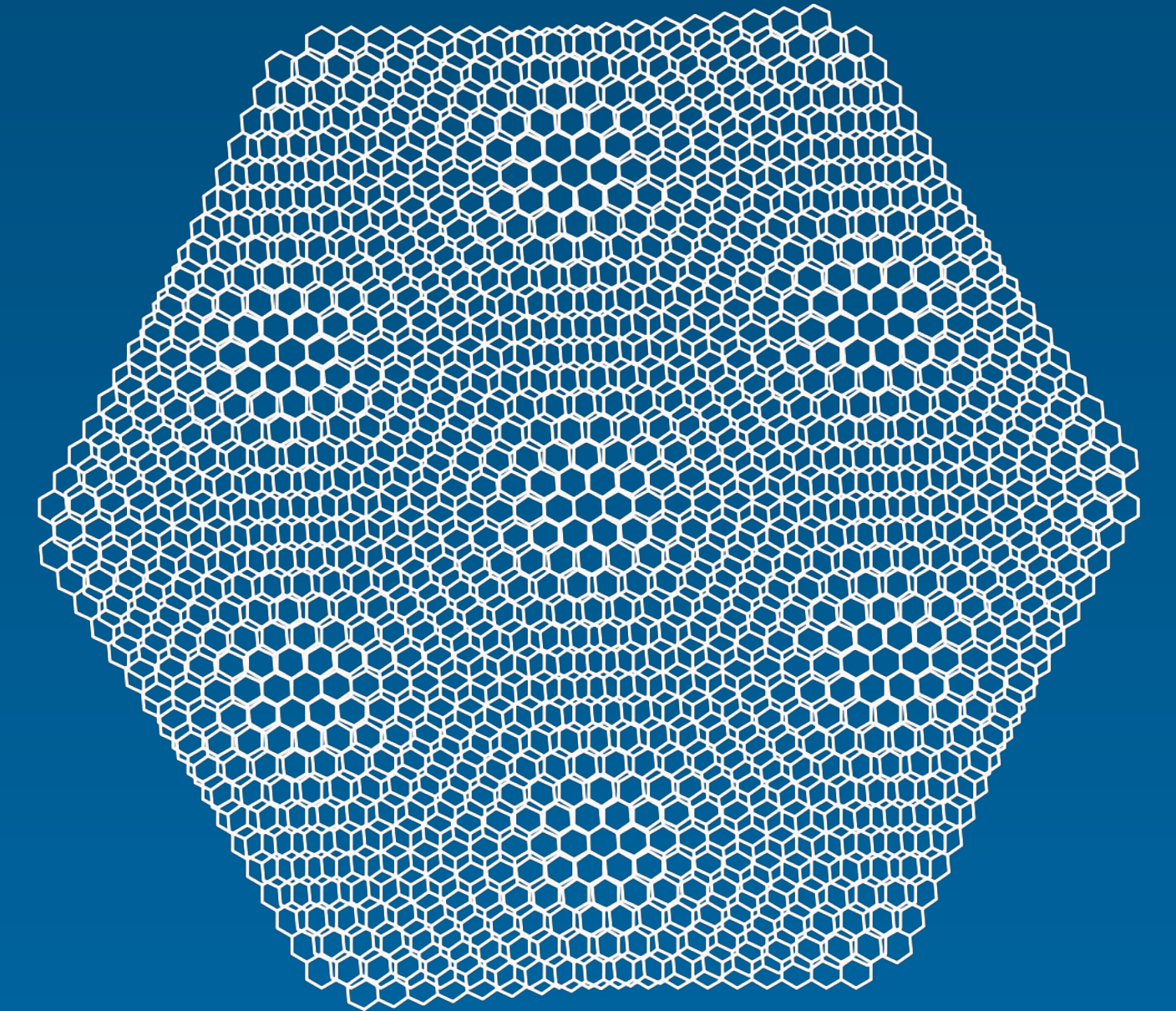


Dirac quantum criticality in moiré systems

Lukas Janssen



Jan Biedermann



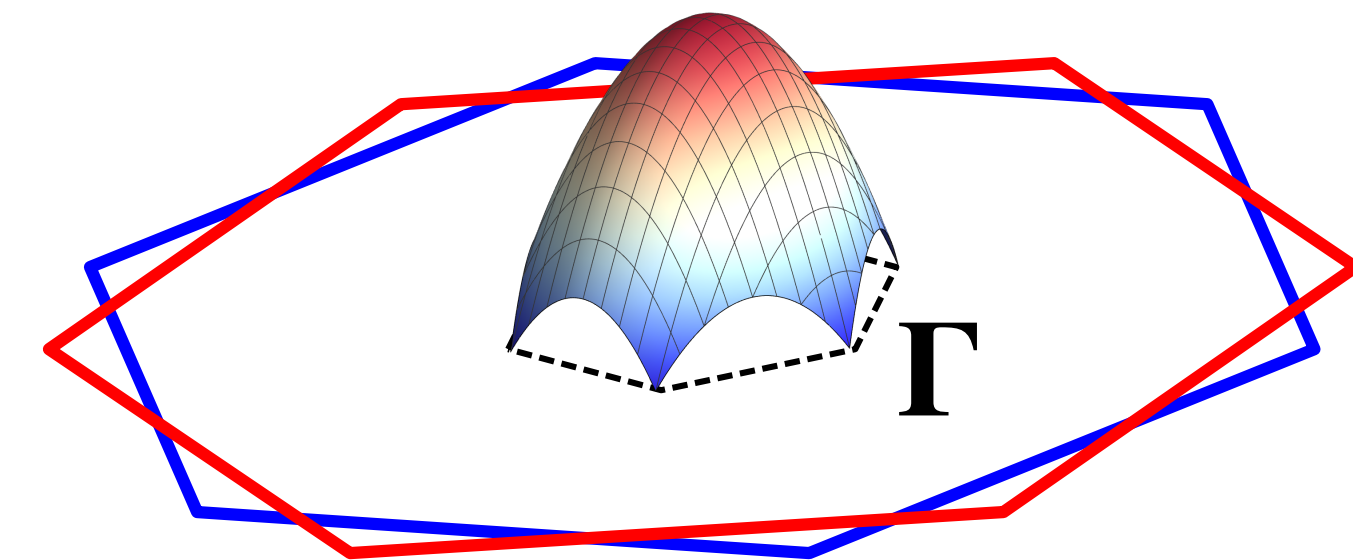
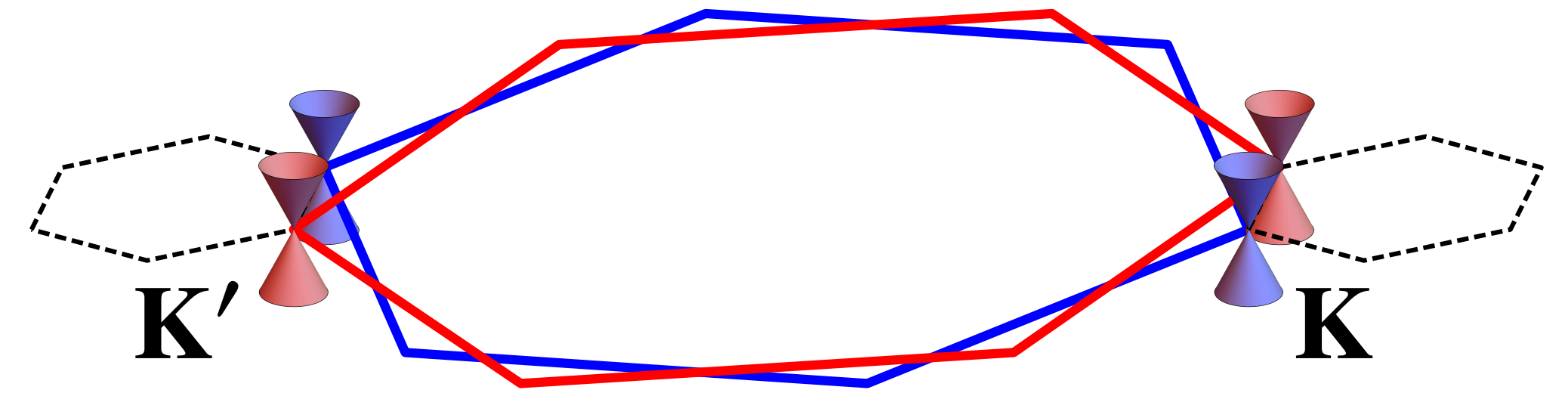
Outline

(1) Introduction

(2) Twisted bilayer graphene

(3) Twisted double bilayer TMDs

(4) Conclusions



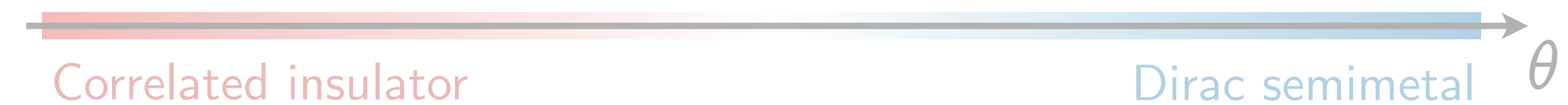
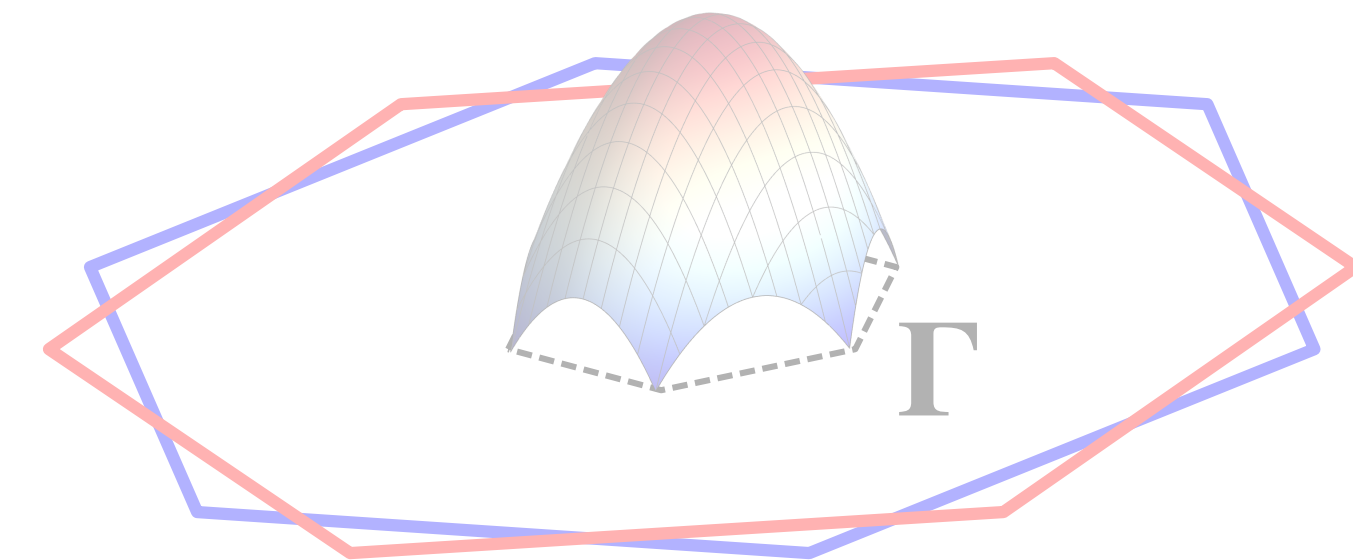
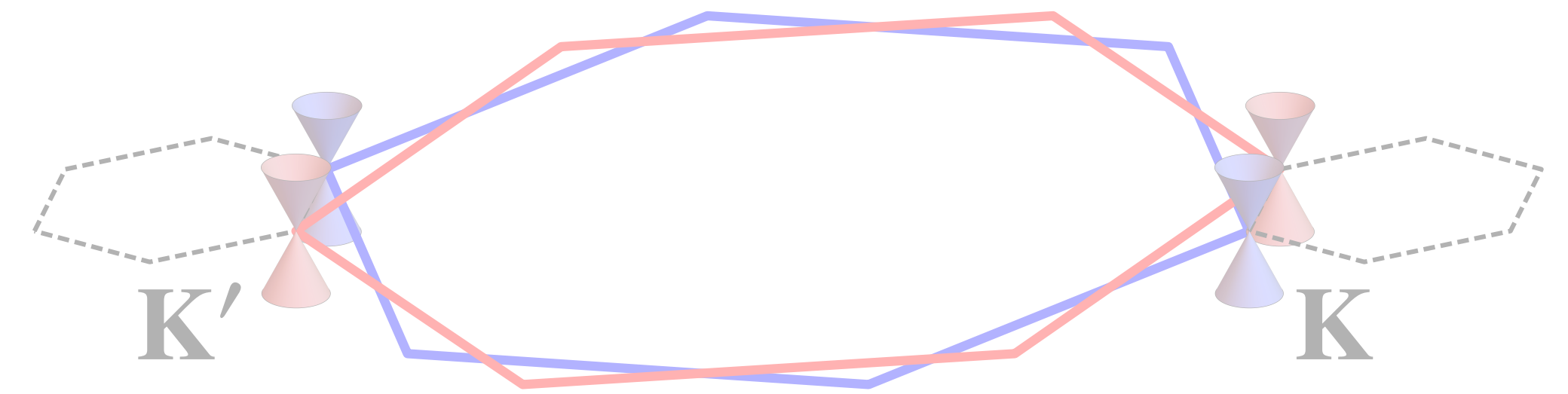
Outline

(1) Introduction

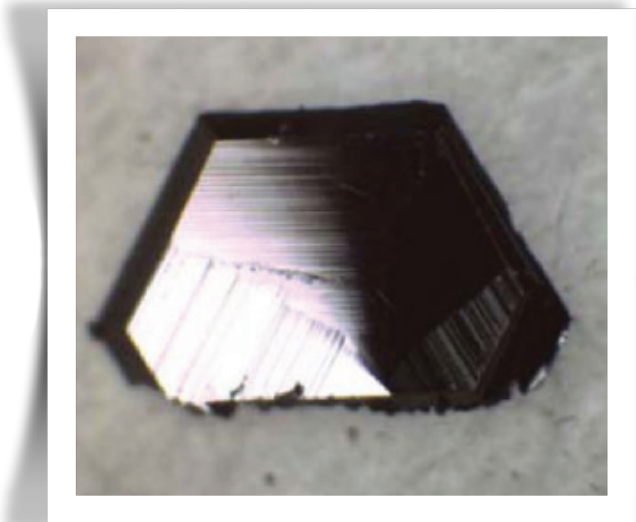
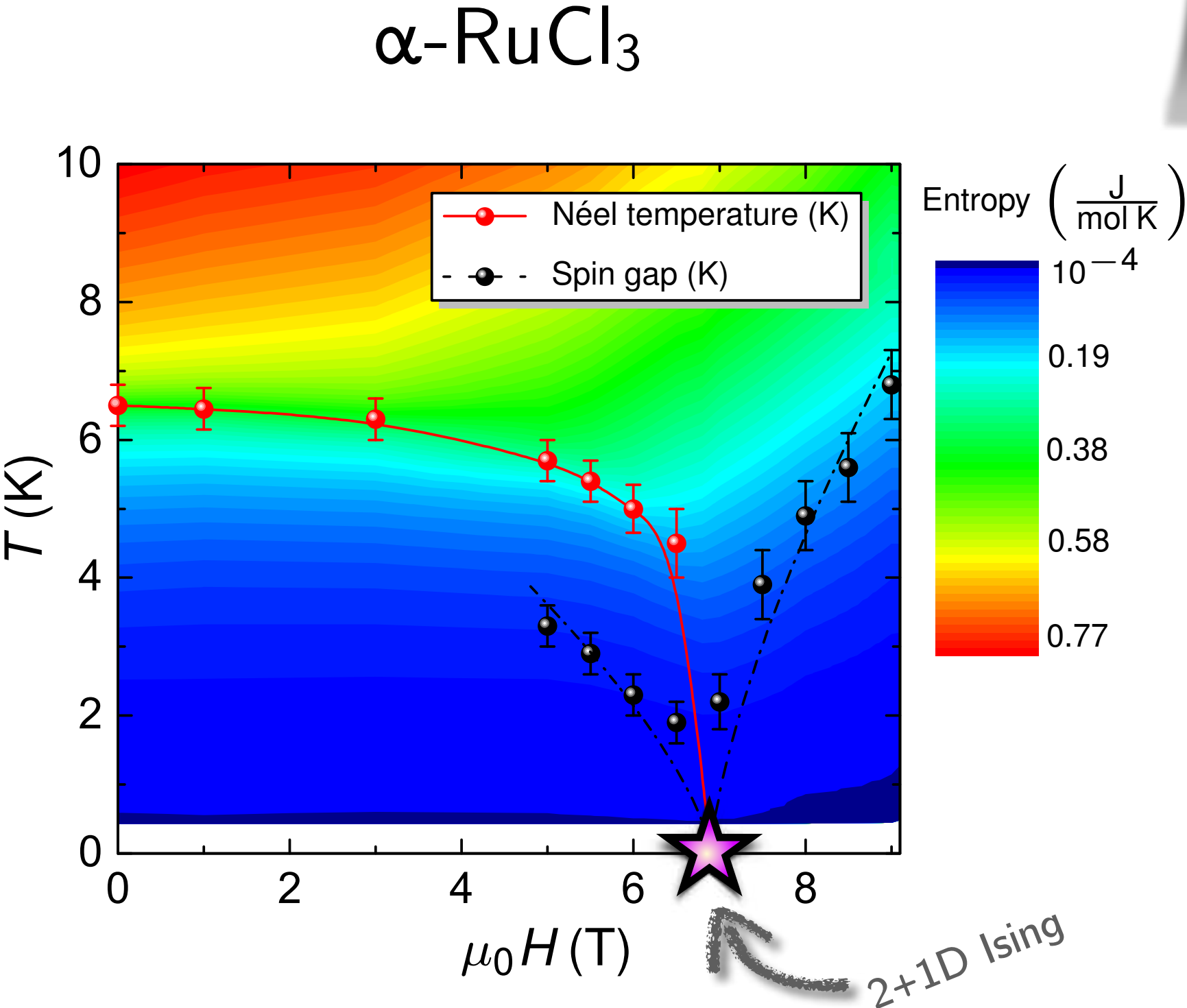
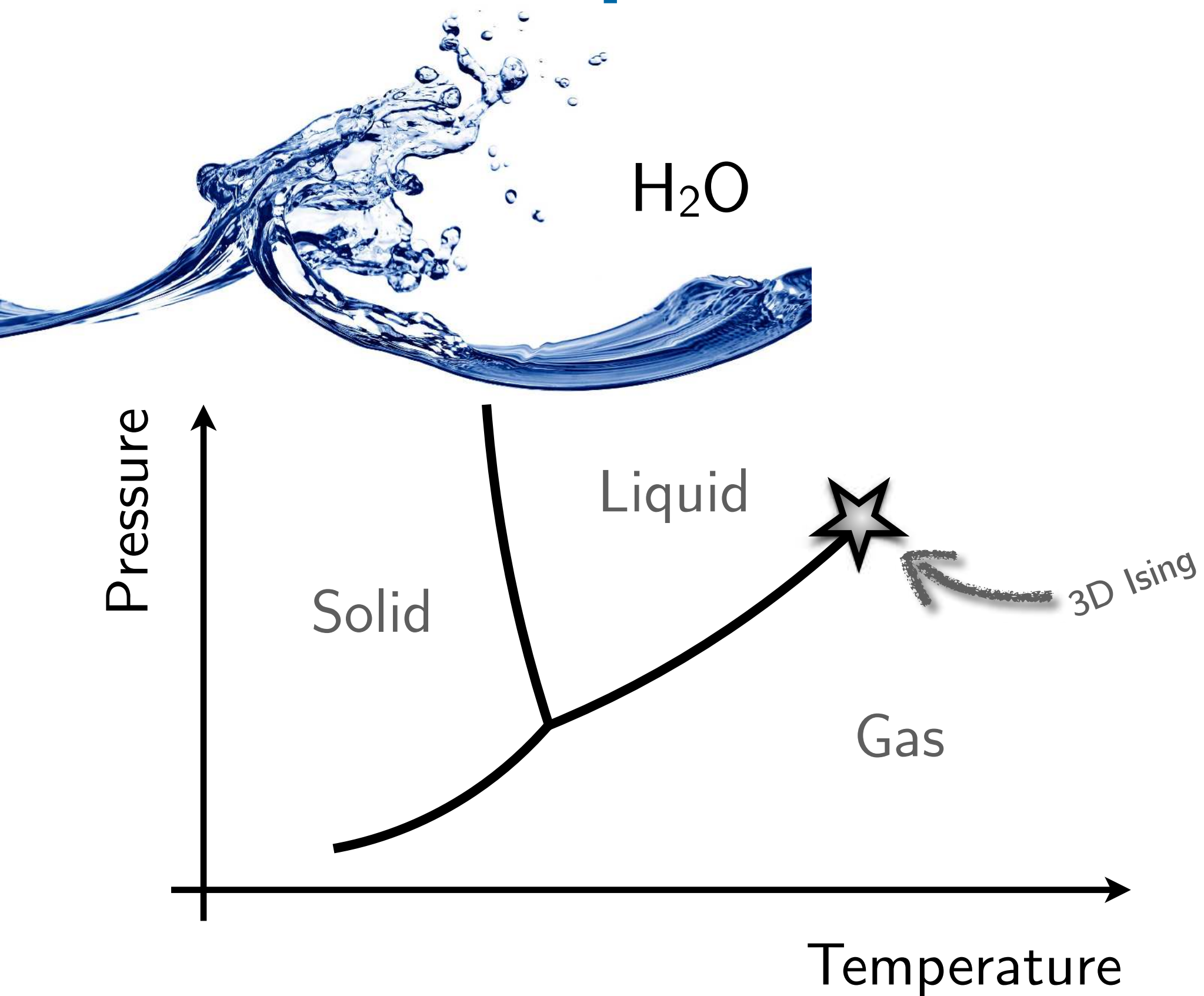
(2) Twisted bilayer graphene

(3) Twisted double bilayer TMDs

(4) Conclusions



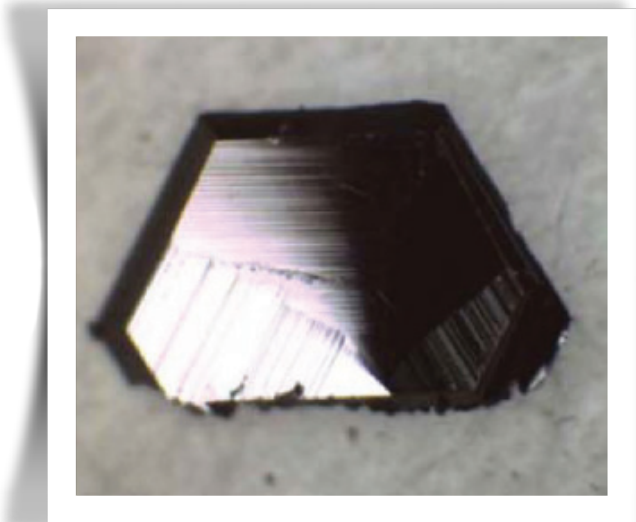
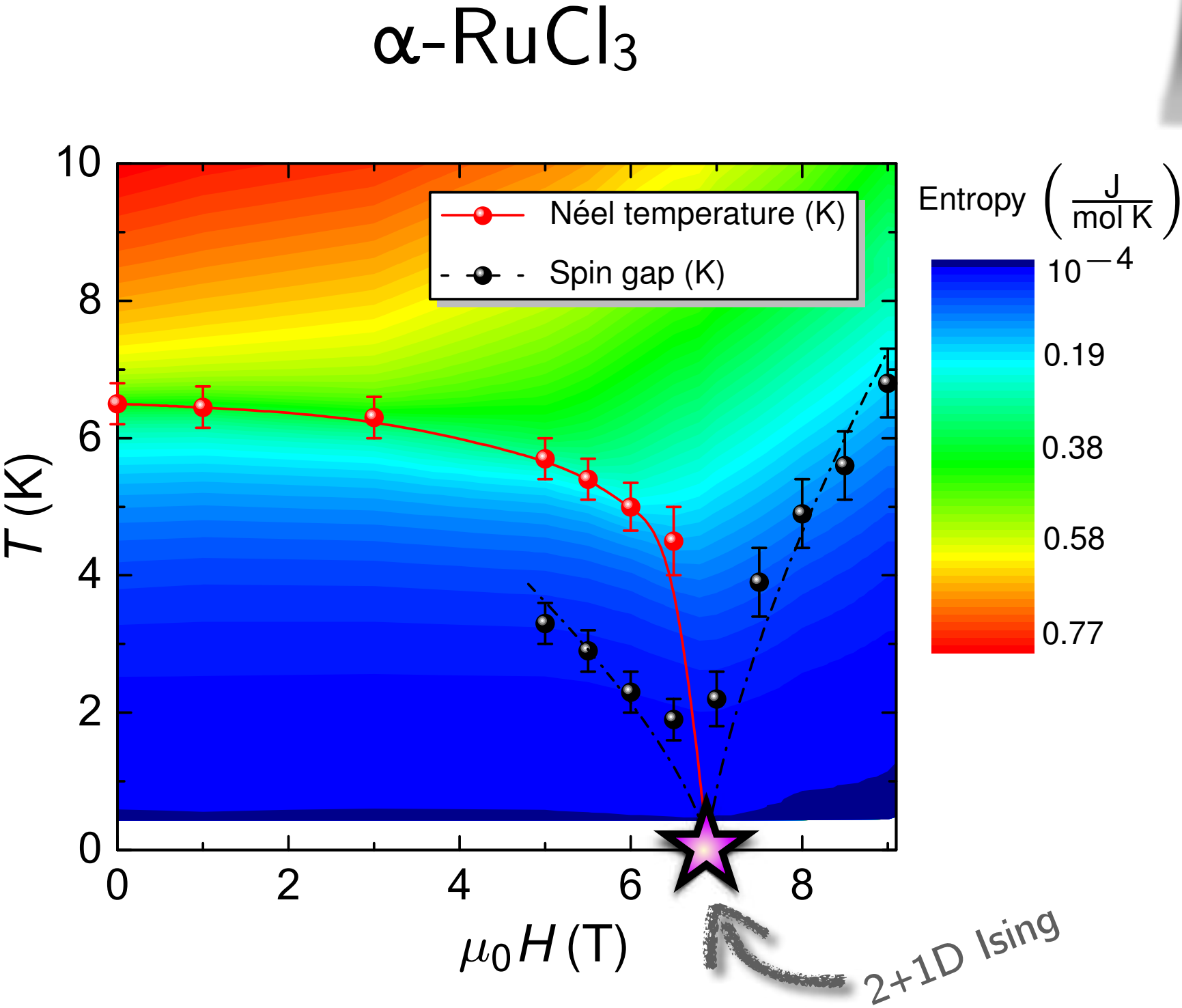
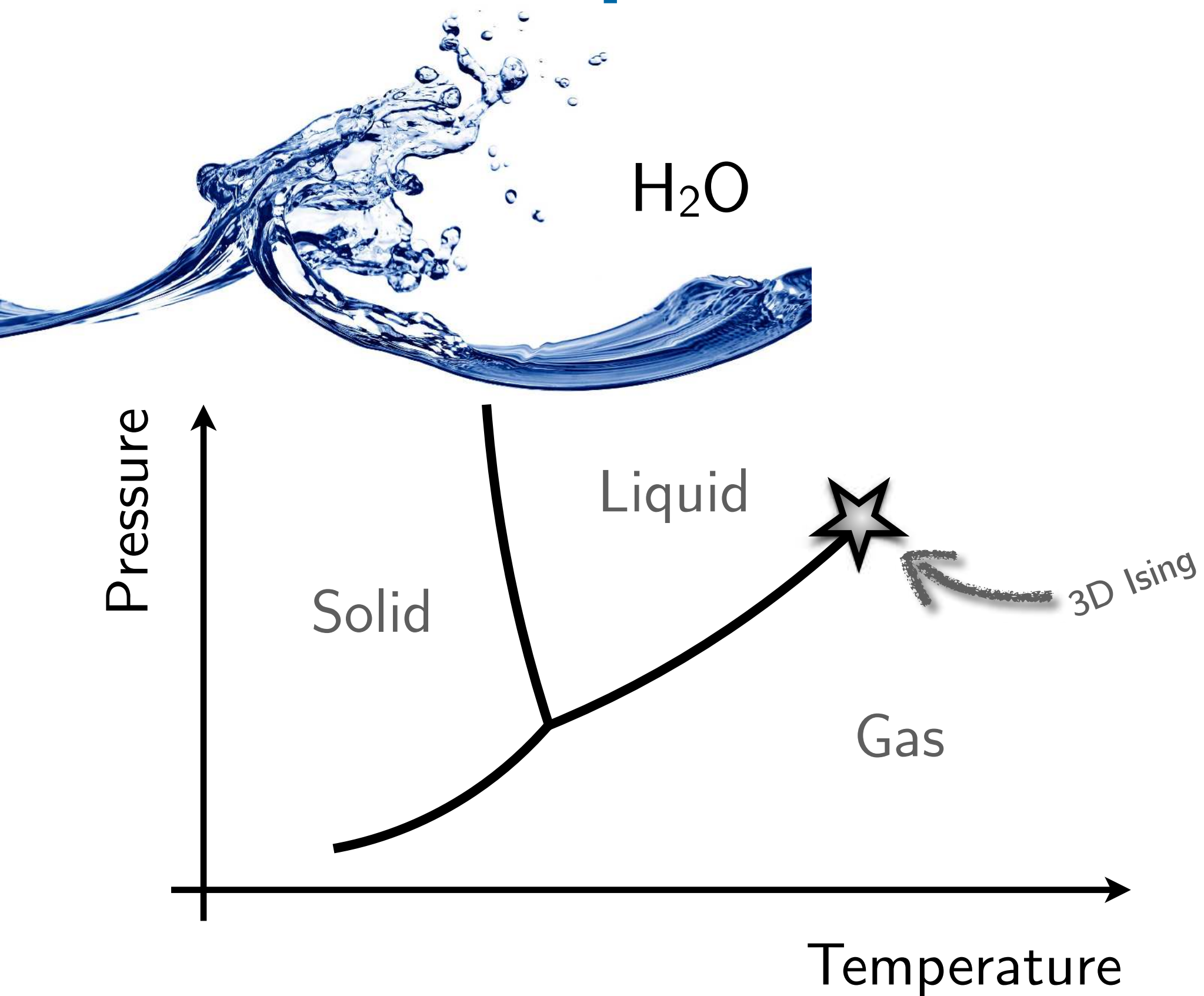
Classical vs quantum criticality



Credit: Jennifer Sears

[Wolter, Corredor, LJ, et al., PRB '17]

