Dualities in field theories for condensed matter

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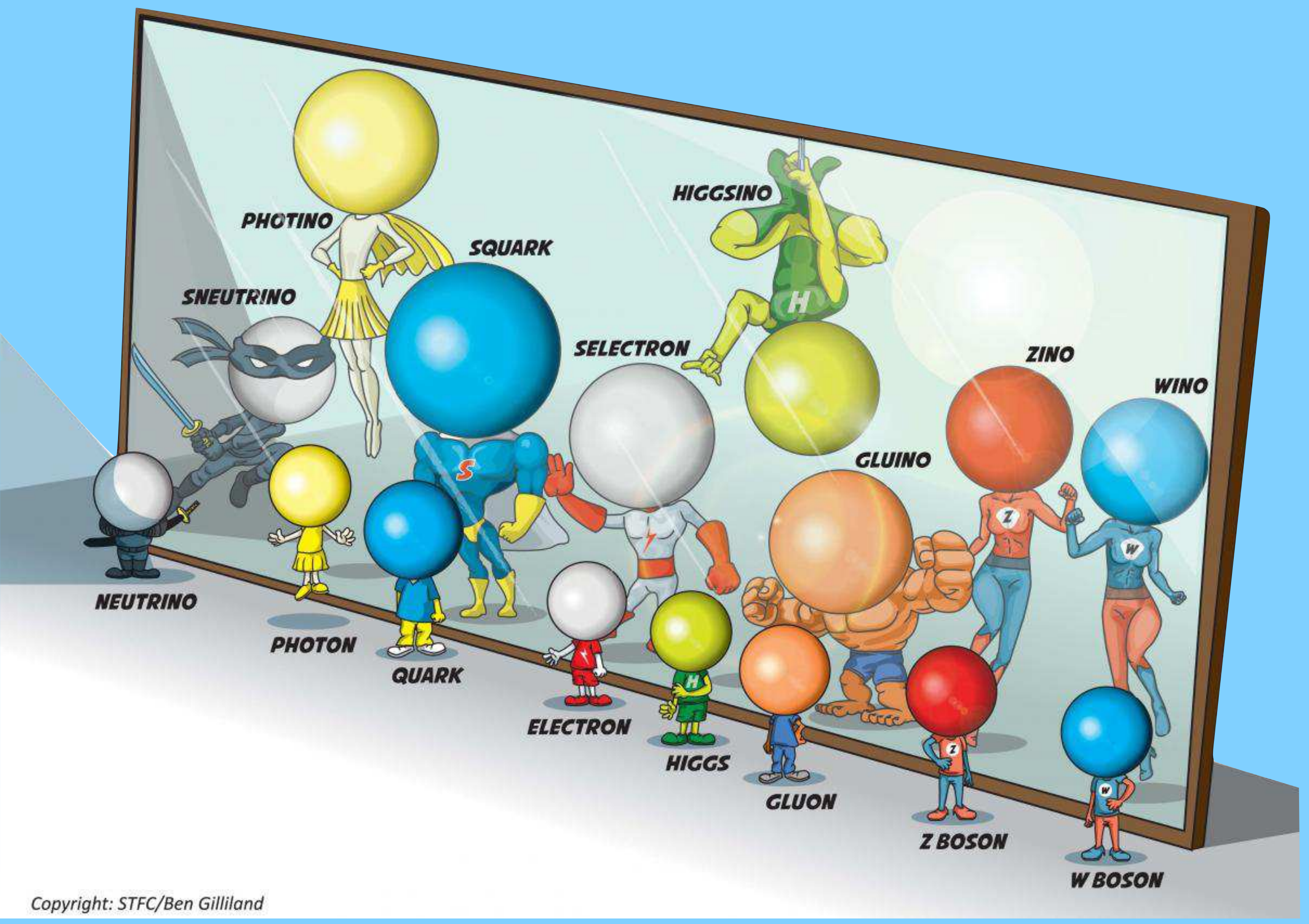
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

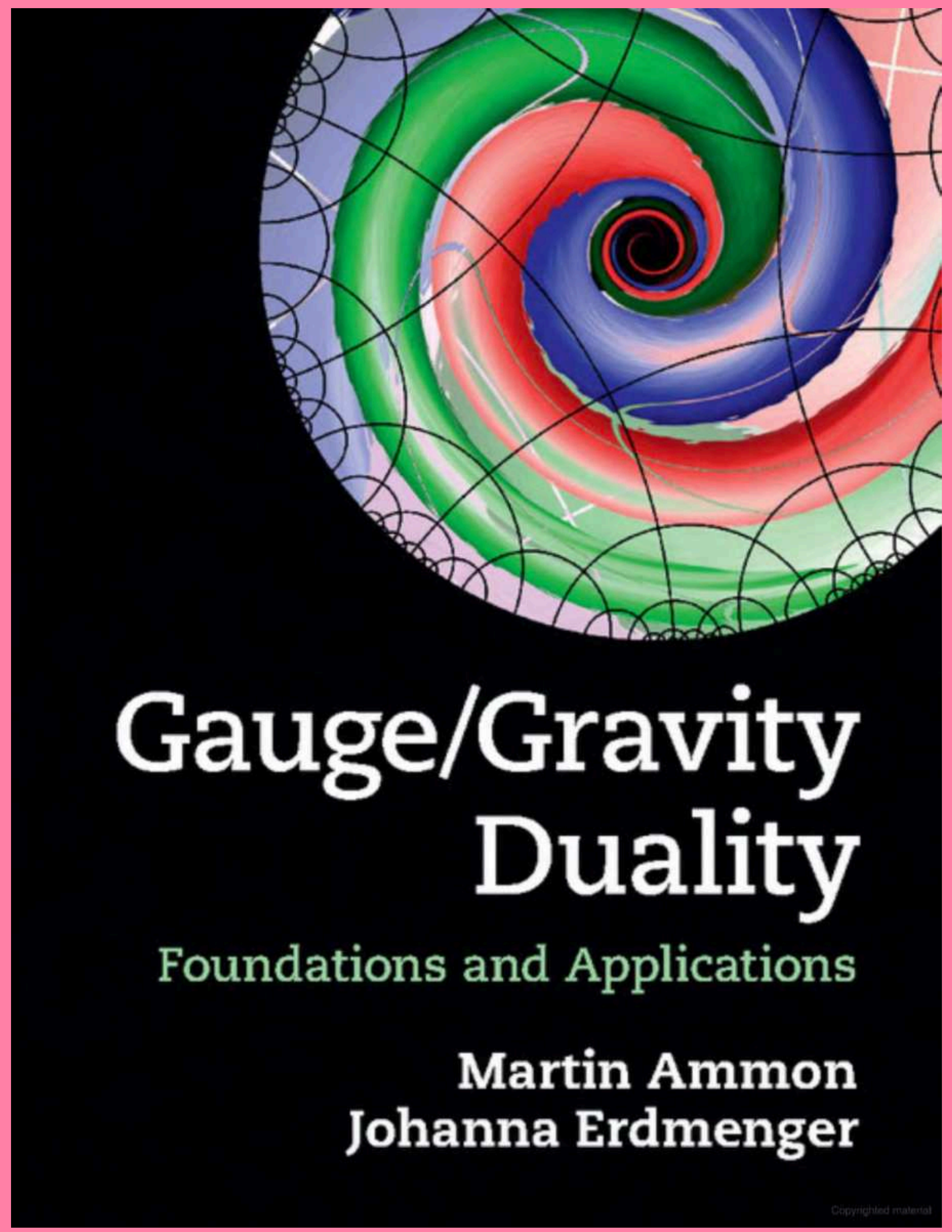
Quantum Critical Matter
JA 2306/4-1
Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Introduction: Dualities in field theories

Dualities in SUSY theories



Gauge/gravity dualities

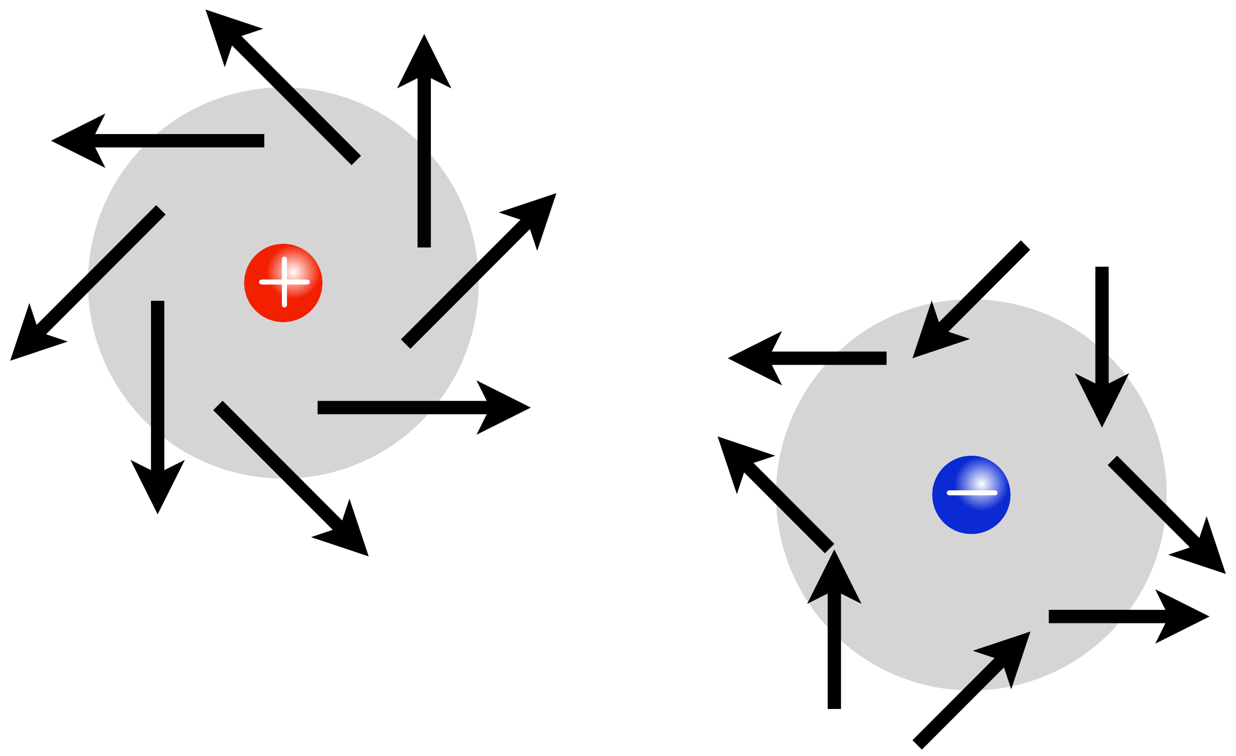


Dualities in cond-mat theories

Magnet

Superconductor

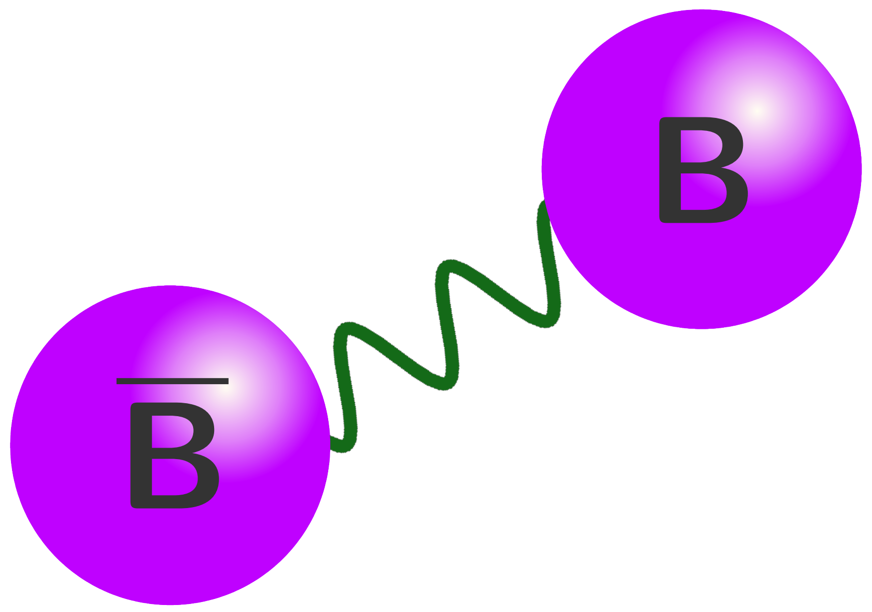
Dualities in condensed-matter field theories: Three examples



Ex #1:

BKT transition in classical planar magnets:

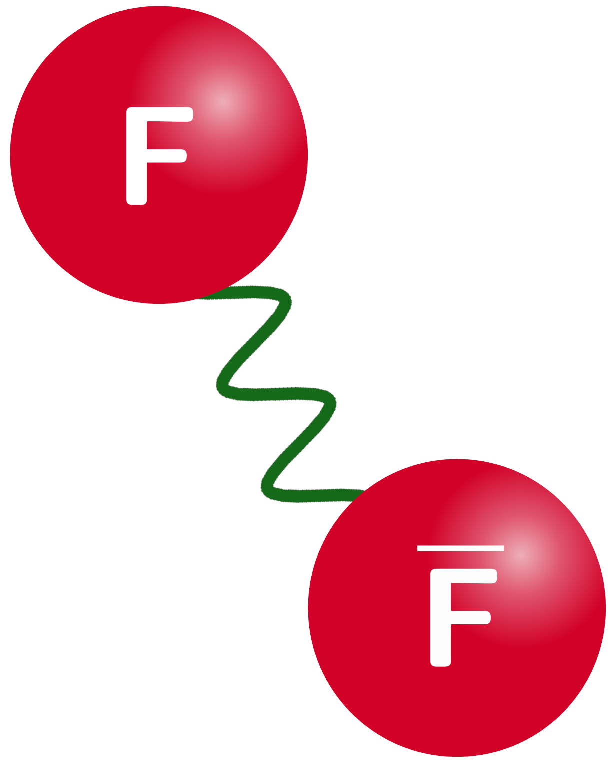
2D XY—Sine-Gordon duality



Ex #2:

Superconducting transition in type-II materials:

3D XY—Abelian-Higgs duality



Ex #3:

Deconfined QCP in quantum planar magnets:

2+1D $NCCP^1$—QED_3-GN duality

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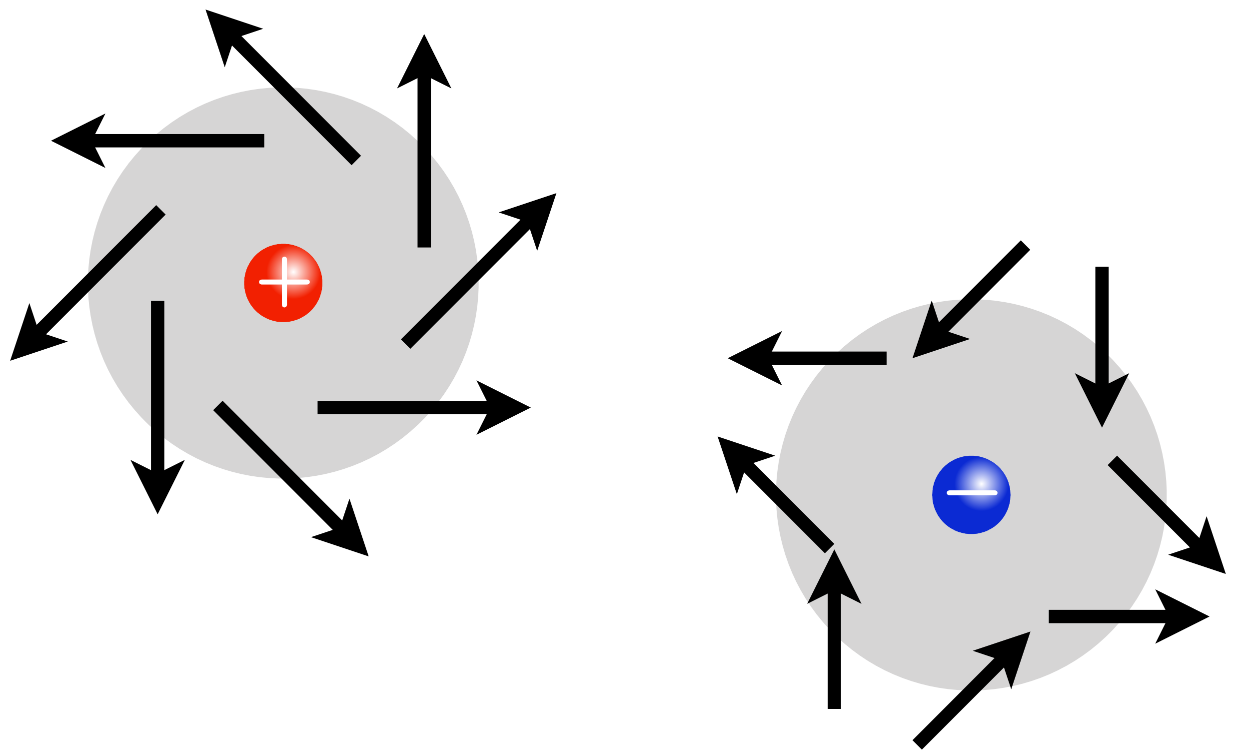

