Emergence of relativistic flatland fermions in systems without fermions

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(TU Dresden)

ECT* Colloquium — Workshop Teaser
Relativistic Fermions in Flatland: theory and application
ECT*, Trento, IT, 5-9 July 2021

Emmy Noether-Programm
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ct.qmat
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

SFB 1143

Outline

(1) Motivation: *Emergence versus constructionism*

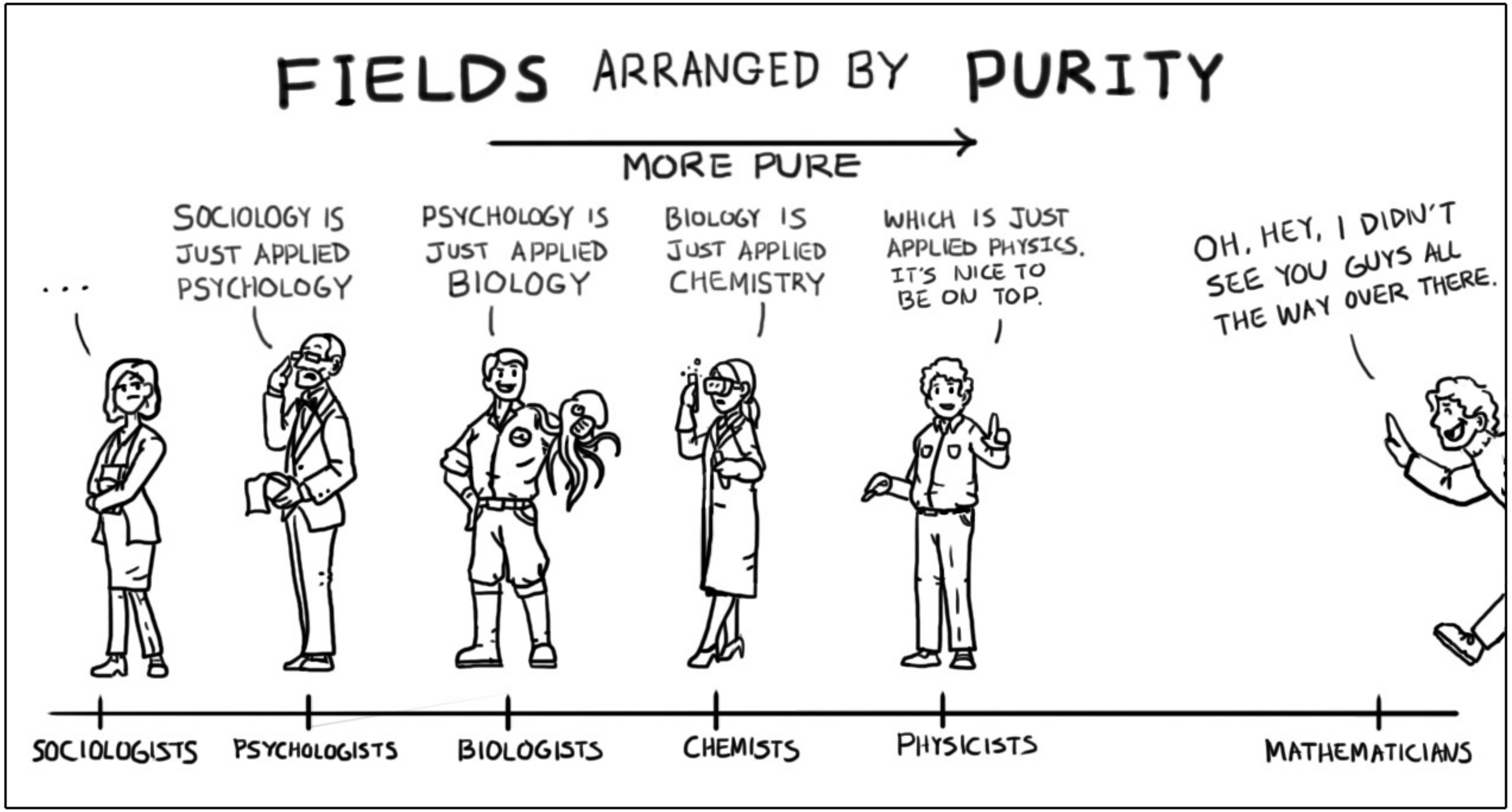
(2) Emergent symmetry: *Relativistic fermions from nonrelativistic electrons*

(3) Emergent topology: *Relativistic fermions from winding numbers*

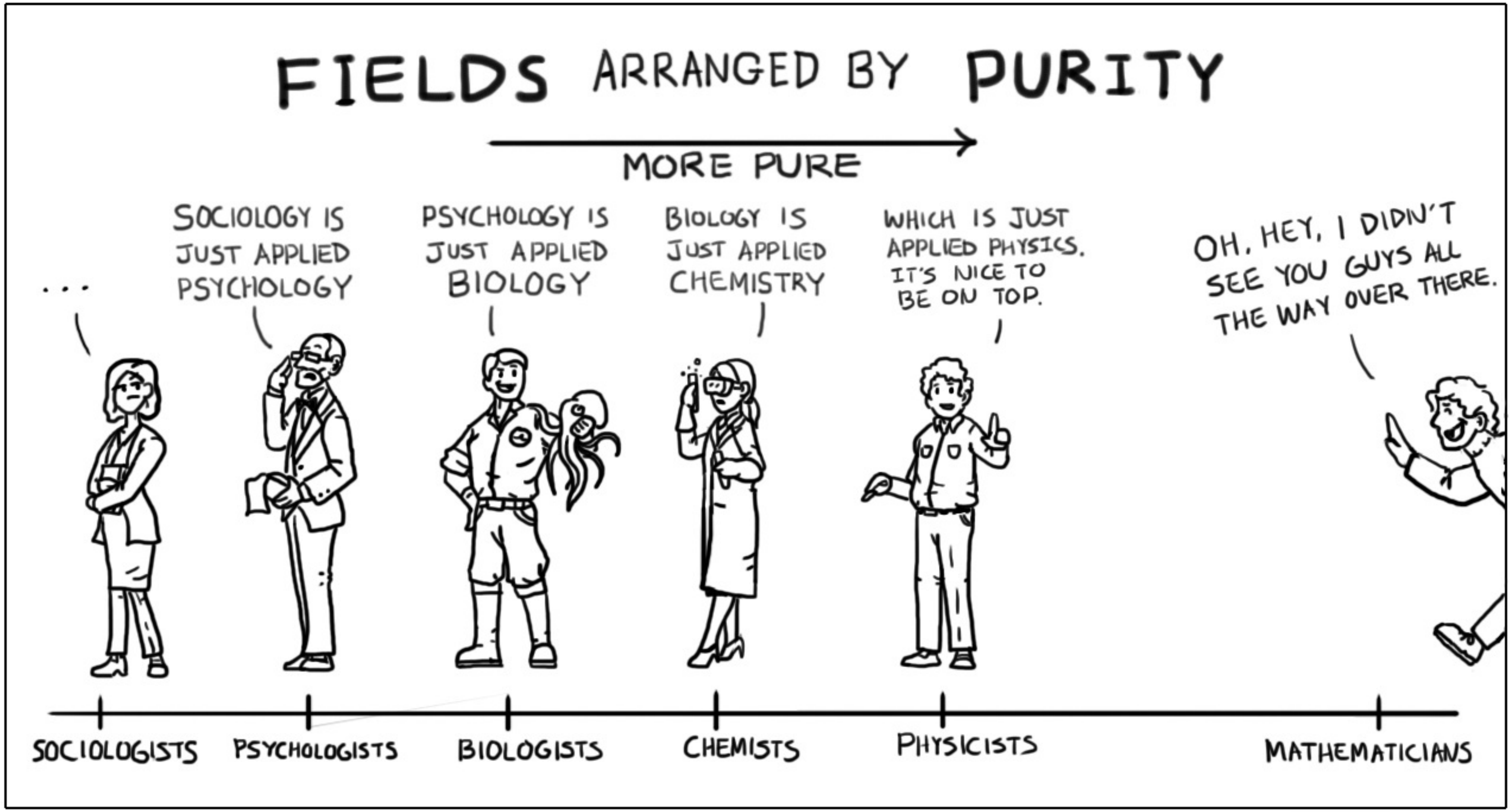
(4) Emergent excitations: *Relativistic fermions from fractionalization*

(5) Conclusions

Fundamental Physics



Fundamental Physics



"Fundamental" science ≡ science of fundamental particles?

Complexity

Emergence

Aristotle, 385-322 BC

Emergence

Aristotle, 385-322 BC

P. Anderson, 1923-2020 AD

[Anderson, Science '72]

More is different: Sports

0 : 1

SET PRICE
352 M€

197 M€

218 M€

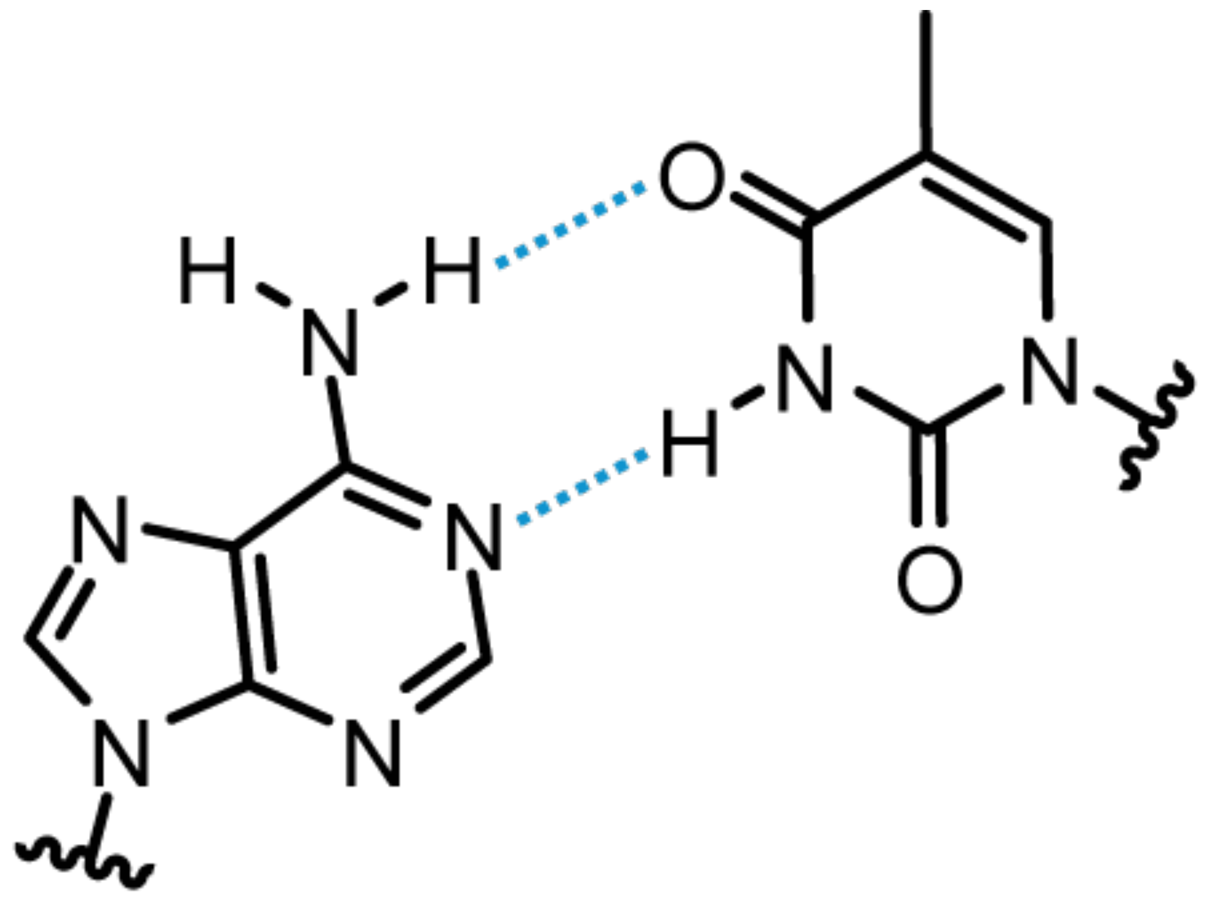
40 M€

The value of a team is greater than the sum of its transfer fees

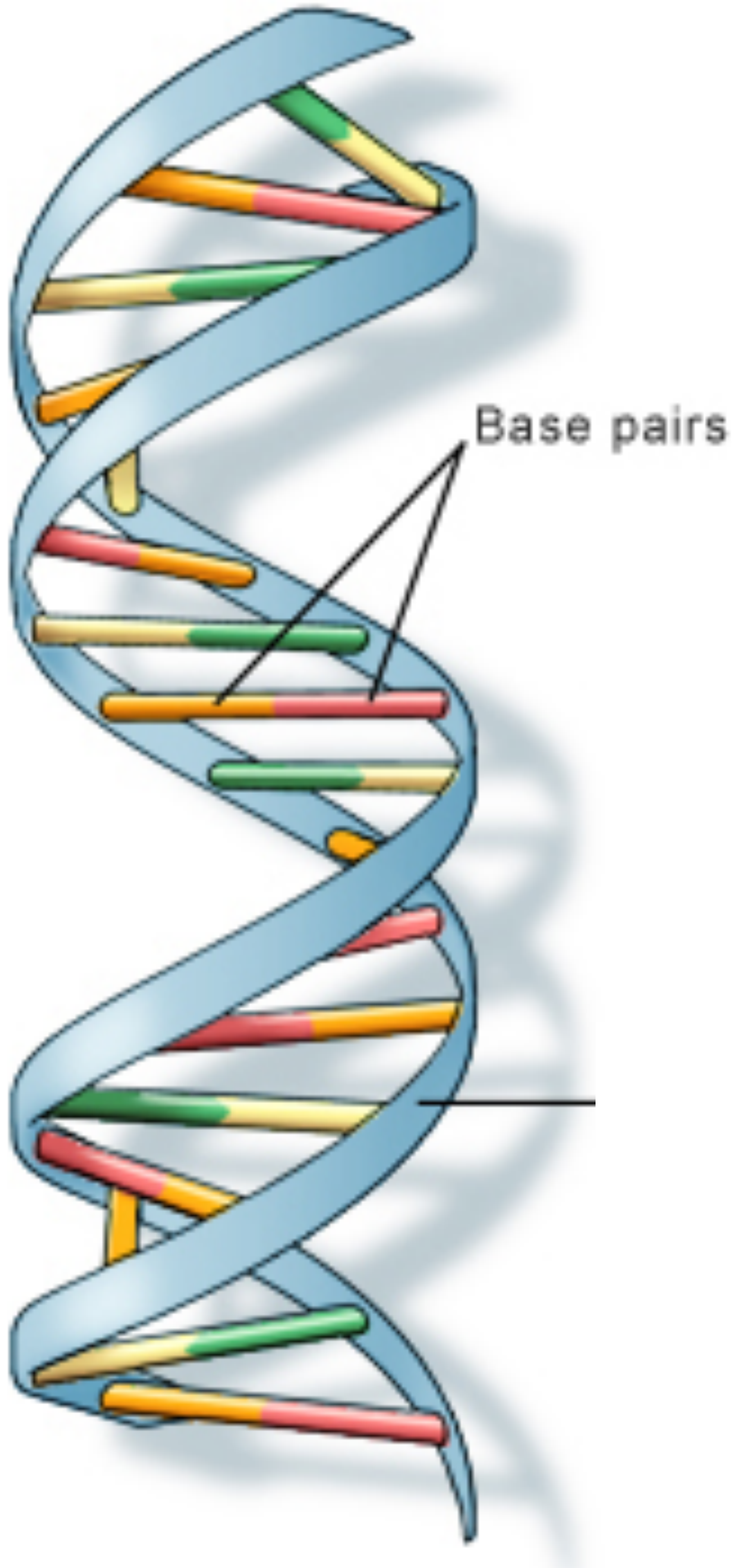
More is different: Big data

The combination of data is greater than the explicit information

More is different: Biology



Base pair



DNA

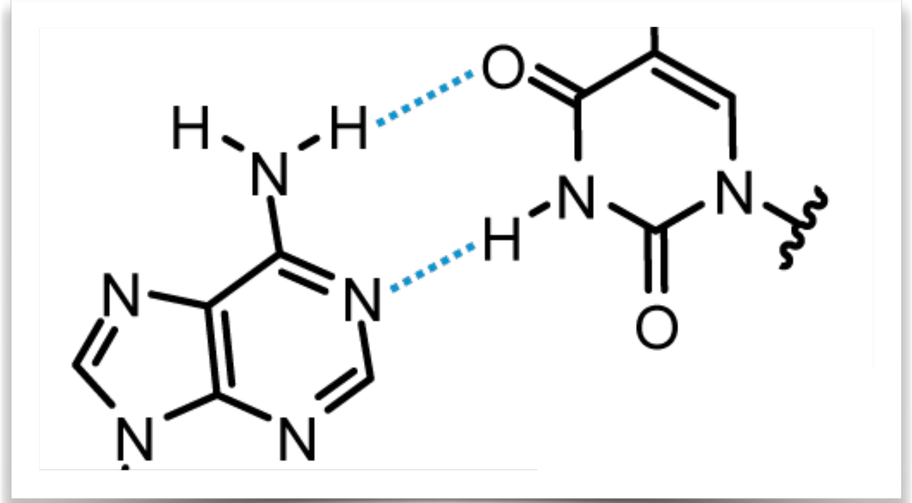
Life

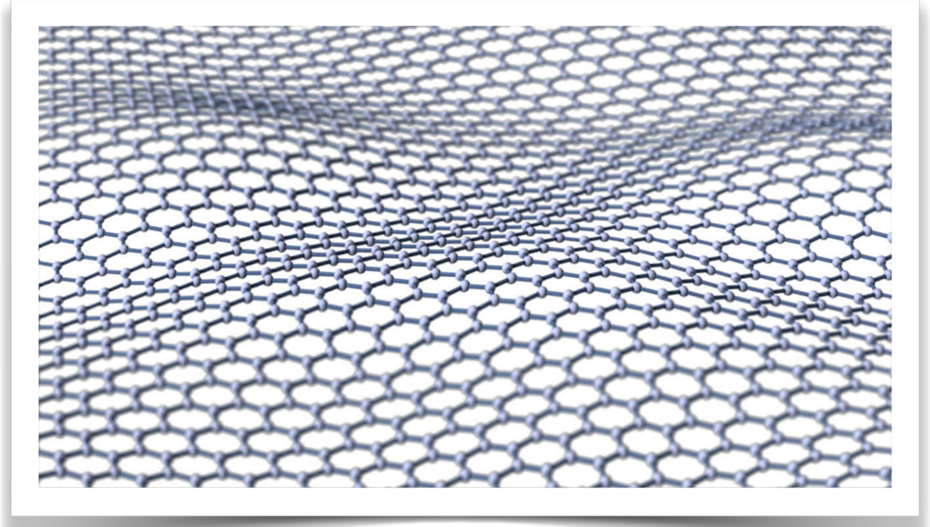
Life is greater than just a conglomerate of carbon compounds

Fundamental vs applied sciences

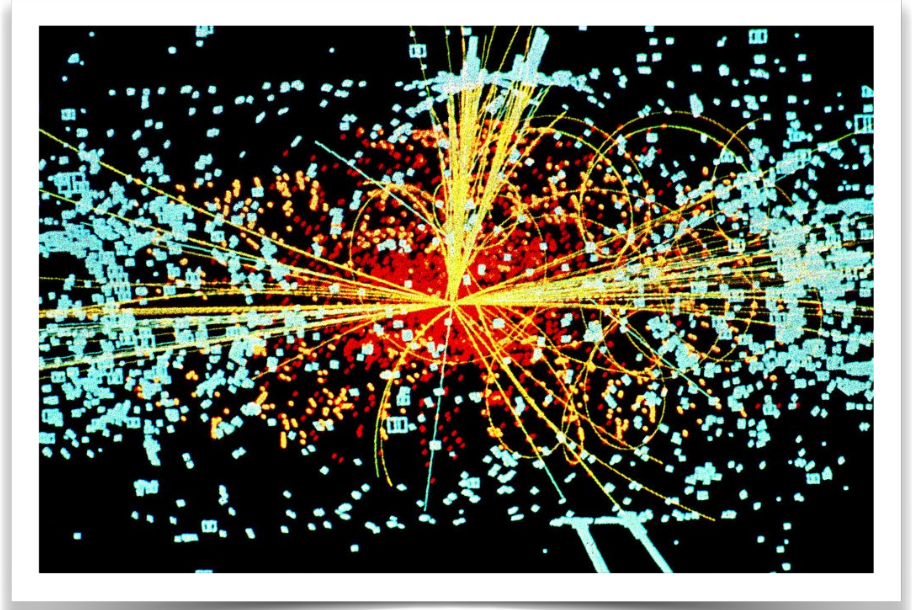
Sociology *not* "just applied psychology"

Biology *not* "just applied chemistry"





Condensed matter *not* "just applied particle physics"



New laws, concepts, and generalizations necessary at each level!

[Anderson, Science '72]

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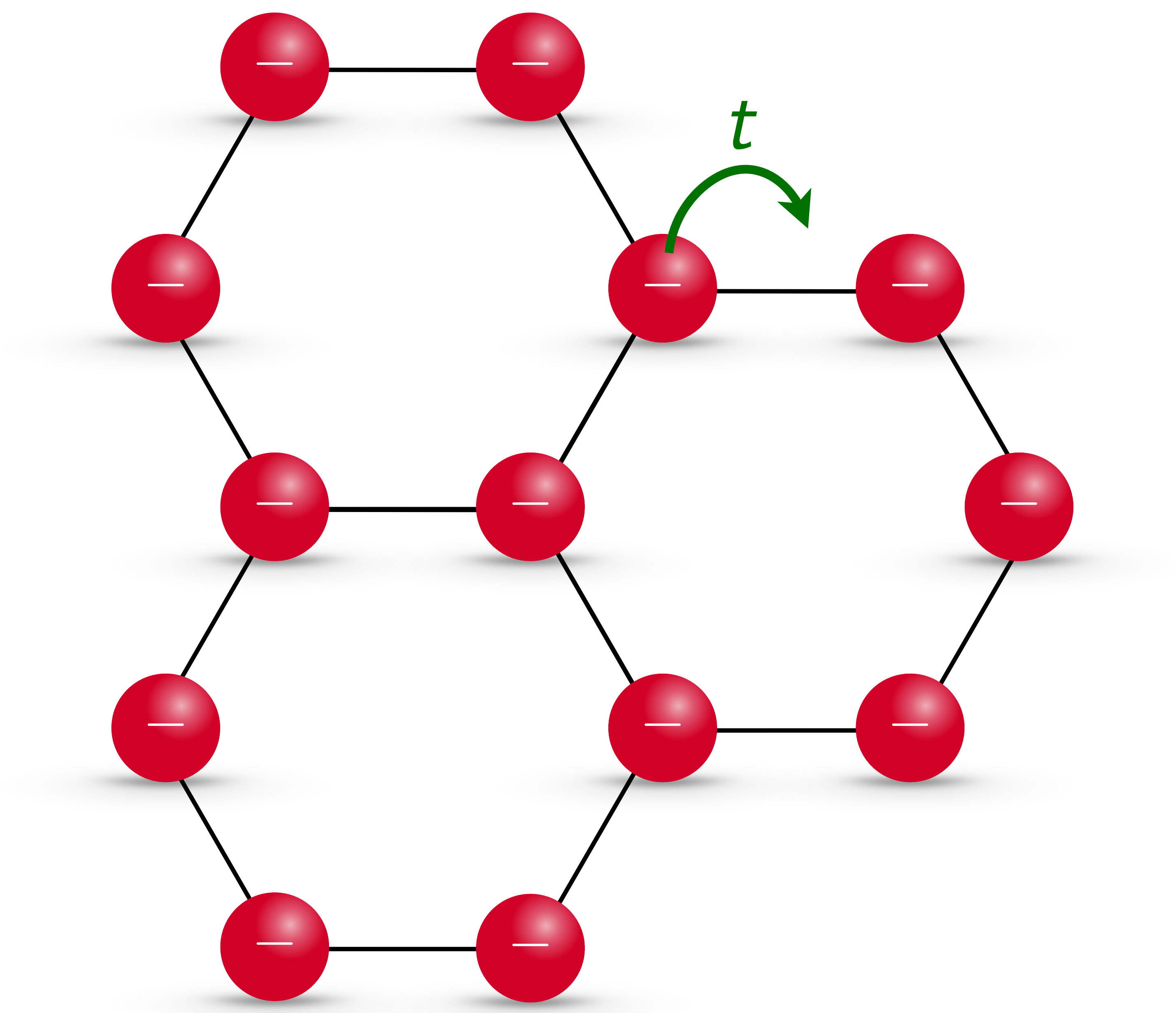
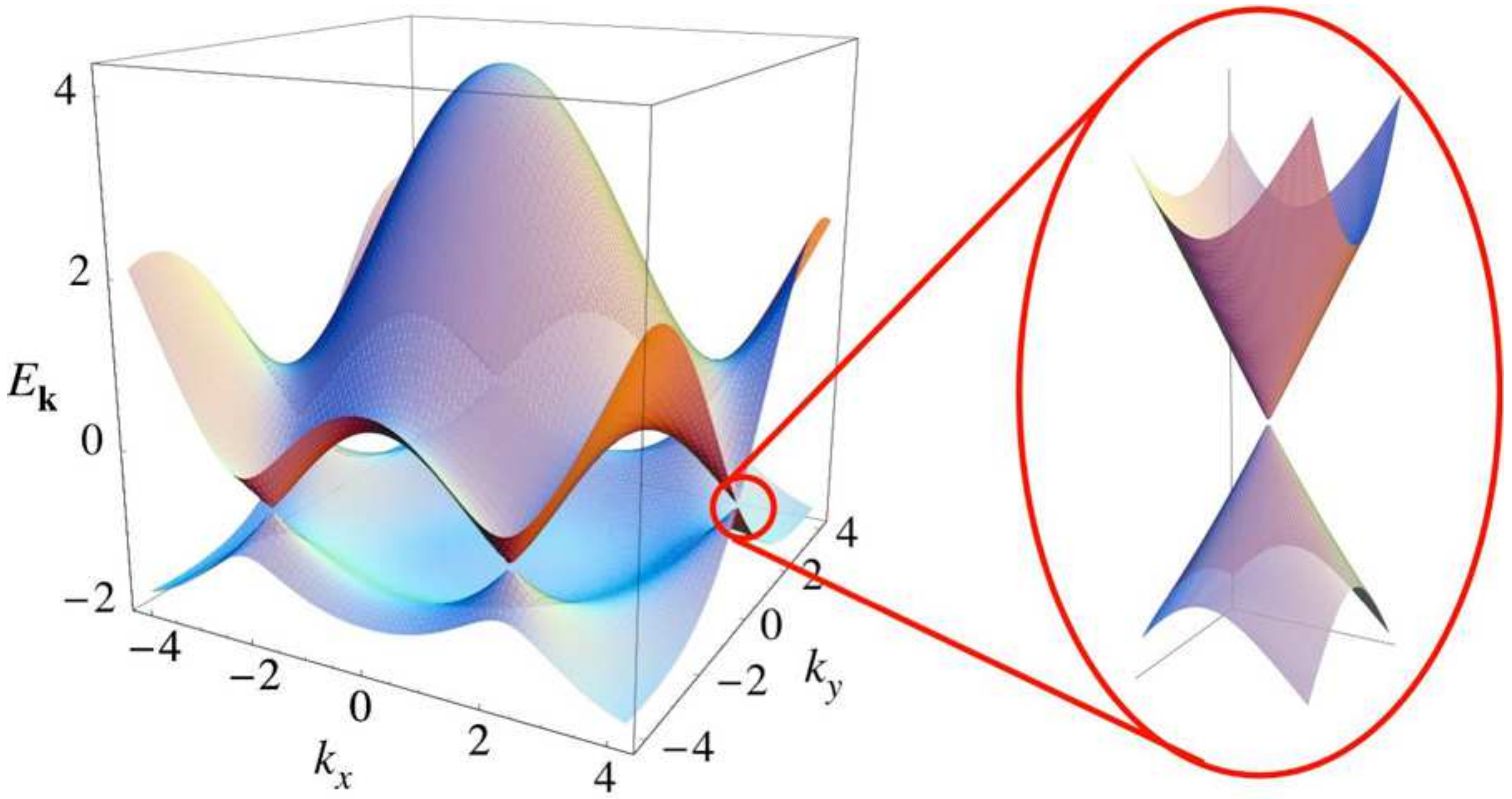
Graphene