Classical vs quantum criticality



Credit: Jennifer Sears

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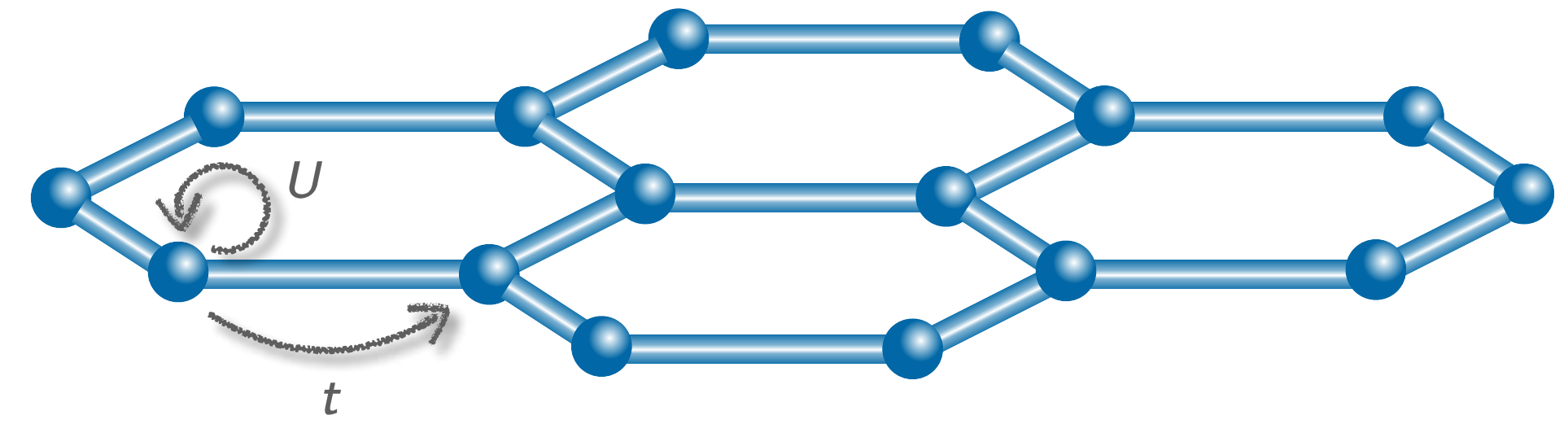
Universal field theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{r}{2}\varphi^2 + \lambda\varphi^4 + \dots$$

Fermionic quantum criticality

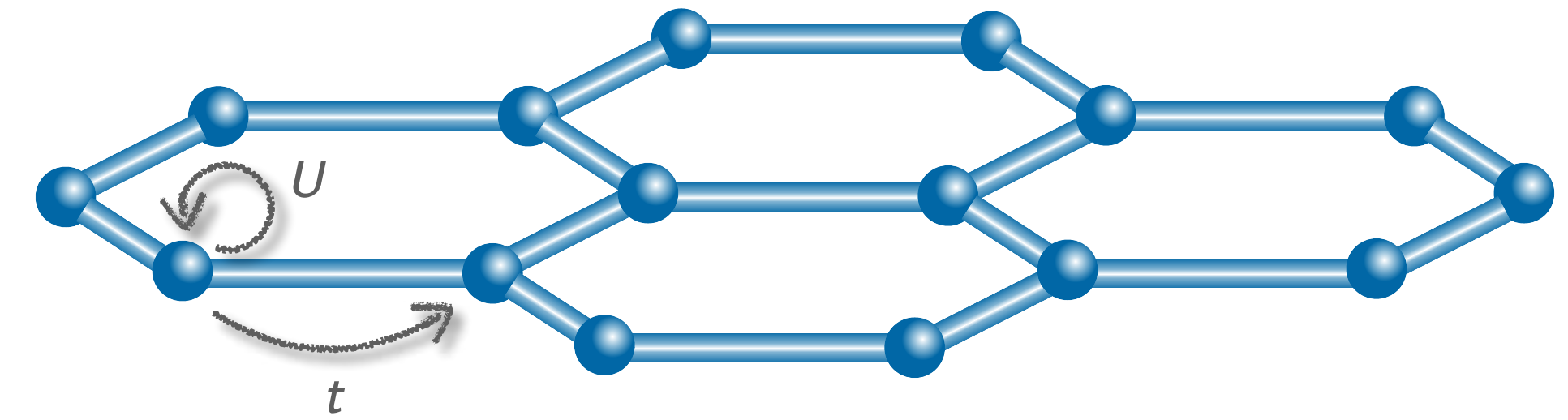
Honeycomb-lattice Hubbard model:

$$\mathcal{H} = -t \sum_{\langle i,j \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



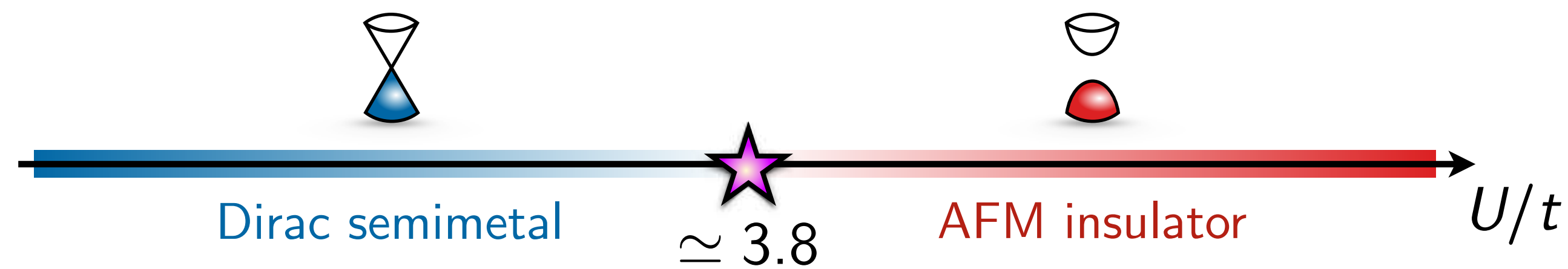
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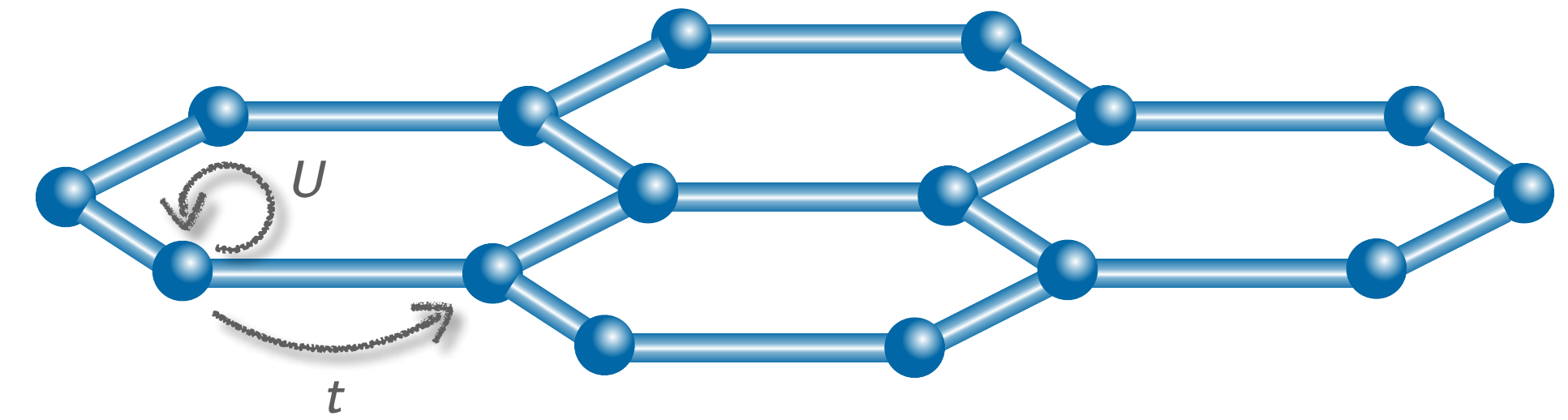
Quantum phase diagram:



[Assaad & Herbut, PRX '13]
[Lang *et al.*, PRL '13]
[Otsuka, Yunoki, Sorella, PRX '16]
[Lang & Läuchli, arXiv:2503.15000]
...

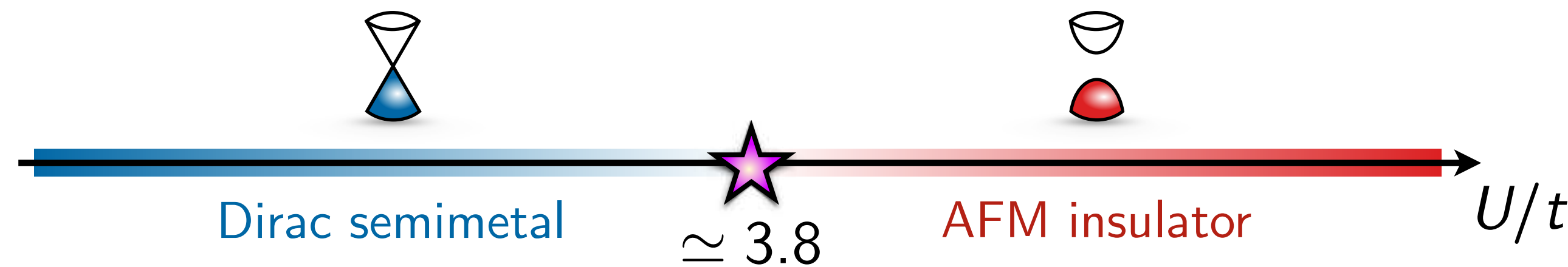
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Quantum phase diagram:



[Assaad & Herbut, PRX '13]
 [Lang *et al.*, PRL '13]
 [Otsuka, Yunoki, Sorella, PRX '16]
 [Lang & Läuchli, arXiv:2503.15000]

Gross-Neveu-Heisenberg field theory:

$$\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi + g(\bar{\psi} \vec{\sigma} \psi) \cdot \vec{\varphi} + \dots$$

→ Talk by T. Lang

[Herbut, Juričić, Vafek, PRB '09]
 [LJ & Herbut, PRB '14]
 [Zerf *et al.*, PRD '17]
 [Ladovrechis, Ray, Meng, LJ, PRB '23]

Experimental realization?

026802 (2009)

PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2009

Is Graphene in Vacuum an Insulator?

Joaquín E. Drut¹ and Timo A. Lähde²

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²Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

(Received 11 July 2008; published 13 January 2009)

We present evidence, from lattice Monte Carlo simulations of the phase diagram of graphene as a function of the Coulomb coupling between quasiparticles, that graphene in vacuum is likely to be an insulator. We find a semimetal-insulator transition at $\alpha_g^{\text{crit}} = 1.11 \pm 0.06$, where $\alpha_g \simeq 2.16$ in vacuum, and $\alpha_g \simeq 0.79$ on a SiO₂ substrate. Our analysis uses the logarithmic derivative of the order parameter, supplemented by an equation of state. The insulating phase disappears above a critical number of four-component fermion flavors $4 < N_f^{\text{crit}} < 6$. Our data are consistent with a second-order transition.

PACS numbers: 73.63.Bd, 05.10.Ln, 71.30.+h

DOI: [10.1103/PhysRevLett.102.026802](https://doi.org/10.1103/PhysRevLett.102.026802)

Graphene, a carbon allotrope with a two-dimensional honeycomb structure, has become an important player at the forefront of condensed matter physics, drawing the attention of theorists and experimentalists alike due to its challenging nature as a many-body problem, its unusual electronic properties, and possible technological applications (see Refs. [1,2] and references therein). Graphene also belongs to a large class of planar condensed matter systems, which includes other graphite-related materials as well as high- T_c superconductors.

A distinctive feature of graphene is that its band structure generates “Dirac points,” in the vicinity of which the quasiparticles behave as relativistic Dirac fermions. These properties resemble QED in a very strongly coupled regime. This provides an exciting opportunity for the study of strongly coupled theories, within a condensed matter analogue that can be experimentally realized with modest equipment.

Notably, Eq. (1) satisfies a chiral $U(2N_f)$ symmetry which can break spontaneously at large enough Coulomb coupling, generating a gap in the quasiparticle spectrum. Whether such an effect occurs in real graphene is an open issue from the experimental point of view (see, however, Ref. [4], where a substrate-induced gap is reported). On the theoretical side, dynamical gap generation is described by a quantum phase transition due to the formation of particle-hole bound states. However, in such a strongly coupled system, even a qualitative analysis should be nonperturbative. The situation for graphene in vacuum, where the Coulomb interaction is partially screened

Experimental realization?

026802 (2009)

PHYSICAL REVIEW LETTERS

PHYSICAL

Is Graphene in Vacuum an Insulator?

1 and Timo A. Lähde²

Joaquín E. Drut¹ and Timo A. Lähde²

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 (Received 11 July 2008; published 13 January 2009)

We present evidence, from lattice Monte Carlo simulations of the p function of the Coulomb coupling between quasiparticles, that graphene is a semimetal-insulator transition at $\alpha_g^{\text{crit}} = 1.11 \pm 0.02$ and $\alpha_g \simeq 0.79$ on a SiO_2 substrate. Our analysis uses the logarithmic supplemented by an equation of state. The insulating phase disappears for component fermion flavors $4 < N_f^{\text{crit}} < 6$. Our data are consistent with

DOI: 10.1103/PhysRevLett.102.026802

Graphene, a carbon allotrope with a two-dimensional honeycomb structure, has become an important player at the forefront of condensed matter physics, drawing the attention of theorists and experimentalists alike due to its challenging nature as a many-body problem, its unusual electronic properties, and possible technological applications (see Refs. [1,2] and references therein). Graphene also belongs to a large class of planar condensed matter systems, which includes other graphite-related materials as high- T_c superconductors.

A distinctive feature of graphene is that its band structure generates "Dirac points," in the vicinity of which relativistic theories apply.

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Over the past decade, graphene has established itself as a remarkable new material with superlative properties [1,2]. However, the early hopes to utilize it as a next-generation transistor have been dashed, mostly because graphene remains metallic—these prototypical Dirac fermions are immune to many of the conventional routes for driving two-dimensional electron gases into an insulating state, including, for example, Anderson localization and percolation transitions (see, e.g., Ref. [3]). Other mechanisms for opening band gaps including hydrogenation [4], application of uniaxial strain [5], and forming nanoribbons [6] are being explored to enhance graphene's mobility. Various other approaches to improve graphene's performance as a transistor are also being investigated.

that it is the nonuniversal, material-specific, and short-range part of the electron-electron interactions that plays the dominant role in determining graphene's ground state. More interestingly, we conclude that the application of isotropic strain is considerably more efficient in approaching the SM-AFM phase transition than substrate manipulation, providing a new route for driving the elusive Mott insulating phase.

PRL 115, 186602 (2015)

PHYSICAL REVIEW LETTERS

REVIEW LETTERS

Interaction-Driven Metal-Insulator Transition in Strained Graphene

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Advanced 2D Materials, National University of Singapore, Singapore

Physics, Faculty of Science, National University of Singapore, Singapore

and Mathematical Sciences, National University of Singapore, Singapore

Tang, L., Laksono, E., Rodrigues, J. N. B., Sengupta, P., & F. (2024). Interaction-Driven Metal-Insulator Transition in Strained Graphene. *Physical Review Letters*, 132(12), 127401. doi:10.1103/PhysRevLett.132.127401

Ho-Kin Tang,^{1,2} E. Laksono,^{1,2} J. N. B. Rodrigues,^{1,2} P. Sengupta,^{1,3} F. F. Assaad,⁴ and S. Adam^{1,2,5}

¹Centre for Advanced 2D Materials, National University of Singapore, 2 Science Drive 2, Singapore 117546, Singapore

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The question of whether electron-electron interactions, as in the case of graphene under realistic experimental conditions, can lead to a ground state with a finite energy gap is a long-standing open problem. We calculate the effective long-range Coulomb interaction in a strained graphene lattice and find that the energy gap opens up for a wide range of strain values. This is in contrast to the case of unstrained graphene, where the energy gap is zero. Our results provide a theoretical basis for the experimental observation of a finite energy gap in strained graphene.

The question of whether electron-electron interactions can drive a metal to insulator transition in graphene under realistic experimental conditions is addressed. Using three representative methods to calculate the effective long-range Coulomb interaction between π electrons in graphene and solving for the ground state using quantum Monte Carlo methods, we argue that, without strain, graphene remains metallic and changing the substrate from SiO_2 to suspended samples hardly makes any difference. In contrast, applying a rather large—but experimentally realistic—uniform and isotropic strain of about 15% seems to be a promising route to making graphene an antiferromagnetic Mott insulator.

DOI: [10.1103/PhysRevLett.115.186602](https://doi.org/10.1103/PhysRevLett.115.186602)

PACS numbers: 73.20.-g, 73.43.-f, 73.40.+c, 73.61.-g, 73.63.-b, 73.63.Lp, 73.63.Nd, 73.63.Jd, 73.63.Hk, 73.63.Db, 73.63.Dr, 73.63.Dg, 73.63.Df, 73.63.De, 73.63.Dc, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df, 73.63.Dg, 73.63.Dh, 73.63.Di, 73.63.Dj, 73.63.Dk, 73.63.Dl, 73.63.Dm, 73.63.Dn, 73.63.Do, 73.63.Dp, 73.63.Dq, 73.63.Dr, 73.63.Ds, 73.63.Dt, 73.63.Du, 73.63.Dv, 73.63.Dw, 73.63.Dx, 73.63.Dy, 73.63.Dz, 73.63.Da, 73.63.Db, 73.63.Dc, 73.63.Dd, 73.63.De, 73.63.Df,

PACS numbers: 72.80.Vp, 71.10.Fd, 71.27.+a, 73.22.Pr

DOI: [10.1103/PhysRevLett.115.186602](https://doi.org/10.1103/PhysRevLett.115.186602)

Experimental realization?

1026802 (2009)

PHYSICAL REVIEW LETTERS

Is Graphene in Vacuum an Insulator?

Joaquín E. Drut¹ and Timo A. Lähde²

¹Department of Physics, The Ohio State University, Columbus, Ohio
²Department of Physics, University of Washington, Seattle, Washington
(Received 11 July 2009; published 13 January 2010)

We present evidence, from lattice Monte Carlo simulations, that the function of the Coulomb coupling between Dirac fermions in a vacuum insulator. We find a semimetal-insulator transition at $\alpha_g \approx 0.79$ on a SiO_2 substrate. Our results are supplemented by an equation of state for the component fermion flavors $4 < N_f < \infty$.

DOI: [10.1103/PhysRevLett.102.026802](https://doi.org/10.1103/PhysRevLett.102.026802)

Graphene, a carbon allotrope with a two-dimensional honeycomb structure, has become an important topic at the forefront of condensed matter physics. The attention of theorists and experimentalists has been drawn to its challenging nature as a many-body problem with unusual electronic properties, and possible technological applications (see Refs. [1,2] and references therein). It also belongs to a large class of planar systems, which includes other graphite-like systems, as well as high- T_c superconductors. A distinctive feature of graphene is the presence of a Dirac semimetal phase, which is characterized by a gapless Dirac cone structure in the Brillouin zone.

PRL 115, 186602 (2015)

PHYSICAL REVIEW LETTERS

Interaction-Driven Metal-Insulator Transition in Strained Graphene

Ho-Kin Tang,^{1,2} E. Laksono,^{1,2} J. N. B. Rodrigues,^{1,2} P. Sengupta,^{1,3} F. F. Assaad,⁴ and S. Adam^{1,2,5}
¹Centre for Advanced 2D Materials, National University of Singapore, 6 Science Drive 2, Singapore 117546, Singapore
²Department of Physics, Faculty of Science, National University of Singapore, 21 Nanyang Link, Singapore 117542, Singapore
³School of Physical and Mathematical Sciences, Nanyang Technological University, 21 Nanyang Link, Singapore 637371, Singapore
⁴Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany
⁵Yale-NUS College, 16 College Avenue West, Singapore 138527, Singapore
(Received 28 May 2015; published 11 November 2015)

RESEARCH

RESEARCH ARTICLE

2D MATERIALS

The role of electron-electron interactions in two-dimensional Dirac fermions

Ho-Kin Tang^{1,2}, J. N. Leaw^{1,2}, J. N. B. Rodrigues^{1,2}, I. F. Herbut³, P. Sengupta^{1,4}, F. F. Assaad⁵, S. Adam^{1,2,6*}

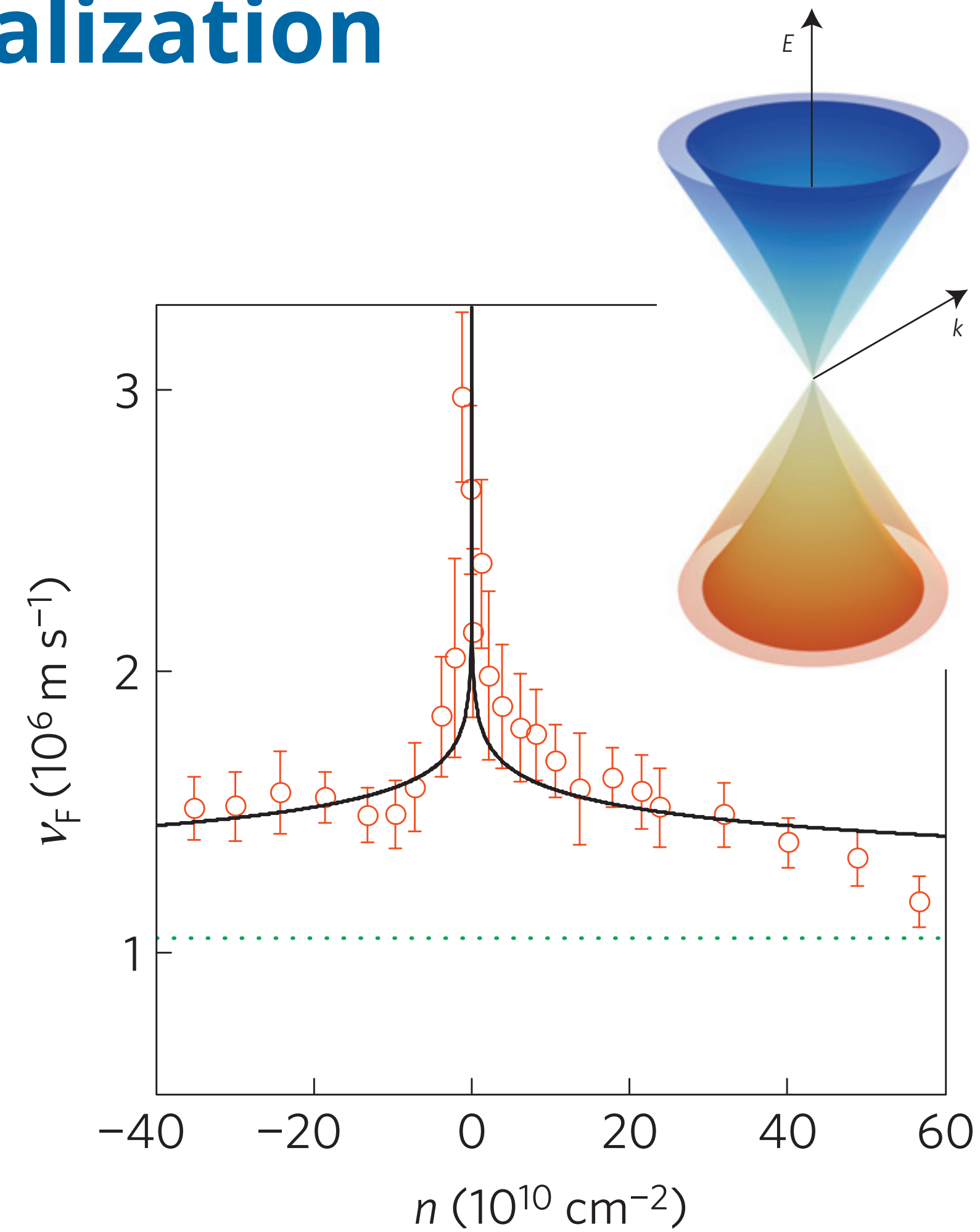
The role of electron-electron interactions in two-dimensional Dirac fermion systems remains enigmatic. Using a combination of nonperturbative numerical and analytical techniques that incorporate both the contact and long-range parts of the Coulomb interaction, we identify the two previously discussed regimes: a Gross-Neveu transition to a strongly correlated Mott insulator and a semimetallic state with a diverging Fermi velocity accurately predicted by the Dirac semimetal phase.

(6, 12) that this pure on-site Hubbard model has limited applicability to experiments done in real materials.

Experimentally, 2D Dirac fermions can be realized in a variety of condensed matter systems, including on the surfaces of 3D topological insulators (13, 14) and in artificial graphene made from quantum corrals of carbon monoxide arranged in a honeycomb lattice on a copper substrate (15), as well as in other systems (16). For concreteness, we focus our attention on graphene, the most studied and versatile realization of 2D Dirac fermions. Experiments have been unable to realize the precise configuration necessary to probe this strange interacting metallic state that features electron quasiparticles moving at the speed of light, despite their quasiparticle character smearing away; however, several probes of ultraclean graphene—including magnetotransport (17), infrared spectroscopy (18), and scanning tunneling spectroscopy (19)—have shown that the Dirac semimetal phase is indeed realized in real materials.

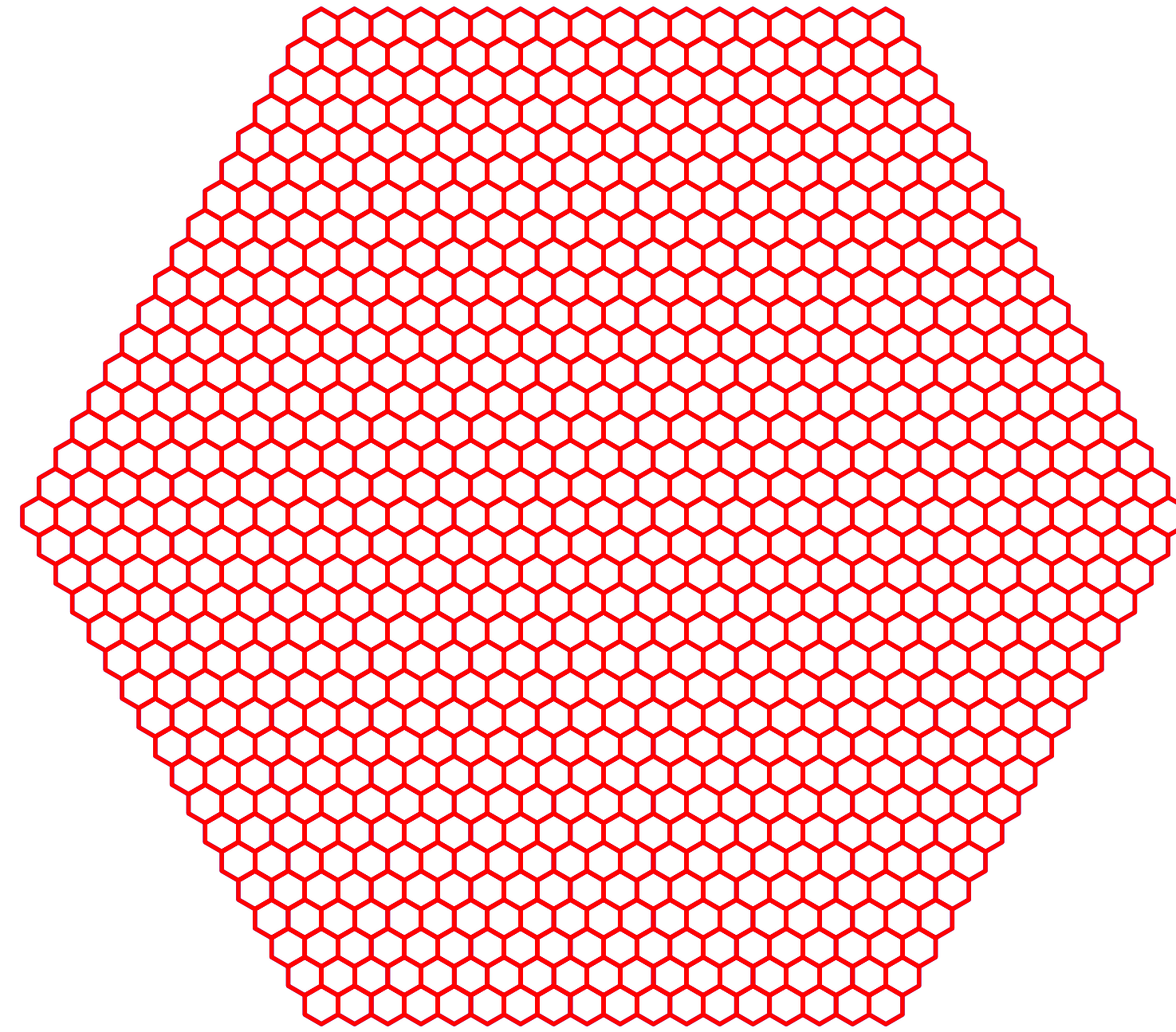
and short-range interactions that play a crucial role in the ground state. The approach of this work is to manipulate the Dirac semimetal phase in real materials.