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Ex #1: Planar magnets

[Herbut, CUP '07]

Classical 2D XY model:

$$\mathcal{H}_{\mathrm{XY}} = -\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad \text{with } \mathbf{S}_i \equiv \mathbf{S}(\mathbf{r}_i) \equiv \begin{pmatrix} \cos\theta(\mathbf{r}_i) \\ \sin\theta(\mathbf{r}_i) \end{pmatrix} : \mathbb{R}^2 \mapsto S^1$$

$$\simeq \frac{1}{2}\int \mathrm{d}^2\mathbf{r}\,(\nabla\theta(\mathbf{r}))^2$$

... in continuum limit

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Closed contour $\mathcal{C} \in \mathbb{R}^2$:

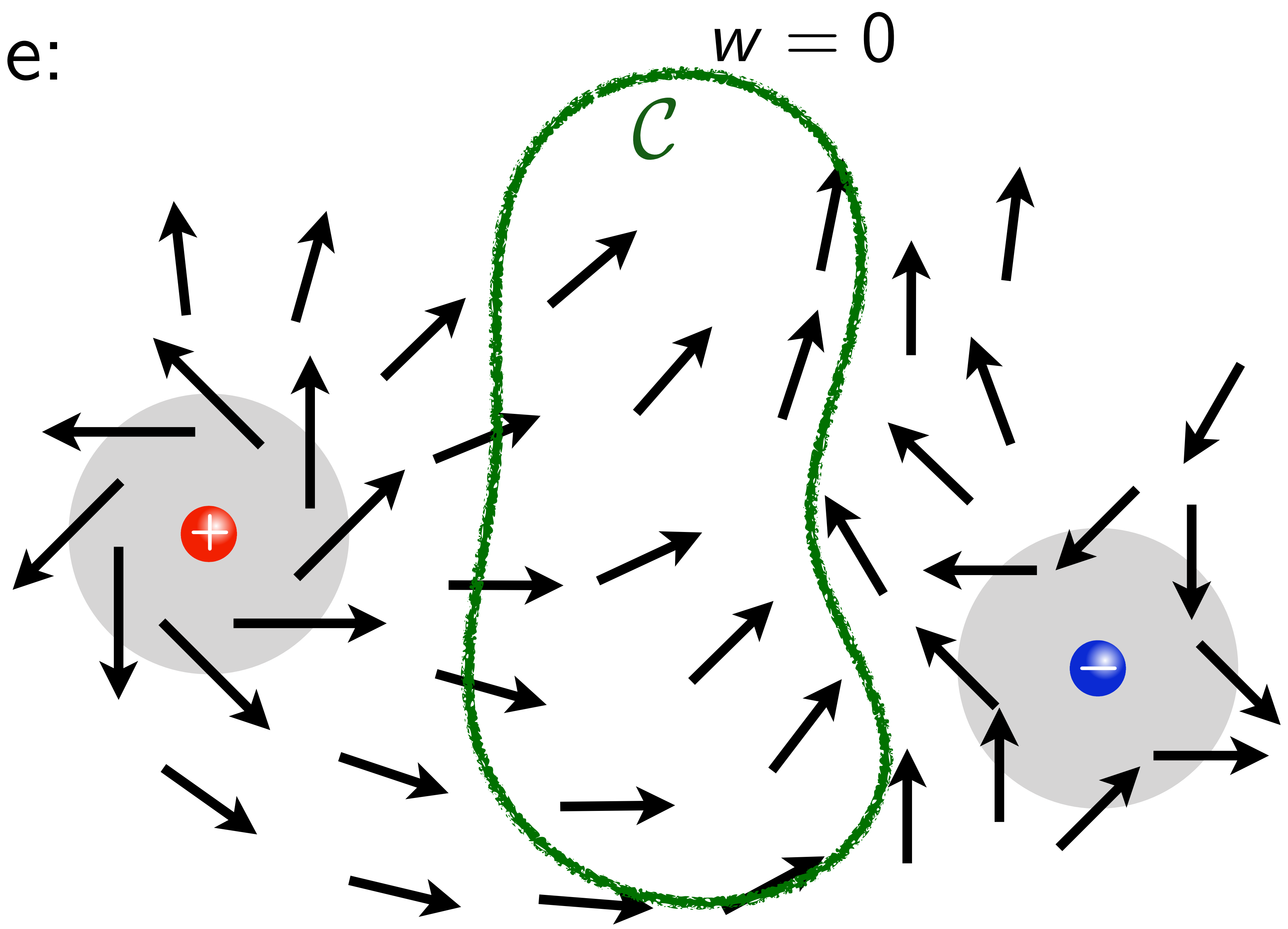
$$w = \frac{1}{2\pi}\oint \mathrm{d}\mathbf{r}\cdot\nabla\theta(\mathbf{r}) \quad \in \mathbb{Z}$$

... winding number

$$= \sum_{\substack{\text{vortices}\\ \text{in } \mathcal{C}}} q_i$$

... q_i: vortex charges

Example:



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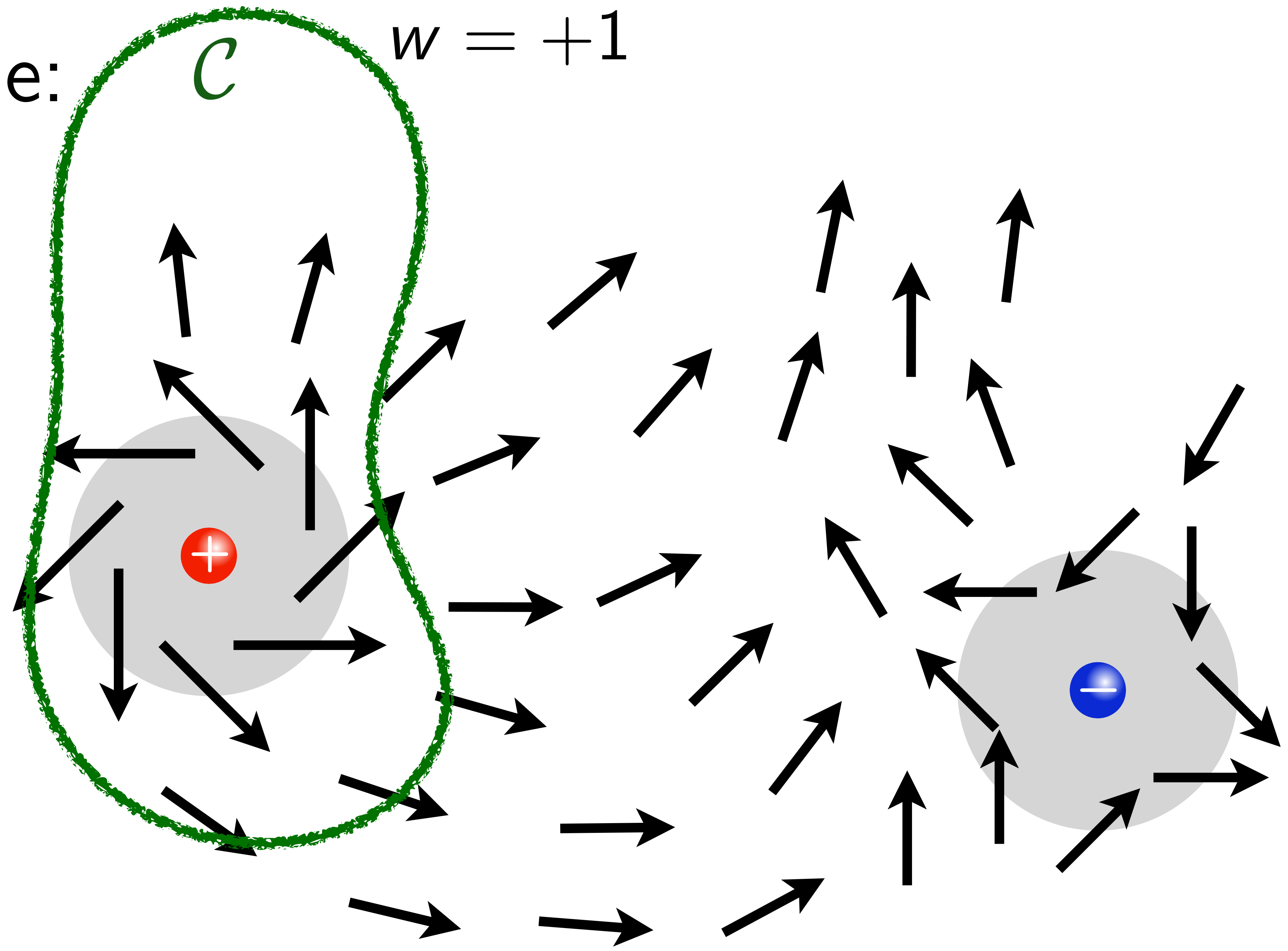

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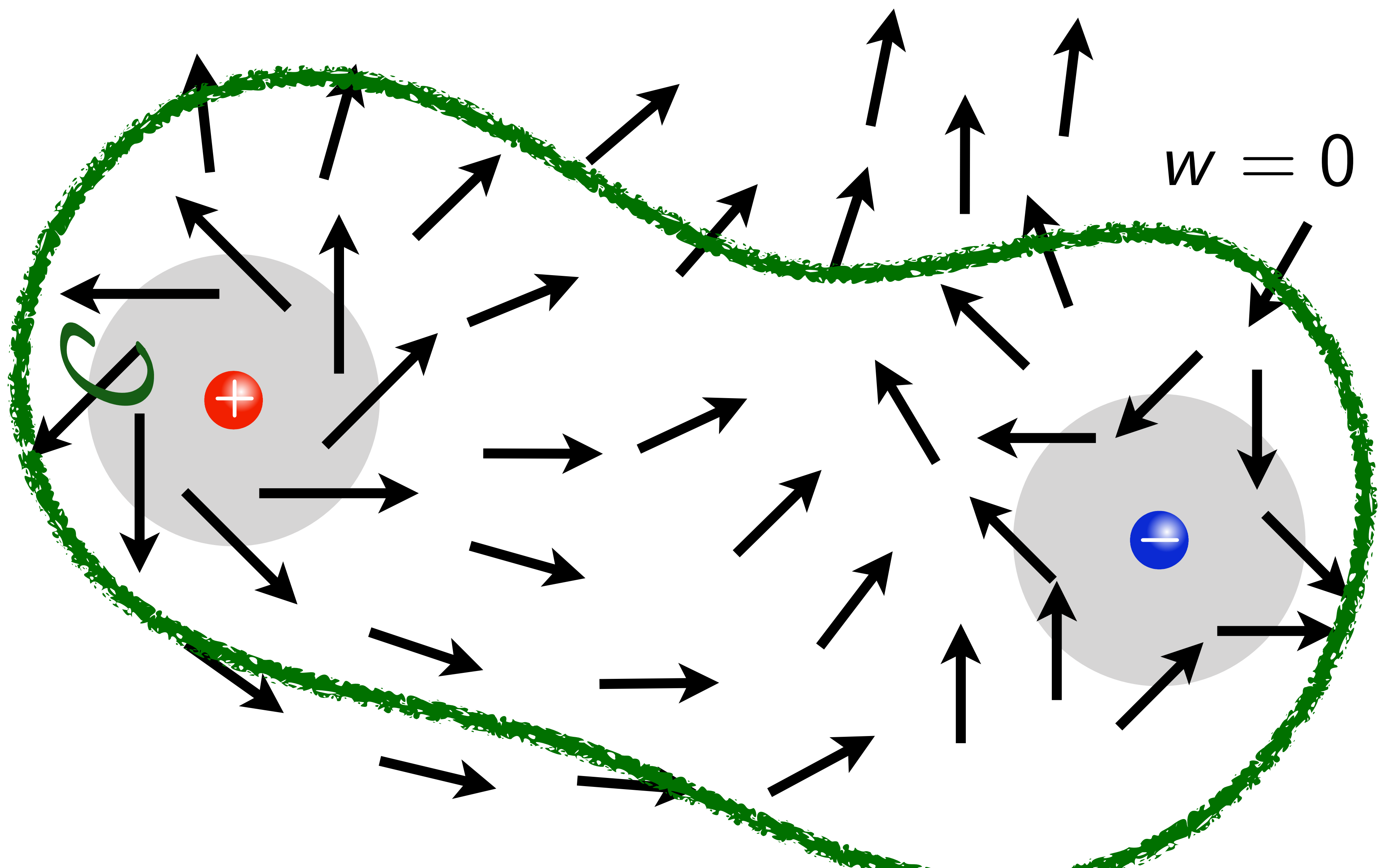
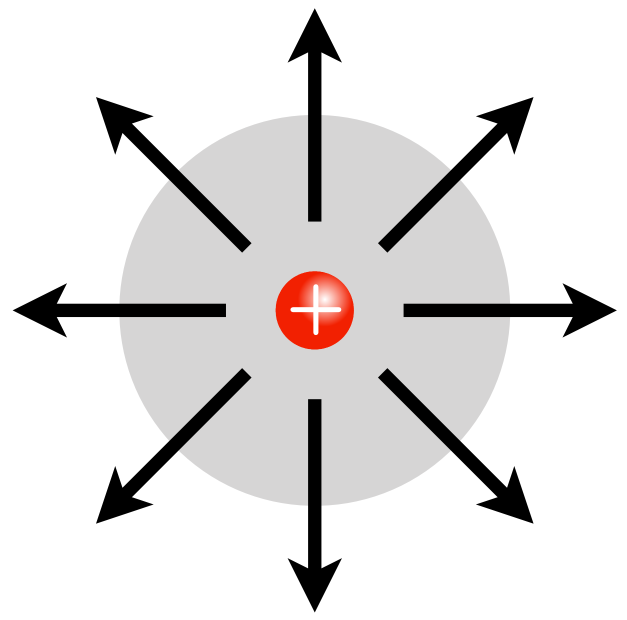


Vortex excitations

Isolated vortex:

$$\theta(\mathbf{r}) = q\alpha \qquad \text{where} \quad \mathbf{r} = r \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$



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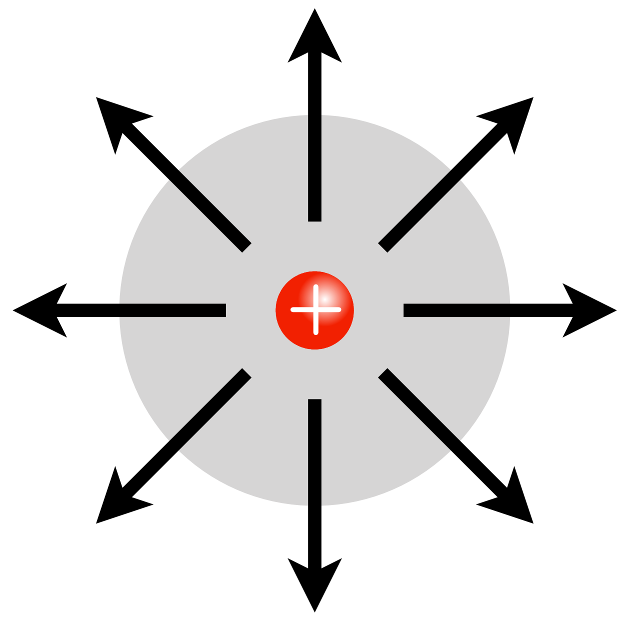

Energy:

system size

$$E_\mathrm{V} = \frac{1}{2}\int \mathrm{d}^2\mathbf{r}\,(\nabla\theta(\mathbf{r}))^2 = \pi q^2 \ln\frac{R}{r_0} \xrightarrow{R\to\infty} \infty$$

... vortices suppressed at low T

“vortex size” ... short-distance cutoff $r_0 \gtrsim a$

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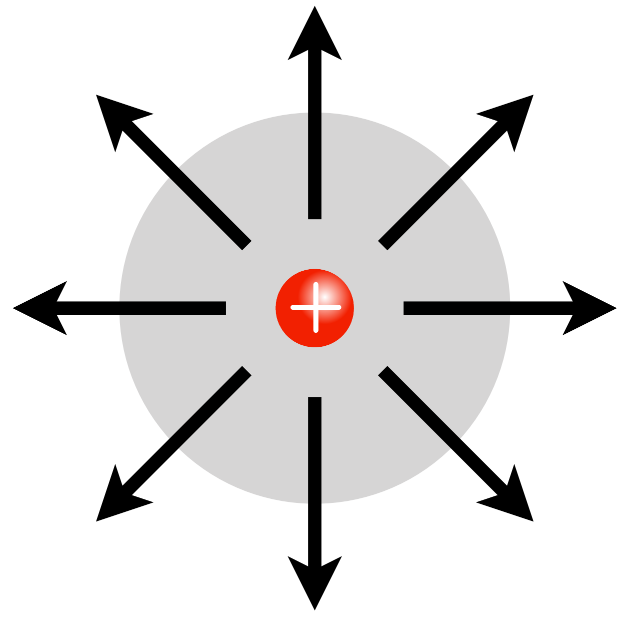