Tight-binding model



Spectrum

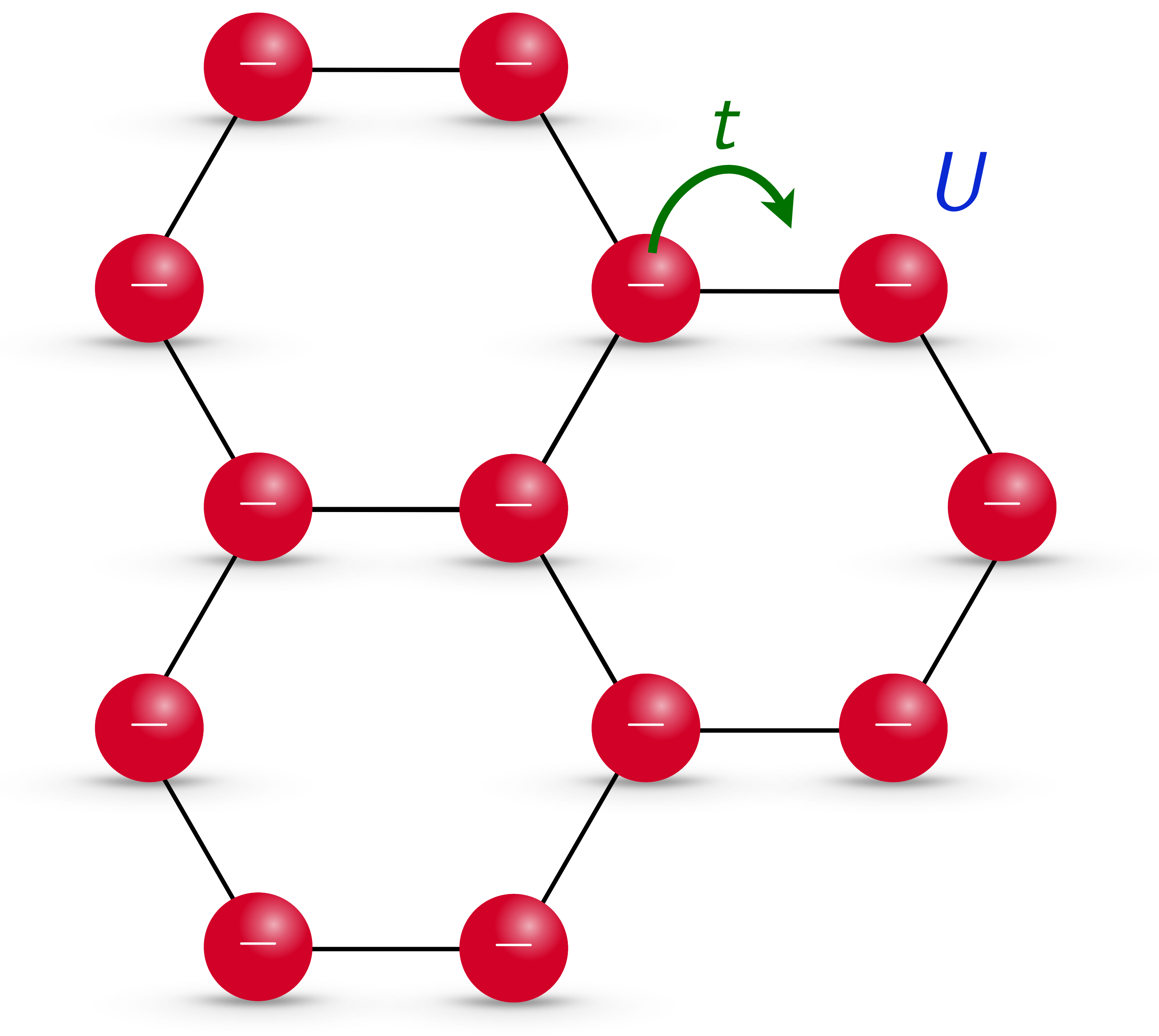
$E_{\mathbf{k}} = \pm v_{\mathrm{F}} |\mathbf{k}|$

... dispersion of massless Dirac fermions

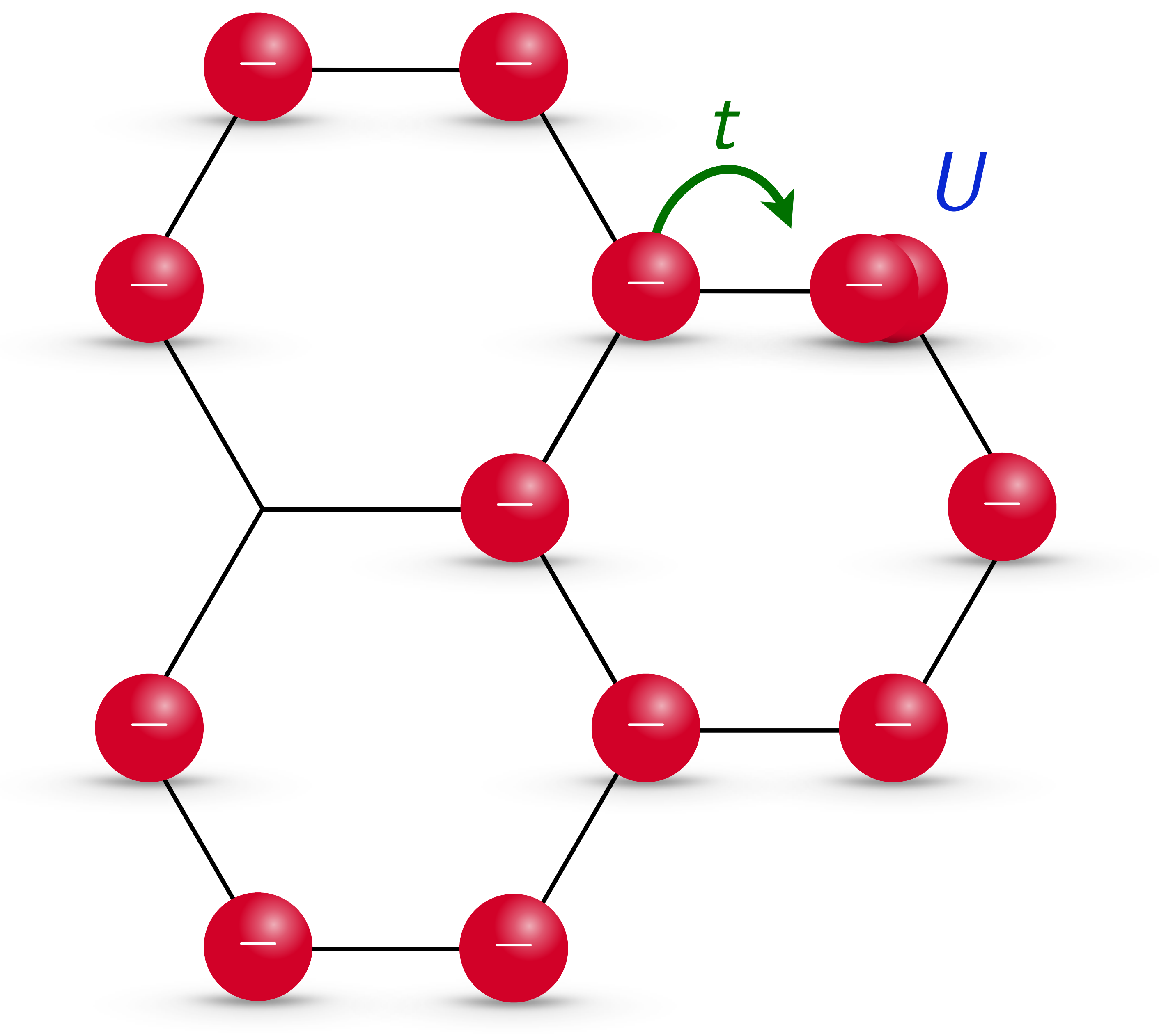
$\boxed{\text{Low energy: Emergent 2+1D Lorentz symmetry!}}$

Review: [Castro Neto *et al.*, RMP '09]

Graphene: Interactions



Graphene: Interactions



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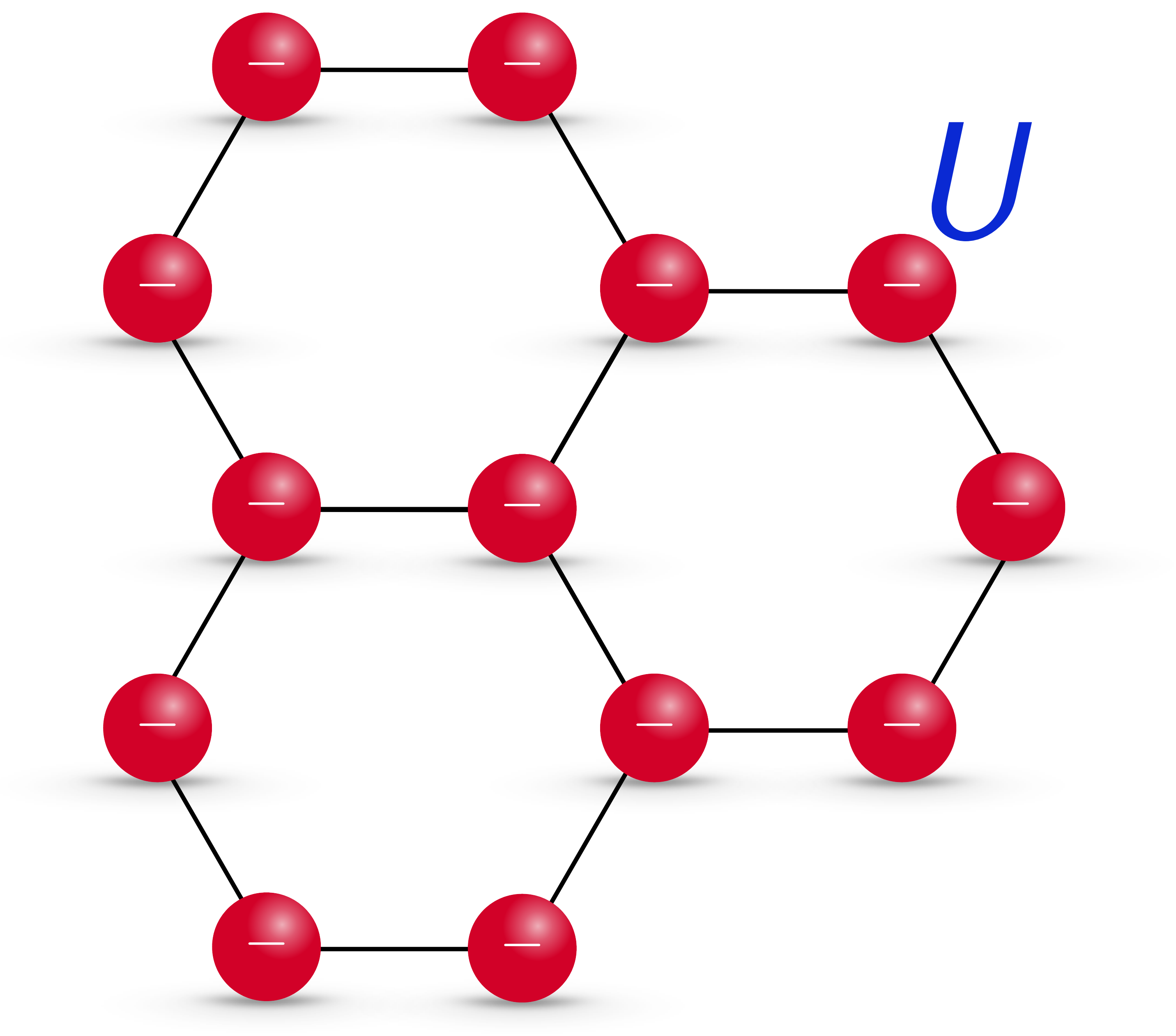

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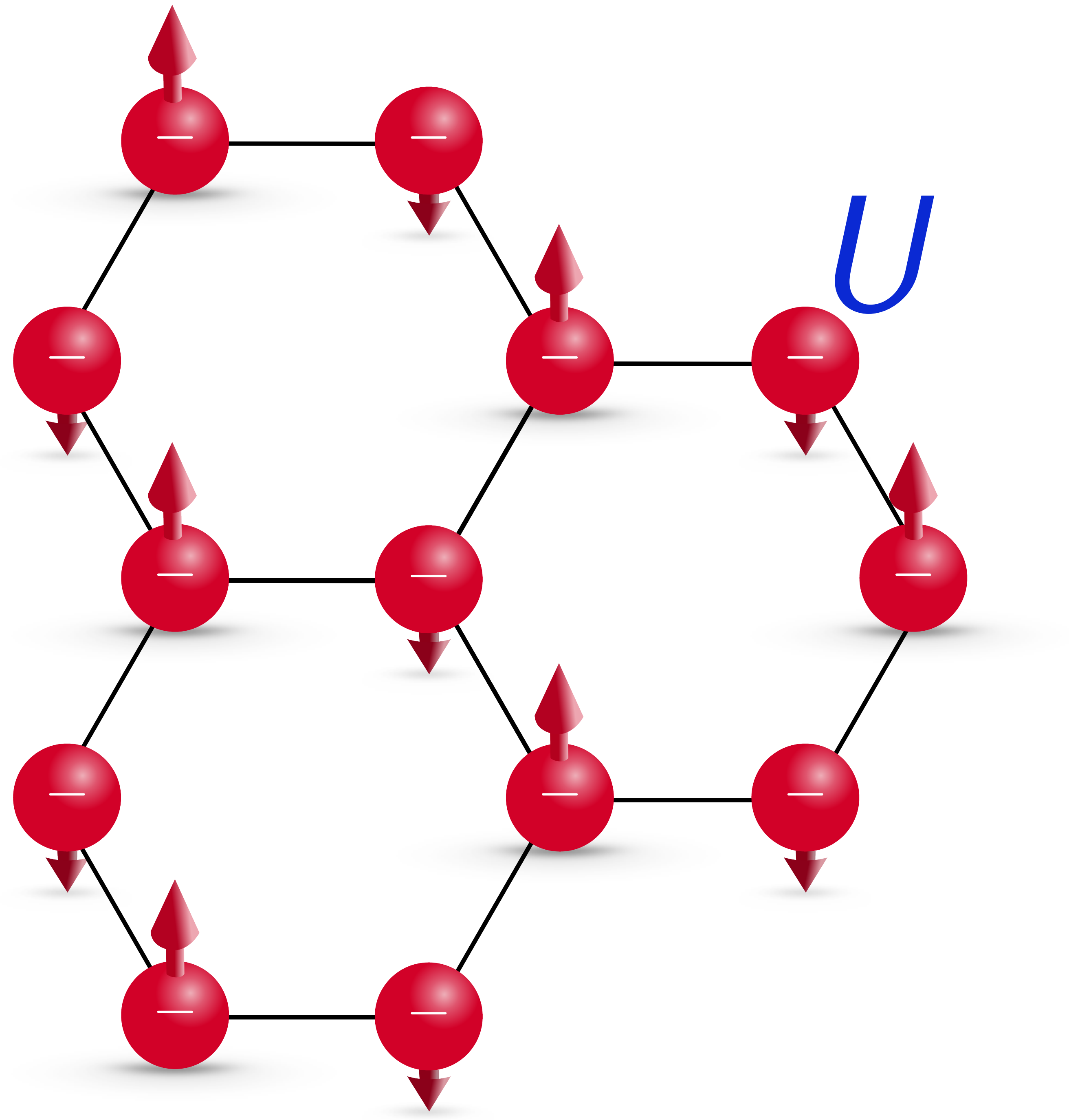

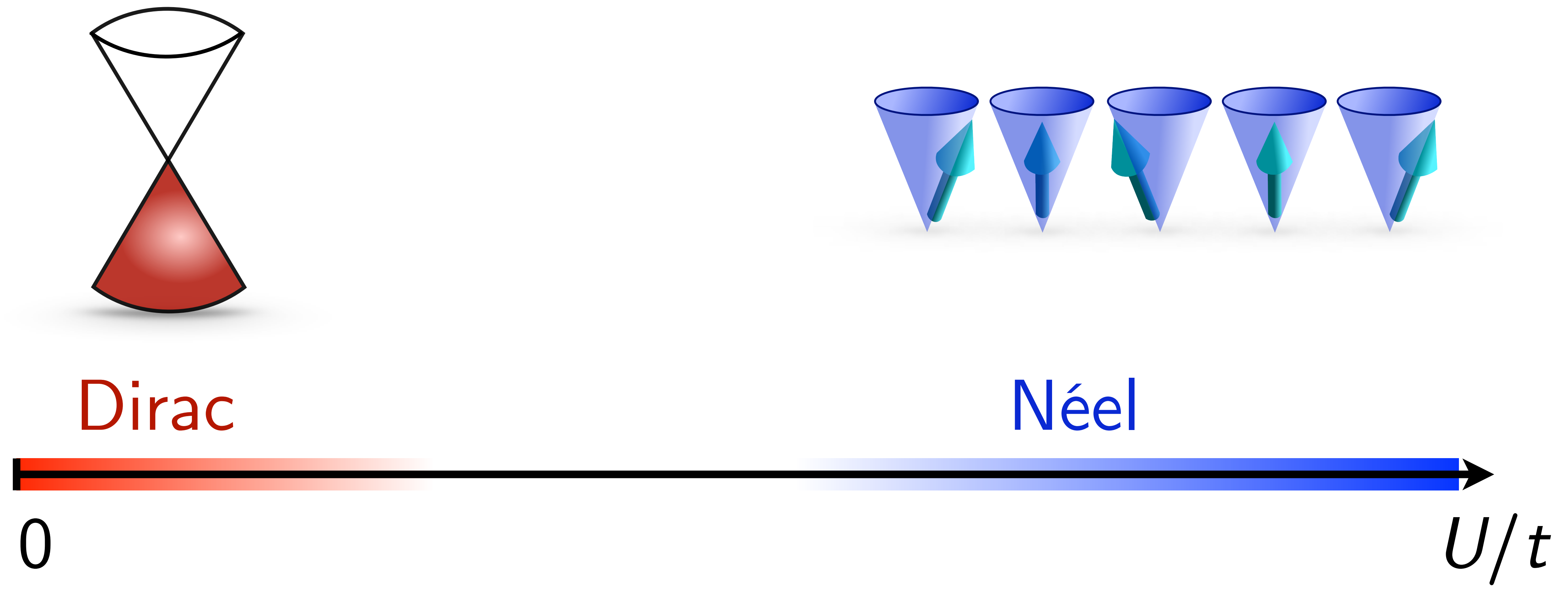

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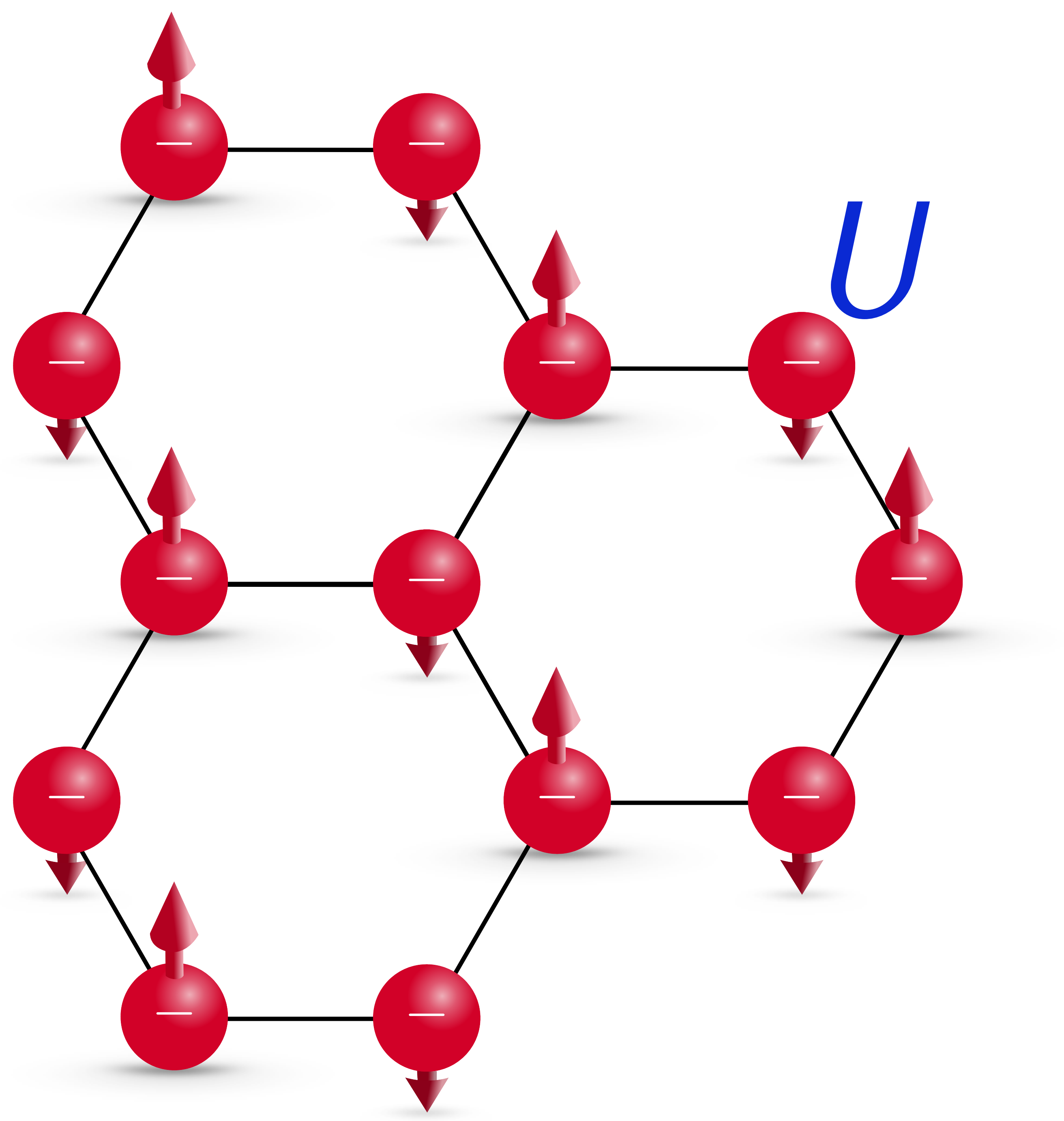
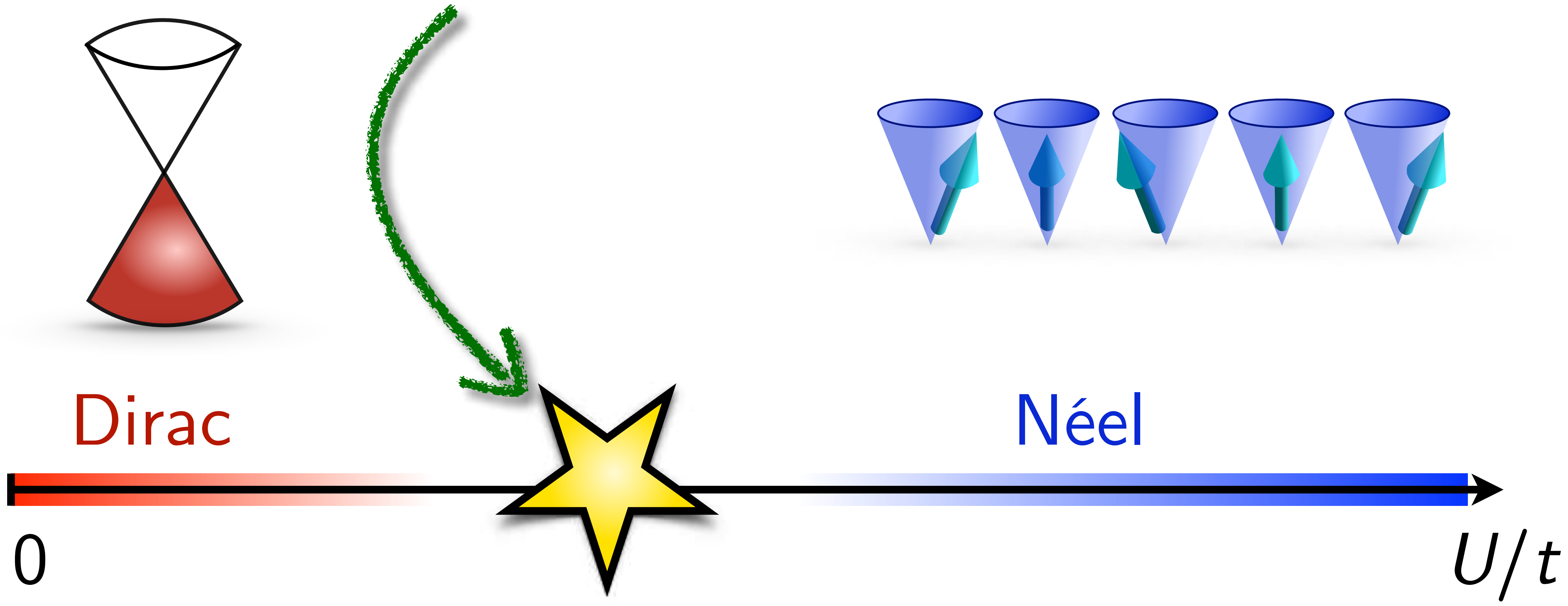