Fermi velocity renormalization

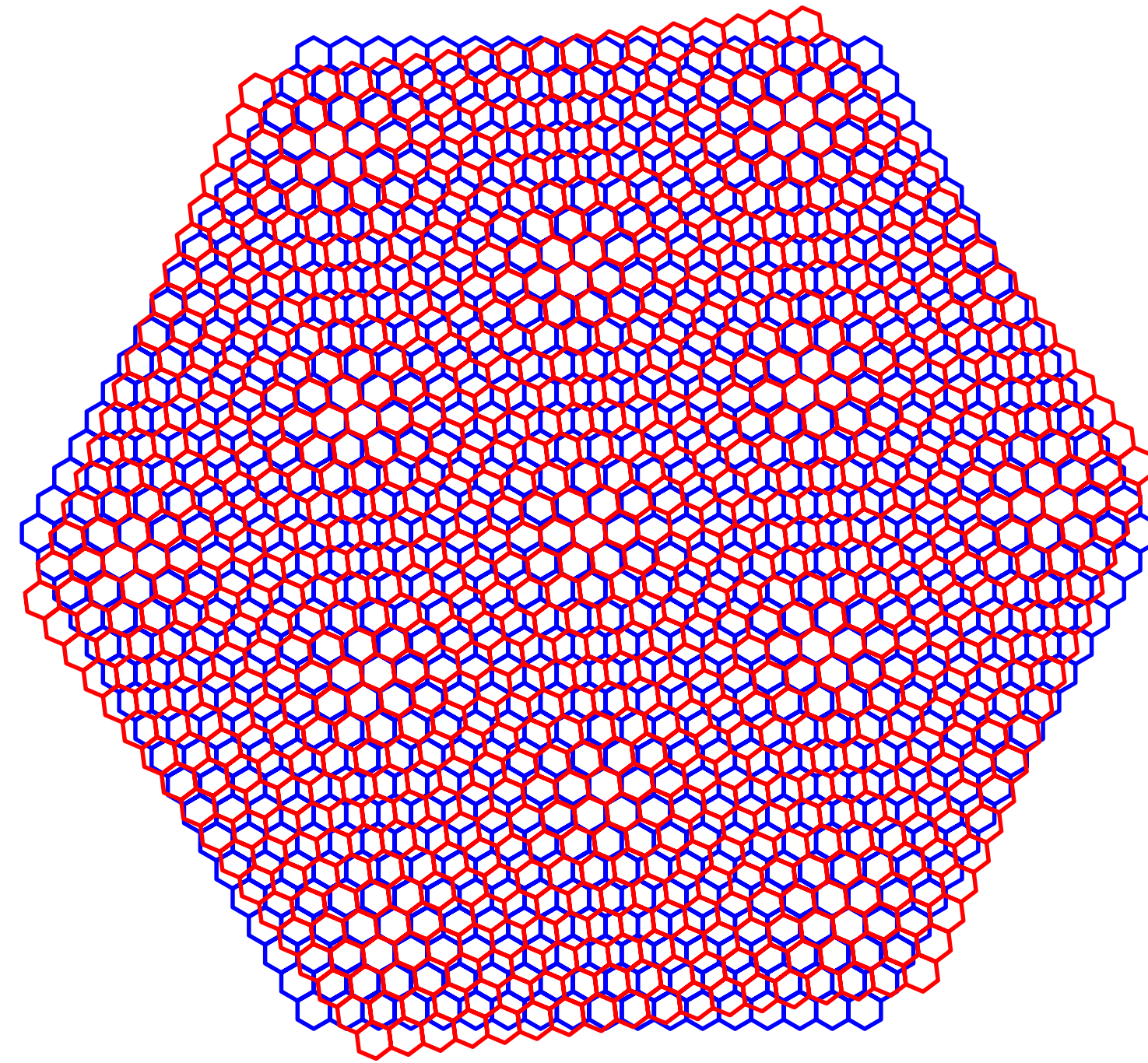


Free-standing graphene remains a Dirac semimetal

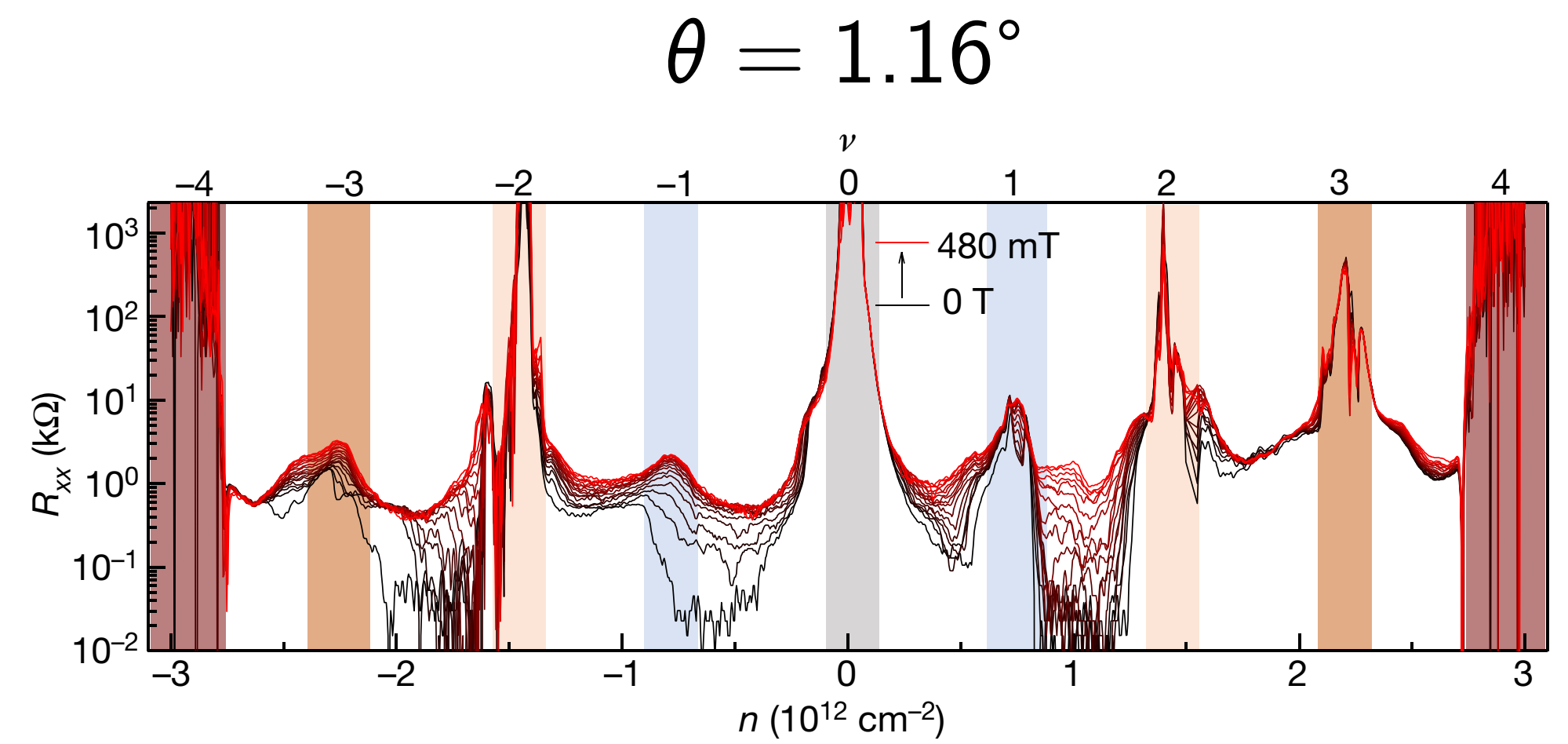
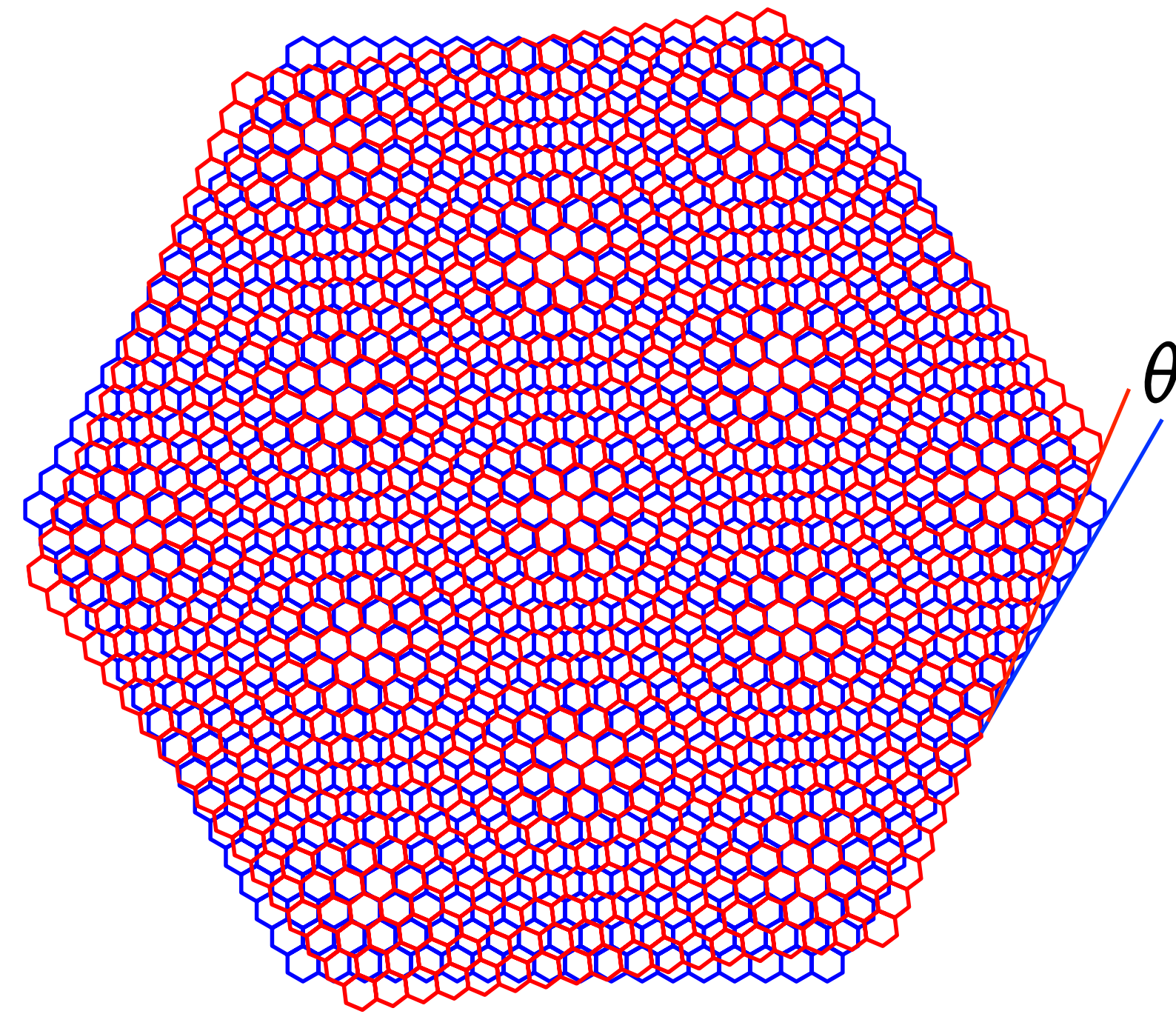
Twisted bilayer graphene



Twisted bilayer graphene

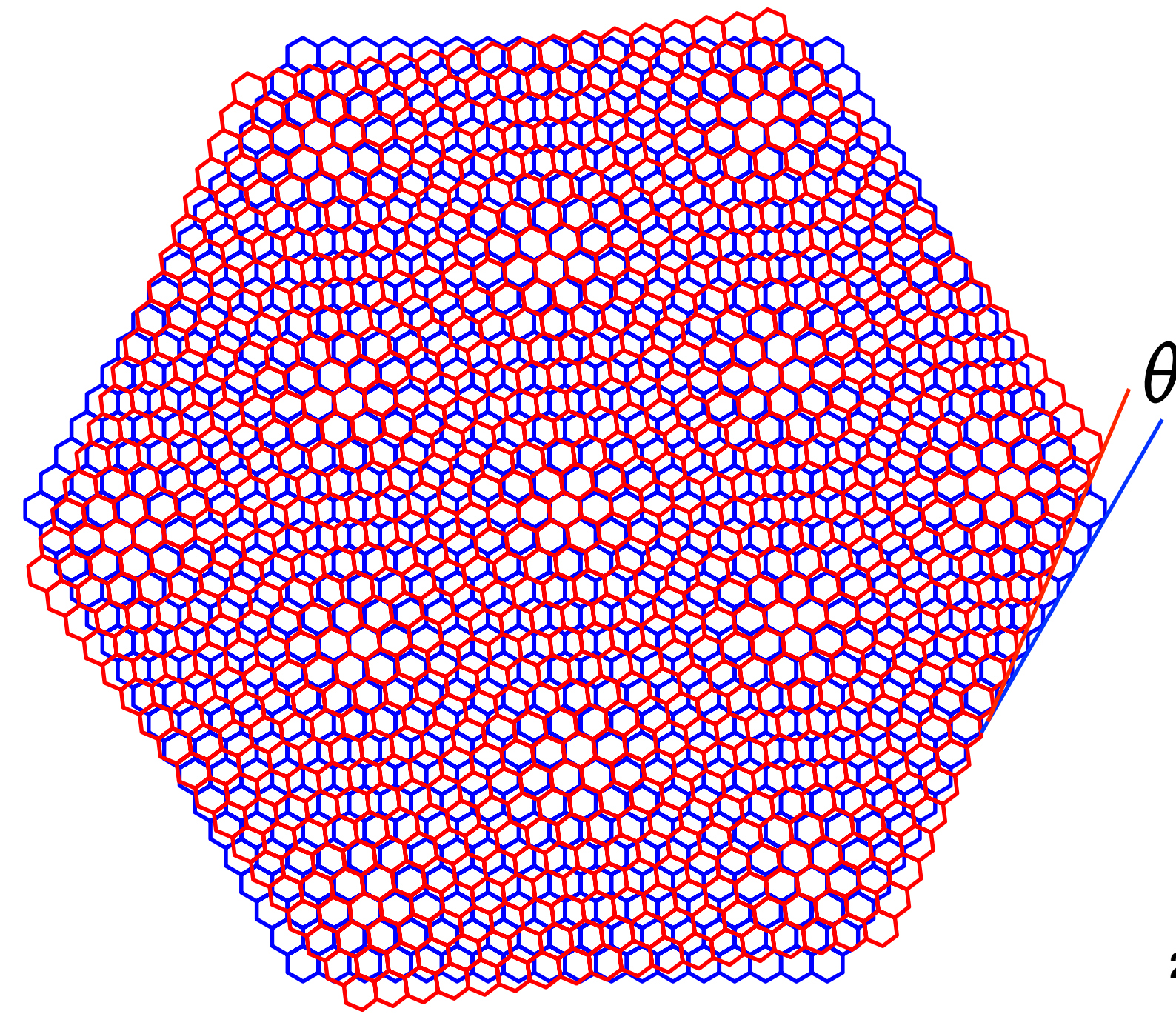


Twisted bilayer graphene

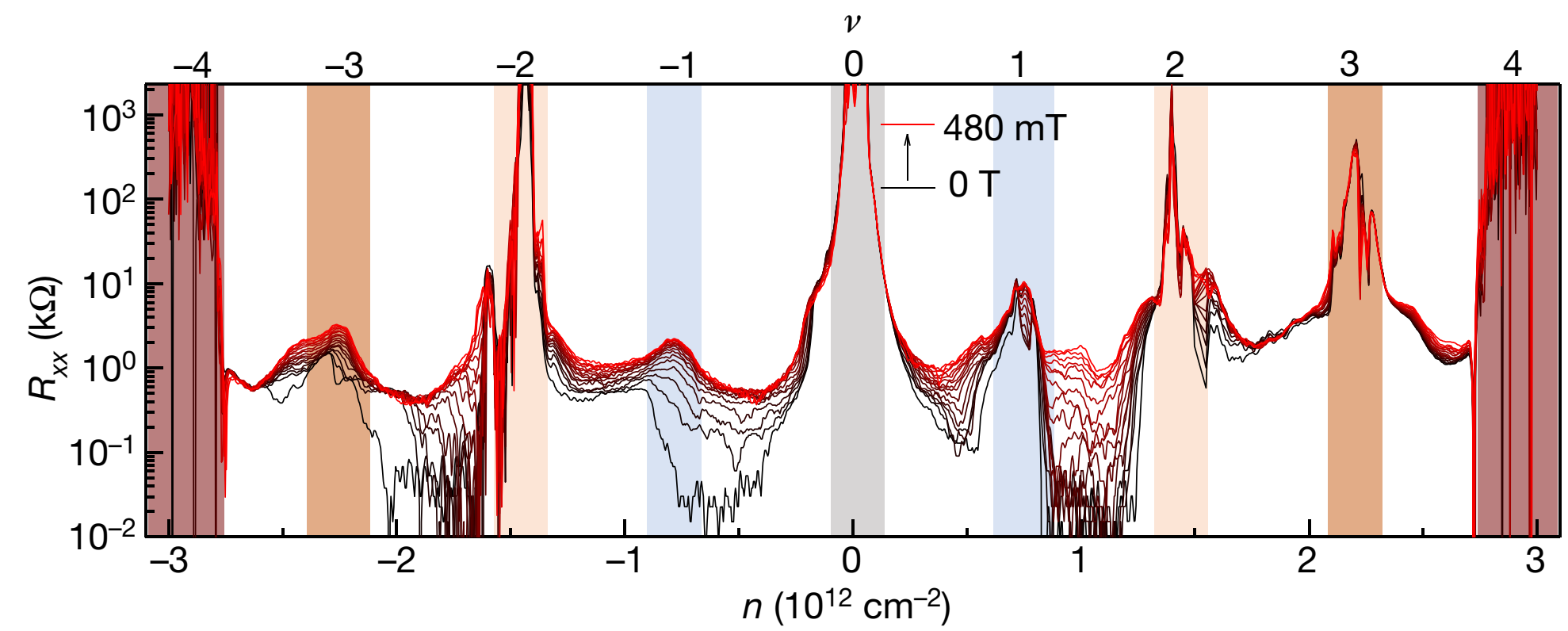


[Lu *et al.*, Nature '19]

Twisted bilayer graphene

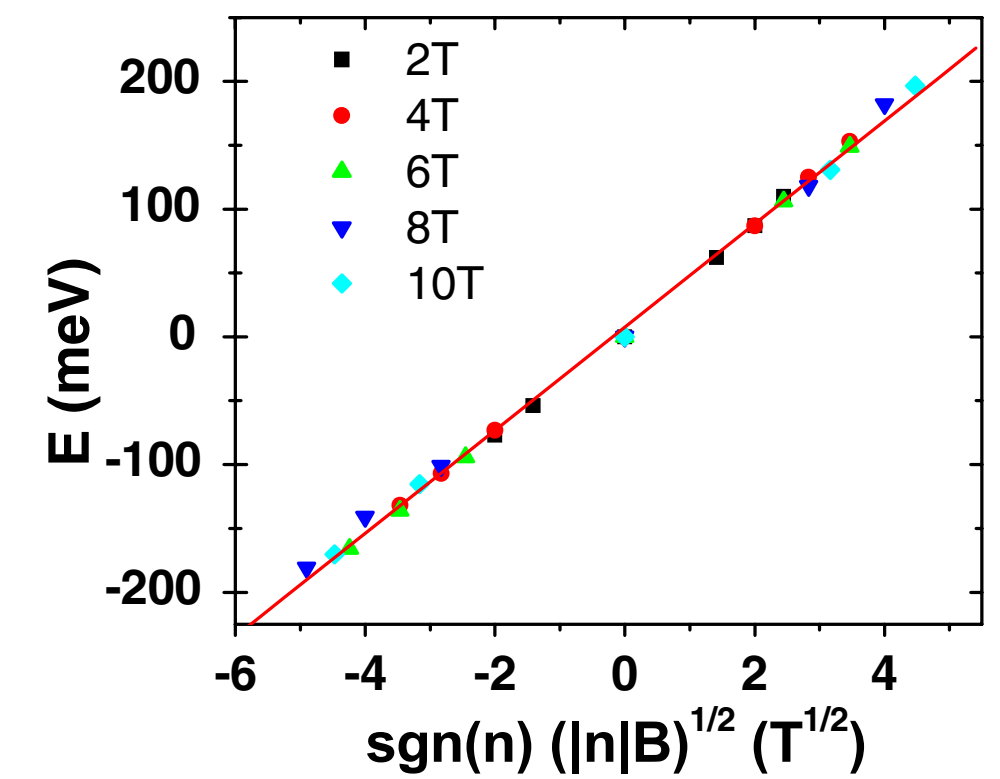


$$\theta = 1.16^\circ$$



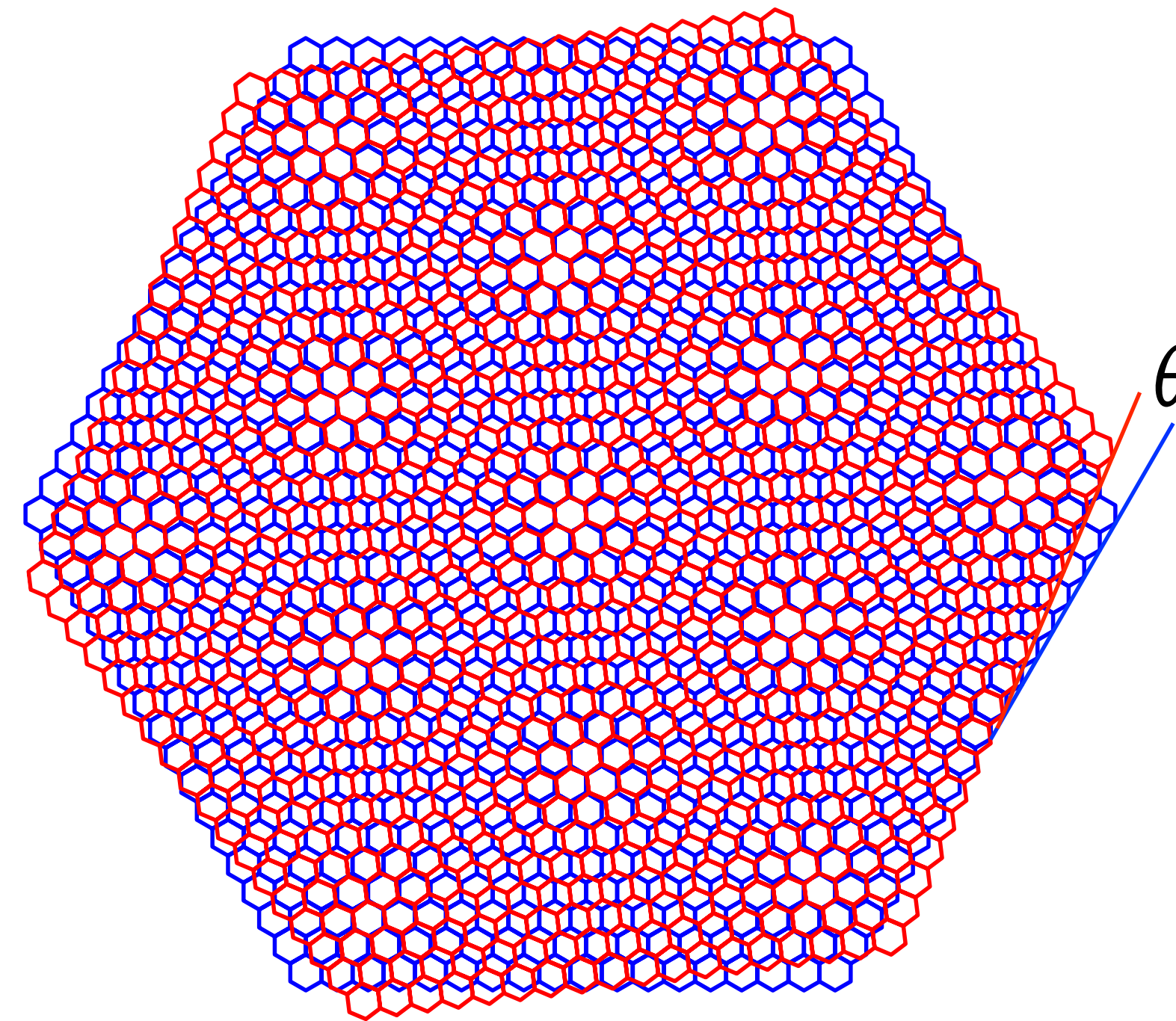
[Lu *et al.*, Nature '19]

$$\theta = 21.8^\circ$$

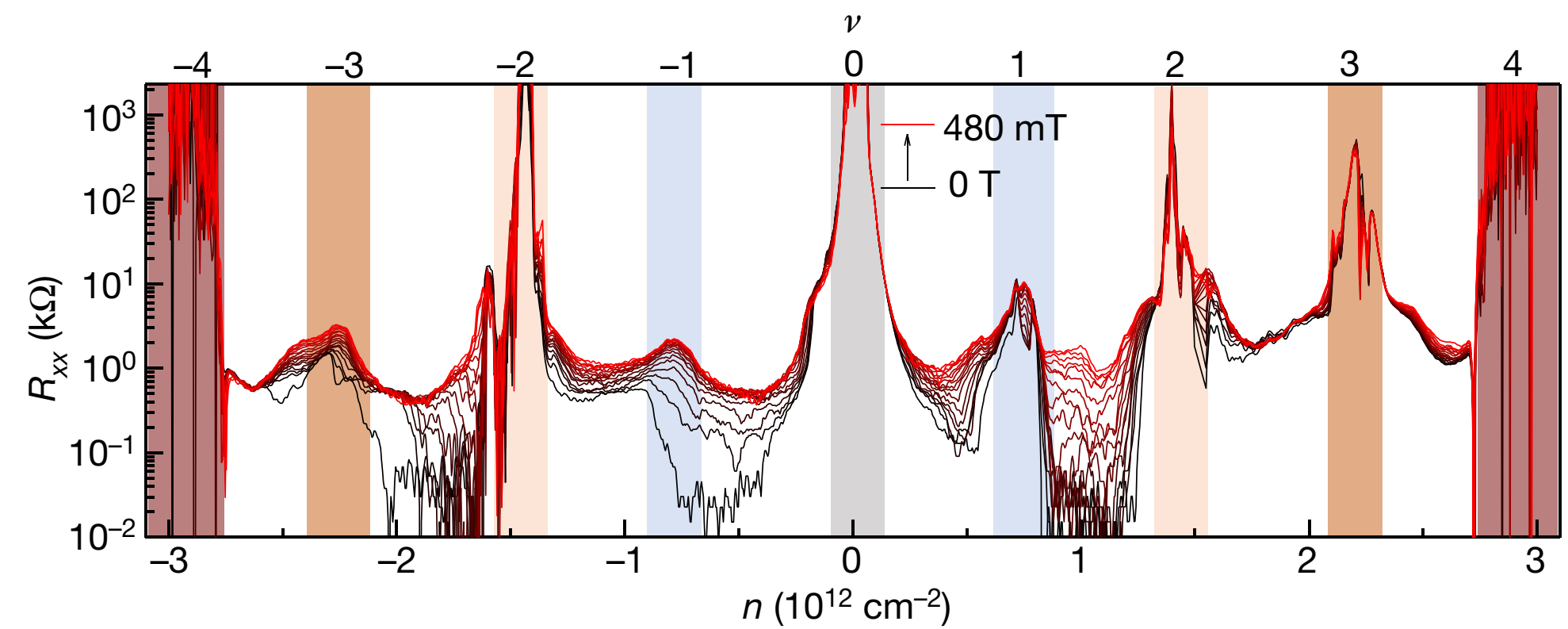


[Luican *et al.*, PRL '11]

Twisted bilayer graphene

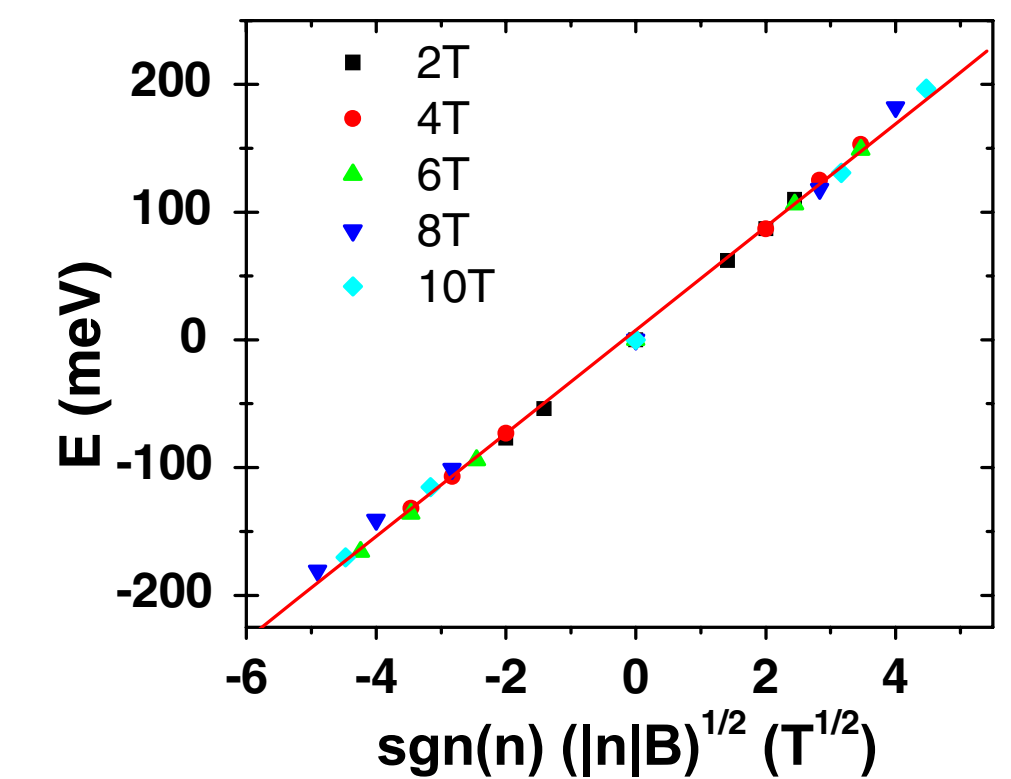


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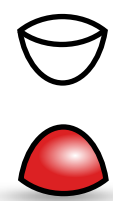


[Lu *et al.*, Nature '19]

$$\theta = 21.8^\circ$$



[Luican *et al.*, PRL '11]