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Entropy:

$$S_{\mathrm{V}} = \ln\Omega \simeq \ln\left(\frac{R}{r_0}\right)^2 \quad \xrightarrow{R\to\infty} \quad \infty$$

... (same) logarithmic divergence
... vortices proliferate at high T

Vortex proliferation

Free energy (isolated vortex):

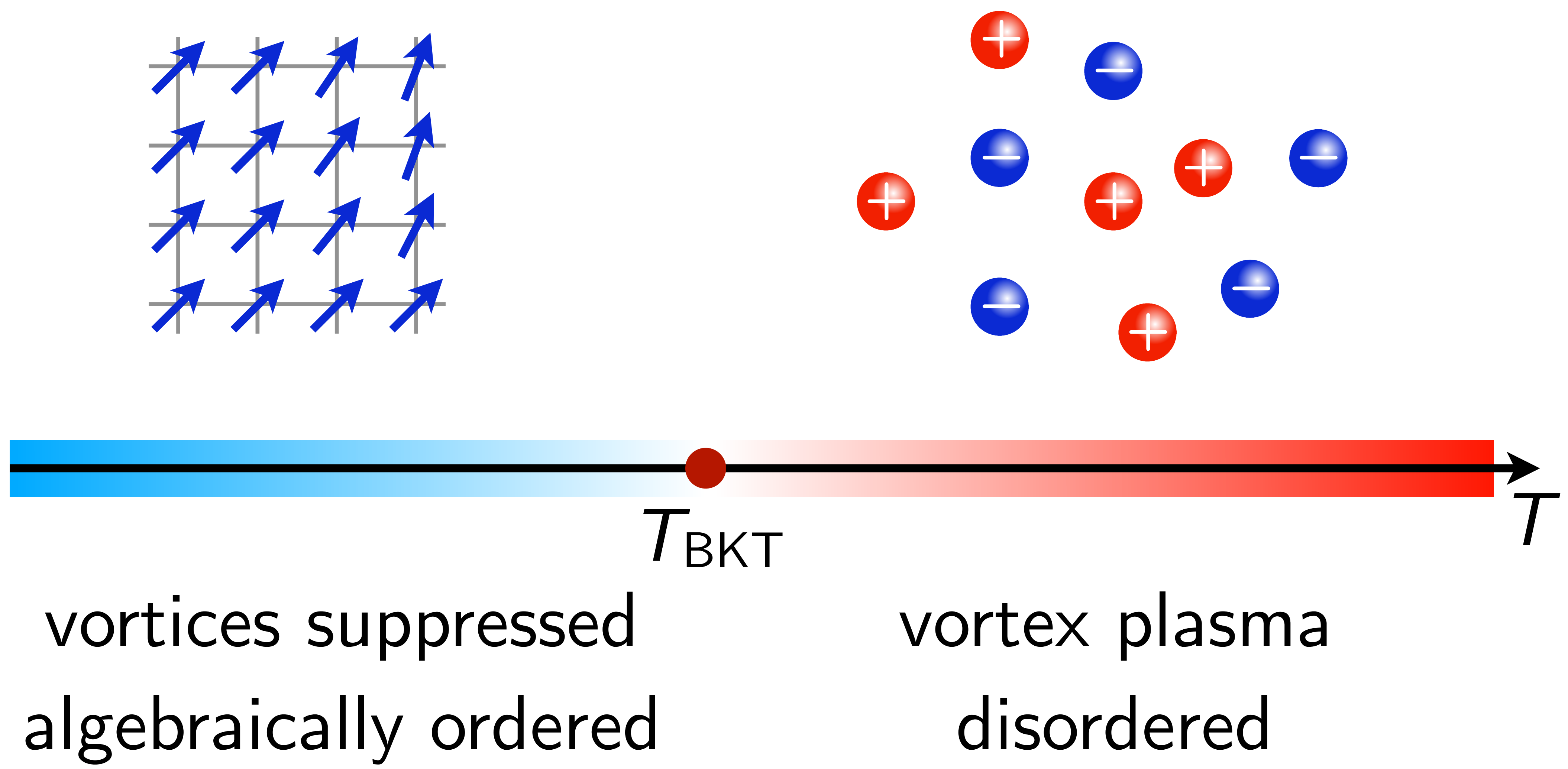
$$\Delta F = E_V - TS_V = \pi q^2 \ln \frac{R}{r_0} - 2T \ln \frac{R}{r_0} \begin{cases} > 0 & \text{for } T < \frac{\pi}{2} q^2 \\ < 0 & \text{for } T > \frac{\pi}{2} q^2 \end{cases}$$

Transition temperature:

$$T_{BKT} = \frac{\pi}{2}$$

... above which $q = \pm 1$ vortices proliferate

Phase diagram:

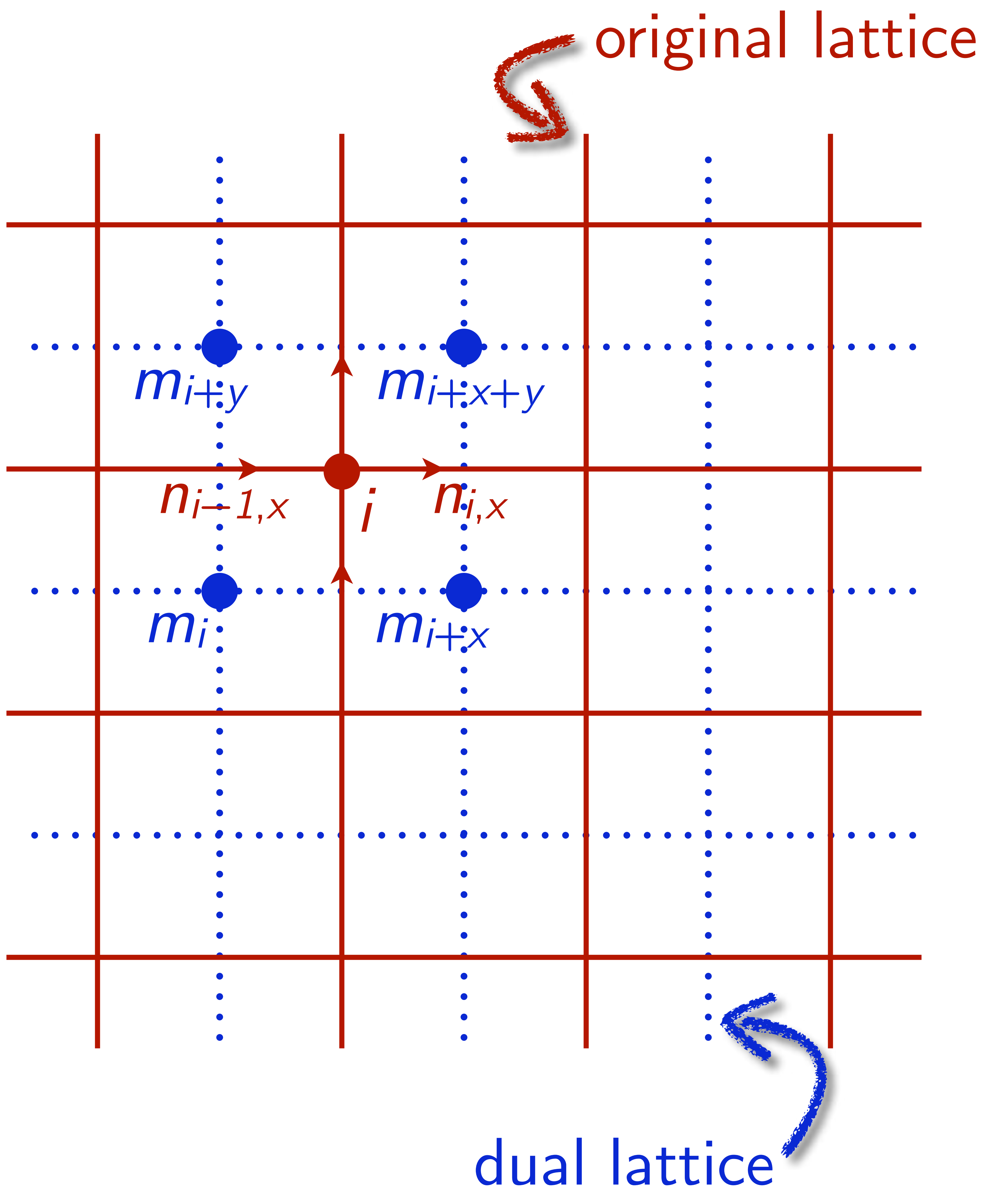


Duality transformation: Sine-Gordon model

XY model:

$$Z_{\mathrm{XY}} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \mathrm{e}^{-\mathcal{H}_{\mathrm{XY}}/T} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \mathrm{e}^{\frac{1}{T}\sum_{i,\hat{\mu}} \underbrace{\cos(\theta_i - \theta_{i+\hat{\mu}})}_{\sim n_{i,\mu}}}$$

... "current" with $\nabla \cdot \mathbf{n} = 0$



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... "current" with $\nabla \cdot \mathbf{n} = 0$

Dual model:

role of T inversed!

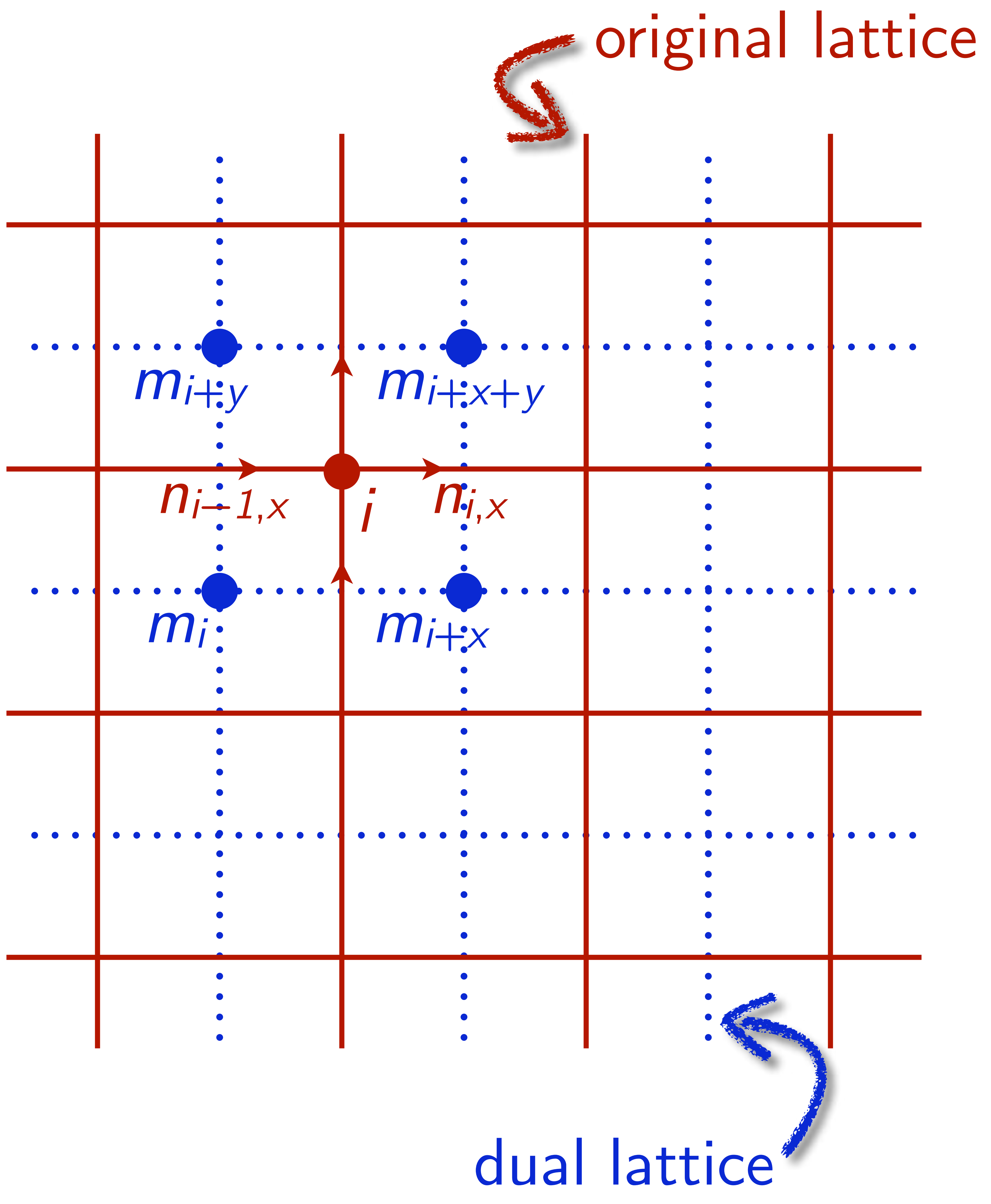
$$Z_{\mathrm{dual}} = \sum_{\{m_i\}} e^{-\frac{T}{2}\sum_{i,\hat{\mu}}(m_{i+\hat{\mu}} - m_i)^2}$$

... using Villain approximation $\theta \approx 2\pi\mathbb{Z} + \delta\theta$
... and Hubbard-Stratonovich $n_{i,\mu} \sim \theta_i - \theta_{i+\hat{\mu}}$
... and $\mathbf{n}(\mathbf{r}) = (\partial_y m(\mathbf{r}), -\partial_x m(\mathbf{r}))$

$$\simeq \int \mathcal{D}\varphi e^{-\int d^2\mathbf{r}\mathcal{L}_{\mathrm{sG}}(\varphi)}$$

with $\mathcal{L}_{\mathrm{sG}} = \frac{T}{2}(\nabla\varphi(\mathbf{r}))^2 - 2y\cos(2\pi\varphi(\mathbf{r}))$

... "sine-Gordon model"
... assuming low "fugacity" $y = e^{\beta\mu} \ll 1$



Renormalization group

Flow equations:

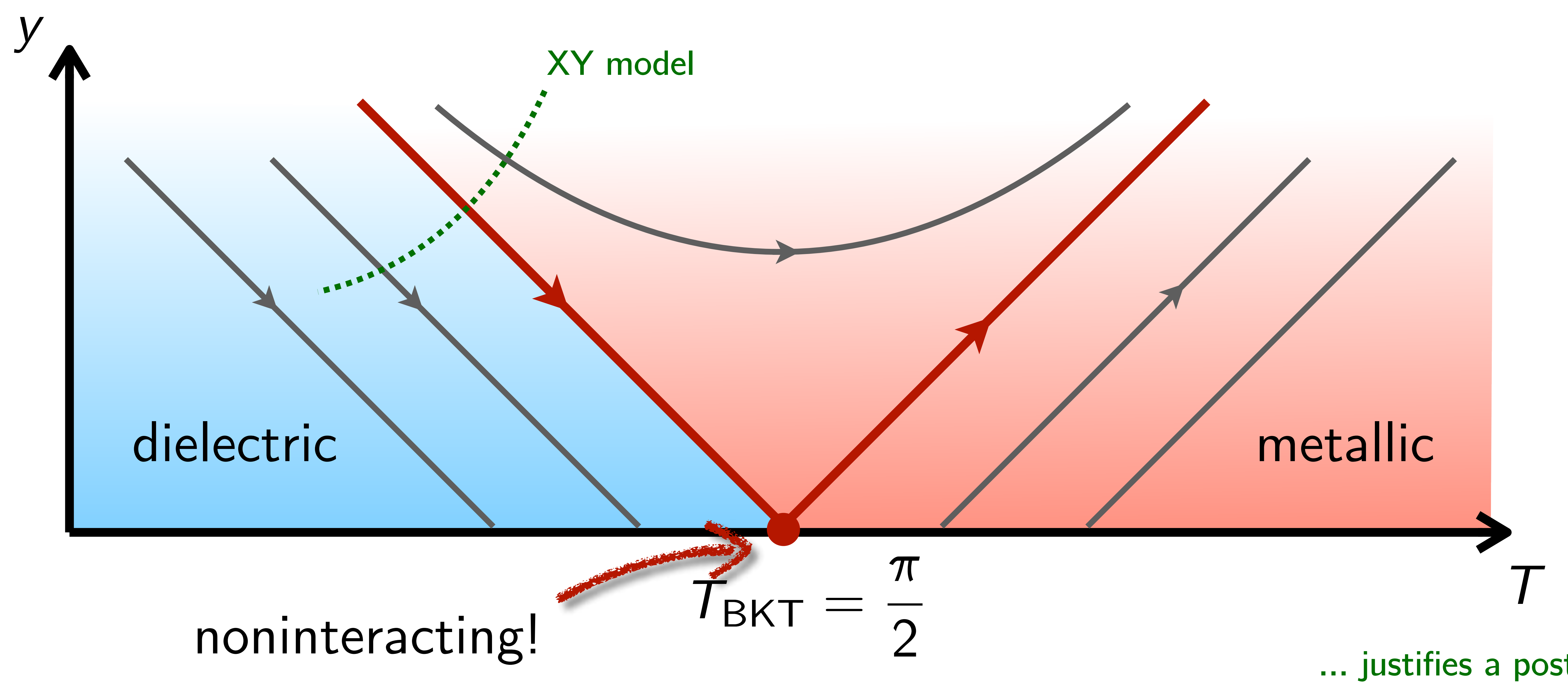
$$\frac{dy}{d\ln b} = \left(2 - \frac{\pi}{T}\right) y + \mathcal{O}(y^3)$$