$$E_F(\mathbf{k}) = \pm v_F|\mathbf{k}|$$
$$E_B(\mathbf{k}) = c|\mathbf{k}|$$

with $c = v_F$!



QCP

$\eta \approx 1.0$
$\nu \approx 1.2$

[Herbut, PRL '06]
[LJ & Herbut, PRB '14]
[Toldin *et al.*, PRB '15]
[Otsuka, Yunoki, Sorella, PRX '16]
[Roy, Juričić, Herbut, JHEP '16]
[Zerf *et al.*, PRD '17]
[Knorr, PRB '18]
[Gracey, PRD '18]
[Buividovich *et al.*, PRB '18]
[Lang, Läuchli, PRL '19]
[Otsuka, Sorella, Yunoki, arXiv:2009.04685]
[Xu, Grover, arXiv:2009.06644]
...
See also: [Hands, Kocic, Kogut, Ann. Phys. '93]

New universality class: 2+1D Gross-Neveu

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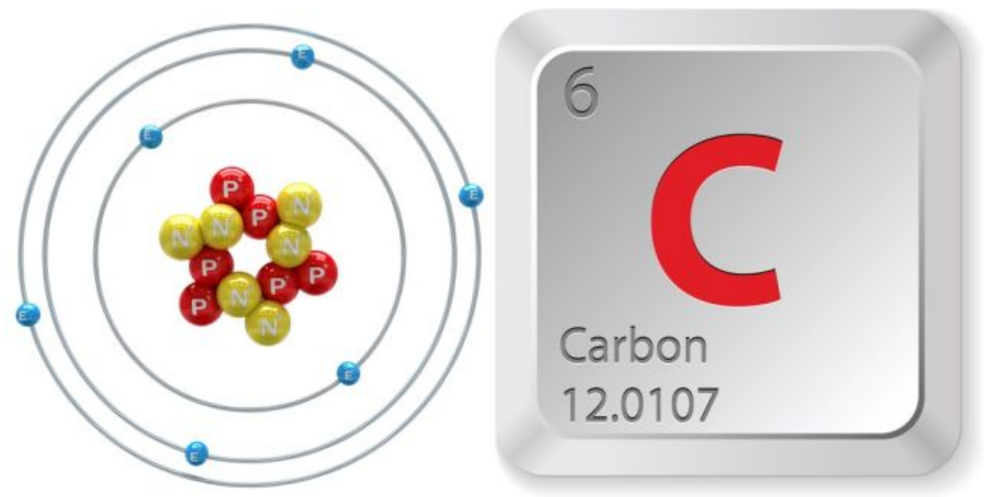
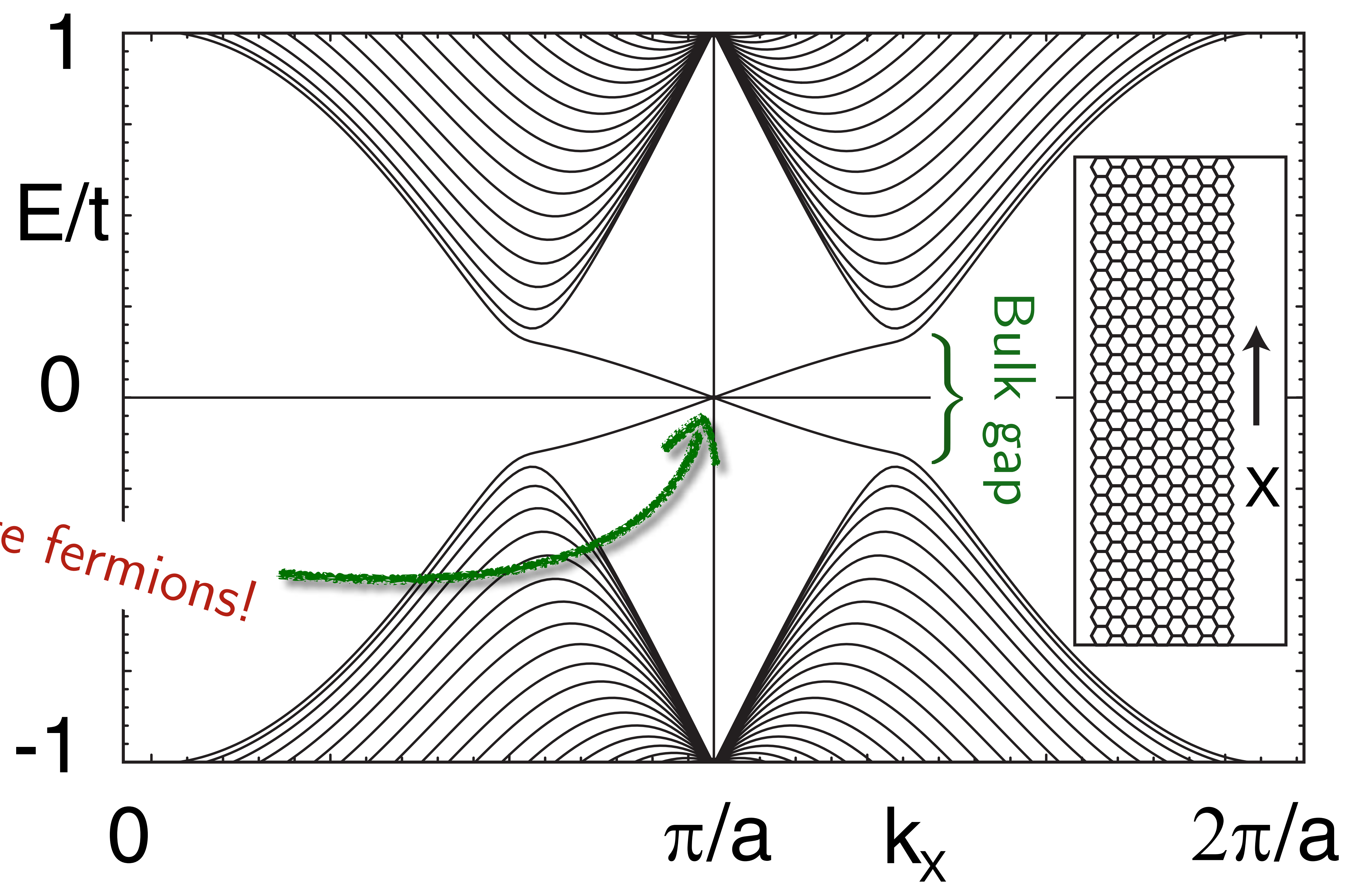
Graphene: Quantum spin Hall insulator



Spin-orbit coupling: $\lambda_{SO} \vec{L} \cdot \vec{S}$ with $\lambda_{SO} \sim 10\,\mathrm{K}$

Spectrum for $\lambda_{SO} > 0$



Relativistic edge fermions!

[Haldane, PRL '88]
[Kane, Mele, PRL '05]

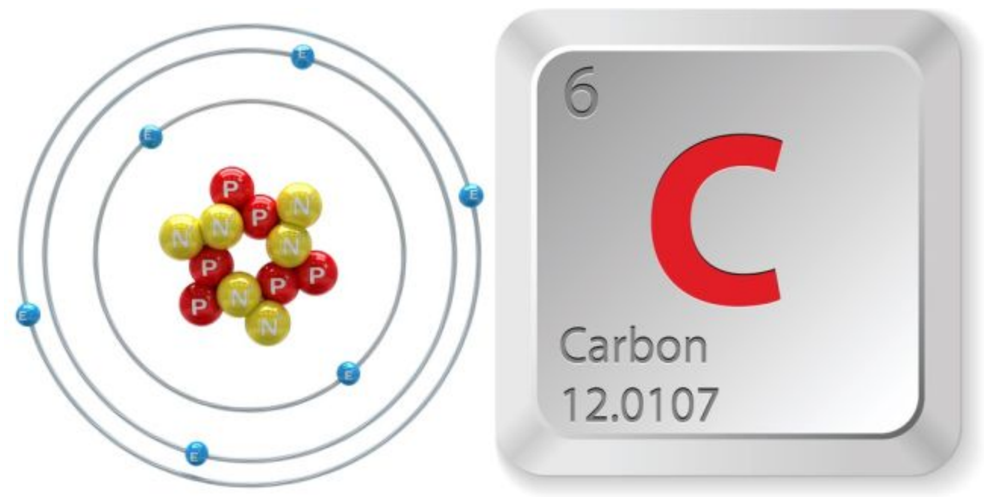
Winding number:

$$C_m = \frac{1}{2\pi} \oint_{BZ} d\vec{k} \cdot \vec{\mathcal{A}}_{m,\vec{k}}$$

Berry phase $i\langle u_m | \nabla_{\vec{k}} | u_m \rangle$

[Thouless, Kohmoto, Nightingale, den Nijs, PRL '82]
[Berry, Proc. R. Soc. '84]

Graphene: Quantum spin Hall insulator



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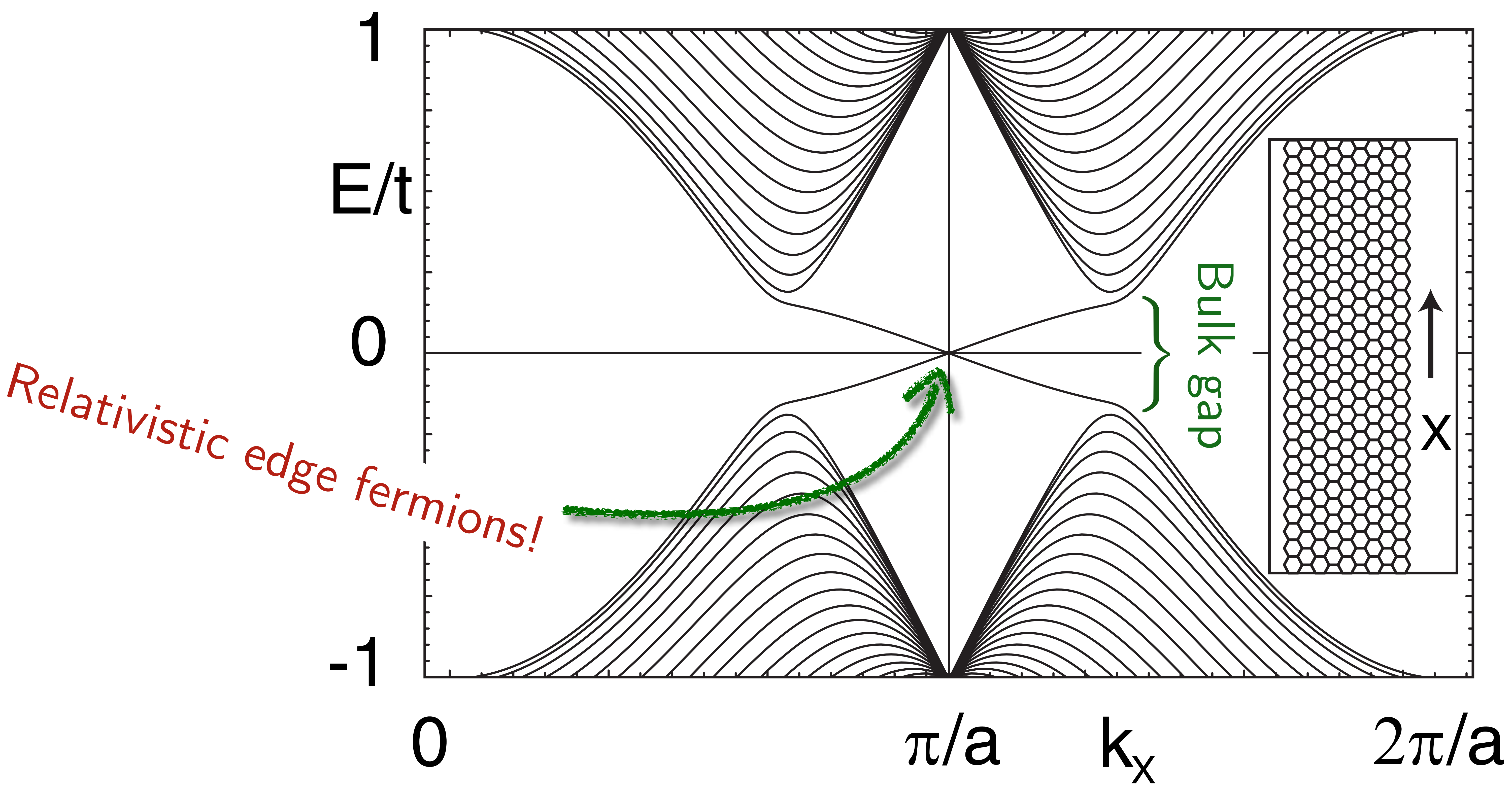

[Haldane, PRL '88]
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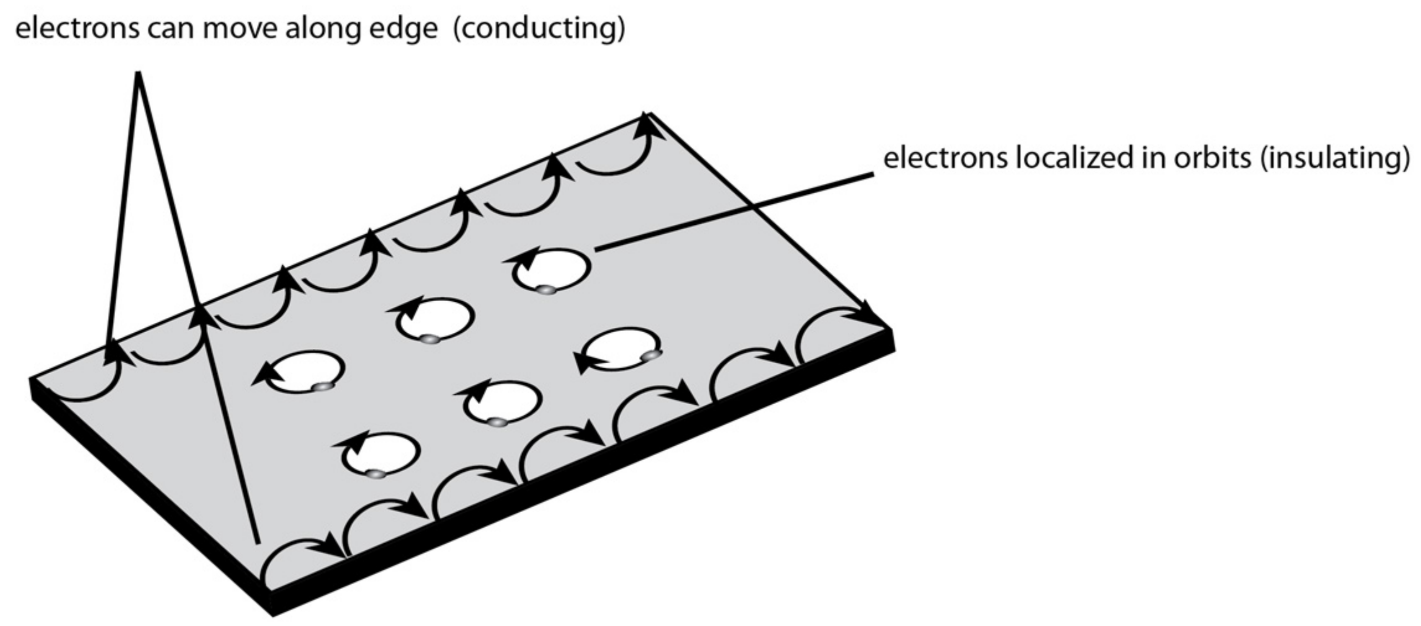
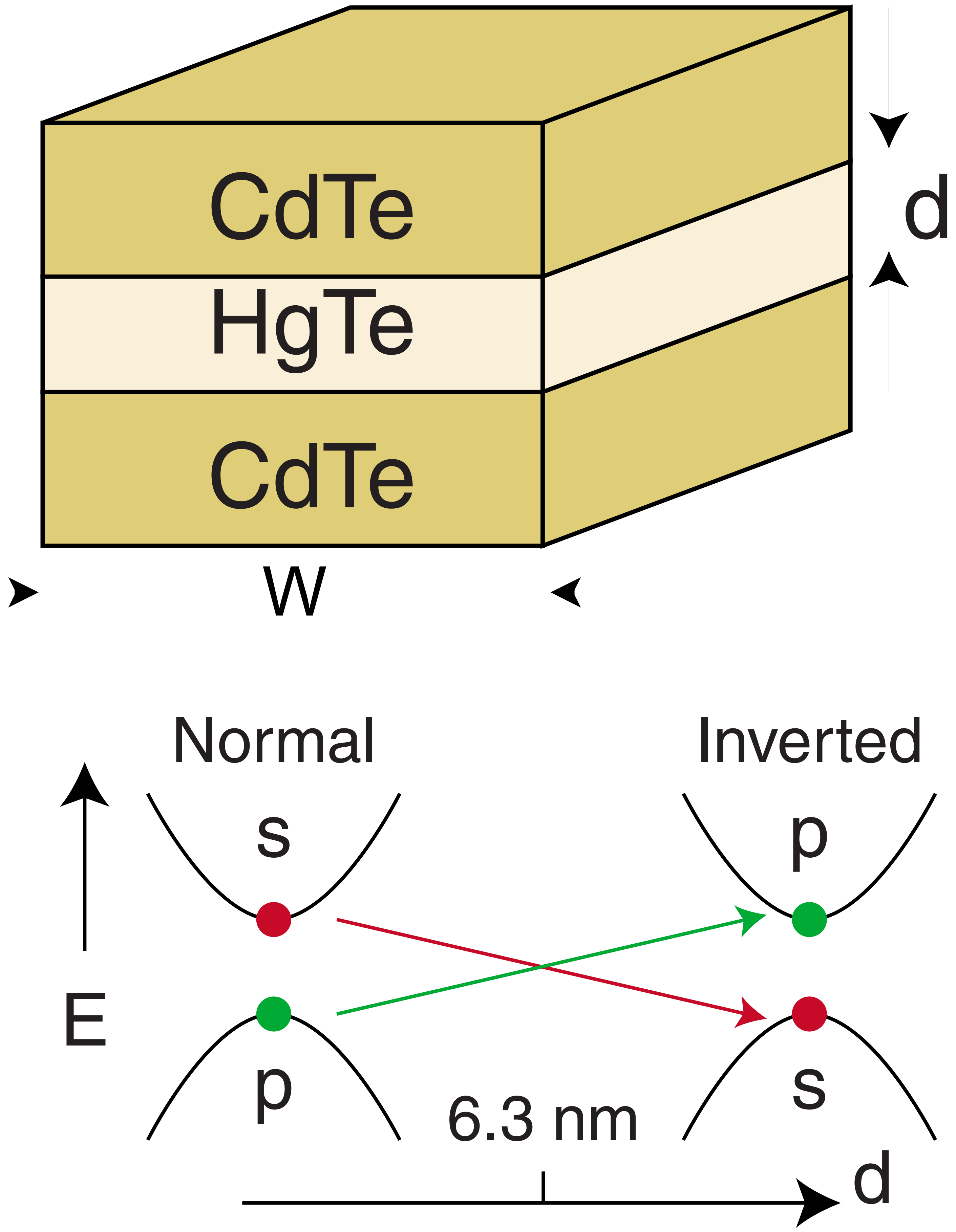
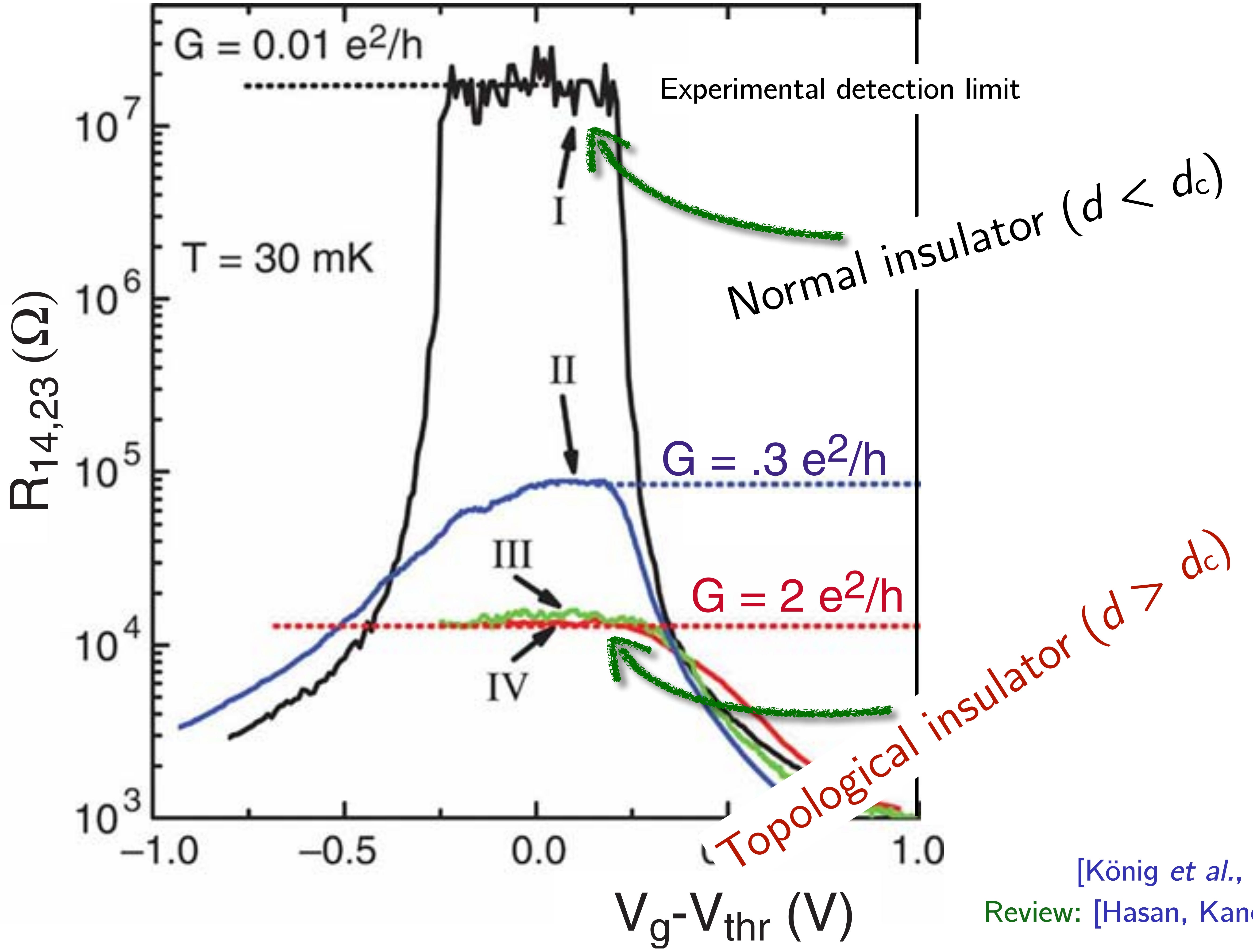