Correlated insulator

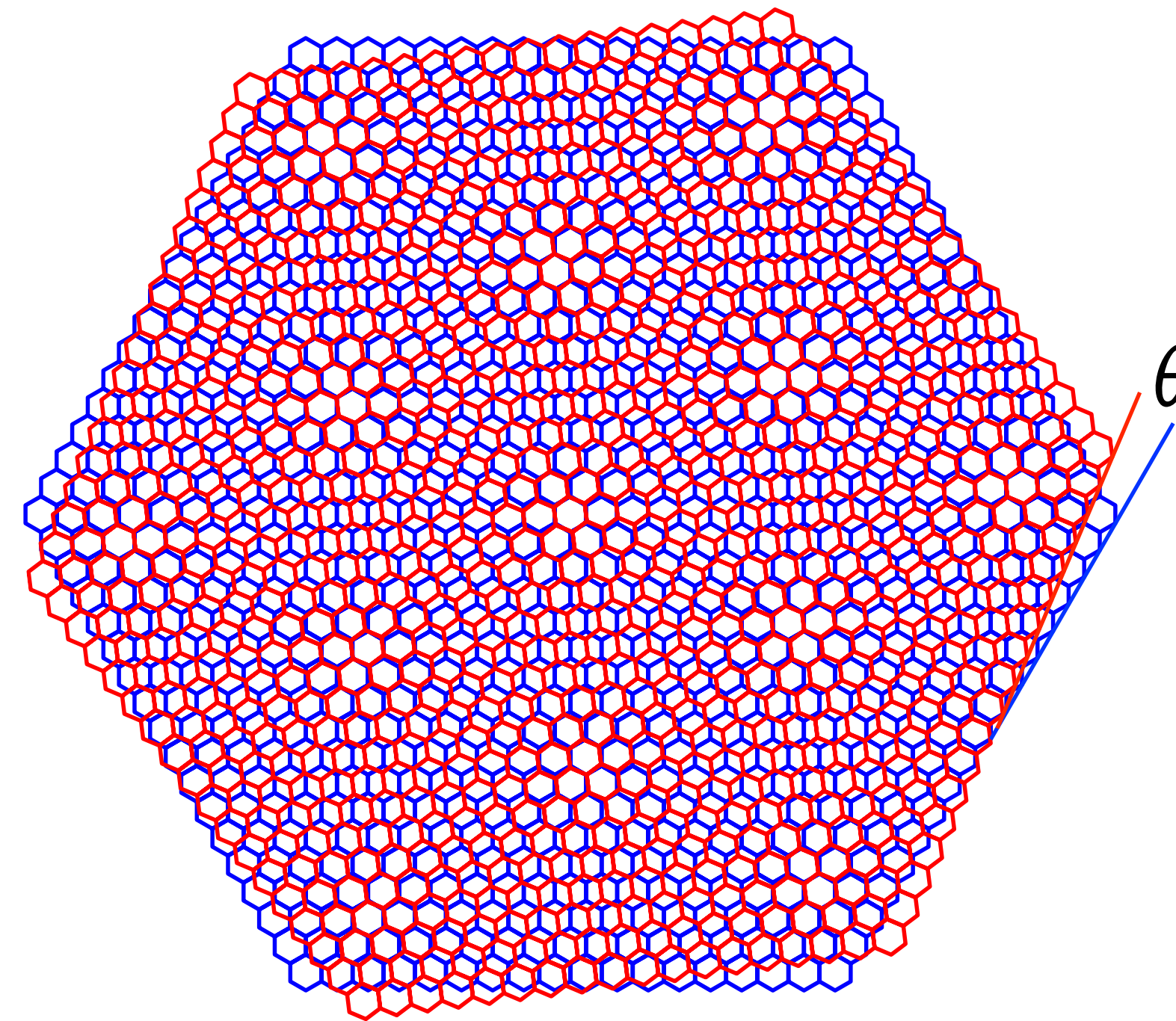


Dirac semimetal

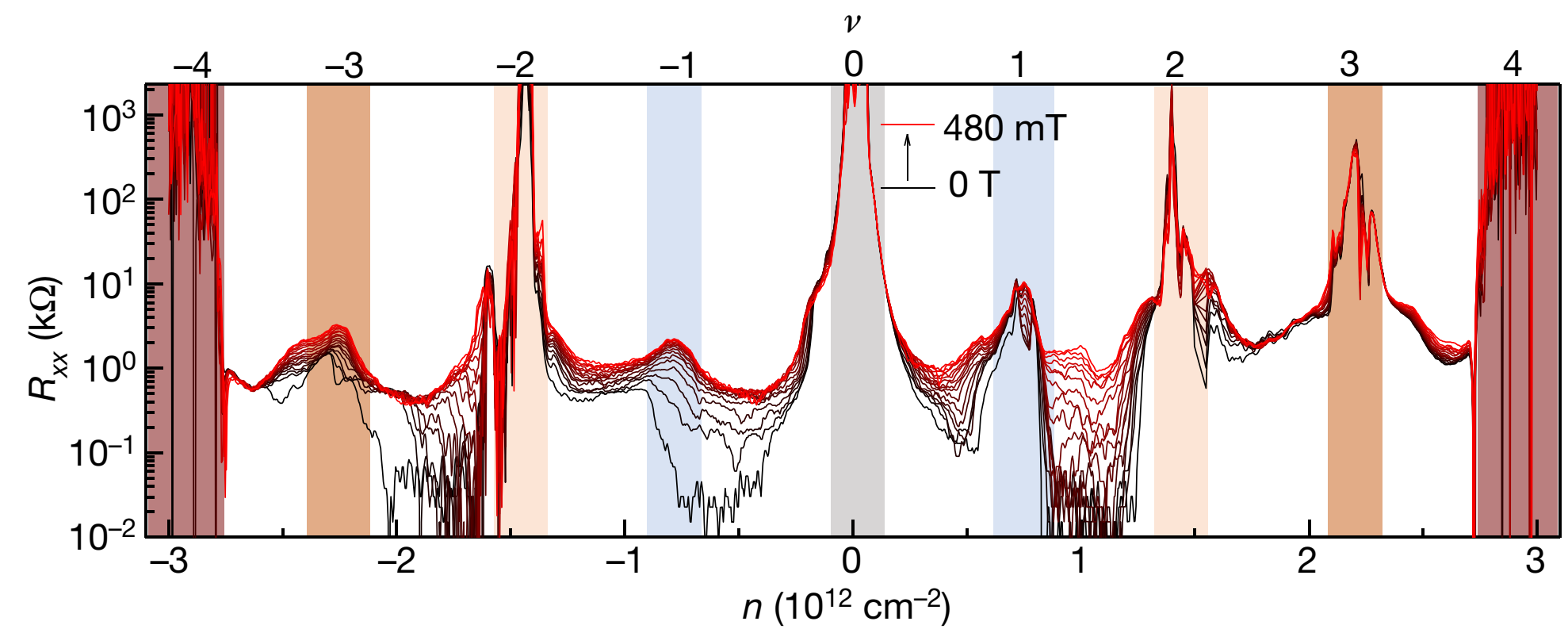
θ

... at charge neutrality $\nu = 0$

Twisted bilayer graphene

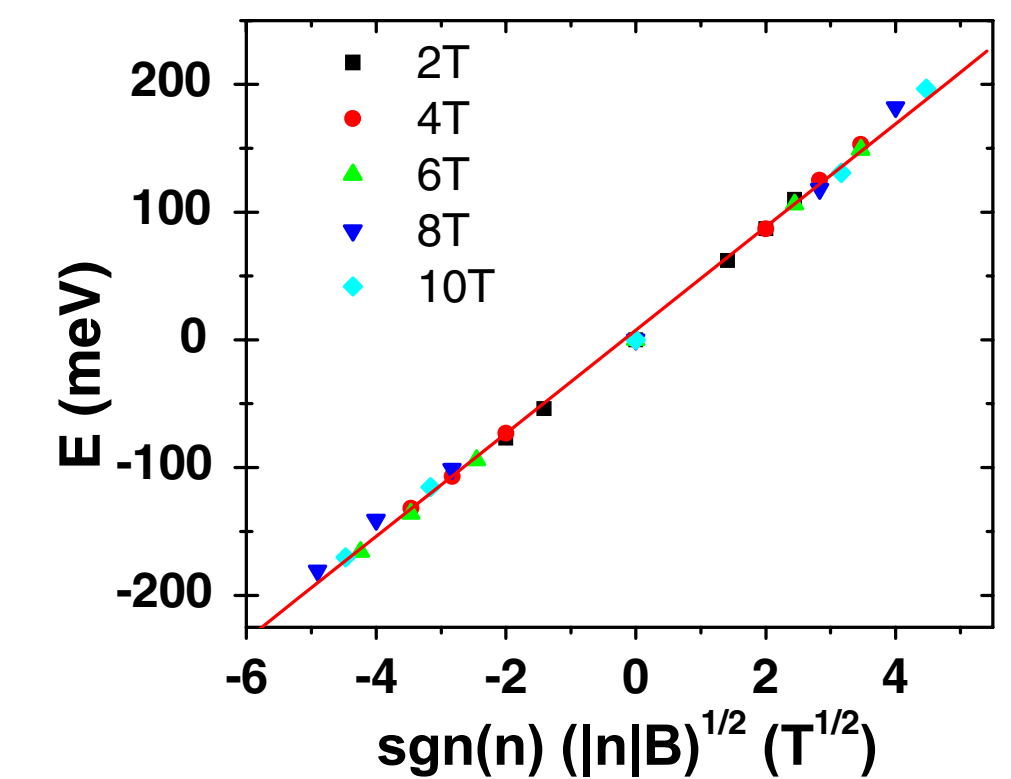


$$\theta = 1.16^\circ$$



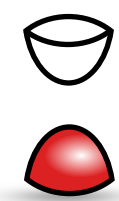
[Lu *et al.*, Nature '19]

$$\theta = 21.8^\circ$$



[Luican *et al.*, PRL '11]

?



Correlated insulator

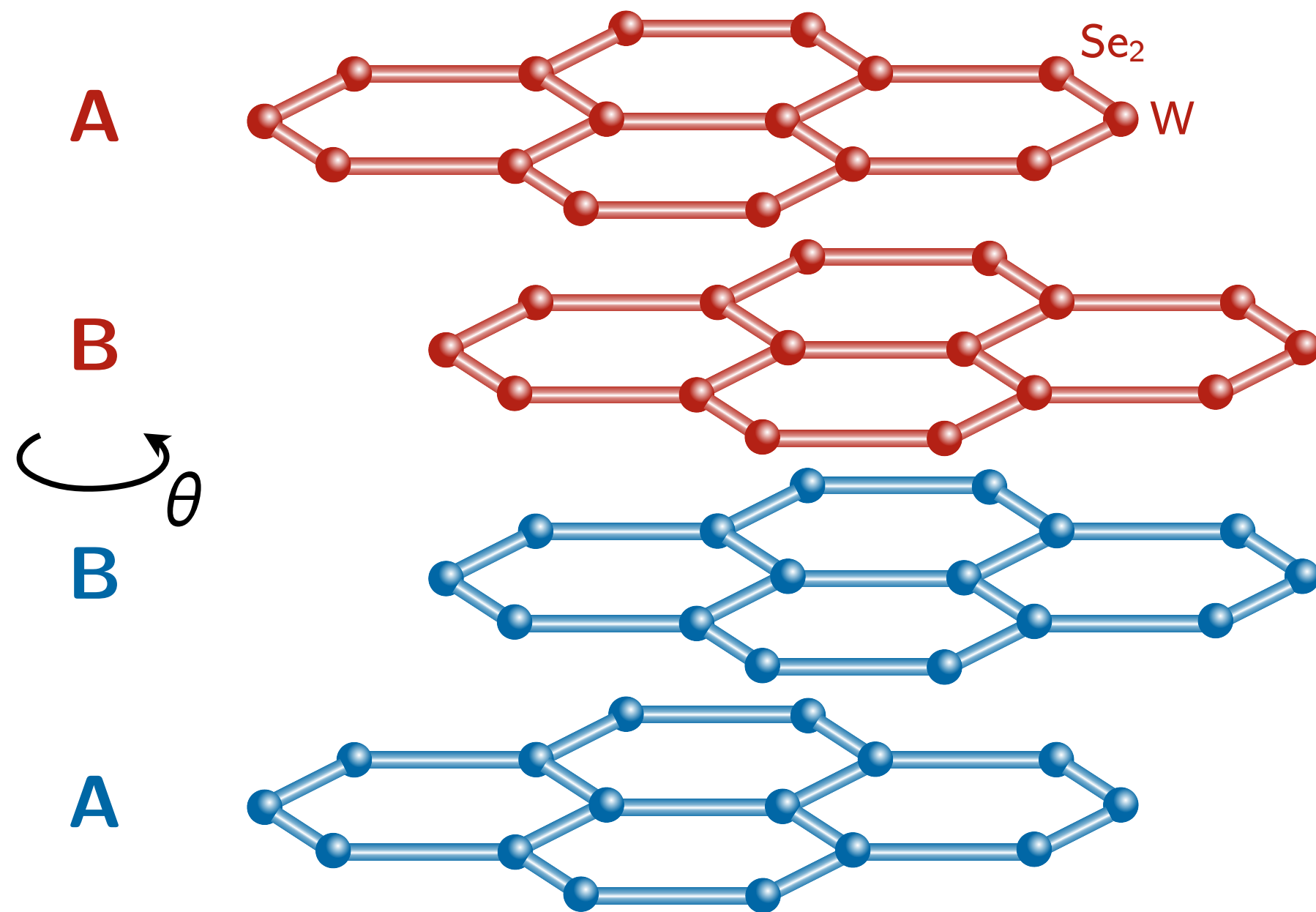


Dirac semimetal

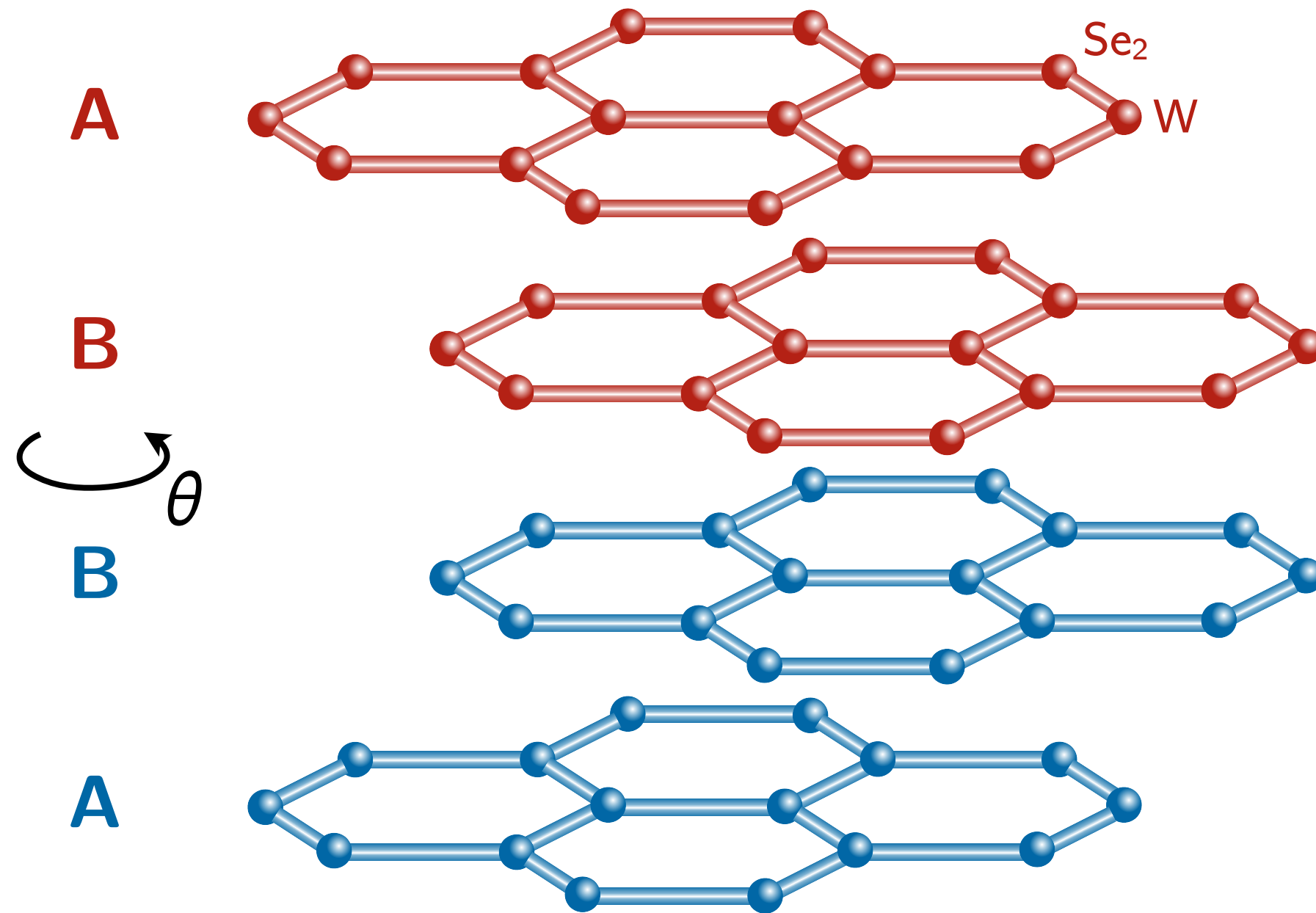
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... at charge neutrality $\nu = 0$

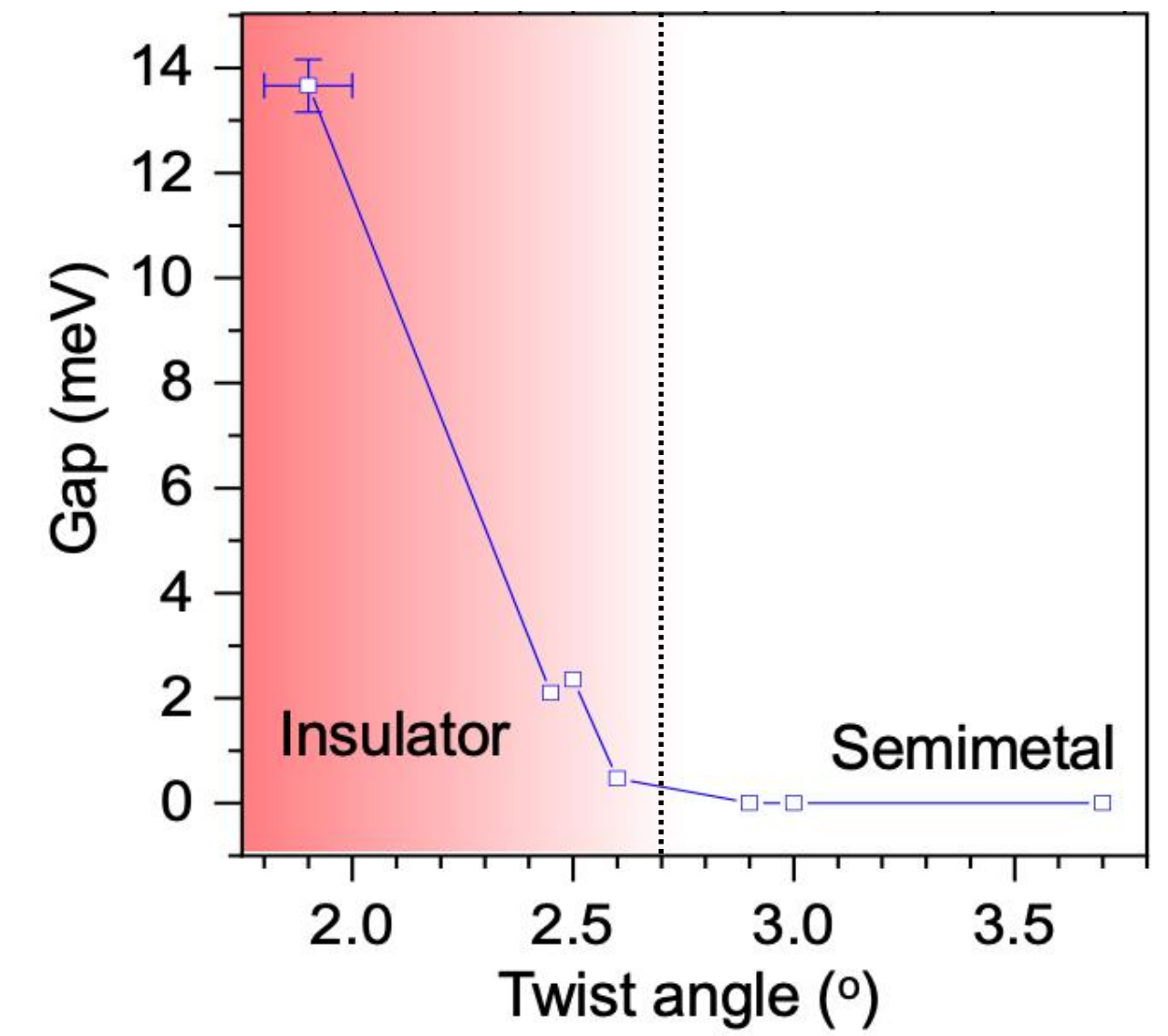
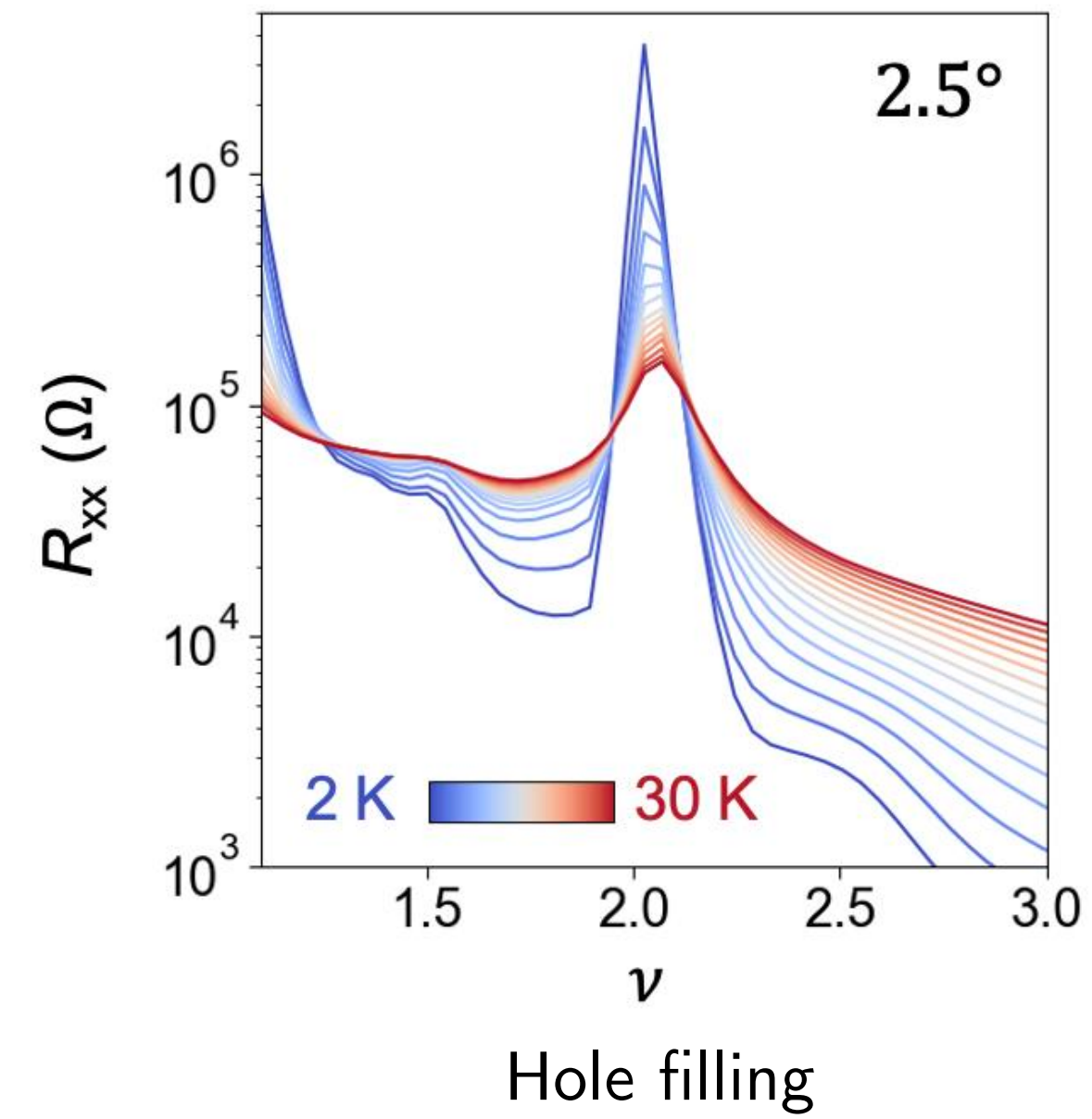
Twisted double bilayer WSe₂



Twisted double bilayer WSe₂



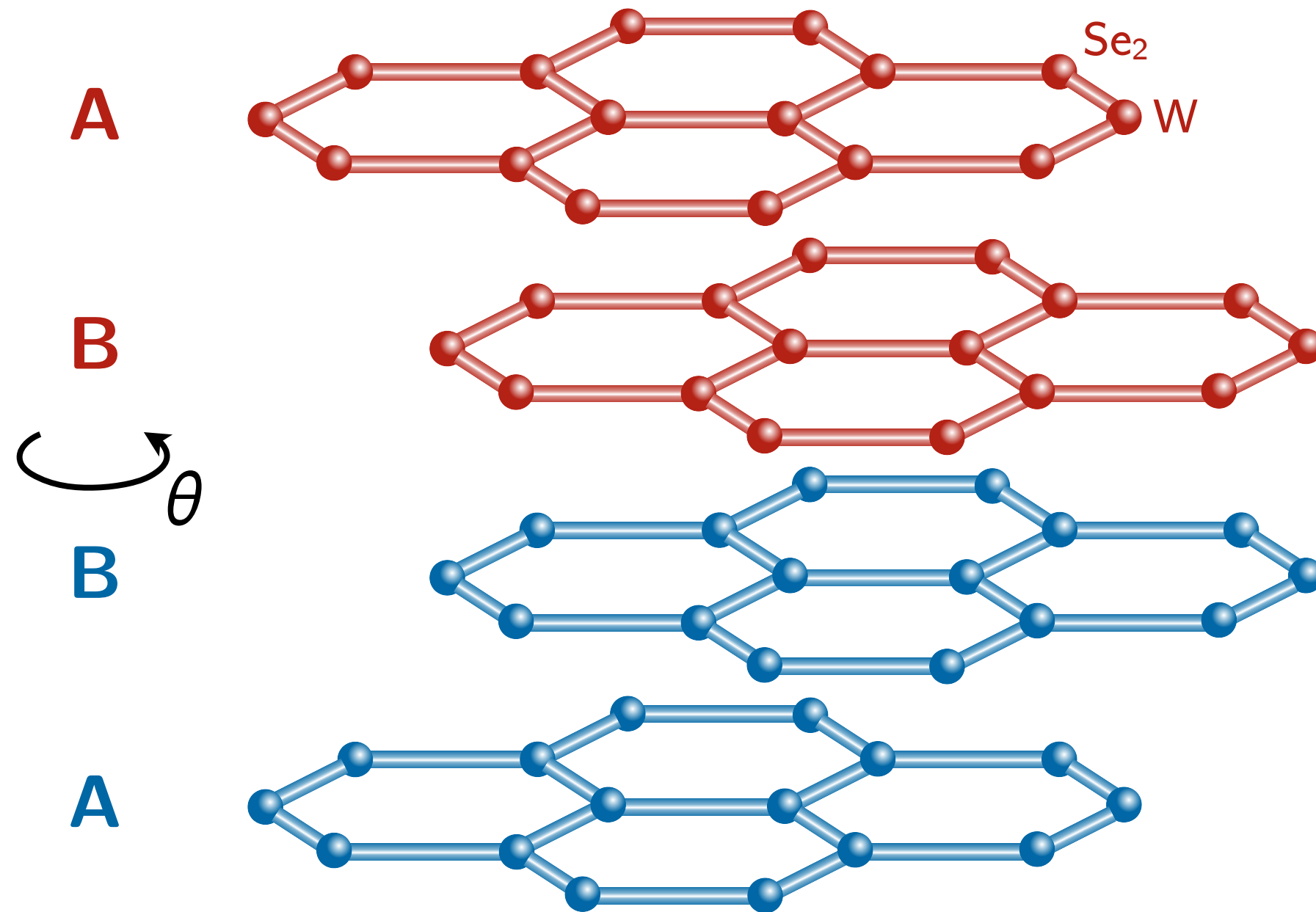
Transport:



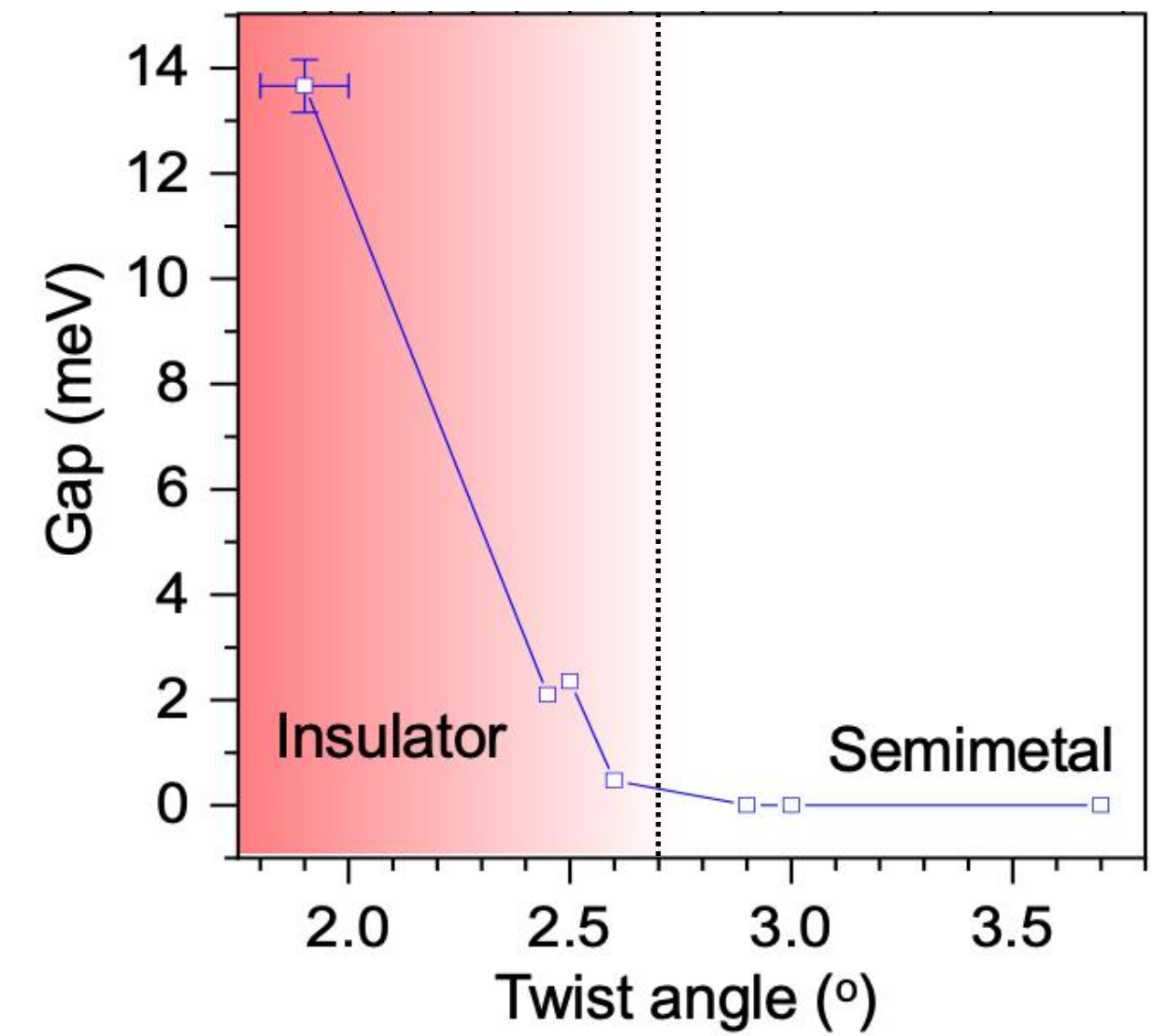
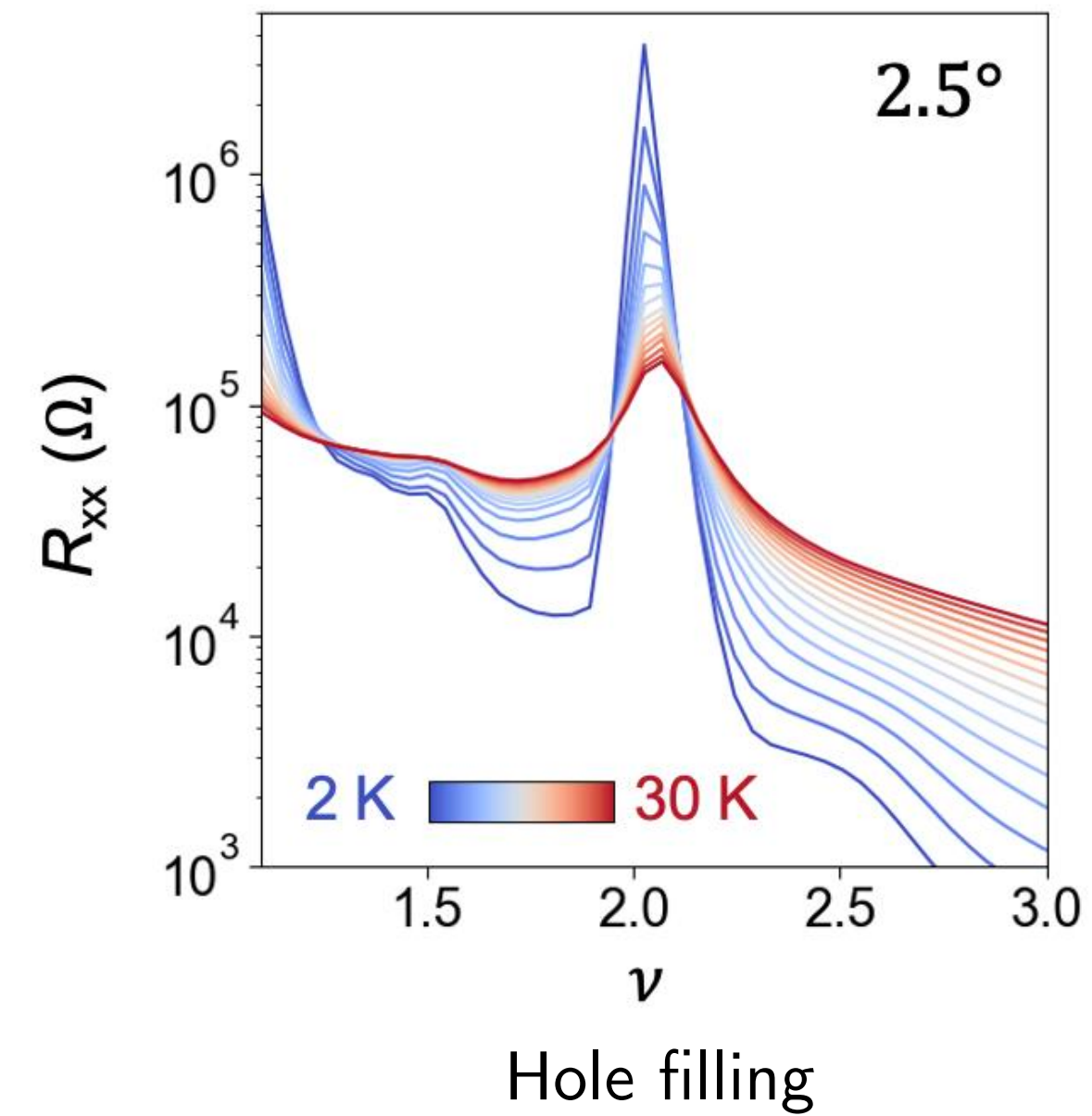
→ Talk by F. Ihssen

[Ma *et al.*, arXiv:2412.07150]

Twisted double bilayer WSe₂



Transport:



Symmetry of ordered state?
Nature of transition?