... irrelevant for $T < \frac{\pi}{2}$
... relevant for $T > \frac{\pi}{2}$

$$\frac{dT}{d\ln b} = \frac{y^2}{2T} + \mathcal{O}(y^4)$$

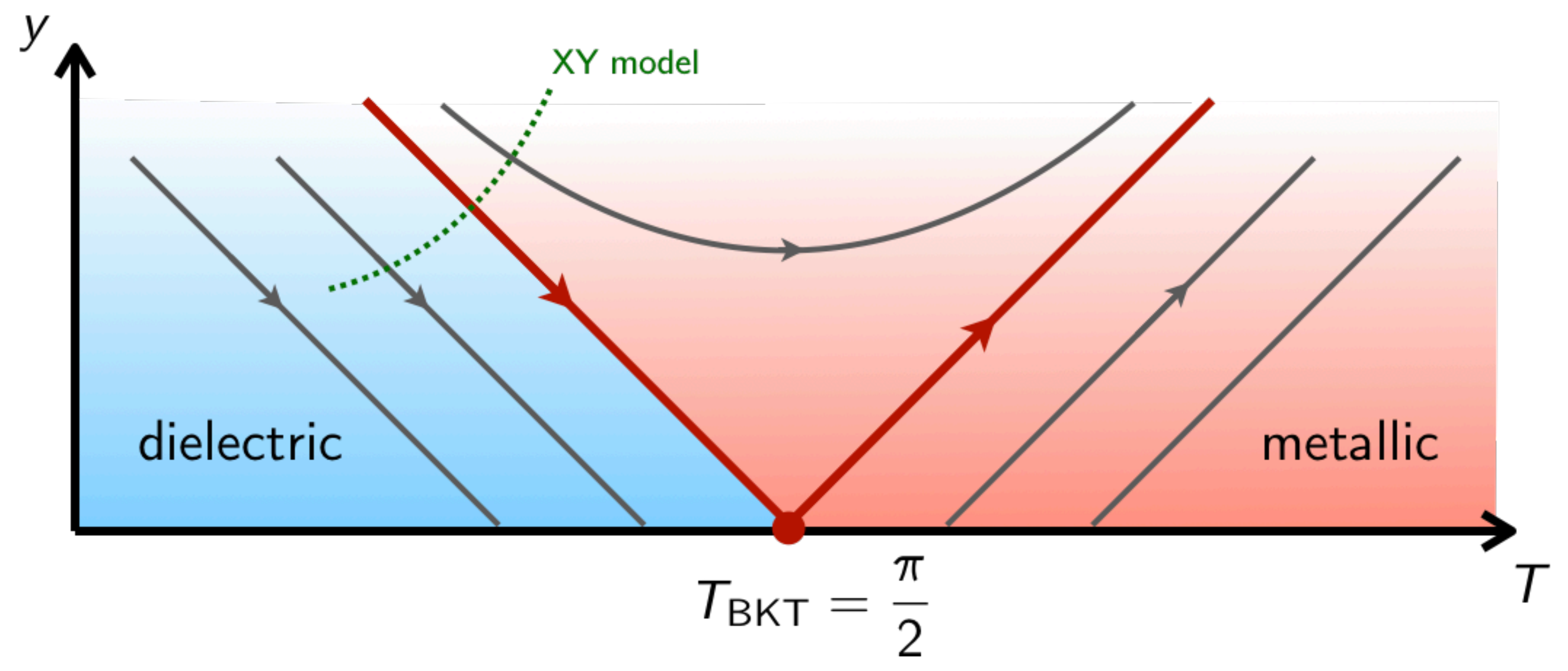
... marginal for $y = 0$
... relevant for $y > 0$

Flow diagram:



... justifies a posteriori simple energy-entropy argument

Critical behavior and algebraic order



For $T < T_c$:

$y \to 0$
$T \to T_\infty < \frac{\pi}{2}$

$$\left\langle e^{i\theta(\mathbf{r})} e^{-i\theta(0)} \right\rangle \propto \frac{1}{|\mathbf{r}|^{T_\infty/(2\pi)}}$$

"algebraic order"

... on line of fixed points

For $T = T_c$:

$y \to 0$
$T \to \frac{\pi}{2}$

$$\left\langle e^{i\theta(\mathbf{r})} e^{-i\theta(0)} \right\rangle \propto \frac{1}{|\mathbf{r}|^{1/4}}$$

... i.e., $\eta = 1/4$

For $T > T_c$:

$$\left\langle e^{i\theta(\mathbf{r})} e^{-i\theta(0)} \right\rangle \propto e^{-|\mathbf{r}|/\xi}$$

with correlation length $\xi \propto e^{C\sqrt{T_c/(T-T_c)}}$

... essential singularity
... since T marginal at $y = 0$

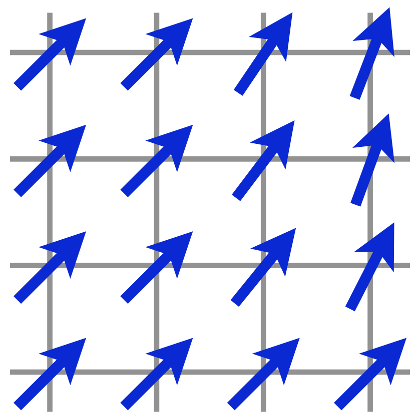
2D XY—Sine-Gordon duality

XY model

$$\mathcal{L}_{\mathrm{XY}} = \frac{1}{2T}(\nabla\theta)^2$$

... with $\theta \equiv \theta + 2\pi$

spin angles



spin picture

... vortices gapped

Sine-Gordon model

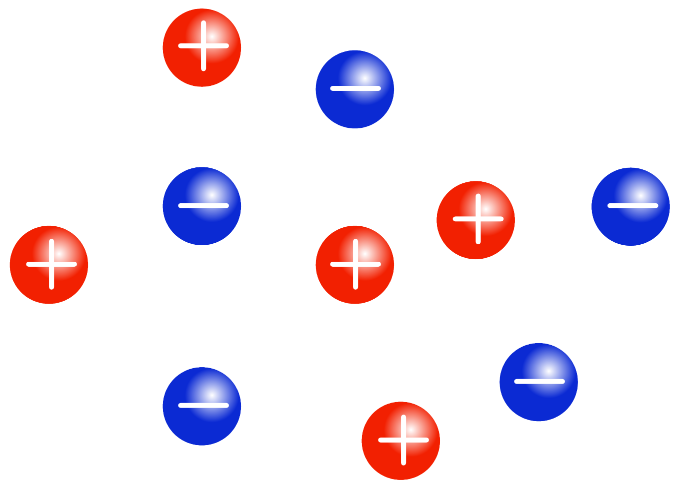
$$\mathcal{L}_{\mathrm{sG}} = \frac{T}{2}(\nabla\varphi)^2 - 2y\cos(2\pi\varphi)$$

... without any constraint on φ

vortex density

vortex fugacity

... $e^{\mu/T}$



vortex picture

... "Coulomb plasma"

Phase diagram:

T

θ ordered
φ disordered
"dielectric phase"

T_{BKT}

θ disordered
φ ordered ... "disorder variables"
"metallic phase"

Dualities in condensed-matter field theories: Three examples

Ex #1:

BKT transition in classical planar magnets:

2D XY—Sine-Gordon duality

Ex #2:

Superconducting transition in type-II materials:

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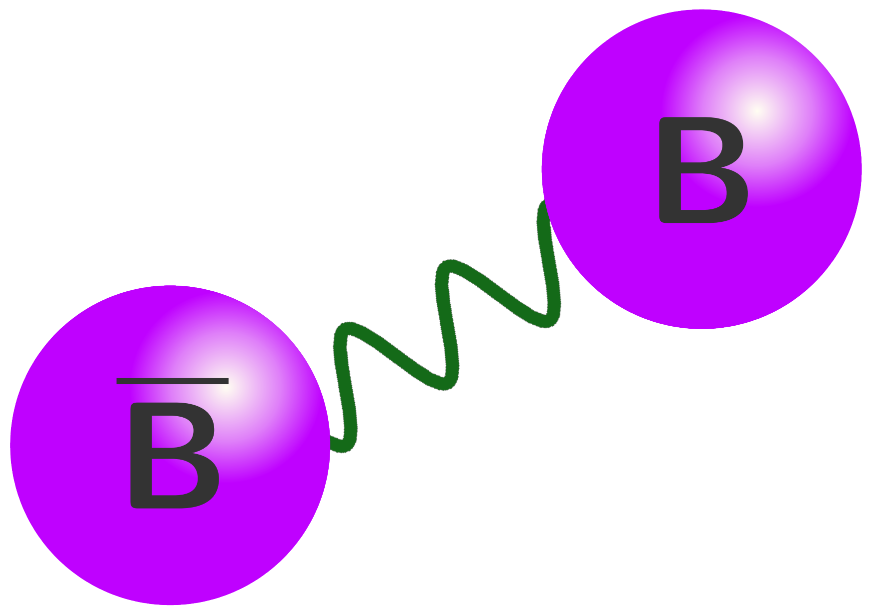


Ex #3:

Deconfined QCP in quantum planar magnets:

2+1D $NCCP^1$—QED_3-GN duality

Ex #2: Superconducting transition in type-II materials

3D Abelian Higgs model:

$$\mathcal{L}_{\text{AH}} = |(\nabla - 2ie\mathbf{a})\phi|^2 + r|\phi|^2 + \lambda|\phi|^4 + \frac{1}{2}(\nabla \times \mathbf{a})^2$$

Cooper pairs

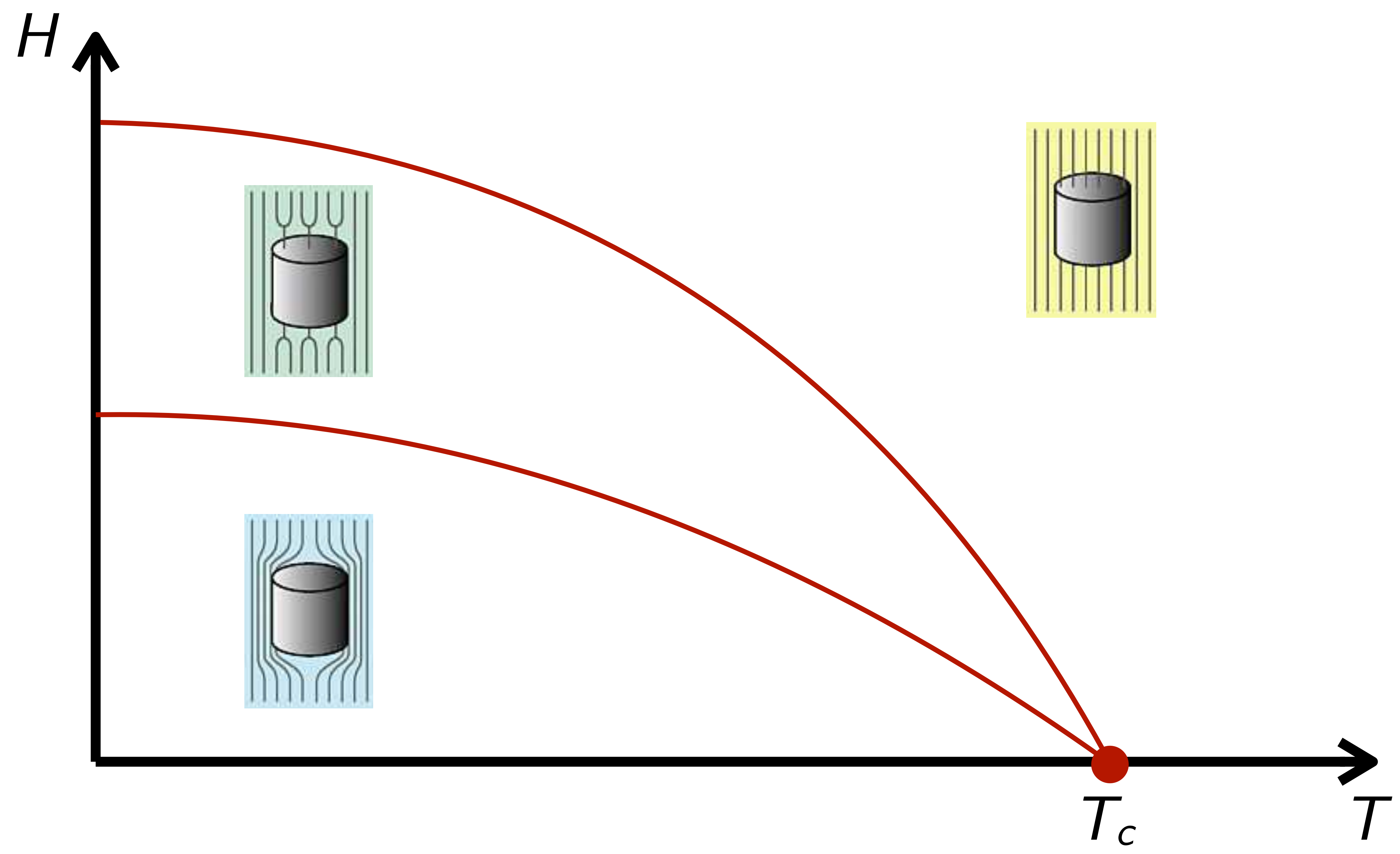
vector potential

$$\lambda > \xi/\sqrt{2}$$

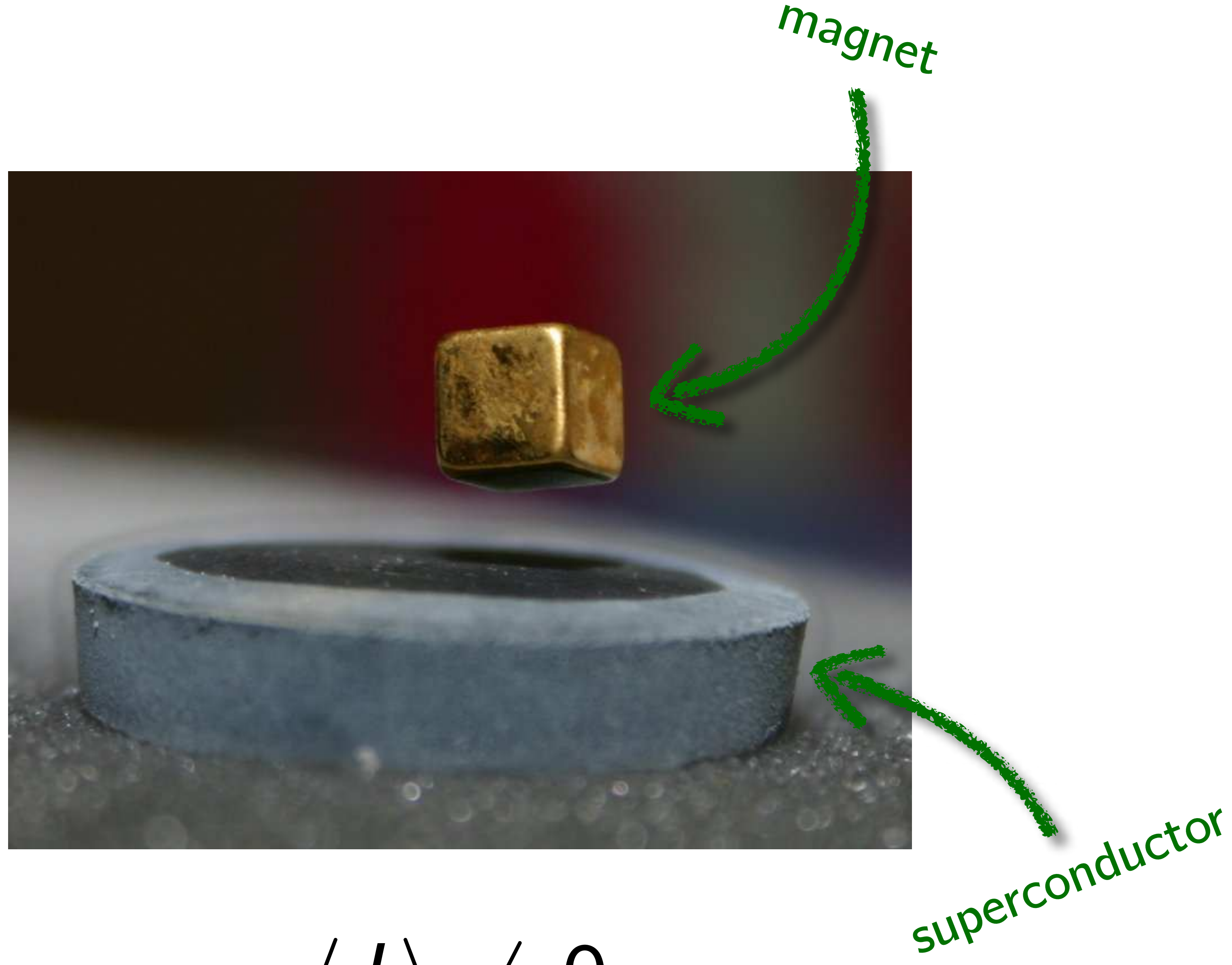
penetration depth

correlation length

Phase diagram (type-II superconductor):