Quantum Hall effect

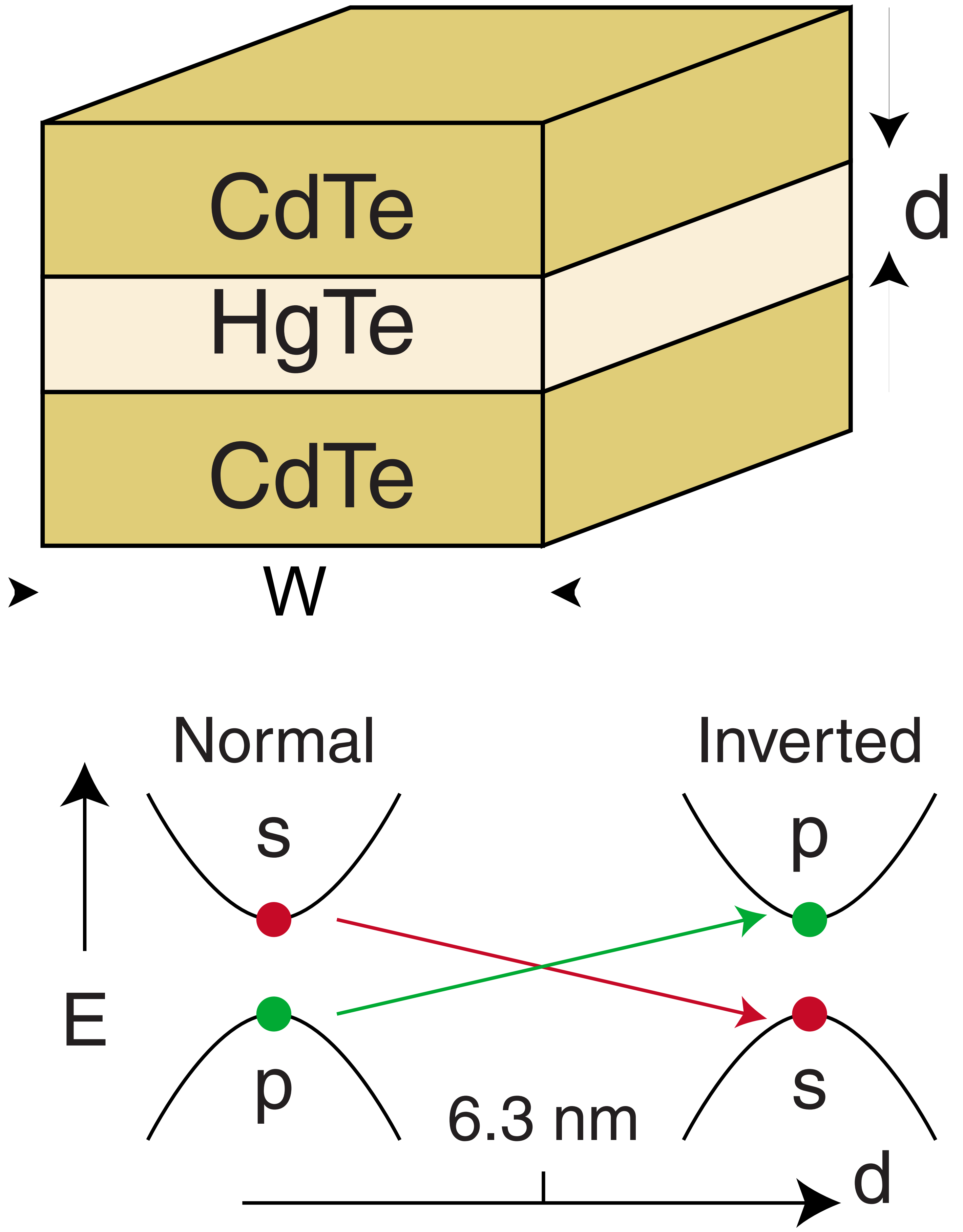
Quantum spin Hall insulator: Experimental verification

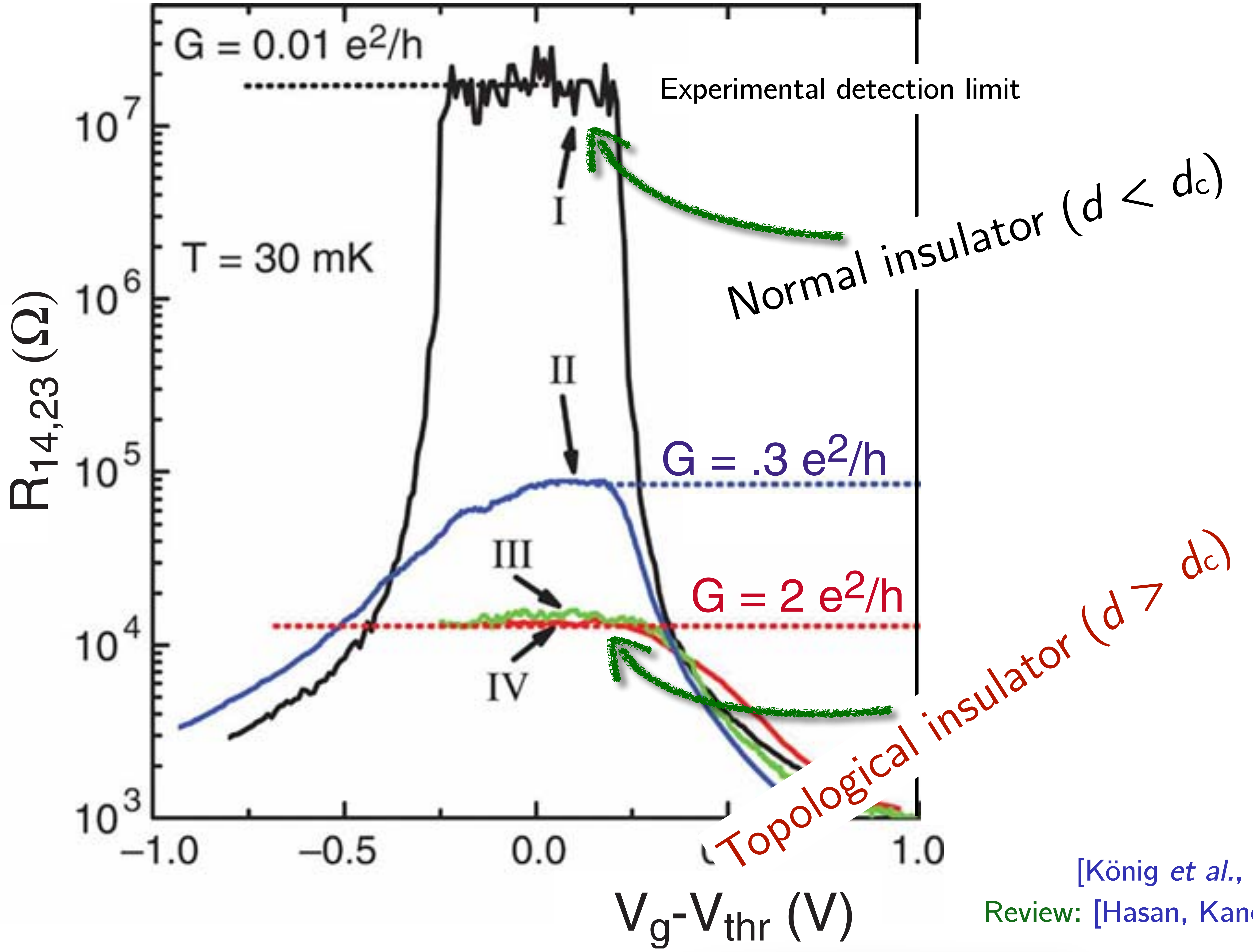




[König *et al.*, Science '07]
Review: [Hasan, Kane, RMP '10]

Quantum spin Hall insulator: Experimental verification





[König *et al.*, Science '07]
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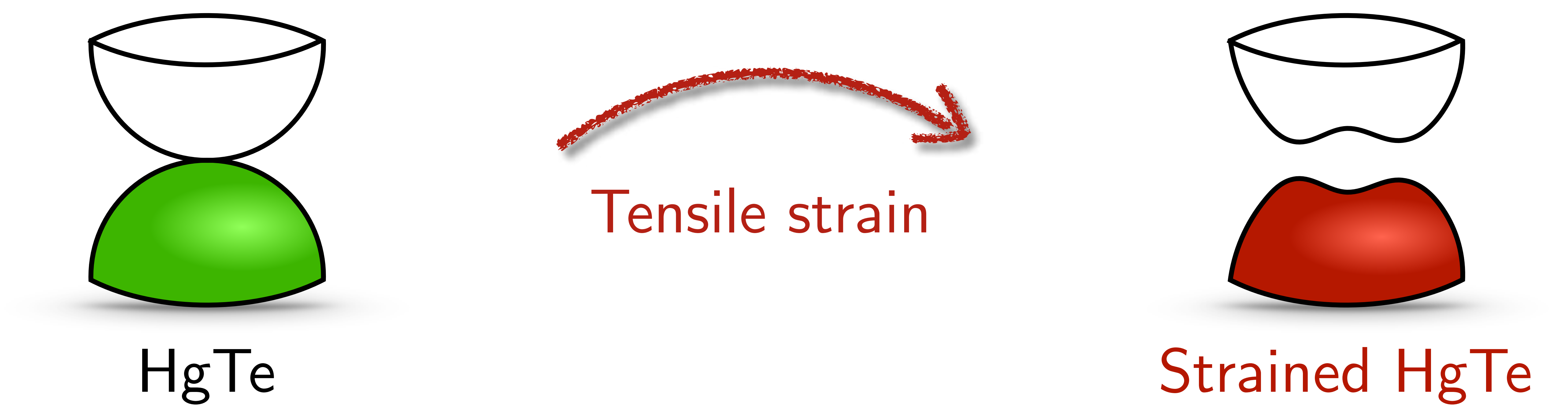
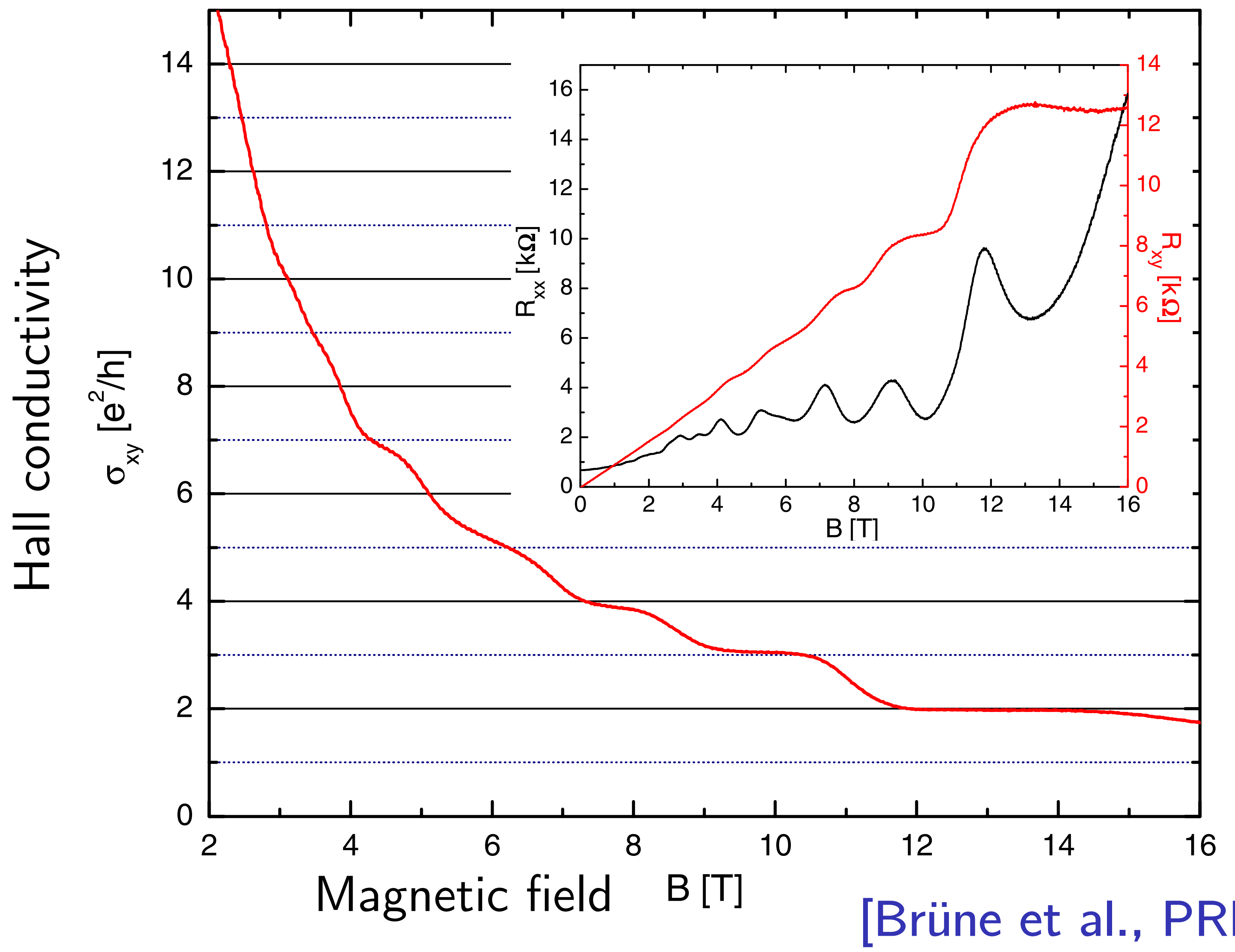
2D topological insulator has 1D gapless edge states

Flatland fermions from 3D topological insulator



QHE for 2D Dirac fermions!

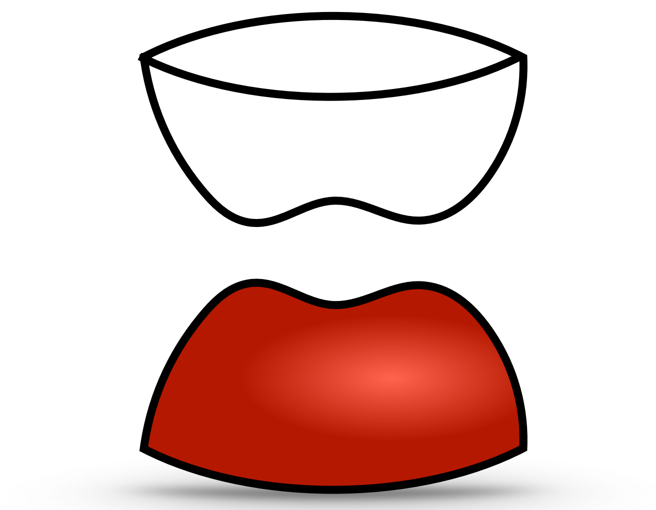


[Brüne et al., PRL '11]

Flatland fermions from 3D topological insulator

HgTe

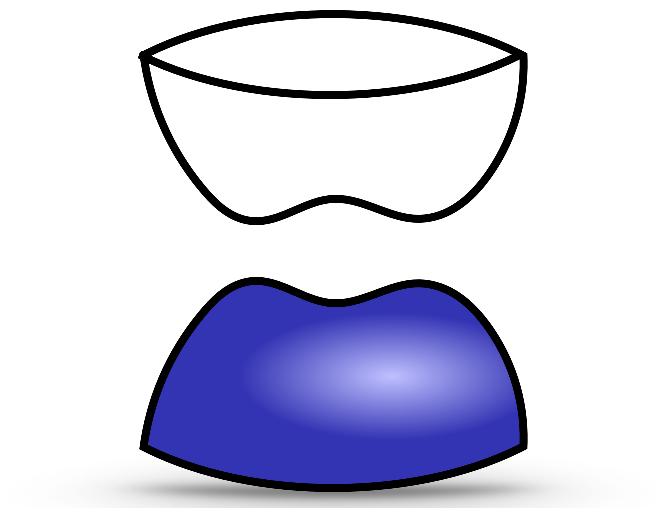
Tensile strain



Strained HgTe

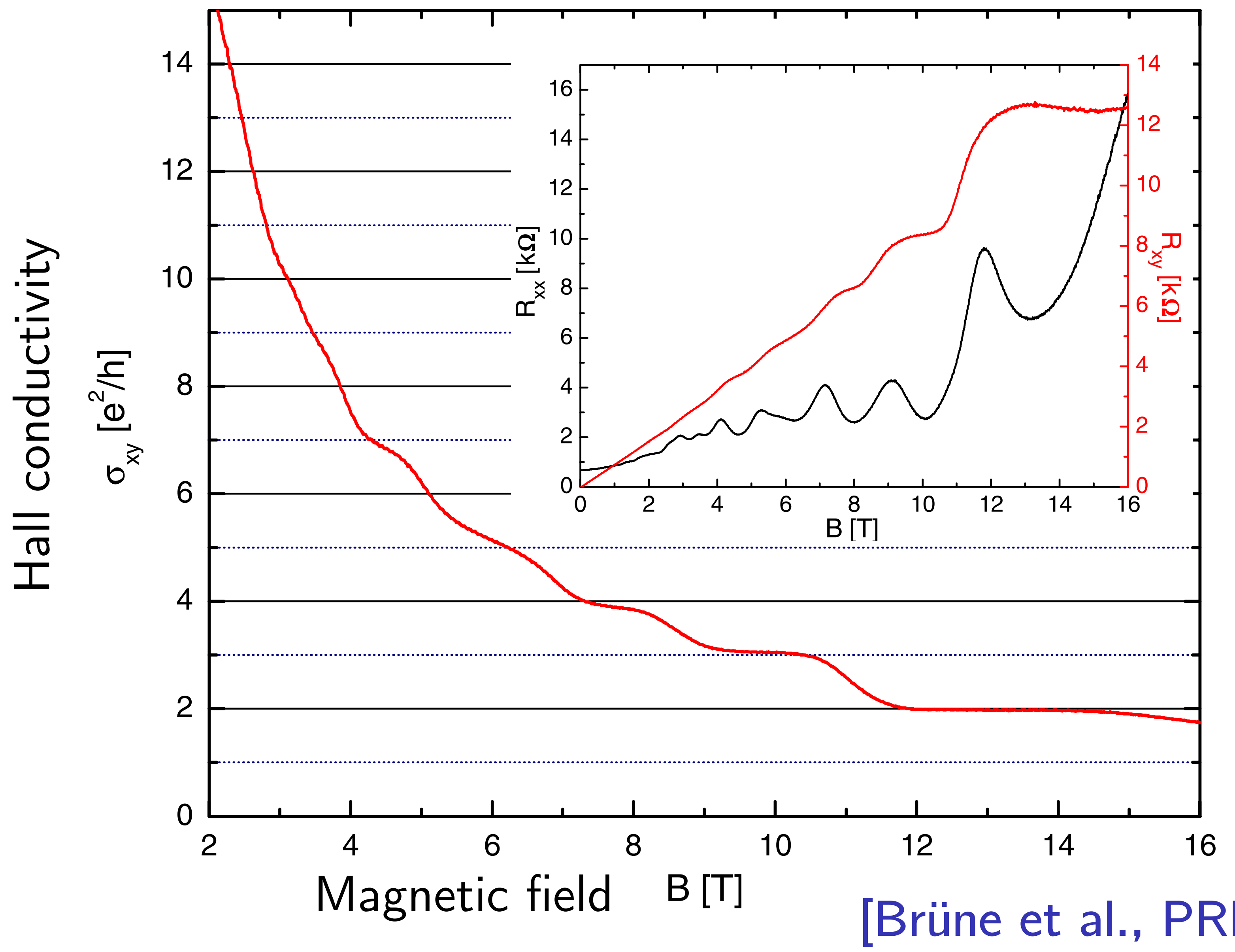
HgTe

Coulomb interaction



Cooled HgTe

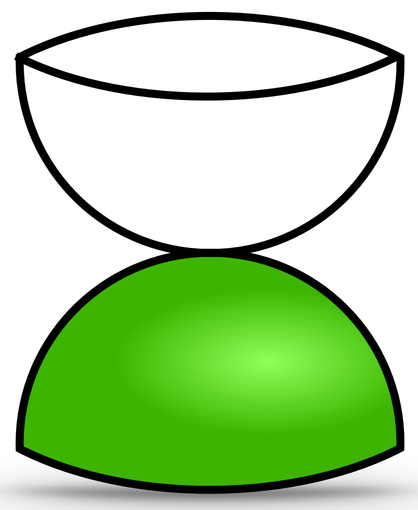
QHE for 2D Dirac fermions!