→ Talk by F. Ihssen

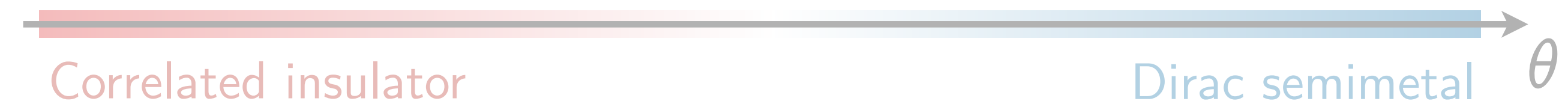
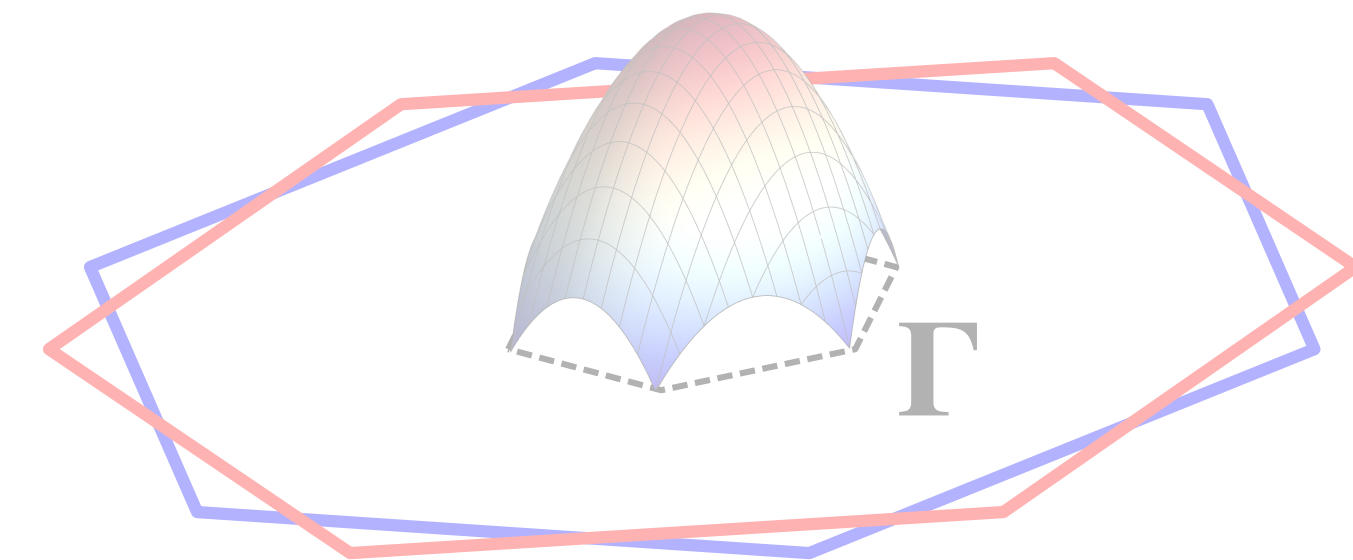
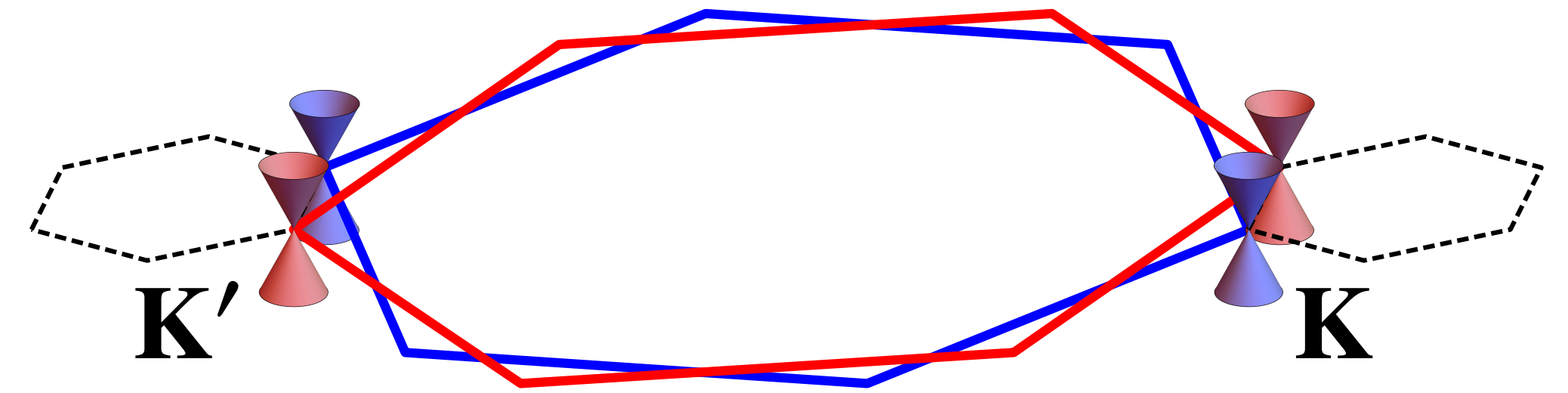
Outline

(1) Introduction

(2) Twisted bilayer graphene

(3) Twisted double bilayer TMDs

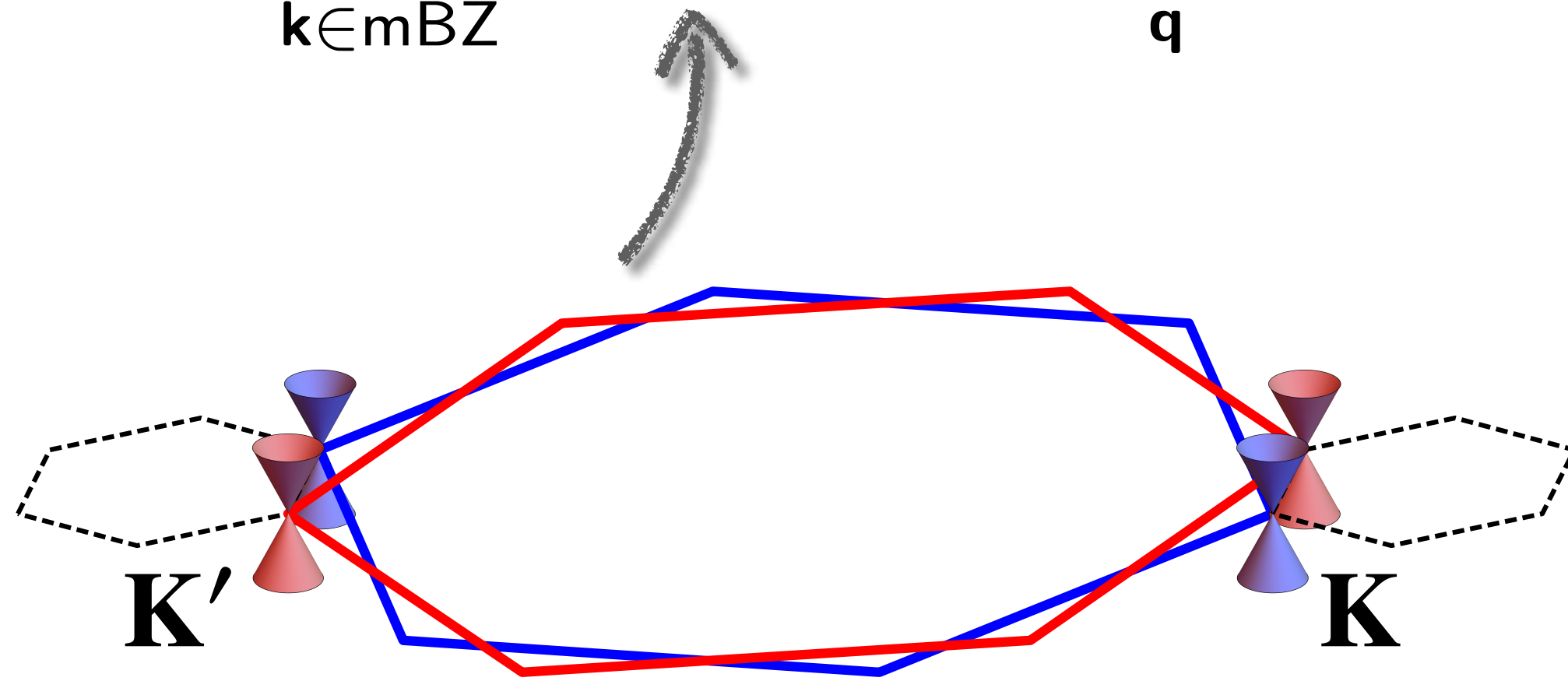
(4) Conclusions



Twisted bilayer graphene: Model

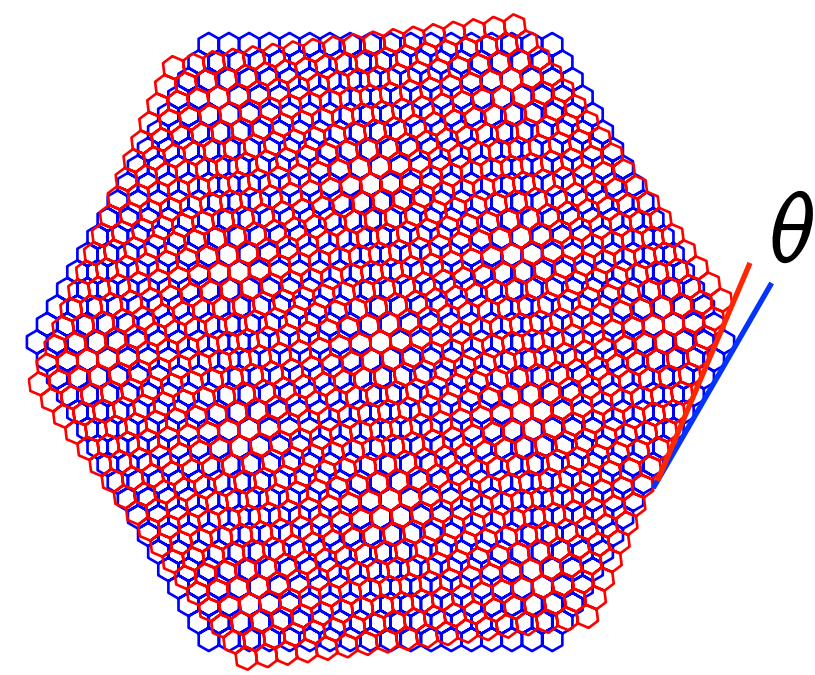
Interacting Bistritzer-MacDonald model:

$$\mathcal{H} = \sum_{\mathbf{k} \in \text{mBZ}} c_{\mathbf{k}}^{\dagger} h(\mathbf{k}) c_{\mathbf{k}} - \frac{1}{2A} \sum_{\mathbf{q}} V_{\mathbf{q}} : \rho_{\mathbf{q}} \rho_{-\mathbf{q}} :$$



[Bistritzer, MacDonald, PNAS '11]

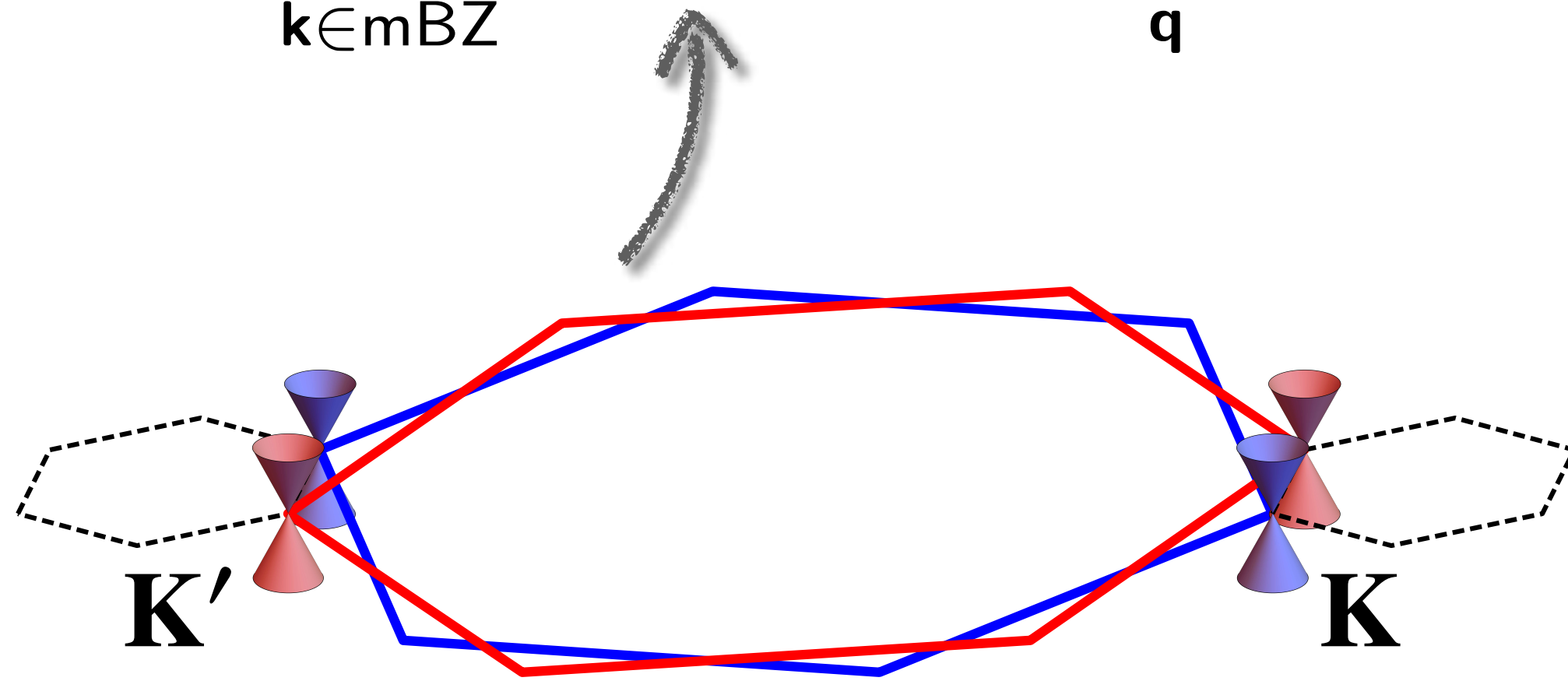
[Bultinck *et al.*, PRX '20]



Twisted bilayer graphene: Model

Interacting Bistritzer-MacDonald model:

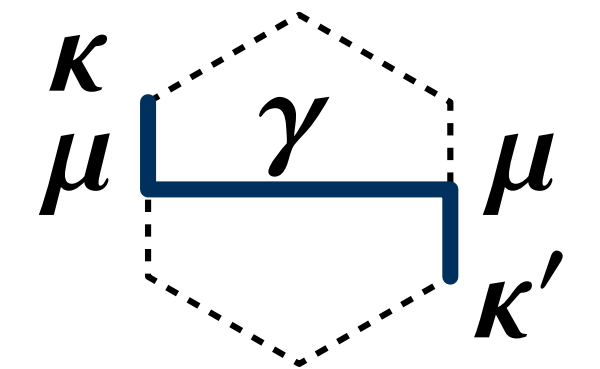
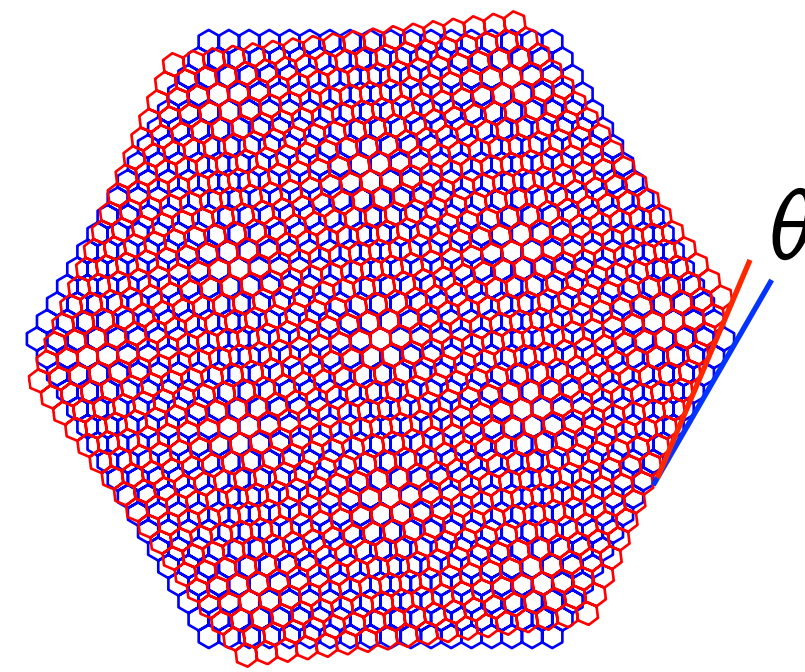
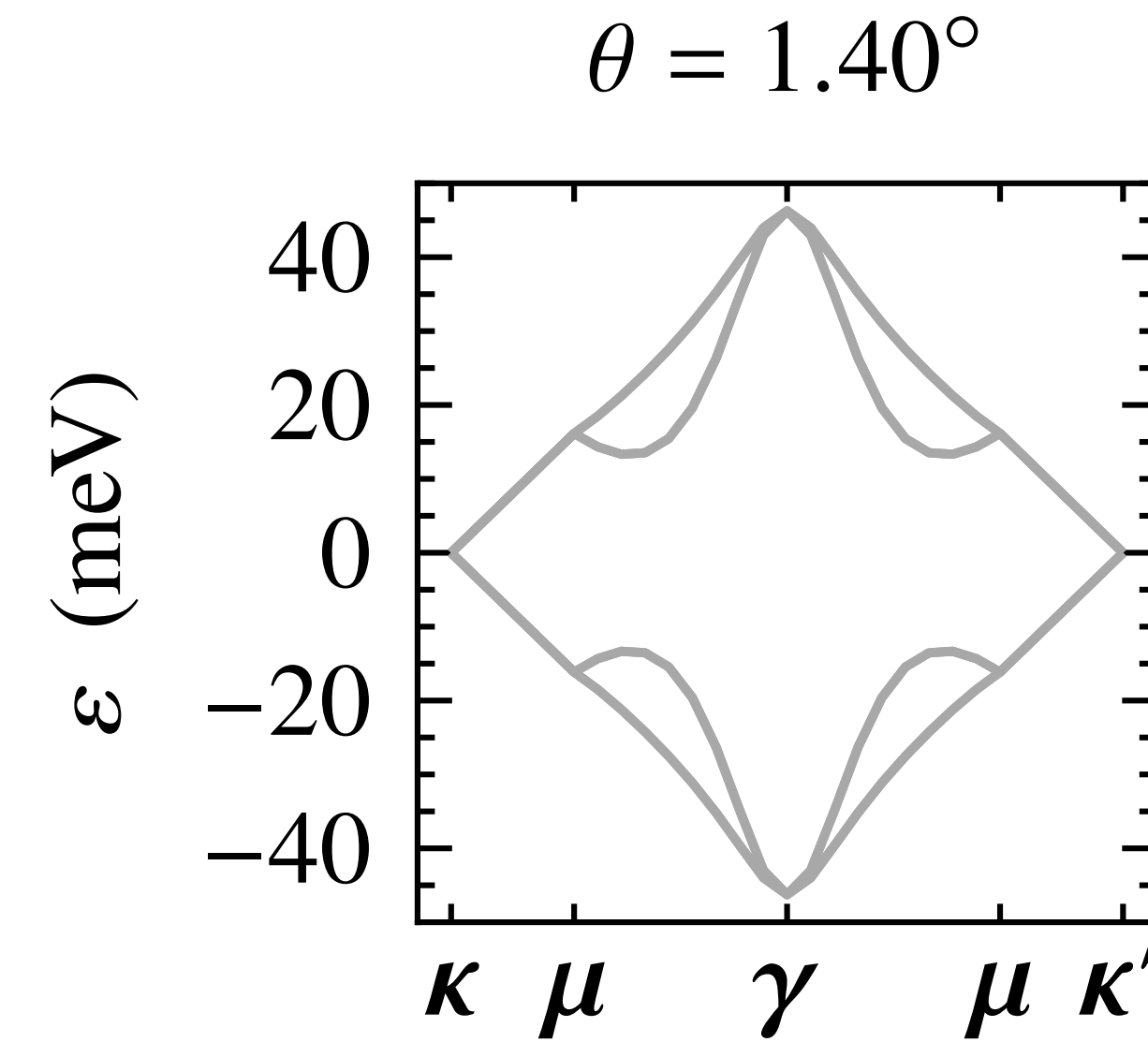
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[Bistritzer, MacDonald, PNAS '11]

[Bultinck *et al.*, PRX '20]

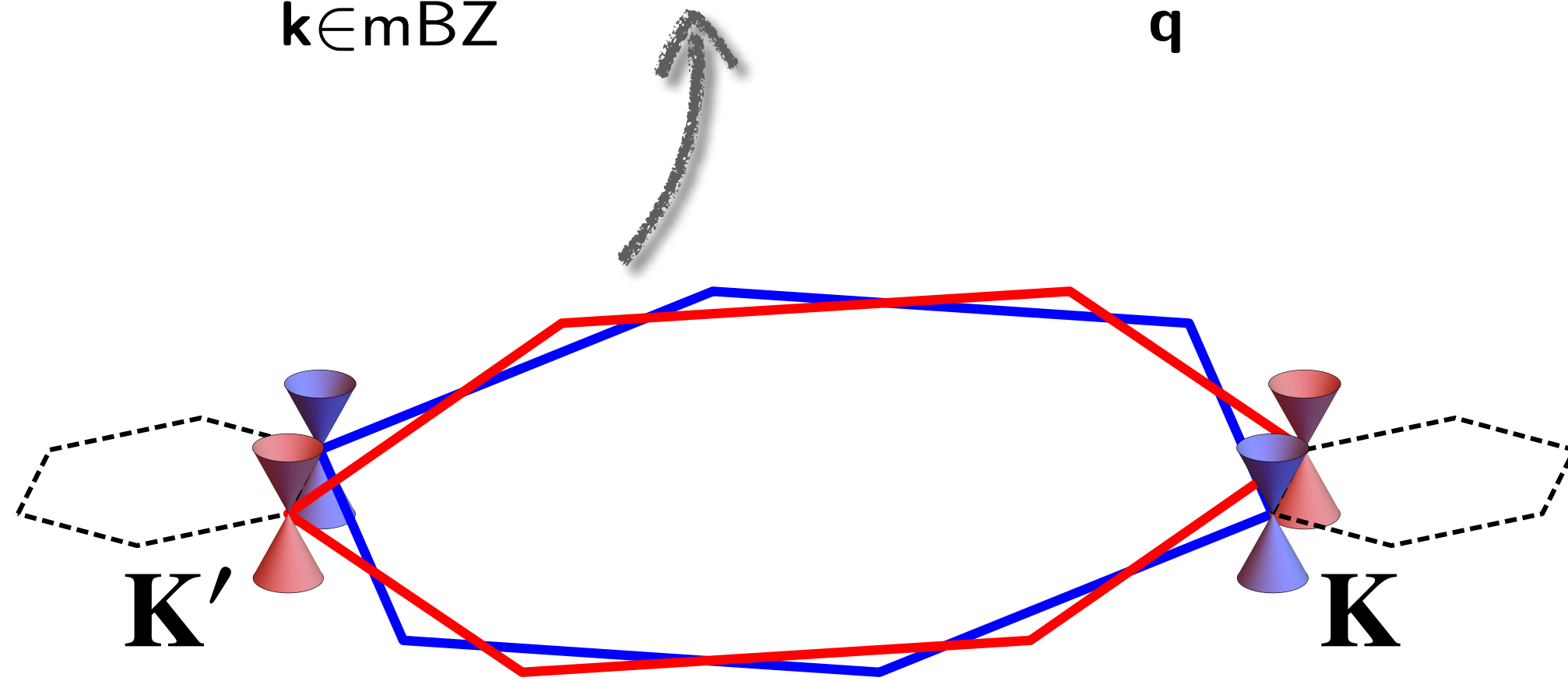
Noninteracting spectrum:



Twisted bilayer graphene: Model

Interacting Bistritzer-MacDonald model:

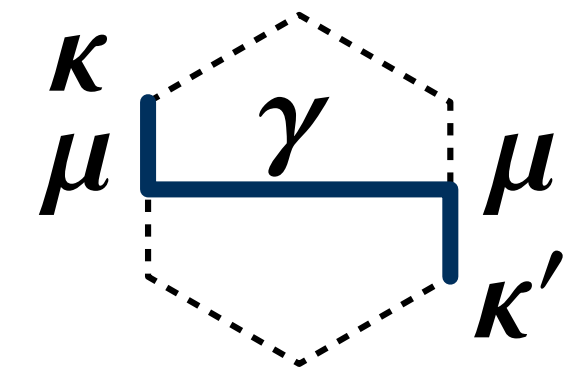
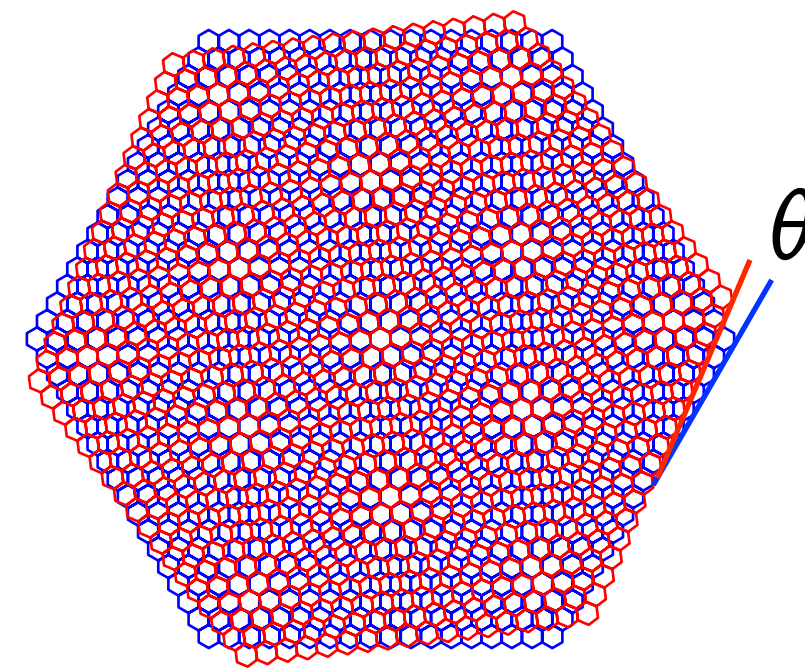
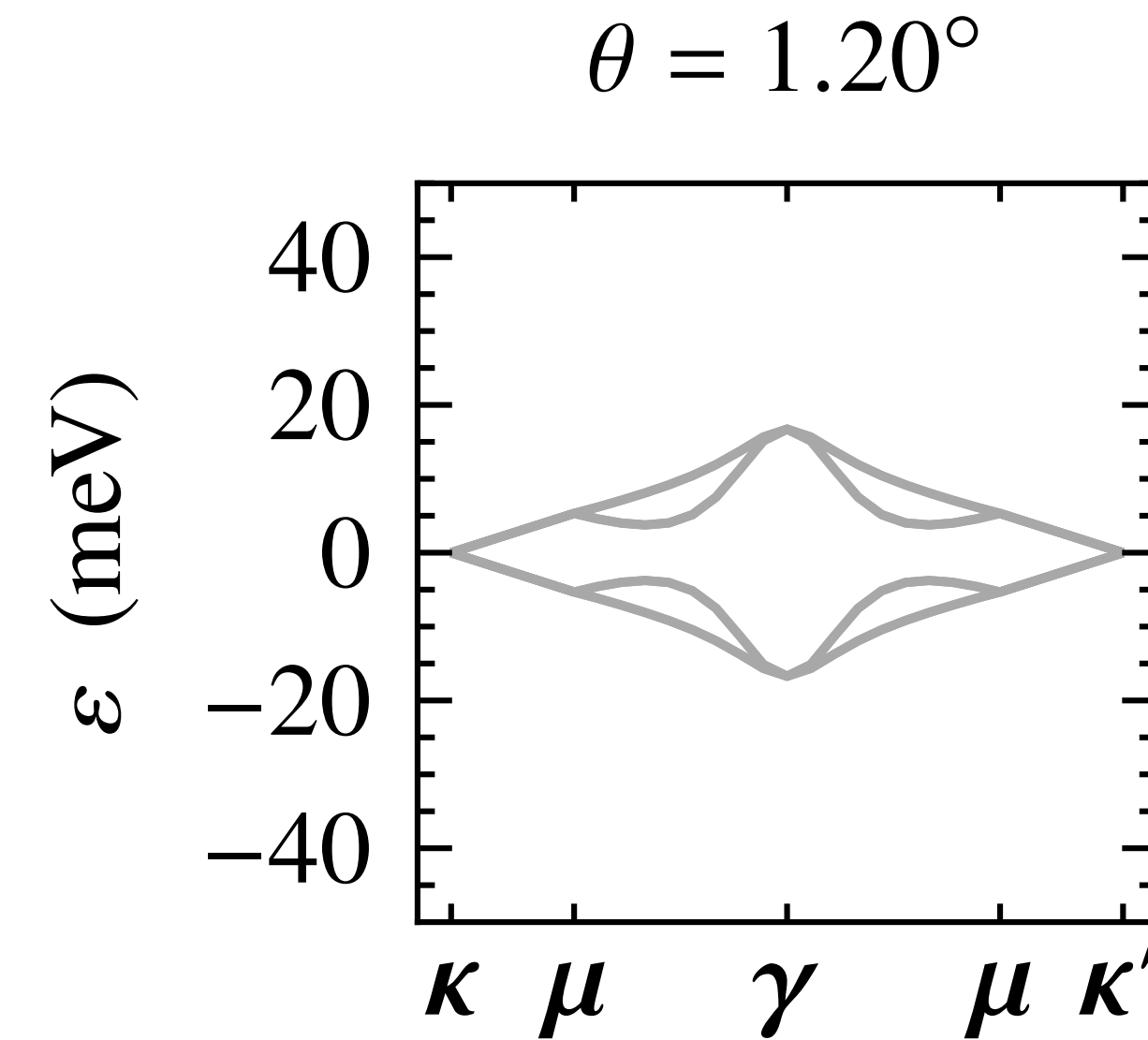
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[Bistritzer, MacDonald, PNAS '11]

