Dirac fermions in cond-mat systems

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Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma_\mu \psi_i + \phi^a \bar{\psi}_i \Upsilon^a_{ij} \psi_j + \dots$$

with Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\mu, \nu = 0, \dots, d$

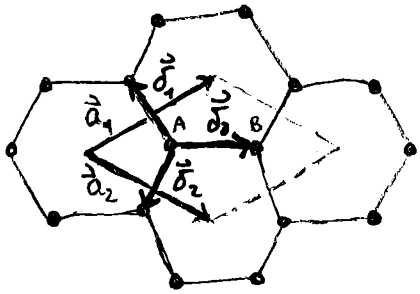
Examples:

(1) Standard model ($d=3$)

(2) Graphene ($d=2$)

(3) Quantum magnets ($d=1,2,3$)

Honeycomb lattice:



$a = 2.46$ Å

Tight-binding Hamiltonian:

$$\hat{\mathcal{H}}_0 = -t \sum_{\langle ij \rangle, \sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + \text{h.c.}$$

with $\sigma = \uparrow, \downarrow$ and $t = 2.7$ eV

Half-filling constraint:

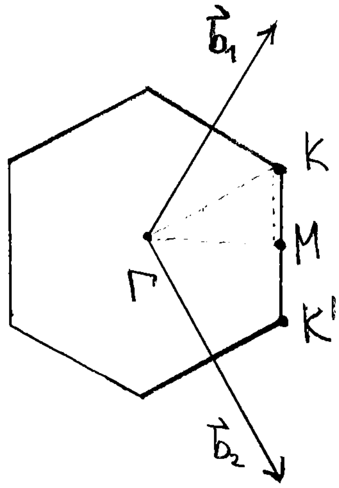
$$\left\langle \sum_{i,\sigma} \hat{n}_{i\sigma} \right\rangle = \frac{1}{2} \underbrace{2N}_{\text{maximal number of electrons}}$$

with $\hat{n}_{i\sigma} = \hat{c}^{+}_{i\sigma}\hat{c}_{i\sigma}$ fermion number operator

Fourier transform:

$$\hat{c}_{i\sigma} = \begin{cases} \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{R}_i} \hat{c}^{A}_{\vec{q}\sigma}, & i \in A \\ \sqrt{\frac{2}{N}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{R}_i} \hat{c}^{B}_{\vec{q}\sigma}, & i \in B \end{cases}$$

with $\vec{k} \in$ Brillouin zone:

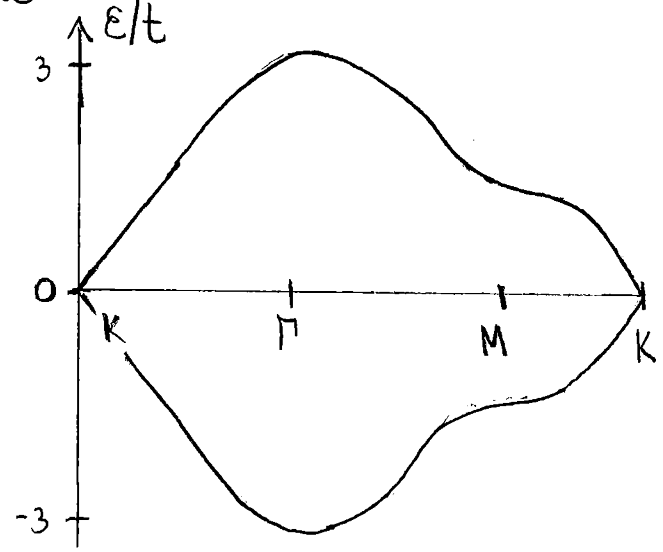


Hamiltonian:

$$\hat{\mathcal{H}}_0 = -t \sum_{\vec{k}} \left(\hat{c}^{A+}_{\vec{k}\sigma}, \hat{c}^{B+}_{\vec{k}\sigma} \right) \begin{pmatrix} 0 & f(\vec{k}) \\ f(\vec{k})^* & 0 \end{pmatrix} \begin{pmatrix} \hat{c}^{A}_{\vec{k}\sigma} \\ \hat{c}^{B}_{k\sigma} \end{pmatrix}$$

with $f(\vec{k}) = \sum_{i=1}^{3} e^{i\vec{k}\cdot\vec{\delta}_i}$

Spectrum:



Low-energy dispersion:

$$\varepsilon(\vec{q}) = \pm v_F \hbar \, |\vec{q}| + \mathcal{O}(q^2)$$

where $\vec{q} = \vec{k} - \vec{K}$ or $\vec{q} = \vec{k} - (-\vec{K})$ and

$$v_F = \frac{\sqrt{3}\, t a}{2\hbar} \approx \frac{c}{300} \qquad \text{"Fermi velocity"}$$