[Brüne et al., PRL '11]

[LJ, Herbut, PRB '17; PRL '14]

Flatland fermions from 3D topological insulator



HgTe

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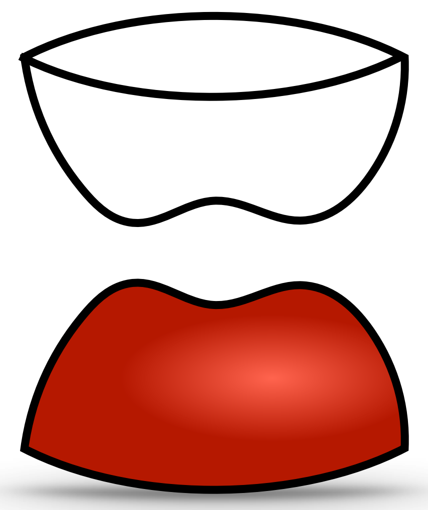
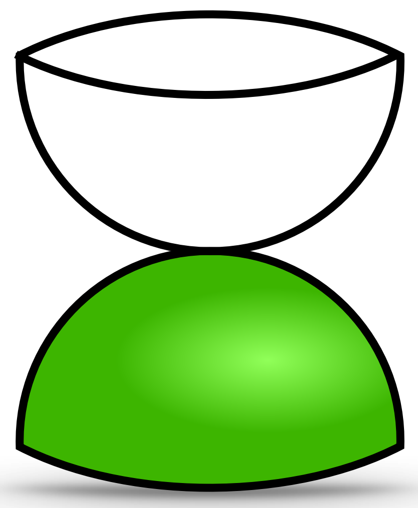


Strained HgTe



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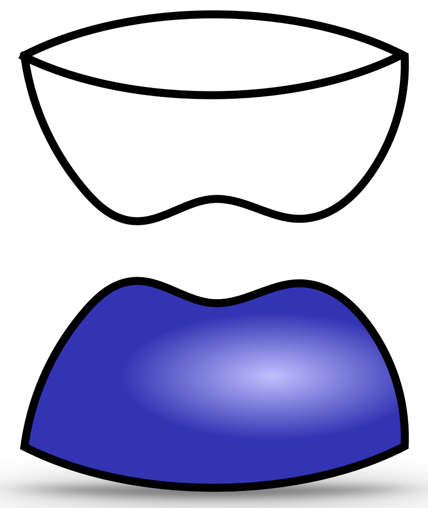

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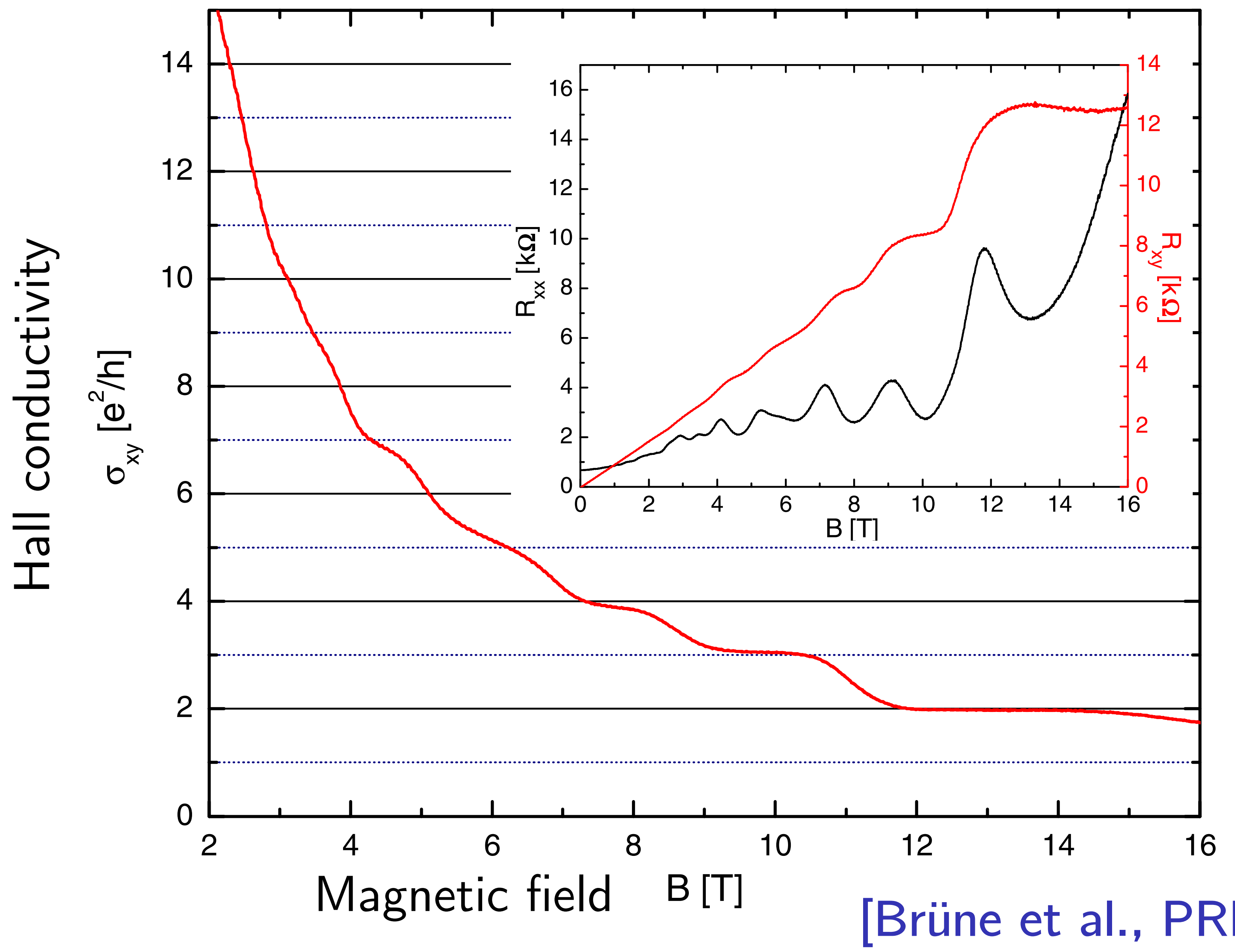

[Brüne et al., PRL '11]

3D TI — T_c — Semimetal — T

[LJ, Herbut, PRB '17; PRL '14]

Relativistic flatland fermions can emerge on surface of 3D TI

Normal insulator

Topological insulator

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Frustrated magnets

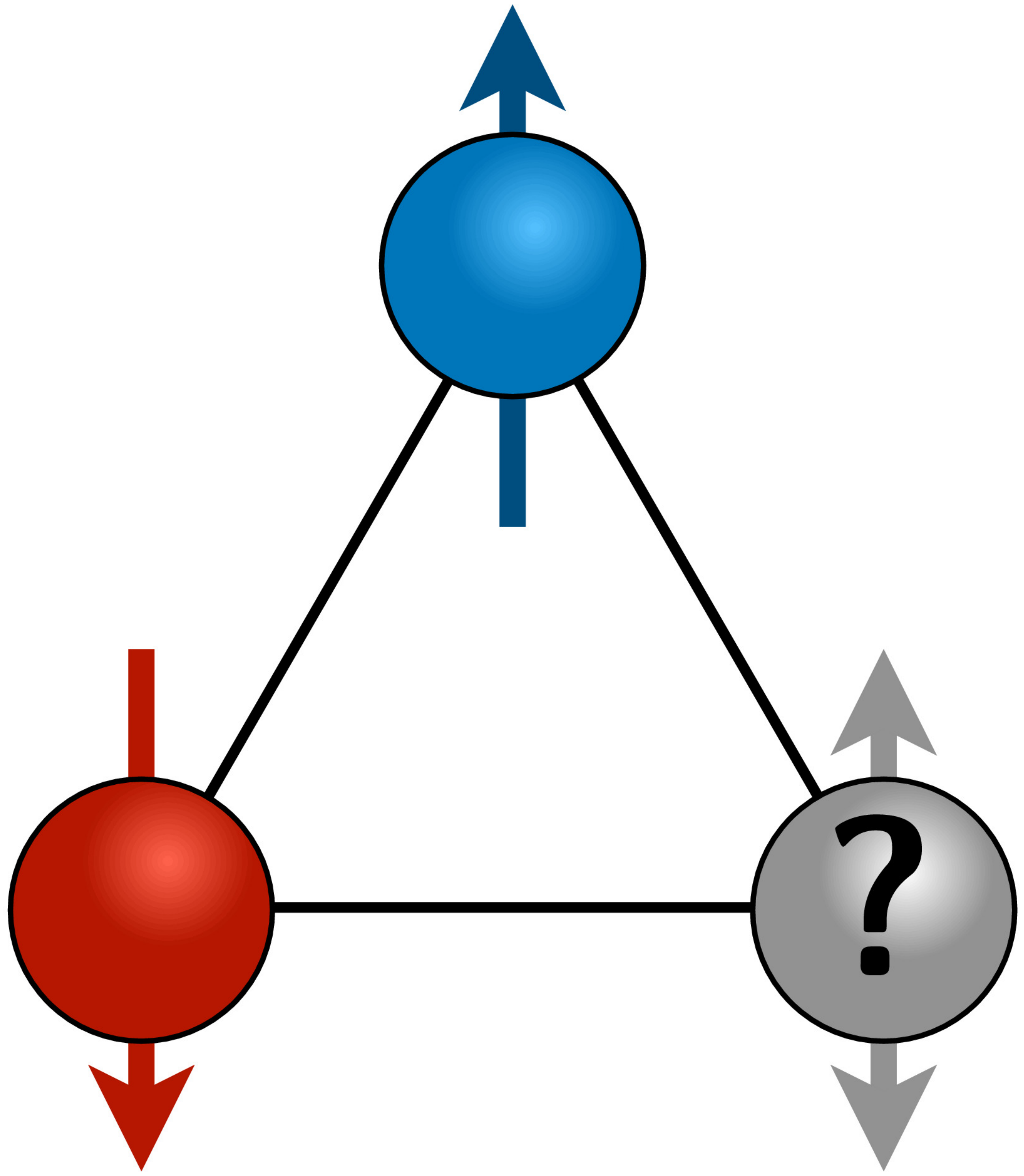
Frustration:

Not all local constraints can be simultaneously satisfied

Consequences:

Classical: Exponentially large ground-state manifold

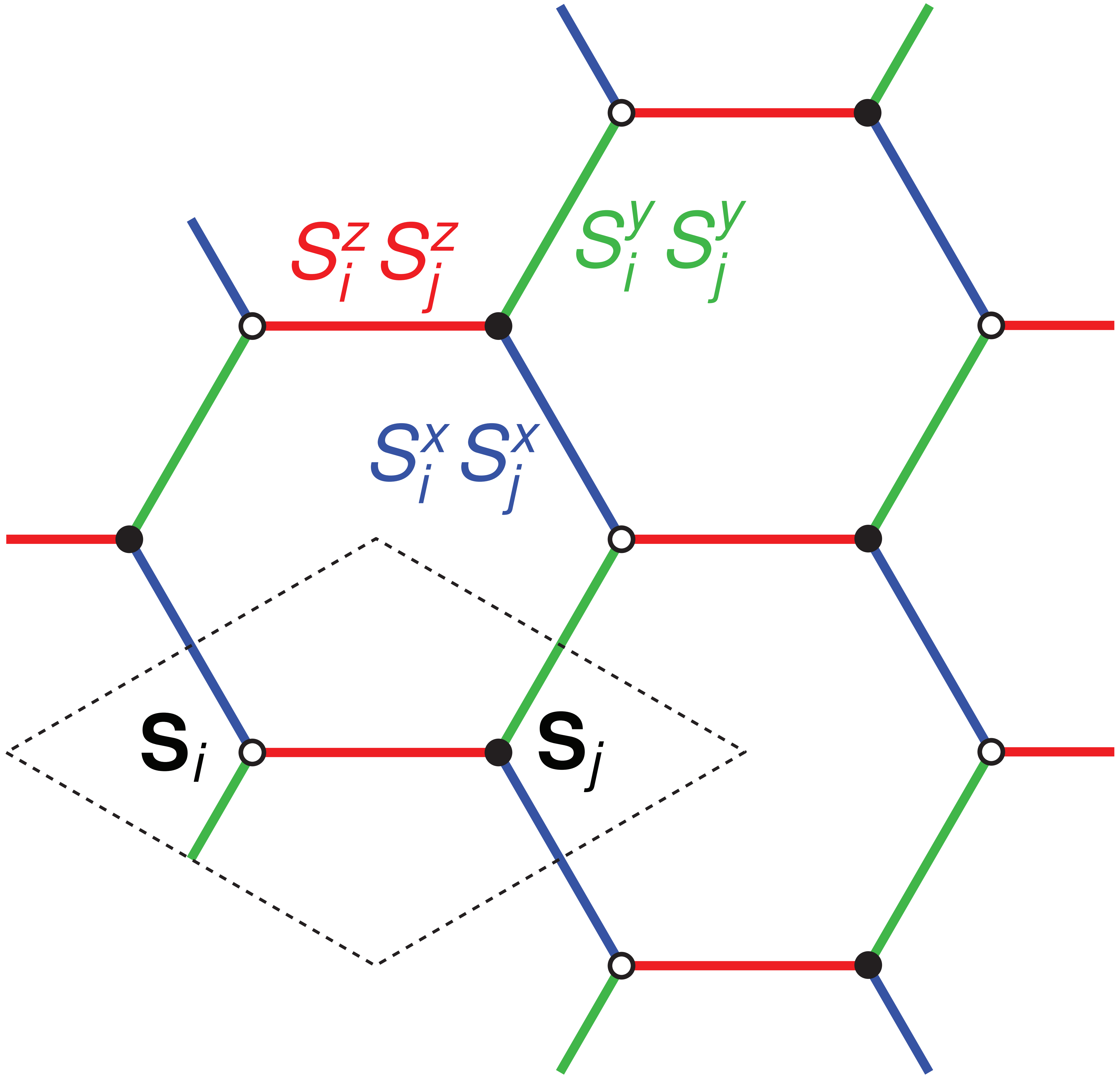
Quantum: New phases of matter?



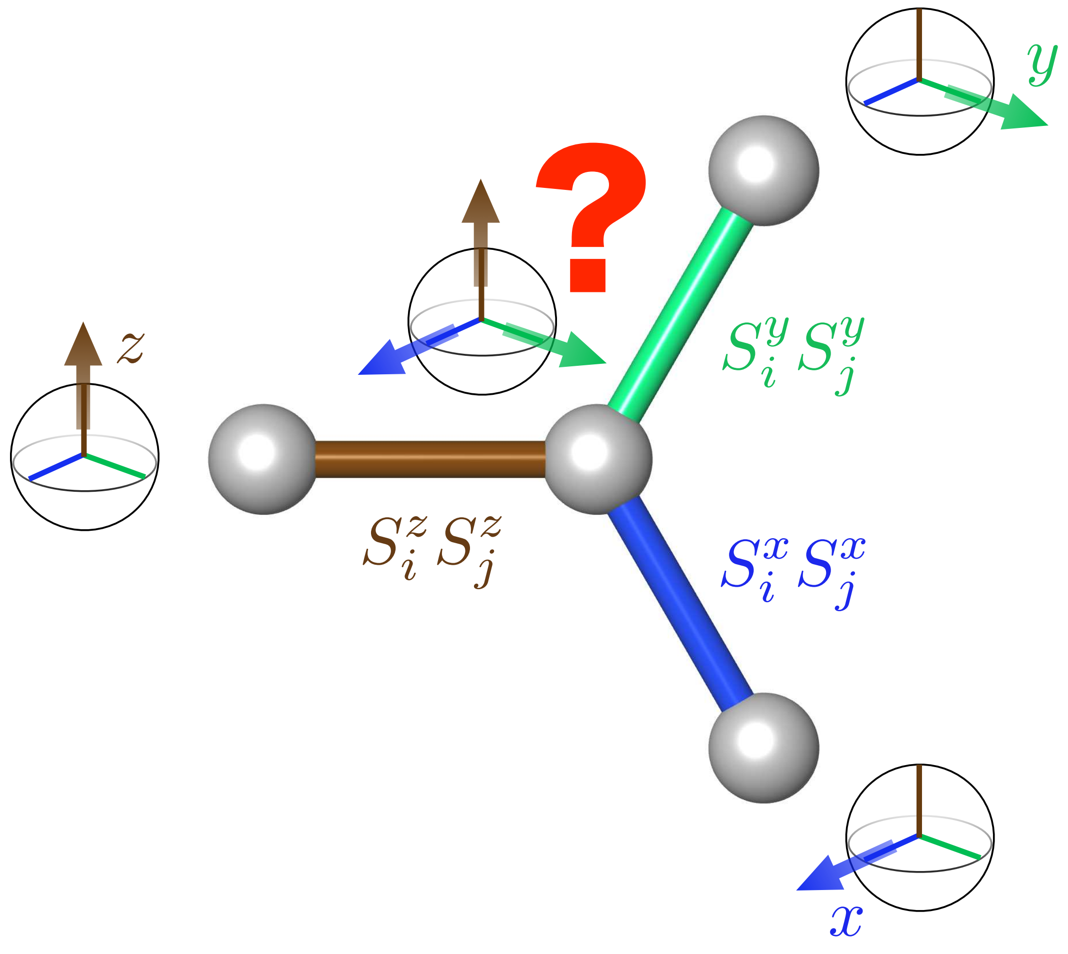
Antiferromagnetic coupling of 3 Ising spins

Kitaev honeycomb model

Spin-1/2 on honeycomb lattice:



Alexei Kitaev



Exchange frustration

Hamiltonian:

$$H = -K_x \sum_{\text{blue links}} \sigma_i^x \sigma_j^x - K_y \sum_{\text{green links}} \sigma_i^y \sigma_j^y - K_z \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$

[Kitaev, Ann. Phys. '06]

Review: [Trebst, arXiv:1701.07056]

Exact solution: Majorana representation


Ettore Majorana

Majorana fermion:

$$c_1 = a + a^\dagger \qquad c_2 = \frac{a - a^\dagger}{i}$$

$$c_1^\dagger = c_1,\, c_2^\dagger = c_2 \qquad c_1 c_2 = -c_2 c_1 \qquad c_1^2 = c_2^2 = \mathbb{1}$$

... Majorana: "half a fermion"

Exact solution: Majorana representation

Ettore Majorana

Majorana fermion:

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... Majorana: "half a fermion"

Majorana spin representation:

Pauli matrices $\in \mathcal{L}(\mathcal{H})$

$$\sigma^x \mapsto \tilde{\sigma}^x = i b^x c$$
$$\sigma^y \mapsto \tilde{\sigma}^y = i b^y c$$
$$\sigma^z \mapsto \tilde{\sigma}^z = i b^z c$$

Majoranas $\in \mathcal{L}(\tilde{\mathcal{H}})$

$$\boxed{\dim \mathcal{H} = 2 \qquad \mapsto \qquad \dim \tilde{\mathcal{H}} = 4}$$

$\mathbb{Z}_2$ gauge transformation

Projection:

$$|\xi\rangle \in \mathcal{H} \subset \tilde{\mathcal{H}} \qquad \Leftrightarrow \qquad D|\xi\rangle = |\xi\rangle, \qquad D = b^x b^y b^z c$$

... $\mathbb{Z}_2$ gauge transformation

Gauge-invariant states

All states

$\mathbb{Z}_2$ gauge transformation

Projection:

$$|\xi\rangle \in \mathcal{H} \subset \tilde{\mathcal{H}} \qquad \Leftrightarrow \qquad D|\xi\rangle = |\xi\rangle, \qquad D = b^x b^y b^z c$$

... $\mathbb{Z}_2$ gauge transformation

Gauge-invariant states

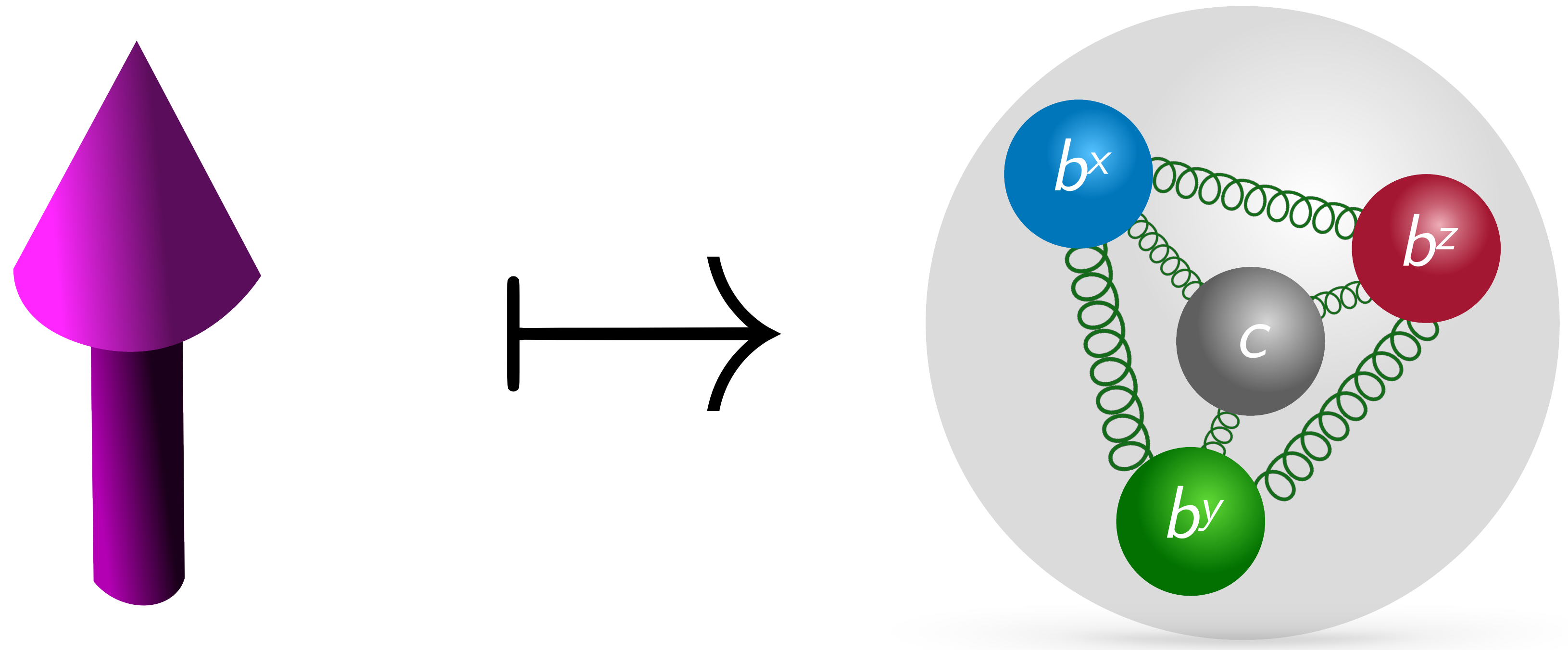
All states

Spin algebra:

$$(\tilde{\sigma}^\alpha)^\dagger = \tilde{\sigma}^\alpha \quad ✔$$

$$(\tilde{\sigma}^\alpha)^2 = \mathbb{1} \quad ✔$$

$$\tilde{\sigma}^x \tilde{\sigma}^y \tilde{\sigma}^z = iD = i \quad \text{for } |\xi\rangle \in \mathcal{H} \quad ✔$$



1 spin

4 Majoranas
with gauge constraint

... "parton" construction

Application to Kitaev model

Hamiltonian:

$$H \mapsto \tilde{H} = -i \sum_{\langle ij\rangle_\alpha} \underbrace{(i b_i^\alpha b_j^\alpha)}_{\equiv\, \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j$$

... on links

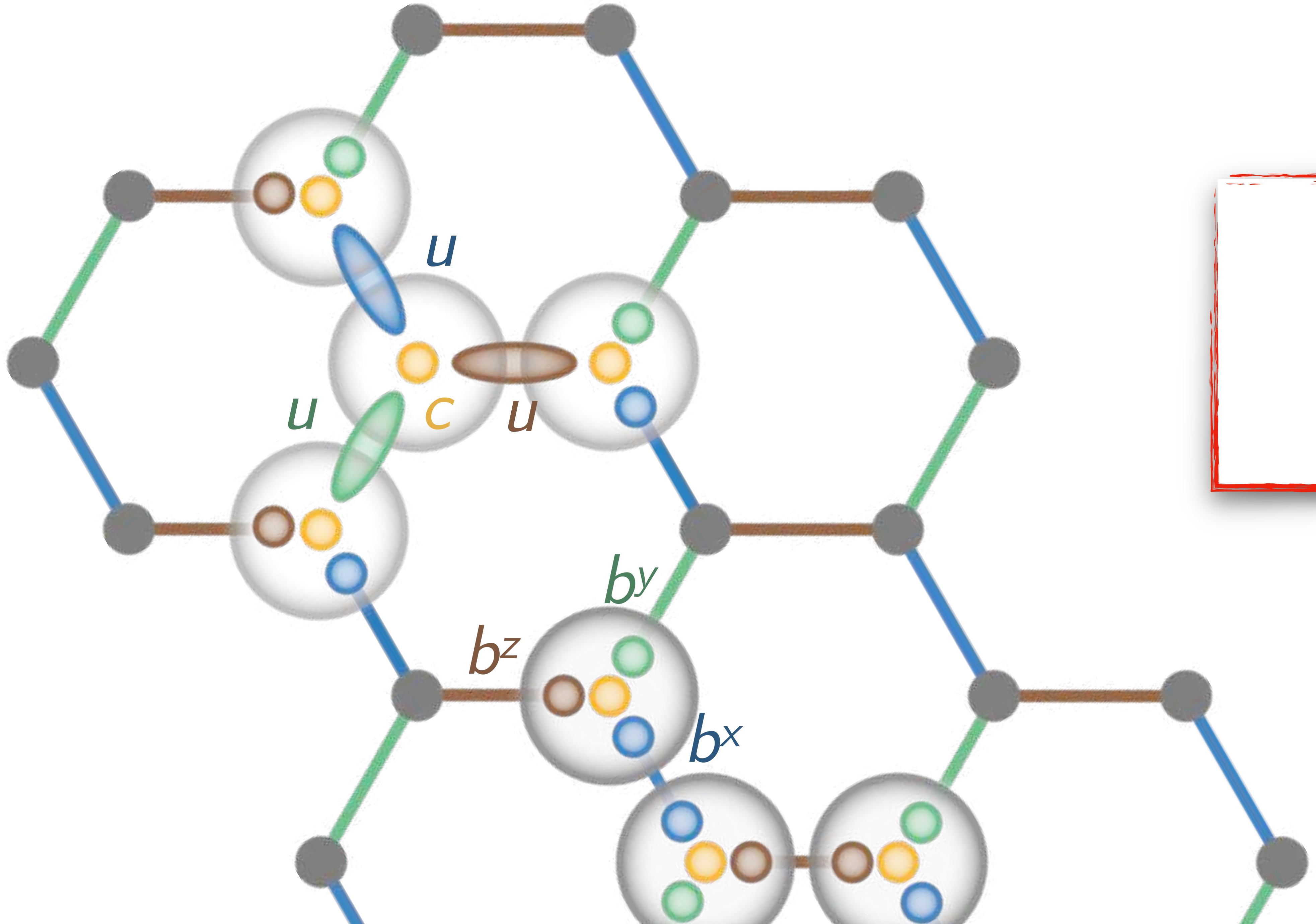
$\mathbb{Z}_2$ gauge field:

$$[\hat{u}_{ij}, \tilde{H}] = 0 = [\hat{u}_{ij}, \hat{u}_{i',j'}]$$

$$\hat{u}_{ij} \mapsto u_{ij} = \pm 1$$

Static!