[Bultinck *et al.*, PRX '20]

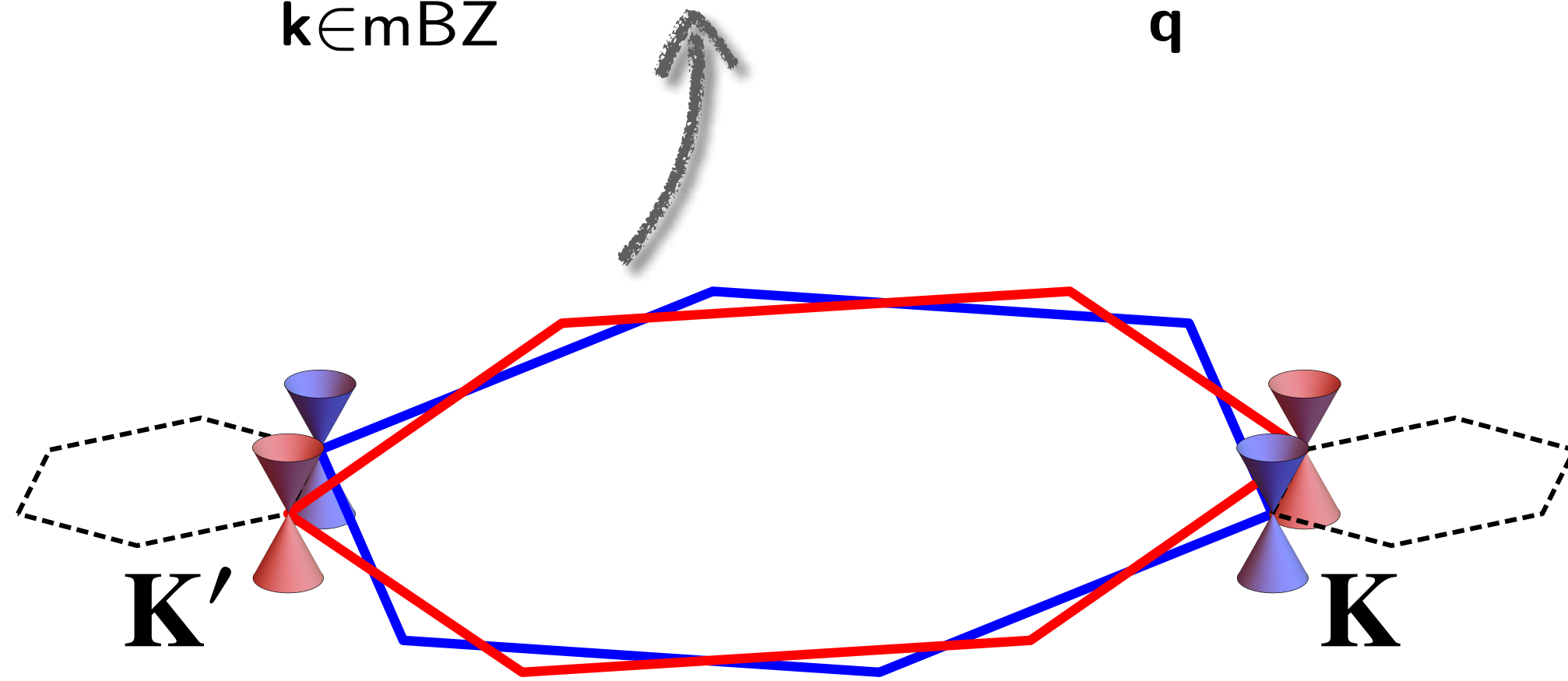
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Twisted bilayer graphene: Model

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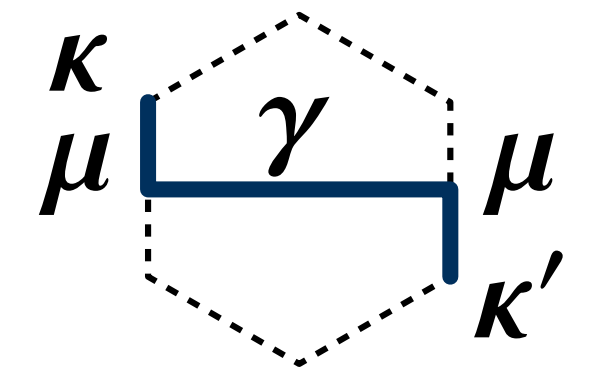
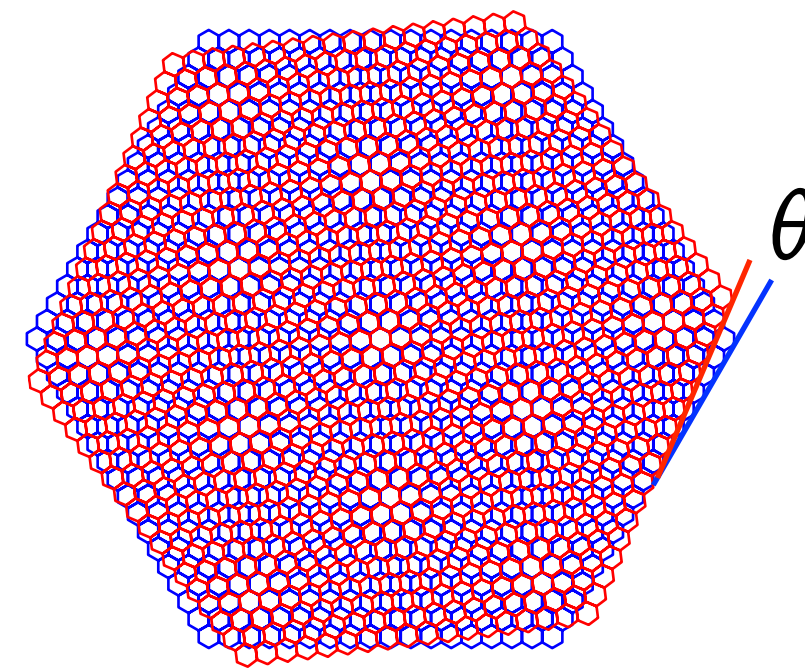
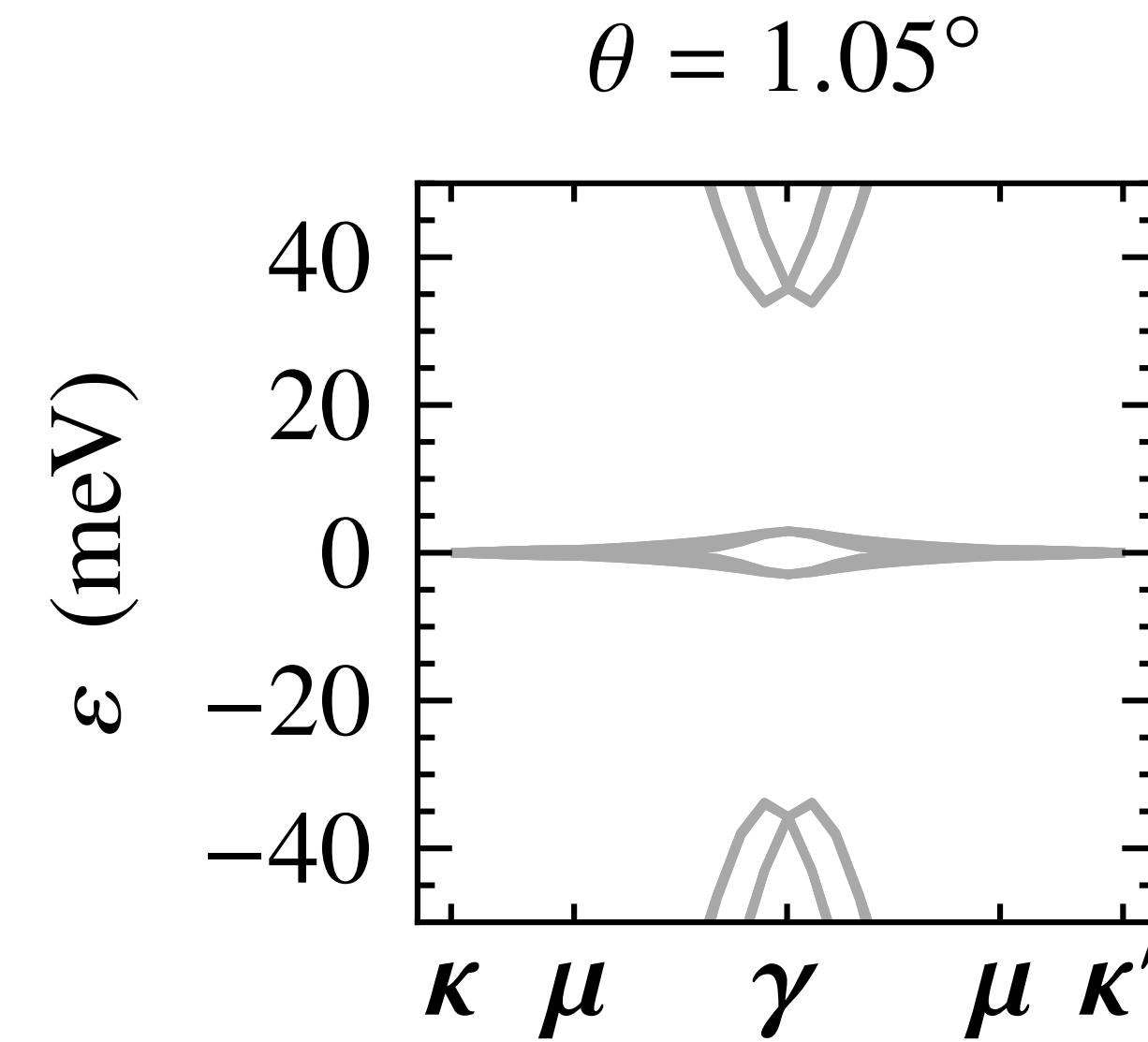
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[Bistritzer, MacDonald, PNAS '11]

[Bultinck *et al.*, PRX '20]

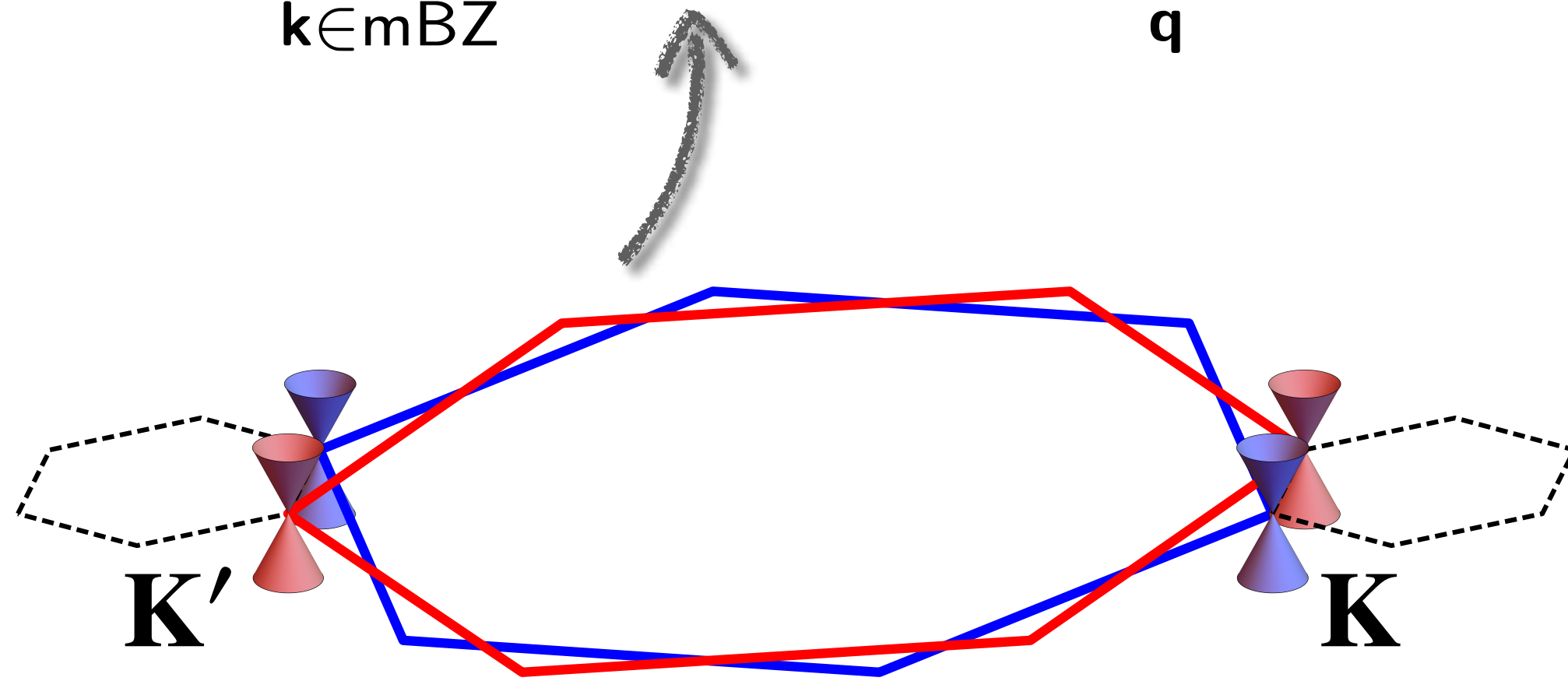
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Twisted bilayer graphene: Model

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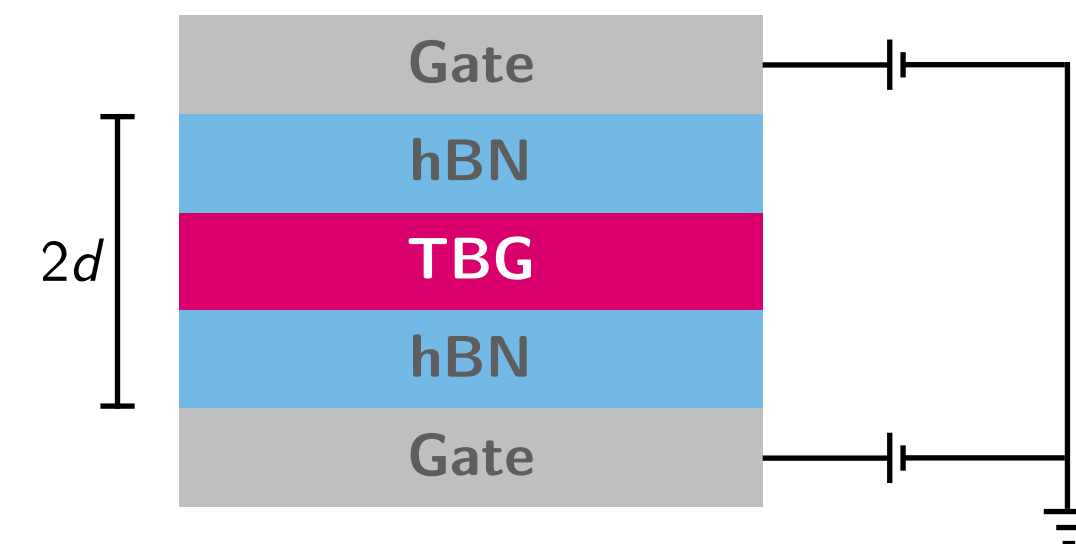
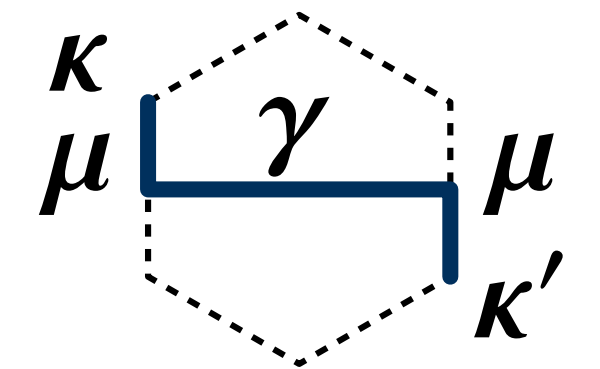
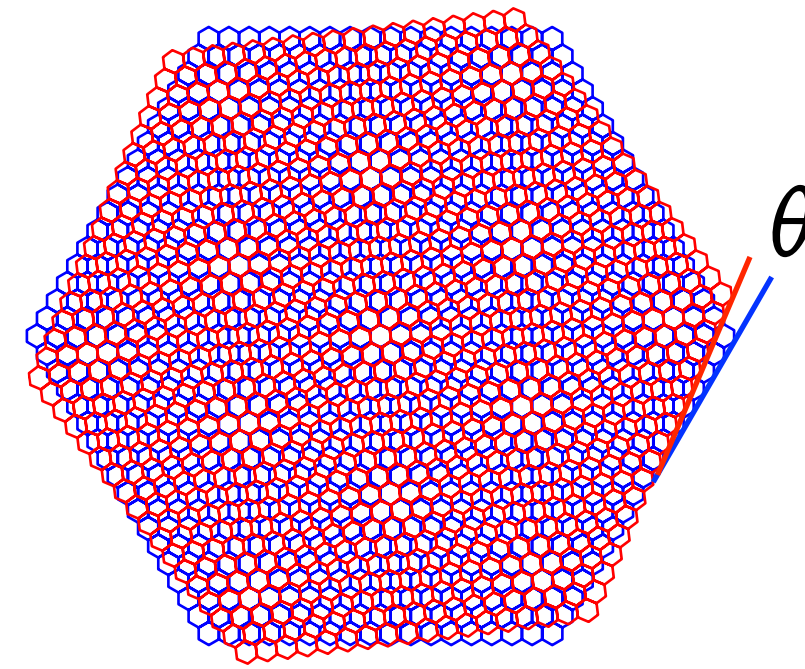
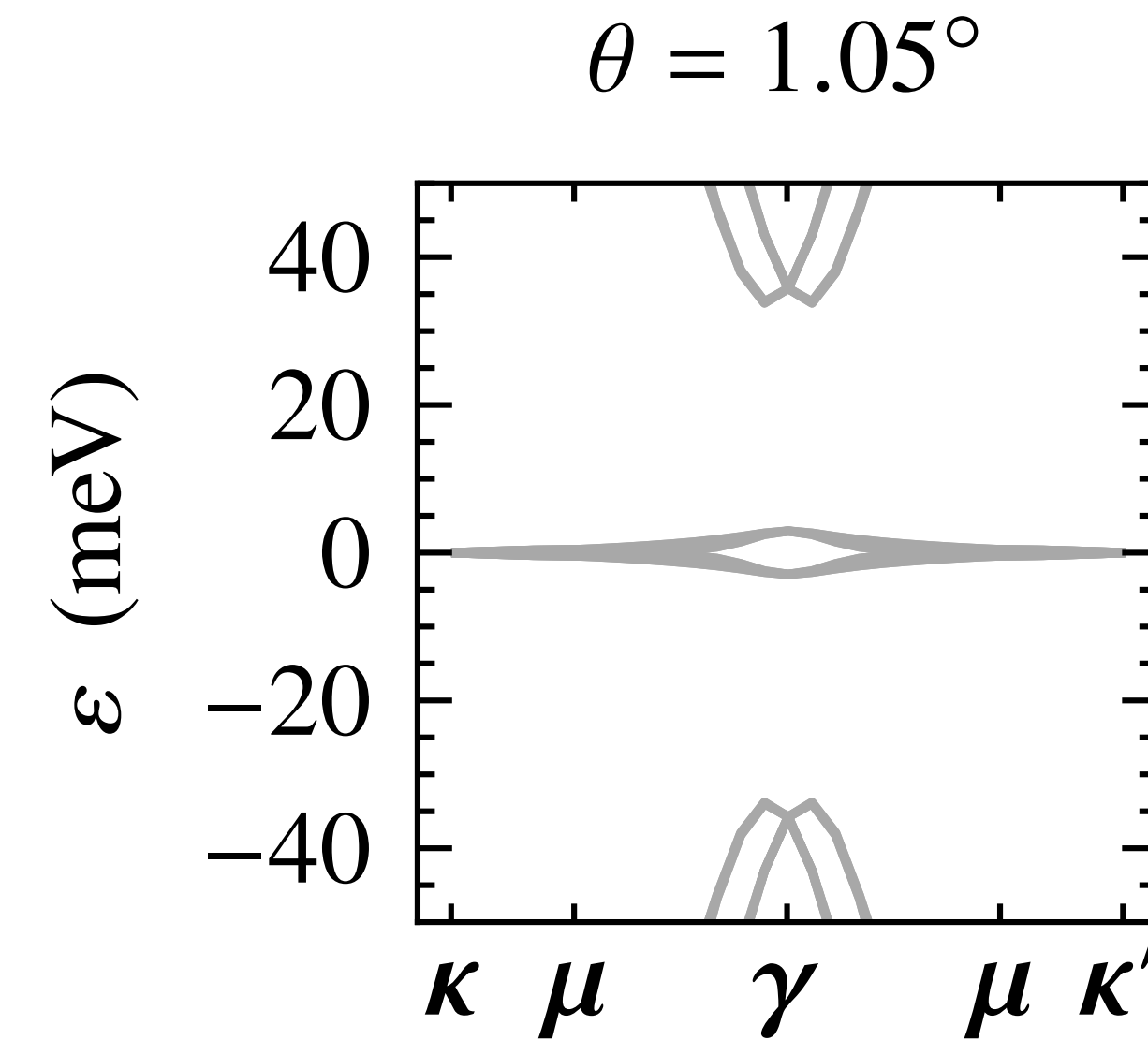
[Bistritzer, MacDonald, PNAS '11]

[Bultinck *et al.*, PRX '20]

Coulomb interaction:

$$V_{\mathbf{q}} = \frac{e^2}{2\epsilon_0 \epsilon(\theta, \mathbf{q}) |\mathbf{q}|} \tanh(|\mathbf{q}|d)$$

Noninteracting spectrum:



Internal screening

Screened Coulomb interaction:

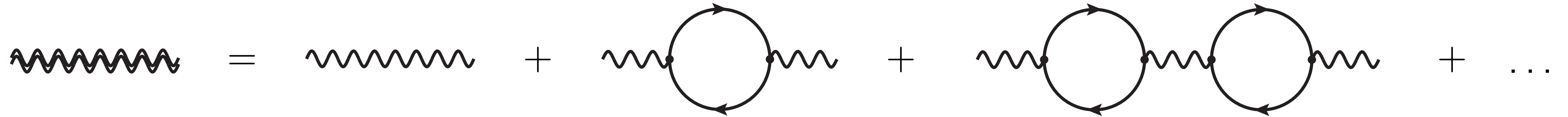
The diagram shows the expansion of the screened Coulomb interaction. On the left, a thick, double-wavy line represents the bare interaction. This is set equal to a series of terms: a single wavy line (the screened interaction), followed by a plus sign and a diagram with a wavy line entering a fermion loop (a circle with two arrows) and a wavy line exiting, followed by another plus sign and a diagram with two such fermion loops connected by a wavy line, and finally a plus sign followed by an ellipsis (...).

... RPA in atomistic tight-binding model

[Goodwin *et al.*, PRB '19]

Internal screening

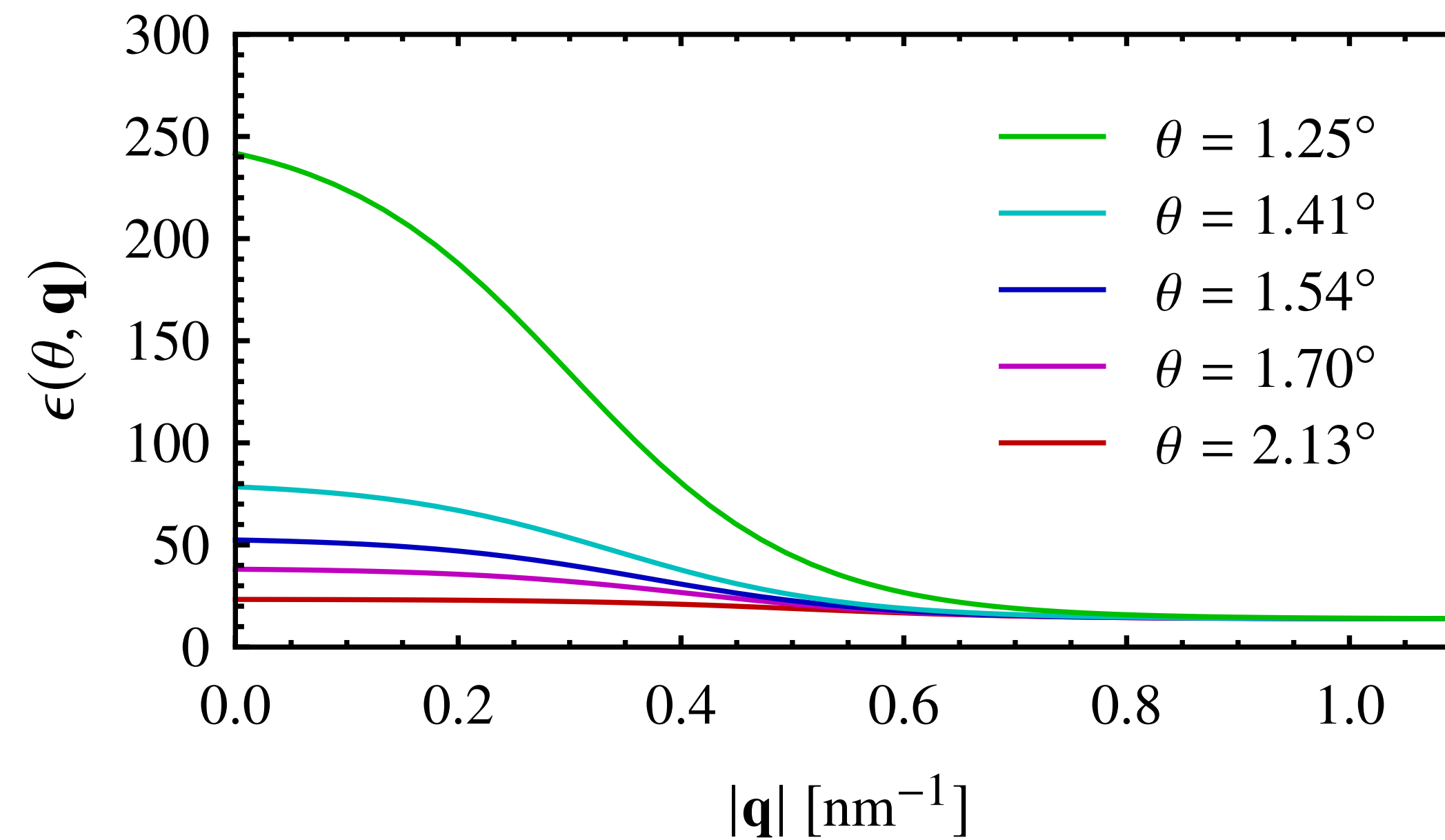
Screened Coulomb interaction:



... RPA in atomistic tight-binding model

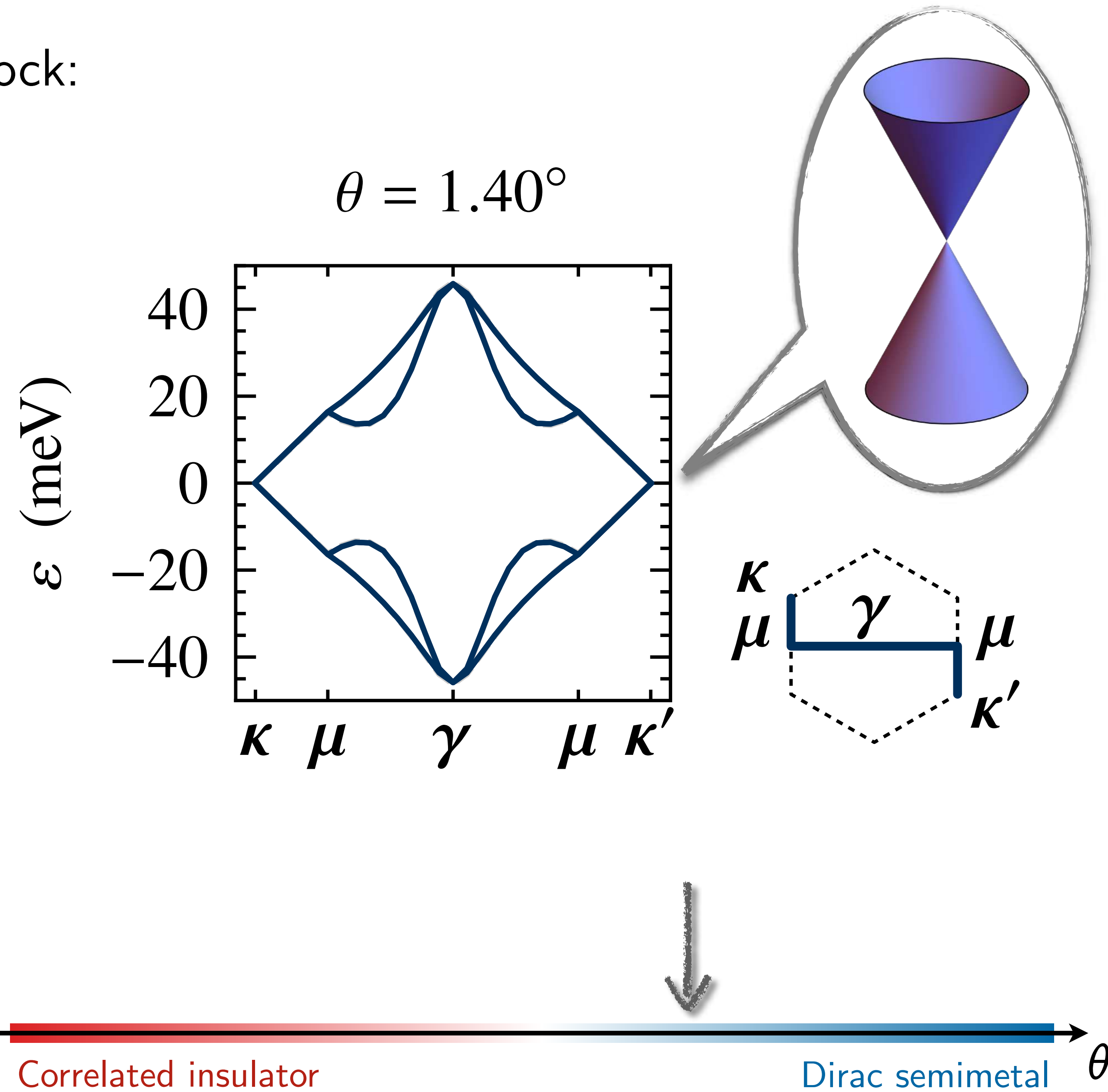
[Goodwin *et al.*, PRB '19]