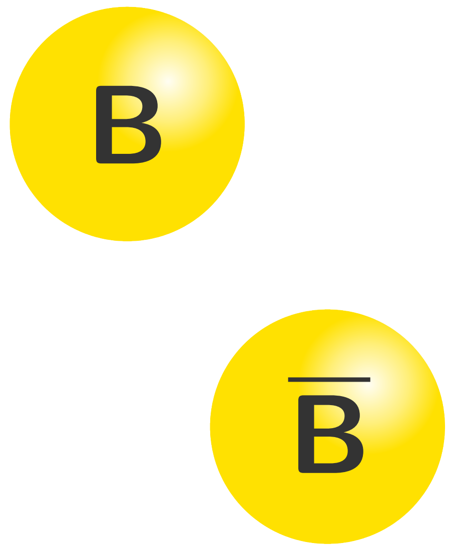


Meissner effect:



$$\langle \phi \rangle \neq 0$$

Duality transformation: 3D XY—Abelian Higgs



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... "current" with $\nabla \cdot \mathbf{n} = 0$

Resolution of constraint:

$$\mathbf{n}(\mathbf{r}) = \nabla \times \mathbf{m}(\mathbf{r}) \qquad \text{with gauge invariance} \qquad \mathbf{m}(\mathbf{r}) \mapsto \mathbf{m}(\mathbf{r}) + \nabla\chi(\mathbf{r})$$

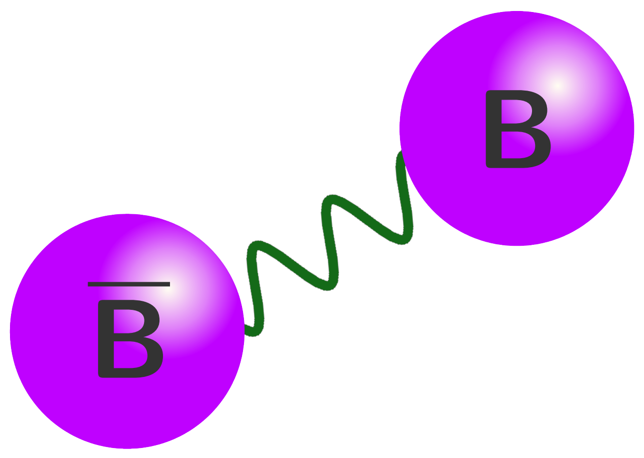
Dual model:

$$Z_{\text{AH}} = \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}a\, \mathrm{e}^{-\mathcal{L}_{\text{AH}}} \qquad \text{with} \qquad \mathcal{L}_{\text{AH}} = |(\nabla - i\frac{2\pi}{\sqrt{T}}\mathbf{a})\phi|^2 + r|\phi|^2 + \lambda|\phi|^4 + \frac{1}{2}(\nabla \times \mathbf{a})^2$$

"particle" "magnetic monopole"

Correlation functions:

$$\left\langle \mathrm{e}^{i\theta(\mathbf{r})}\mathrm{e}^{-i\theta(\mathbf{r}')} \right\rangle_{\text{XY}} = \frac{Z_{\text{AH}}[\mathcal{M}_a(\mathbf{r}), \mathcal{M}_a^\dagger(\mathbf{r}')]}{Z_{\text{AH}}[0,0]}$$



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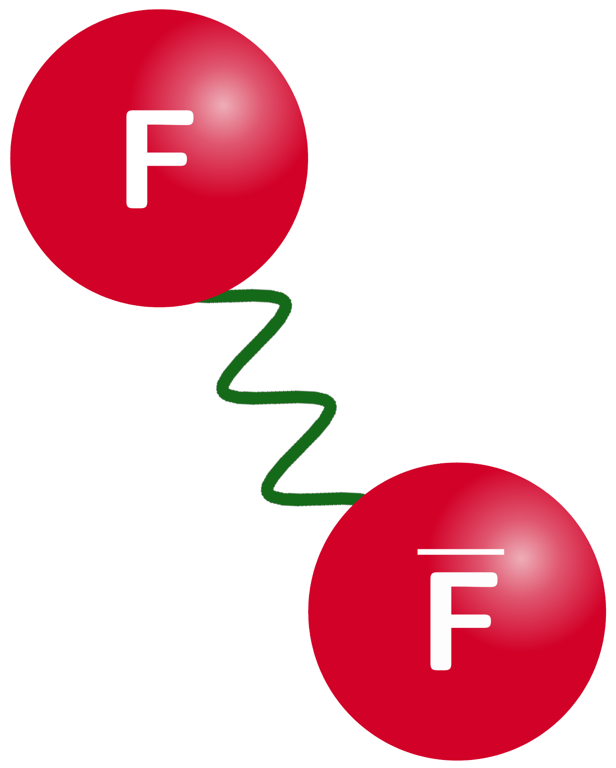


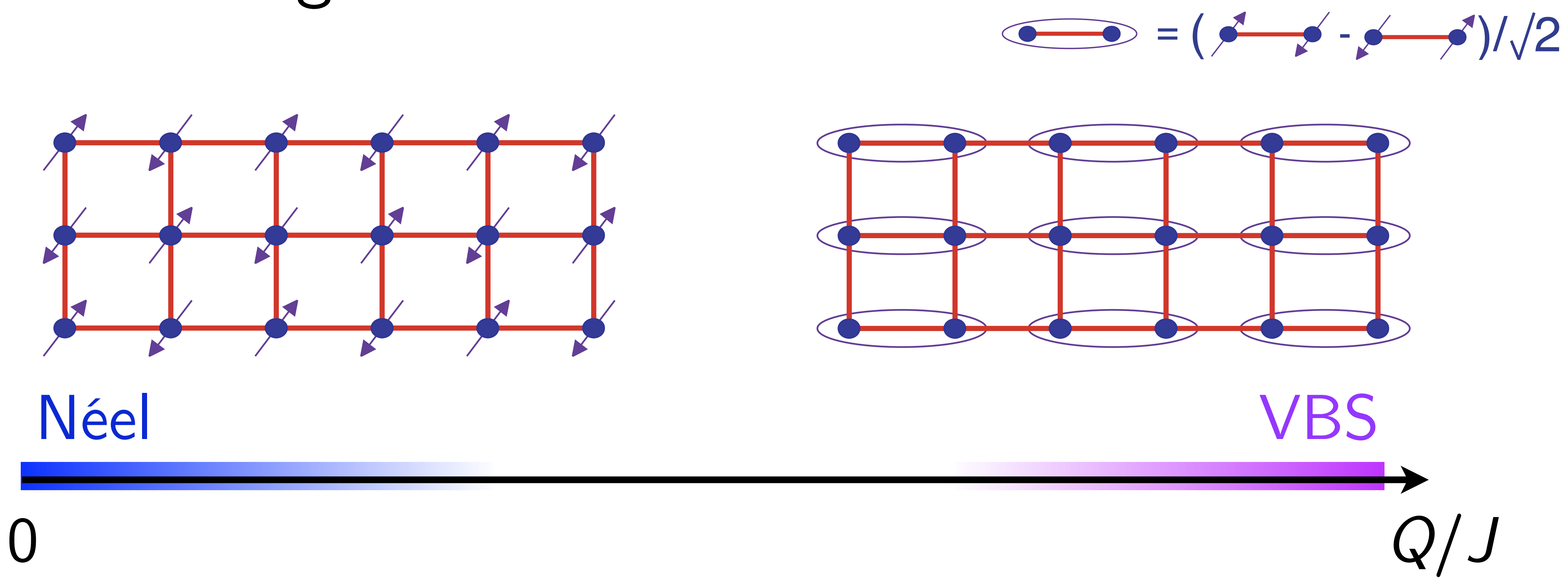
Ex #3: Deconfined QCP in quantum planar magnets

Toy model (spin-1/2 on square lattice):

[Sandvik, PRL '07; PRL '10]

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle ijkl \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \frac{1}{4} \right)$$

Phase diagram:



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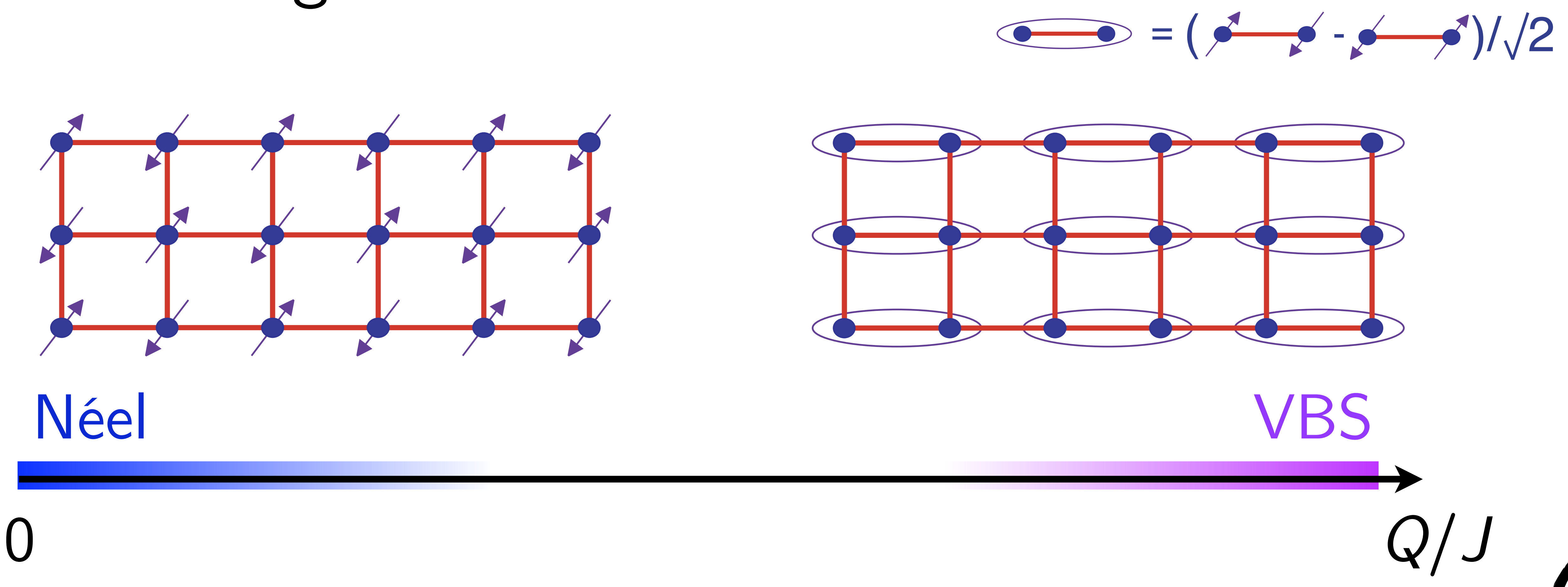
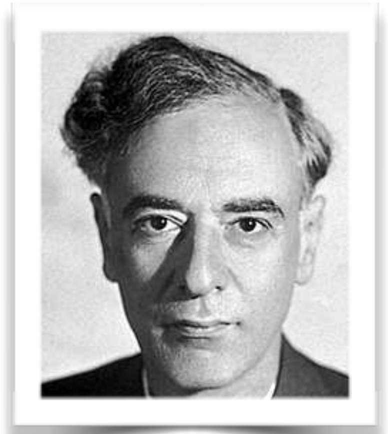
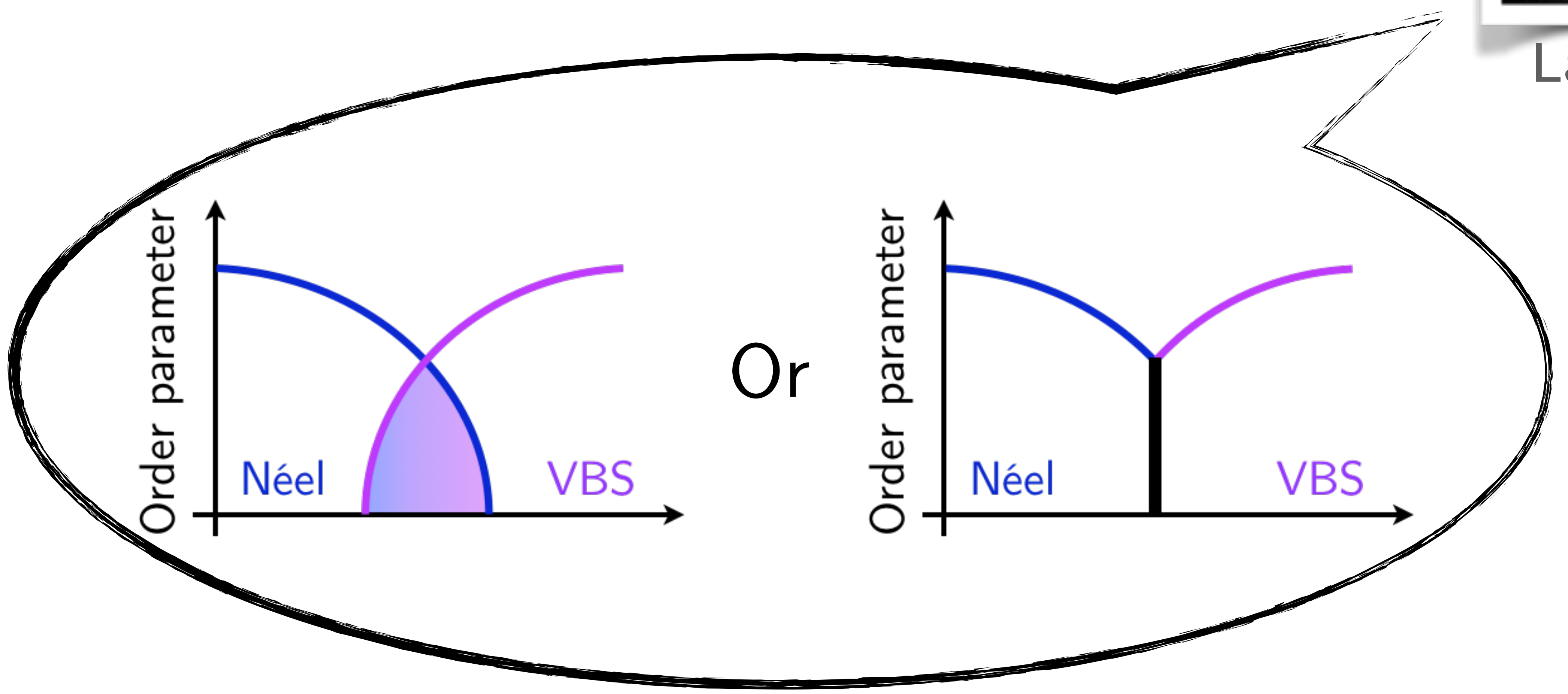