Fractionalization:



Spins fractionalize into fermions and gauge fields

Review: [Trebst, arXiv:1701.07056]

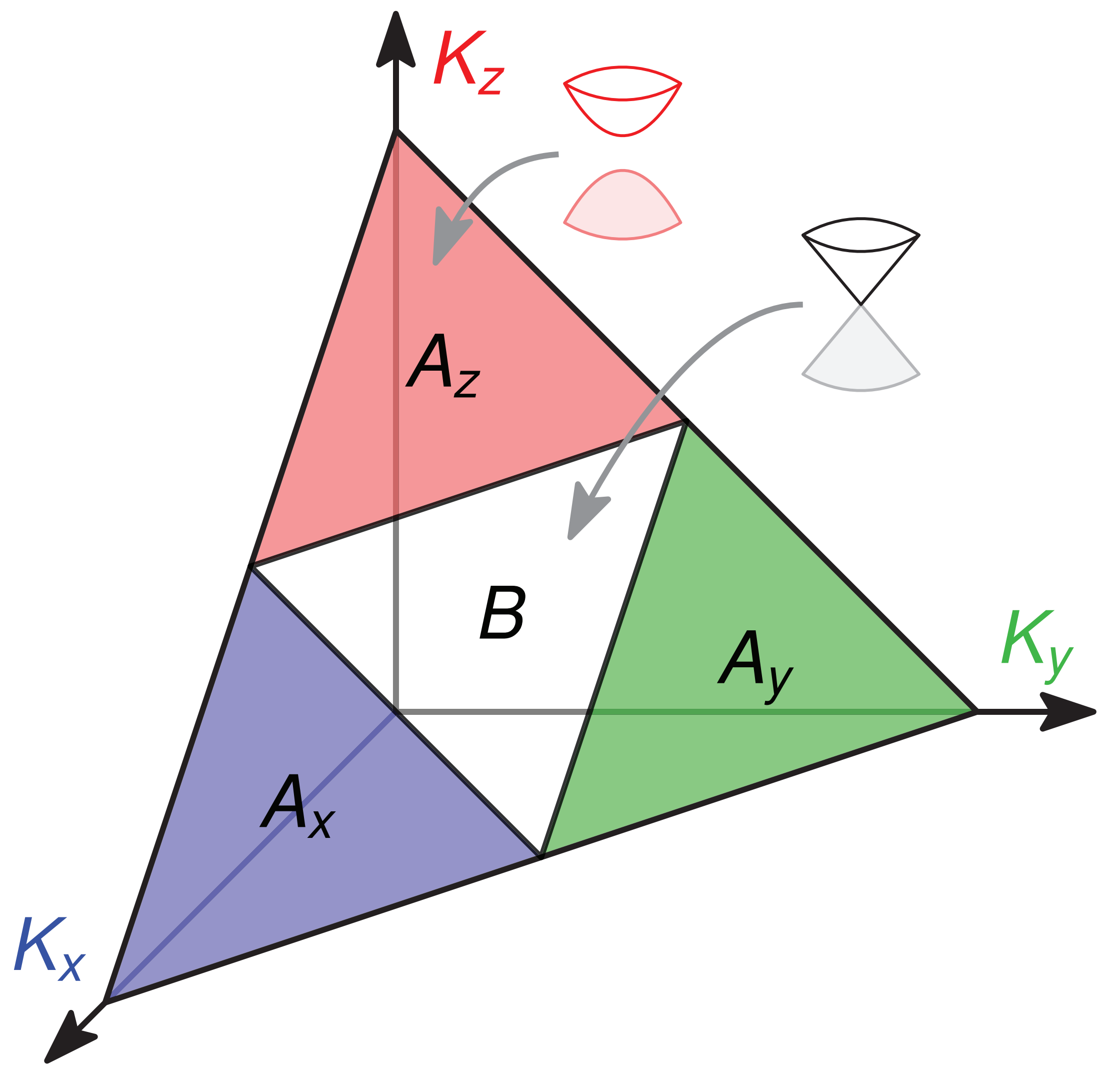
Kitaev quantum spin liquid

Gauge field:

$$u_{ij} \equiv 1 \qquad \text{on all links } \langle ij \rangle$$

[Lieb, PRL '94]

Fermion spectrum:



Quantum spin liquid: Ground state with fractionalized excitations

[Kitaev, Ann. Phys. '06]

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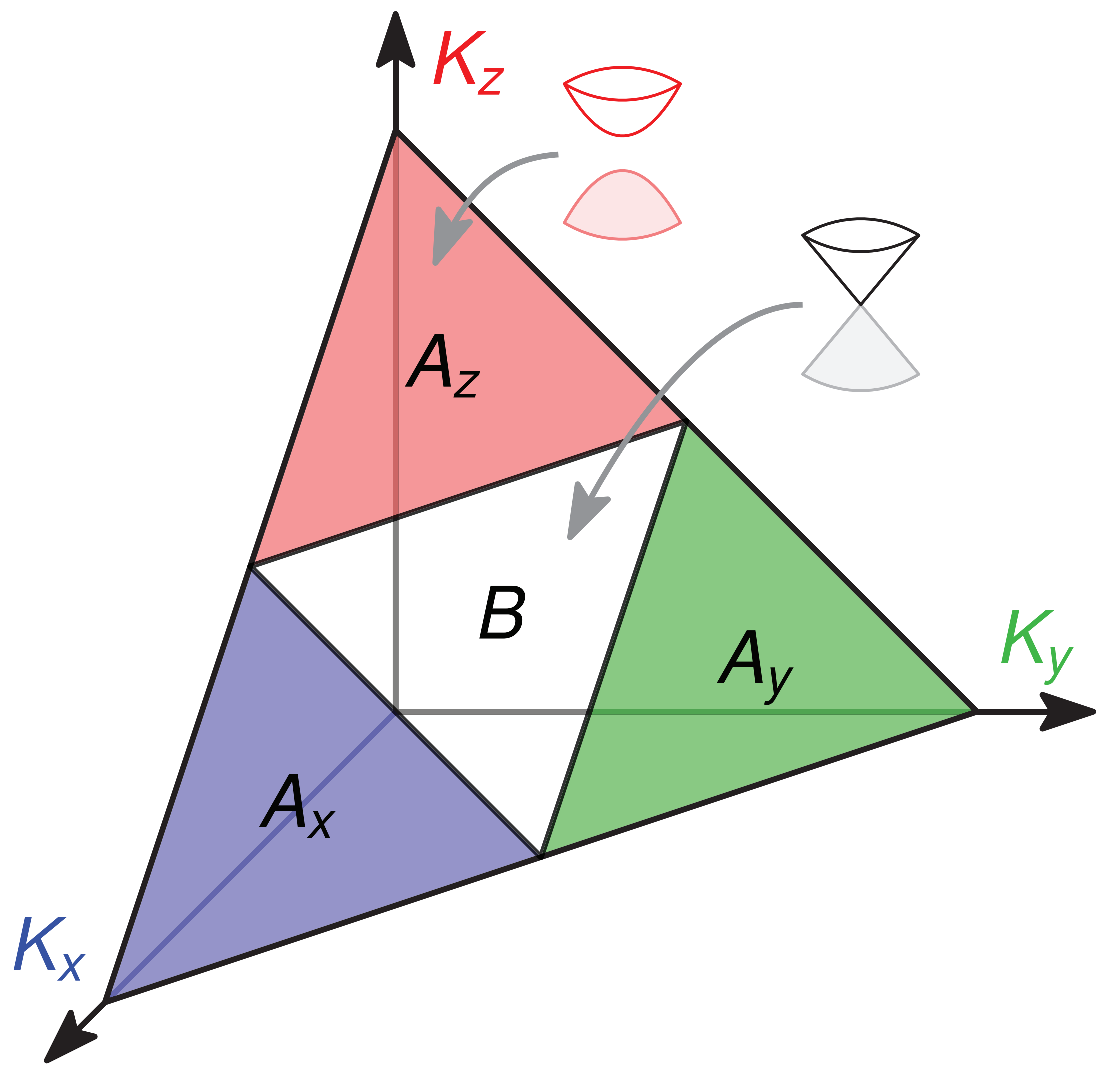
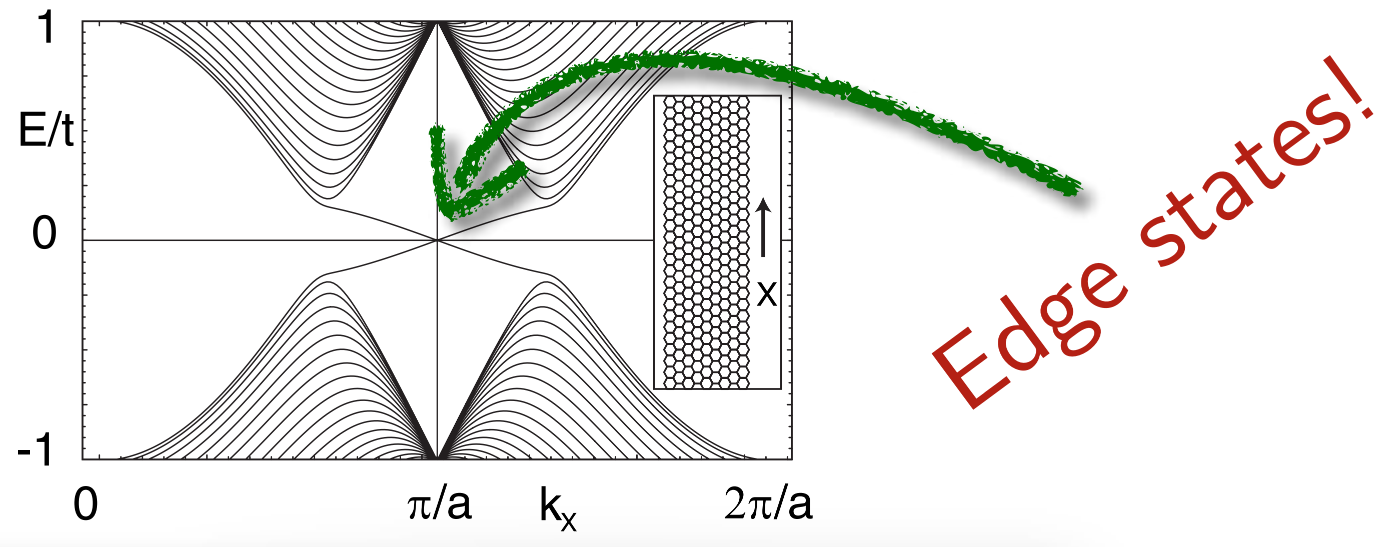


Quantum spin liquid: Ground state with fractionalized excitations

External magnetic field:

$$H_{\text{Zeeman}} = -\vec{h} \cdot \sum_i \vec{\sigma}_i$$

Spectrum for $h > 0$

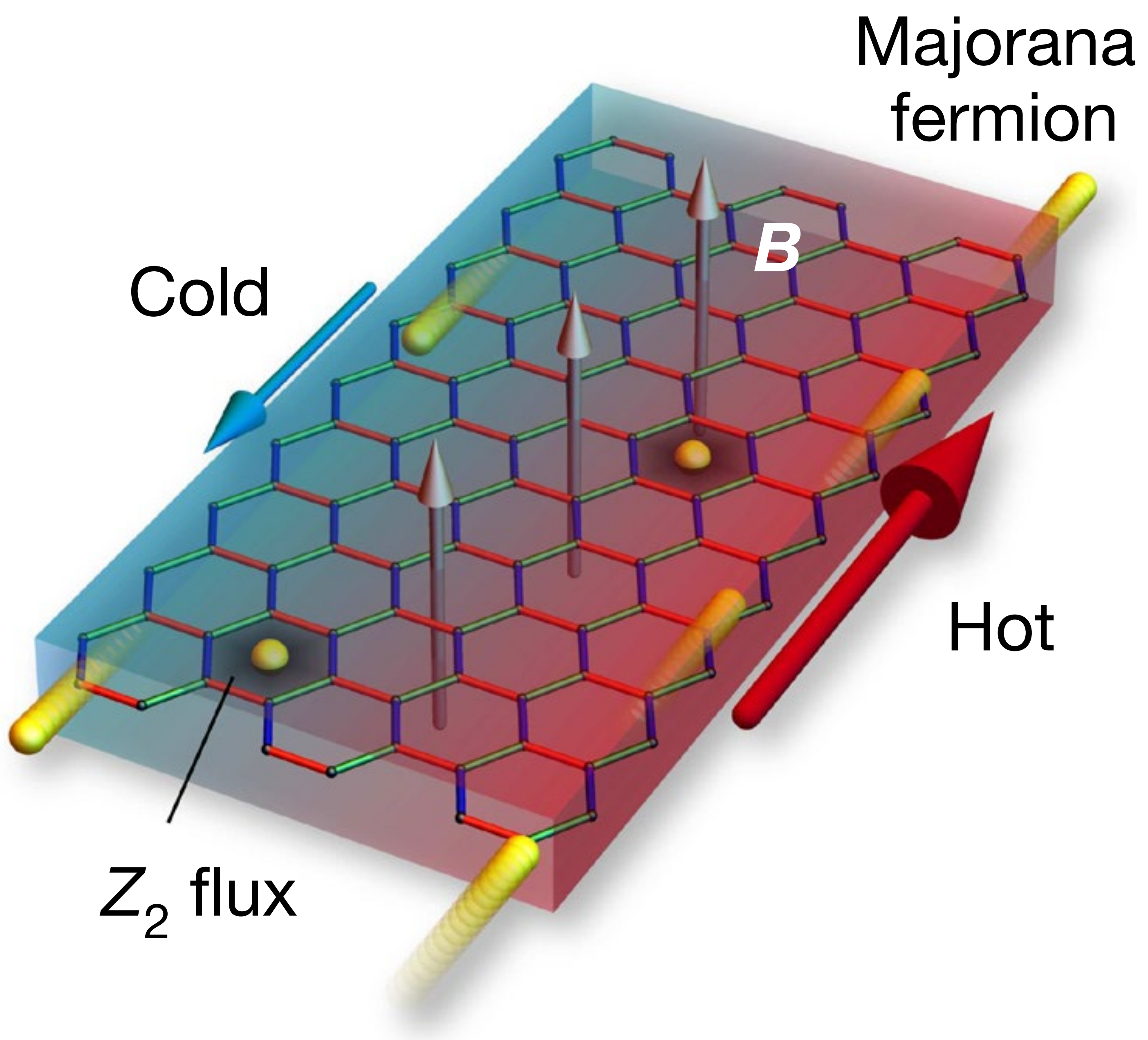


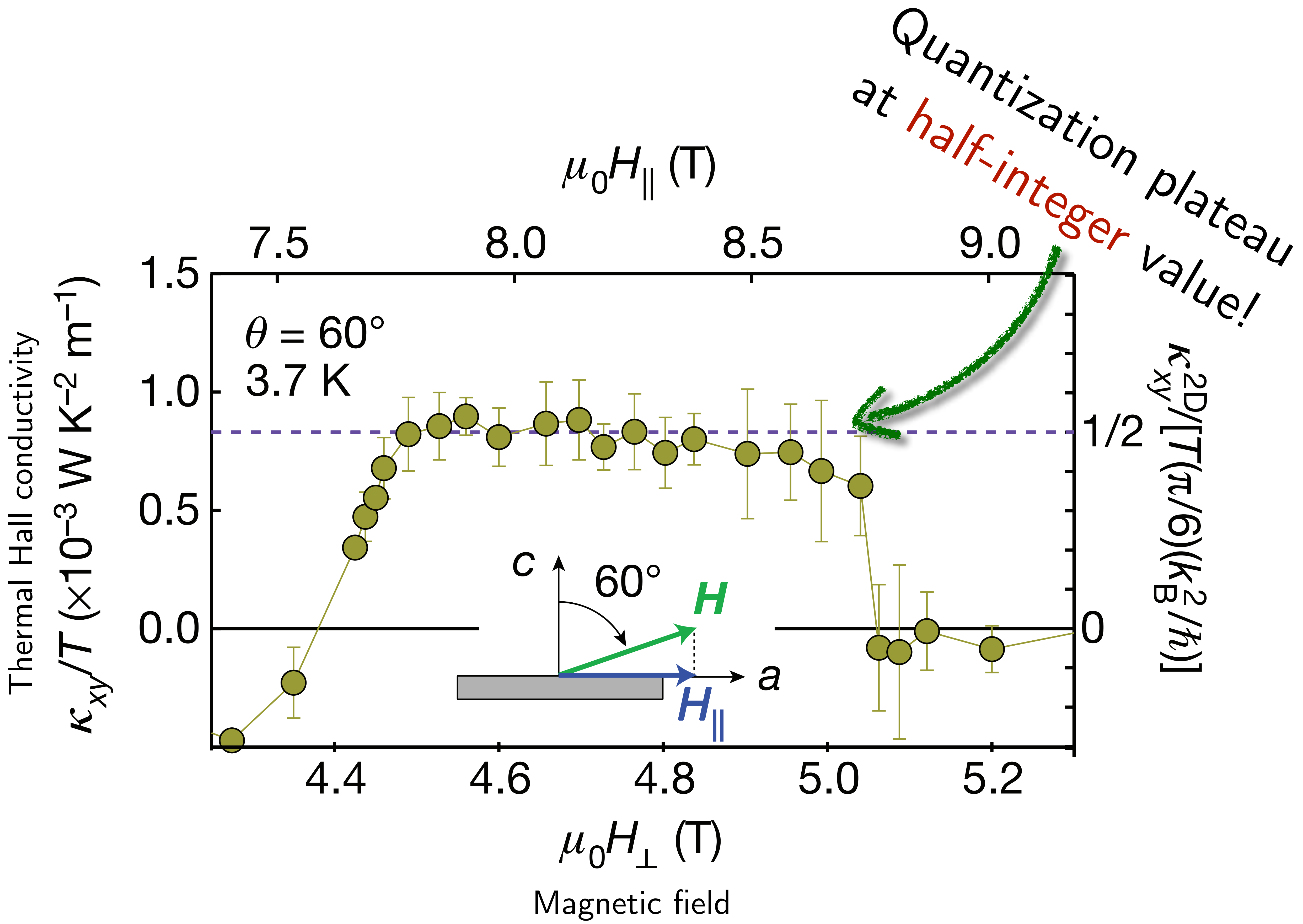
Kitaev model in field: Fractionalized version of a 2D TI

[Kitaev, Ann. Phys. '06]

Experimental search: α-$RuCl_3$

Half-integer thermal Quantum Hall effect:



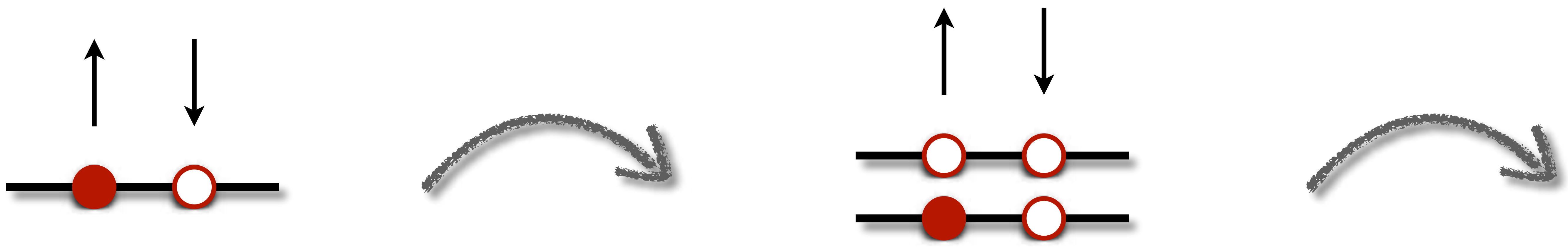


[Kasahara *et al.*, Nature '18]
Topical Review: [LJ & Vojta, JPCM '19]

Smoking-gun signature of Majorana edge states?

Generalizations of Kitaev model: Spin-orbital liquids

Spin + orbital + ... degrees of freedom:



...

$\sigma^\alpha \quad 2 \times 2$

$\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$

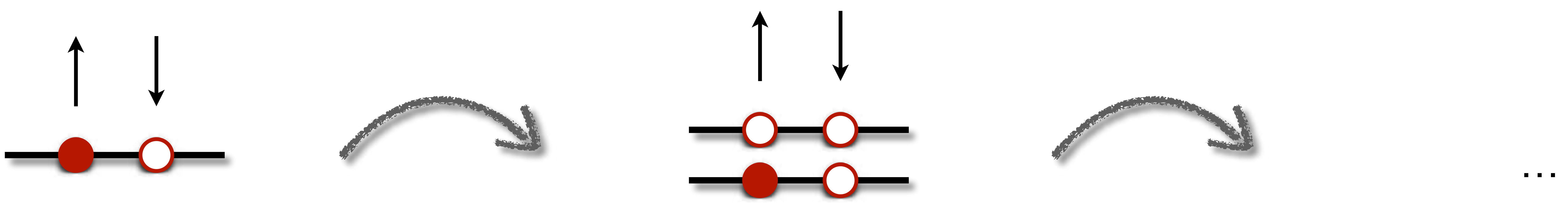
$\Gamma^\mu \quad 8 \times 8$

... can realize all 16 $\mathbb{Z}_2$ topological superconductors

[Chulliparambil, ..., LJ, Tu, arXiv:2005.13683]

Generalizations of Kitaev model: Spin-orbital liquids

Spin + orbital + ... degrees of freedom:

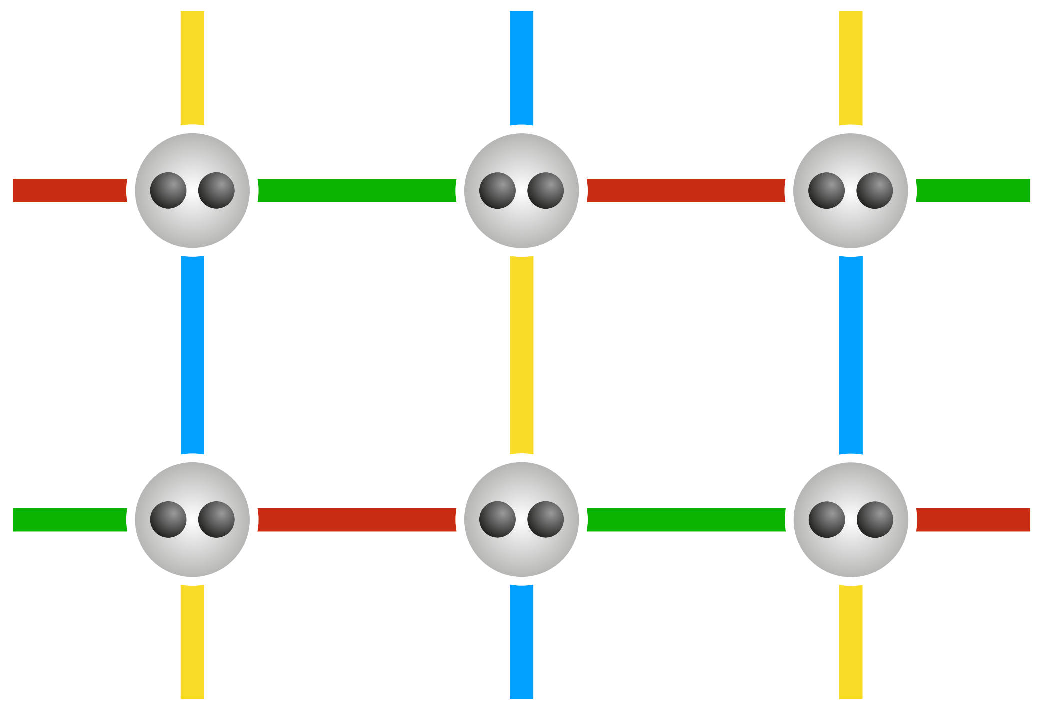


$\sigma^\alpha \quad 2 \times 2$ $\qquad$ $\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$ $\qquad$ $\Gamma^\mu \quad 8 \times 8$

... can realize all 16 $\mathbb{Z}_2$ topological superconductors
[Chulliparambil, ..., LJ, Tu, arXiv:2005.13683]

Example: $j = 3/2$

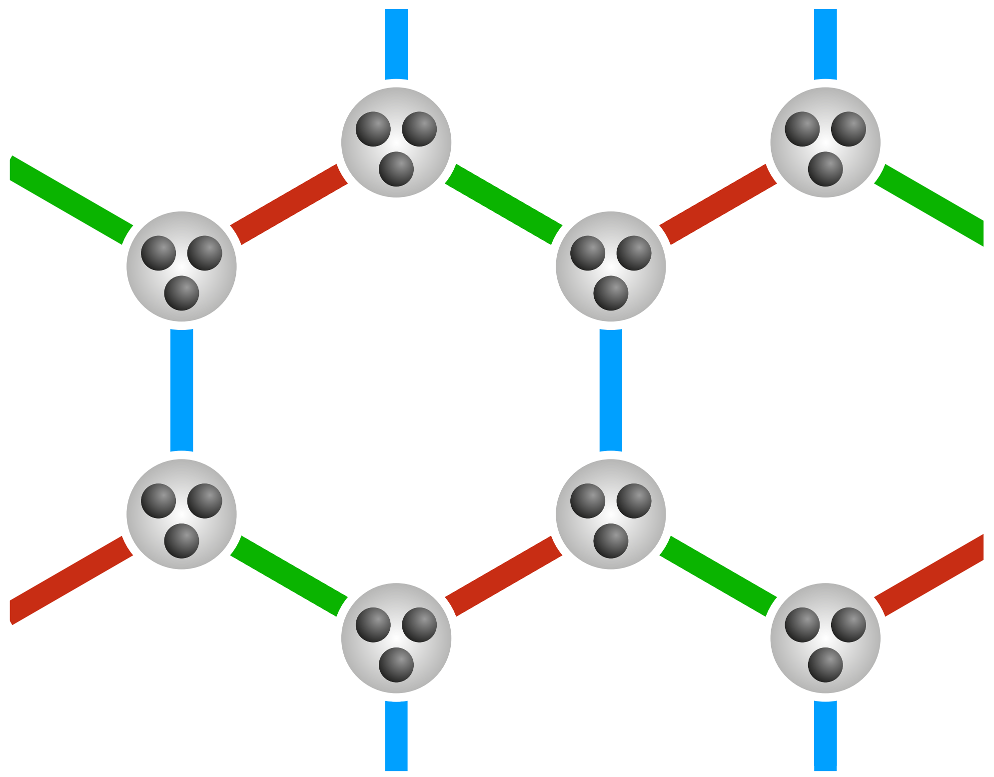
$$\gamma^i = i b^i c, \qquad i = 1, \ldots, 5 \quad \curvearrowright \quad \text{6 Majoranas}$$



