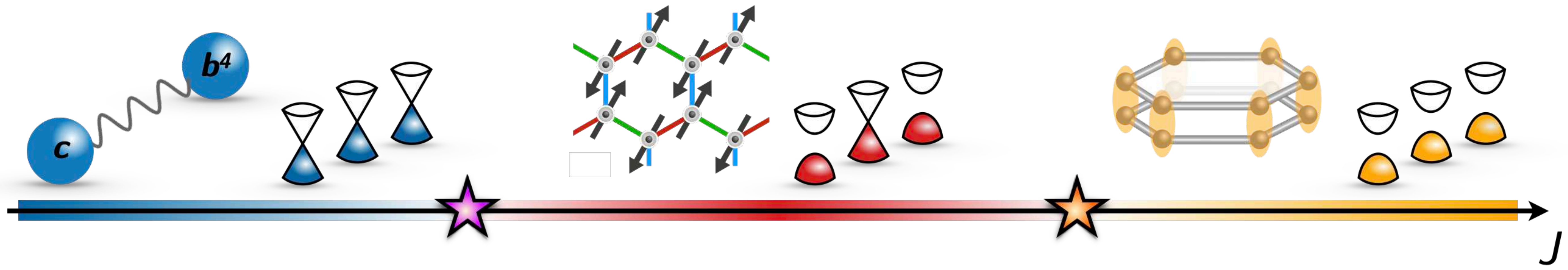
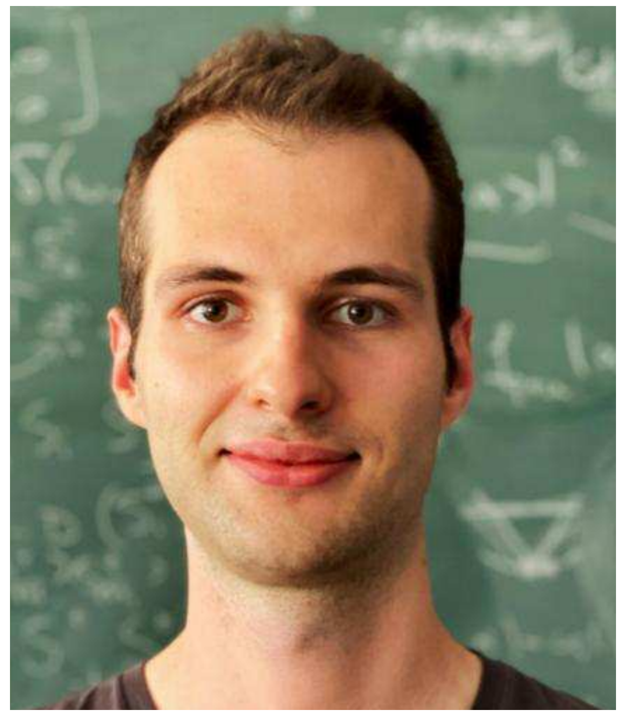
Fractionalized fermionic quantum criticality



Lukas Janssen
TU Dresden


Urban Seifert, Santa Barbara


Zihong Liu, Würzburg

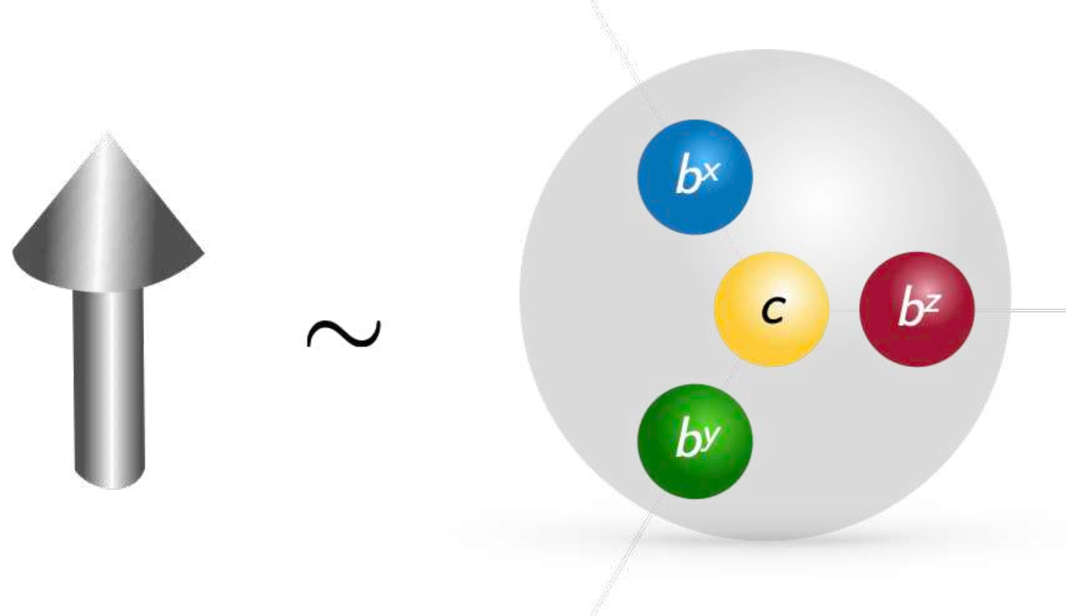
Fakher Assaad, Würzburg
Sreejith Chulliparambil, Dresden
Xiao-Yu Dong, Ghent
Hong-Hao Tu, Dresden
Matthias Vojta, Dresden
Shouryya Ray, Odense
Michael Scherer, Bochum

Quantum critical matter
JA 2306/4-1
Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

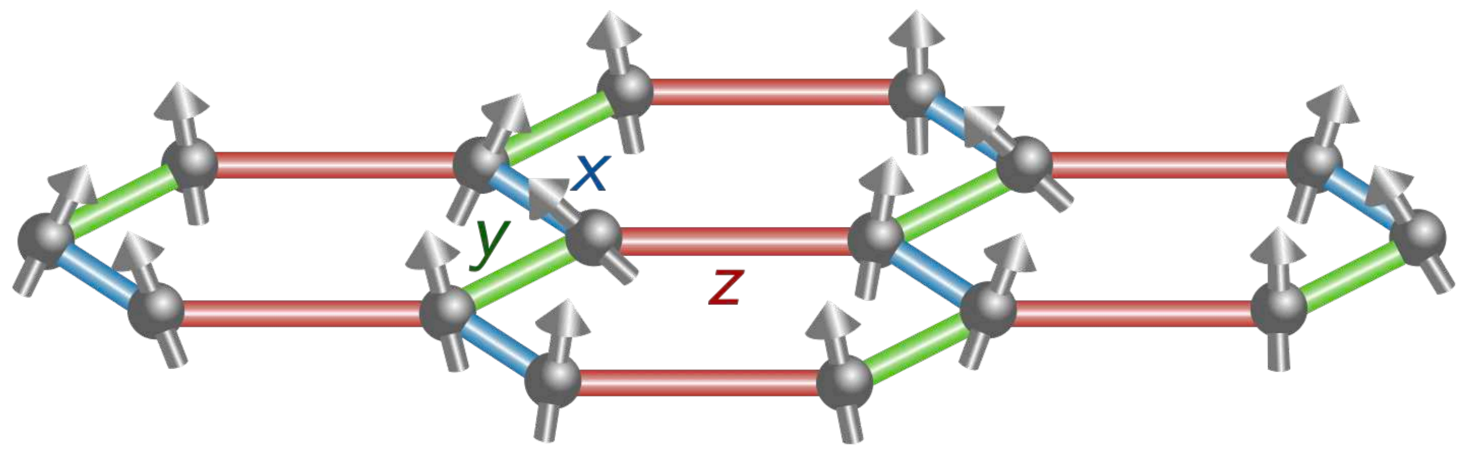
ct.qmat
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

SFB 1143

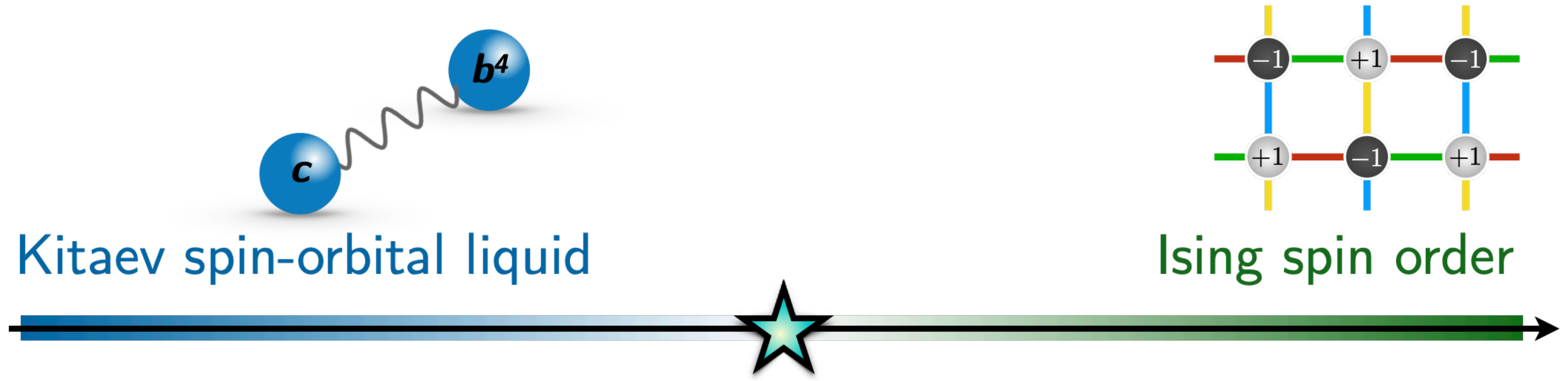
Outline



(1) Fractionalized quantum criticality



(2) Frustrated spins and spin-orbitals

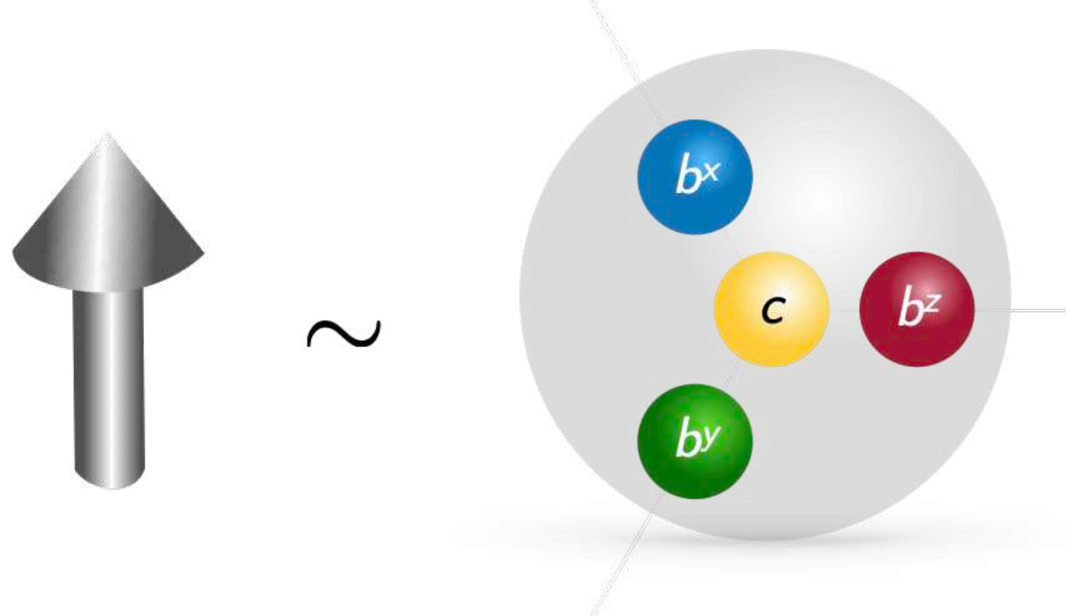


(3) Square-lattice Kitaev-Ising spin-orbital model

(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model

(5) Conclusions

Outline

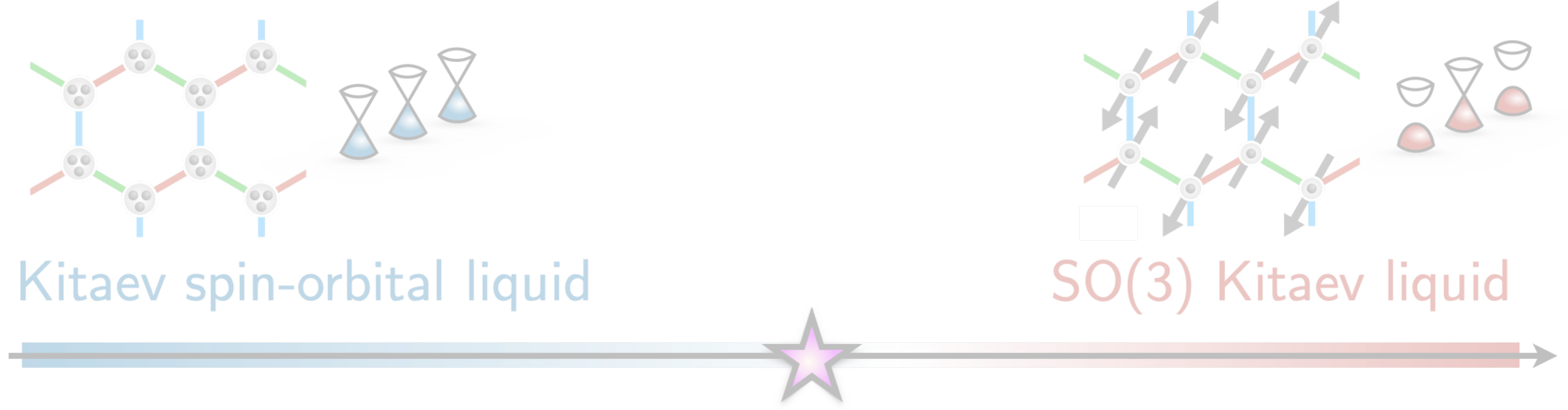


(1) Fractionalized quantum criticality

(2) Frustrated spins and spin-orbitals

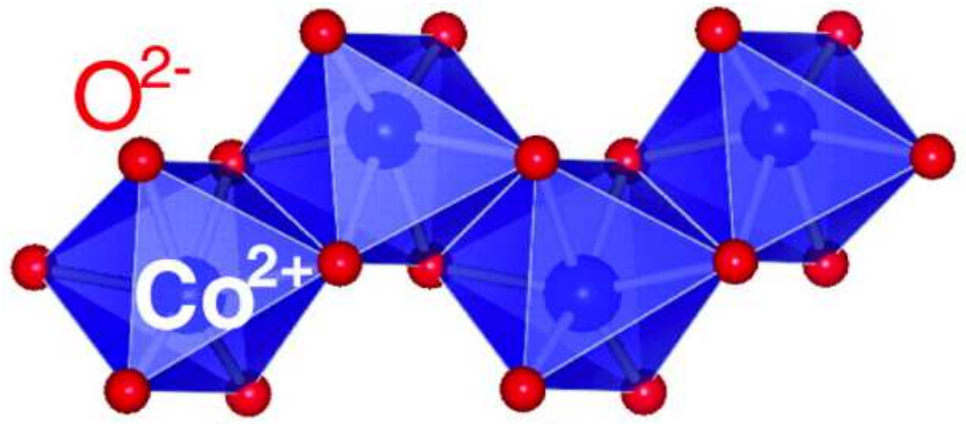
(3) Square-lattice Kitaev-Ising spin-orbital model

(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model

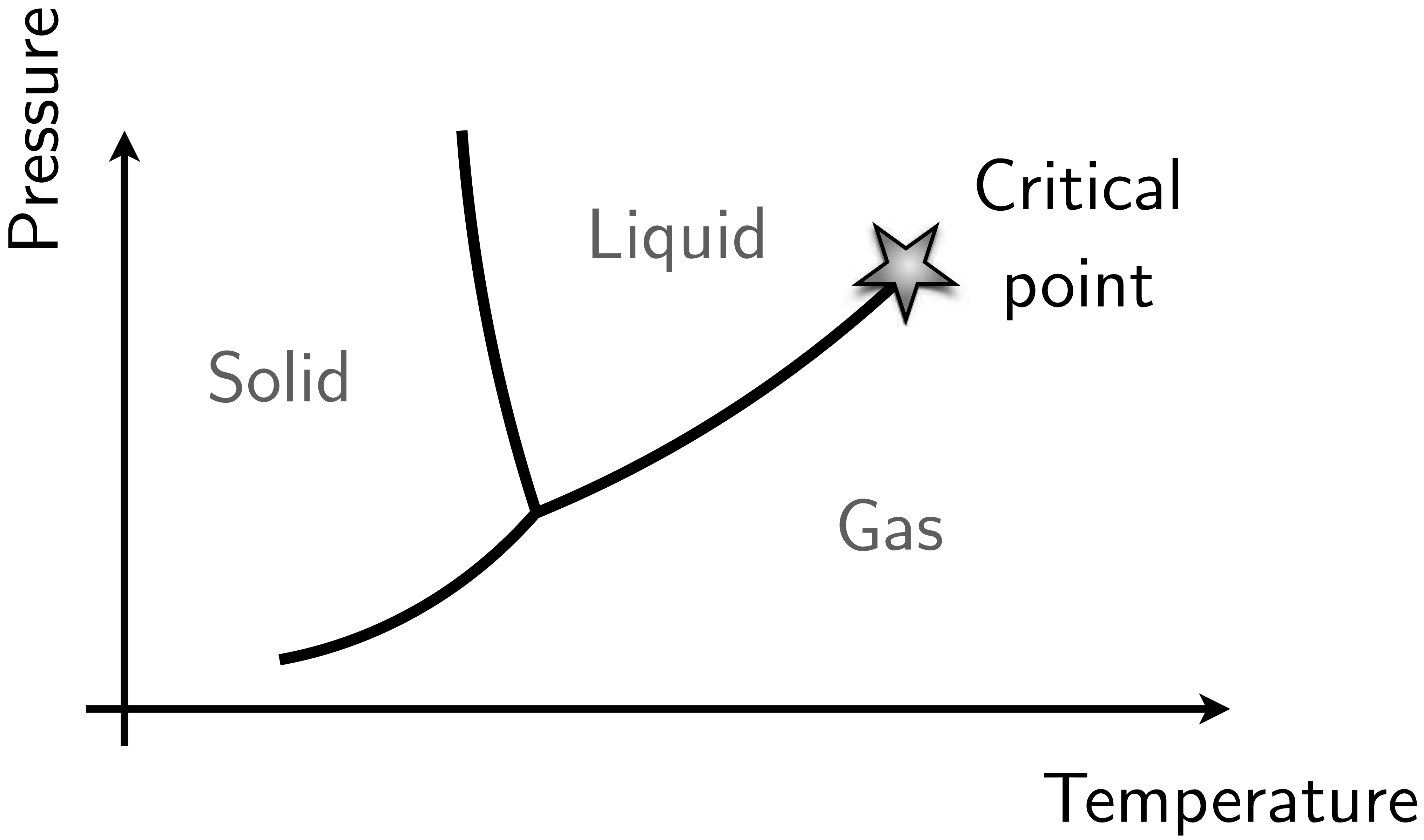


(5) Conclusions

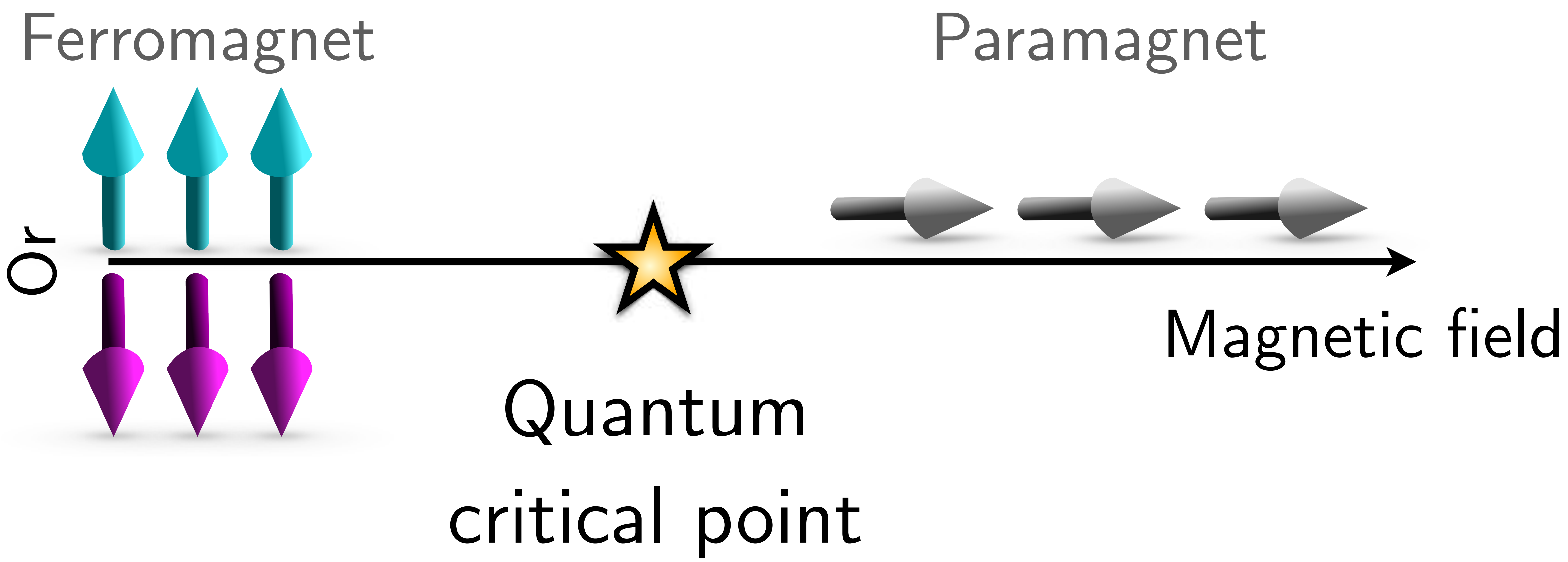
Classical vs quantum criticality



H_2O $\boldsymbol{T > 0}$

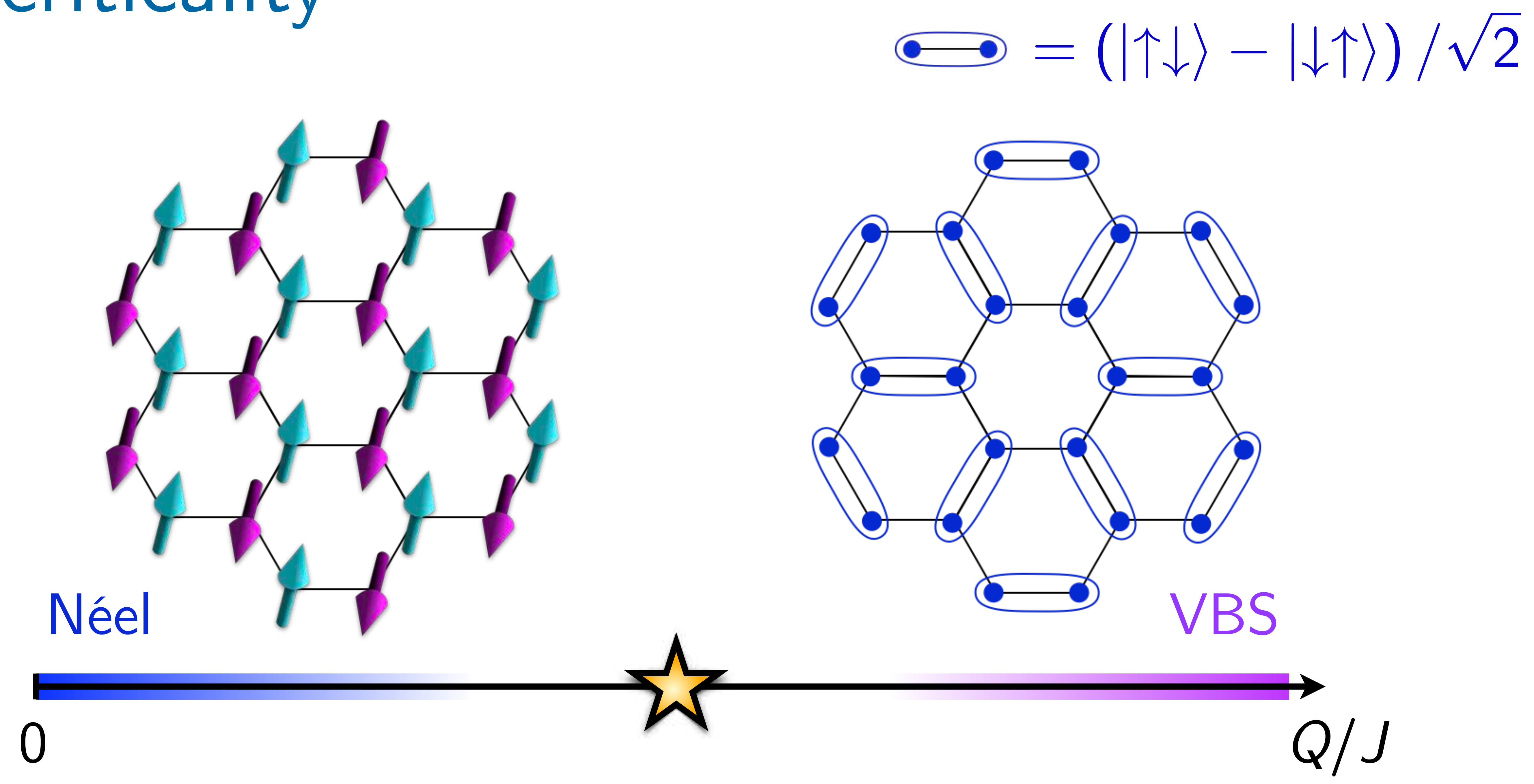


$CoNb_2O_6$ $\boldsymbol{T \to 0}$

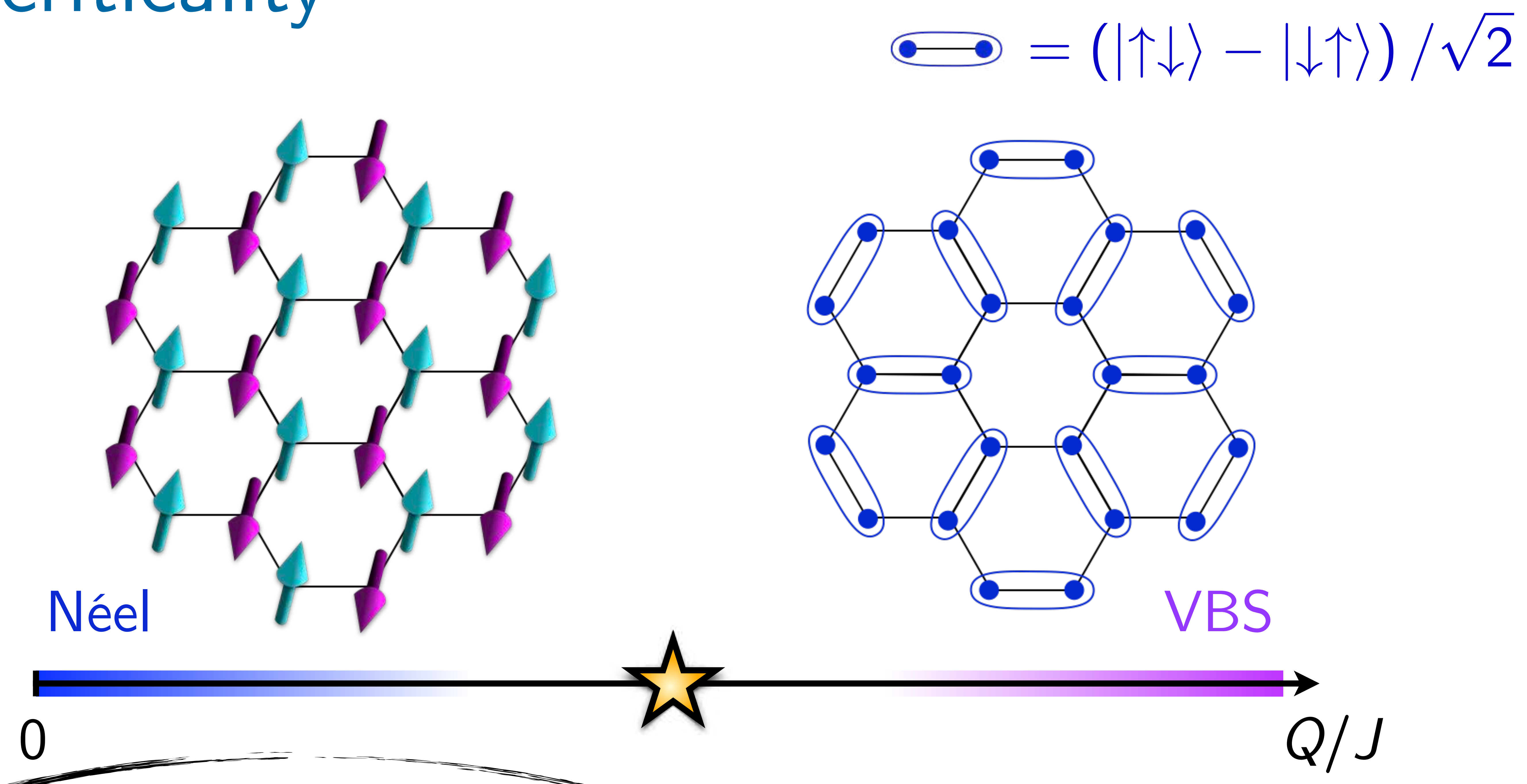


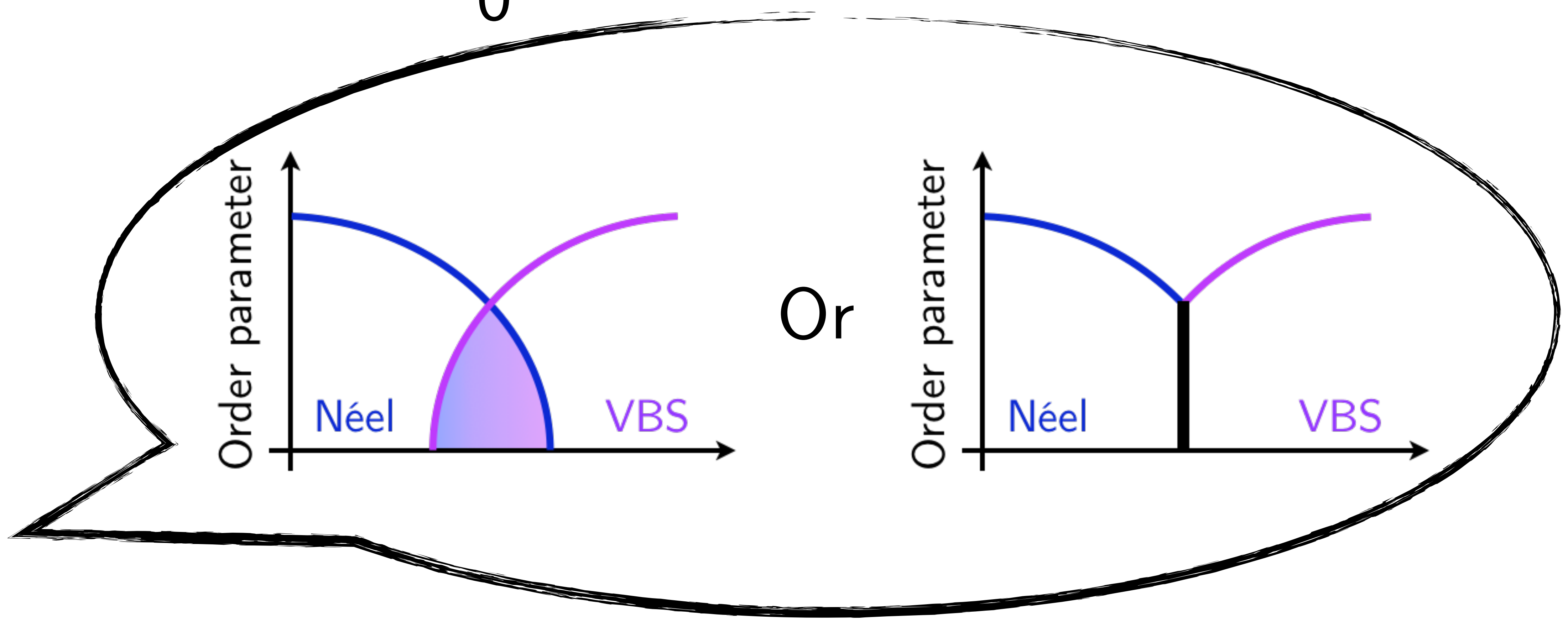
[Coldea *et al.*, Science '10]
[Kinross *et al.*, PRX '14]
[Morris *et al.*, Kaul, Armitage, Nat. Phys. '21]
...

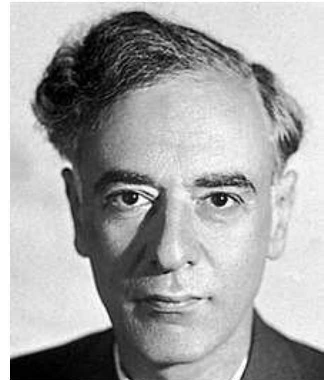
Deconfined quantum criticality



Deconfined quantum criticality

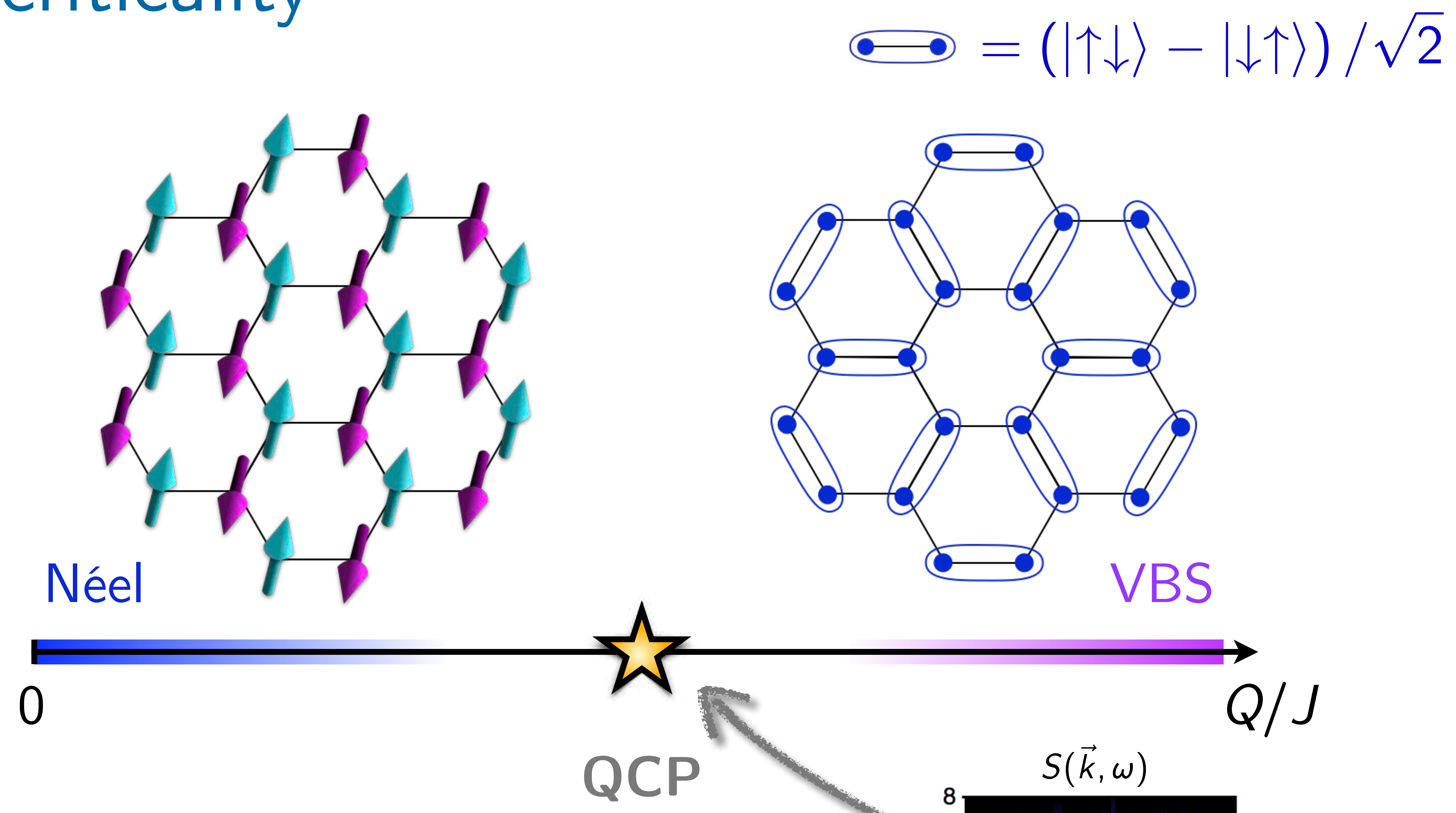


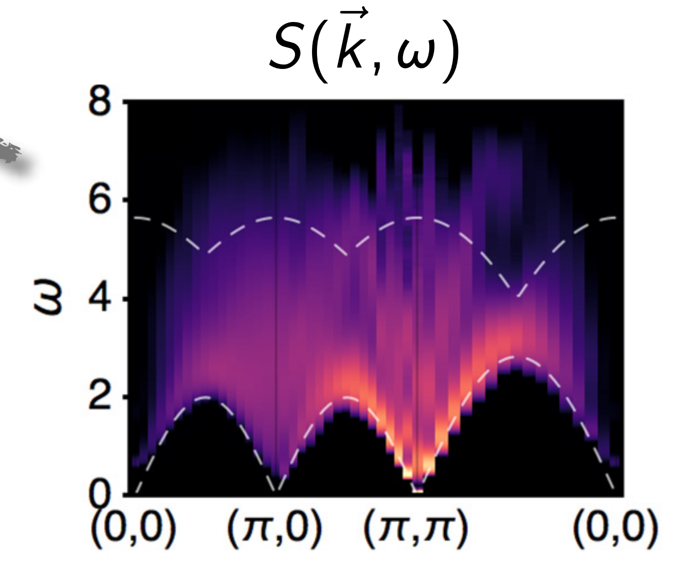




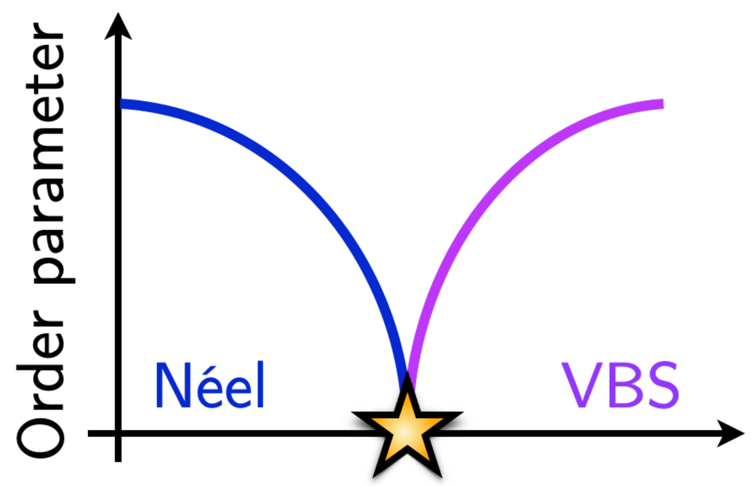
Landau

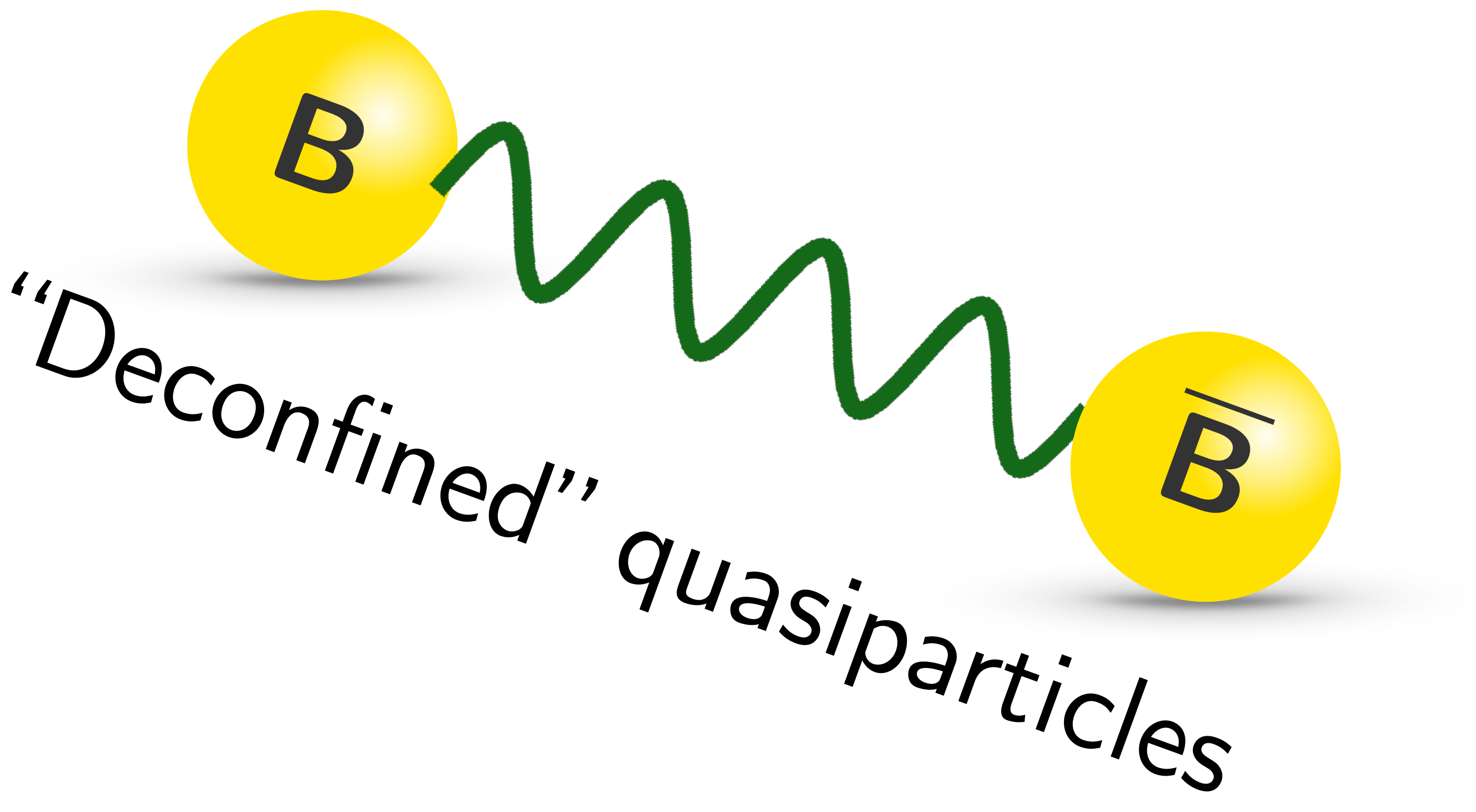
Deconfined quantum criticality





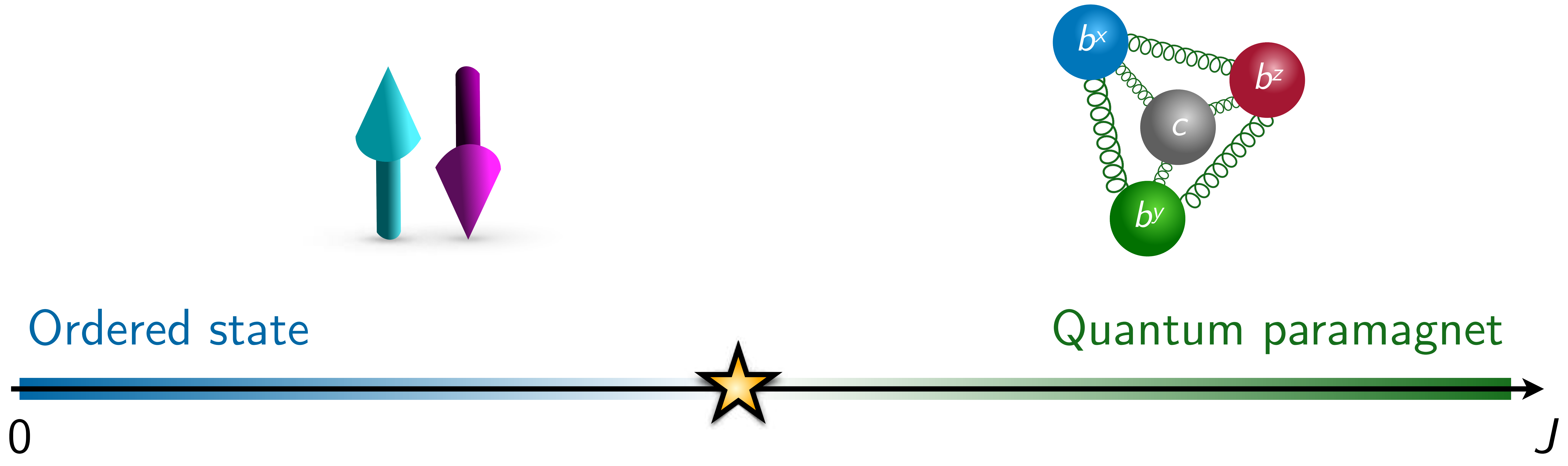
[Ma *et int.*, Meng, PRB '18]



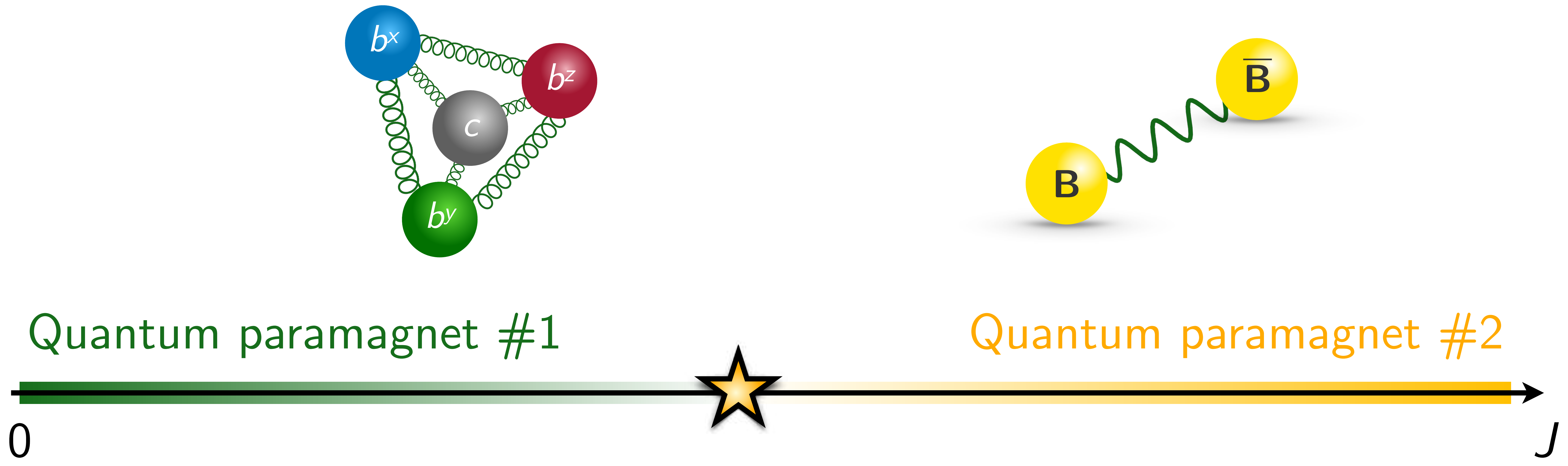


[Senthil *et al.*, Science '04]
[Pujari, Damle, Alet, PRL '13]
[Block, Melko, Kaul, PRL '13]
[Shao, Guo, Sandvik, Science '16]
...