Landau



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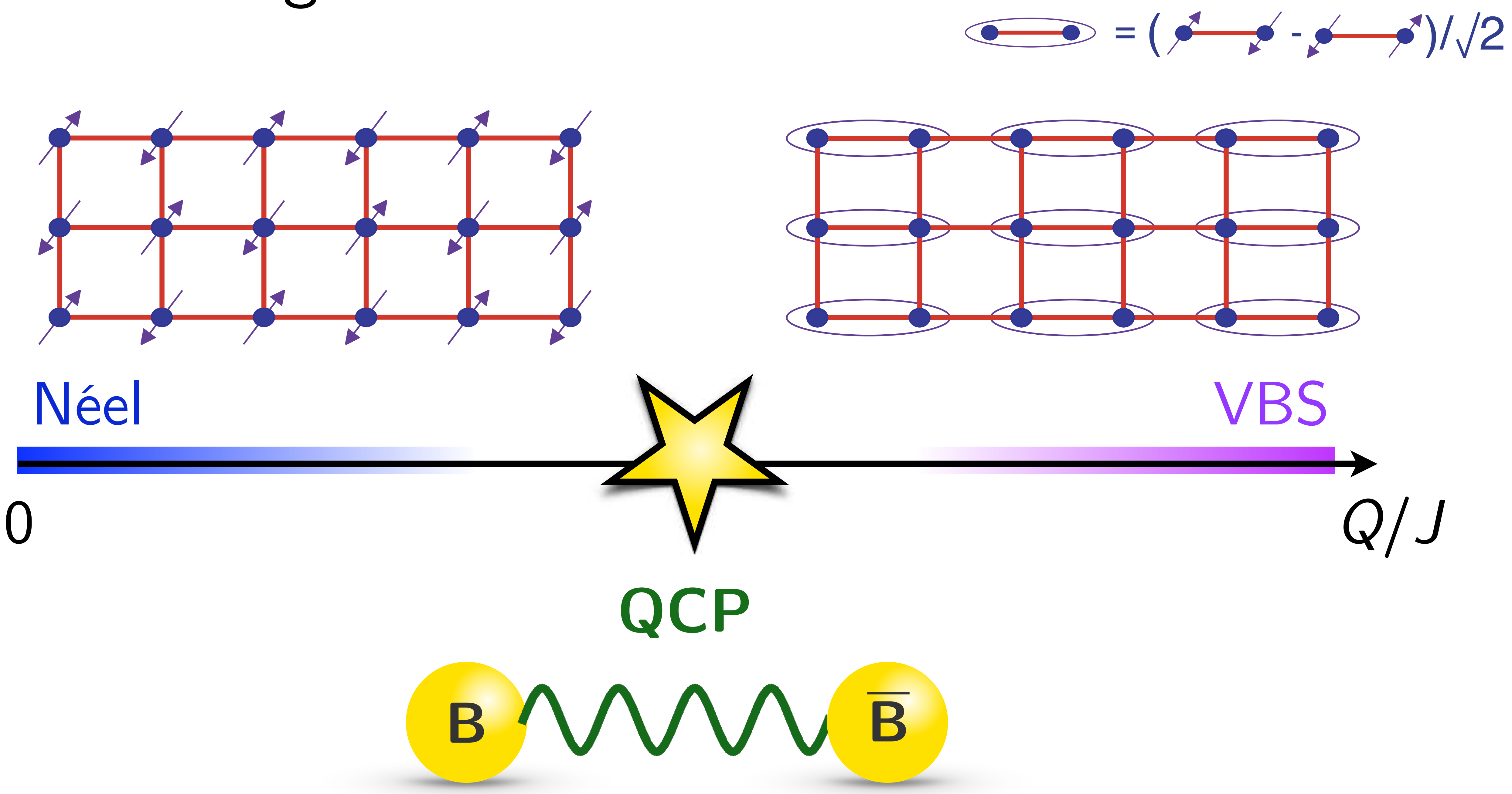
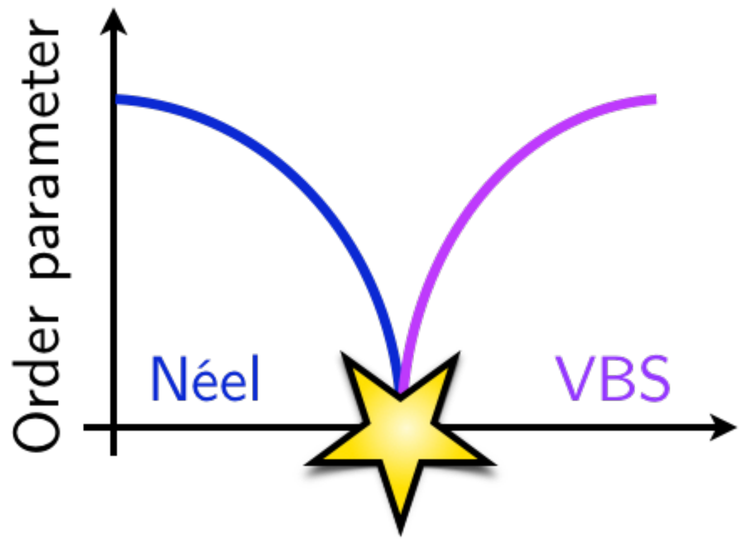


“Deconfined” quasiparticles



[Senthil *et al.*, Science ’04; PRB ’04]

Field theory for deconfined criticality

Fractionalization:

... CP^1 parametrization

$$\vec{n} = z^\dagger \vec{\sigma} z$$

$z = (z_1, z_2)$ ... complex "spinon"

Continuum field theory:

$$S_z = \int d^2\vec{r}d\tau \left[\sum_{\alpha=1,2} |(\partial_\mu - ib_\mu)z_\alpha|^2 - \left(|z_1|^2 + |z_2|^2\right)^2 \right]$$

b_μ ... "photon"

Monopoles irrelevant at critical point!

[Senthil *et al.*, Science '04; PRB '04]

→ "noncompact CP^1 model"

... with conserved flux (but monopole operators exist)

Deconfined QCP = critical point with fractionalized excitations

... with fractionalized excitations being "confined" in either phase

Field theory for deconfined criticality

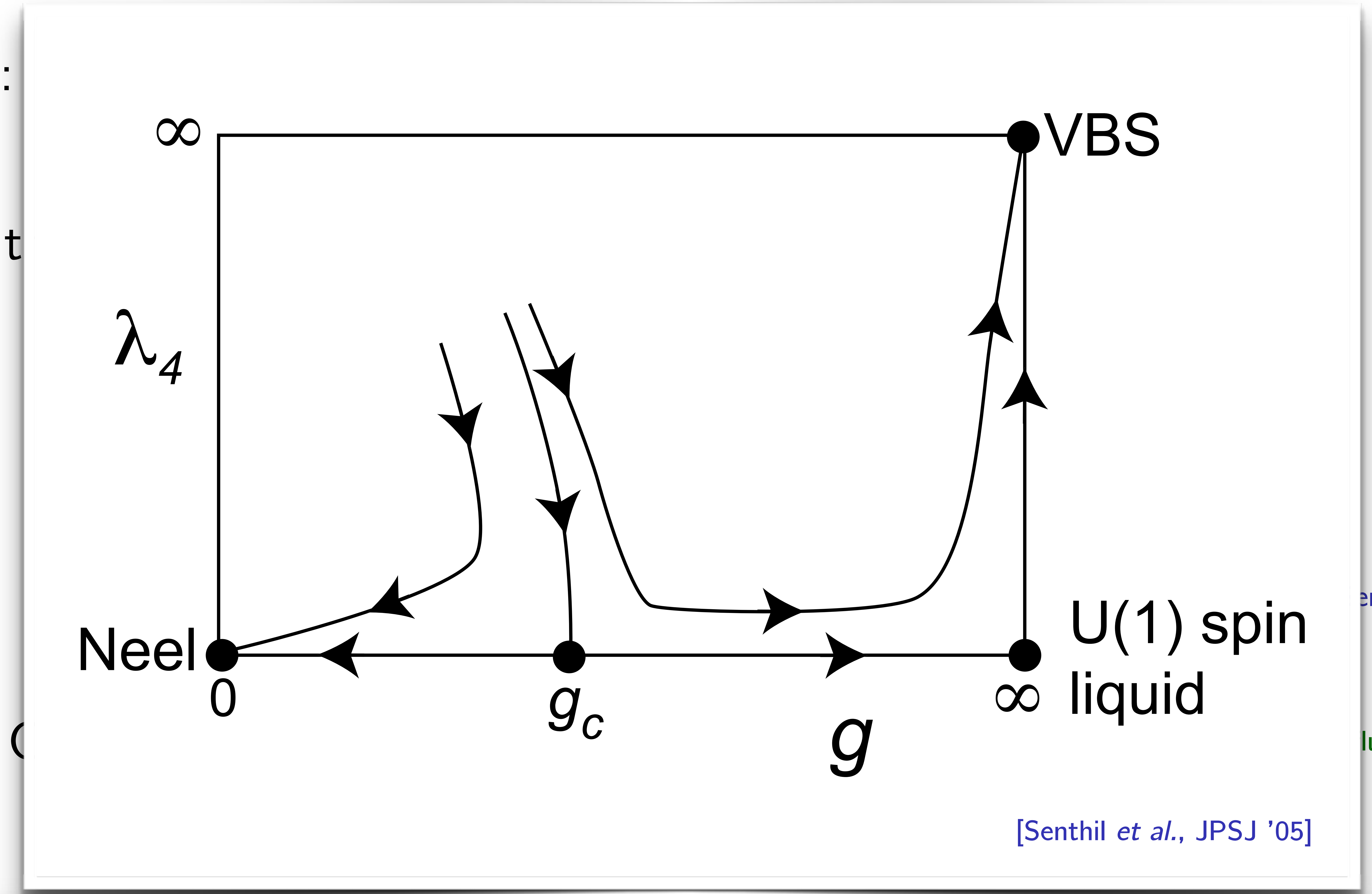
Fractionalization:

... CP^1 parametrization

$z = (z_1, z_2)$ ... complex "spinon"

Continuum field t

b_μ ... "photon"



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lux (but monopole operators exist)

Deconfined QCP = critical point with fractionalized excitations

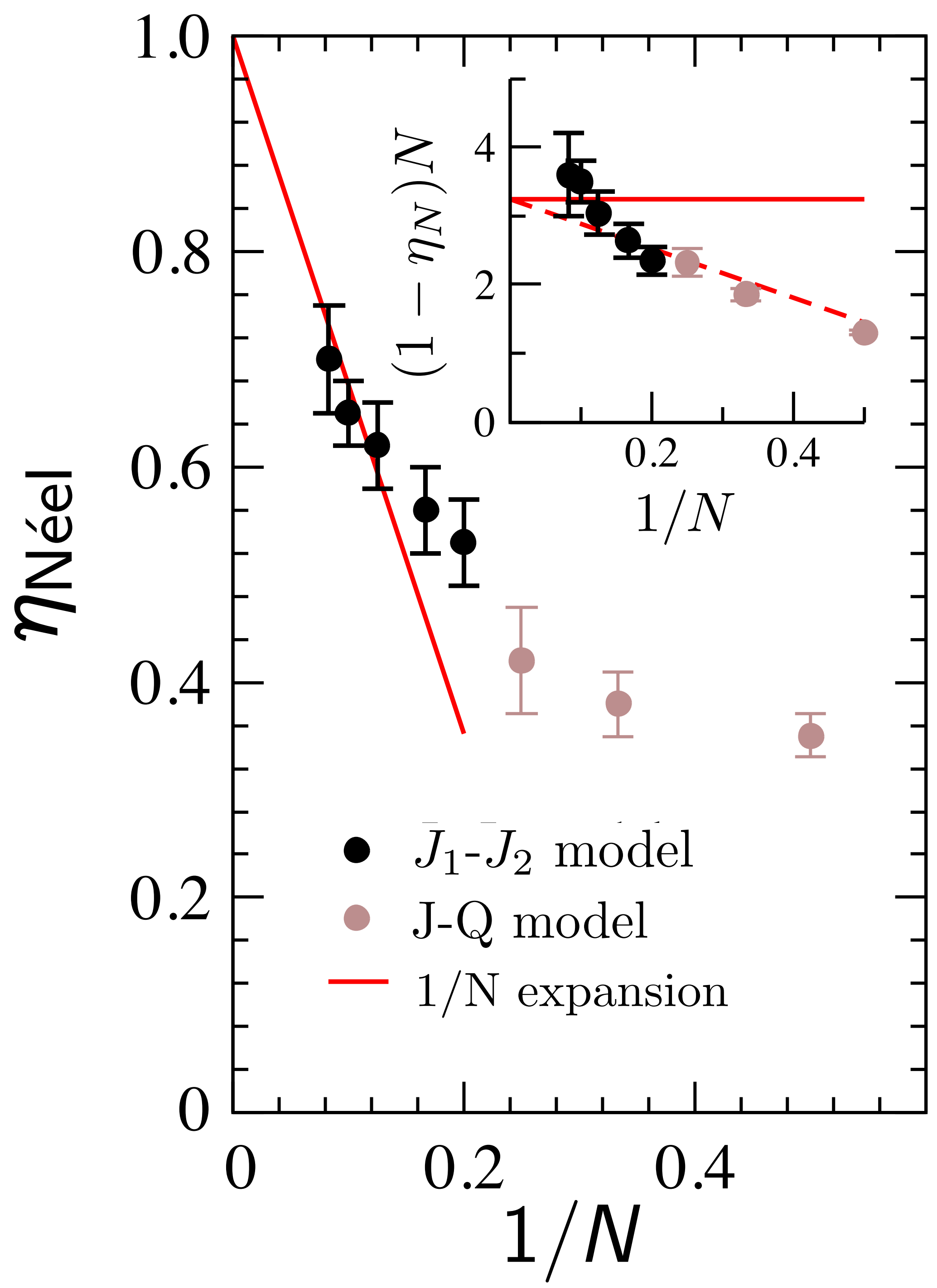
... with fractionalized excitations being "confined" in either phase

Evidence for deconfined criticality: Large N

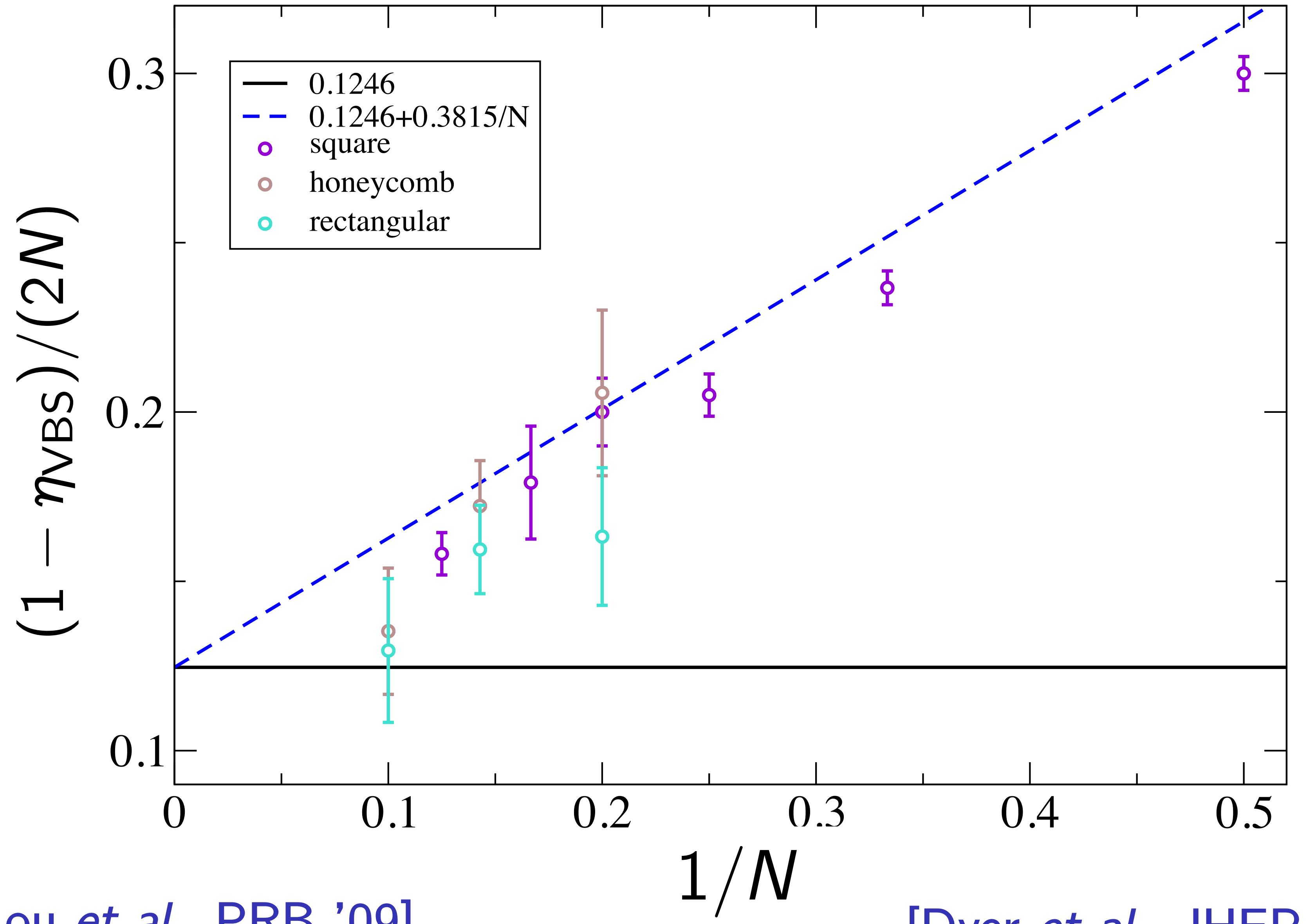
Field theory (noncompact CP$^{N\text{-}1}$ model):

$$S_z = \int d^2\vec{r}d\tau \left[\sum_{\alpha=1,2} |(\partial_\mu - i b_\mu) z_\alpha|^2 - \left(|z_1|^2 + |z_2|^2\right)^2 \right]$$

MC [SU(N) J_1-J_2 Heisenberg model]:



[Kaul & Sandvik, PRL '12]



[Lou *et al.*, PRB '09]