Effective model (non-interacting):

$$\mathcal{H}_0 = v_F \sum_{\vec{q}} \psi^\dagger_{\vec{q}\sigma} \Big(q_x (\sigma_z \otimes \sigma_x) + q_y (\mathbb{1} \otimes (-\sigma_y)) \Big) \psi_{\vec{q}\sigma} + \mathcal{O}(q^2)$$

with

$$\psi_{\vec{q}\sigma} = \begin{pmatrix} c^A_\sigma(\vec{K}+\vec{q}) \\ c^B_\sigma(\vec{K}+\vec{q}) \\ c^A_\sigma(-\vec{K}+\vec{q}) \\ c^B_\sigma(-\vec{K}+\vec{q}) \end{pmatrix} \qquad \text{4-component spinor}$$

Lagrangian ($T \to 0$):

$$\mathcal{L} = \overline{\Psi}_\sigma(\vec{x},\tau)\, \gamma_\mu \partial_\mu \Psi_\sigma(\vec{x},\tau) \qquad , \ \mu = 0,1,2$$

with $\gamma_0 = \mathbb{1}_2 \otimes \sigma_z$, $\gamma_1 = \sigma_z \otimes \sigma_y$, $\gamma_2 = \mathbb{1} \otimes \sigma_x$ [in units of $\hbar = v_F = 1$]
and $\overline{\Psi}_\sigma = \psi^\dagger \gamma_0$

→ electronic excitations in graphene described by Dirac Lagrangian

Interactions (Hubbard):

$$\hat{\mathcal{H}}_{int} = U \sum_i (\hat{n}_{i\uparrow} - \tfrac{1}{2})(\hat{n}_{i\downarrow} - \tfrac{1}{2}) \qquad , \quad n_{i\sigma} = \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma}$$

$$= U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \text{const.} \qquad \text{[at half filling]}$$

Strong-coupling limit ($\frac{t}{U} \to 0$):

$[\hat{\mathcal{H}}_{int}, \hat{n}_{i\sigma}] = 0 \Rightarrow (n_{i\uparrow}, n_{i\downarrow}) = (1,0)$ or $(0,1)$ in ground state

Charge excitation gap $\propto U$ "Mott insulator"

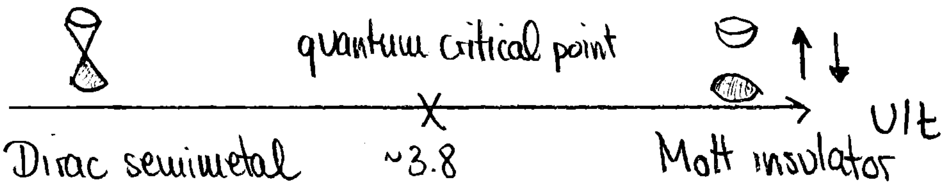
Effective model (strong interactions):

$$\hat{\mathcal{H}}_{eff} = \frac{t^2}{U} \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j + \mathcal{O}\left(\frac{t^4}{U^3}\right) \qquad \text{"antiferromagnetic H'berg model"}$$

Mean-field ground state: $|\uparrow\downarrow\uparrow\downarrow\uparrow \dots\rangle$ Néel order

Excitations: Magnons (bosons!) [collective excitations associated with spin fluctuations]

Phase diagram (AF-QMC):



[Assaad & Herbut, PRX'13]

Effective model (critical point):

$$\mathcal{L} = \bar{\psi}_\sigma \gamma_\mu \partial_\mu \psi_\sigma + g\, \vec{\phi} \cdot \bar{\psi}_\sigma \vec{\sigma}_{\sigma\sigma'} \psi_{\sigma'}$$

"Gross-Neveu-SU(2) model"

Critical behavior:

$$\xi \sim (U-U_c)^{-\nu} \quad \text{with} \quad \nu \approx \begin{cases} 0.83 & \text{GN-SU(2) model, } \varepsilon \text{ expansion [Ladovrechis et al., PRB'23]} \\ 0.86 & \text{Hubbard model, HMC [Buividovich et al., PRB'18]} \\ \dots & \end{cases}$$

$$\langle \phi(\vec{x},\tau)\,\phi(0,0)\rangle \sim \frac{1}{(\vec{x}^2+\tau^2)^{\frac{1+\eta_\phi}{2}}} \quad \text{with} \quad \eta_\phi \approx \begin{cases} 1.01 & \text{GN-SU(2) model} \\ 0.87 & \text{Hubbard model} \\ \dots & \end{cases}$$