Spin-liquid transitions

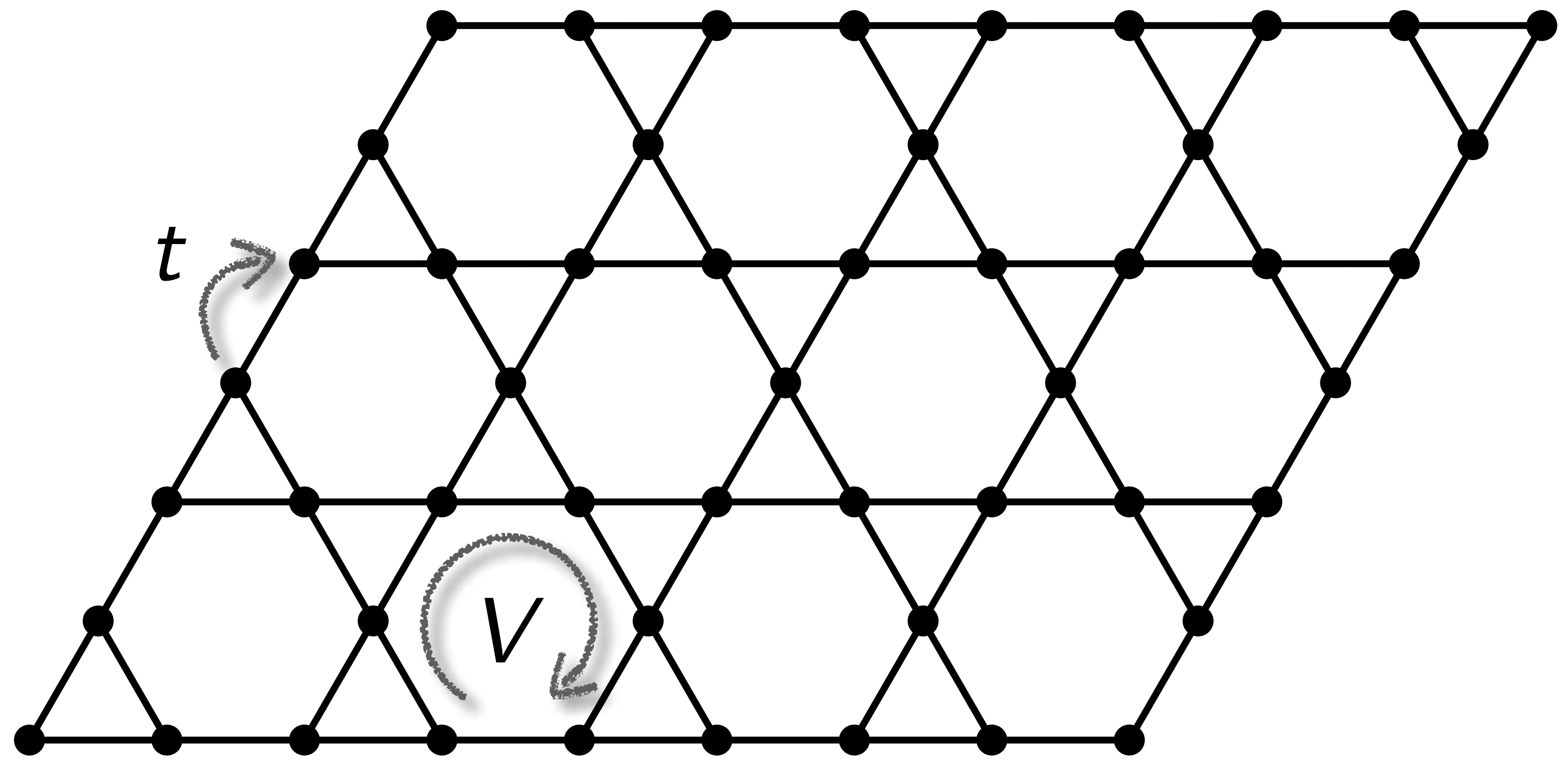


[Assaad & Grover, PRX '16]
[Xu,, Qi, Zhang, Assaad, Xu, Meng, PRX '19]
[LJ, Wang, Scherer, Meng, Xu, PRB '20]
...



[Metlitski, Mross, Sachdev, Senthil, PRB '15]
[LJ & He, PRB '17]
[Boyack, Lin, Zerf, Rayyan, Maciejko, PRB '18]
...

Example: Kagome-lattice Bose-Hubbard model



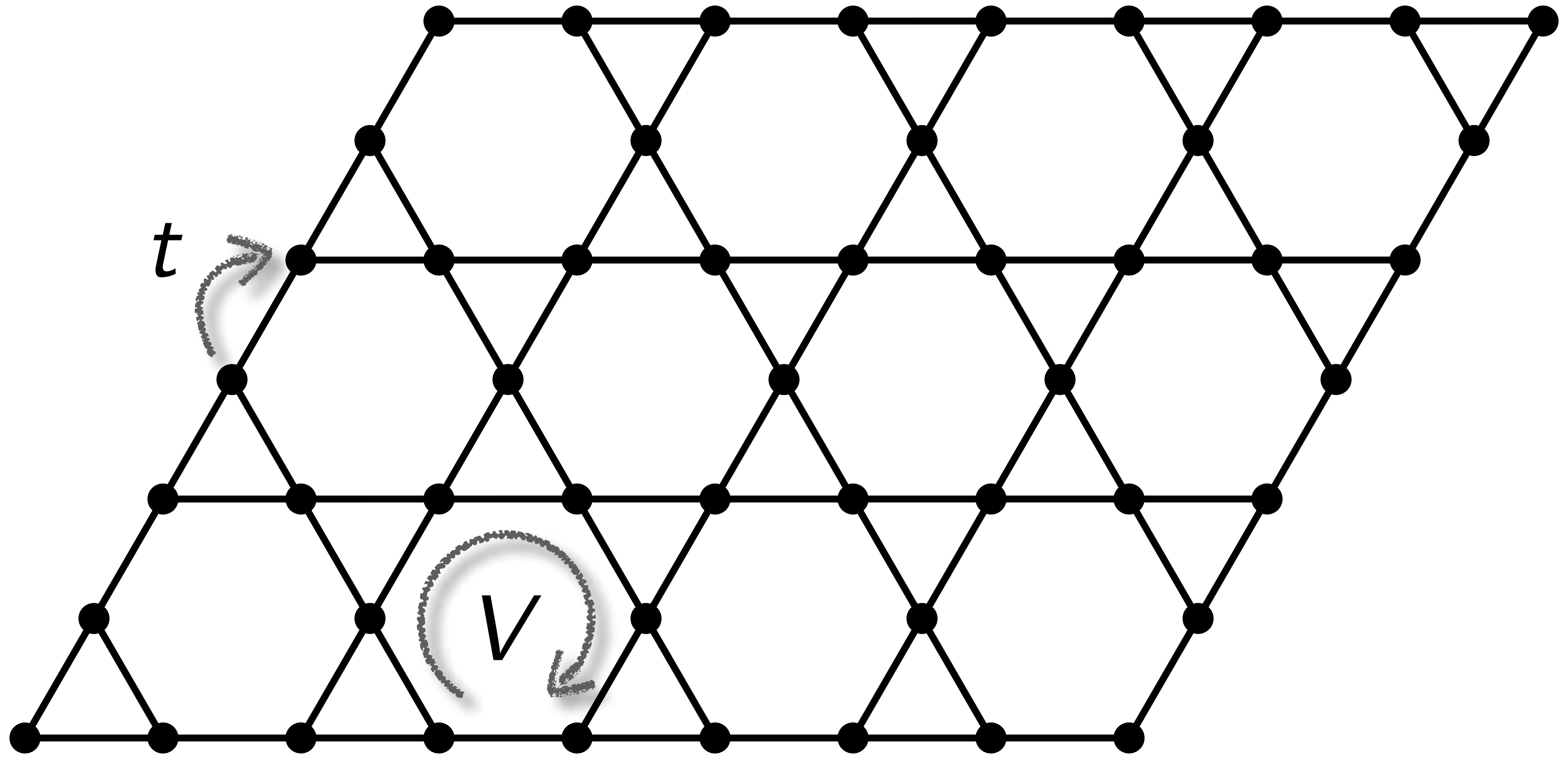
Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\hexagon} (n_{\hexagon})^2$$

... b_i hard-core bosons

[Isakov, Hastings, Melko, Nat. Phys. '11]

Example: Kagome-lattice Bose-Hubbard model

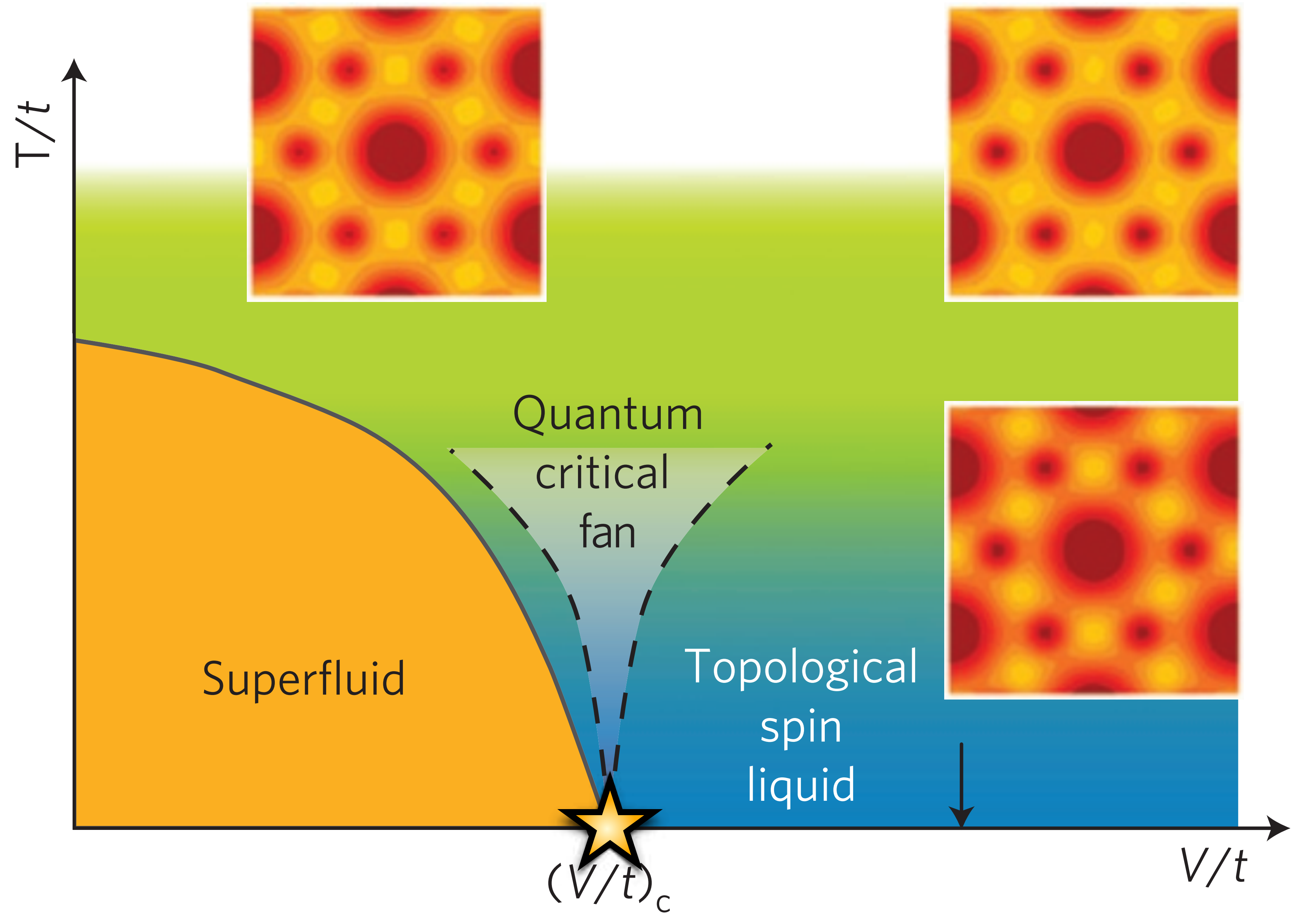


Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\hexagon} (n_{\hexagon})^2$$

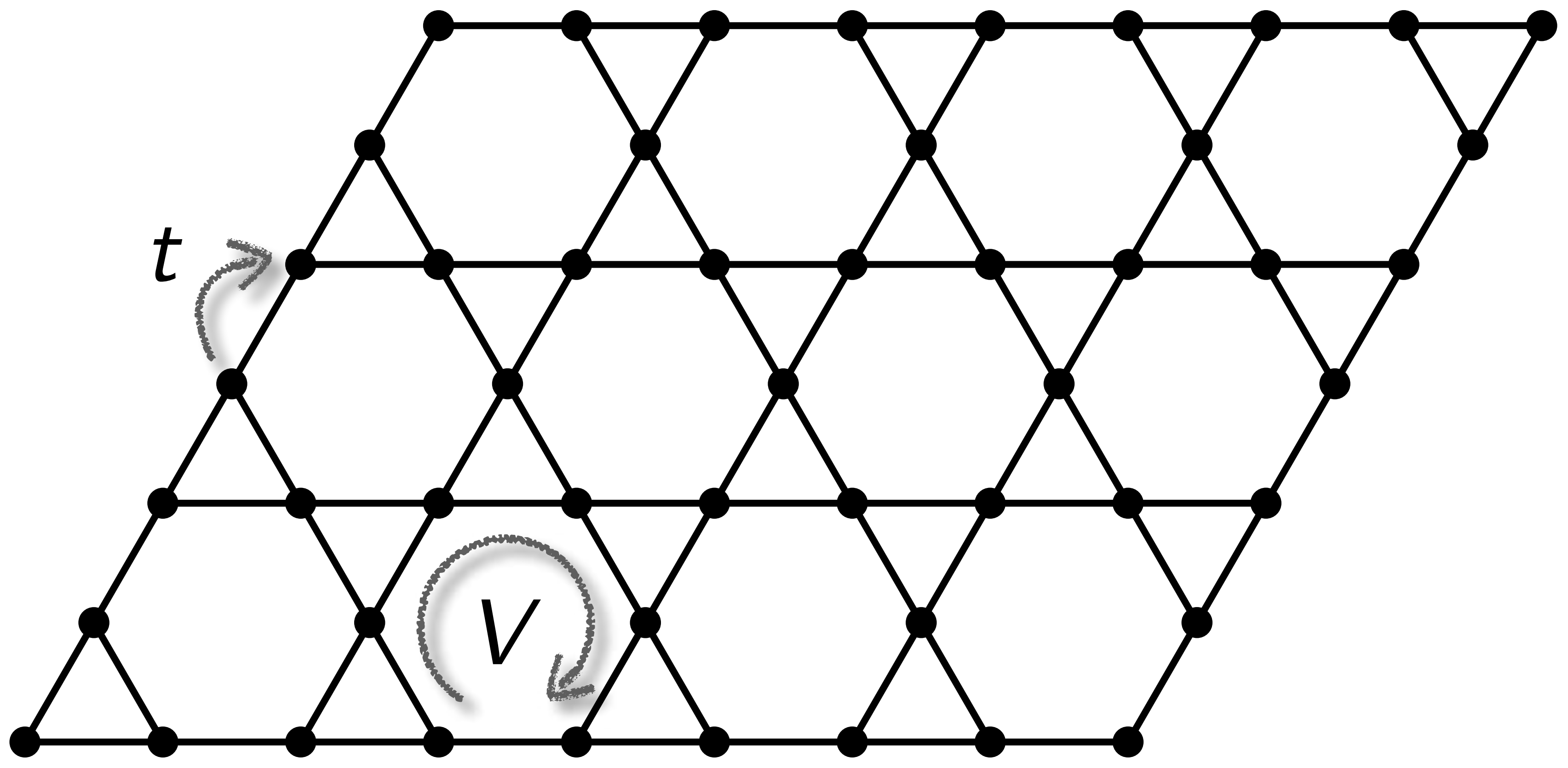
... b_i hard-core bosons

Phase diagram:



[Isakov, Hastings, Melko, Nat. Phys. '11]

Example: Kagome-lattice Bose-Hubbard model

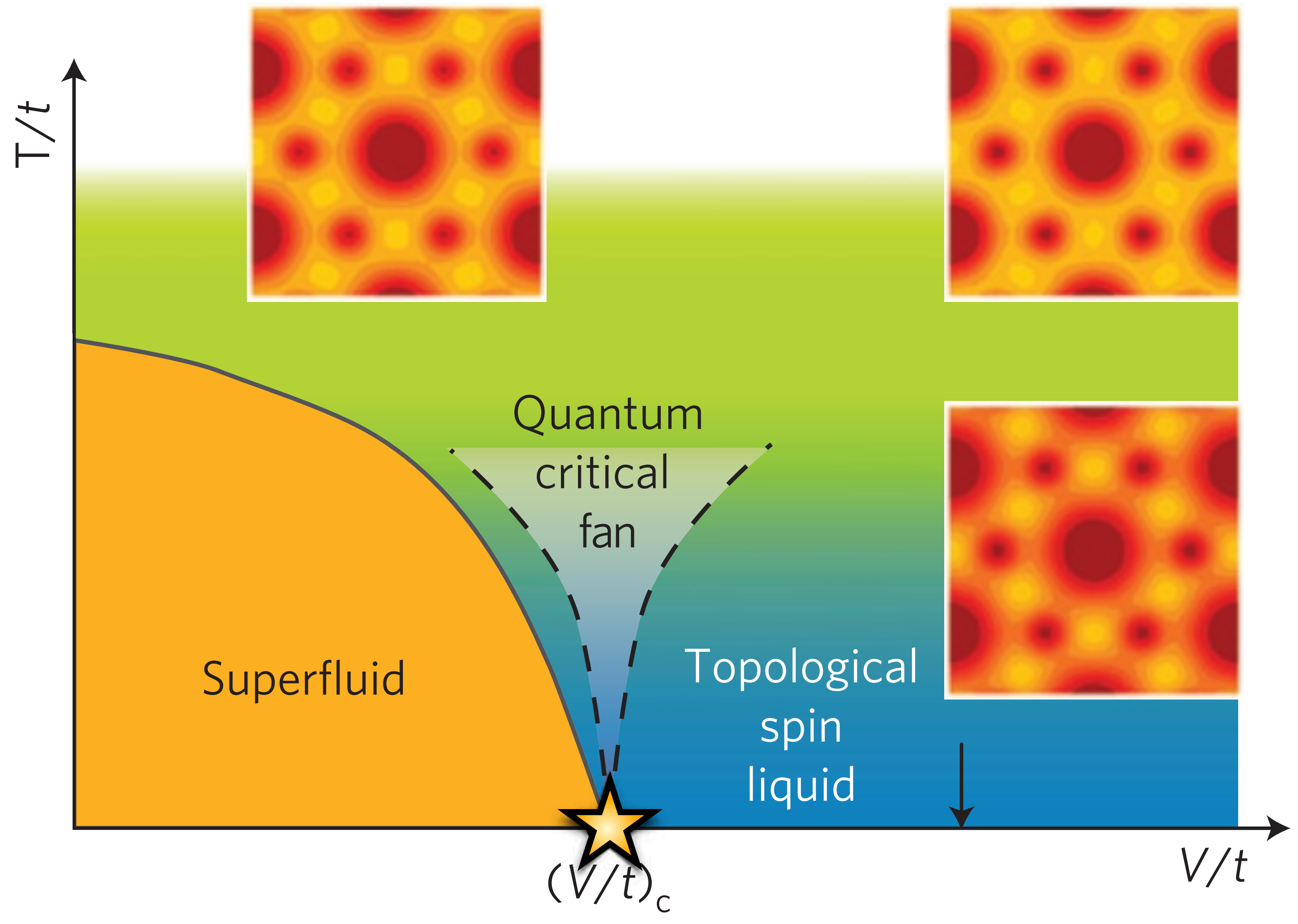


Hamiltonian:

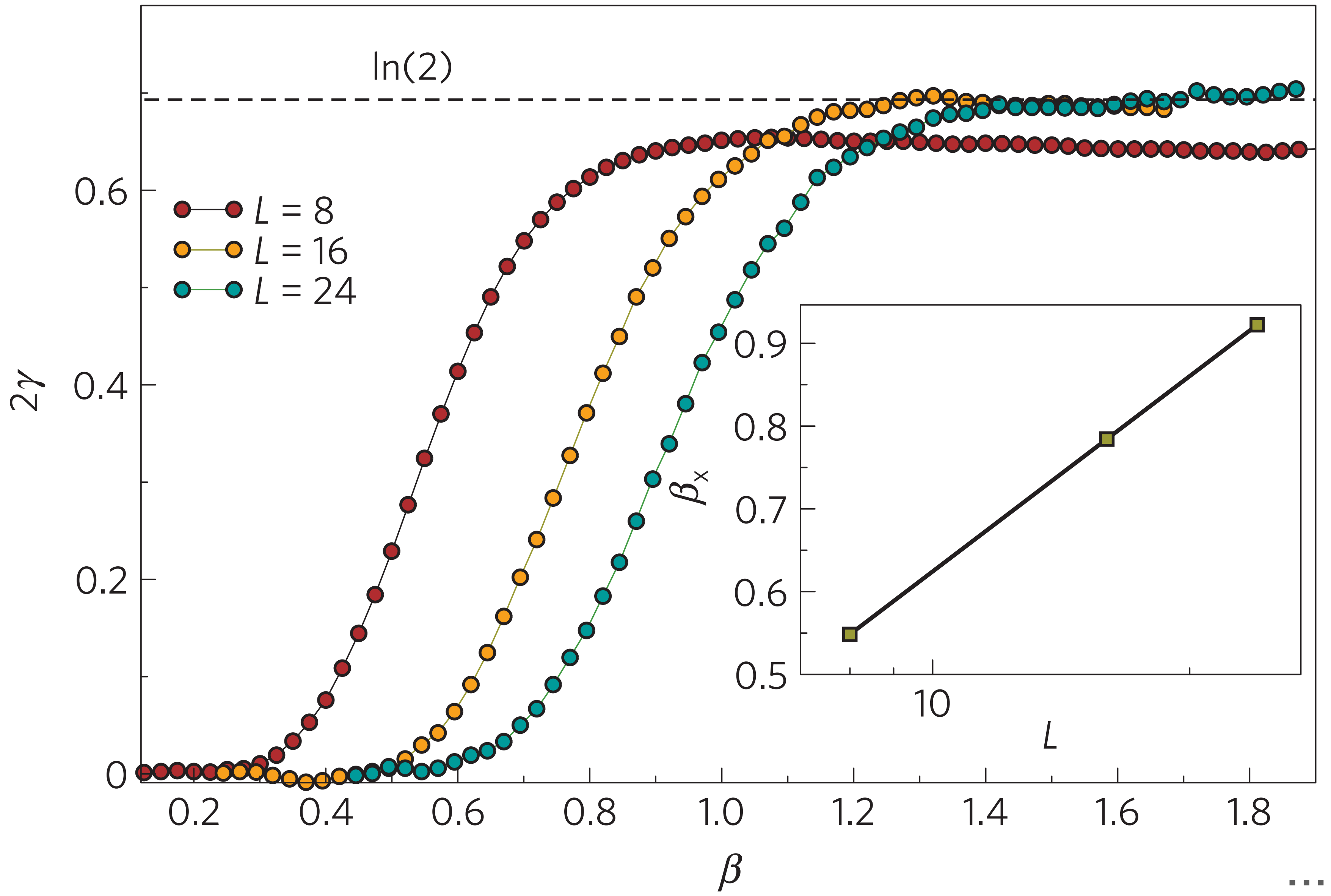
$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\hexagon} (n_{\hexagon})^2$$

... b_i hard-core bosons

Phase diagram:



Entanglement entropy: $S_n(A) = a\ell - \gamma + \ldots$

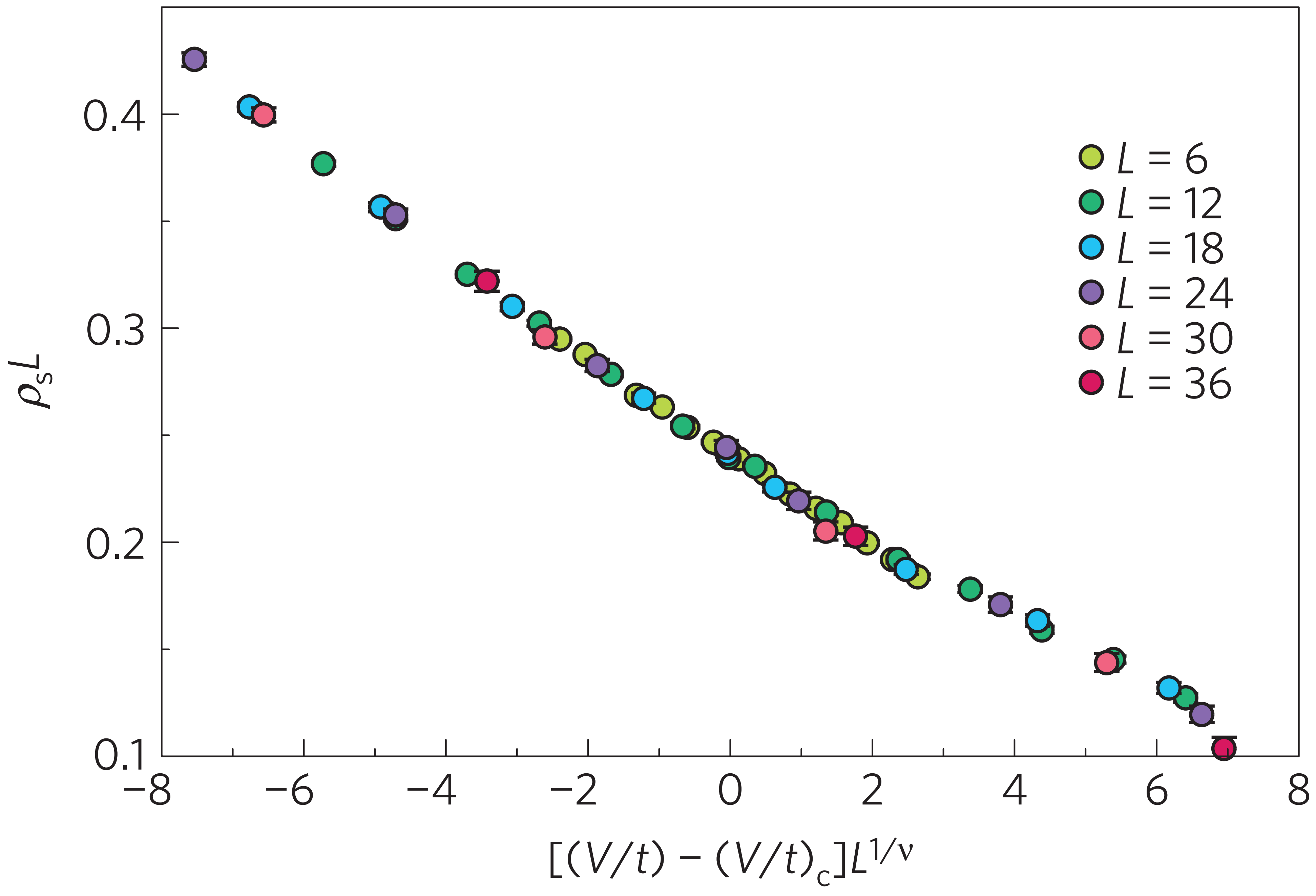


... in spin liquid phase

[Isakov, Hastings, Melko, Nat. Phys. '11]

Quantum critical scaling: XY*

Superfluid density:

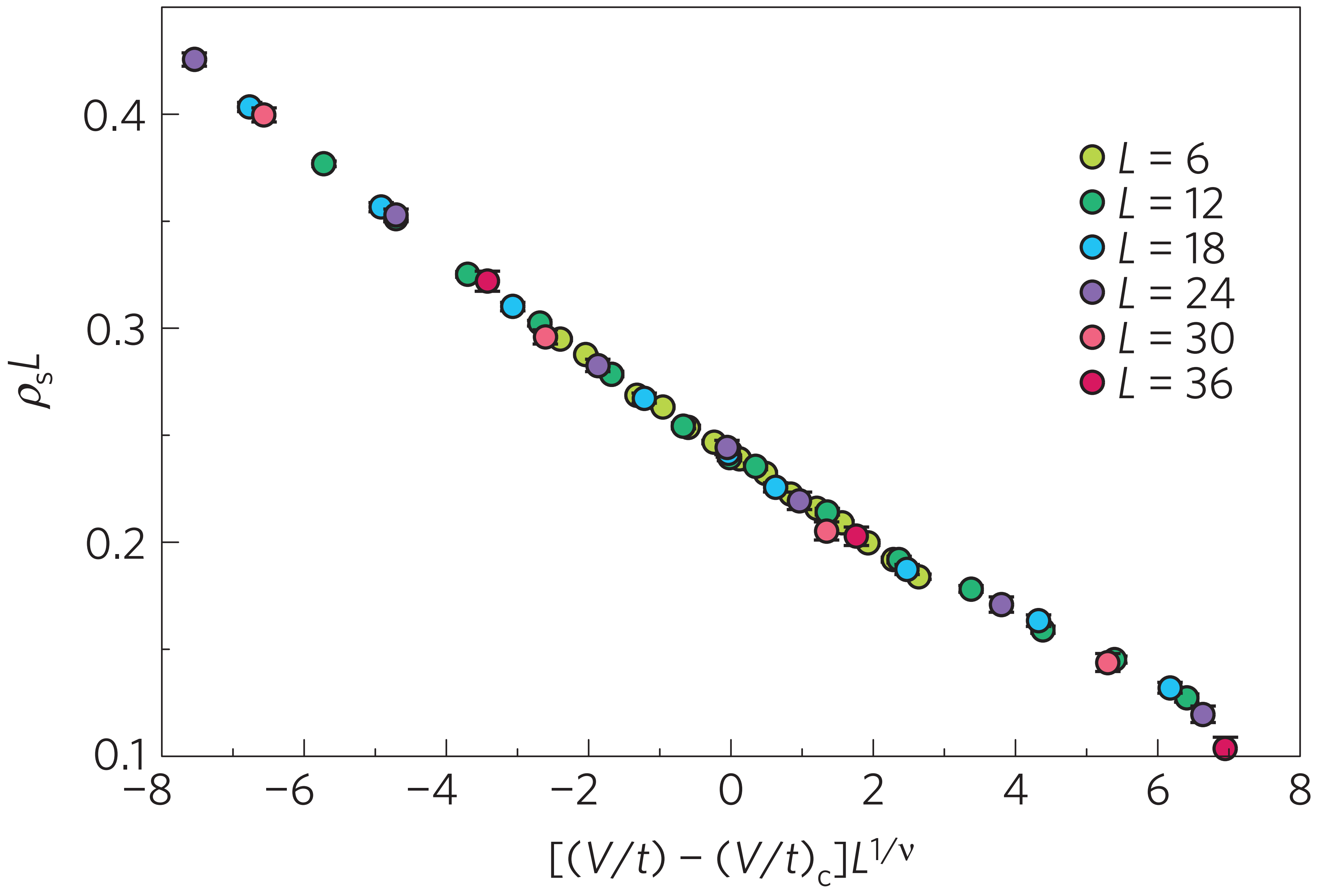


[Isakov, Hastings, Melko, Nat. Phys. ’11]

$$\nu \approx 0.67 = \nu_{XY}$$

Quantum critical scaling: XY*

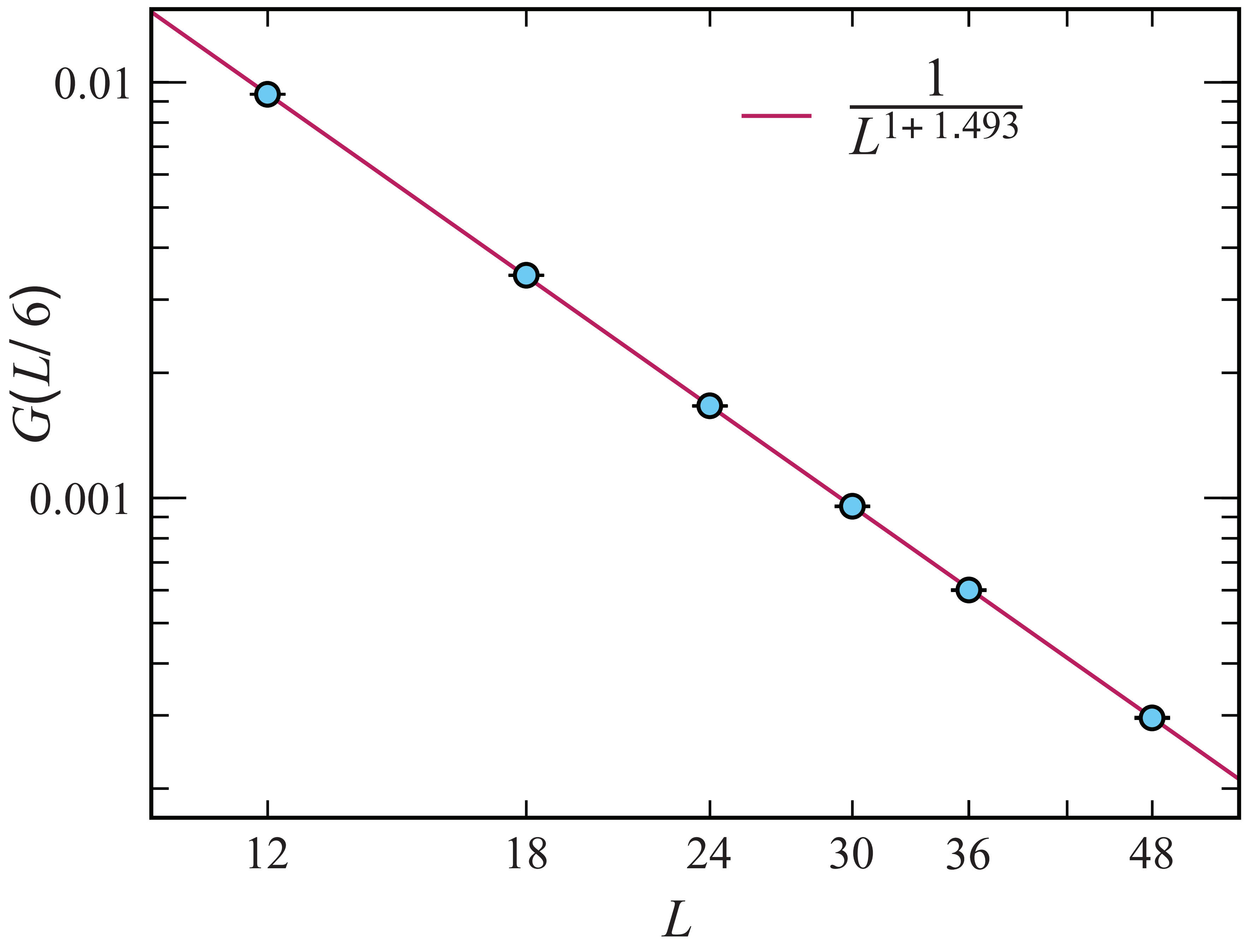
Superfluid density:



[Isakov, Hastings, Melko, Nat. Phys. '11]

$$\nu \approx 0.67 = \nu_{XY}$$

Two-point superfluid correlator:



[Isakov, Melko, Hastings, Science '12]

$$\eta \approx 1.49 \neq \eta_{XY} \approx 0.038$$

Order parameter *composite* of fractionalized particles!