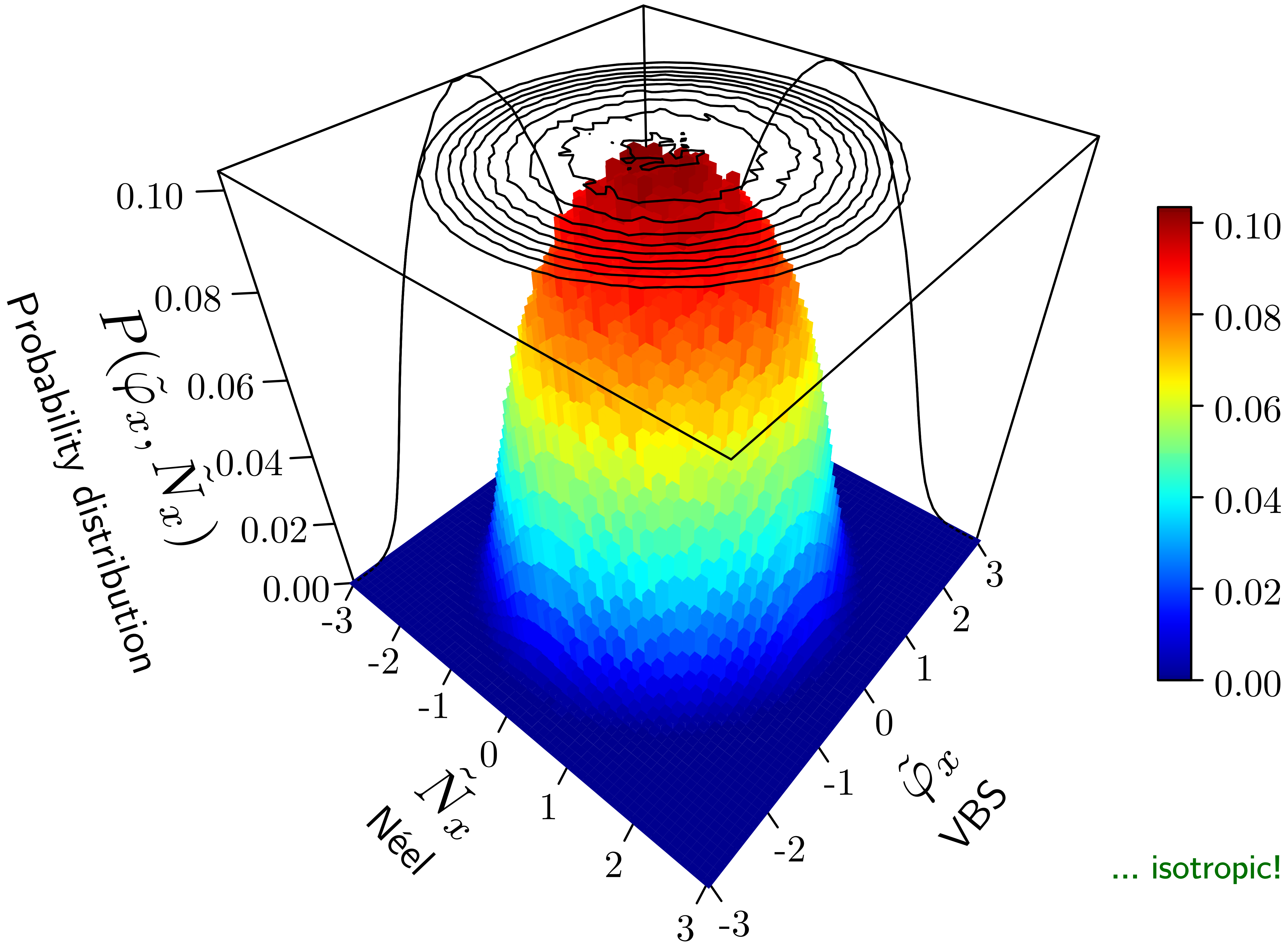
[Kaul & Sandvik, PRL '12]

[Block *et al.*, PRL '13]

[Dyer *et al.*, JHEP '16]

... excellent agreement

$N = 2$: Emergent symmetry?



... isotropic!

J-*Q* model:

$$\vec{n} = (\varphi_x, \varphi_y, N_x, N_y, N_z)$$

Noncompact CP¹ model:

$$\vec{n} = (2\,\mathrm{Re}\,\mathcal{M}_b, 2\,\mathrm{Im}\,\mathcal{M}_b, z^\dagger\sigma_x z, z^\dagger\sigma_y z, z^\dagger\sigma_z z)$$

monopole operators

Néel order parameter

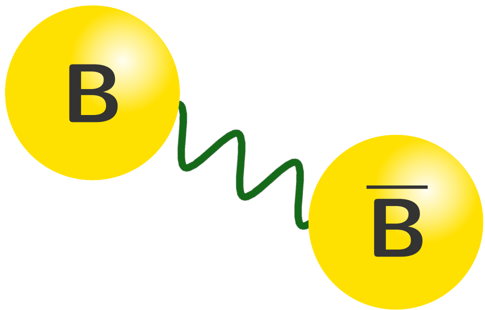
Emergent SO(5)?

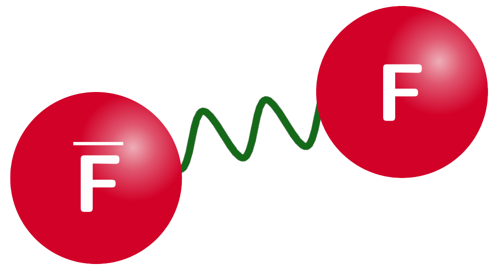
[Nahum *et al.*, PRL '15]

Duality conjecture

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]

Noncompact CP^1 model $\Longleftrightarrow$ QED_3-Gross-Neveu model





$$\sum_{\alpha=1,2} |D_b z_\alpha| - \left(|z_1|^2 + |z_2|^2\right)^2 \quad \Leftrightarrow \quad \sum_{i=1,2} \left(\bar{\psi}_i \not{D}_a \psi_i + \phi \bar{\psi}_i \psi_i\right) + V(\phi)$$

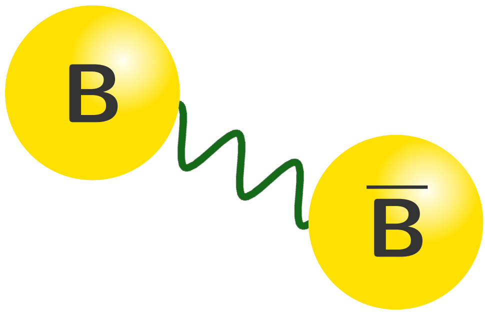
... with $V(\Phi)$ tuned to criticality

... part of "duality web" in 2+1D:

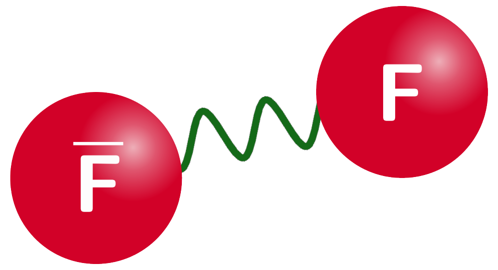
[Seiberg, Senthil, Wang, Witten, Ann. Phys. '16]
[Karch & Tong, PRX '16]
[Thomson & Sachdev, PRX '17]
...

Duality conjecture

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]



Noncompact CP¹ model $\Longleftrightarrow$ QED$_3$-Gross-Neveu model



$$\sum_{\alpha=1,2} |D_b z_\alpha| - \left(|z_1|^2 + |z_2|^2\right)^2 \quad \Leftrightarrow \quad \sum_{i=1,2} \left(\bar{\psi}_i \not{D}_a \psi_i + \phi \bar{\psi}_i \psi_i\right) + V(\phi)$$

... with $V(\Phi)$ tuned to criticality

... part of "duality web" in 2+1D:

[Seiberg, Senthil, Wang, Witten, Ann. Phys. '16]
[Karch & Tong, PRX '16]
[Thomson & Sachdev, PRX '17]
...

Explicitly:

monopole in b → U(1); Néel order parameter → O(3); Ising order parameter → ϕ

$$(n_1, n_2, n_3, n_4, n_5) \sim (\underbrace{2\,\mathrm{Re}\,\mathcal{M}_b, 2\,\mathrm{Im}\,\mathcal{M}_b}_{\mathrm{U}(1)}, \underbrace{z^\dagger \sigma_x z, z^\dagger \sigma_y z, z^\dagger \sigma_z z}_{\mathrm{O}(3)})$$

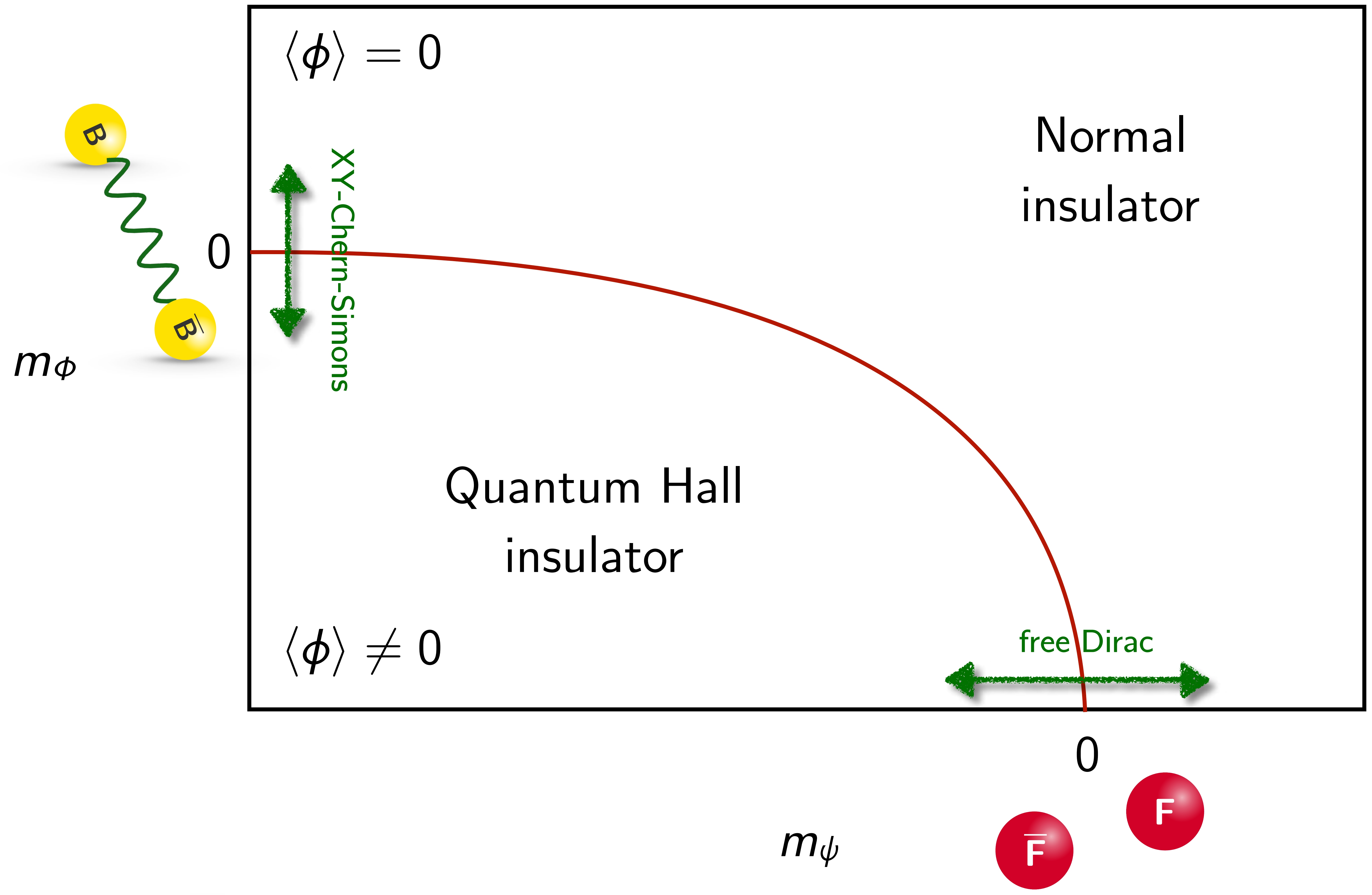
$$\sim \left[\underbrace{\mathrm{Re}\,(\psi_1^\dagger \mathcal{M}_a), -\mathrm{Im}\,(\psi_1^\dagger \mathcal{M}_a), \mathrm{Re}\,(\psi_2^\dagger \mathcal{M}_a), \mathrm{Im}\,(\psi_2^\dagger \mathcal{M}_a)}_{\mathrm{U}(2)}, \phi\right]$$

zero mode → $\psi_1^\dagger$; monopole in a → $\mathcal{M}_a$

... naturally explains emergent SO(5)!

Idea of a "derivation" of the duality

Phase diagram of a "mother theory":

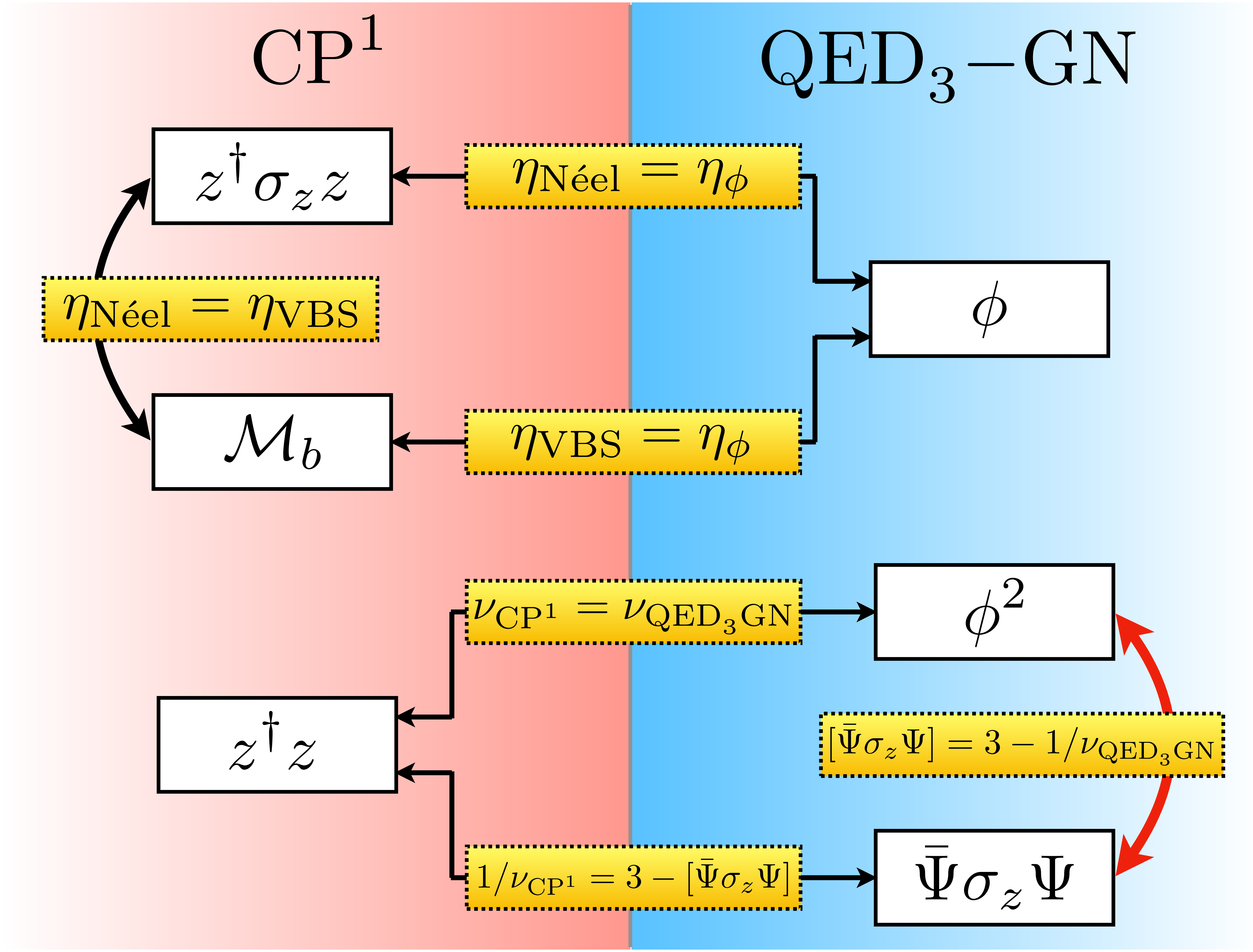


Free Dirac vs. XY-Chern-Simons: Two limits of the same transition?

