistent with SO(5) scaling relation
- inconsistent with AFM-VBS numerics ... large anomalous dimension η_ϕ

... however: large η_ϕ necessary for emergent SO(5)

[Nakayama & Ohtsuki, PRL '16]

QED$_3$-GN model: Bilinear scaling dimension

Scaling dimension of $[\bar{\psi}\sigma^z\psi]$

N	$1/N$ exp. @ $O(1/N^2)$					ε exp. @ $O(\varepsilon^4)$
3	1.761967	1.750715	1.751288	1.750017	1.757367	1.76 ± 0.05
4	1.816001	1.809349	1.809603	1.809035	1.812634	1.81 ± 0.04
5	1.850041	1.845652	1.845787	1.845491	1.847514	1.84 ± 0.03
6	1.873453	1.870342	1.870421	1.870257	1.871506	1.86 ± 0.02
10	1.922100	1.920932	1.920950	1.921027	1.921335	1.917 ± 0.007

[Gracey, arXiv:1808.07697] [Zerf *et al.*, arXiv:1808.00549]

... excellent agreement

QED_3-GN vs. $NCCP^1$ duality: Critical exponents

CP^1		QED$_3$-GN ($N=1$)	
$\eta_{\text{Néel}}$	$\approx 0.26(3)$ [64]	η_ϕ	$\approx 2.1(1)$ [this work]
	$\approx 0.35(3)$ [10]		$\approx 1.3(9)$ [26]
	$\approx 0.30(5)$ [65]		
	≈ 0.22 [66]		
	$\approx 0.259(6)$ [4]		
η_{VBS}	$\approx 0.28(8)$ [65]		
	$\approx 0.25(3)$ [4]		
$3-1/\nu$	$\approx 1.72(5)$ [64]	$3-1/\nu$	$\approx 2.33(1)$ [this work]
	$\approx 1.53(9)$ [10]		$\approx 2.7(4)$ [26]
	$\approx 1.15(19)$ [65]		
	≈ 1.21 [66]	$[\bar\Psi\sigma_z\Psi]$	$\approx 2.12(50)$ [this work]
	≈ 1 [4]		$\approx 1.8(5)$ [26]
	$\approx 0.76(4)$ [5]		

[Sandvik, PRL '07]
[Melko & Kaul, PRL '08]
[Pujari *et al.*, PRL '13]
[Bartosch, PRB '13]
[Nahum *et al.*, PRX '15]
[Shao *et al.*, Science '16]

[Ihrig, LJ, Mihaila, Scherer, arXiv:1807.04958]
[LJ & He, PRB '17]

... no agreement

... honest answer: We don't know, but ...

Scaling dimension of $[\bar{\psi}\sigma^z\psi]$

N	$1/N$ exp. @ $O(1/N^2)$					ε exp. @ $O(\varepsilon^4)$
3	1.761967	1.750715	1.751288	1.750017	1.757367	1.76 ± 0.05
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