... cf. $\eta_T \approx 1.47$ from field theory
[Calabrese, Pelissetto, Vicari, PRE '02]

Finite-size spectroscopy: Ising vs Ising*

Transverse-field Ising:

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Transverse-field toric code:

$$\mathcal{H} = -J \sum_s \prod_{i \in s} \sigma_i^x - J \sum_p \prod_{i \in p} \sigma_i^z - h \sum_i \sigma_i^x$$

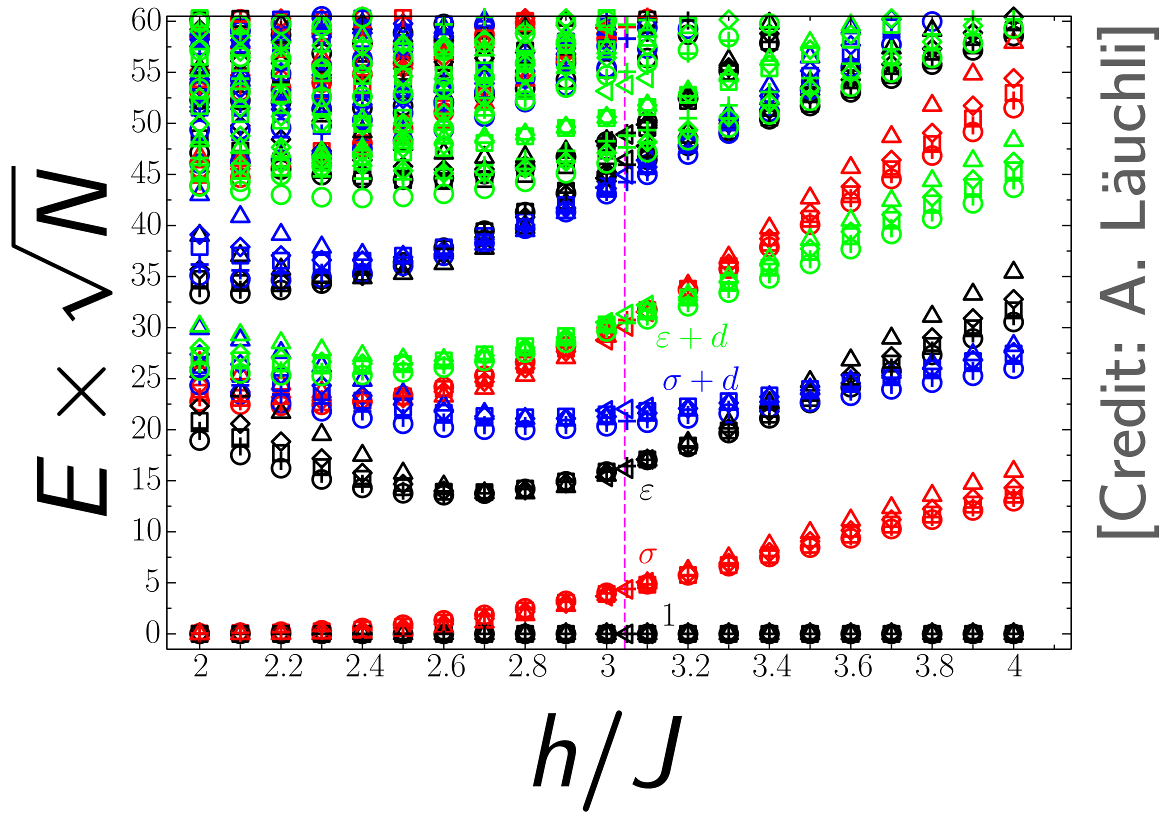
Finite-size spectroscopy: Ising vs Ising*

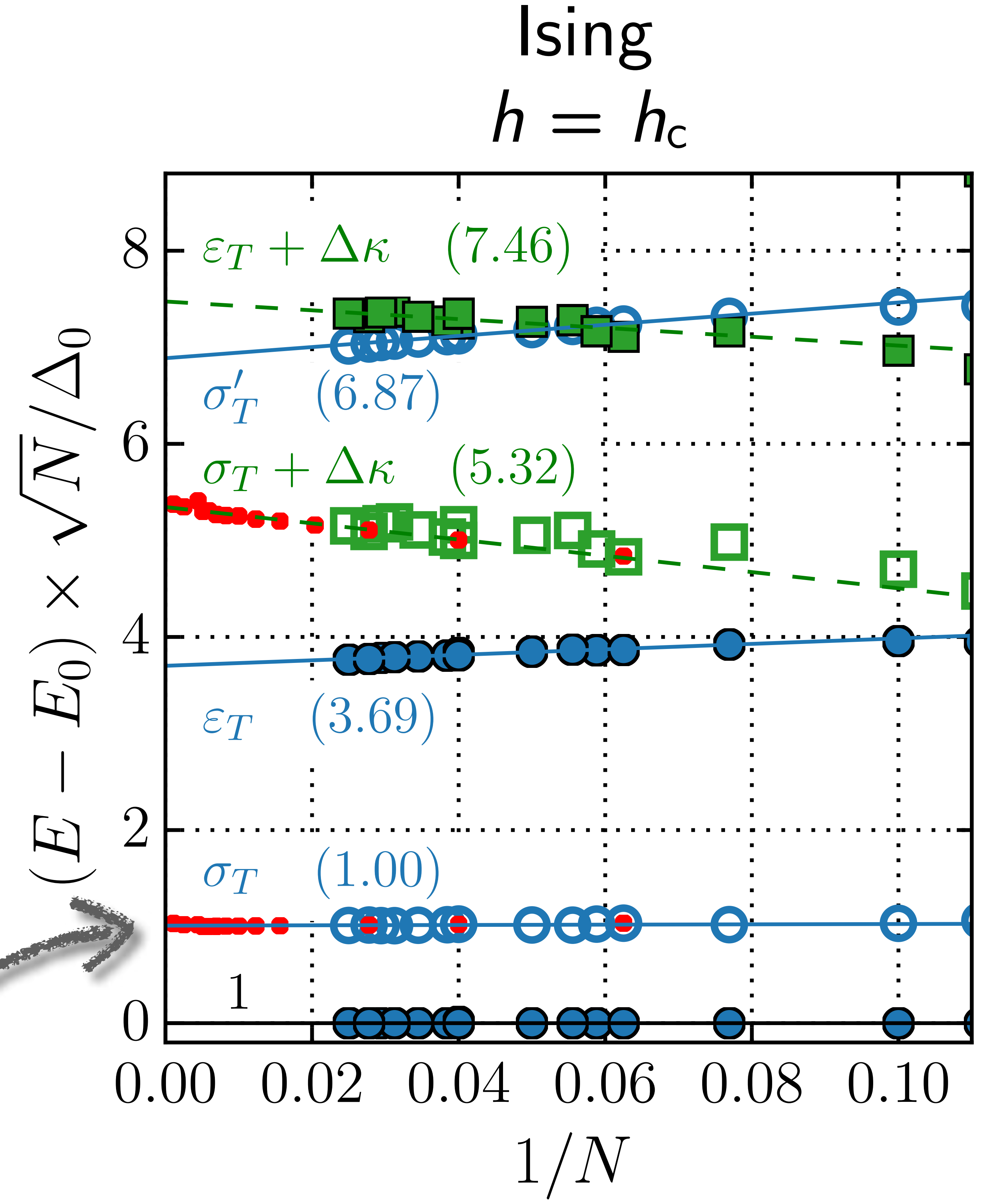
Transverse-field Ising:

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

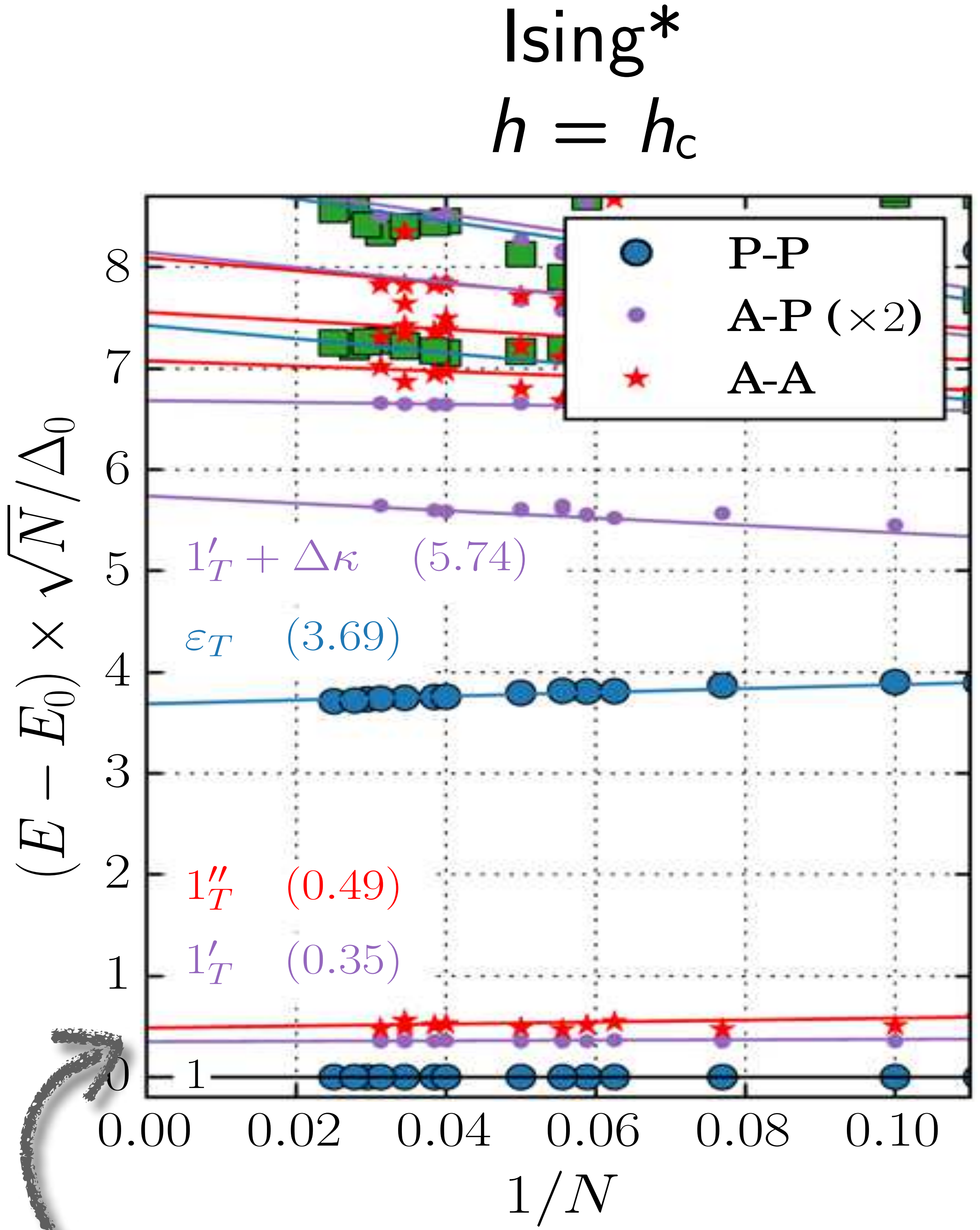
Transverse-field toric code:

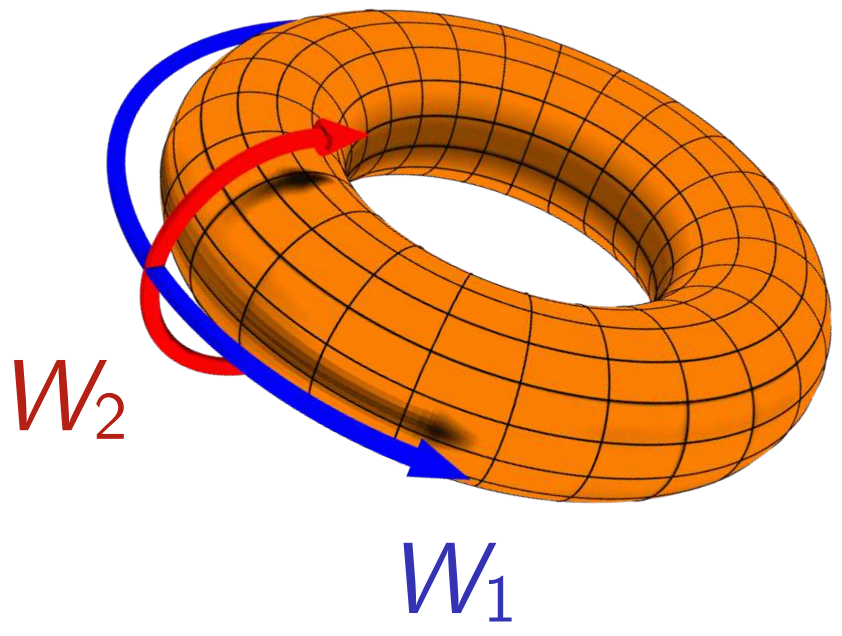
$$\mathcal{H} = -J \sum_s \prod_{i \in s} \sigma_i^x - J \sum_p \prod_{i \in p} \sigma_i^z - h \sum_i \sigma_i^x$$





missing in Ising*





topological "copies"

[Schuler, Whitsitt, Henry, Sachdev, Läuchli, PRL '16]

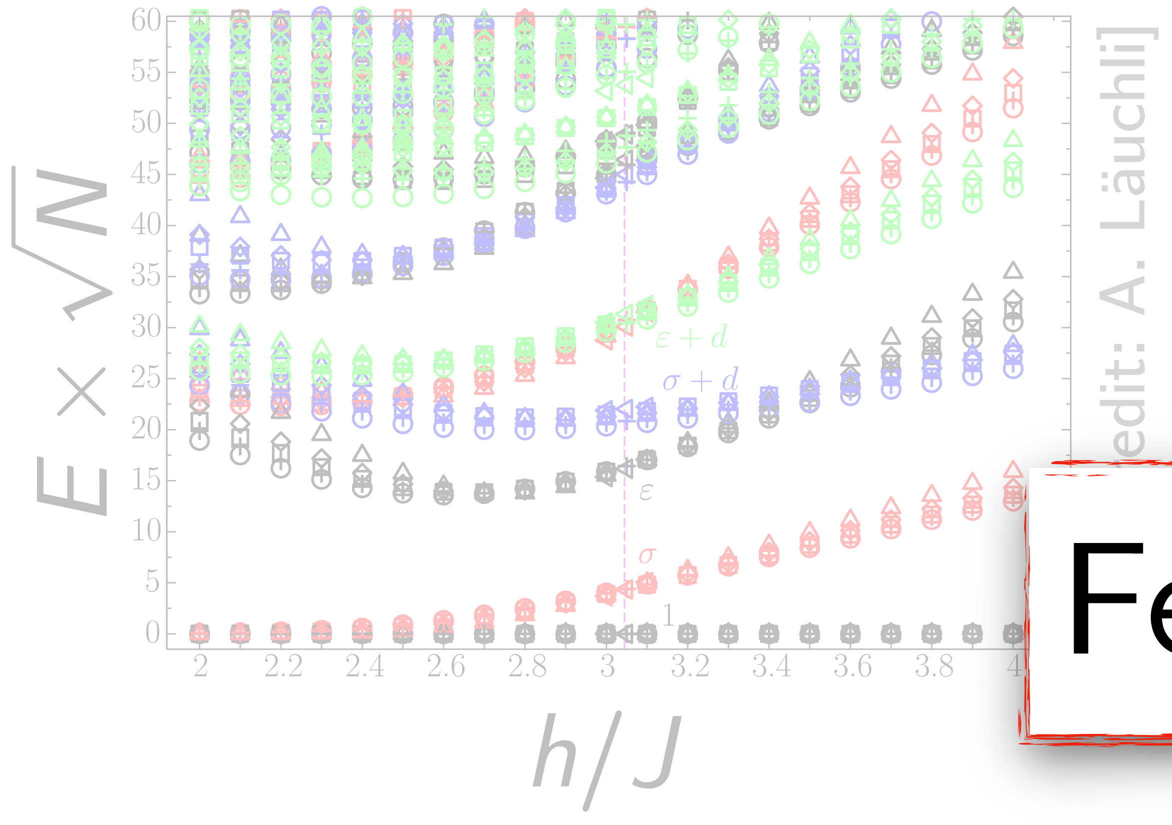
Finite-size spectroscopy: Ising vs Ising*

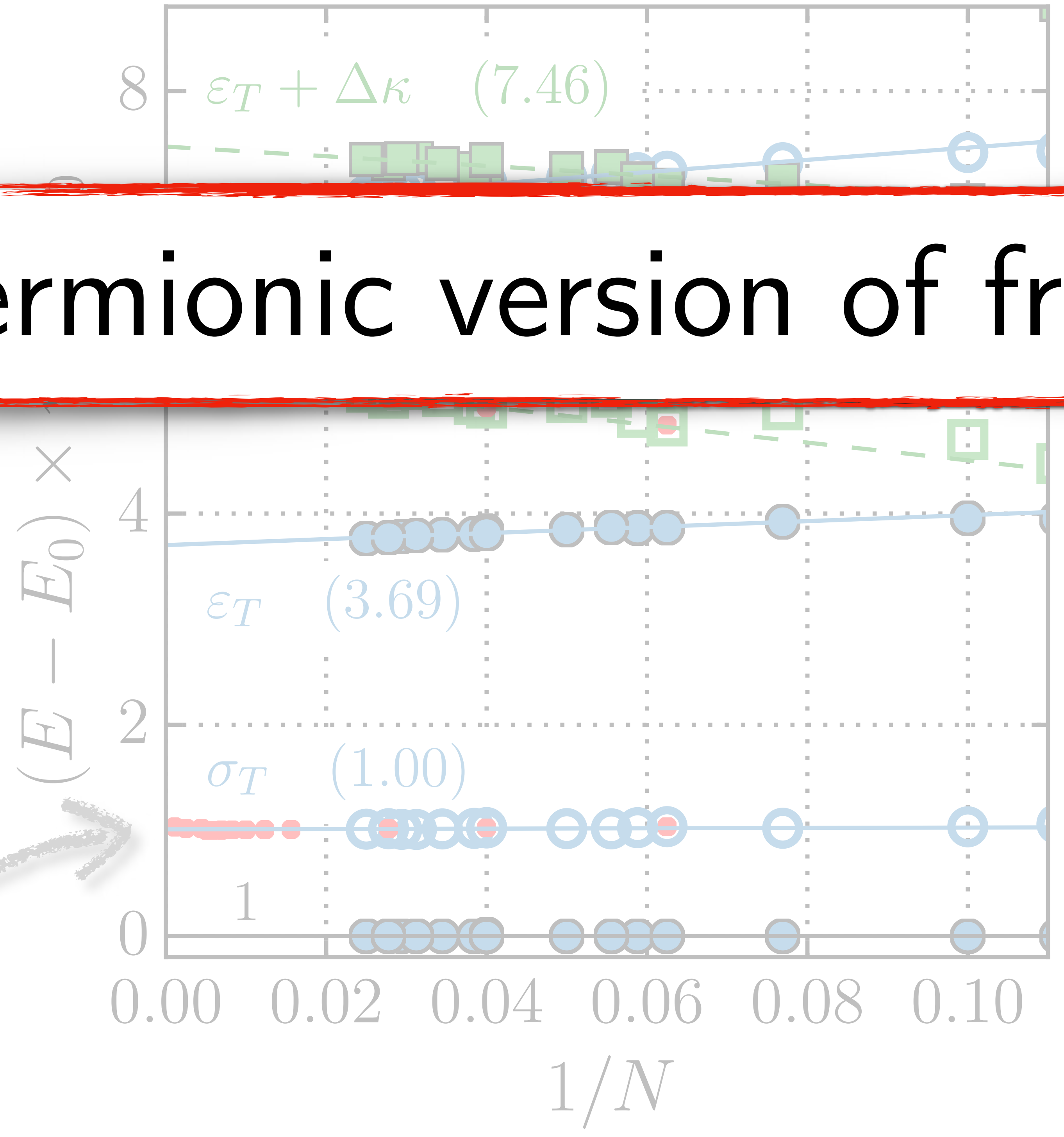
Transverse-field Ising:

$$\mathcal{H} = -J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x$$

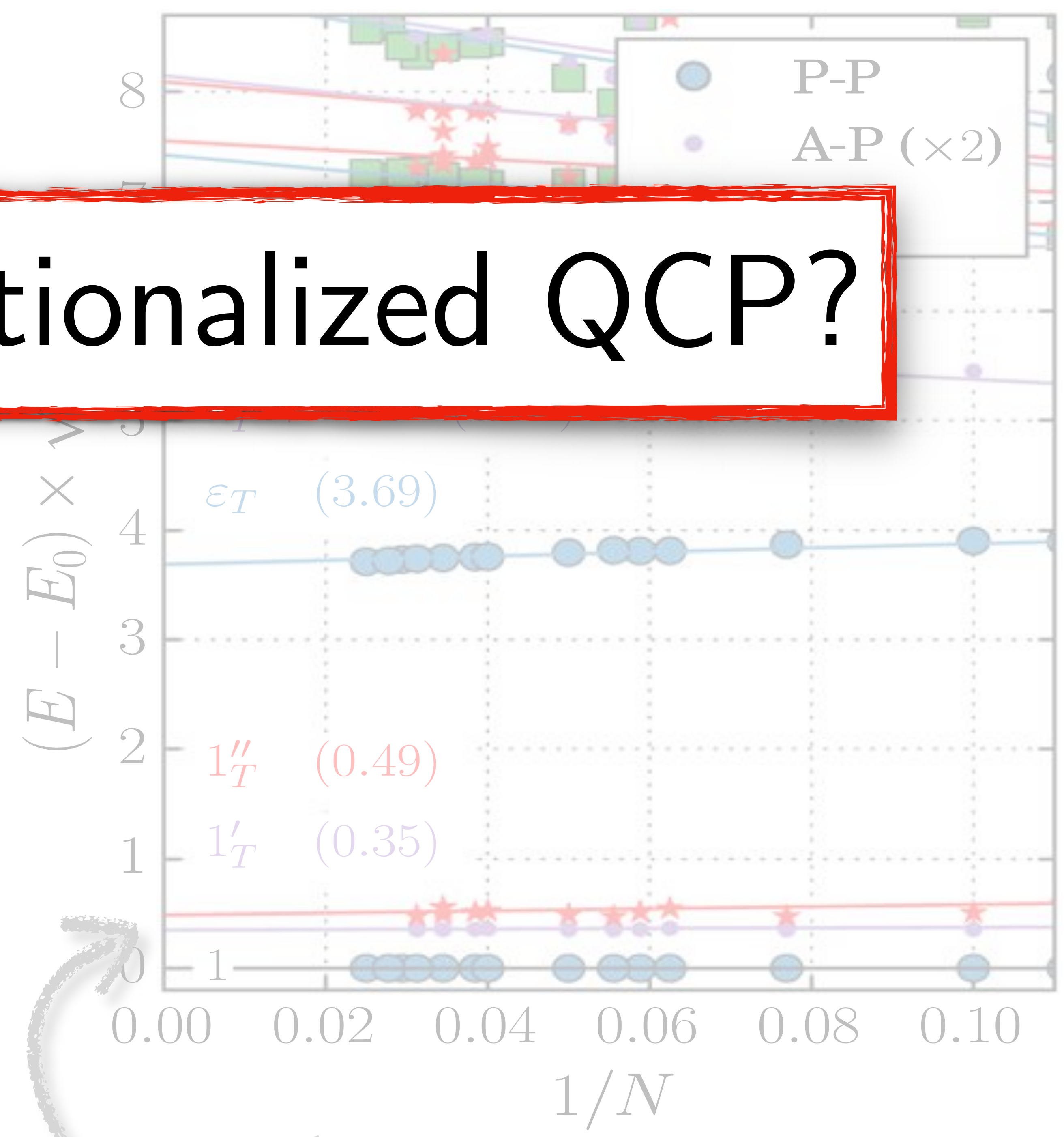
Transverse-field toric code:

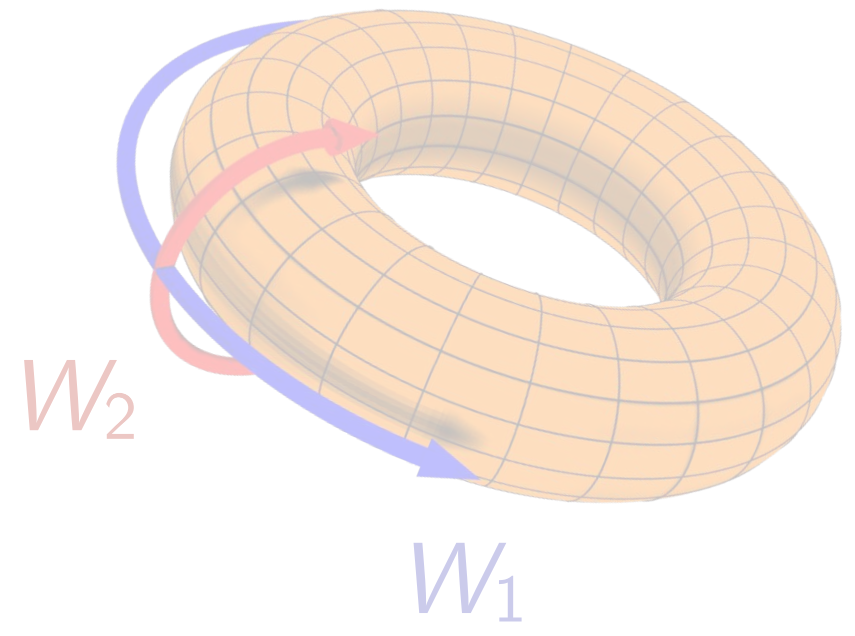
$$\mathcal{H} = -J\sum_s\prod_{i\in s}\sigma_i^x - J\sum_p\prod_{i\in p}\sigma_i^z - h\sum_i\sigma_i^x$$





Fermionic version of fractionalized QCP?





[Schuler, Whitsitt, Henry, Sachdev, Läuchli, PRL '16]

Outline

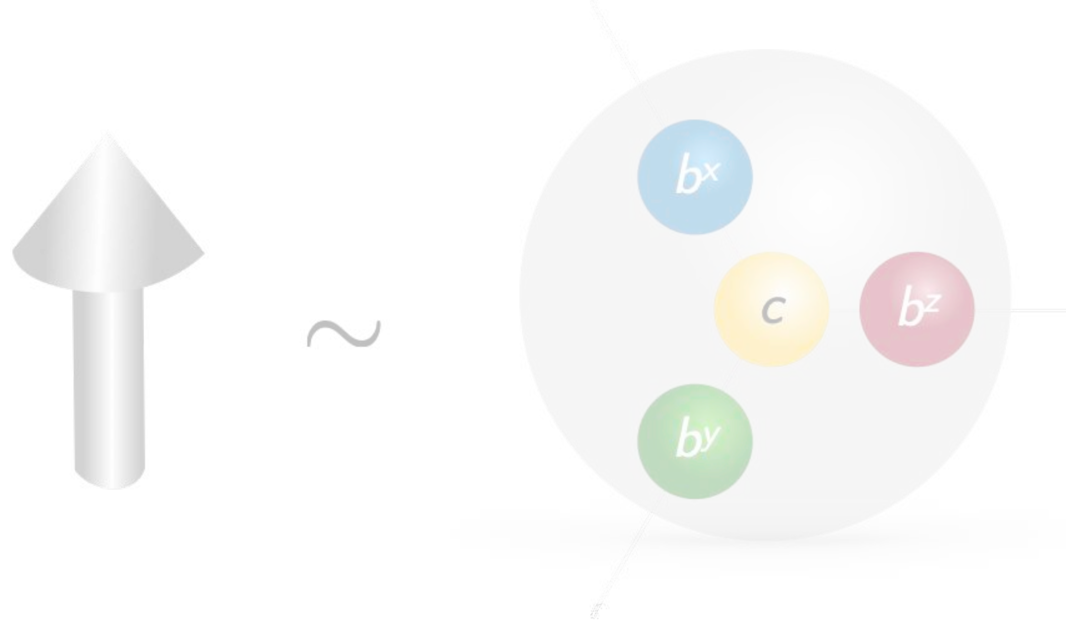
(1) Fractionalized quantum criticality

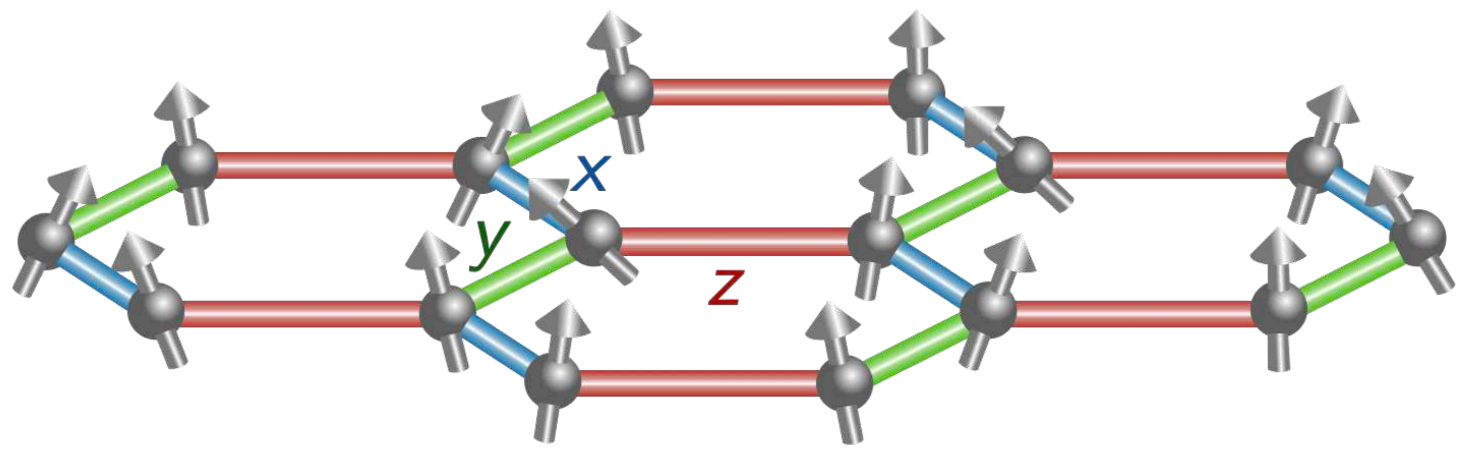
(2) Frustrated spins and spin-orbitals

(3) Square-lattice Kitaev-Ising spin-orbital model

(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model

(5) Conclusions

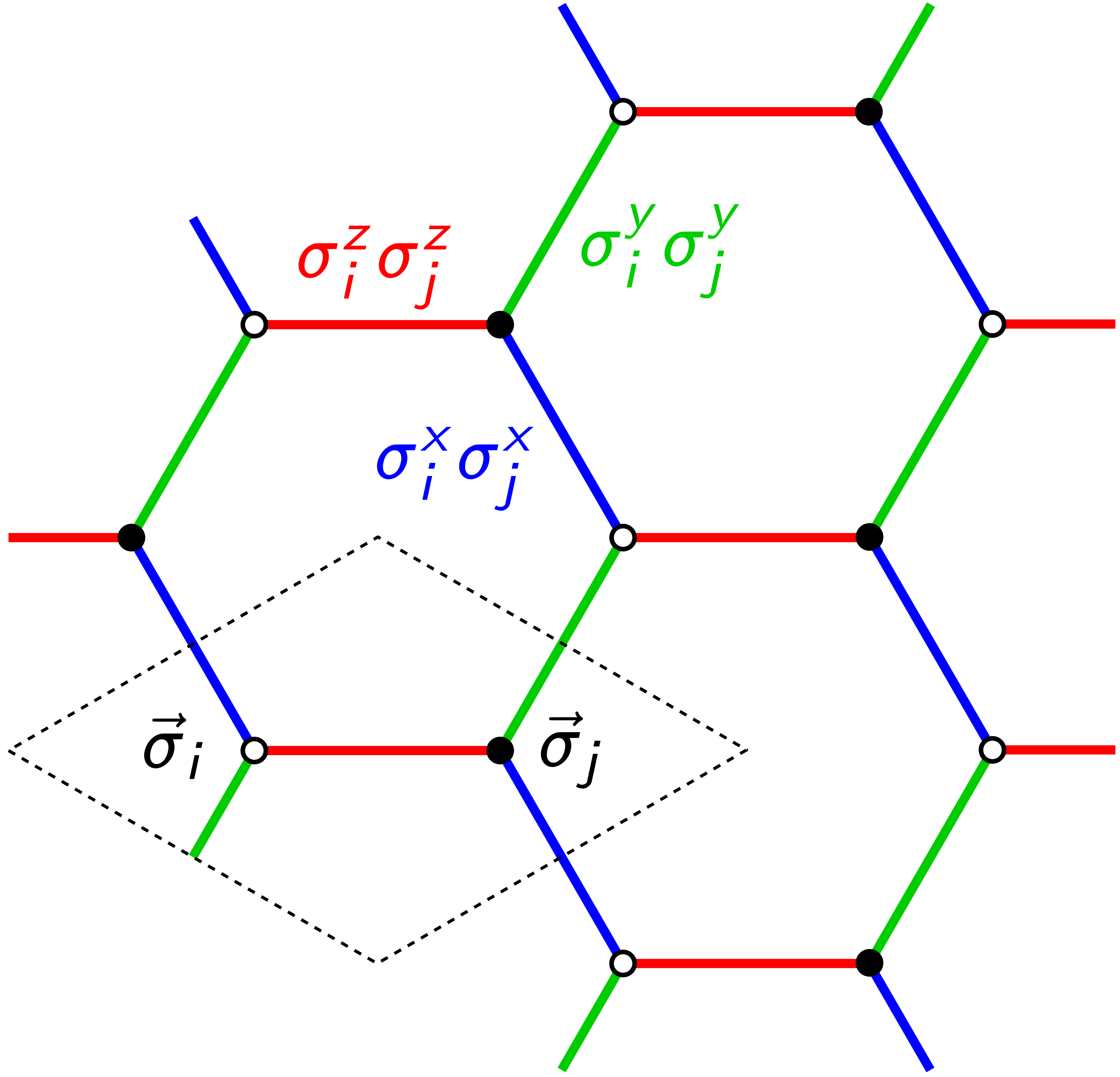




Kitaev spin-1/2 model

Hamiltonian:

$$\mathcal{H} = K \sum_{\text{blue links}} \sigma_i^x \sigma_j^x + K \sum_{\text{green links}} \sigma_i^y \sigma_j^y + K \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$

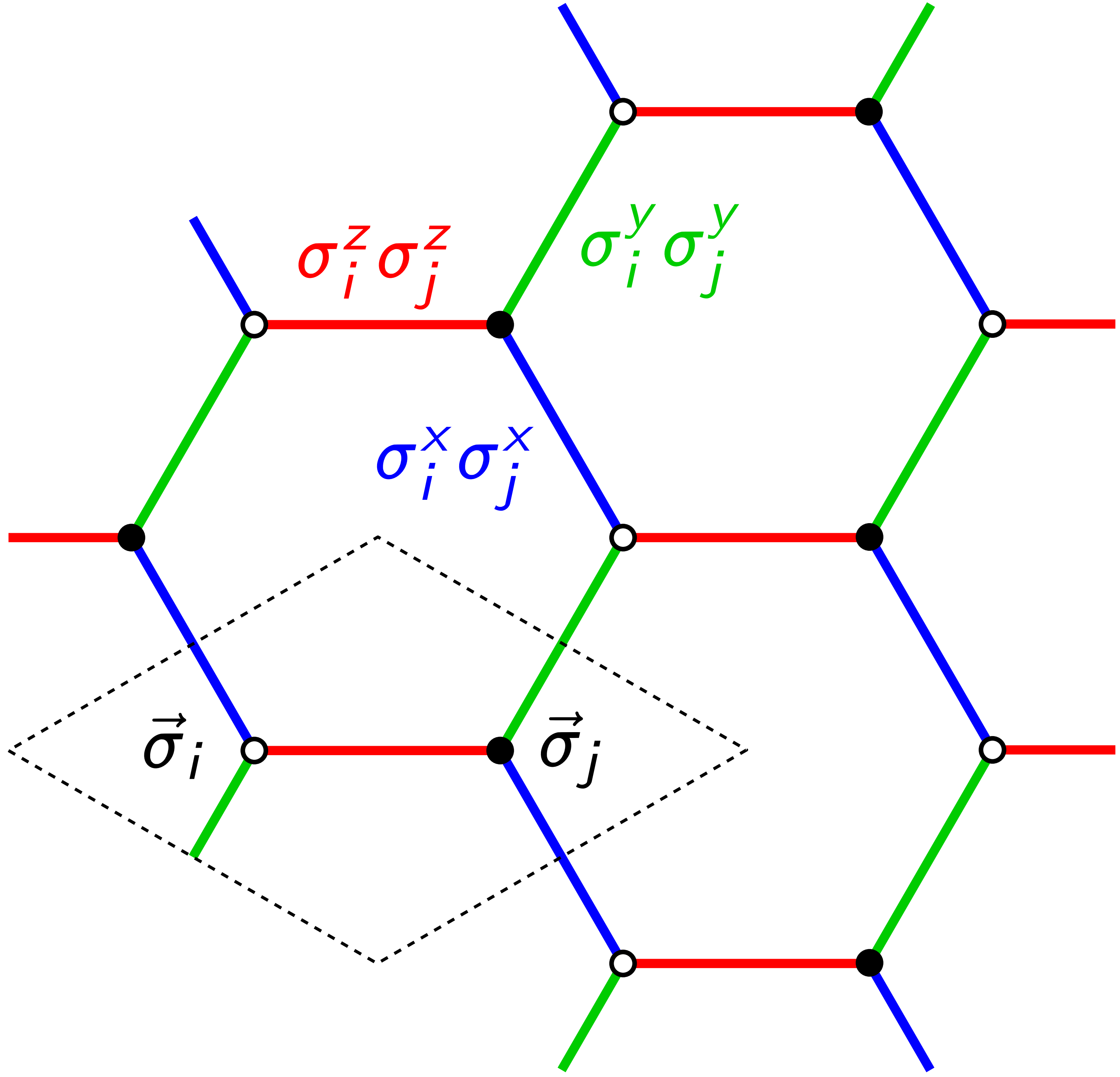


[Kitaev, Ann. Phys. '06]

Kitaev spin-1/2 model

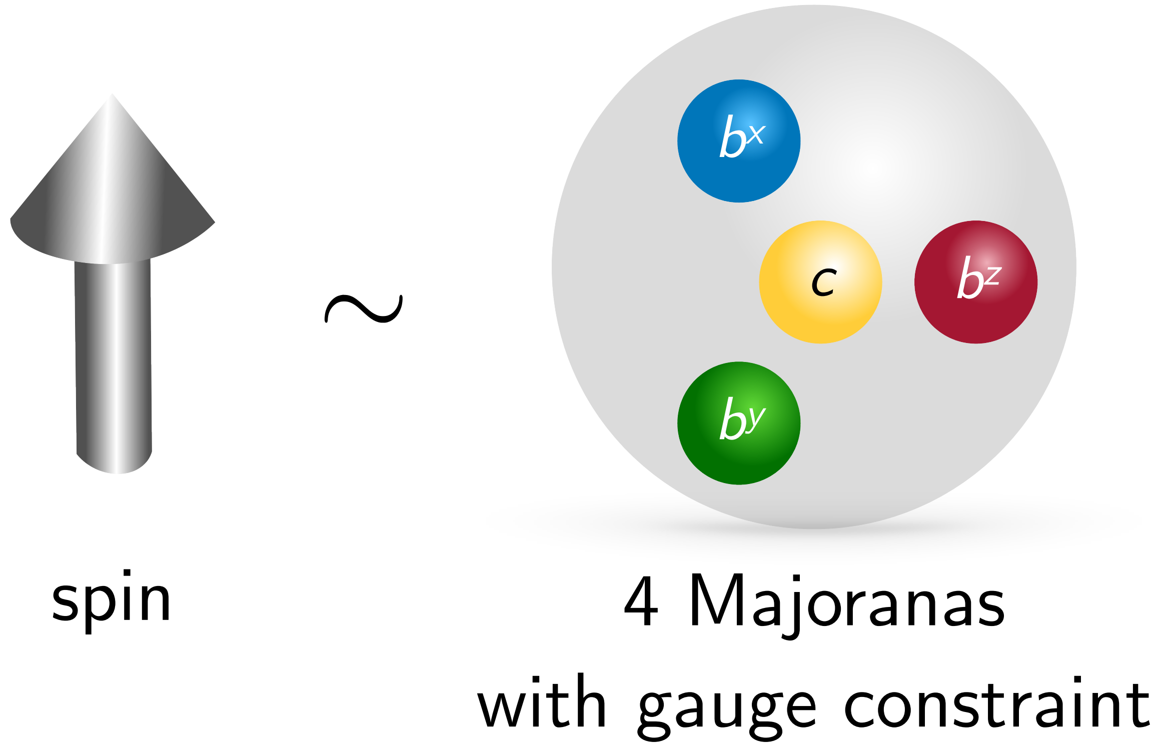
Hamiltonian:

$$\mathcal{H} = K \sum_{\text{blue links}} \sigma_i^x \sigma_j^x + K \sum_{\text{green links}} \sigma_i^y \sigma_j^y + K \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$