2 itinerant fermions
$C = 2$

Square lattice

[Nakai, Ryu, Furusaki, PRB '12]

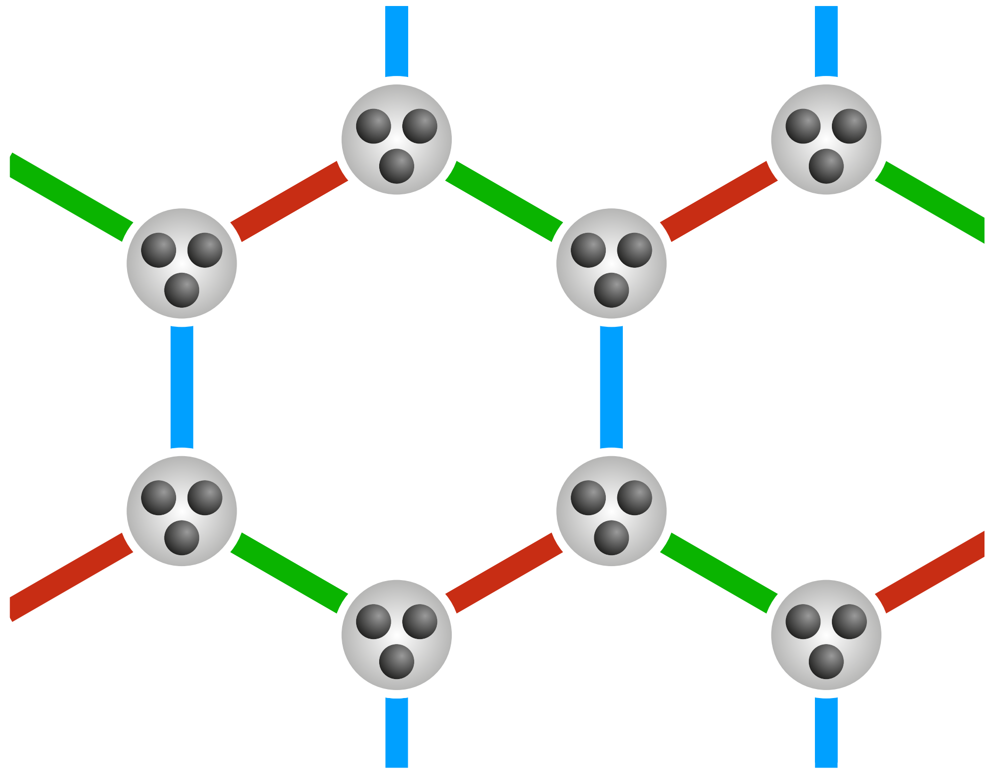


3 itinerant fermions
$C = 3$

Honeycomb lattice

[Yao, Lee, PRL '11]

Short-range Majorana interactions

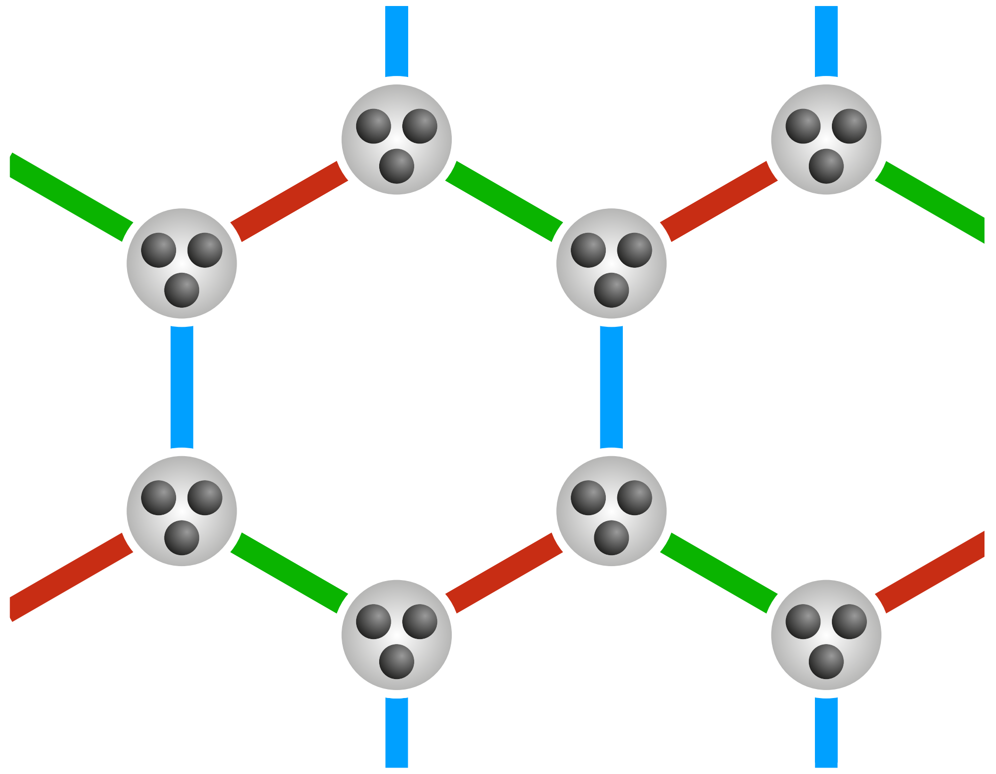


Kitaev + perturbations:

$$H = -K \sum_{\langle ij \rangle_\alpha} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\alpha \tau_j^\alpha \qquad \text{“Kitaev”}$$

$$+ J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j \qquad \text{“Heisenberg”}$$

Short-range Majorana interactions



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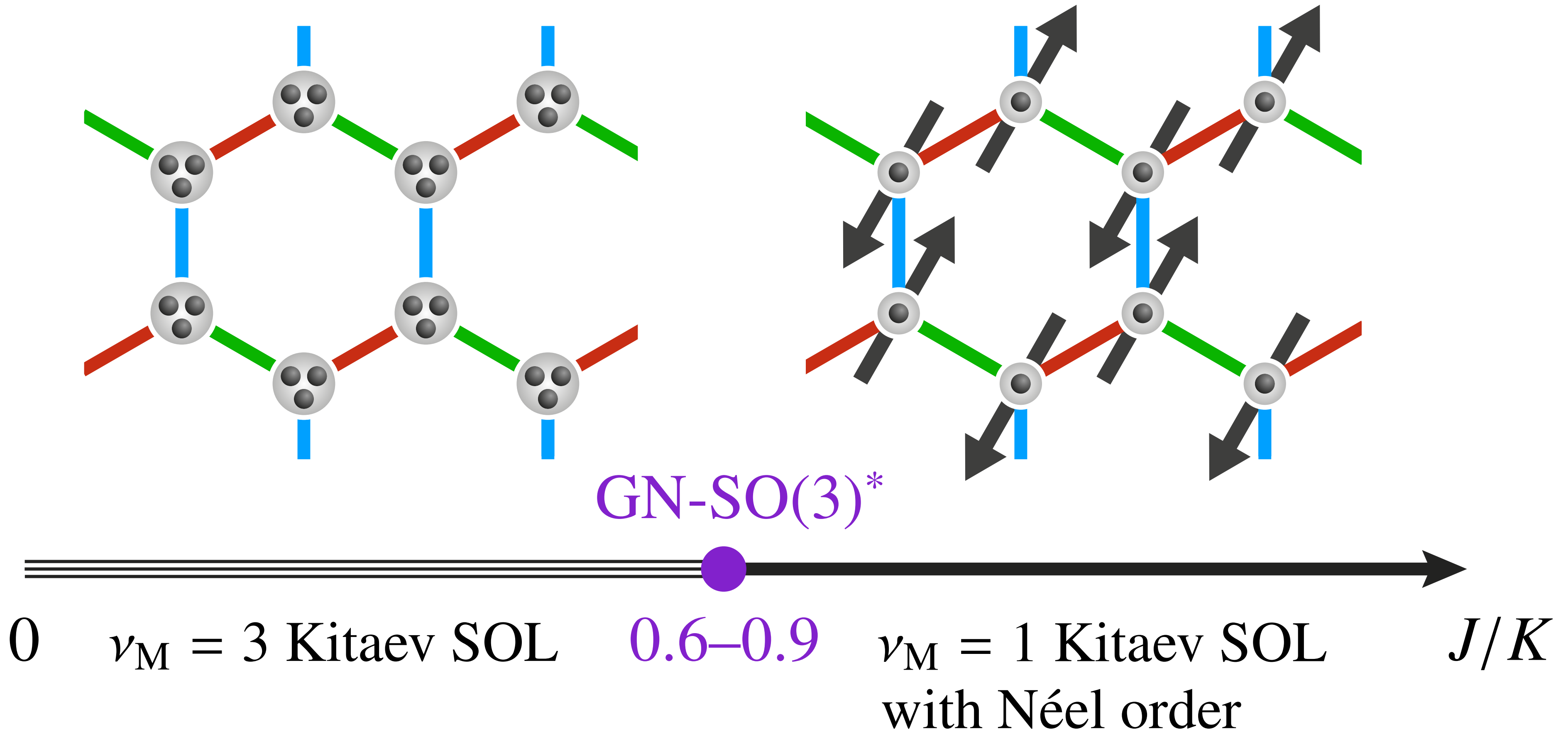
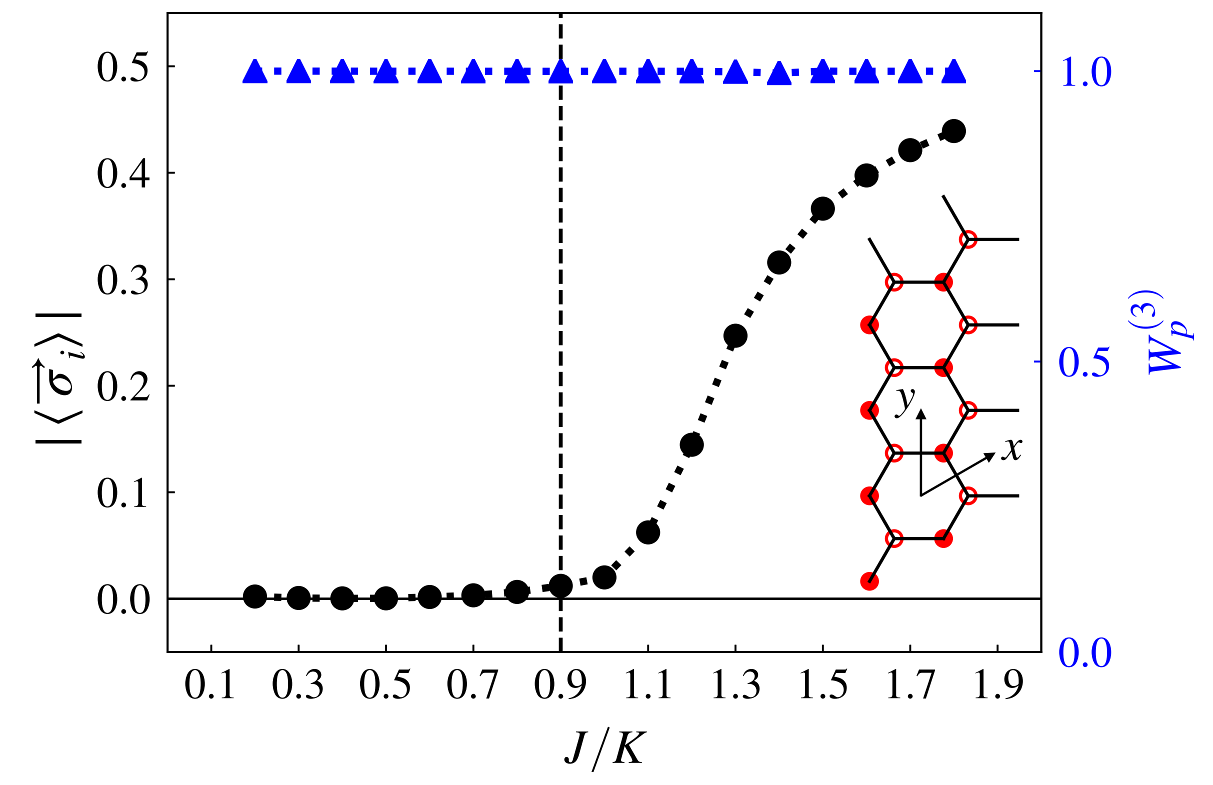
Phase diagram:

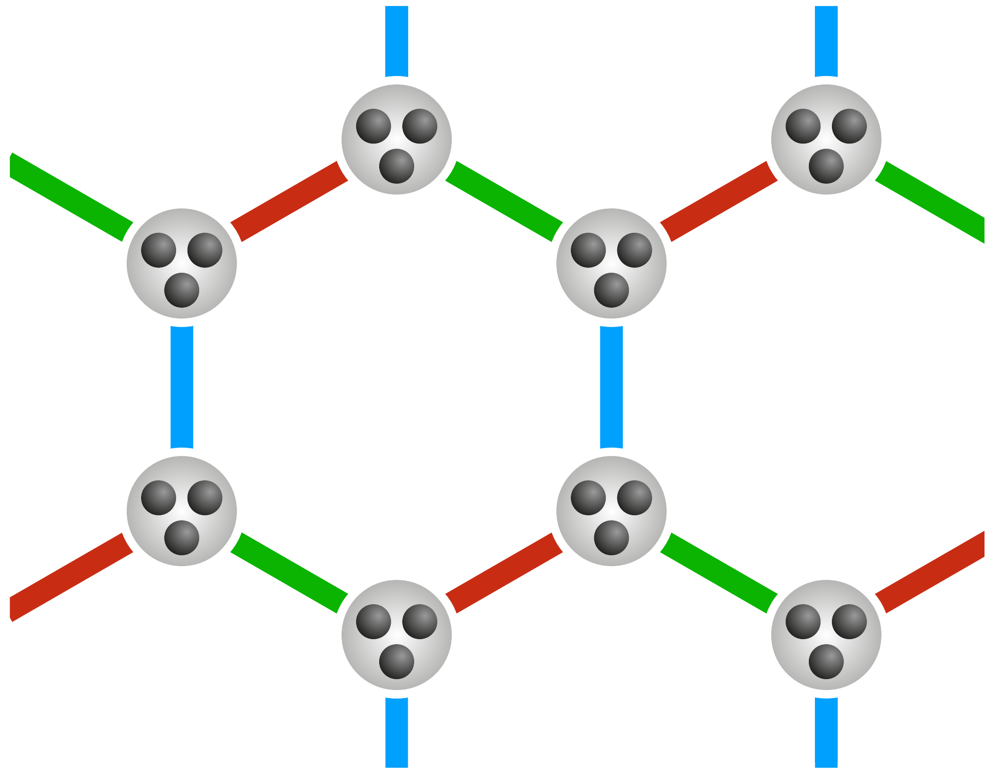


DMRG:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, arXiv:2009.05051]

Short-range Majorana interactions



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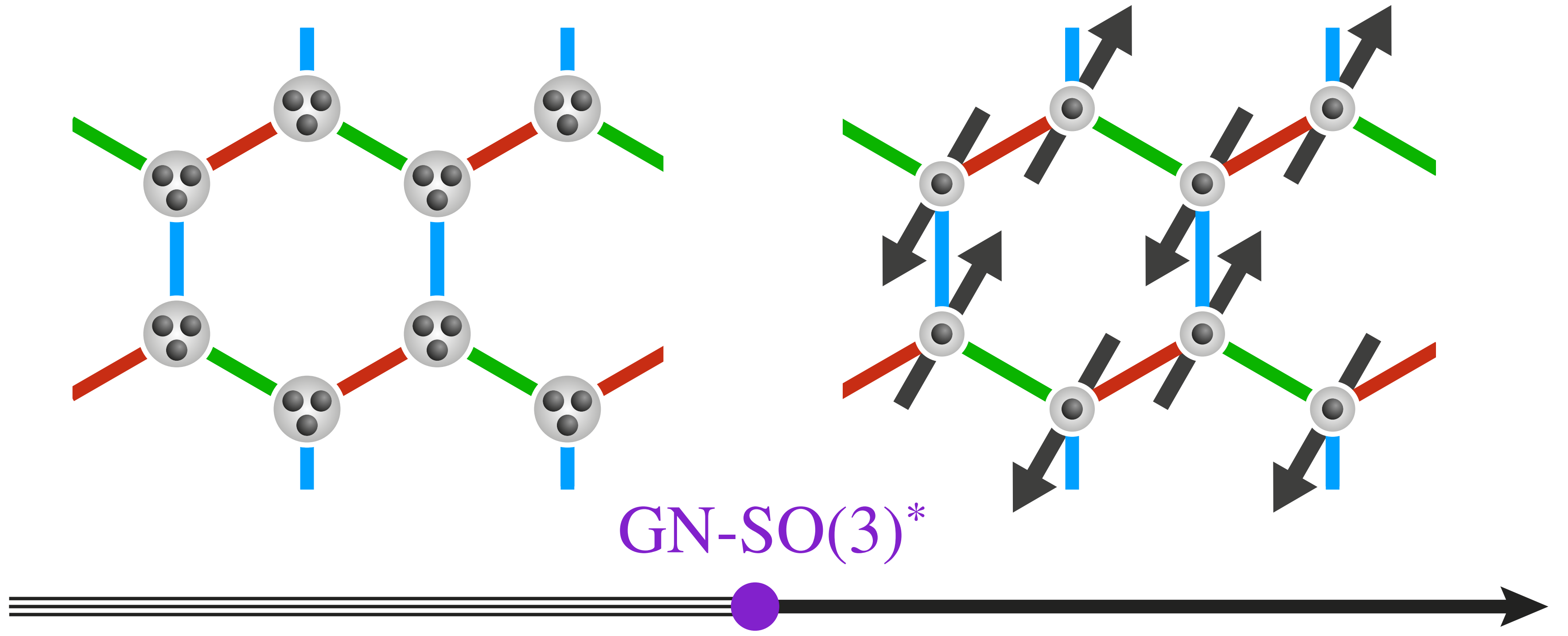

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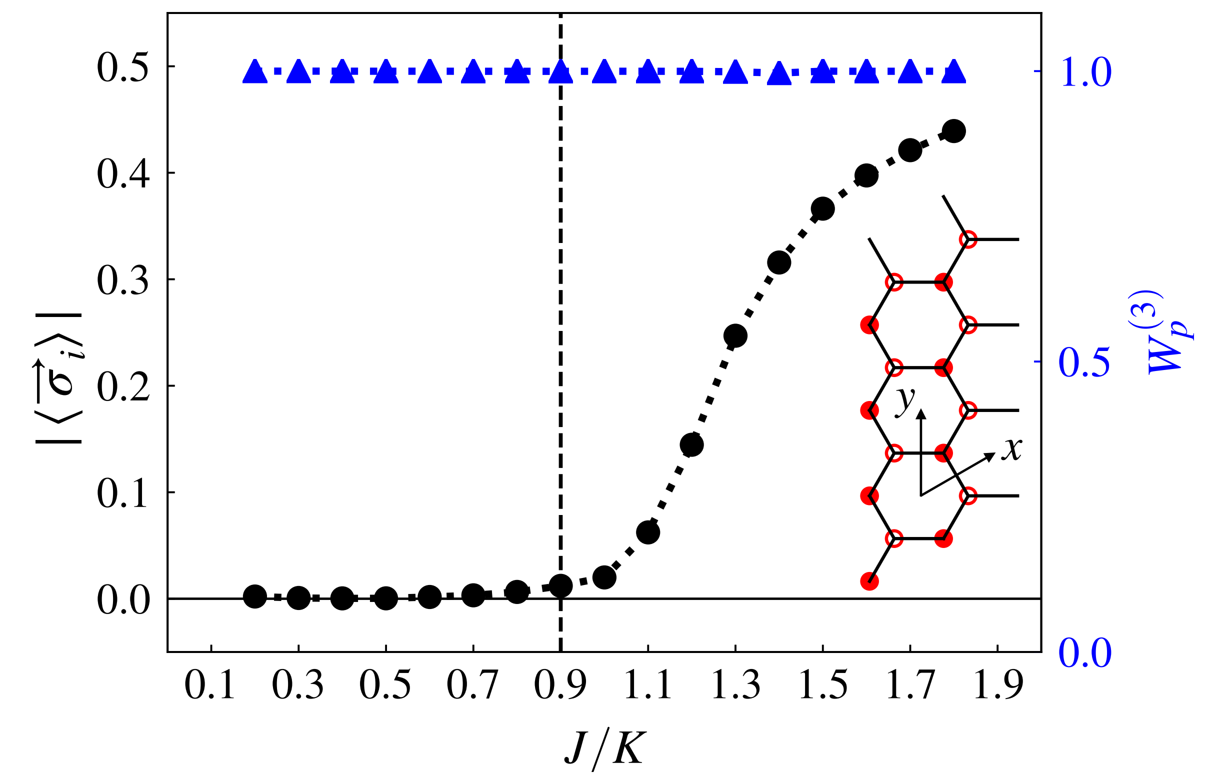

Fractionalized version of 2+1D Gross-Neveu

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, arXiv:2009.05051]

Gross-Neveu-SO(3)* criticality

Gross-Neveu* versus Gross-Neveu:

- Adjacent phases topological: four topological sectors
- Quasiparticles fractionalized: “missing” states

... cf. Ising*: [Schuler *et al.*, PRL ’16]

Universal fingerprints in finite-size spectra

Gross-Neveu-SO(3)* criticality

Gross-Neveu* versus Gross-Neveu:

- Adjacent phases topological: ↷ four topological sectors
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... cf. Ising*: [Schuler *et al.*, PRL '16]

$\boxed{\text{Universal fingerprints in finite-size spectra}}$

Gross-Neveu-SO(3) vs Gross-Neveu-SU(2): $\mathcal{L}_{\mathrm{FB}} = g\vec{\varphi} \cdot \bar{\psi}(\mathbb{1} \otimes \vec{L})\psi$

Spin-1 vs Spin-1/2

$\boxed{\text{New member of Gross-Neveu family}}$

... with $\eta \approx 0.32...0.33$, $\nu \approx 1...2$
... from $1/N$ and $4-\epsilon$ expansion

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, arXiv:2009.05051]

Outline

(1) Motivation: *Emergence versus constructionism*

(2) Emergent symmetry: *Relativistic fermions from nonrelativistic electrons*

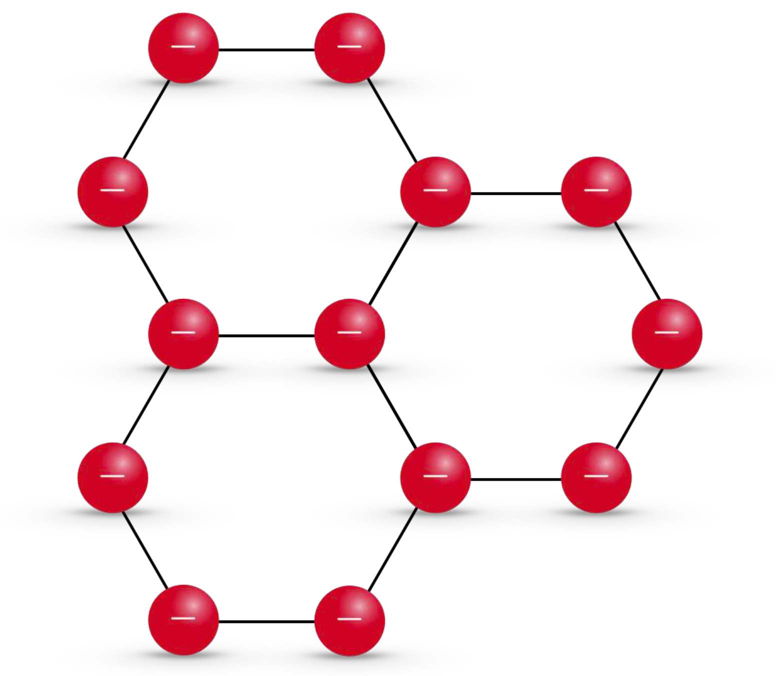
(3) Emergent topology: *Relativistic fermions from winding numbers*