[Seiberg, Senthil, Wang, Witten, Ann. Phys. '16]

Consequences of $CP^1 \Longleftrightarrow QED_3$-Gross-Neveu

[Wang, Nahum, Metlitski, Xu, Senthil, PRX '17]
[LJ & He, PRB '17]
[Ihrig, LJ, Mihaila, Scherer, PRB '18]



QED$_3$-GN model: 4-ε expansion

Lagrangian:

[LJ & He, PRB '17]

$$\mathcal{L}_{\psi\phi} = \sum_{i=1,2} \left[\bar{\psi}_i(\partial_\mu - iea_\mu)\gamma_\mu\psi_i + g\phi\bar{\psi}_i\psi_i\right] + \frac{1}{2}\phi(r - \partial_\mu^2)\phi + \lambda\phi^4$$

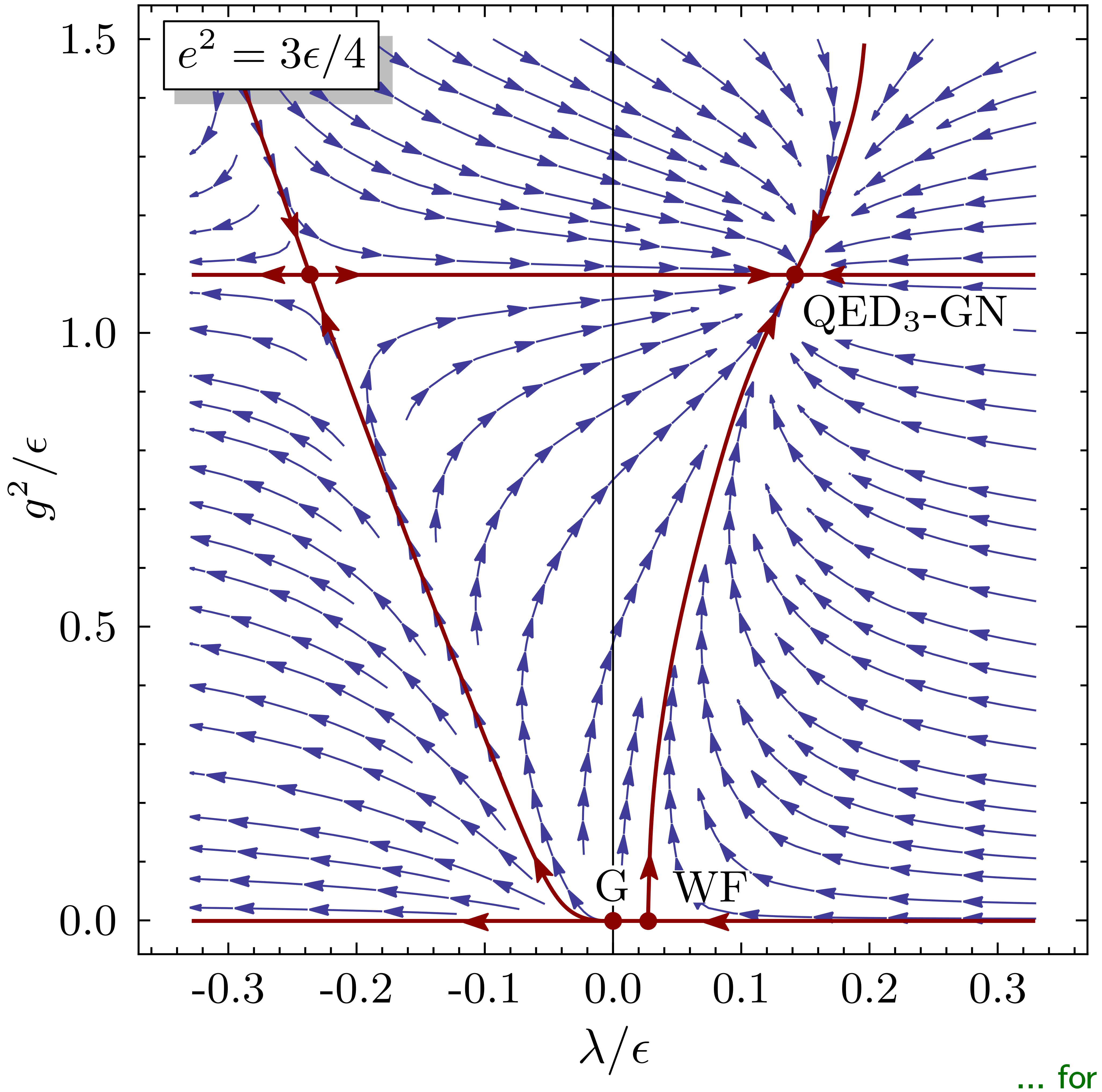
Engineering dimensions:

$$[e^2] = 4 - D, \qquad [g] = \frac{4-D}{2}, \qquad [\lambda] = 4 - D$$

... become simultaneously marginal near $D = 3{+}1$!

ε expansion in $D = 4$-ε possible!

QED$_3$-GN model: Flow diagram in $D = 4$-ε



... for $N = 2$

... fully IR stable fixed point

[LJ & He, PRB '17]

QED$_3$-GN model at three loops

[Ihrig, LJ, Mihaila, Scherer, PRB '18]

Critical exponents ($N = 2$):

$$\eta_\phi = 2.2\epsilon - 0.222725\epsilon^2 + 16.8838\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\nu^{-1} = 2 - 3.90514\epsilon + 7.47146\epsilon^2 - 90.5962\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$[\bar{\psi}\sigma^z\psi] = 3 - 1.6\epsilon + 1.987\epsilon^2 - 17.46\epsilon^3 + \mathcal{O}(\epsilon^4)$$

... large $\mathcal{O}(\epsilon^3)$ corrections

Padé approximant:

$$[m/n] = \frac{a_0 + a_1\epsilon + \ldots a_m\epsilon^m}{1 + b_1\epsilon + \cdots + b_n\epsilon^n}$$

Mean values:

$$1/\nu = 0.67(1) \qquad [\bar{\psi}\sigma^z\psi] \approx 2.12(50)$$

QED$_3$-GN vs. CP^1 duality: SO(5) scaling relation

[Ihrig, LJ, Mihaila, Scherer, PRB '18]

Scaling relation from SO(5) symmetry:

$$[\bar{\psi}\sigma^z\psi] = 3 - 1/\nu$$

Our estimates:

$$[\bar{\psi}\sigma^z\psi] \approx 2.12(50) \qquad \text{vs.} \qquad 3 - 1/\nu \approx 2.33(1)$$

... not inconsistent with duality prediction!

Comparison: Large-N (QED$_3$-GN) — numerics (J-Q model)

$$\eta_\phi \approx 0.17\text{–}0.30 \qquad \text{vs.} \qquad \eta_{\text{Néel}} \approx \eta_{\text{VBS}} \approx 0.25(3)$$

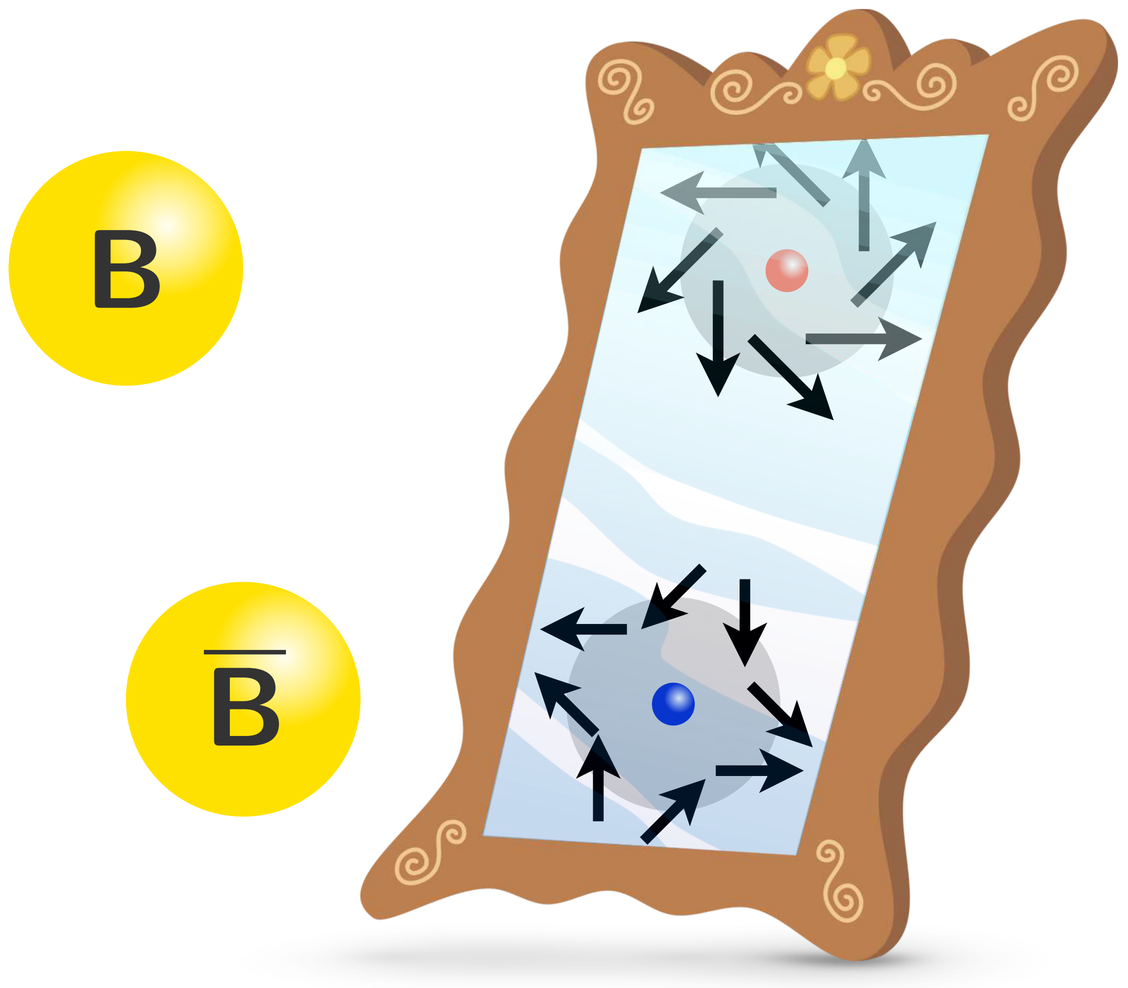
remarkably small!
(fermionic QCP)

remarkably large!
(bosonic QCP)

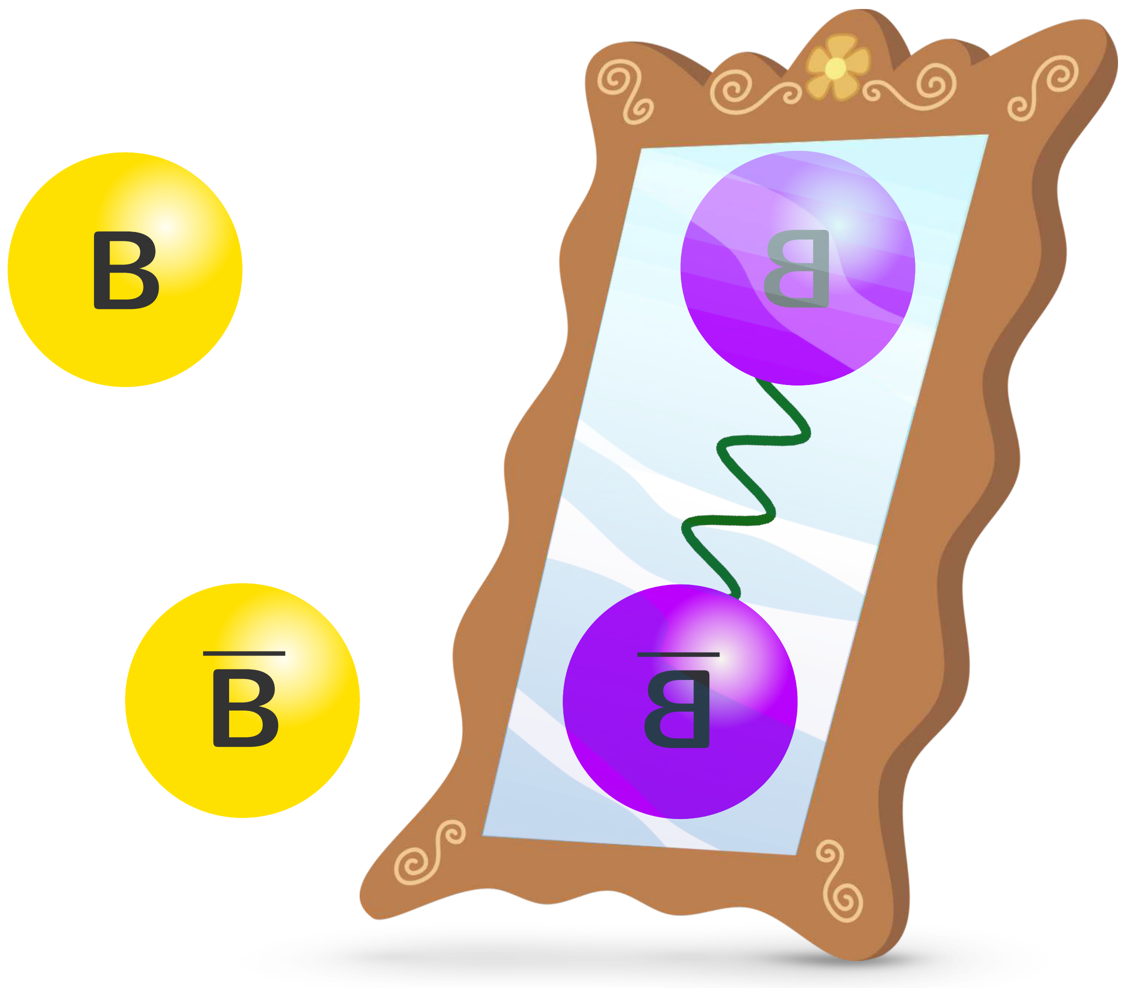
[Boyack *et al.*, PRB '19]

[Nahum *et al.*, PRX '15]

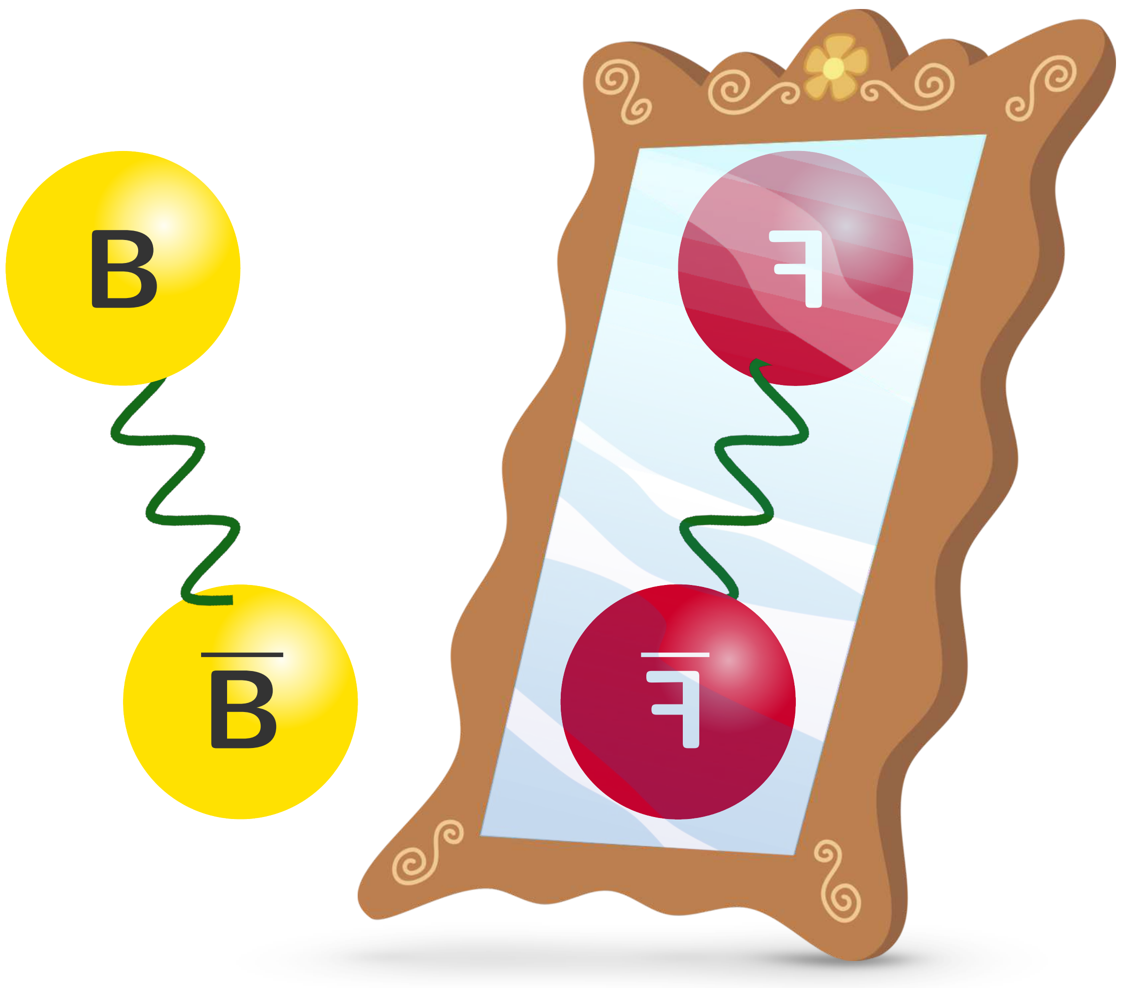
Conclusions: Three examples for dualities in cond-mat field theories



2D XY—Sine-Gordon



3D XY—Abelian-Higgs



2+1D NCCP^1—QED$_3$-GN