Effective dielectric permittivity:



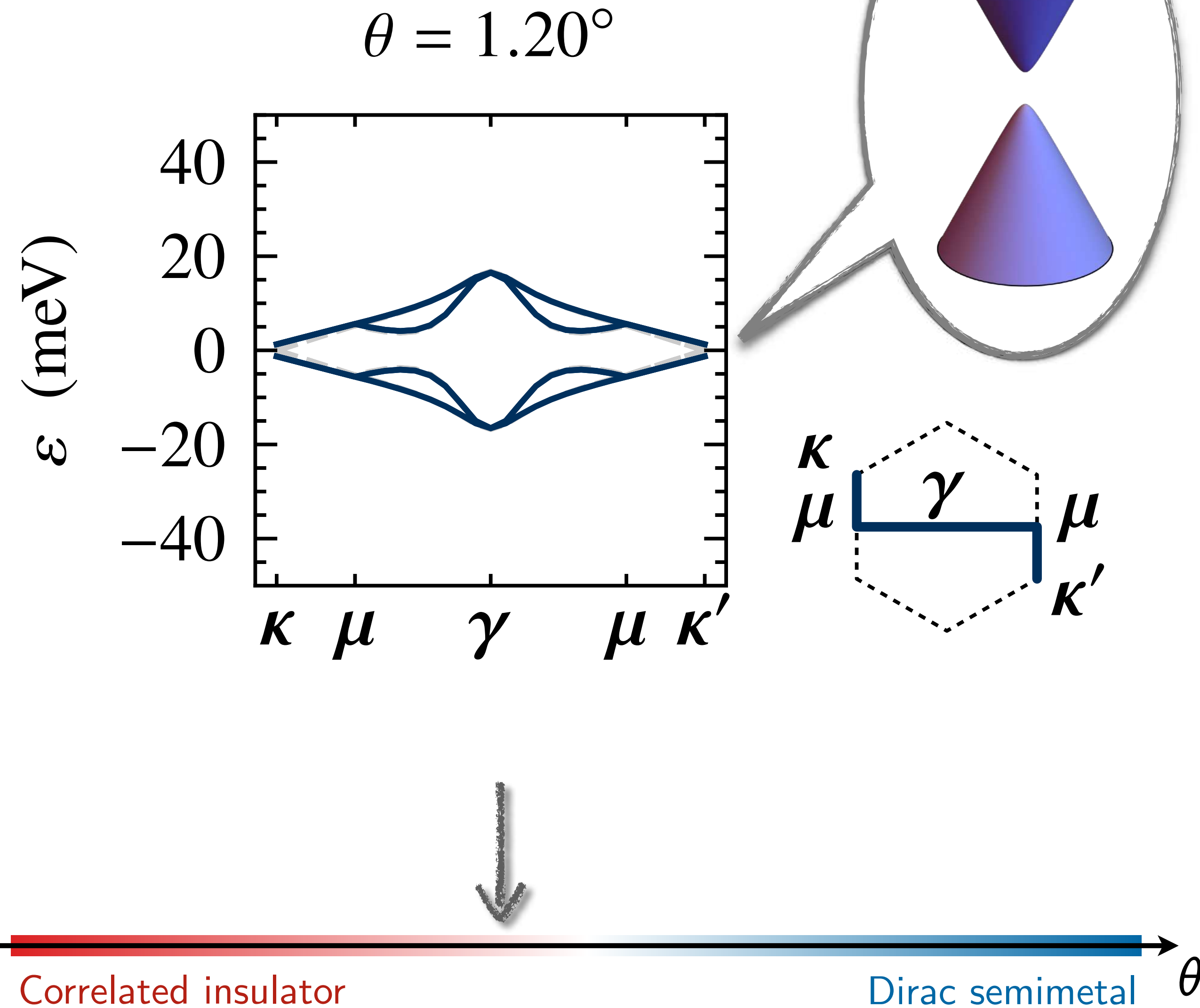
Twisted bilayer graphene: Interacting spectrum

Selfconsistent Hartree-Fock:



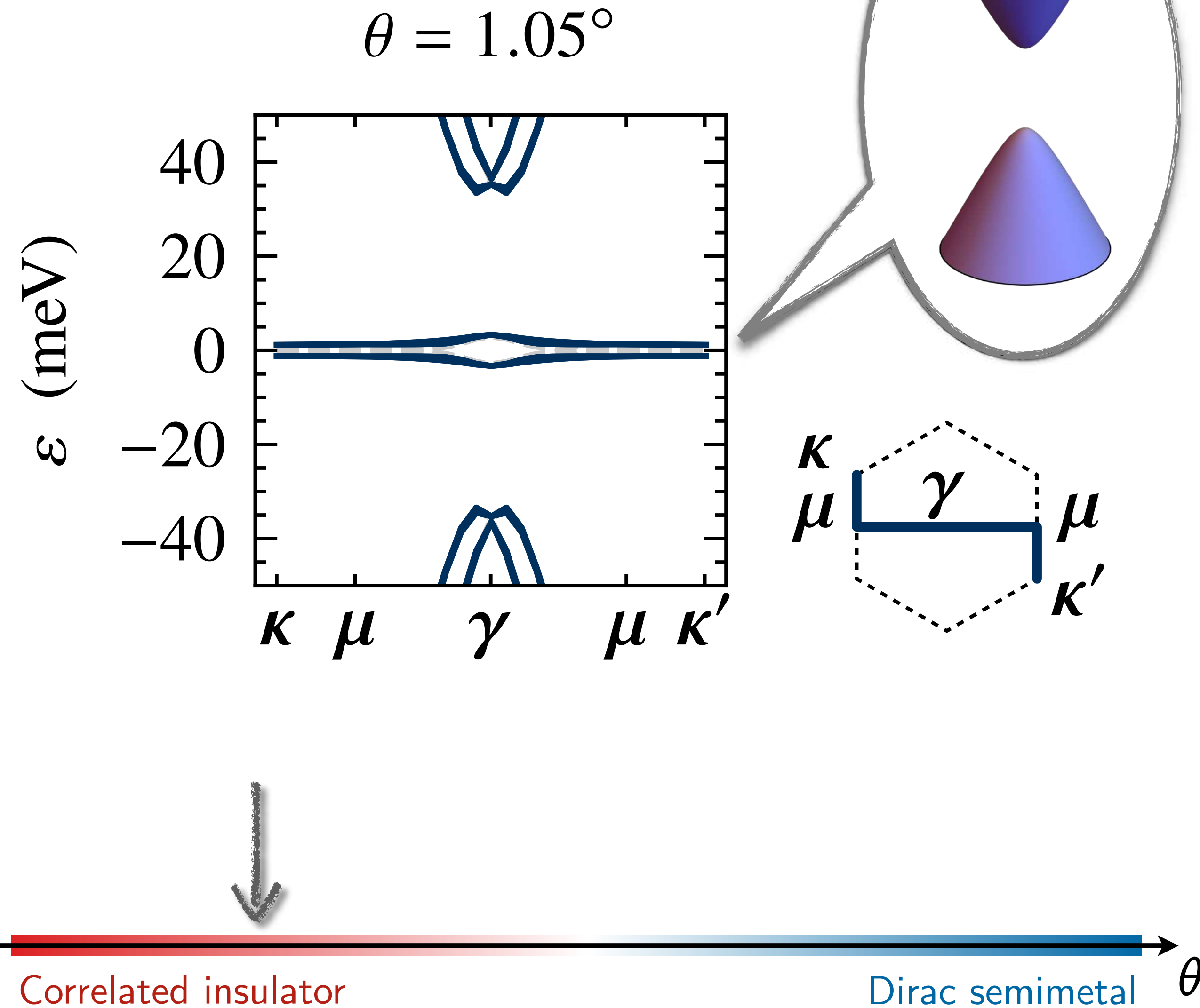
Twisted bilayer graphene: Interacting spectrum

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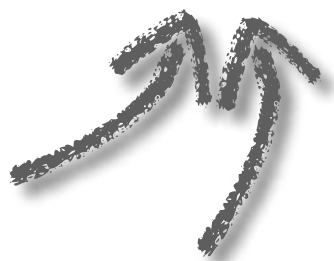
Twisted bilayer graphene: Interacting spectrum

Selfconsistent Hartree-Fock:



Twisted bilayer graphene: Kramers intervalley-coherent insulator

Density matrix:

$$\langle c_{\tau\sigma}^\dagger c_{\tau'\sigma'} \rangle$$


valley

sublattice

The diagram illustrates the mapping of physical properties to the indices in the density matrix expression. Two arrows originate from the labels 'valley' and 'sublattice' below. The 'valley' arrow points to the valley index τ in the creation operator $c_{\tau\sigma}^\dagger$. The 'sublattice' arrow points to the sublattice index σ in the same operator. A second arrow points from the 'sublattice' label to the sublattice index σ' in the annihilation operator $c_{\tau'\sigma'}$.

Twisted bilayer graphene: Kramers intervalley-coherent insulator

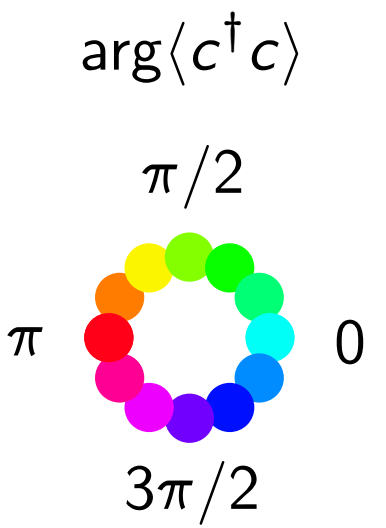
Density matrix:

$$\left\langle c_{\tau\sigma}^\dagger c_{\tau'\sigma'} \right\rangle \xrightarrow{\mathbf{k}=0} \frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & \bullet & & \\ \hline \end{array} \right)$$

Dirac semimetal

valley

sublattice



Twisted bilayer graphene: Kramers intervalley-coherent insulator

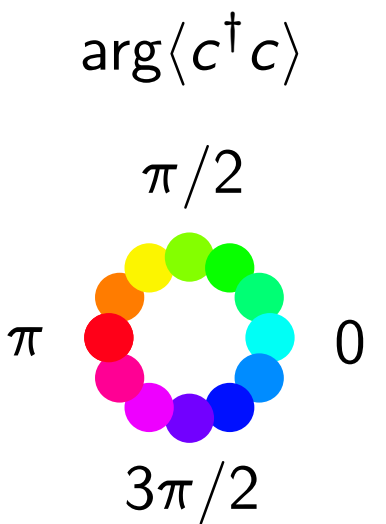
Density matrix:

$$\langle c_{\tau\sigma}^\dagger c_{\tau'\sigma'} \rangle \xrightarrow{\mathbf{k}=0} \begin{cases} \frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & \bullet & & \\ \hline \end{array} \right) \\ \frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \bullet & & & \\ \hline \end{array} \right) \end{cases}$$

valley sublattice

Dirac semimetal

KIVC insulator @ strong coupling



... breaks time reversal $\mathcal{T}$ & $U(1)_{\text{valley}}$
 ... preserves combination $(-i)\tau_z\mathcal{T}$

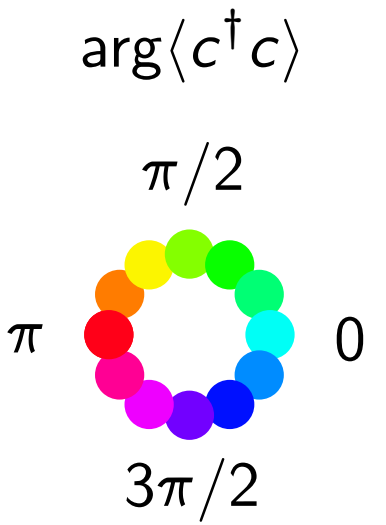
Twisted bilayer graphene: Kramers intervalley-coherent insulator

Density matrix:

$$\langle c_{T\sigma}^\dagger c_{T'\sigma'} \rangle \xrightarrow{\mathbf{k}=0} \left\{ \begin{array}{l} \frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \\ \frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \end{array} \right.$$

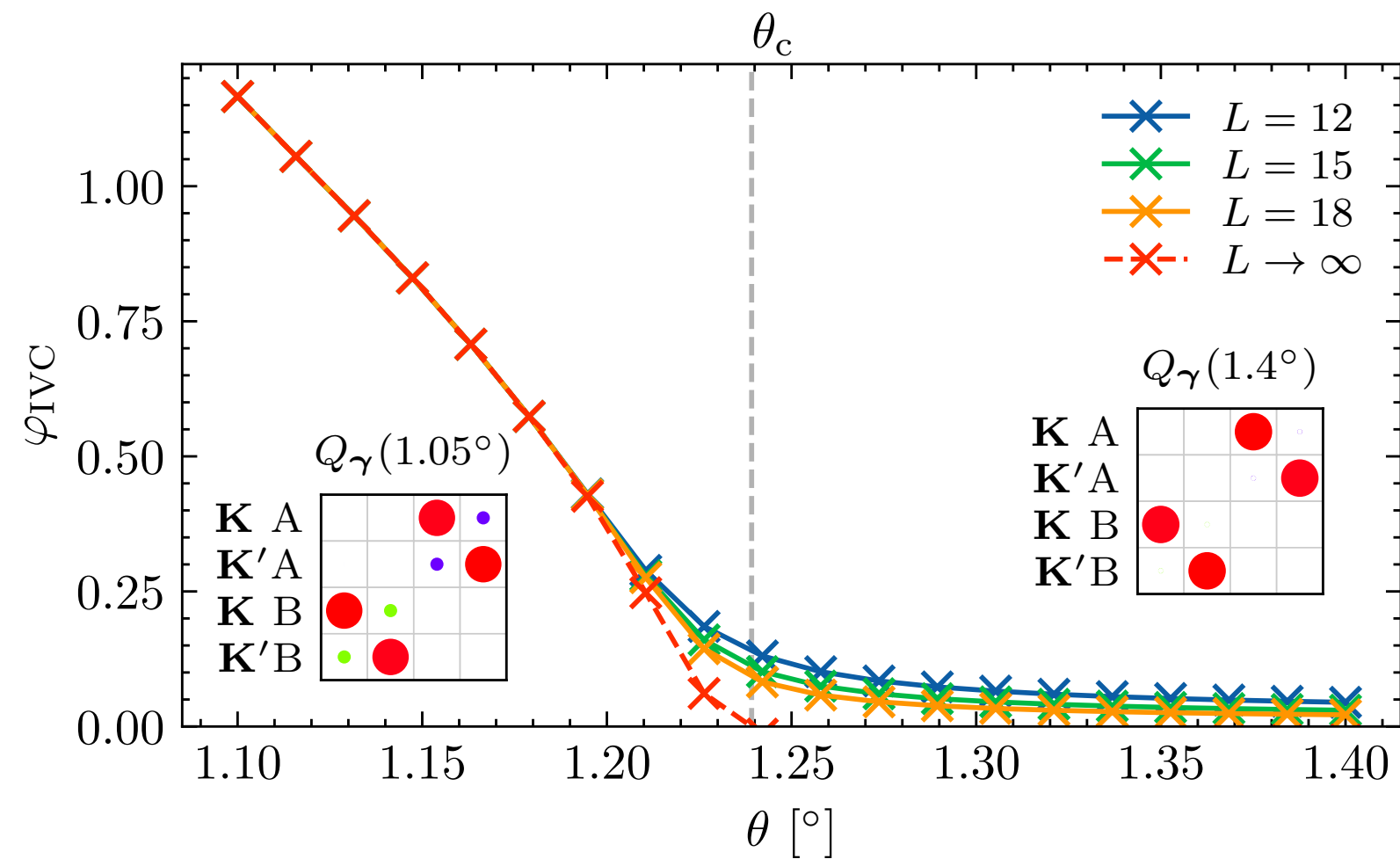
Dirac semimetal

KIVC insulator @ strong coupling



- ... breaks time reversal $\mathcal{T}$ & $U(1)_{\text{valley}}$
- ... preserves combination $(-i)\tau_z\mathcal{T}$

Intervalley-coherence order parameter:



... for $d = 20\text{nm}$

Twisted bilayer graphene: Kramers intervalley-coherent insulator

Density matrix:

valley

sublattice

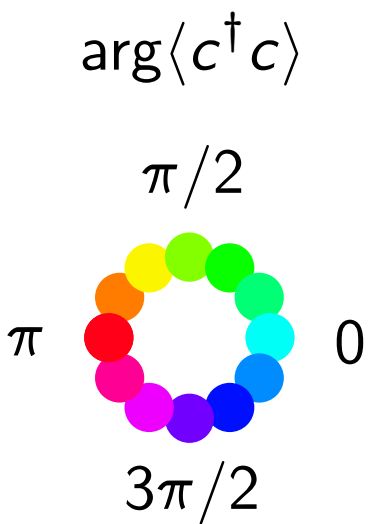
$\langle c_{\tau\sigma}^\dagger c_{\tau'\sigma'} \rangle \xrightarrow{\mathbf{k}=0}$

$\frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & \bullet & & \\ \hline \end{array} \right)$

$\frac{1}{2} \left(\mathbb{1} + \begin{array}{|c|c|c|c|} \hline & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \bullet & & & \\ \hline \end{array} \right)$

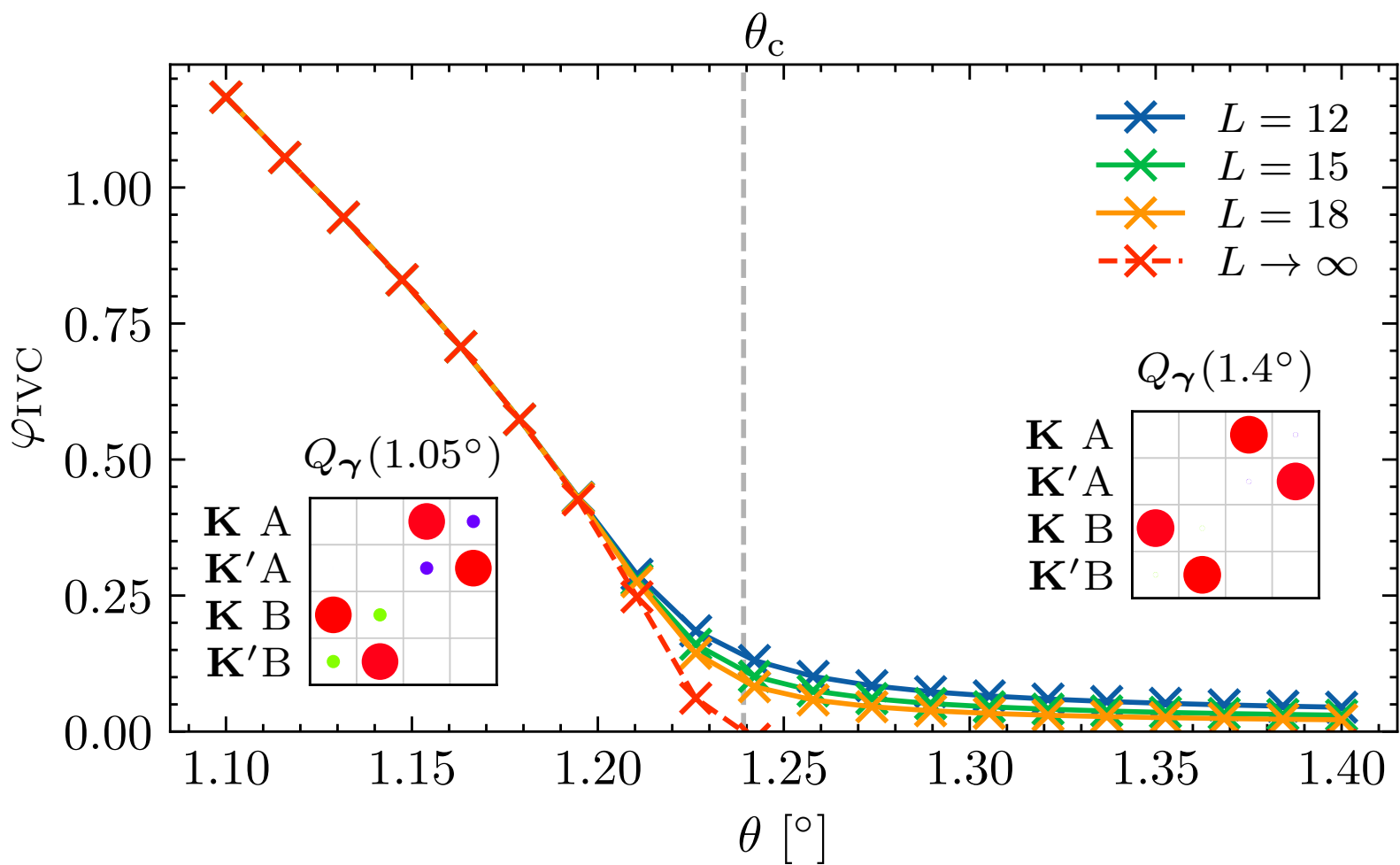
Dirac semimetal

KIVC insulator @ strong coupling



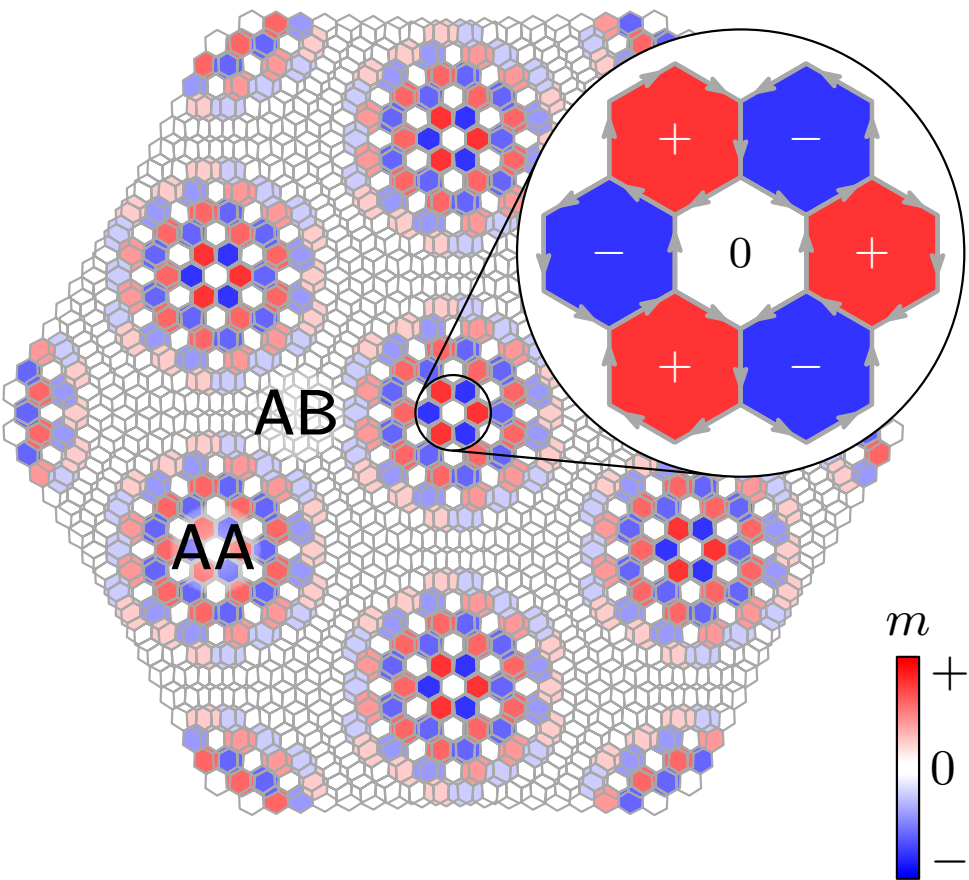
... breaks time reversal $\mathcal{T}$ & $U(1)_{\text{valley}}$
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Intervalley-coherence order parameter:



... for $d = 20\text{nm}$

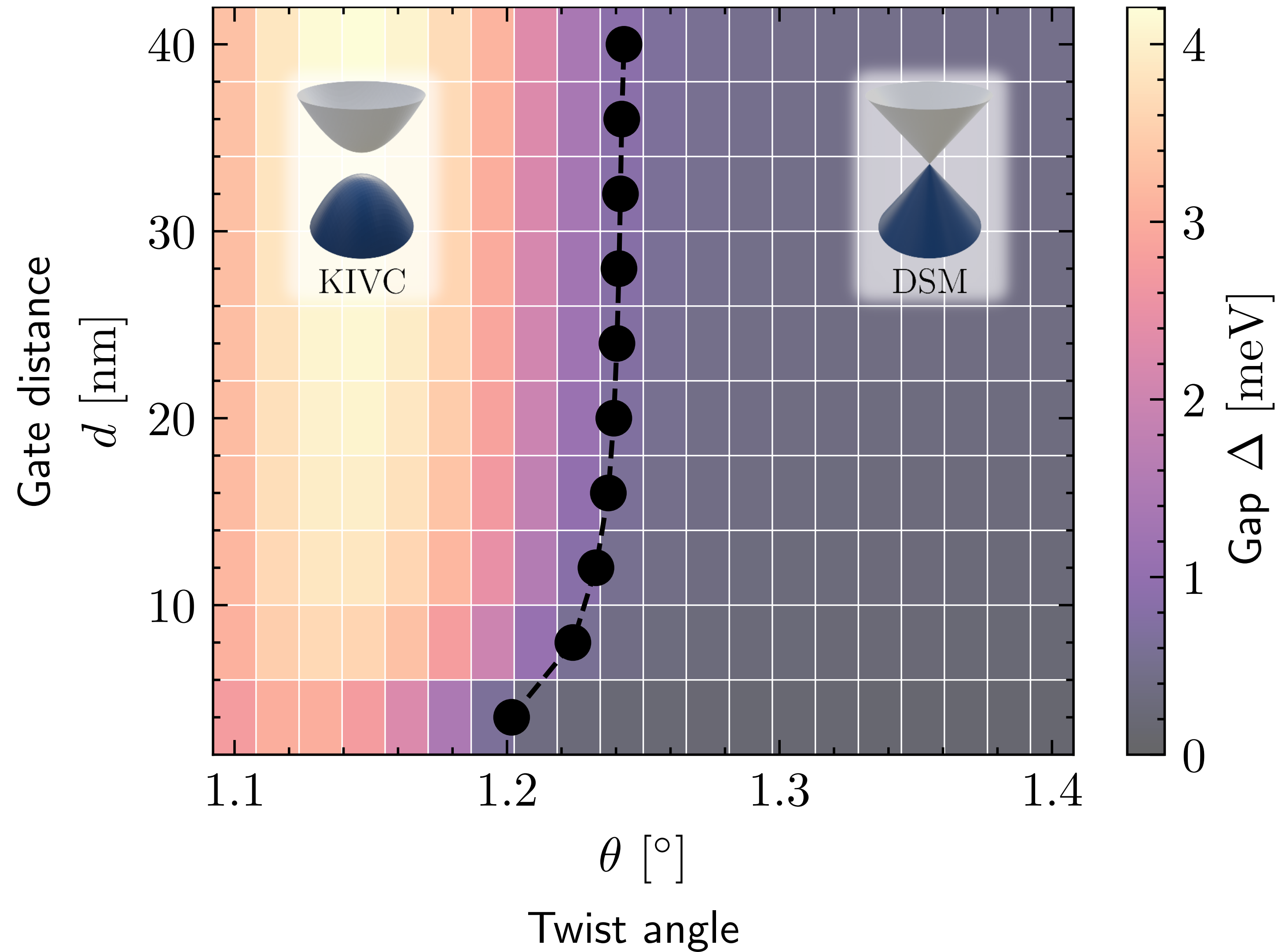
Real-space representation:



... in agreement with
[Bultinck *et al.*, PRX '20]
[Kwan *et al.*, PRX '21]
[Wagner *et al.*, PRL '22]
[Hofmann *et al.*, PRX '22]
[Rai *et al.*, PRX '24]

→ Talk by J. Hofmann

Twisted bilayer graphene: Phase diagram

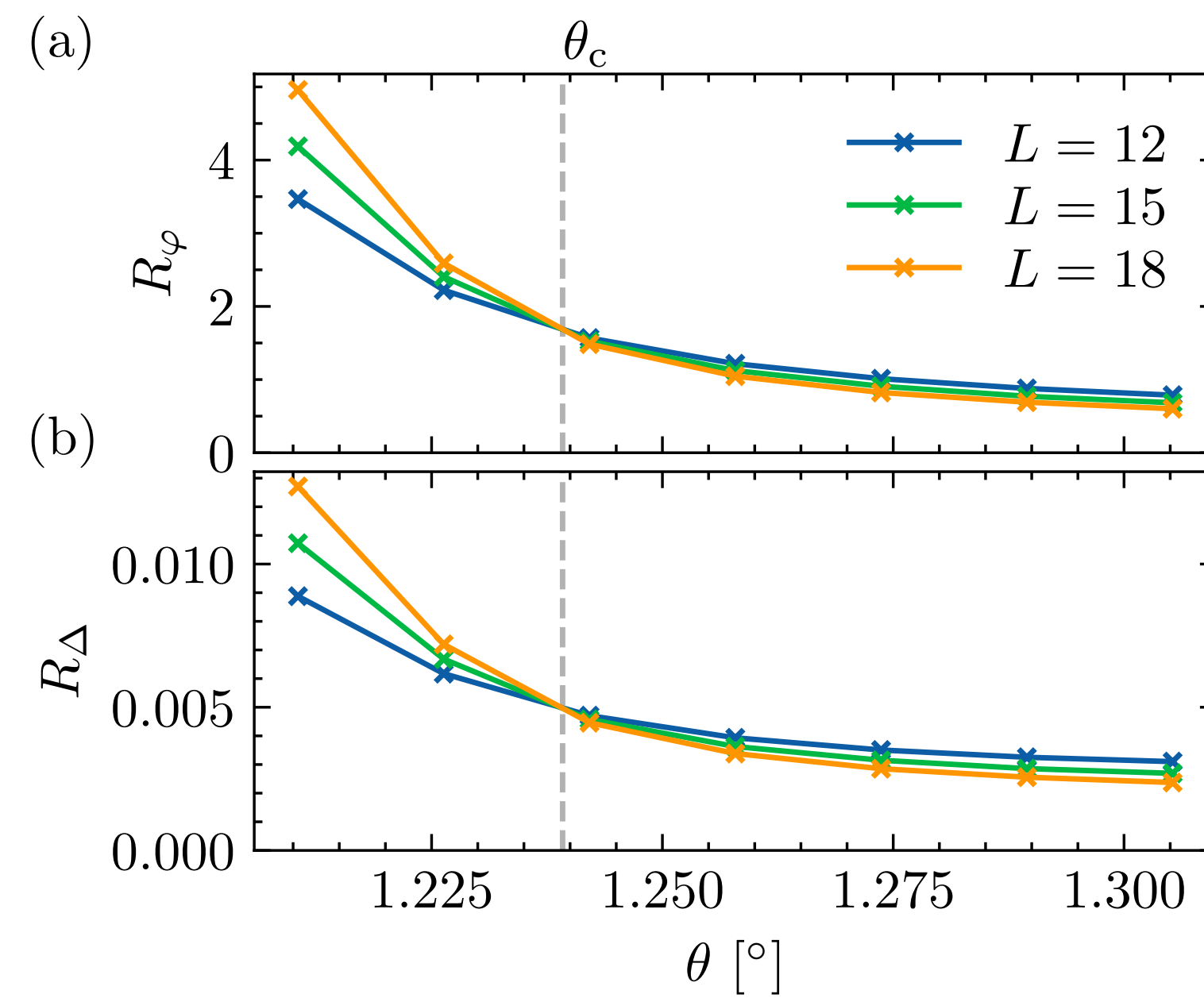


[Biedermann & LJ, PRB '25]