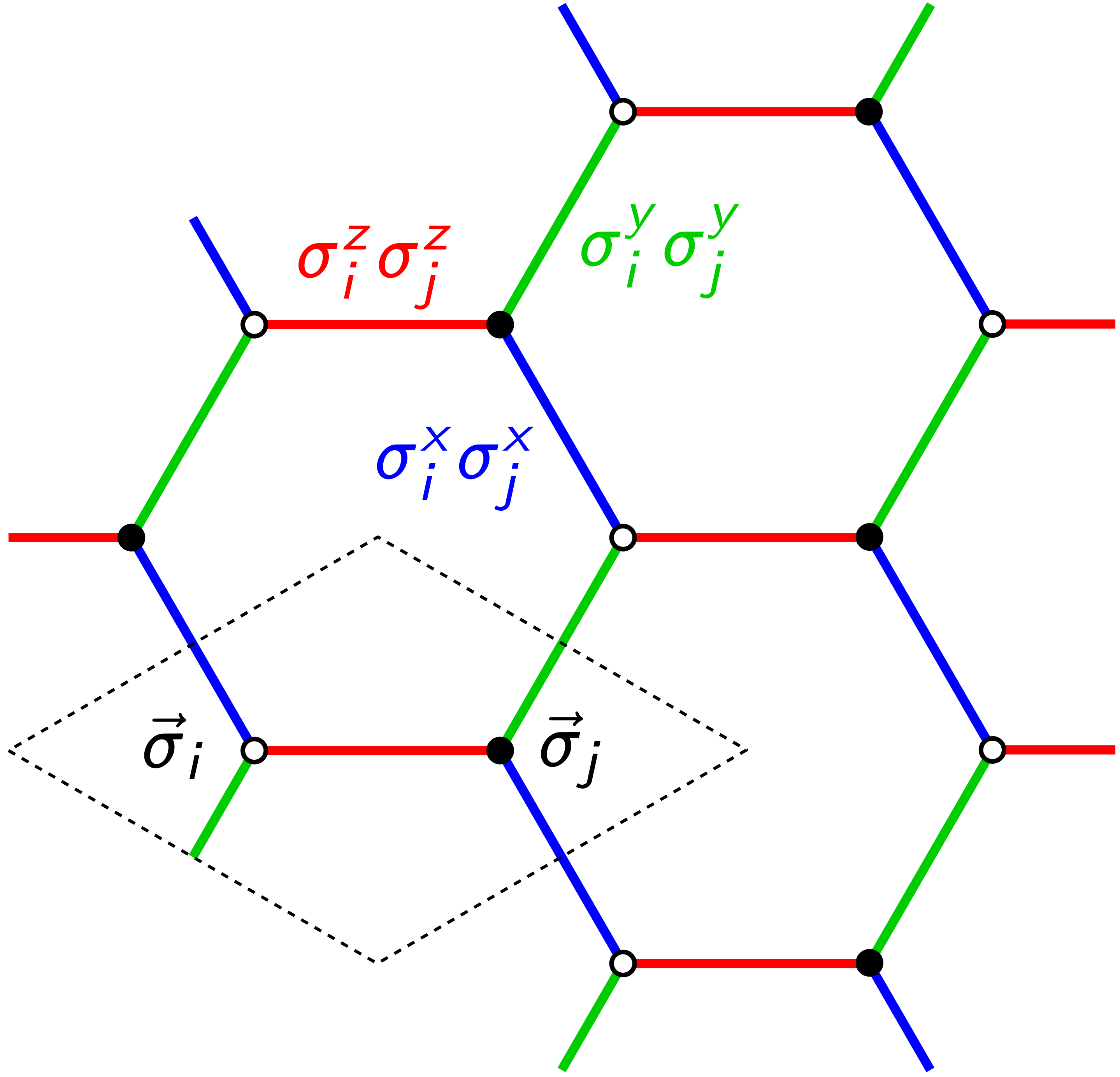
Majorana representation:

$$\sigma^x \mapsto \tilde{\sigma}^x = i b^x c$$
$$\sigma^y \mapsto \tilde{\sigma}^y = i b^y c$$
$$\sigma^z \mapsto \tilde{\sigma}^z = i b^z c$$



[Kitaev, Ann. Phys. '06]

Kitaev spin-1/2 model

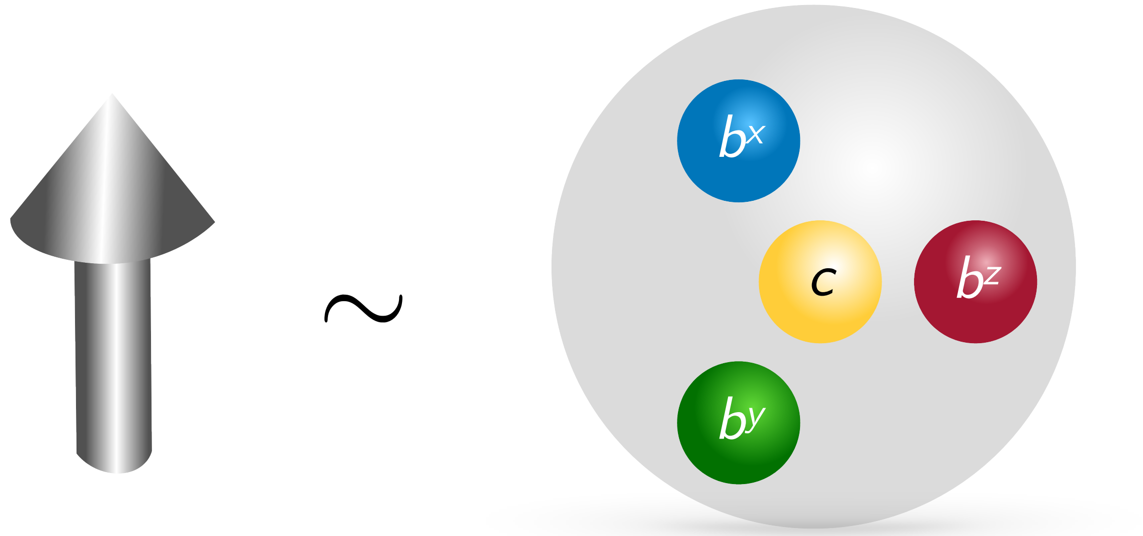


Hamiltonian:

$$\mathcal{H} = K \sum_{\text{blue links}} \sigma_i^x \sigma_j^x + K \sum_{\text{green links}} \sigma_i^y \sigma_j^y + K \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$

Majorana representation:

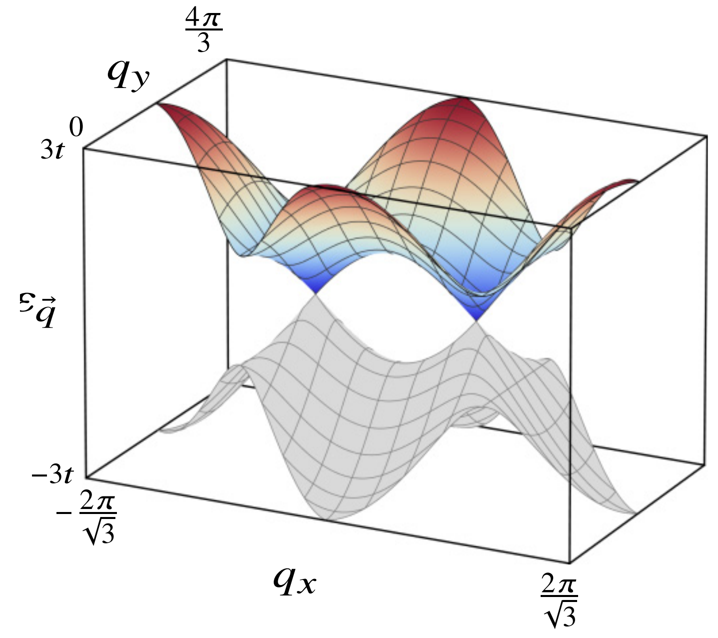
$$\sigma^x \mapsto \tilde{\sigma}^x = i b^x c$$
$$\sigma^y \mapsto \tilde{\sigma}^y = i b^y c$$
$$\sigma^z \mapsto \tilde{\sigma}^z = i b^z c$$



spin ~ 4 Majoranas with gauge constraint

Fractionalization:

$$\mathcal{H} \mapsto \tilde{\mathcal{H}} = iK \sum_{\langle ij \rangle_\gamma} \underbrace{(i b_i^\gamma b_j^\gamma)}_{\equiv \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j$$

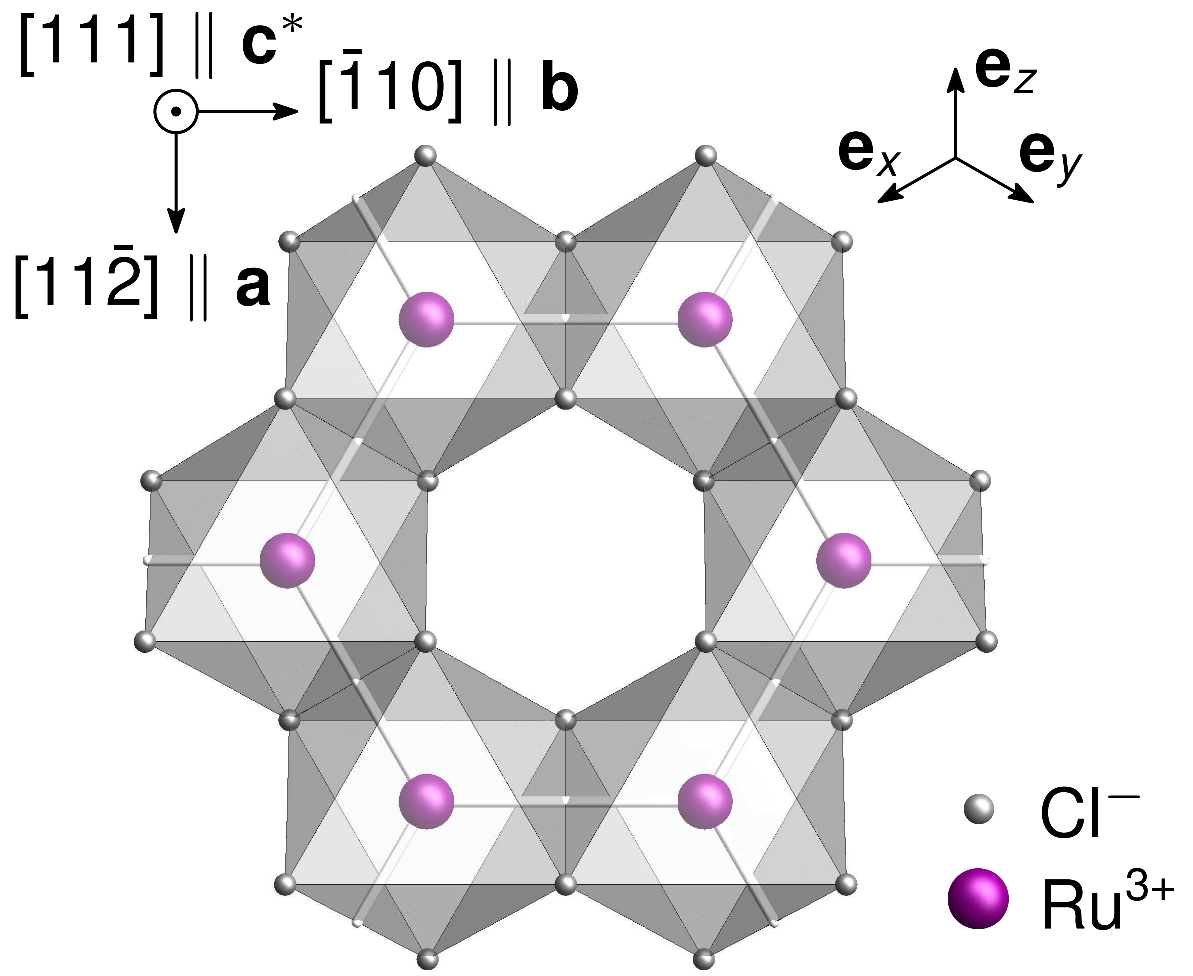


with $\left[\hat{u}_{ij}, \tilde{\mathcal{H}}\right] = 0 \quad \Rightarrow \quad$ static $\mathbb{Z}_2$ gauge field!

Ground-state flux pattern: $u \equiv 1$
[Lieb, PRL '94]

[Kitaev, Ann. Phys. '06]

Kitaev-Heisenberg spin-1/2 model

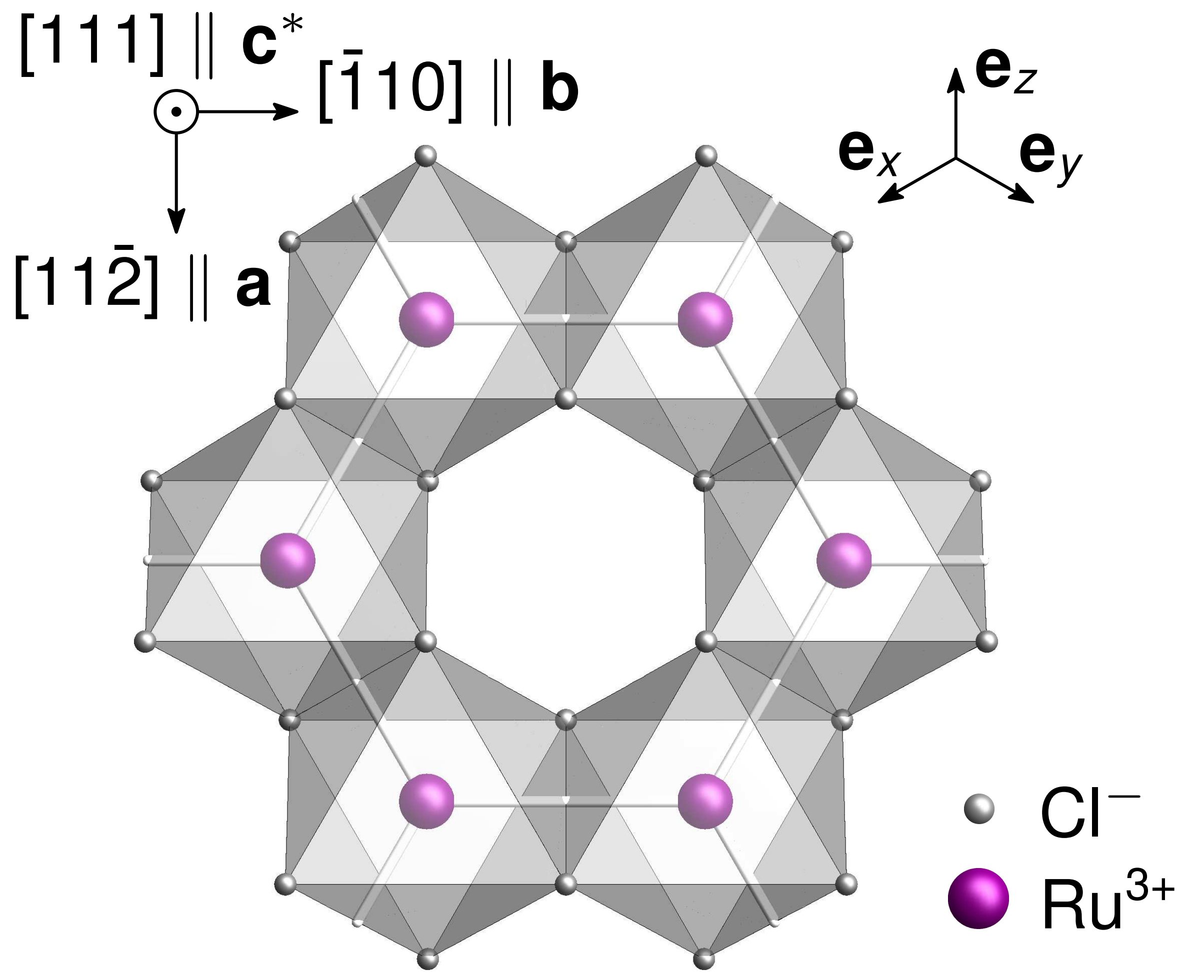


... possible relevance to α-$RuCl_3$, Na_2IrO_3, $Na_2Co_2TeO_6$, ...

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \sigma_i^\gamma \sigma_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Kitaev-Heisenberg spin-1/2 model

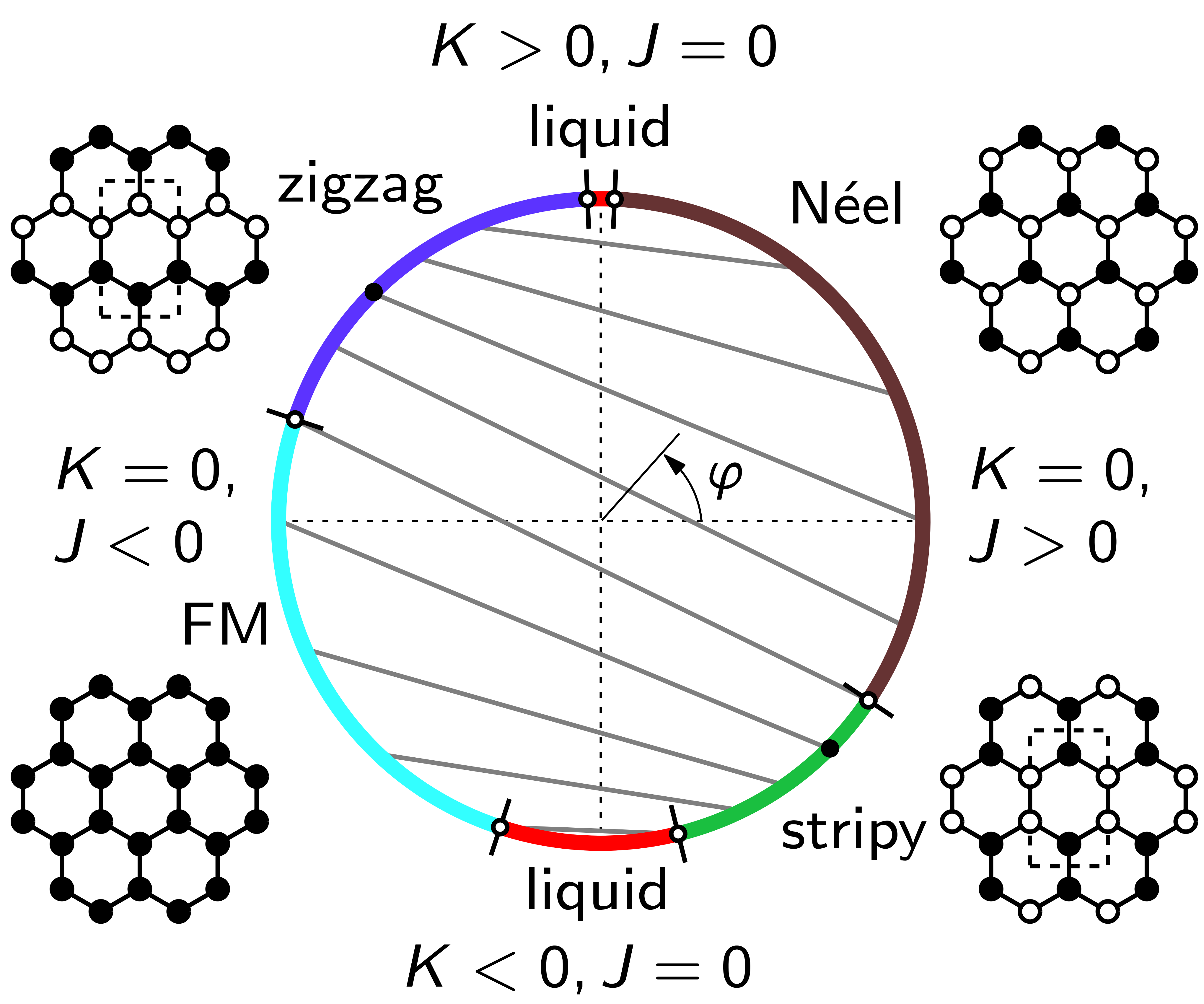


Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \sigma_i^\gamma \sigma_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

... possible relevance to α-$RuCl_3$, Na_2IrO_3, $Na_2Co_2TeO_6$, ...

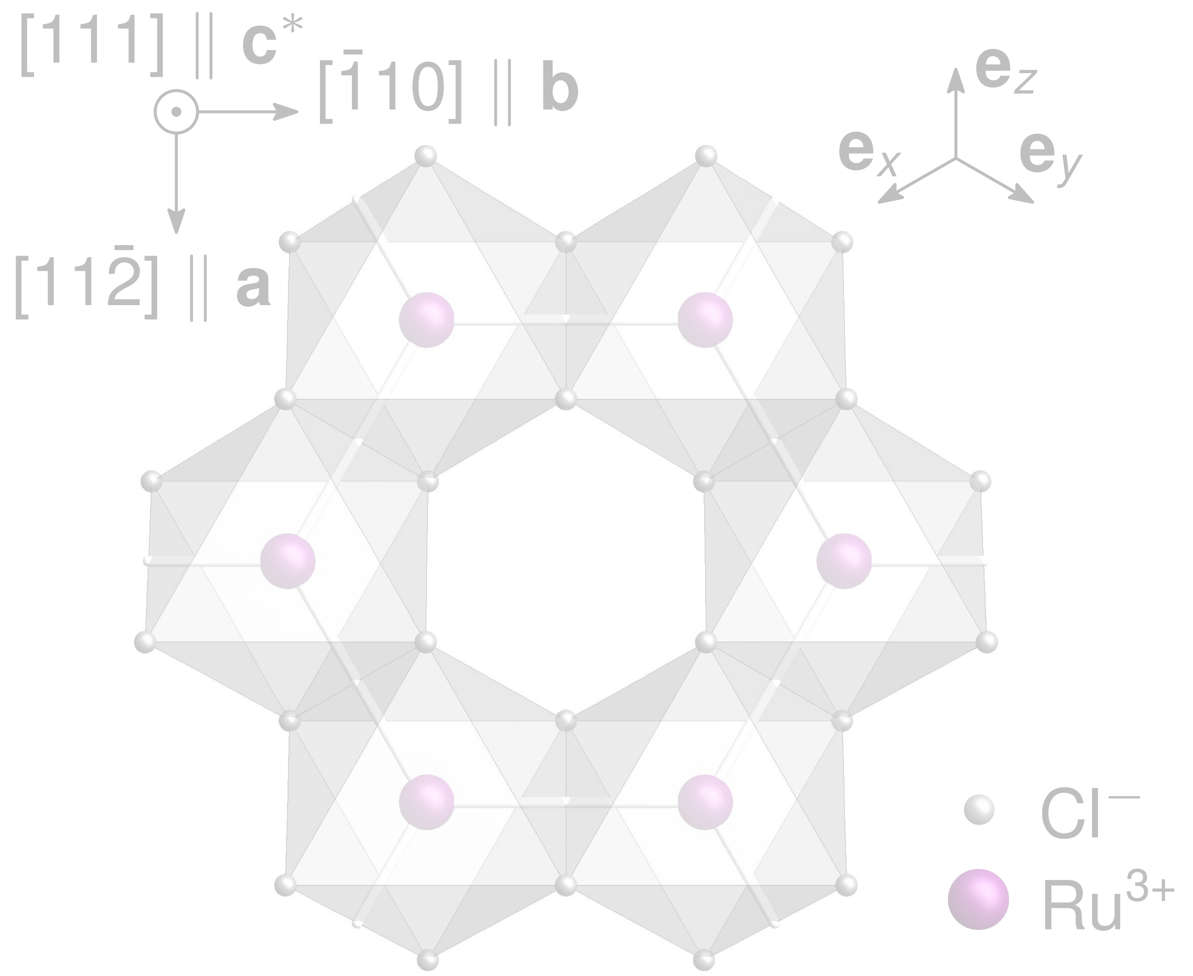
Phase diagram:



$$J = A \cos \varphi$$
$$K = 2A \sin \varphi$$

... from 24-site ED: [Chaloupka, Jackeli, Khaliullin, PRL '13]

Kitaev-Heisenberg spin-1/2 model

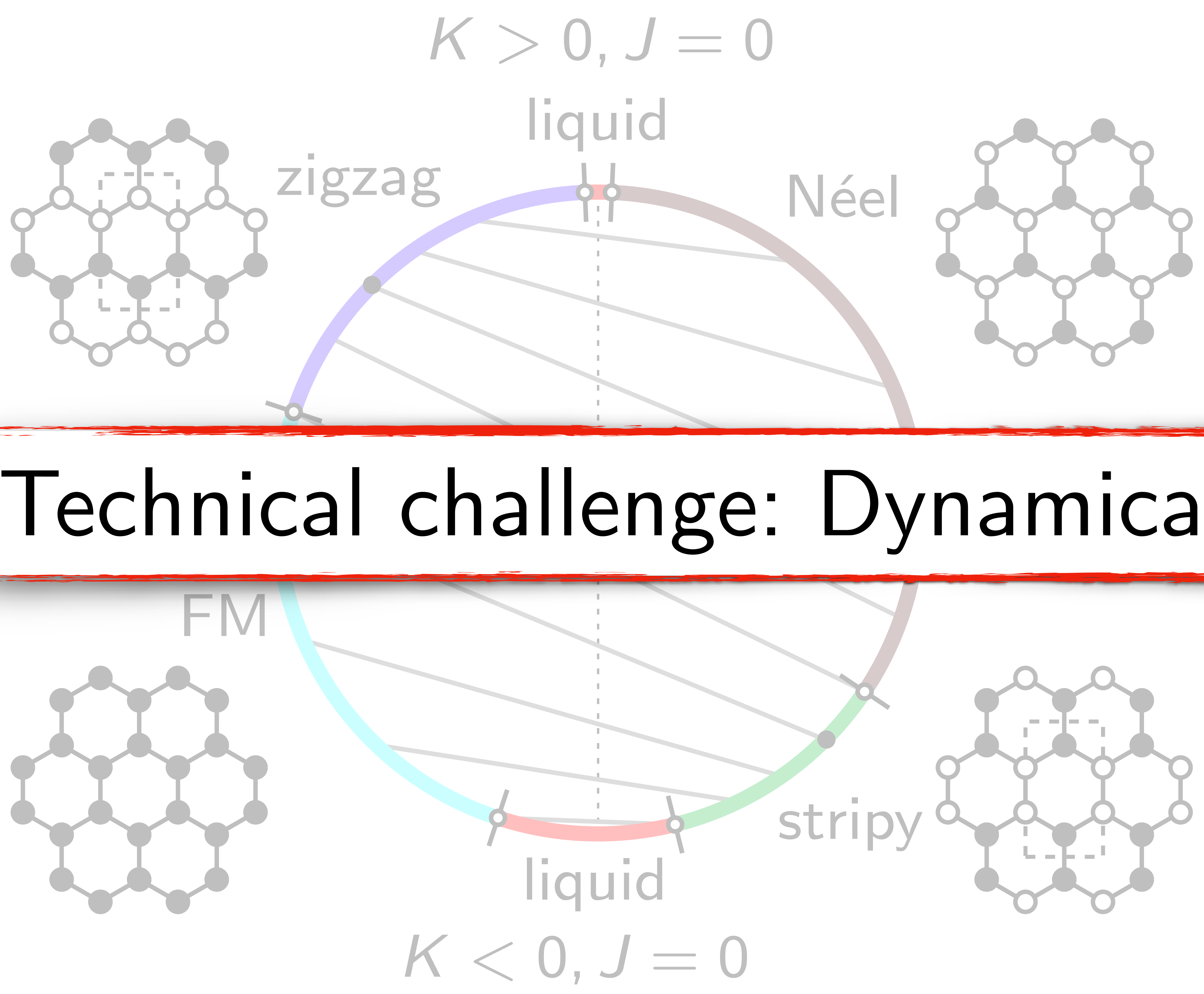


Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \sigma_i^\gamma \sigma_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

... possible relevance to α-$RuCl_3$, Na_2IrO_3, $Na_2Co_2TeO_6$, ...

Phase diagram:



Technical challenge: Dynamical $\mathbb{Z}_2$ gauge field!

... no sign-problem-free QMC available: [Sato & Assaad, PRB '21]

... from 24-site ED: [Chaloupka, Jackeli, Khaliullin, PRL '13]

Kitaev's 16-fold way of anyon theories

Clifford algebra:

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\delta^{\mu\nu}$$

Representations:

$$\sigma^\alpha \quad 2 \times 2 \quad \curvearrowright \quad \gamma^i \quad 4 \times 4 \quad \curvearrowright \quad \Gamma^\mu \quad 8 \times 8 \quad \curvearrowright \quad \cdots$$

... can realize all 16 topological SCs:
[Chulliparambil, ..., LJ, Tu, PRB '20]

Kitaev's 16-fold way of anyon theories

Clifford algebra:

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\delta^{\mu\nu}$$

Representations:

$\sigma^\alpha \quad 2\times 2 \quad \rightarrow \quad \gamma^i \quad 4\times 4 \quad \rightarrow \quad \Gamma^\mu \quad 8\times 8 \quad \rightarrow \quad \dots$

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij\rangle_\gamma} \left(\Gamma_i^\gamma \Gamma_j^\gamma + \sum_{\beta=\gamma_{\mathrm{m}}+1}^{2q+3} \Gamma_i^{\gamma\beta} \Gamma_j^{\gamma\beta} \right)$$

... can realize all 16 topological SCs:
[Chulliparambil, ..., LJ, Tu, PRB '20]

Kitaev's 16-fold way of anyon theories

Clifford algebra:

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\delta^{\mu\nu}$$

Representations:

$\sigma^\alpha \quad 2 \times 2 \quad \rightarrow \quad \gamma^i \quad 4 \times 4 \quad \rightarrow \quad \Gamma^\mu \quad 8 \times 8 \quad \rightarrow \quad \dots$

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \left(\Gamma_i^\gamma \Gamma_j^\gamma + \sum_{\beta=\gamma_\mathrm{m}+1}^{2q+3} \Gamma_i^{\gamma\beta} \Gamma_j^{\gamma\beta} \right)$$

Majorana representation:

$$\Gamma^\alpha = i b^\alpha c$$
$$\Gamma^{\alpha\beta} = i b^\alpha b^\beta$$

... can realize all 16 topological SCs:
[Chulliparambil, ..., LJ, Tu, PRB '20]

Kitaev's 16-fold way of anyon theories

Clifford algebra:

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\delta^{\mu\nu}$$

Representations:

$$\sigma^\alpha \quad 2\times 2 \quad \curvearrowright \quad \gamma^i \quad 4\times 4 \quad \curvearrowright \quad \Gamma^\mu \quad 8\times 8 \quad \curvearrowright \quad \ldots$$

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij\rangle_\gamma} \left(\Gamma_i^\gamma \Gamma_j^\gamma + \sum_{\beta=\gamma_m+1}^{2q+3} \Gamma_i^{\gamma\beta} \Gamma_j^{\gamma\beta} \right)$$

Majorana representation:

$$\Gamma^\alpha = i b^\alpha c$$
$$\Gamma^{\alpha\beta} = i b^\alpha b^\beta$$

Fractionalization:

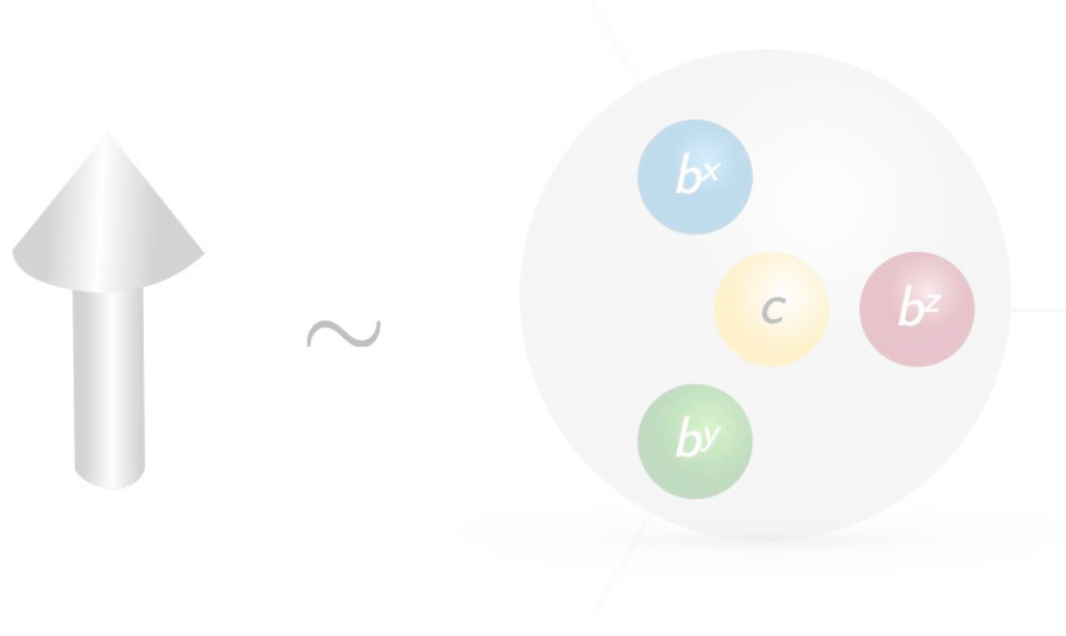
$$\mathcal{H} \mapsto \tilde{\mathcal{H}} = iK \sum_{\langle ij\rangle_\gamma} \hat{u}_{ij} \left(c_i c_j + \sum_{\beta=\gamma_m+1}^{2q+3} b_i^\beta b_j^\beta \right)$$

with $\left[\hat{u}_{ij}, \tilde{\mathcal{H}}\right] = 0 \quad \Rightarrow \quad$ static $\mathbb{Z}_2$ gauge field!

... can realize all 16 topological SCs:
[Chulliparambil, ..., LJ, Tu, PRB '20]

Outline

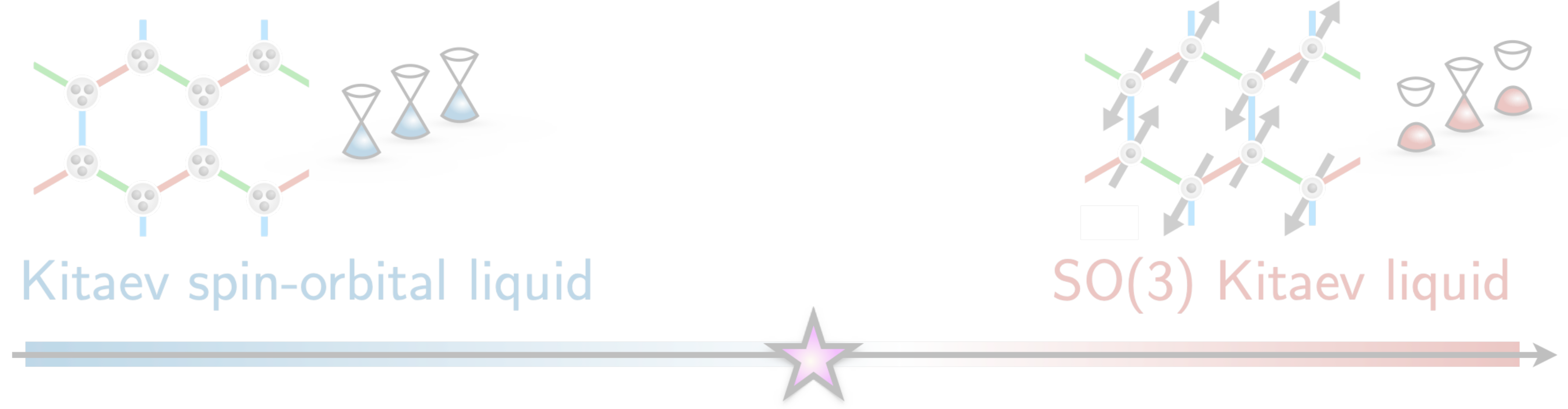
(1) Fractionalized quantum criticality



(2) Frustrated spins and spin-orbitals

(3) Square-lattice Kitaev-Ising spin-orbital model

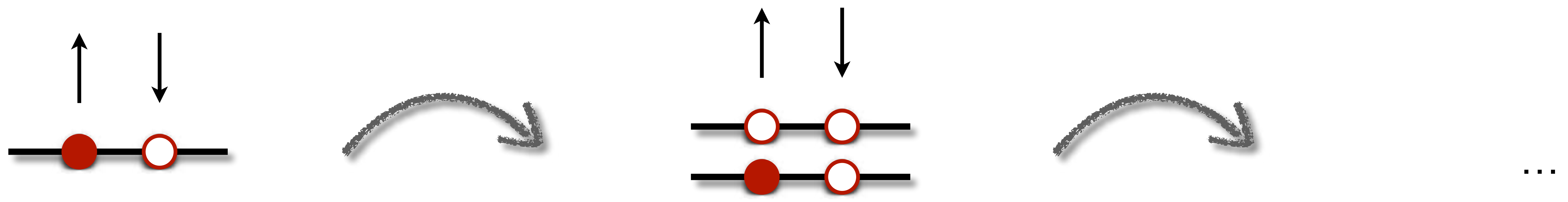
(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model



(5) Conclusions

Kitaev spin-orbital models

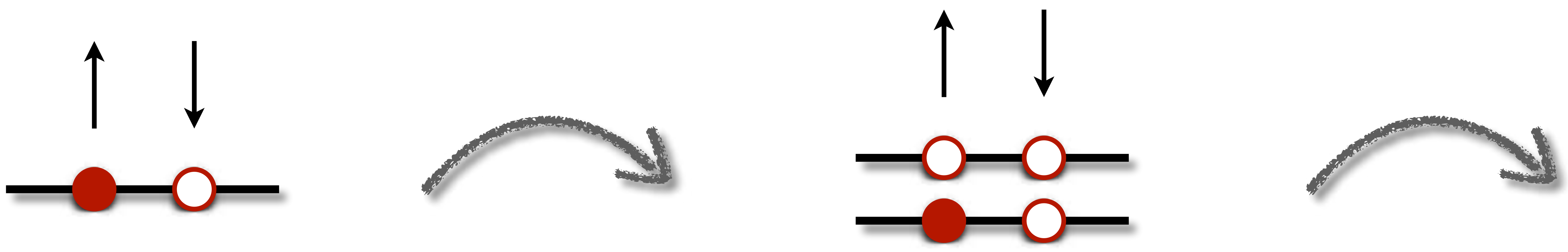
Spin + orbital + ... degrees of freedom:



σ^{α} 2×2 $\sigma^{\alpha} \otimes \tau^{\beta} = \gamma^{i}$ 4×4 Γ^{μ} 8×8

Kitaev spin-orbital models

Spin + orbital + ... degrees of freedom:



...