(4) Emergent excitations: *Relativistic fermions from fractionalization*

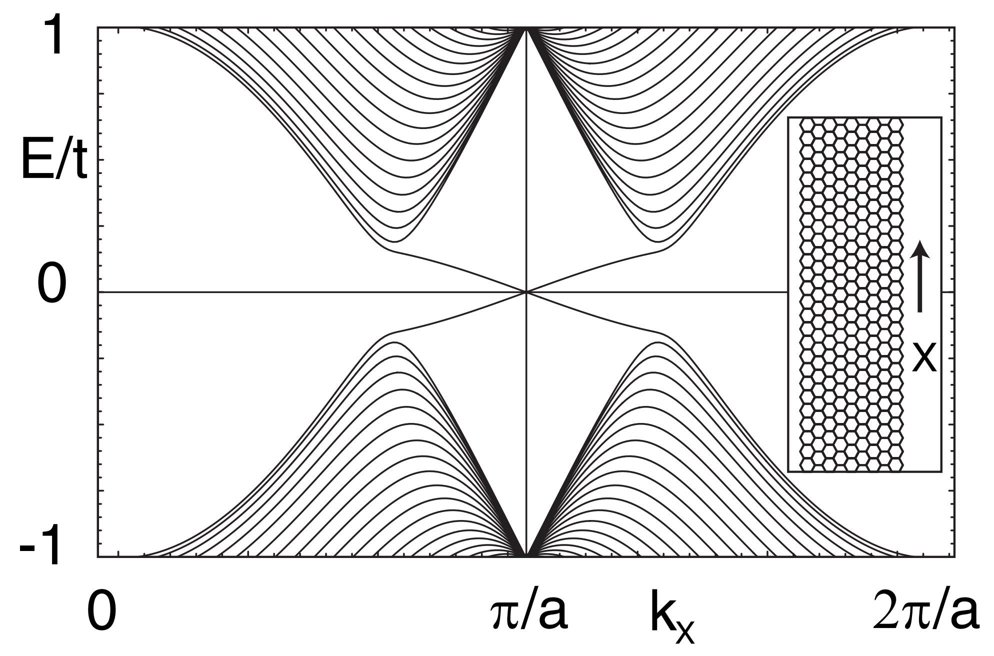
(5) Conclusions

Conclusions

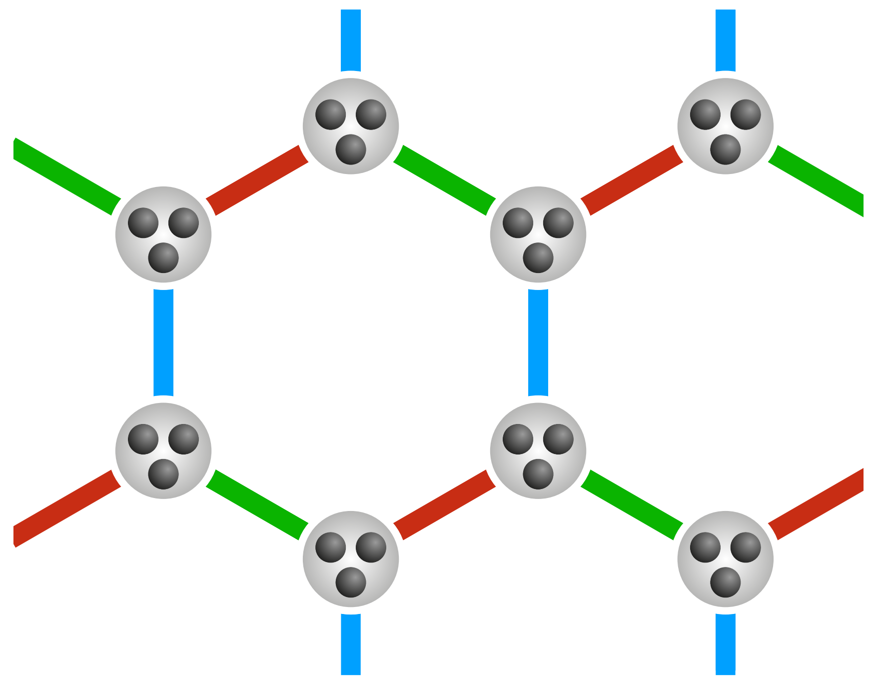
Emergent phenomena in condensed matter:



Emergent symmetry



Emergent topology

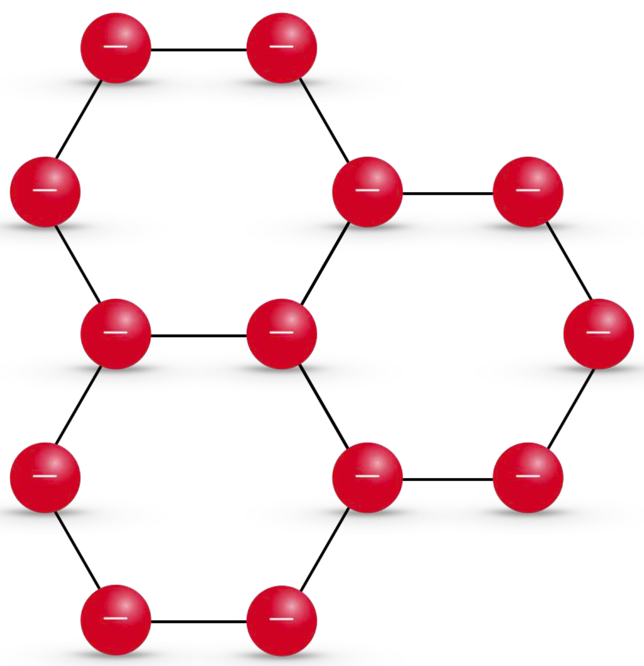


Emergent particles

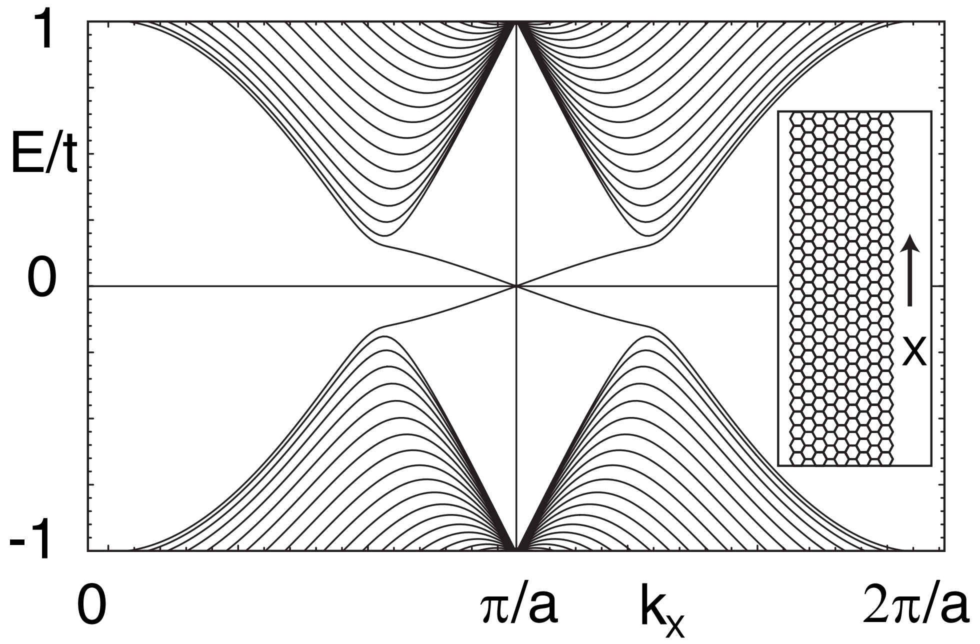
P. Anderson

Conclusions

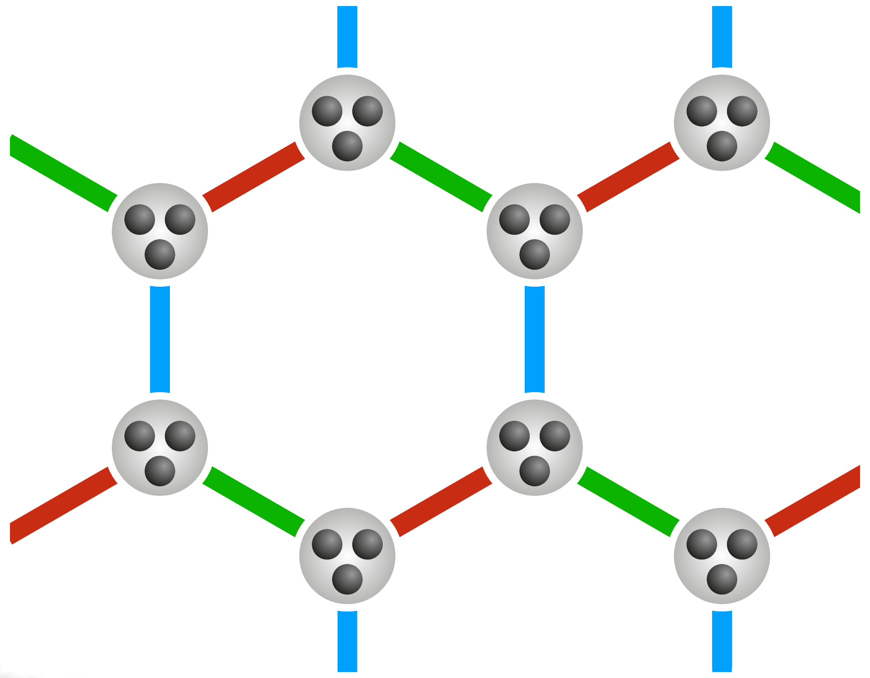
Emergent phenomena in condensed matter:



Emergent symmetry

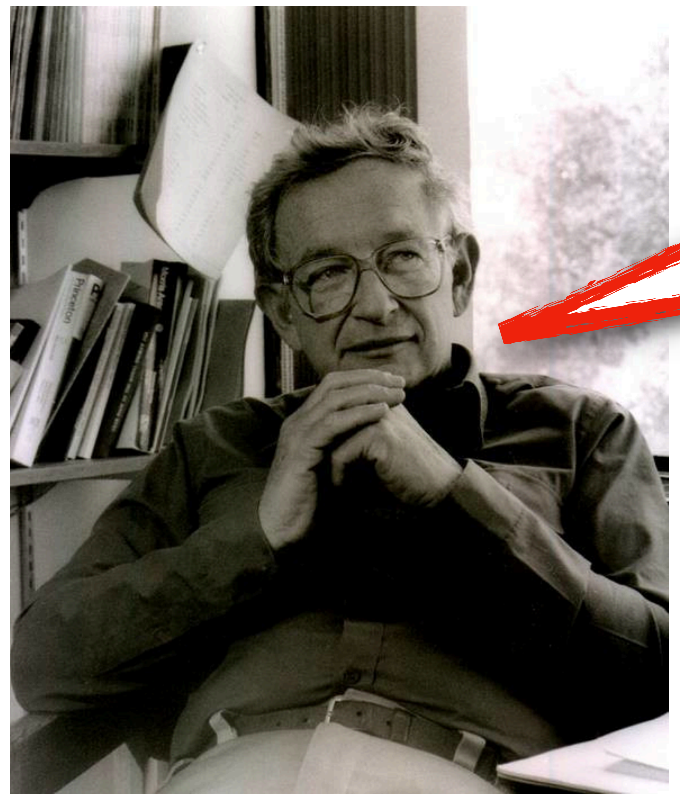


Emergent topology



Emergent particles

More is different!

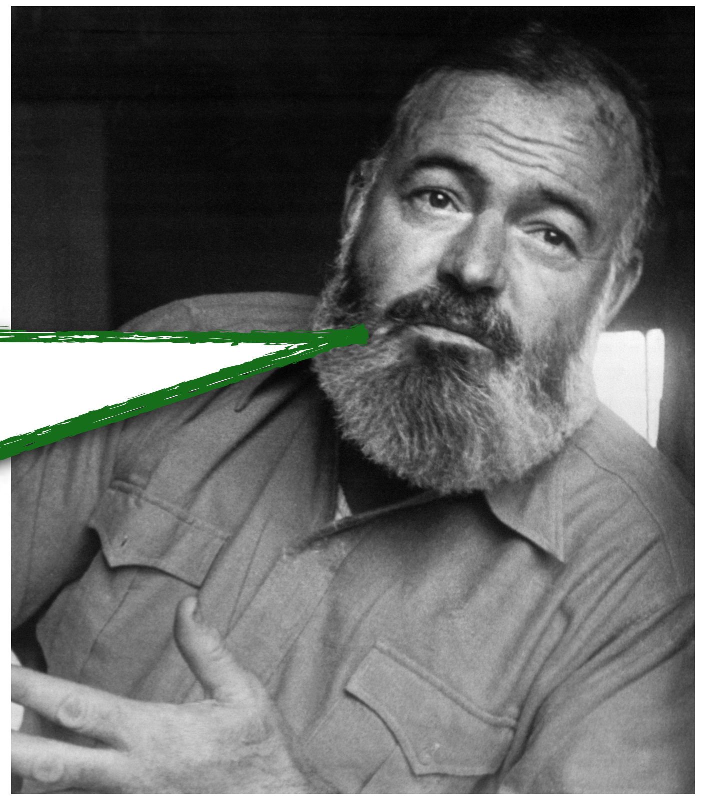


P. Anderson

You know, Ernest, the rich are different from you and me.

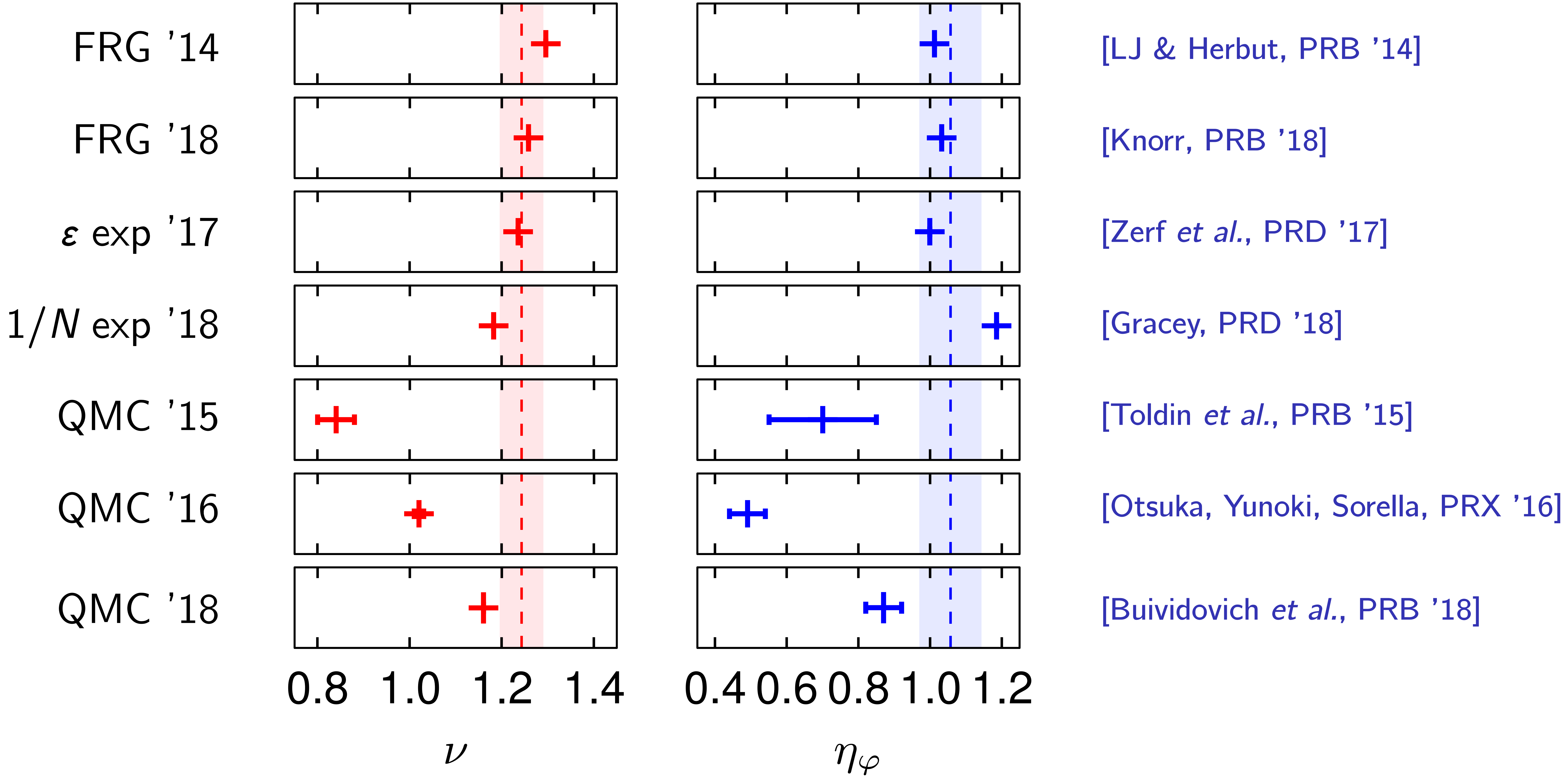
F. S. Fitzgerald

Yes, they have more money.



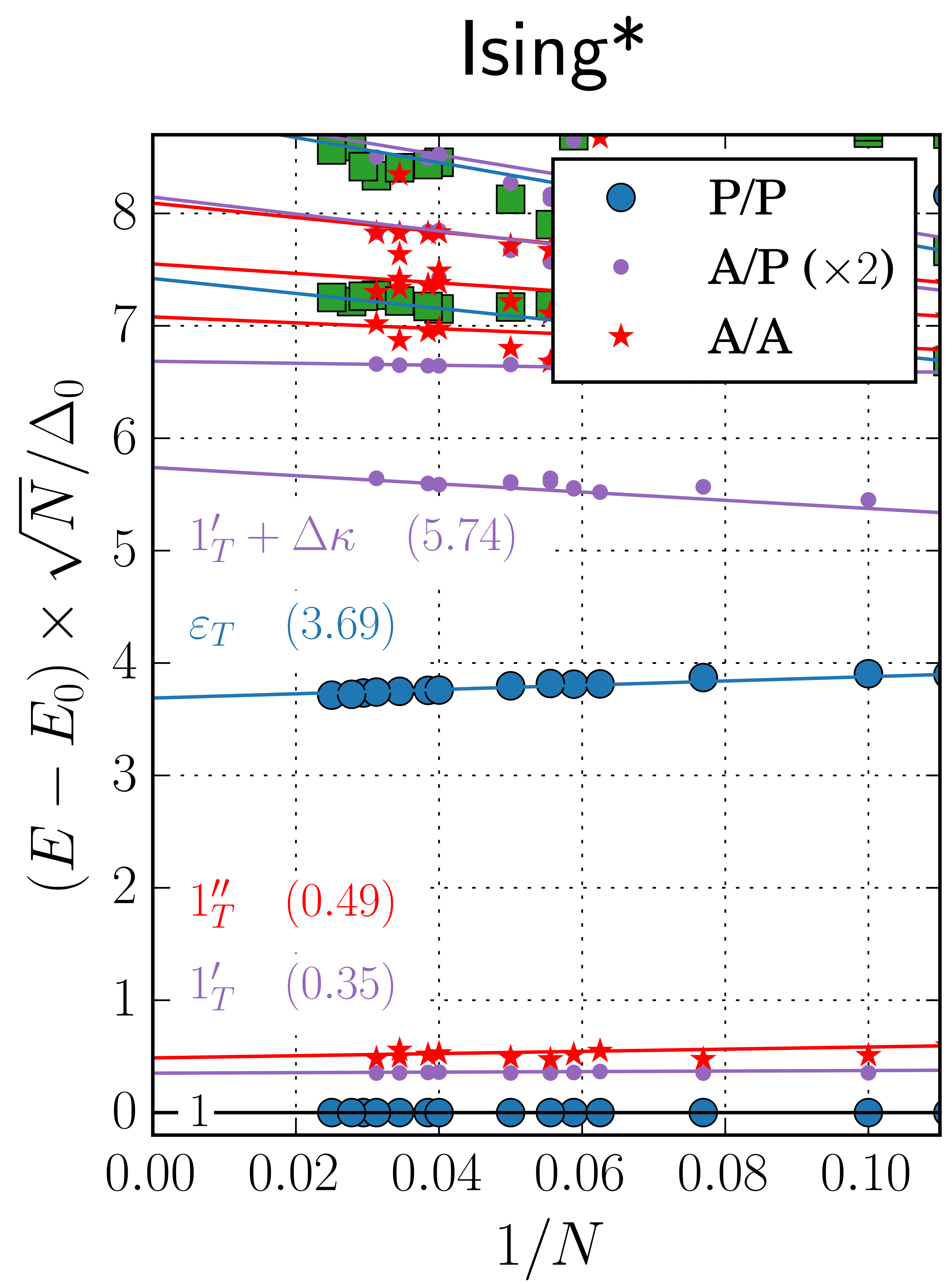
E. Hemingway

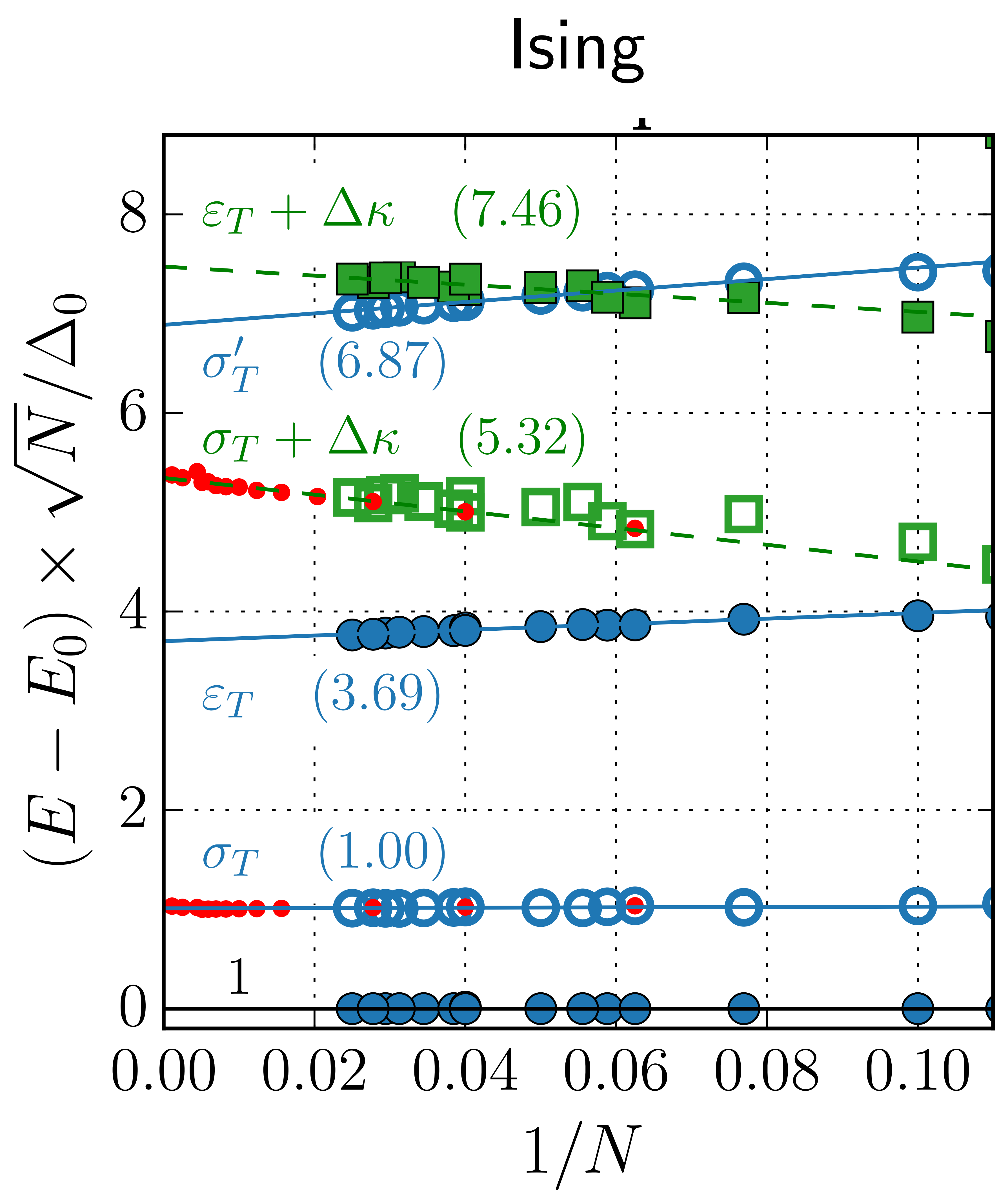
Gross-Neveu-SU(2) universality class



Classical Heisenberg universality: $\nu = 0.7112(5)$ $\eta = 0.0375(5)$ [Campostrini *et al.*, PRB '02]

Ising* vs Ising criticality





[Schuler, Whitsitt, Henry, Sachdev, Laeuchli, PRL '16]