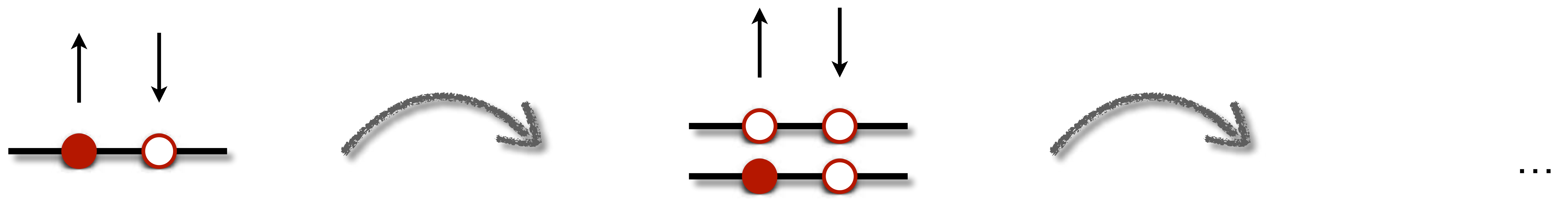
$\sigma^\alpha \quad 2 \times 2$ $\qquad$ $\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$ $\qquad$ $\Gamma^\mu \quad 8 \times 8$

Spin-orbital representation:

$$\gamma^1 = \sigma^y \otimes \tau^x = i b^1 c$$
$$\gamma^2 = \sigma^y \otimes \tau^y = i b^2 c$$
$$\gamma^3 = \sigma^y \otimes \tau^z = i b^3 c$$
$$\gamma^4 = \sigma^x \otimes \mathbb{1} = i b^4 c$$
$$\gamma^5 = \sigma^z \otimes \mathbb{1} = i b^5 c$$

Kitaev spin-orbital models

Spin + orbital + ... degrees of freedom:



$\sigma^\alpha \quad 2\times 2$ $\qquad$ $\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4\times 4$ $\qquad$ $\Gamma^\mu \quad 8\times 8$

Spin-orbital representation:

$$\gamma^1 = \sigma^y \otimes \tau^x = i b^1 c$$
$$\gamma^2 = \sigma^y \otimes \tau^y = i b^2 c$$
$$\gamma^3 = \sigma^y \otimes \tau^z = i b^3 c$$
$$\gamma^4 = \sigma^x \otimes \mathbb{1} = i b^4 c$$
$$\gamma^5 = \sigma^z \otimes \mathbb{1} = i b^5 c$$

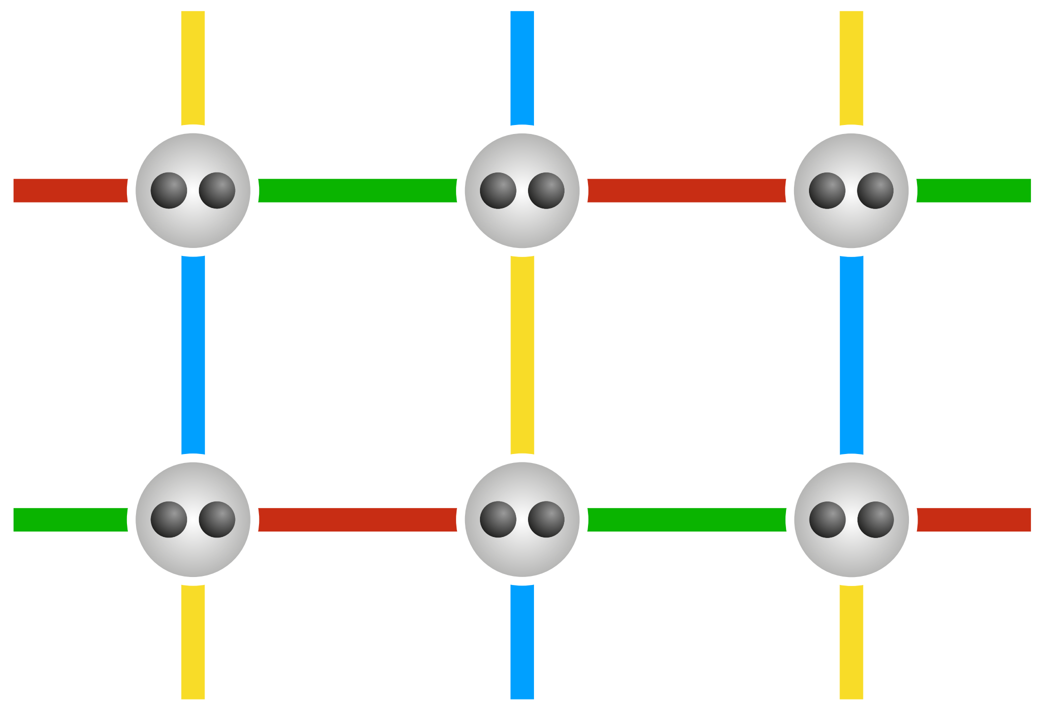
Example (square lattice):

$$H_K = K \sum_{\langle ij\rangle_\gamma} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \otimes \tau_i^\gamma \tau_j^\gamma$$

XY spin

Kitaev orbital

$$\mapsto iK \sum_{\langle ij\rangle_\gamma} \hat{u}_{ij} \left(c_i c_j + b_i^5 b_j^5\right)$$



2 itinerant fermions

... recover known model for $j = 3/2$ spin liquid:

[Yao, Zhang, Kivelson, PRL '09]

[Nakai, Ryu, Furusaki, PRB '12]

Kitaev-Ising spin-orbital model

Ising perturbation:

$$H = H_K + J^z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z \otimes \mathbb{1}_i \mathbb{1}_j$$

Kitaev-Ising spin-orbital model

Ising perturbation:

$$H = H_K + J^z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z \otimes \mathbb{1}_i \mathbb{1}_j$$

Parton representation:

hopping parameter $t = 2K$

π flux

nearest-neighbor repulsion $V = 4J^z$

$$H \mapsto \sum_{\langle ij \rangle} \left[2K\, u_{ij} (f_i^\dagger f_j + f_j^\dagger f_i) + 4J^z (n_i - \tfrac{1}{2})(n_j - \tfrac{1}{2}) \right]$$

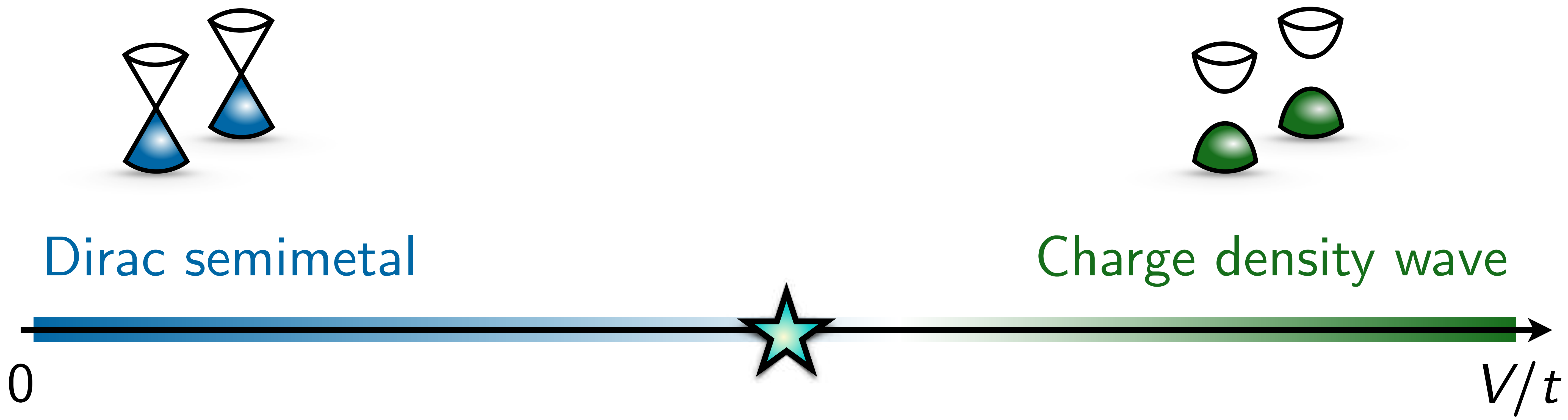
$f = \frac{1}{2}(c + ib^5)$

electron density $f^\dagger f$

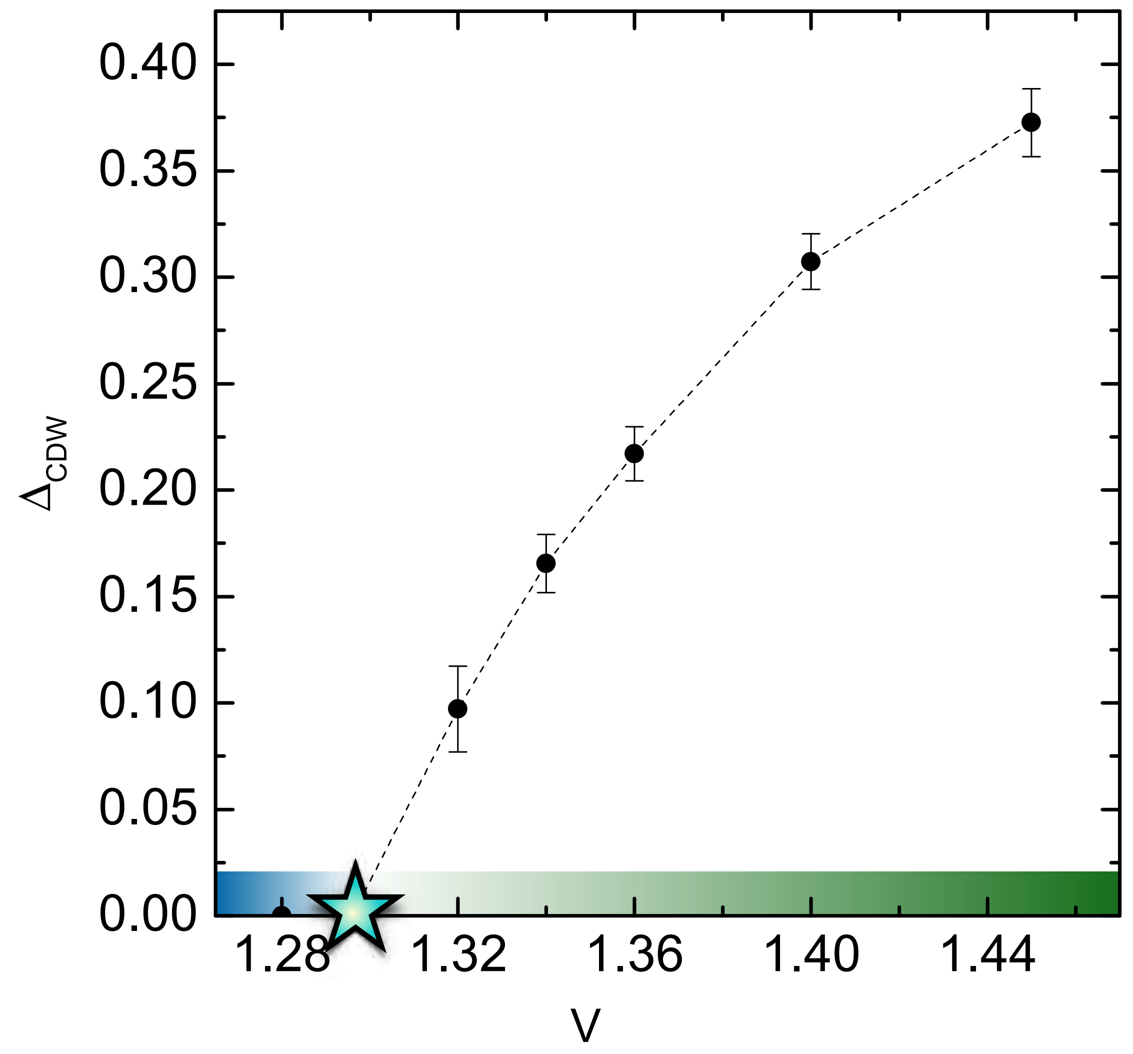
Ground-state flux pattern: [Lieb, PRL '94]

Spin-orbital model $\mapsto$ interacting fermions on π-flux lattice

Spinless fermions on π-flux lattice: QMC

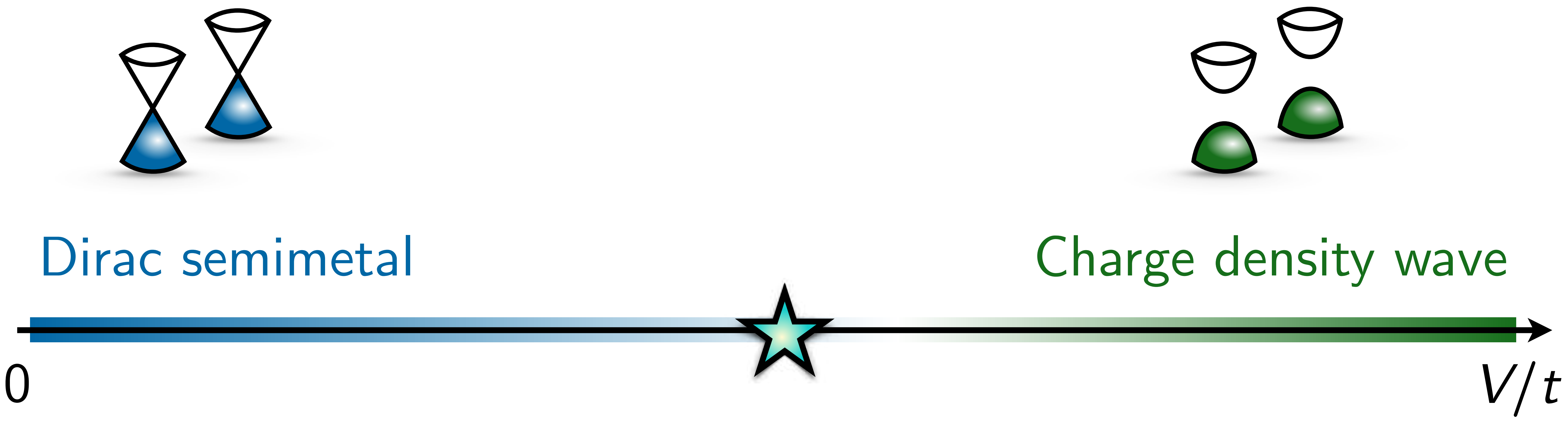


$(V/t)_c = 1.281(2)$

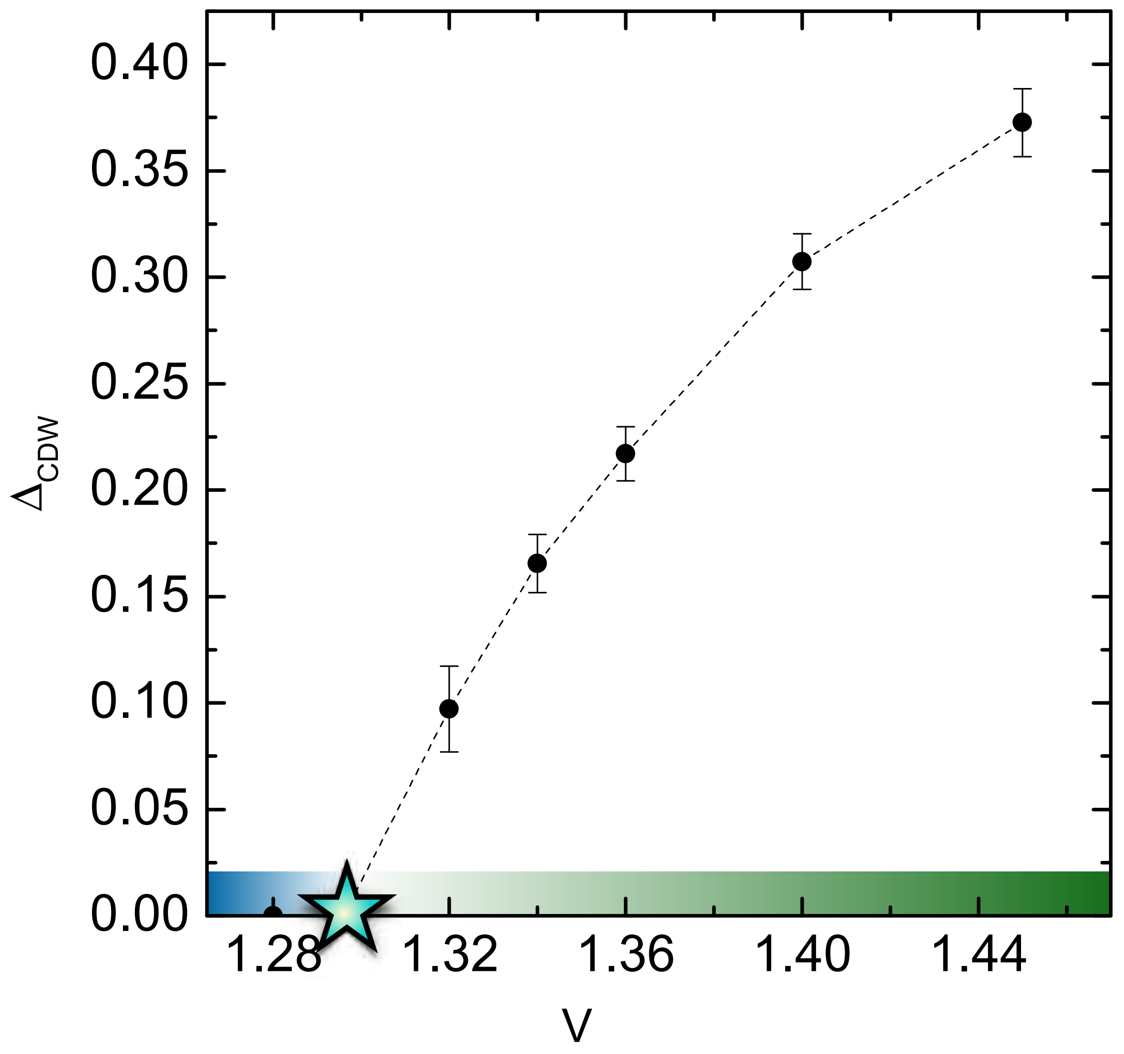


[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]
...

Spinless fermions on π-flux lattice: QMC

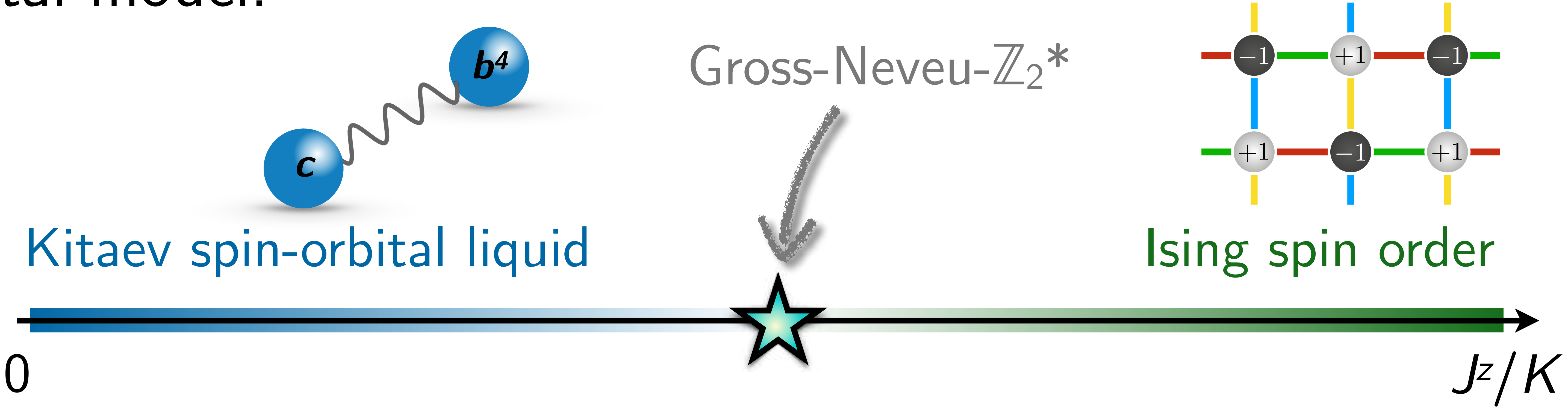


$$(V/t)_c = 1.281(2)$$



[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]
...

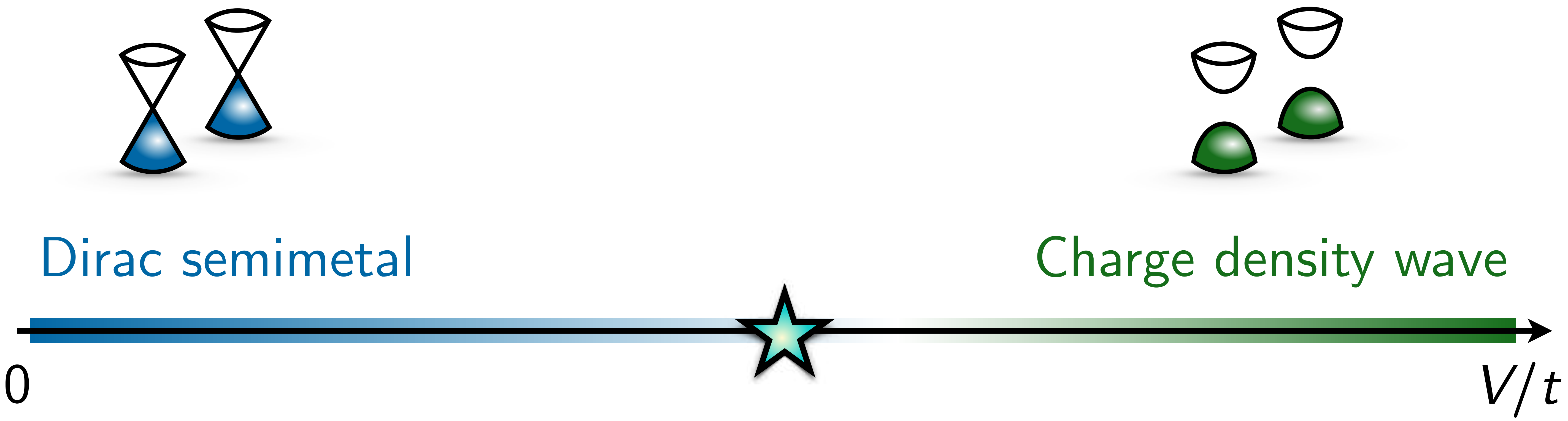
Spin-orbital model:



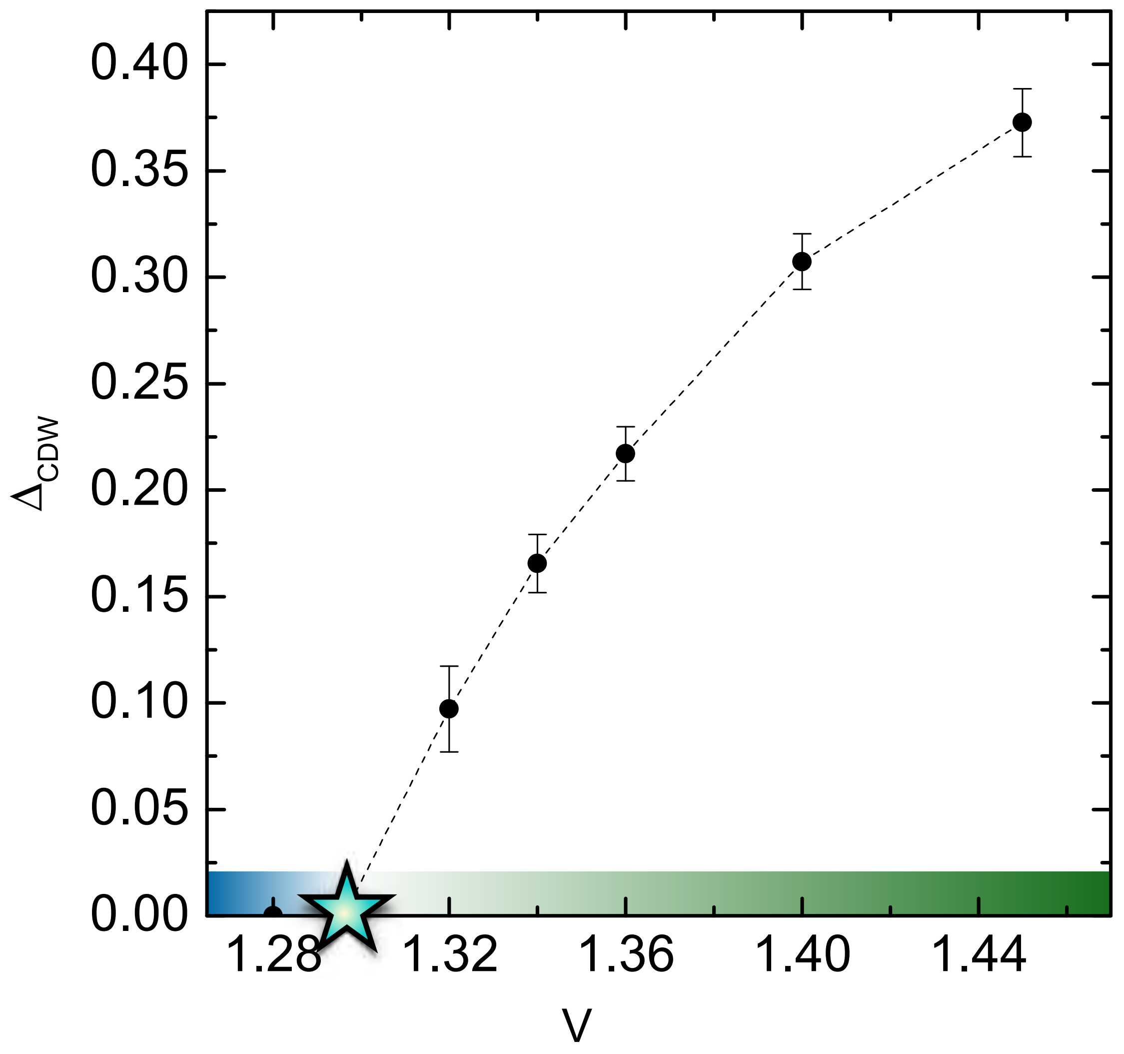
$$(J^z/K)_c = 0.642(2)$$

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Spinless fermions on π-flux lattice: QMC



$(V/t)_c = 1.281(2)$

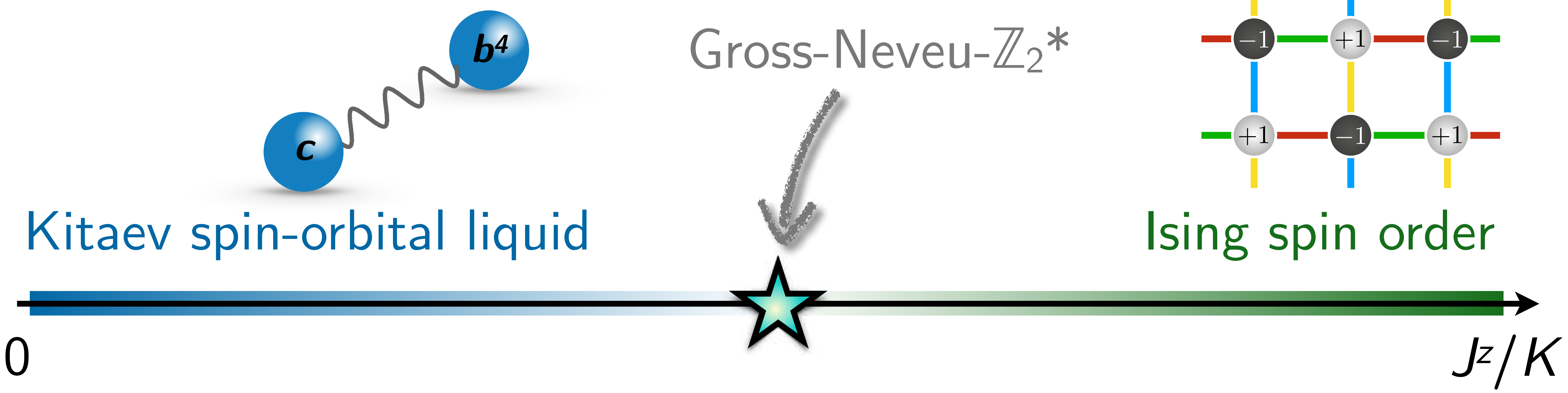


[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]
...

Gross-Neveu-$\mathbb{Z}_2$ universality: $1/\nu = 1.12(1)$, $\eta_\phi = 0.51(3)$

[LJ & Herbut, PRB '14]
[Ihrig, Mihaila, Scherer, PRB '18]
[Erramilli *et al.*, JHEP '23]
...
$N = 8$: [Wang & Meng, arXiv:2304.00034]

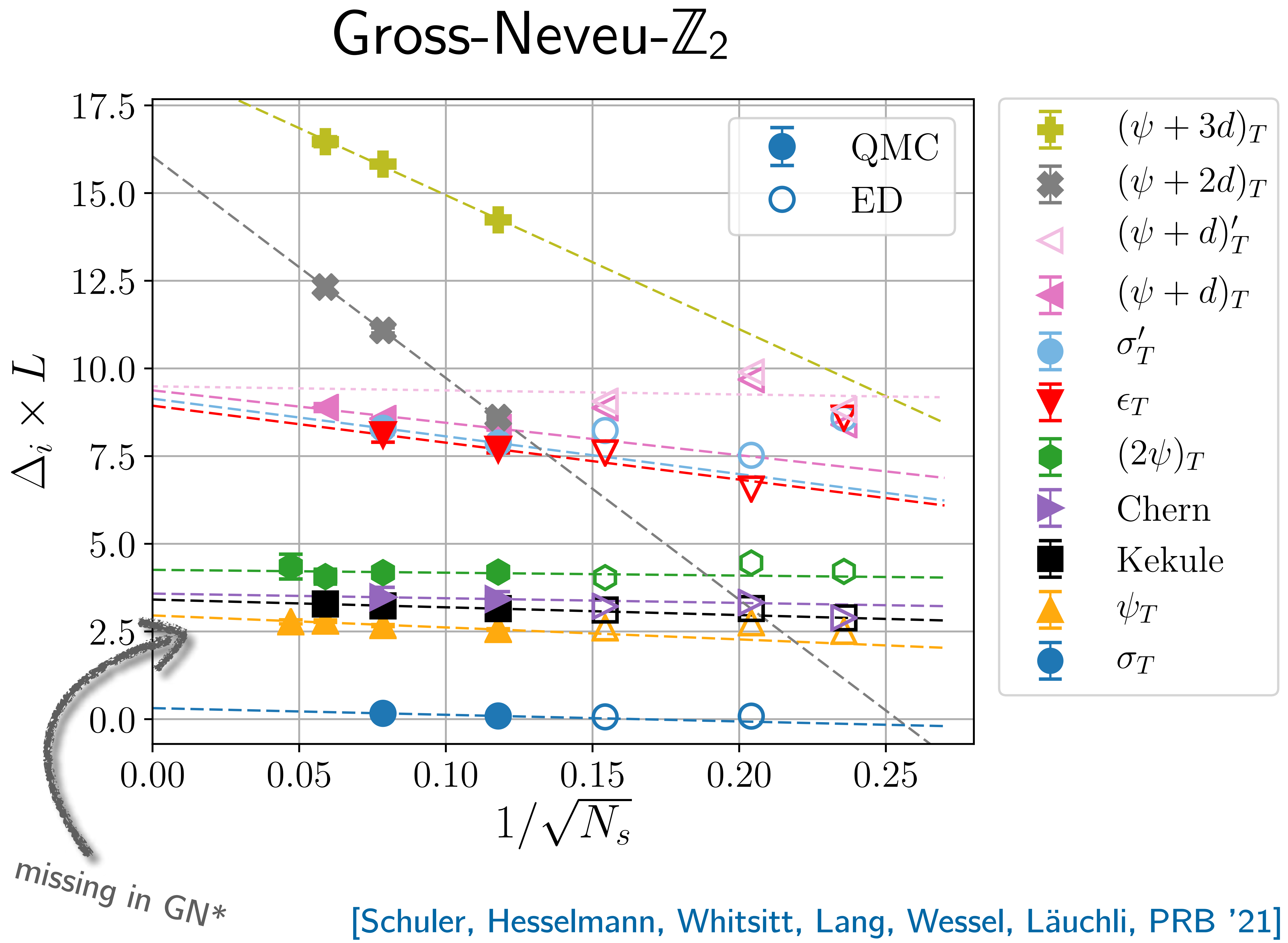
Spin-orbital model:



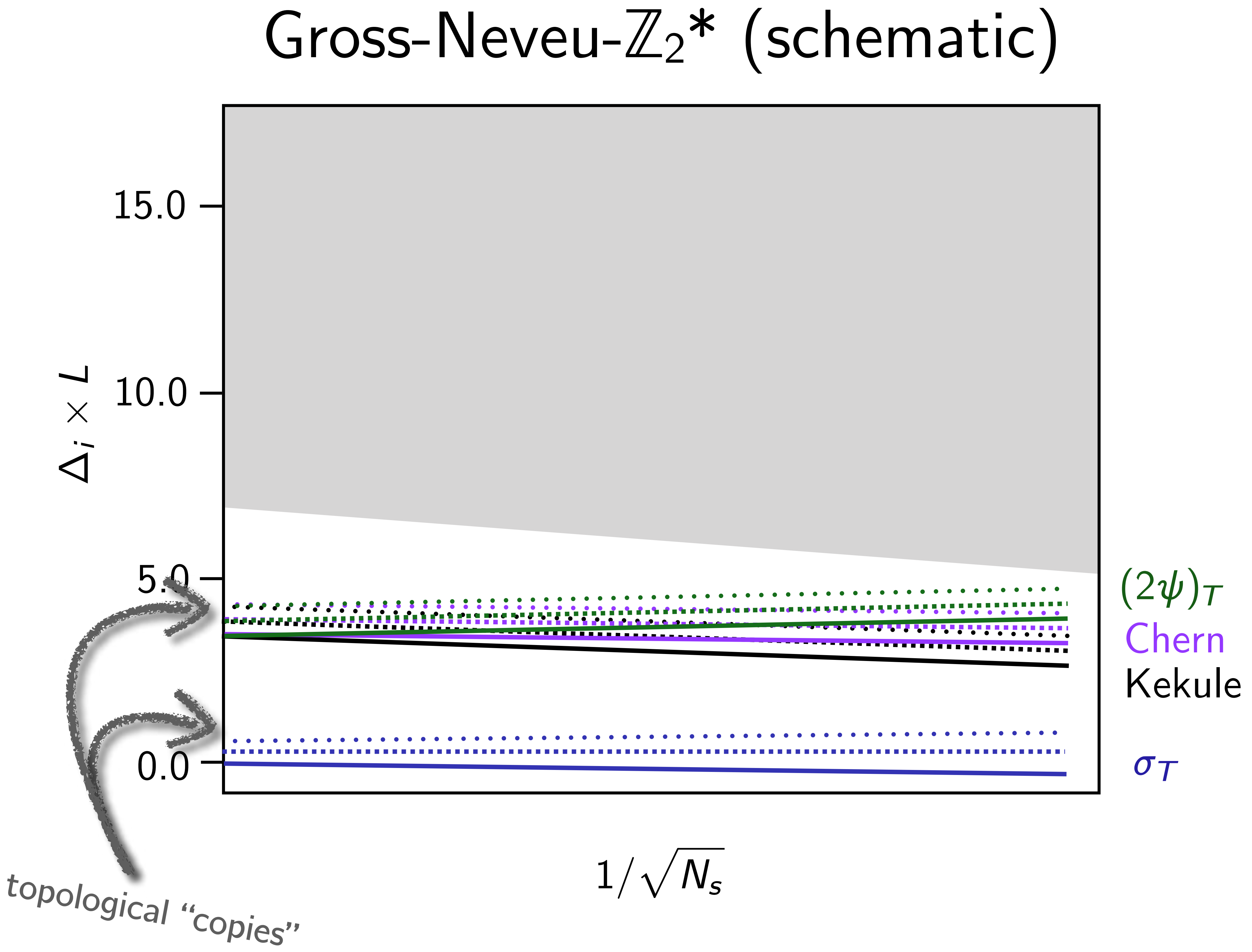
$(J^z/K)_c = 0.642(2)$

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Gross-Neveu vs Gross-Neveu*



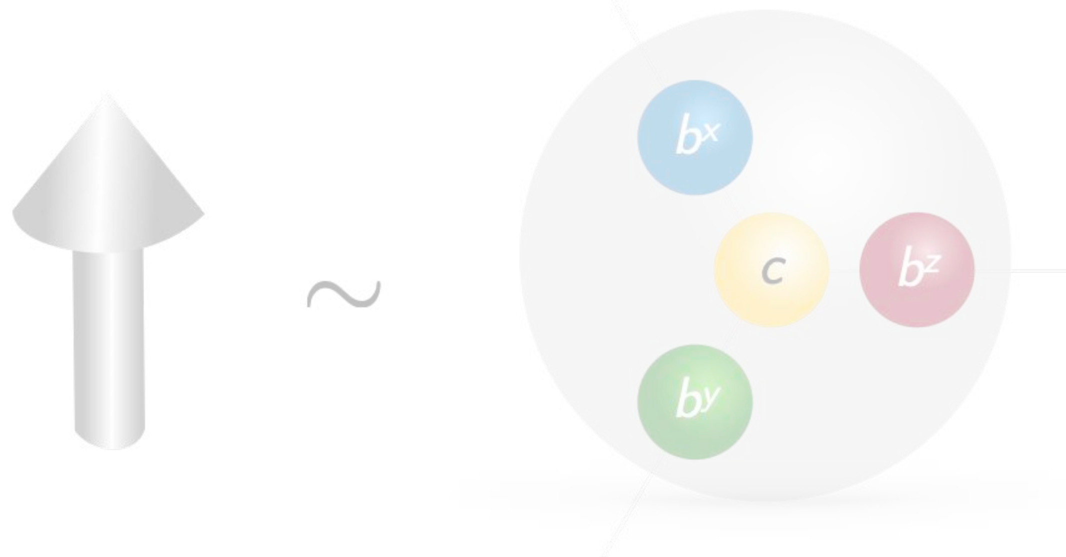
[Schuler, Hesselmann, Whitsitt, Lang, Wessel, Läuchli, PRB '21]



... testable in future simulations

Outline

(1) Fractionalized quantum criticality



(2) Frustrated spins and spin-orbitals

(3) Square-lattice Kitaev-Ising spin-orbital model

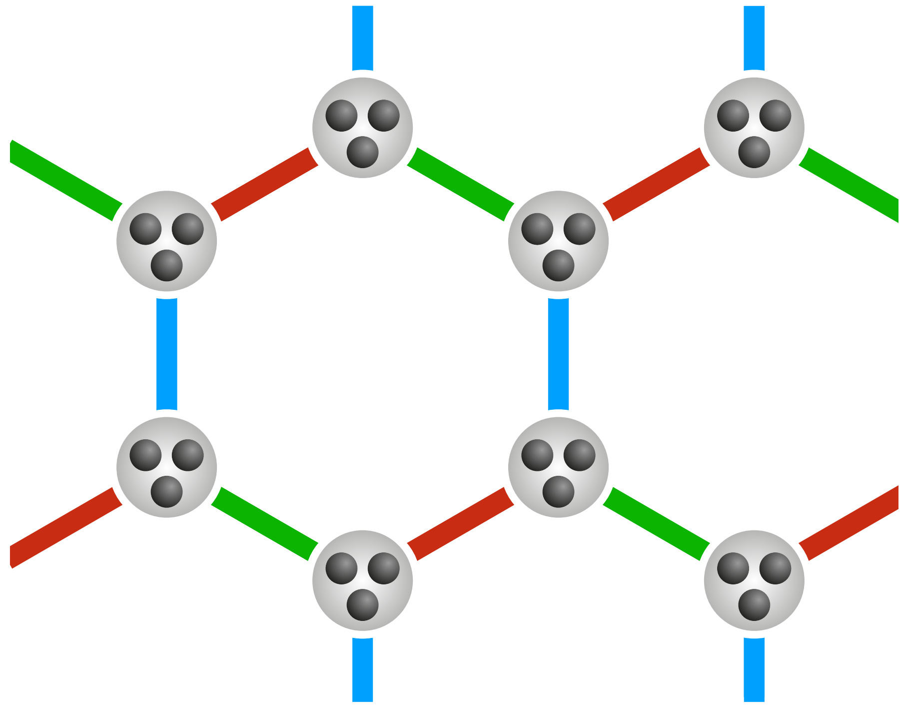
(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model

(5) Conclusions

Kitaev-Heisenberg spin-orbital model

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j$$



3 itinerant fermions

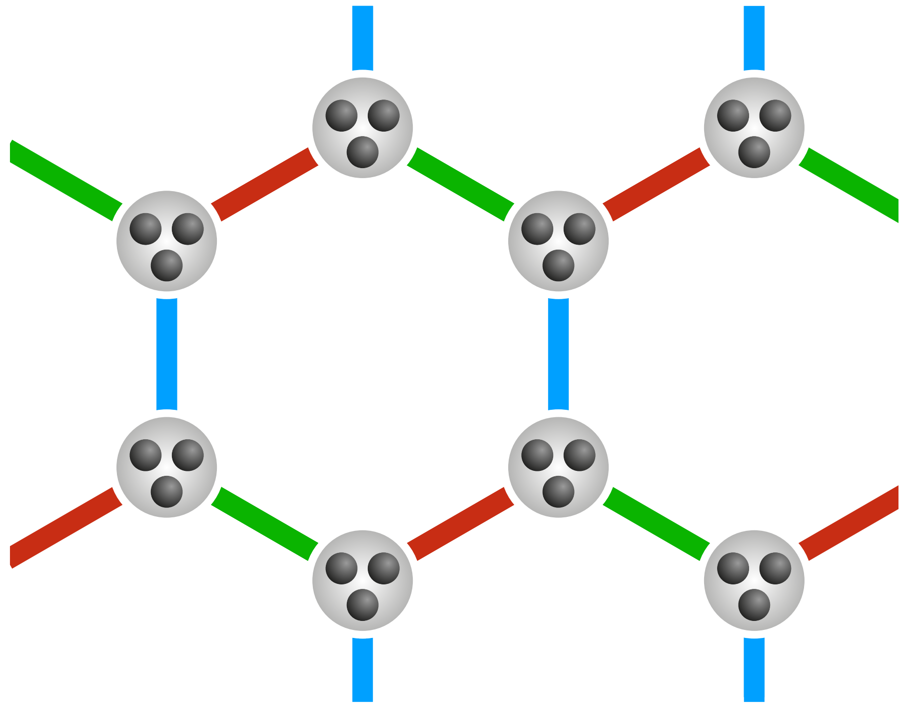
Kitaev-Heisenberg spin-orbital model

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma}_{} + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j$$

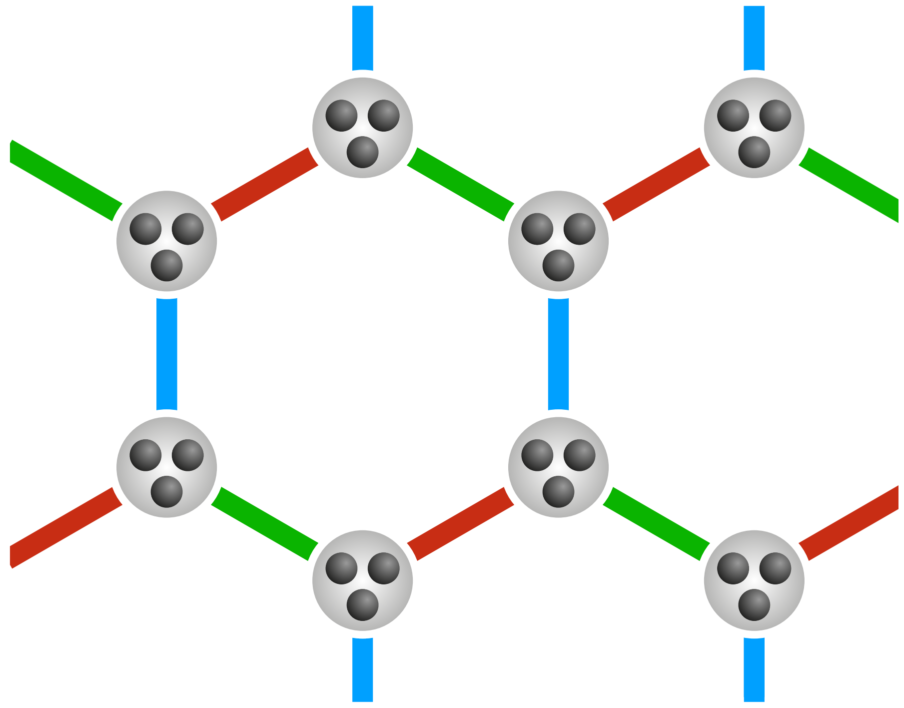
$$\mapsto \quad \hat{u}_{ij} \left(c_i ,\ b_i^4 ,\ b_i^5 \right) \cdot \begin{pmatrix} c_j \\ b_j^4 \\ b_j^5 \end{pmatrix}$$

$$\equiv \hat{u}_{ij} c_i^\top c_j$$



3 itinerant fermions

Kitaev-Heisenberg spin-orbital model



3 itinerant fermions

Hamiltonian:

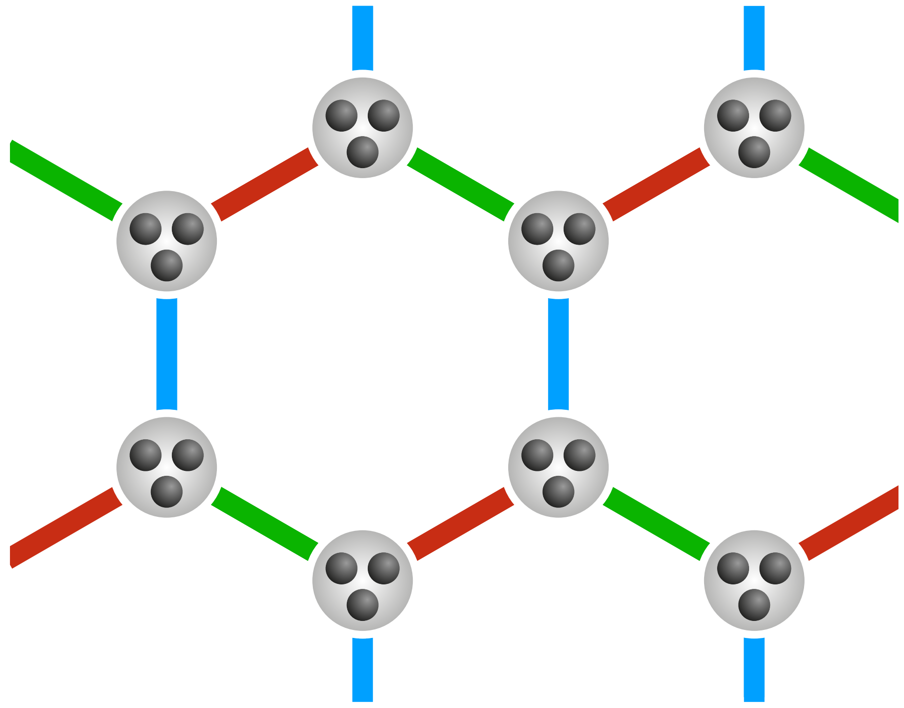
$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma} + J \sum_{\langle ij \rangle} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j}$$

$$\mapsto \quad \hat{u}_{ij} \left(c_i, \, b_i^4, \, b_i^5 \right) \cdot \begin{pmatrix} c_j \\ b_j^4 \\ b_j^5 \end{pmatrix} \qquad \mapsto \quad \frac{1}{4}(c_i^\top \vec{L} c_i) \cdot (c_j \vec{L} c_j)$$

spin-1 matrices

$$\equiv \hat{u}_{ij} c_i^\top c_j \qquad \text{with } [\hat{u}_{ij}, \mathcal{H}] = 0 \text{ still static!}$$

Kitaev-Heisenberg spin-orbital model



3 itinerant fermions

Hamiltonian:

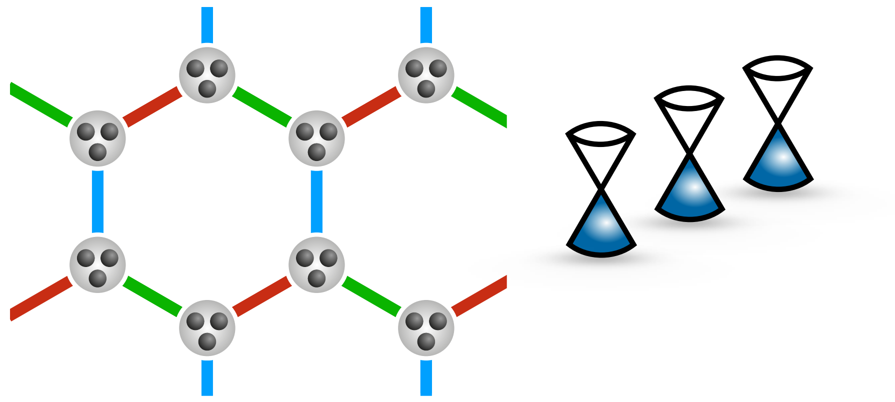
$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma} + J \sum_{\langle ij \rangle} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j}$$

$$\mapsto \hat{u}_{ij} \left(c_i,\ b_i^4,\ b_i^5 \right) \cdot \begin{pmatrix} c_j \\ b_j^4 \\ b_j^5 \end{pmatrix} \qquad \mapsto \frac{1}{4} (c_i^\top \vec{L} c_i) \cdot (c_j \vec{L} c_j)$$

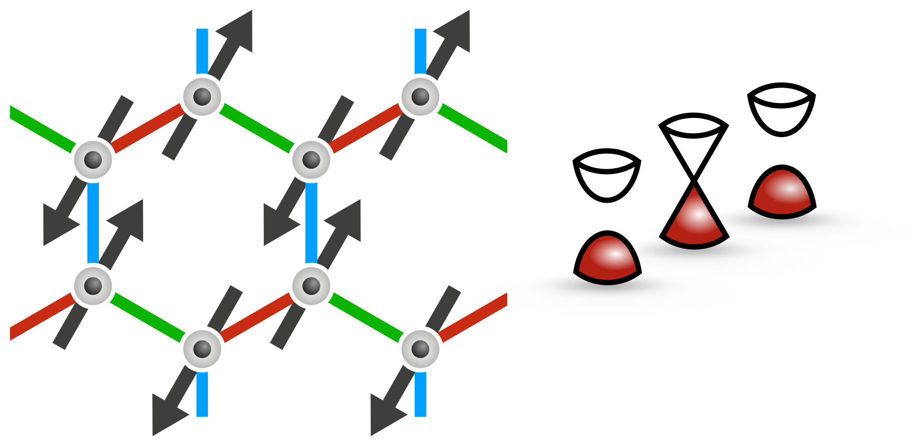
spin-1 matrices

$$\equiv \hat{u}_{ij} c_i^\top c_j \qquad \text{with } [\hat{u}_{ij}, \mathcal{H}] = 0 \text{ still static!}$$

Phase diagram:



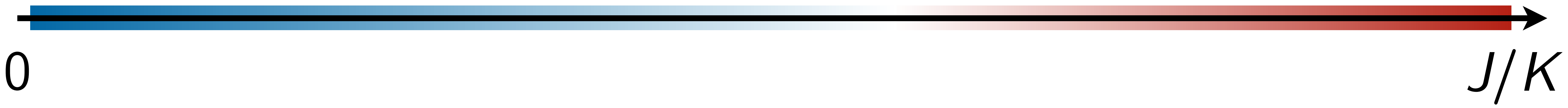
Kitaev spin-orbital liquid



SO(3) Kitaev liquid

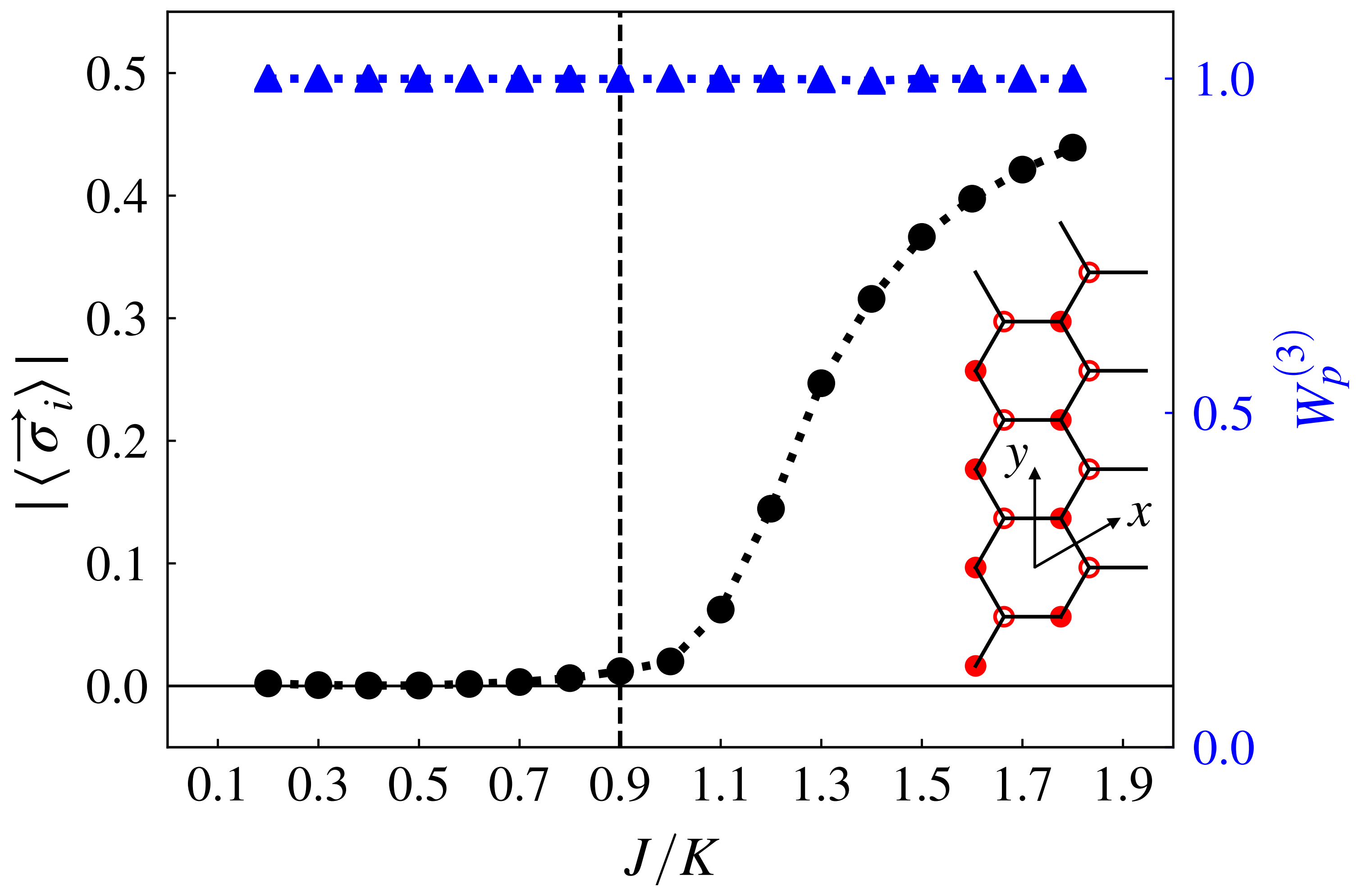
$\langle c_{iA}^\top \vec{L} c_{iA} \rangle \neq \langle c_{jB}^\top \vec{L} c_{jB} \rangle$

"spin density wave"



Gross-Neveu-SO(3)* transition

iDMRG:

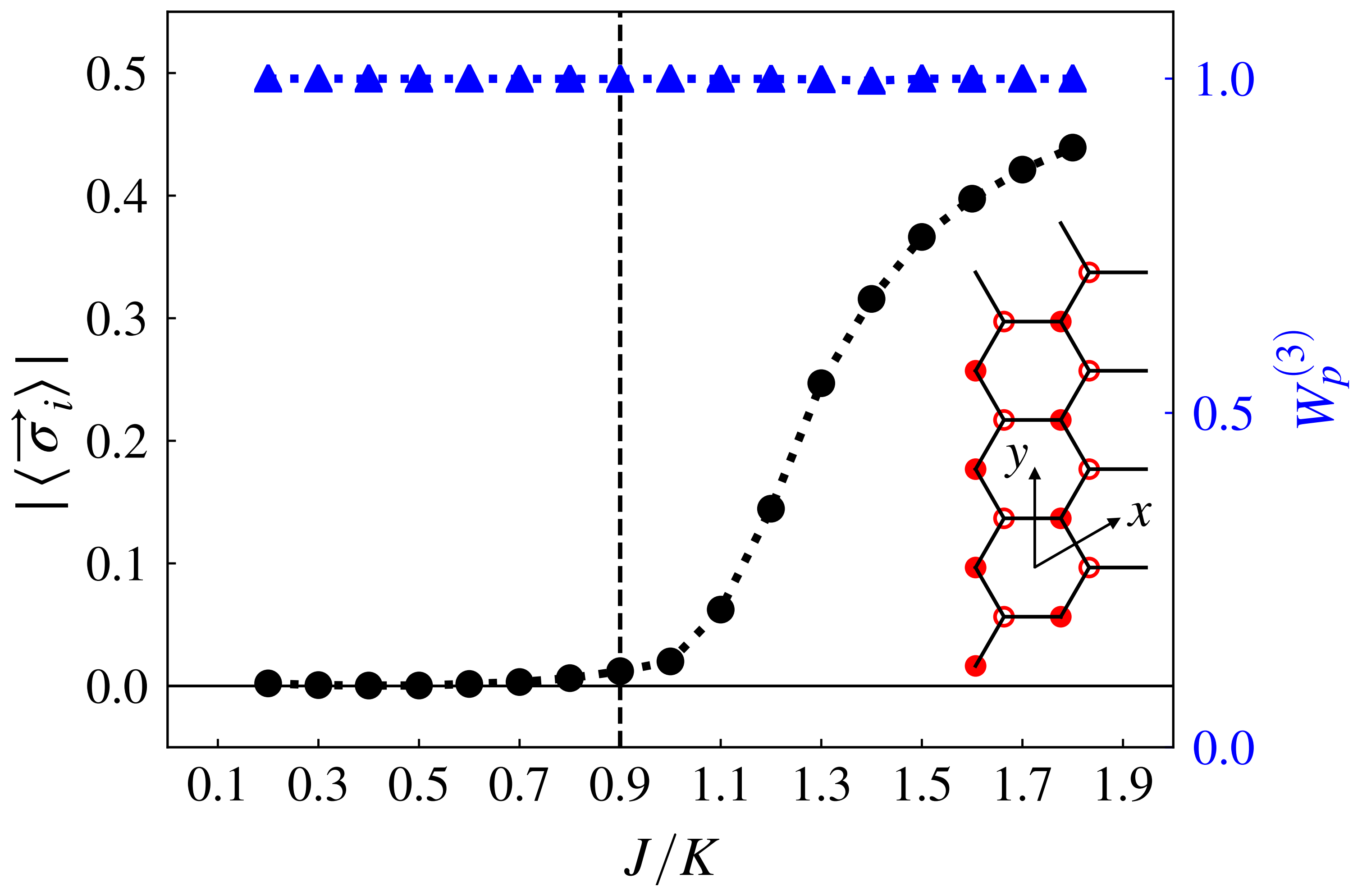


... on cylinder with $L_y = 4$ unit cells

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

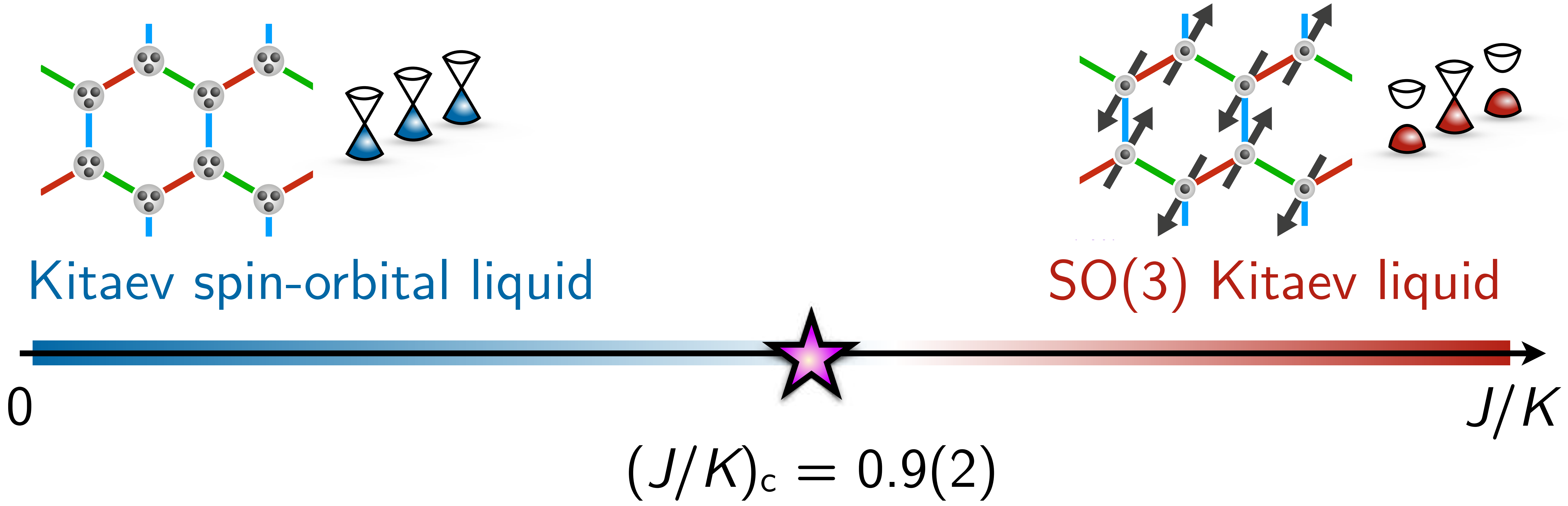
Gross-Neveu-SO(3)* transition

iDMRG:



... on cylinder with $L_y = 4$ unit cells

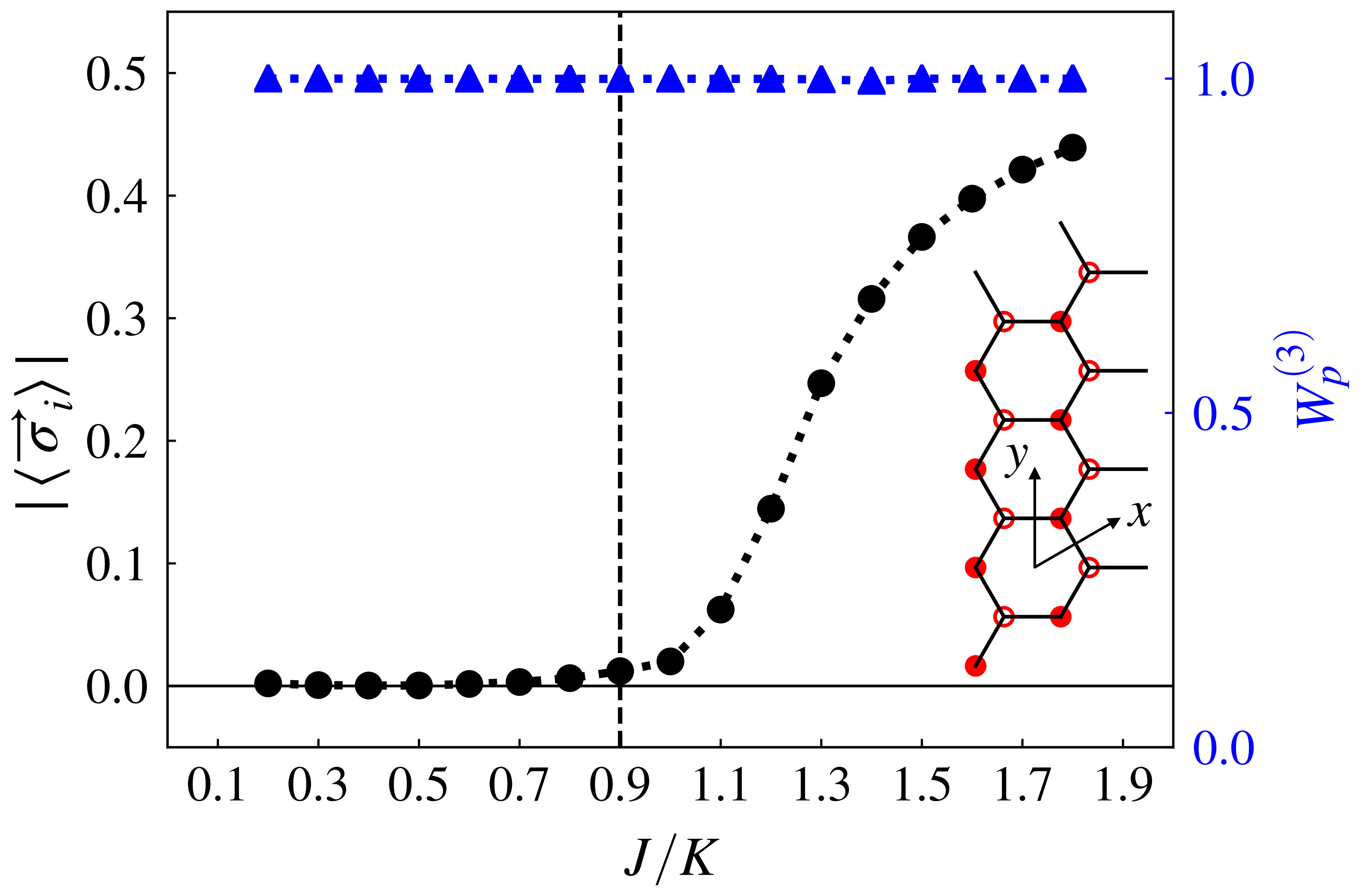
Phase diagram:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

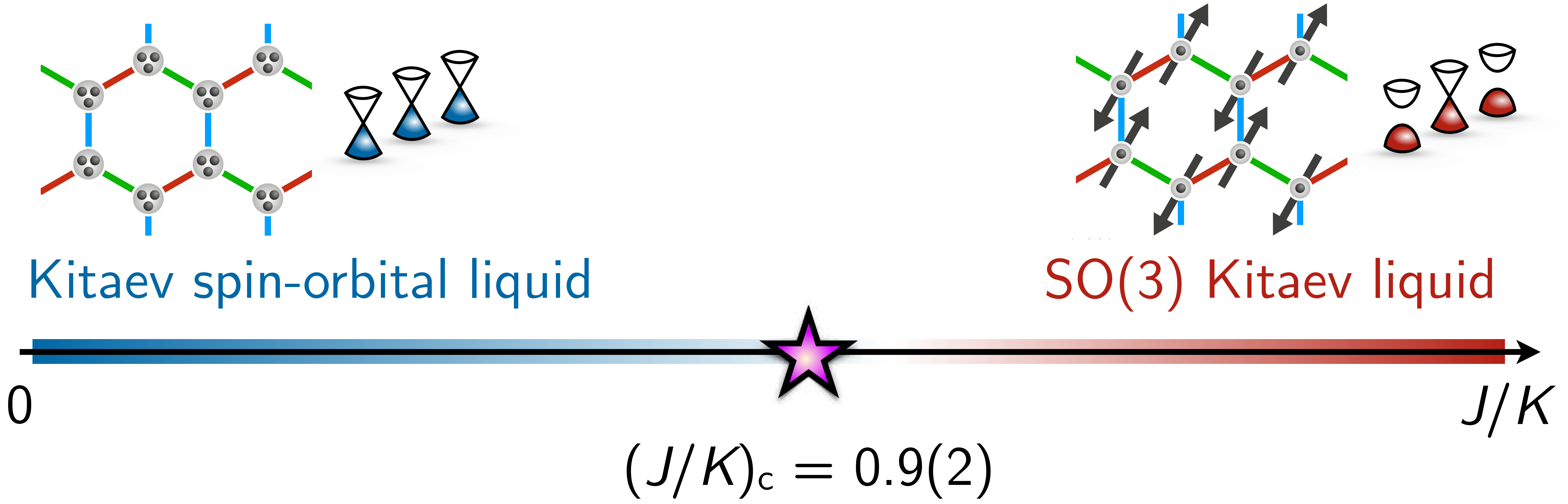
Gross-Neveu-SO(3)* transition

iDMRG:



... on cylinder with $L_y = 4$ unit cells

Phase diagram:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Effective field theory:

$$\mathcal{S} = \int d^2\vec{x}d\tau \left[\bar{\psi}\gamma^\mu\partial_\mu\psi + g\vec{\varphi}\cdot\bar{\psi}(\mathbb{1}_2\otimes\vec{L})\psi\right]$$

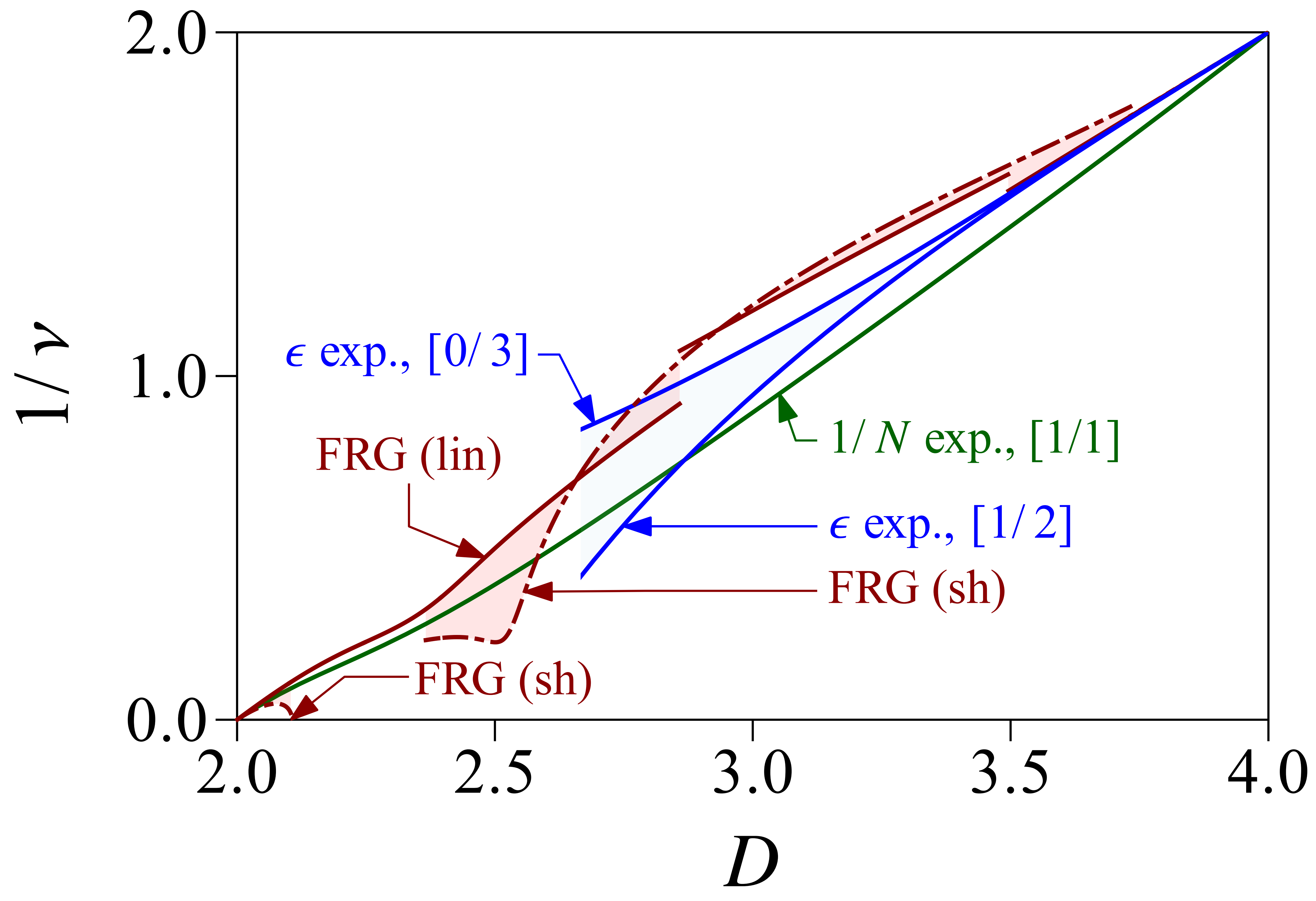
"Gross-Neveu-SO(3)"

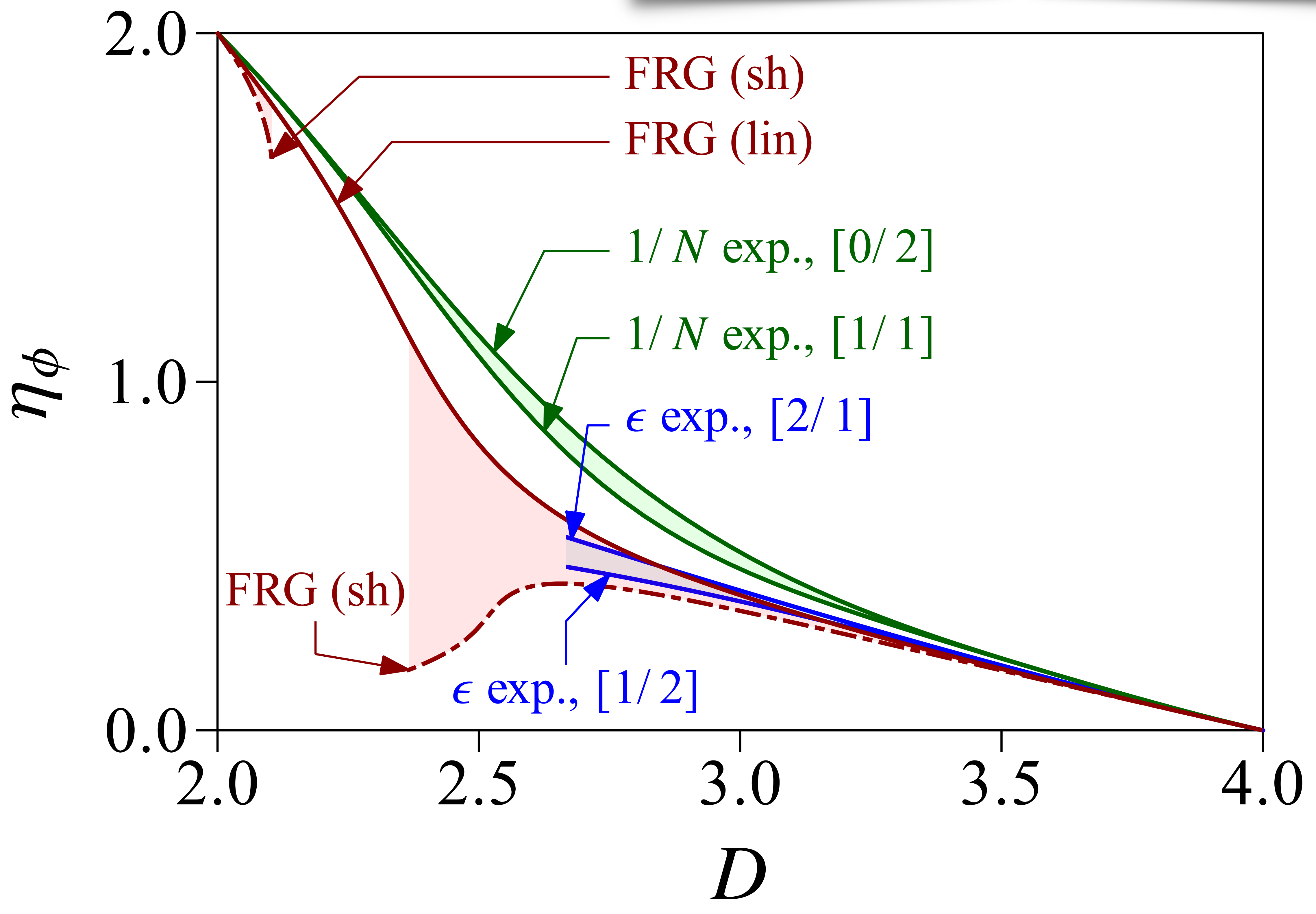
Gross-Neveu-SO(3)* criticality

Critical exponents from ...

- 4 - ε expansion @ 3 loop
- $1/N$ expansion @ $O(1/N^2)$
- Functional RG @ LPA'

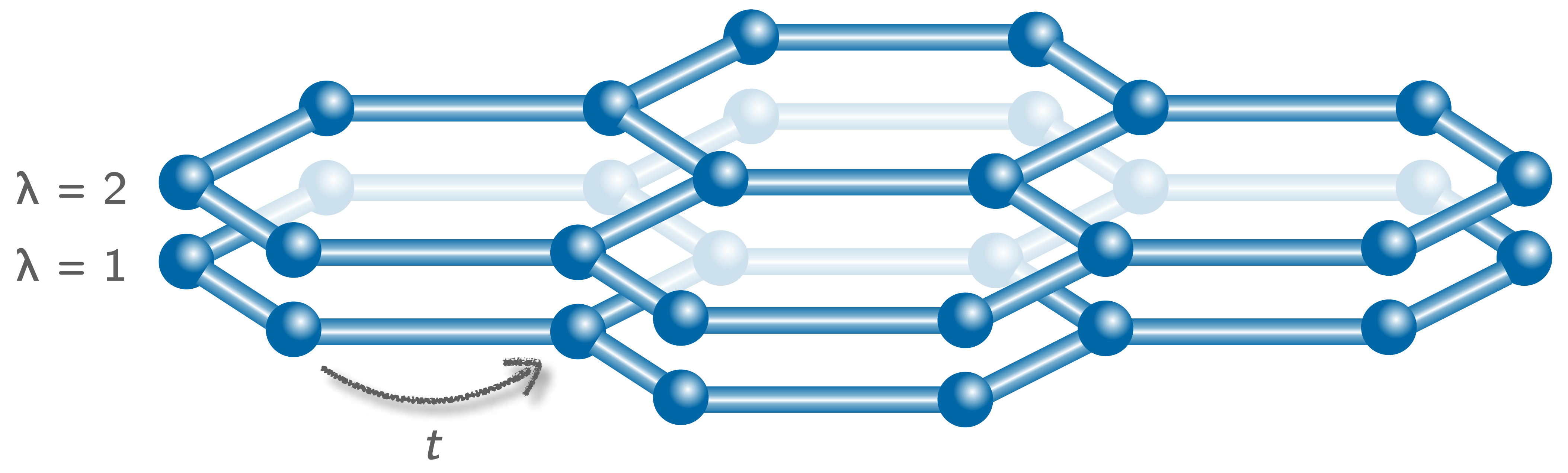
$N = 3$			$1/\nu$	η_ϕ	η_ψ
$4-\epsilon$ expansion		naïve	0.97516	0.39181	0.17234
		[1/2]	0.94472	0.40086	0.16458
		[2/1]	sing.	0.36989	0.18622
		[0/3]	1.09000	n.-e.	n.-e.
$1/N$ expansion		naïve	2.67318	0.49833	—
		[1/1]	0.89397	0.46276	—
		[0/2]	sing.	0.51074	n.-e.
		naïve	—	—	0.22116
		[1/2]	—	—	0.12337
		[2/1]	—	—	0.22716
		[0/3]	—	—	n.-e.
FRG	Taylor	linear	1.1901(10)	0.38781(6)	0.15068(8)
		sharp	1.209(4)	0.3434(5)	0.1966(6)
	pseudospectral	linear	1.18974	0.38781	0.15072
		sharp	1.20465	0.34340	0.19649





[Ray, Ihrig, Kruti, Gracey, Scherer, LJ, PRB '21]

Sign-problem-free bilayer model



Hamiltonian:

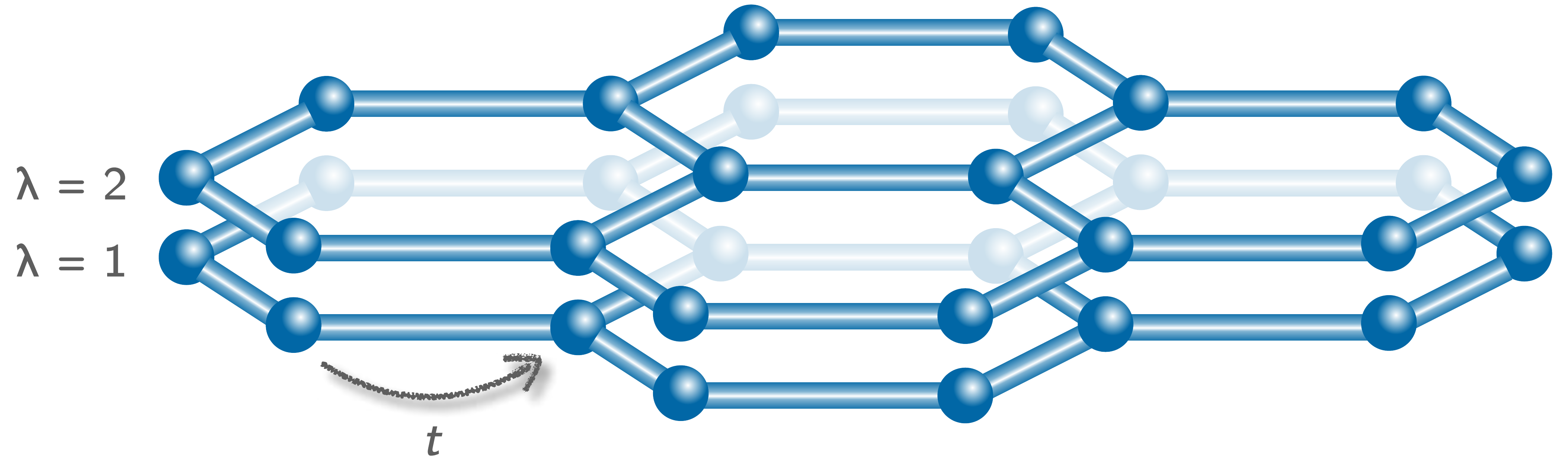
$$\mathcal{H} = -t \sum_{\langle ij \rangle} c_{i\lambda}^{\dagger} c_{j\lambda} - J \sum_{i} \left(c_{i\lambda}^{\dagger} \vec{L} \tau_{\lambda\lambda'}^{z} c_{i\lambda'} \right)^2$$

... with $SO(3) \times U_{\lambda}(1) \times U_{c} \times \mathbb{Z}_2$ symmetry

Sign-problem-free bilayer model

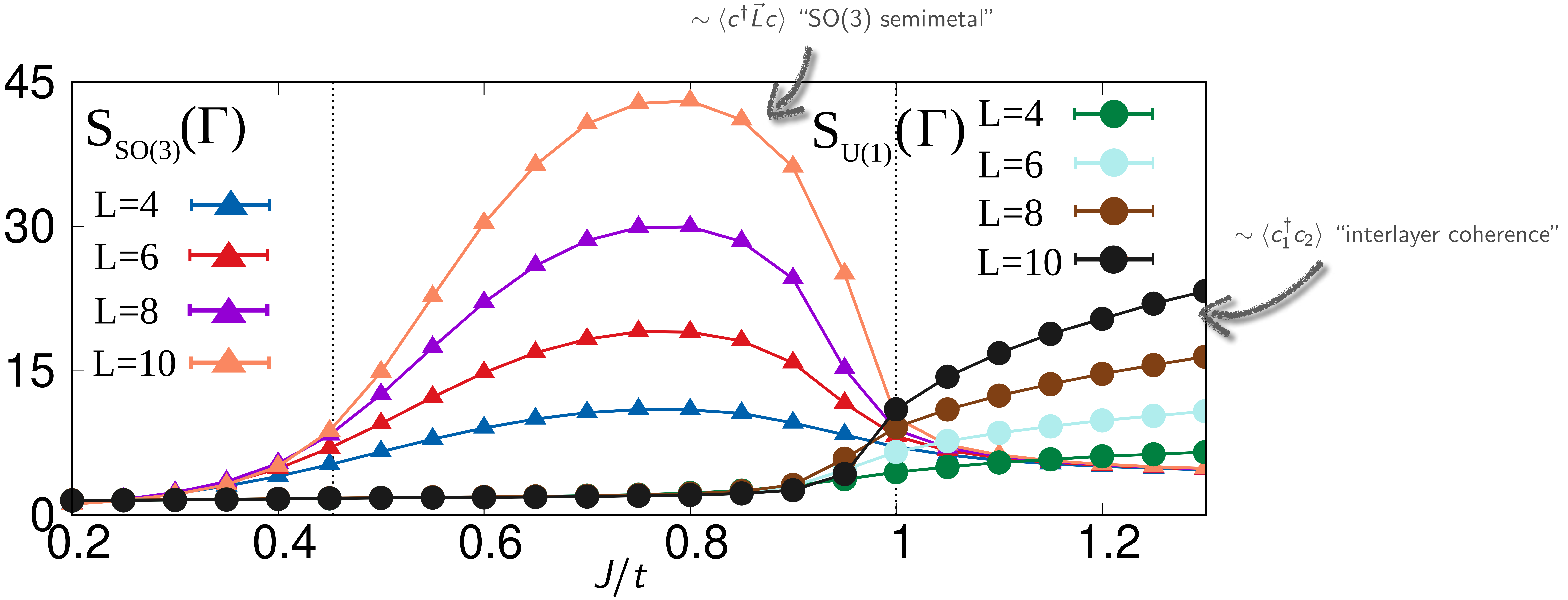
Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} c^\dagger_{i\lambda} c_{j\lambda} - J \sum_i \left(c^\dagger_{i\lambda} \vec{L} \tau^z_{\lambda\lambda'} c_{i\lambda'} \right)^2$$



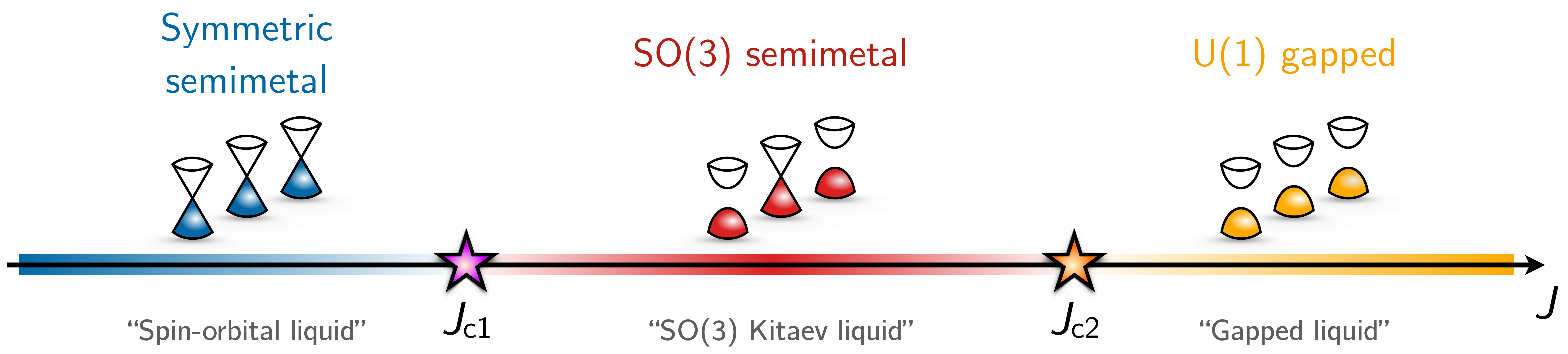
… with SO(3) × $U_\lambda(1)$ × U_c × $\mathbb{Z}_2$ symmetry

QMC structure factors:



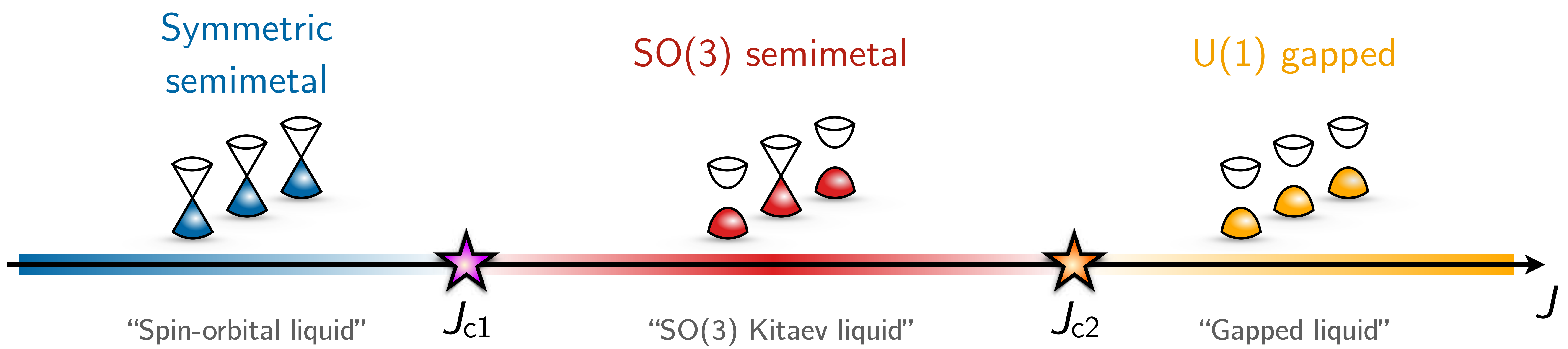
Sign-problem-free bilayer model

Phase diagram:

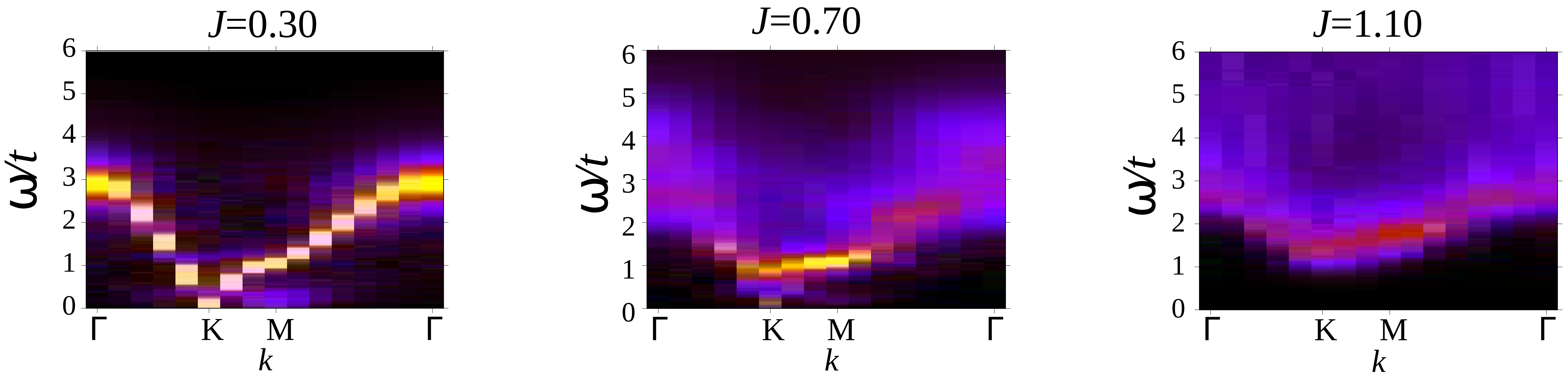


Sign-problem-free bilayer model

Phase diagram:

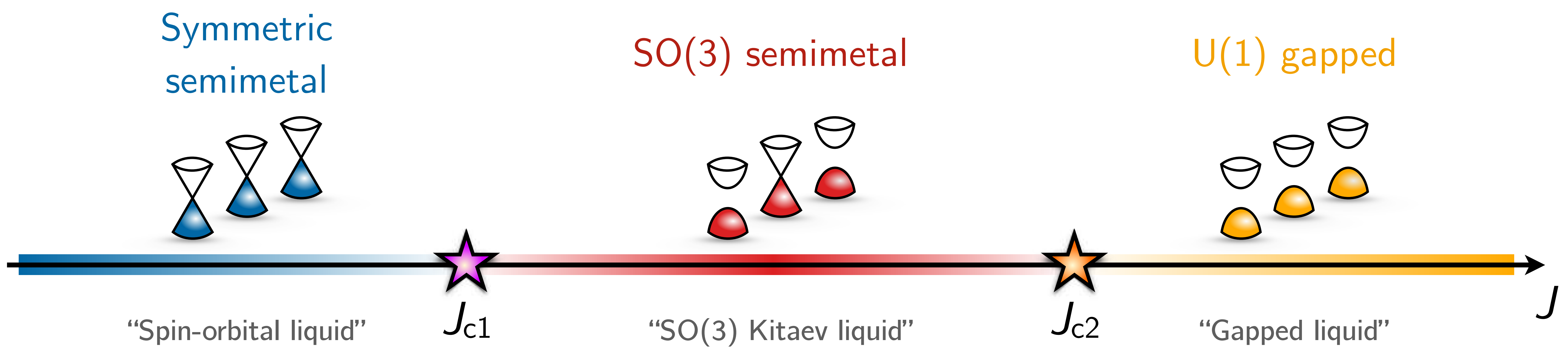


Fermion spectral function:



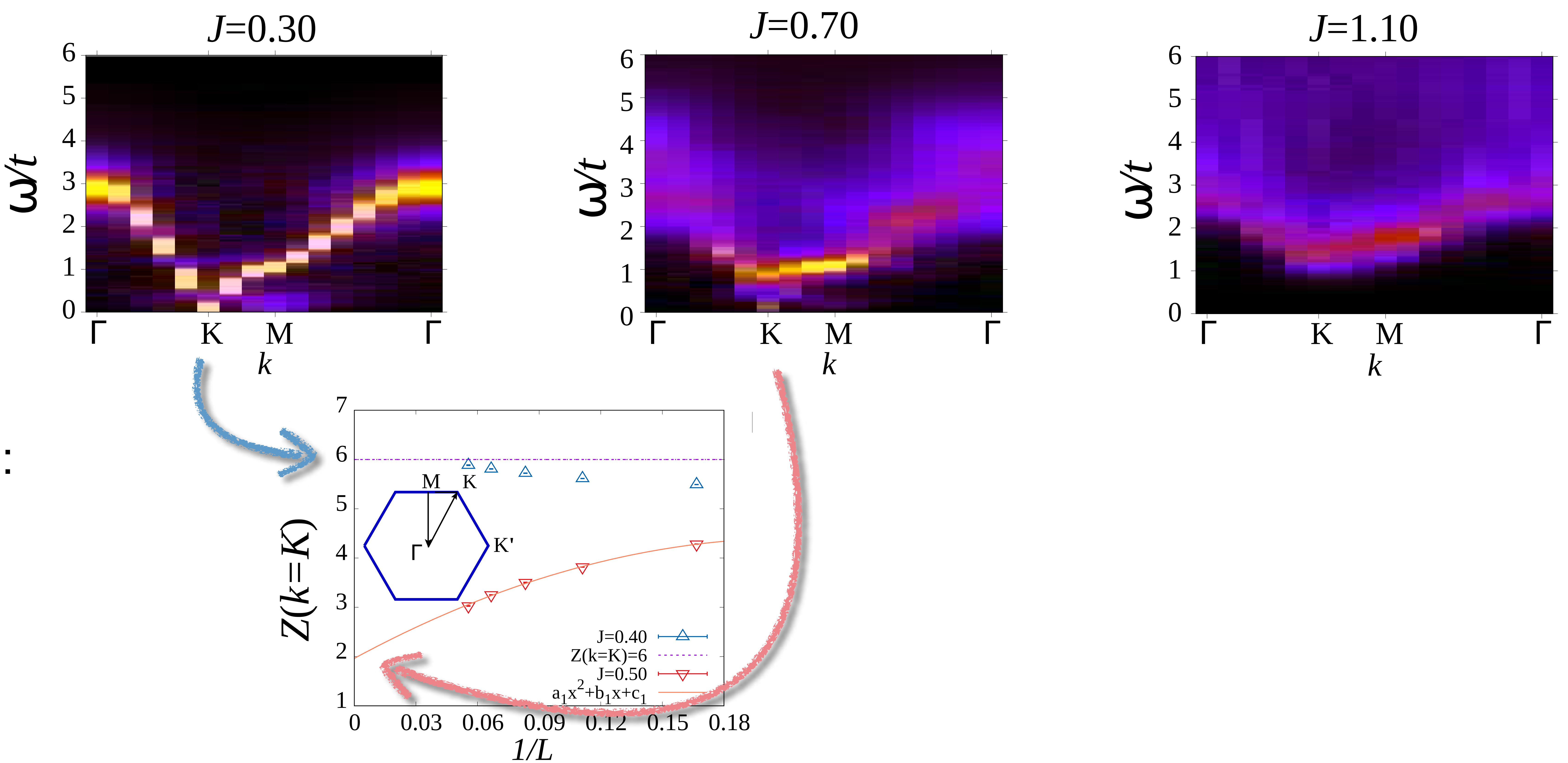
Sign-problem-free bilayer model

Phase diagram:

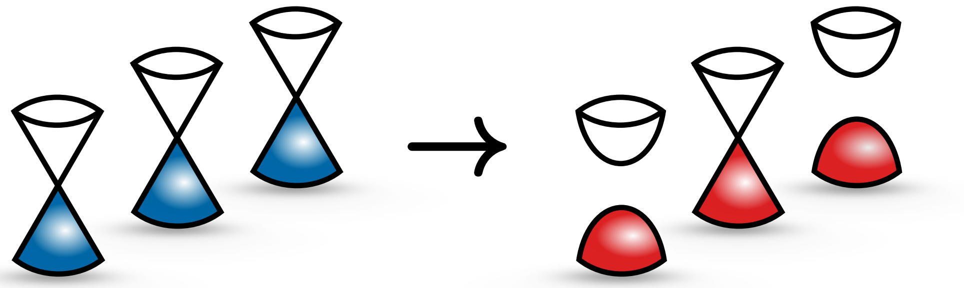


Fermion spectral function:

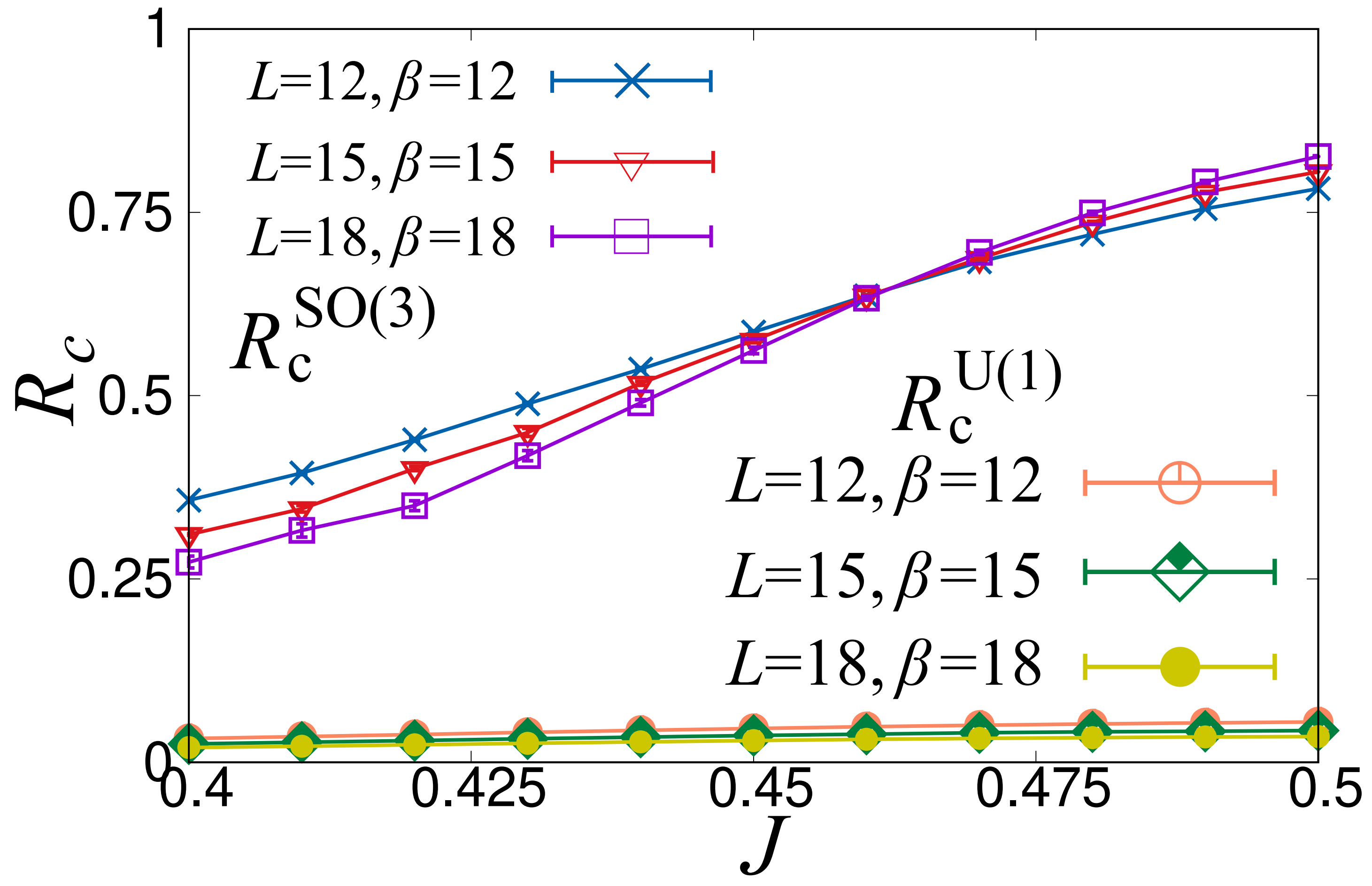
Quasiparticle weight:

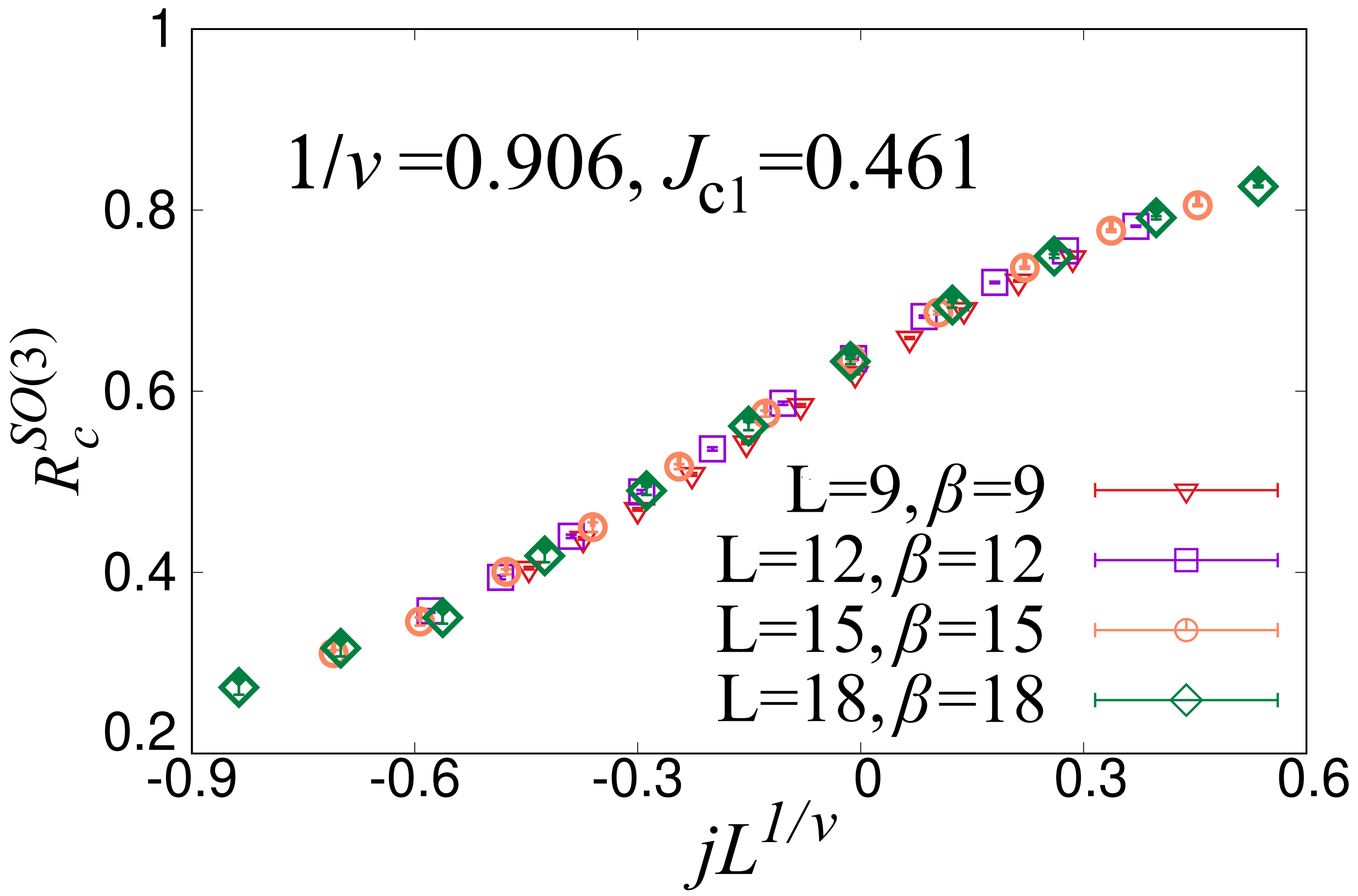


Gross-Neveu-SO(3) transition at J_{c1}



Correlation ratio: $$R_c = 1 - \frac{S(\Gamma + d\vec{k})}{S(\Gamma)}$$

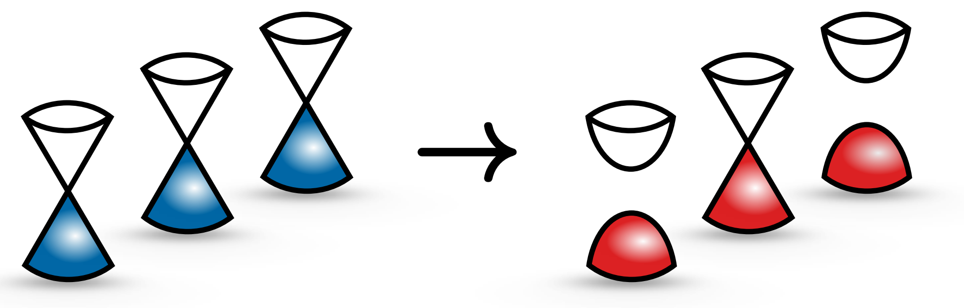




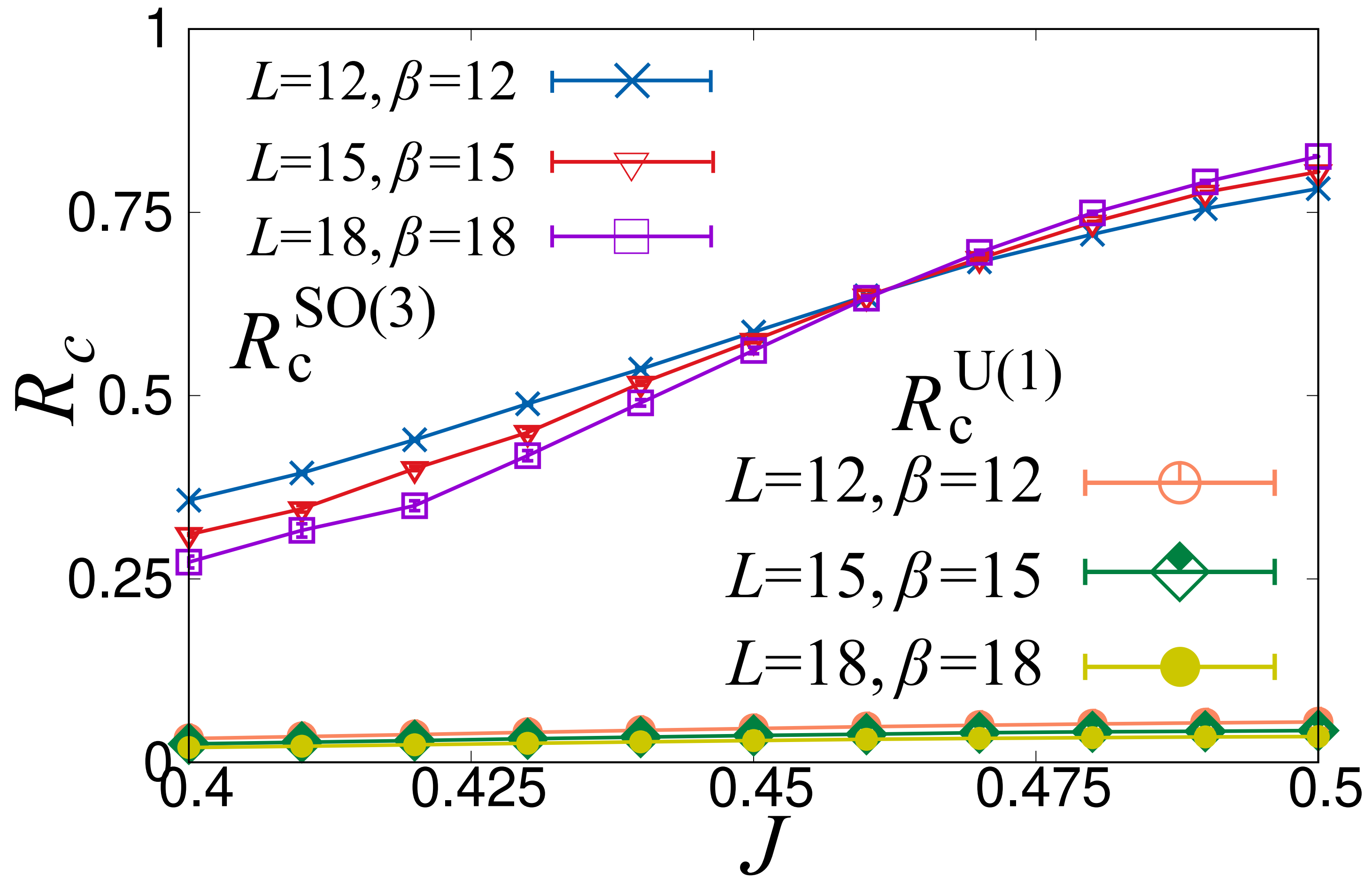
$\Rightarrow \quad 1/\nu = 0.906(35)$

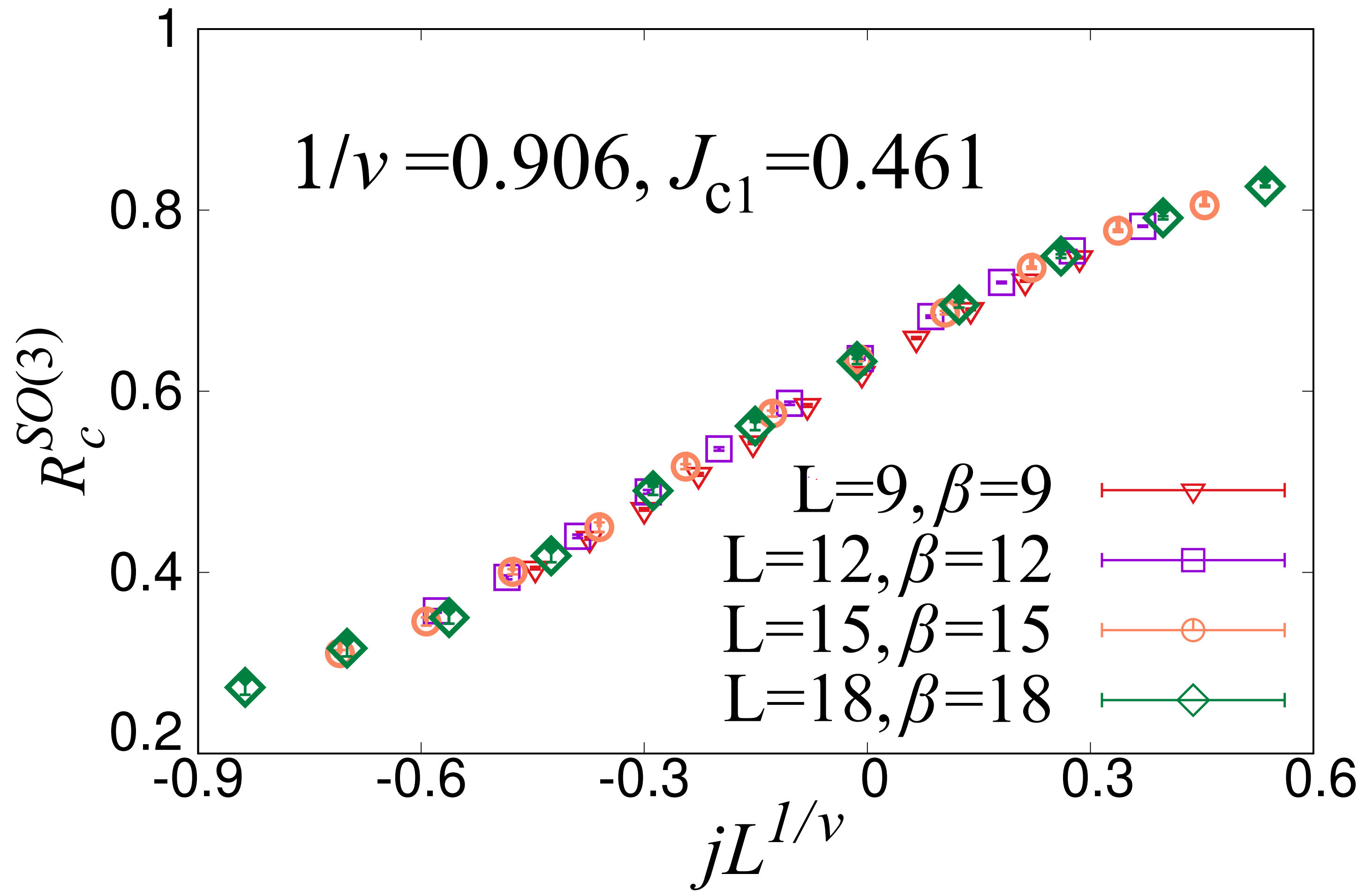
... cf. $1/\nu = 0.93(4)$ and $\eta_\phi = 0.83(4)$ from field theory
[Ray, Ihrig, Kruti, Gracey, Scherer, LJ, PRB '21]

Gross-Neveu-SO(3) transition at J_{c1}



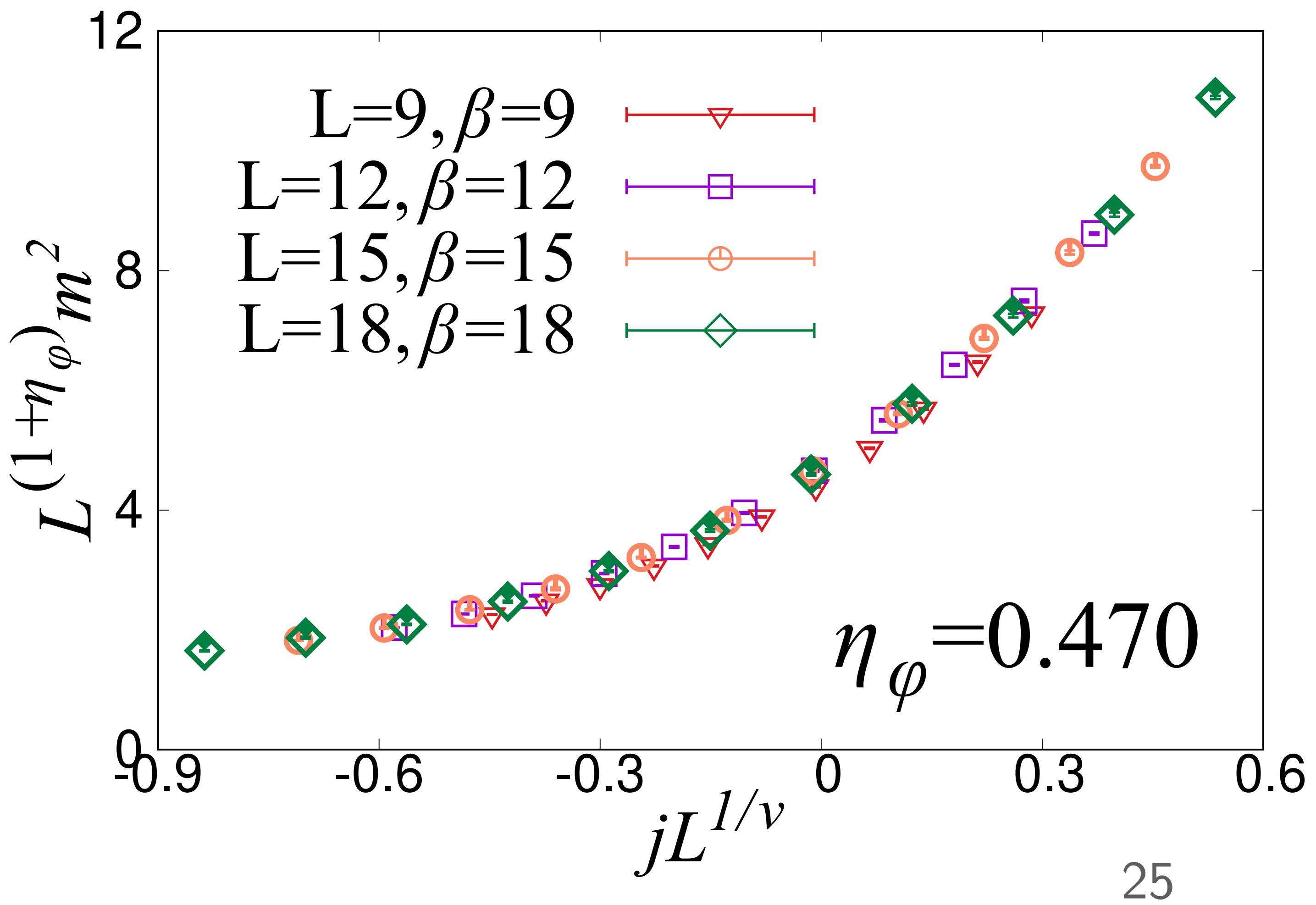
Correlation ratio: $$R_c = 1 - \frac{S(\Gamma + d\vec{k})}{S(\Gamma)}$$





$\Rightarrow \quad 1/\nu = 0.906(35)$

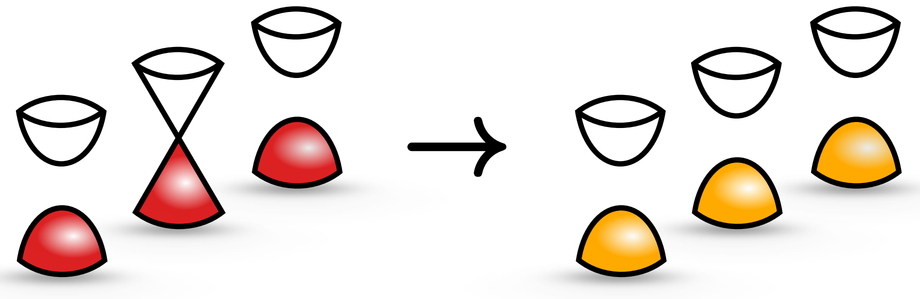
Order parameter:



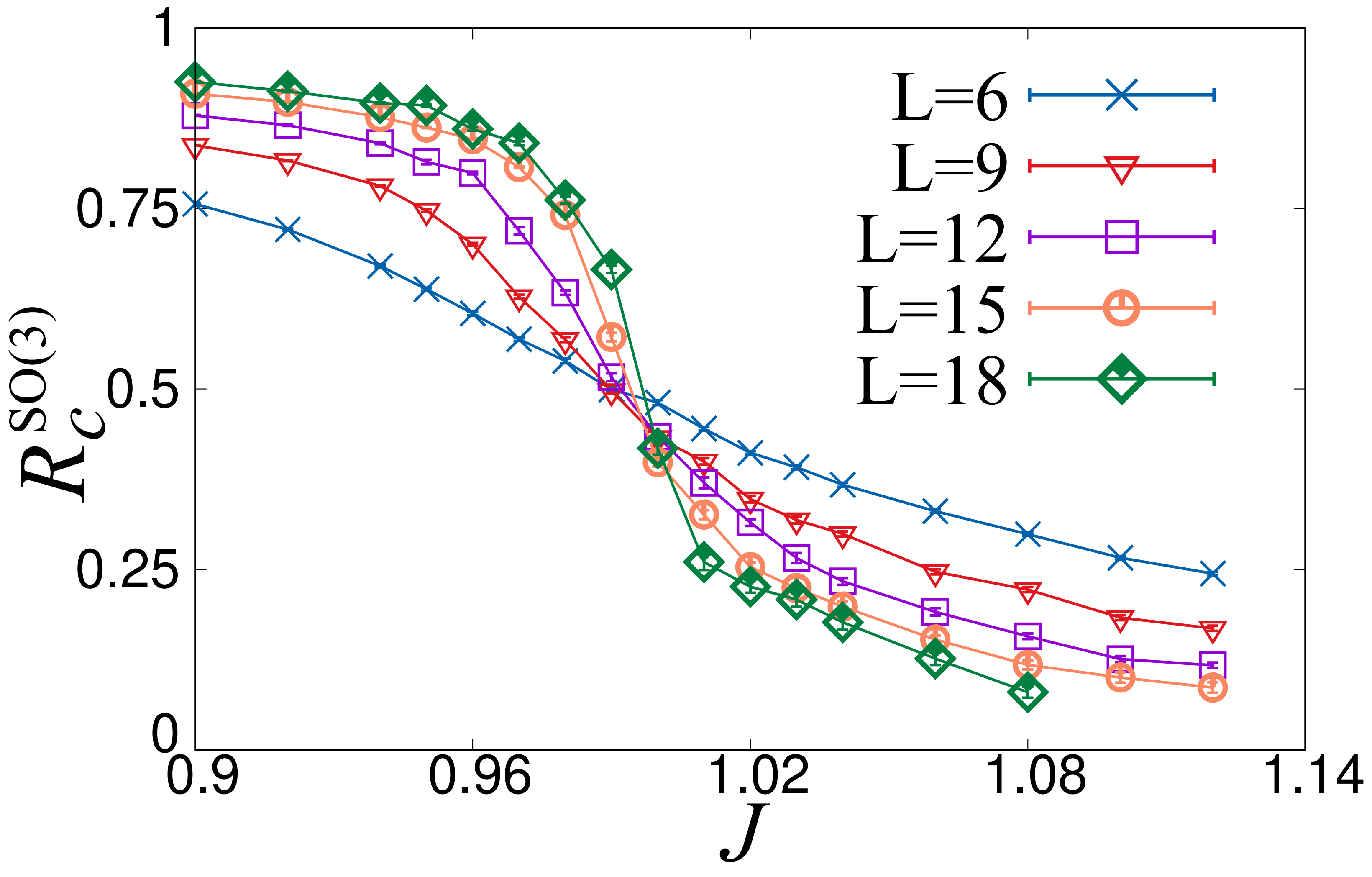
$\Rightarrow \quad \eta_\phi = 0.470(13)$

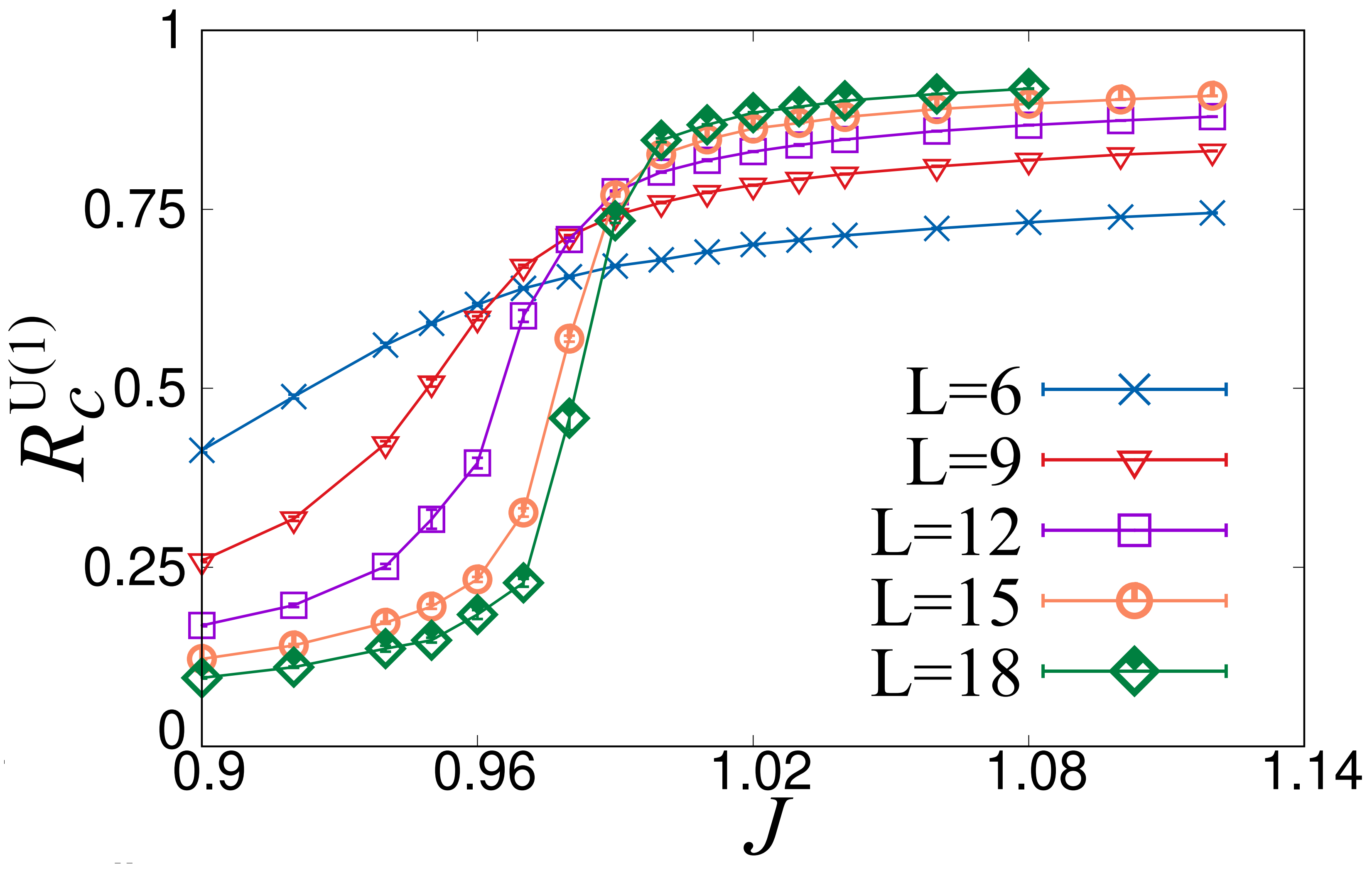
... cf. $1/\nu = 0.93(4)$ and $\eta_\phi = 0.83(4)$ from field theory
[Ray, Ihrig, Kruti, Gracey, Scherer, LJ, PRB '21]

SO(3)-U(1) transition at J_{c2}



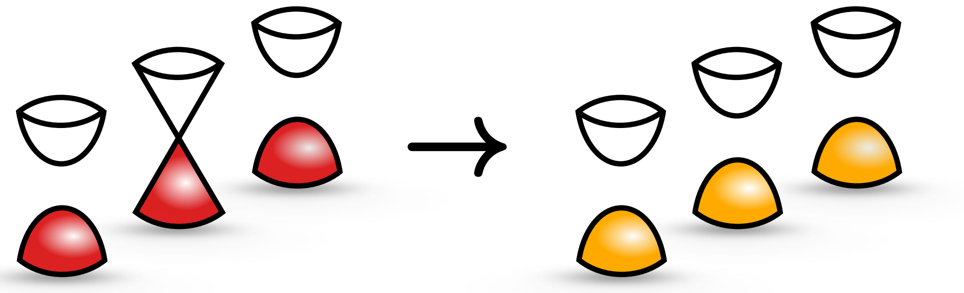
Correlation ratios:



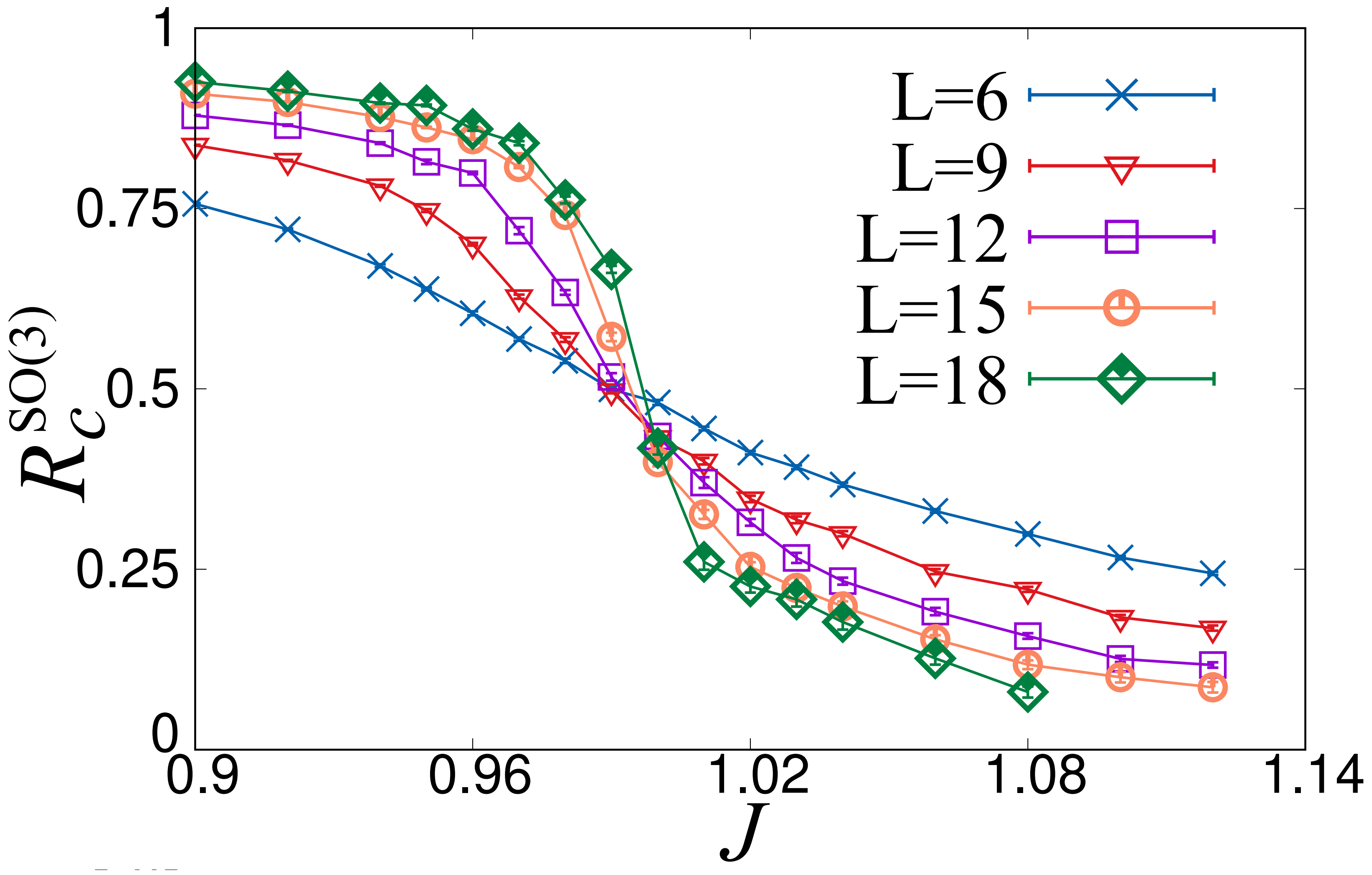


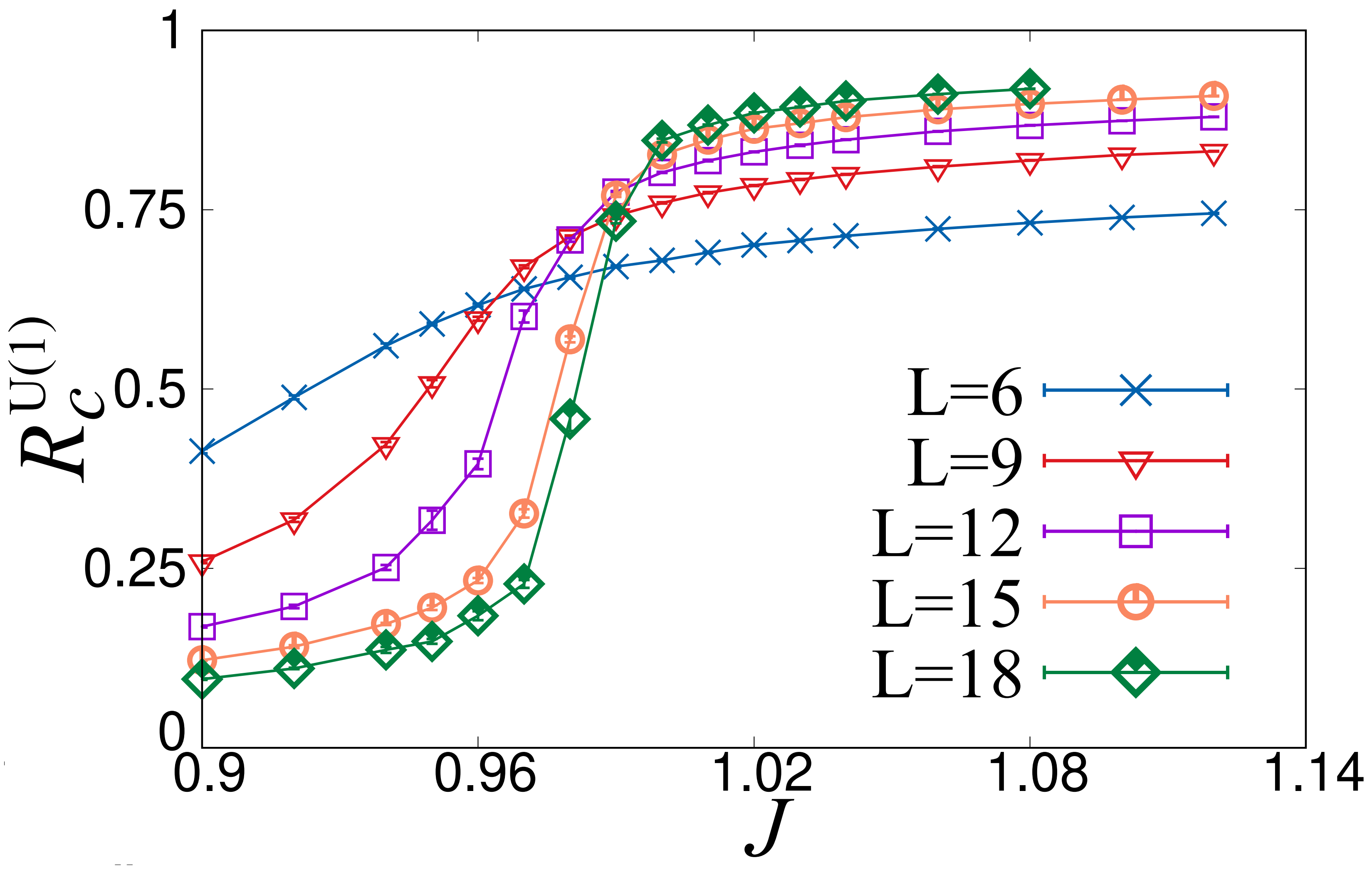
[Liu, Vojta, Assaad, LJ, PRL ’22 (Editors’ Suggestion)]

SO(3)-U(1) transition at J_{c2}

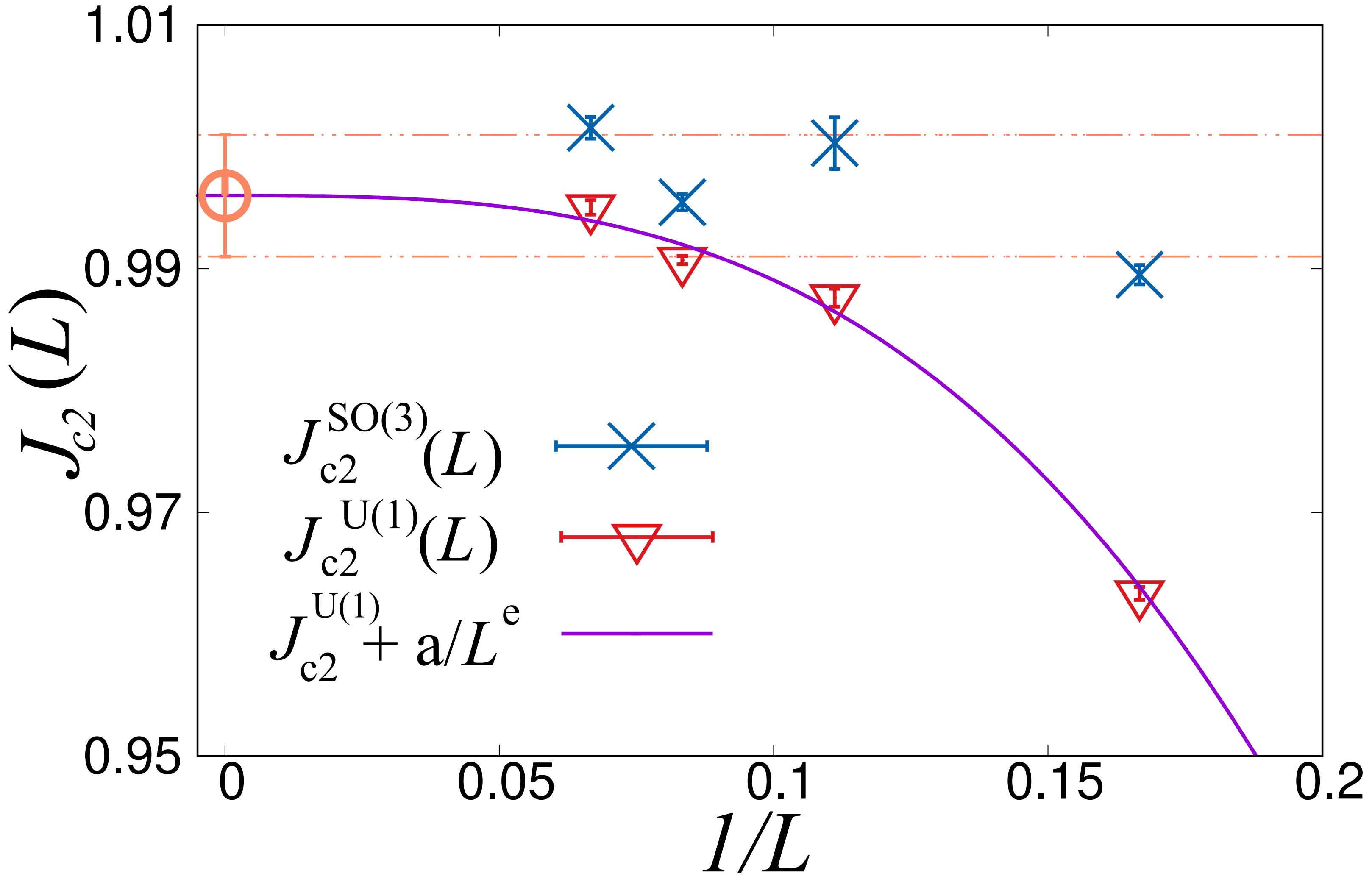


Correlation ratios:





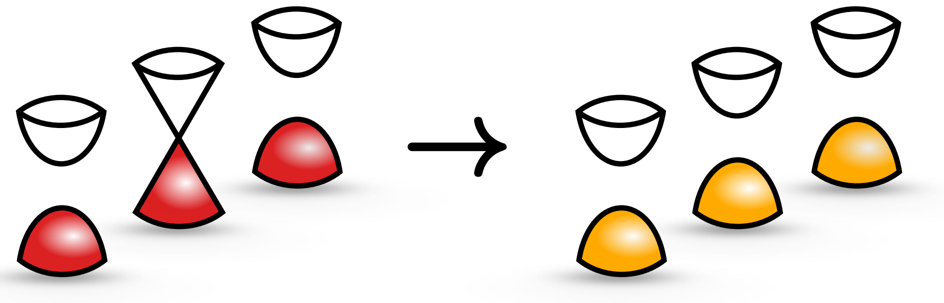
Critical couplings:



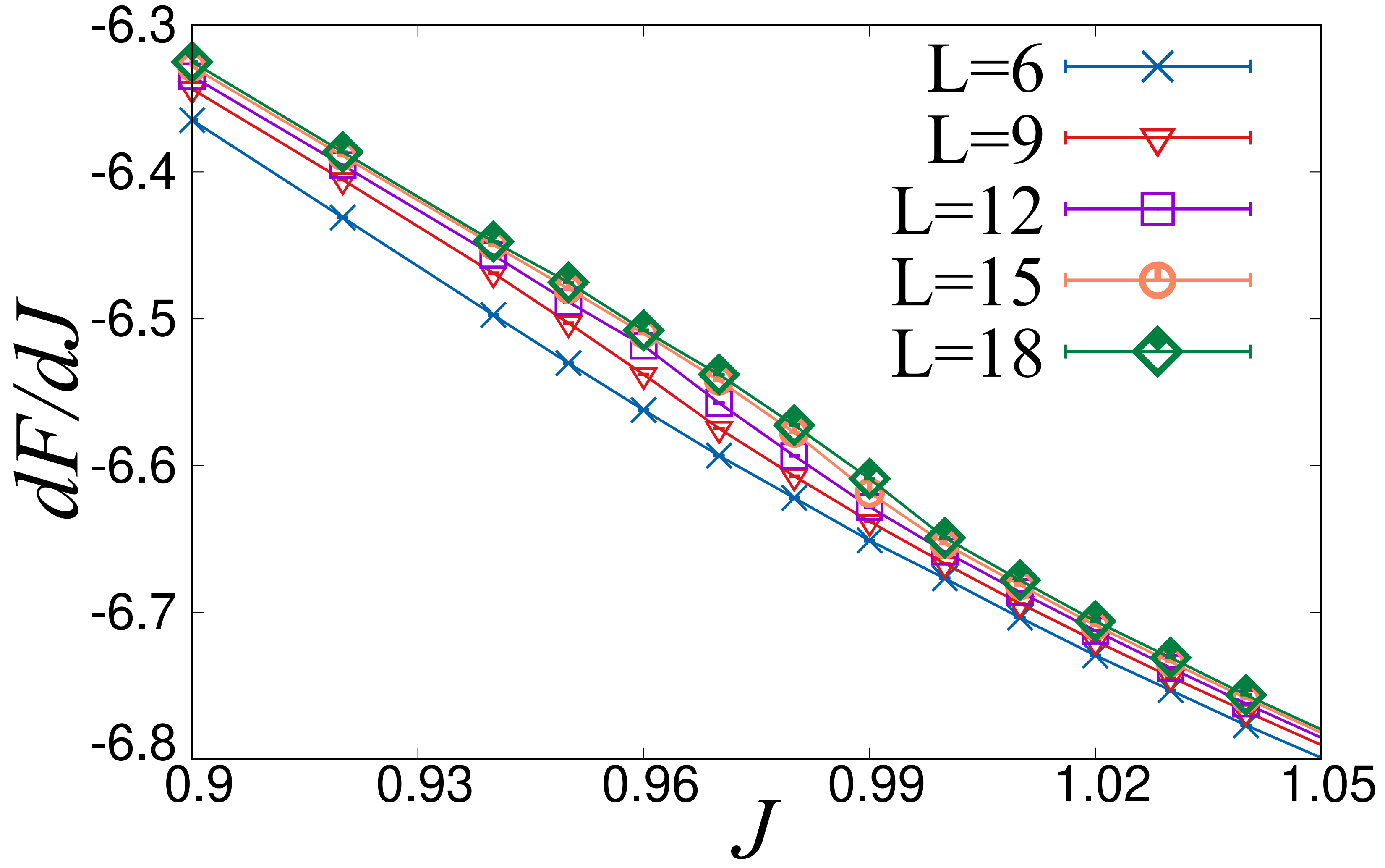
$\Rightarrow \quad J_{c2}^{SO(3)} = J_{c2}^{U(1)}$ unique!

[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]

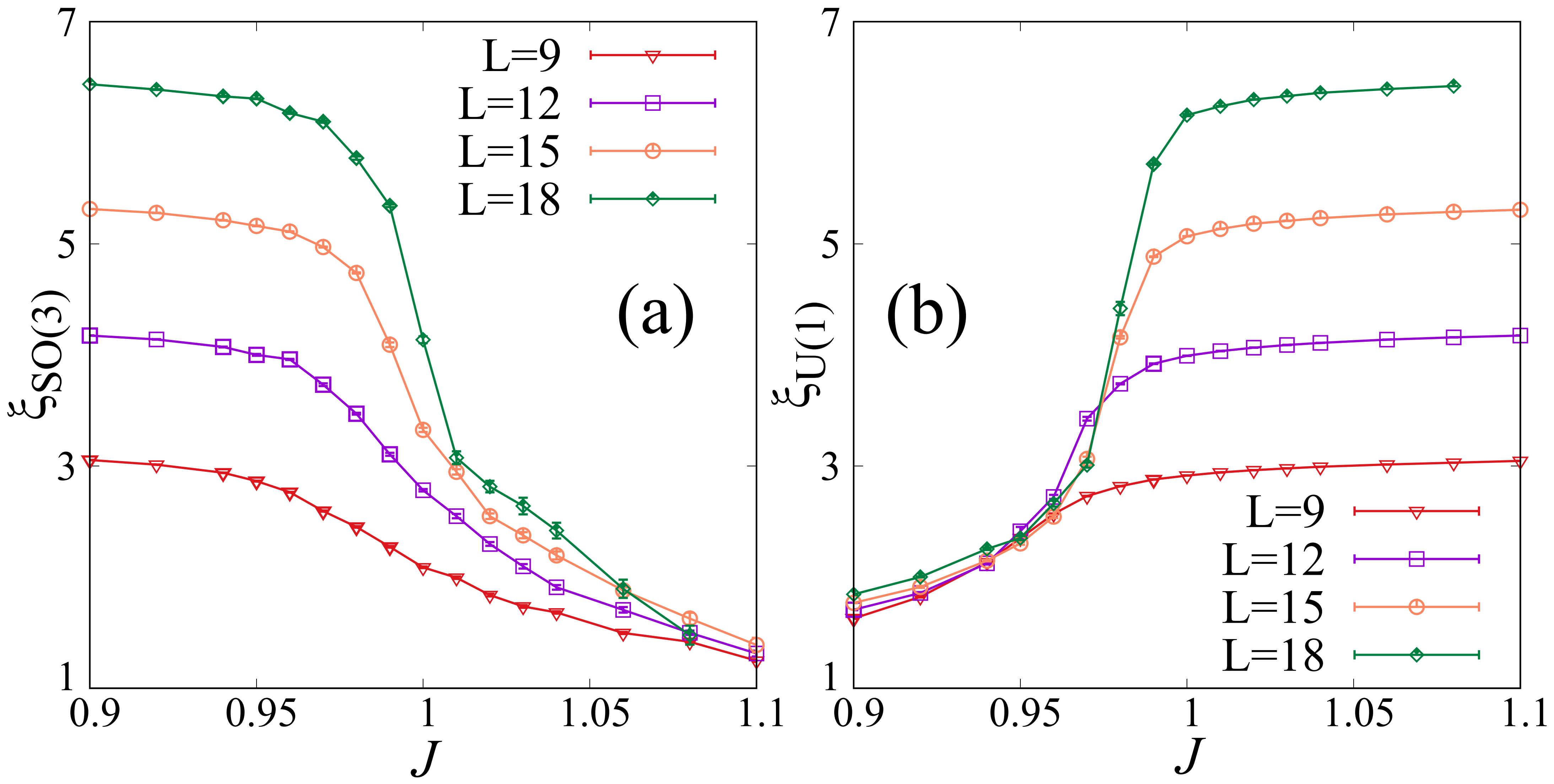
SO(3)-U(1) transition at J_{c2}



Free energy:



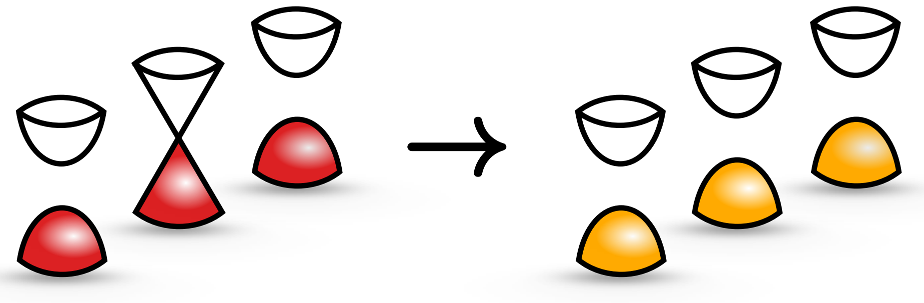
Correlation lengths:



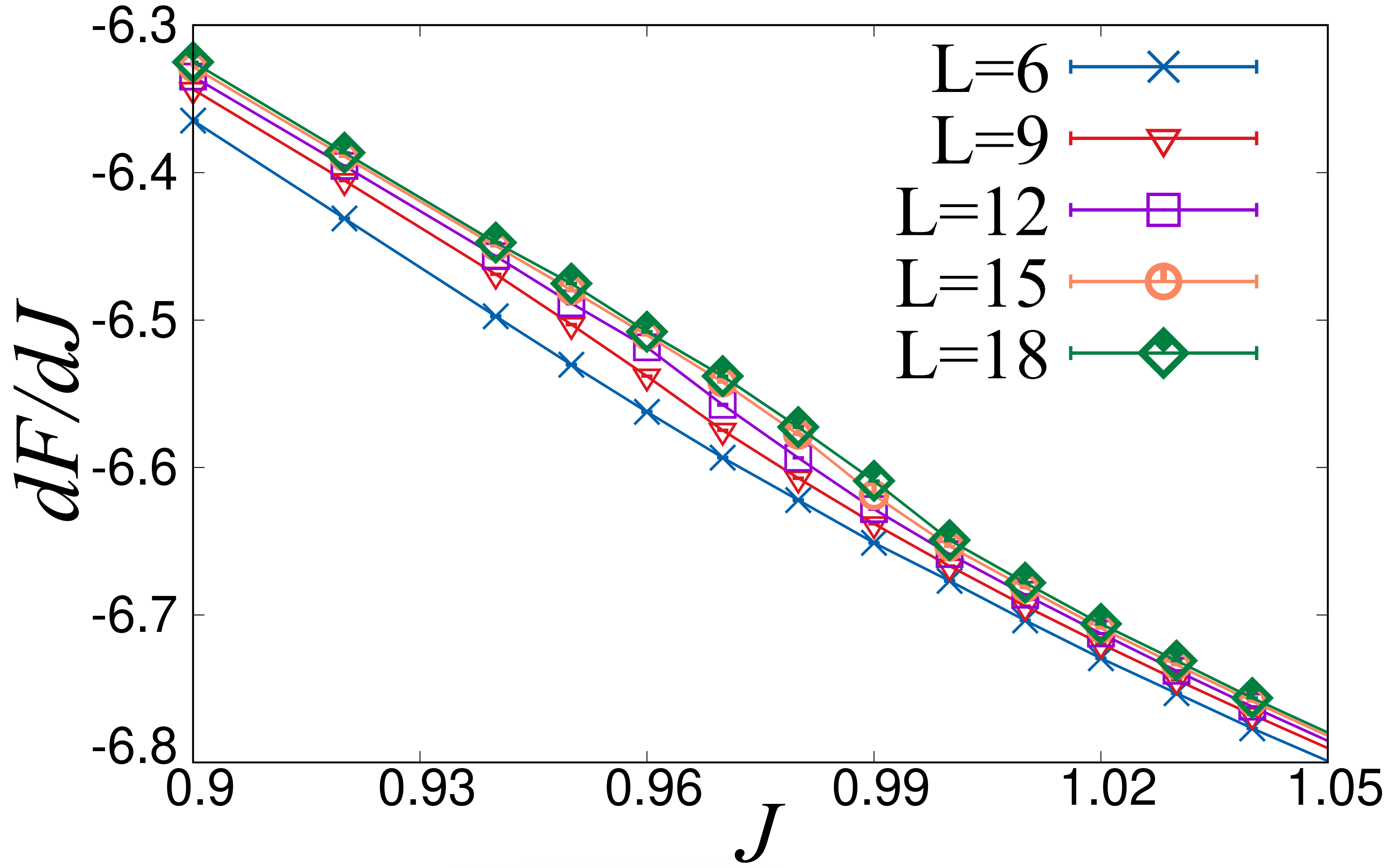
$$\xi^2 = \frac{1}{2d} \frac{\sum_{\vec{r}} |\vec{r}|^2 S(\vec{r})}{\sum_{\vec{r}} S(\vec{r})}$$

[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]

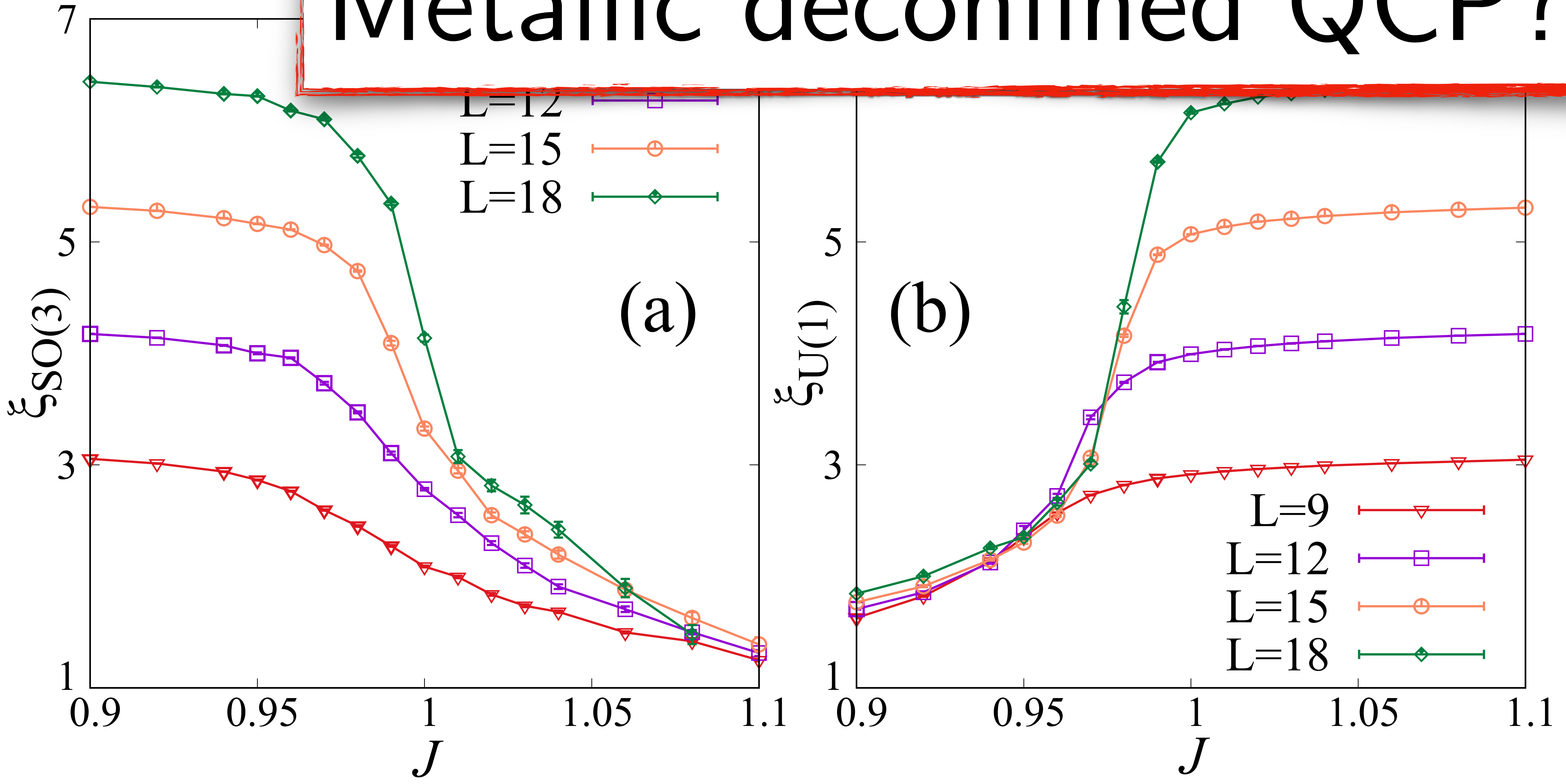
SO(3)-U(1) transition at J_{c2}



Free energy:



Correlation lengths:

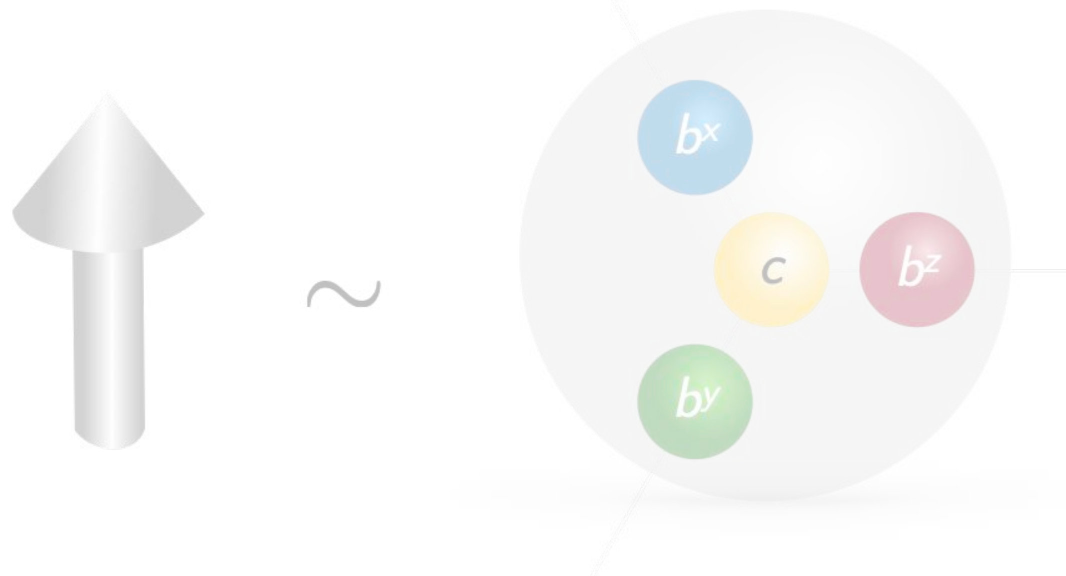


Metallic deconfined QCP?

$$\xi^2 = \frac{1}{2d}\frac{\sum_{\vec{r}} |\vec{r}|^2 S(\vec{r})}{\sum_{\vec{r}} S(\vec{r})}$$

[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]

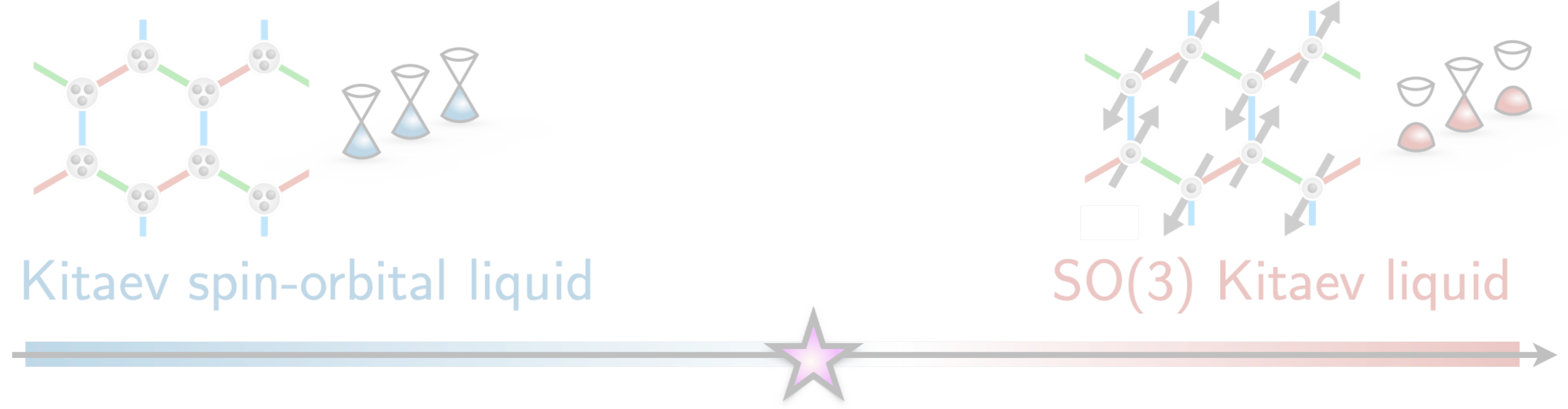
Outline



(1) Fractionalized quantum criticality

(2) Frustrated spins and spin-orbitals

(3) Square-lattice Kitaev-Ising spin-orbital model

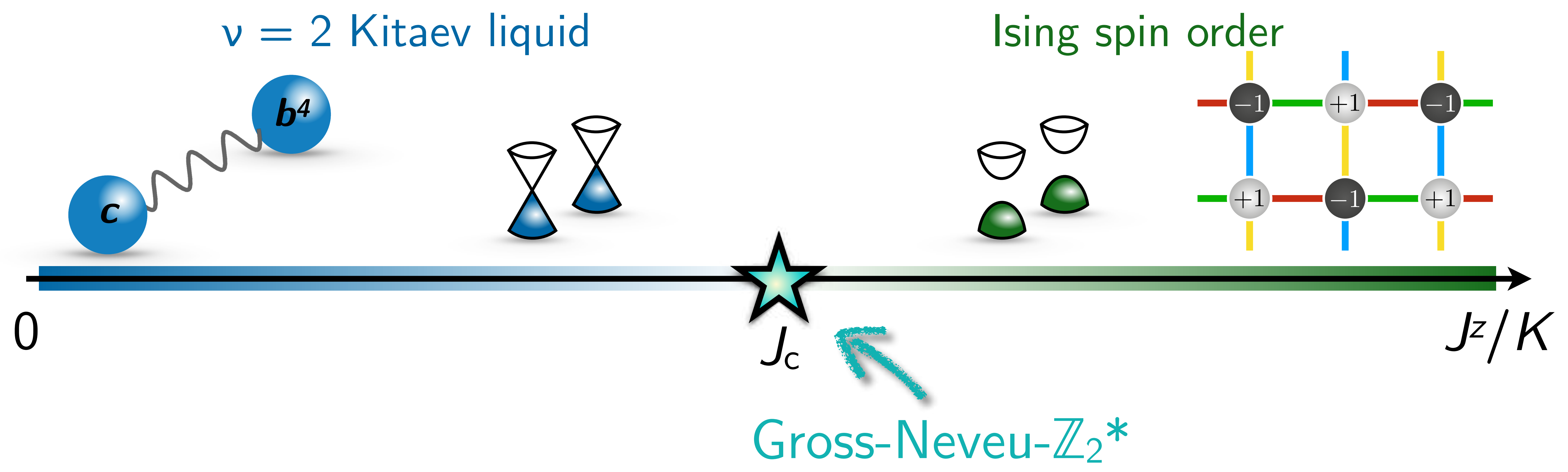


(4) Honeycomb-lattice Kitaev-Heisenberg spin-orbital model

(5) Conclusions

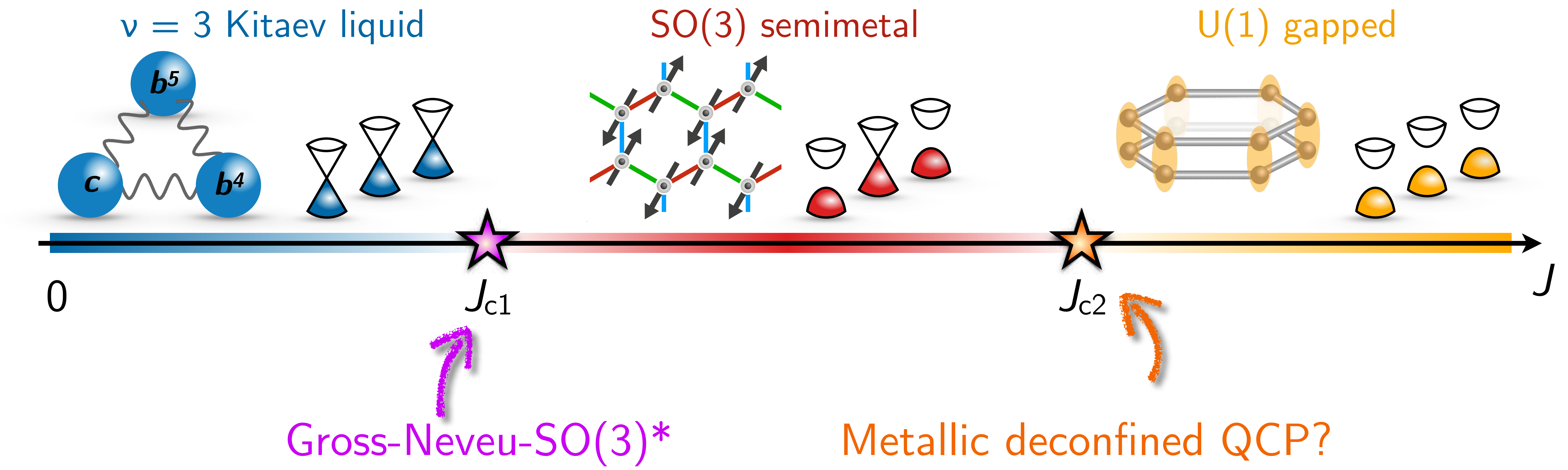
Conclusions

Square-lattice Kitaev-Ising spin-orbital model:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

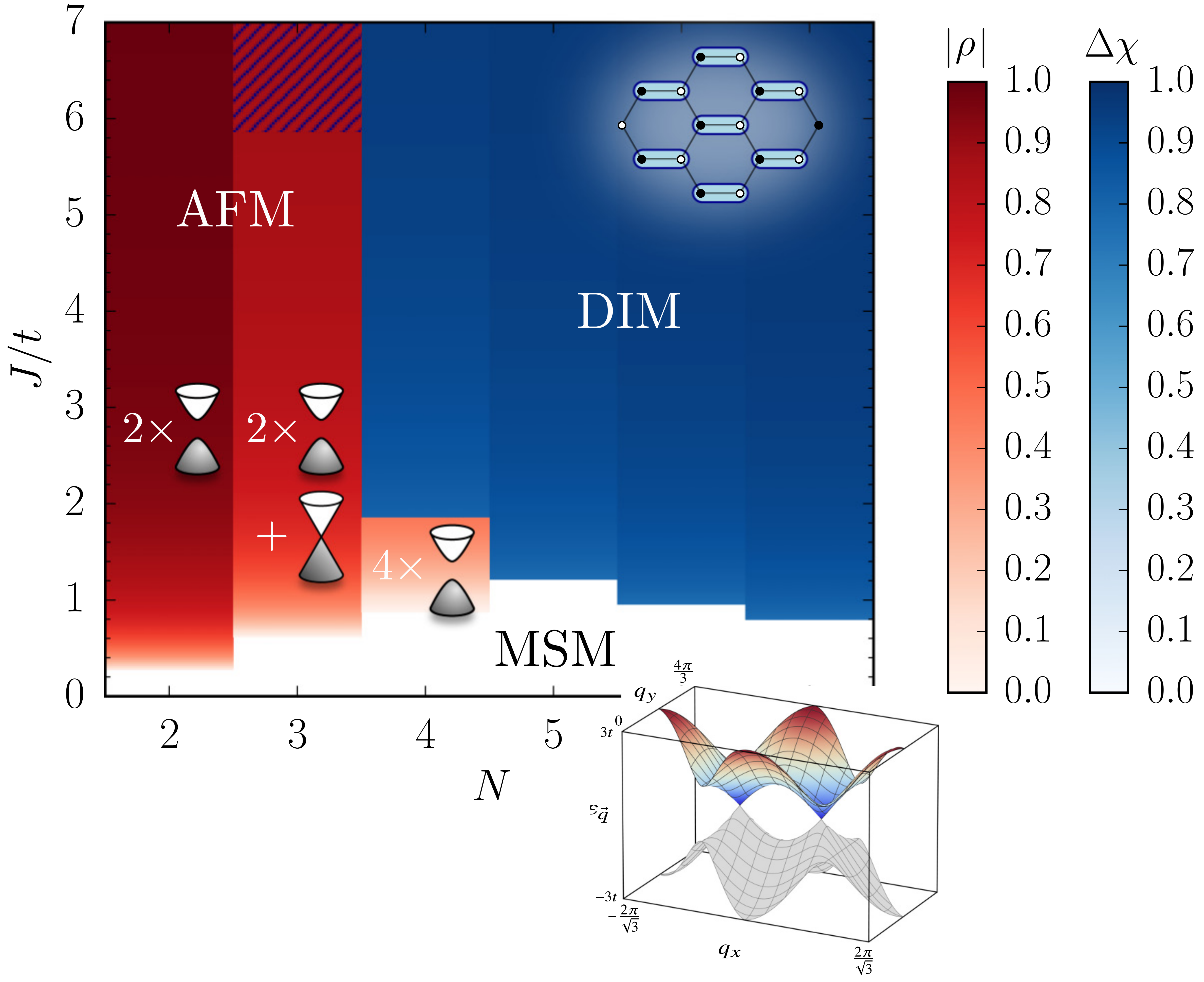
Honeycomb-lattice Kitaev-Heisenberg spin-orbital model:



[Liu, Vojta, Assaad, LJ, PRL '22]

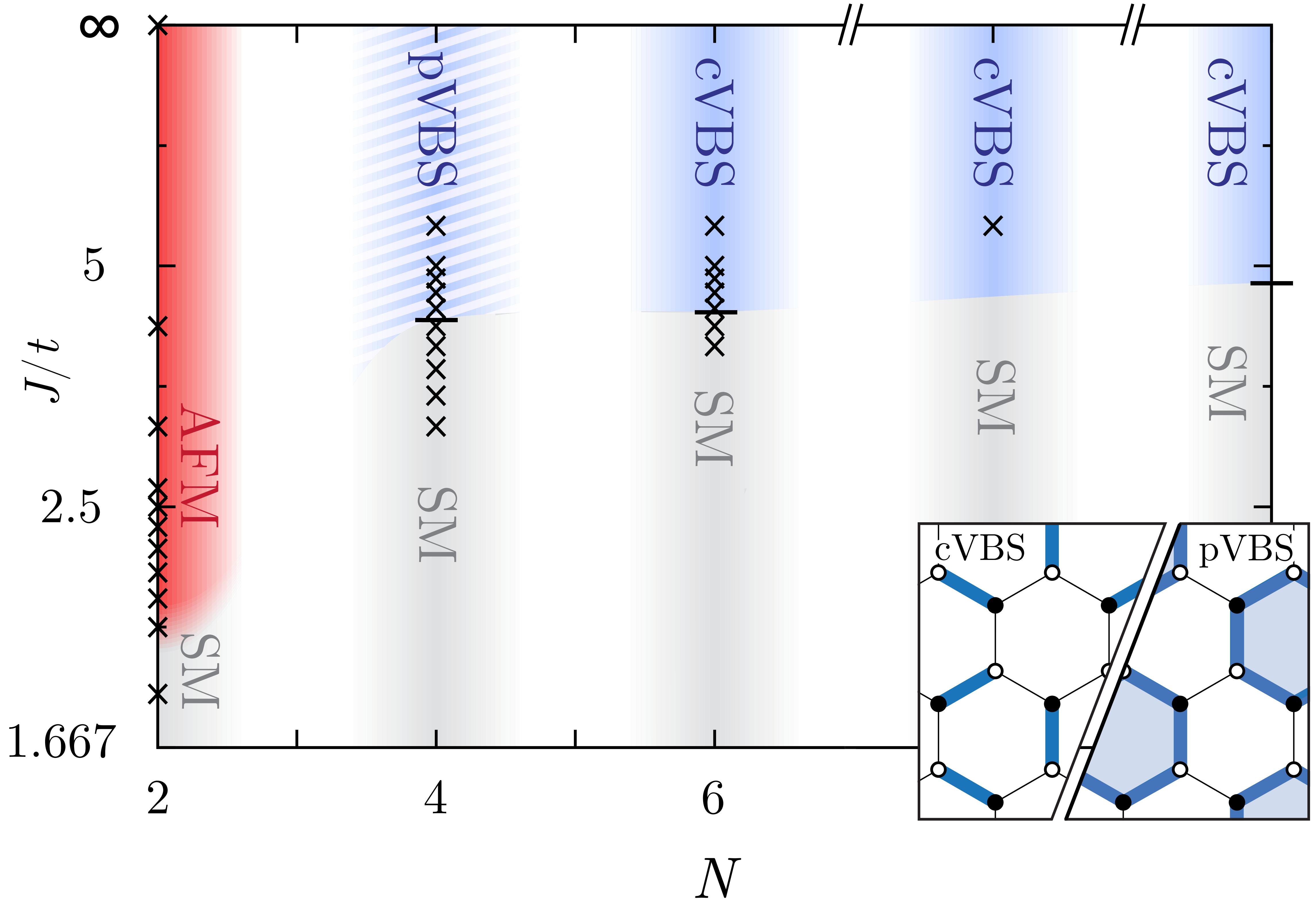
Conclusions

SO(N) Majorana-Hubbard models



[LJ & Seifert, PRB '22]

SU(N) Hubbard-Heisenberg models



[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]