... see also [Huang *et al.*, Nat. Commun. '25]

Twisted bilayer graphene: Quantum criticality

Crossing-point analysis:



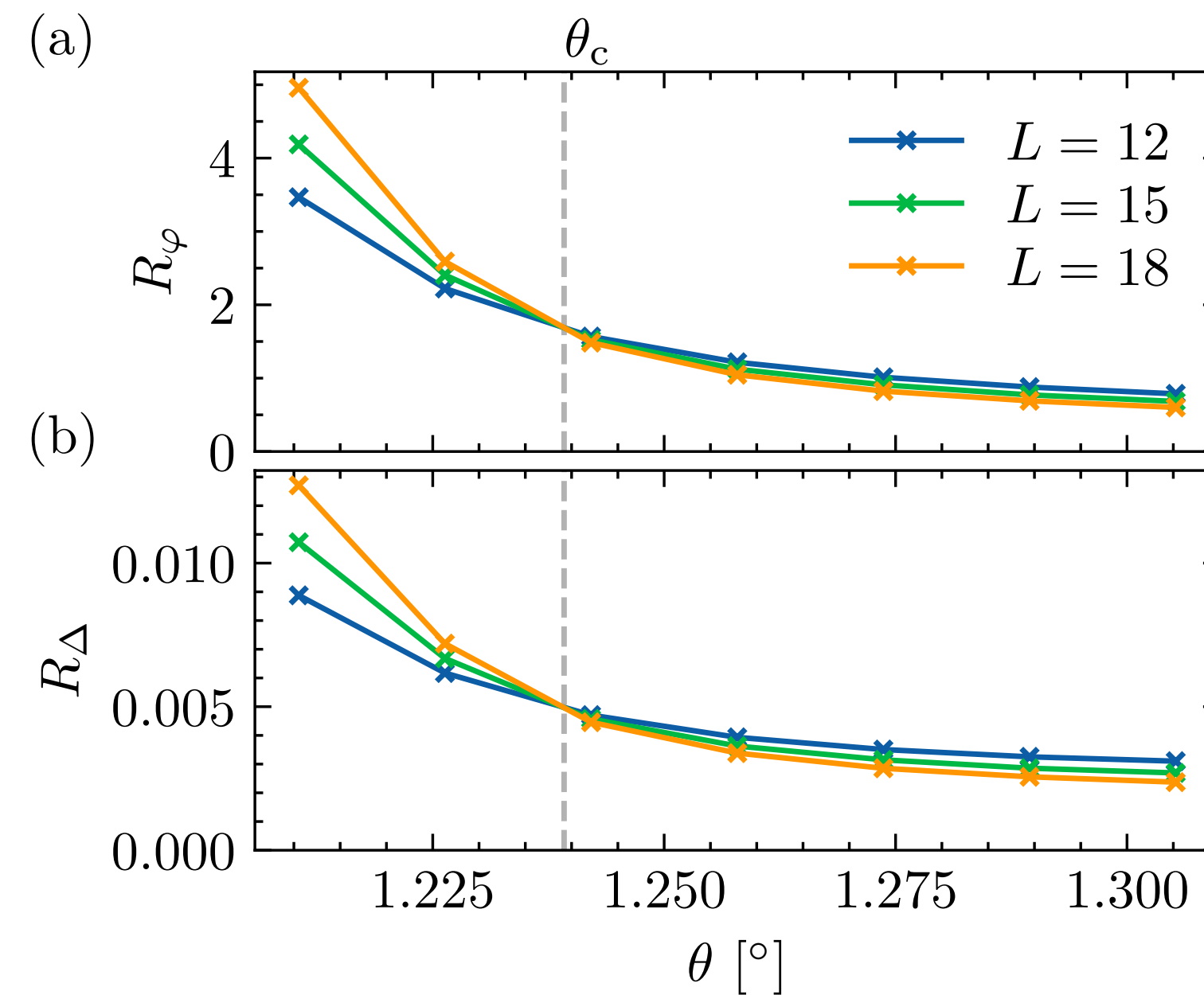
$$R_\varphi = L^{(1+\eta_\varphi)/2} \varphi_{\text{IVC}}$$
$$R_\Delta = L^z \Delta / t_0$$

Annotations:

- anomalous dimension (pointing to η_φ)
- dynamical exponent (pointing to z)
- nearest-neighbor hopping amplitude (pointing to t_0)

Twisted bilayer graphene: Quantum criticality

Crossing-point analysis:



$$R_\varphi = L^{(1+\eta_\varphi)/2} \varphi_{\text{IVC}}$$

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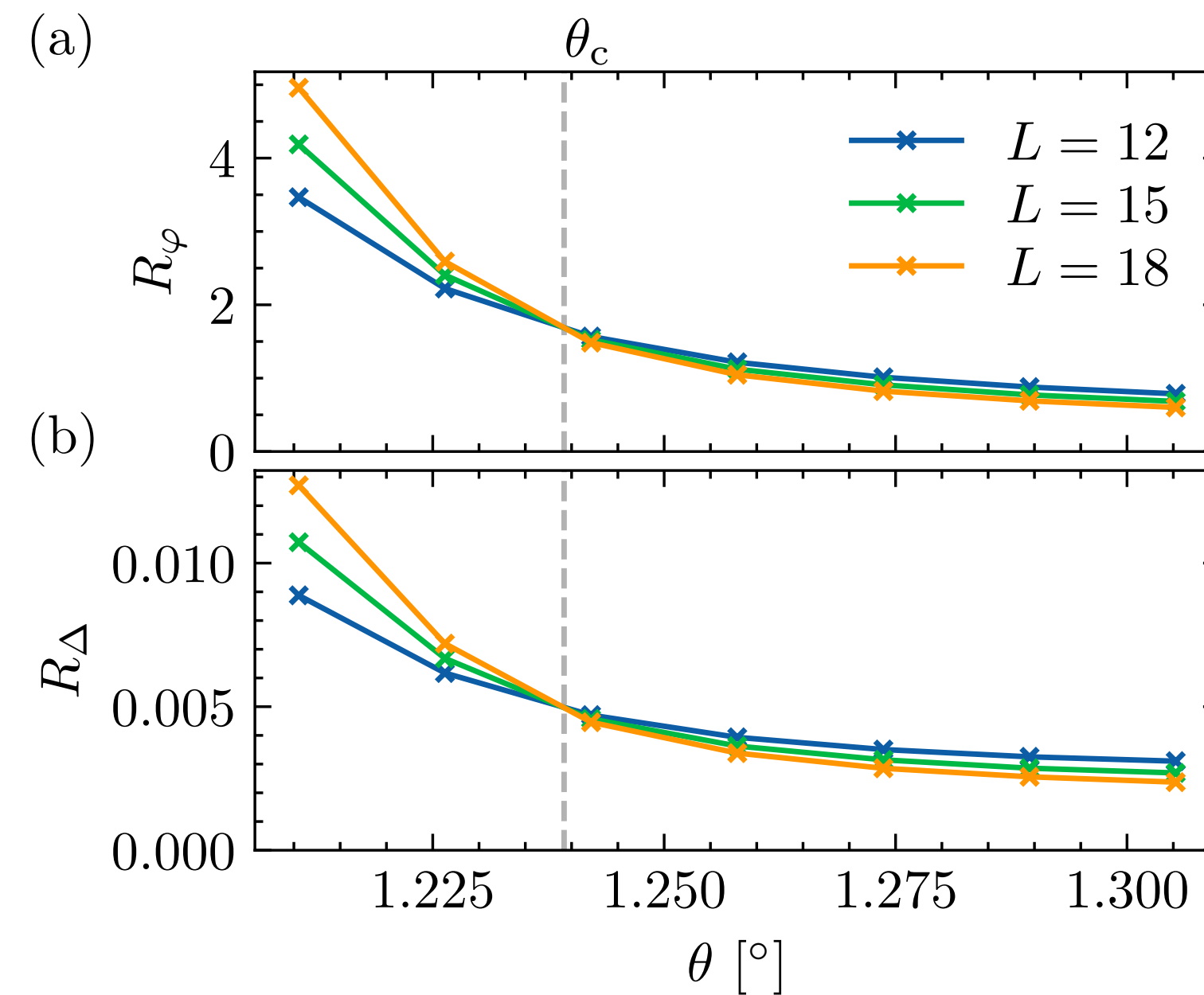
$$\mathcal{L} = \bar{\psi} \gamma_\mu \partial_\mu \psi + g \begin{pmatrix} i\bar{\psi} \gamma_3 \psi \\ i\bar{\psi} \gamma_5 \psi \end{pmatrix} \cdot \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} + \dots$$

... with emergent relativistic symmetry

[Biedermann & LJ, PRB '25]

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[Biedermann & LJ, PRB '25]

Critical exponents:

$$z = 1$$

$$\eta_\varphi \approx 0.93$$

$$\beta \approx 1.05$$

... from $4 - \epsilon$ and $2 + \epsilon$ expansions

[Hawashin, Scherer, LJ, PRB '25]

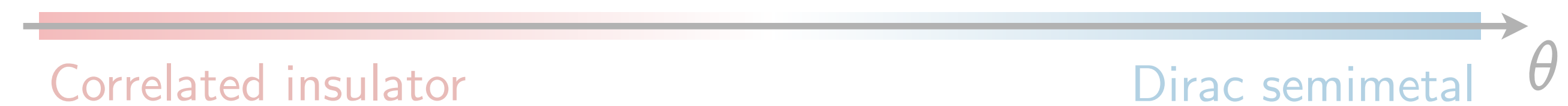
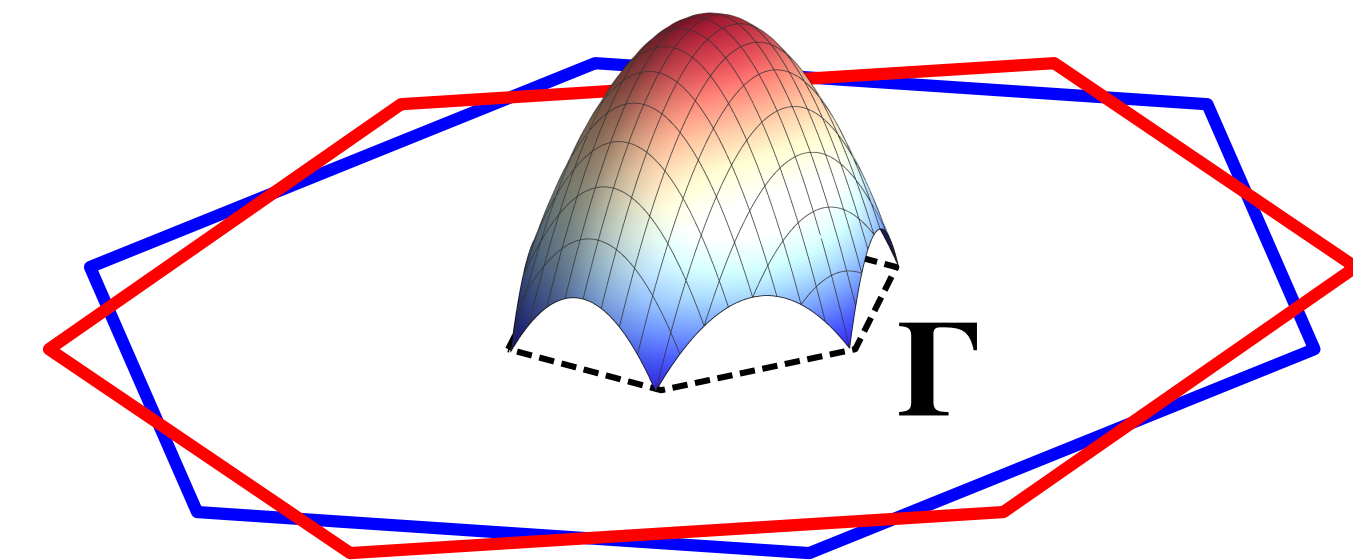
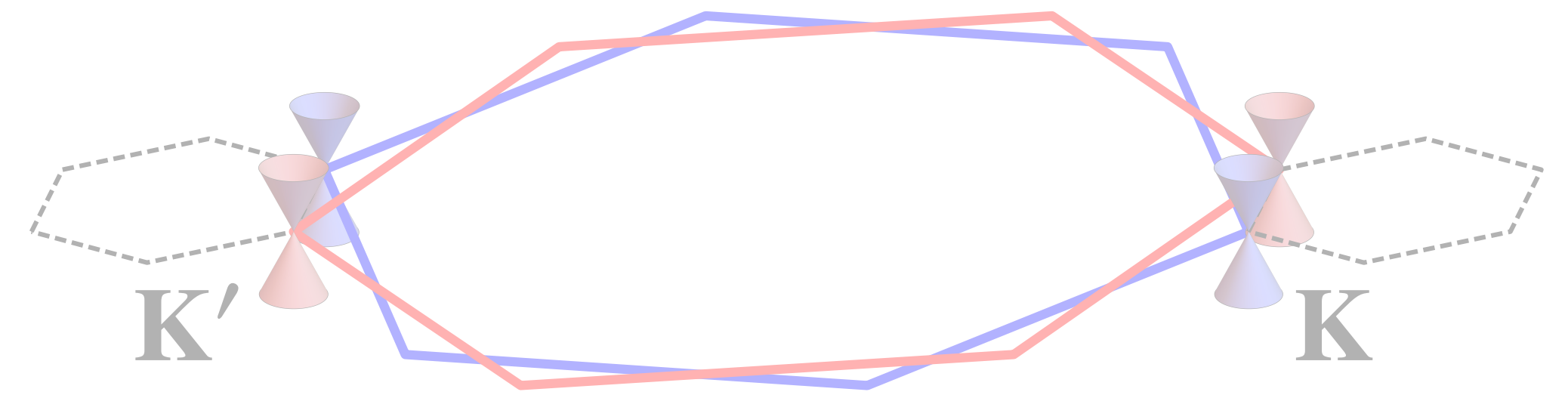
Outline

(1) Introduction

(2) Twisted bilayer graphene

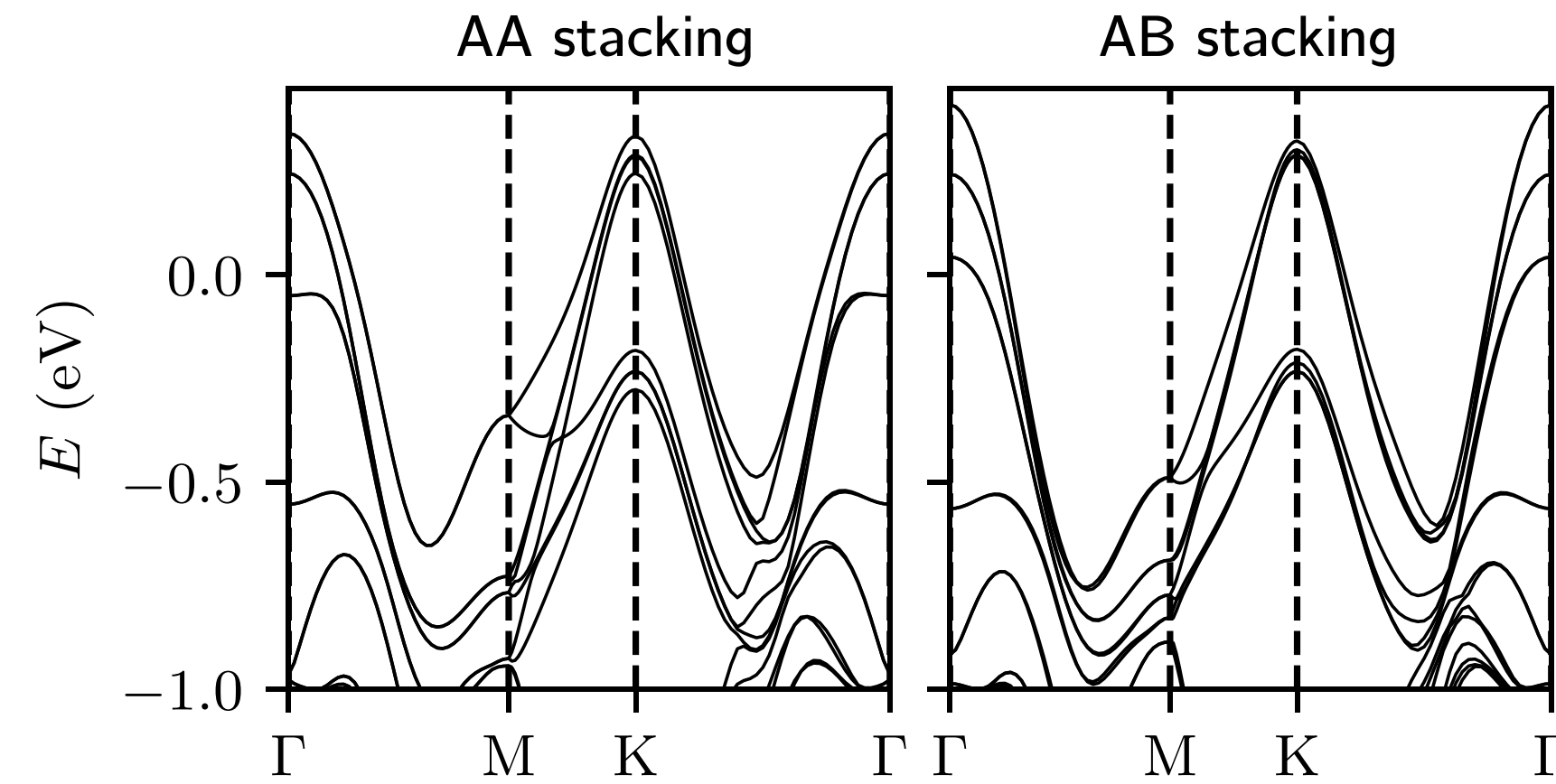
(3) Twisted double bilayer TMDs

(4) Conclusions



Double bilayer transition metal dichalcogenides (TMDs)

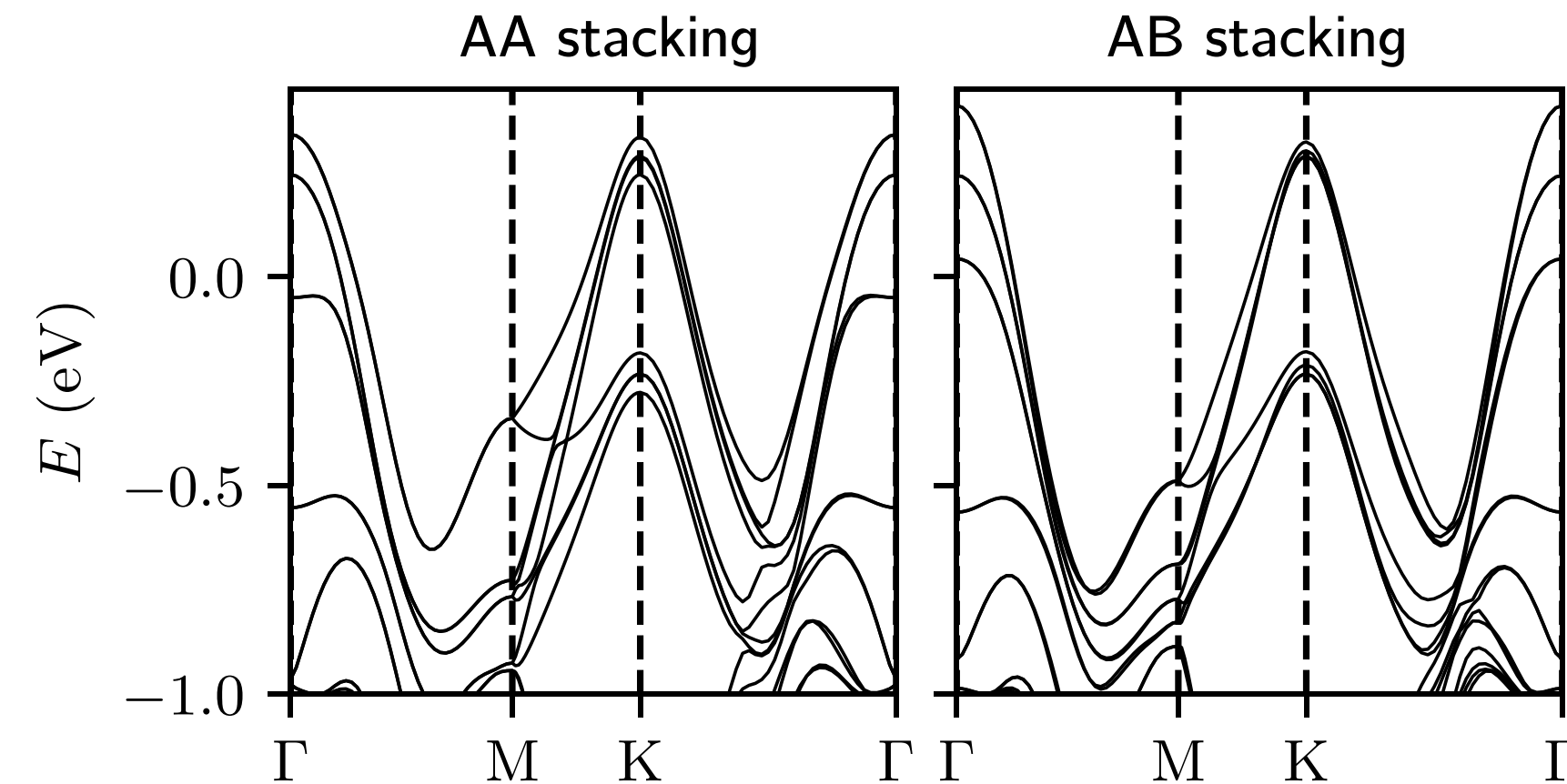
Untwisted noninteracting spectrum:



... for double bilayer WSe₂
[Pan *et al.*, PRRes '23]

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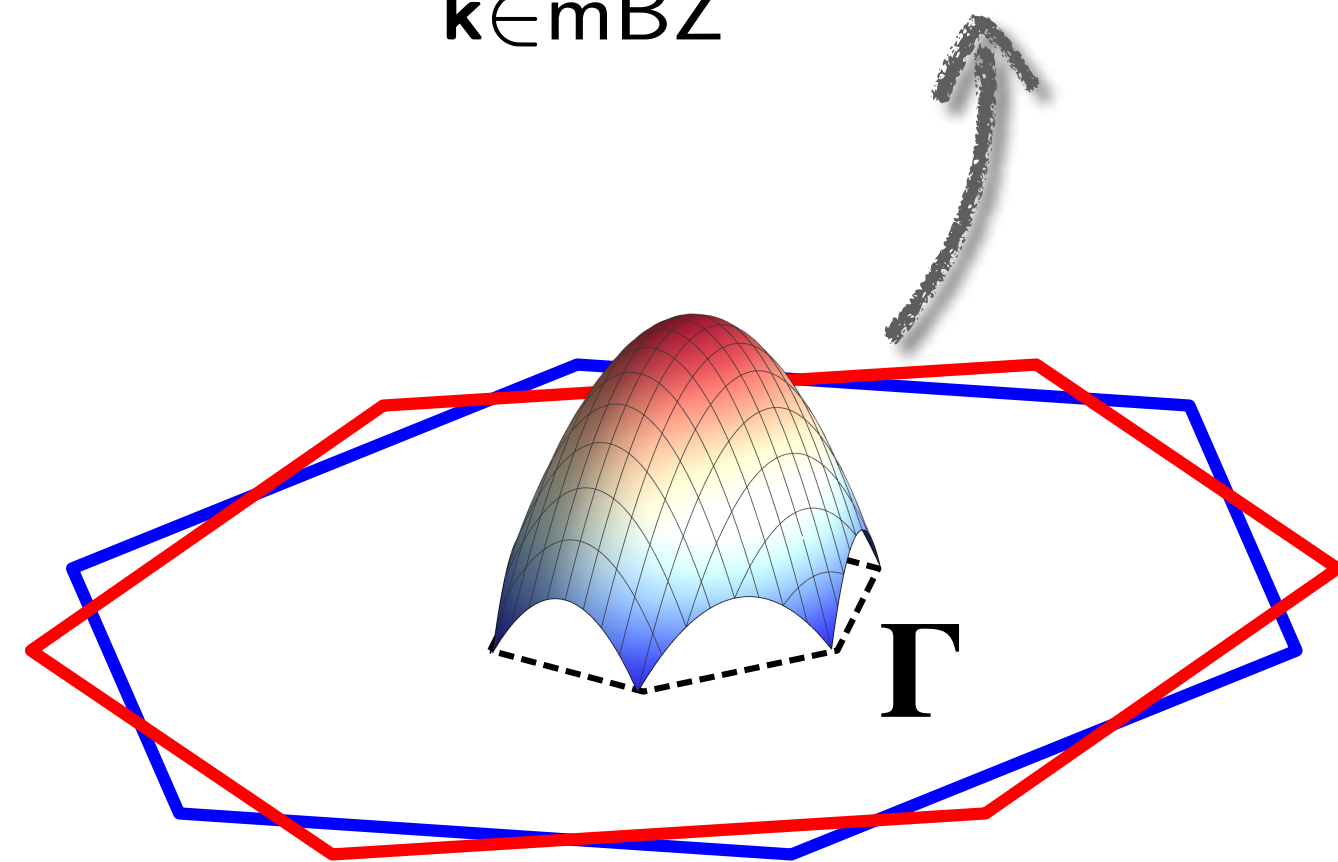
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Interacting Bistritzer-MacDonald model:

$$\mathcal{H} = \sum_{\mathbf{k} \in \text{mBZ}} c_{\mathbf{k}}^{\dagger} h(\mathbf{k}) c_{\mathbf{k}} - \frac{1}{2A} \sum_{\mathbf{q}} V_{\mathbf{q}} : \rho_{\mathbf{q}} \rho_{-\mathbf{q}} :$$

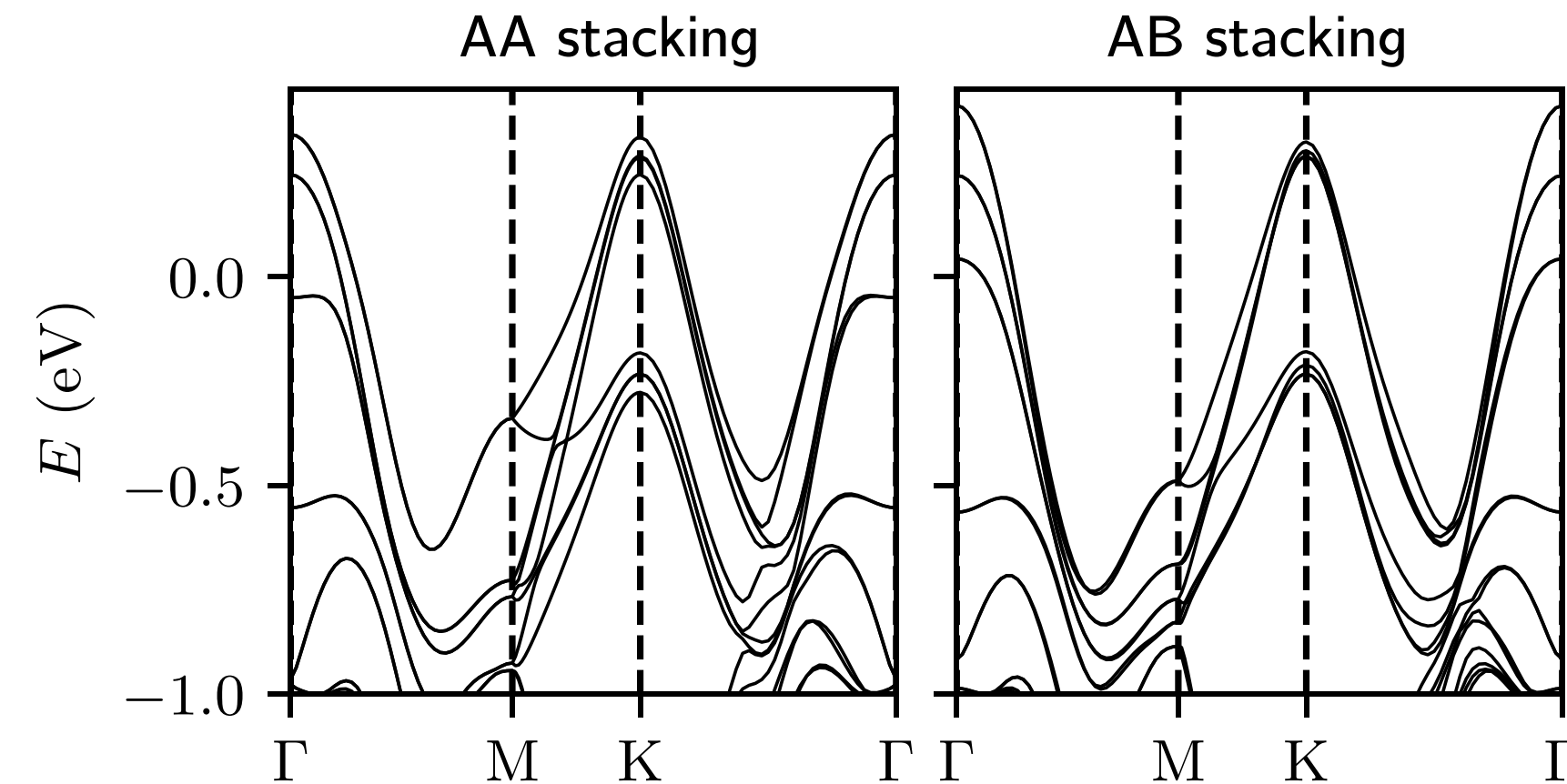


Coulomb interaction with
effective permittivity ϵ_{eff}

[Angeli & MacDonald, PNAS '21]

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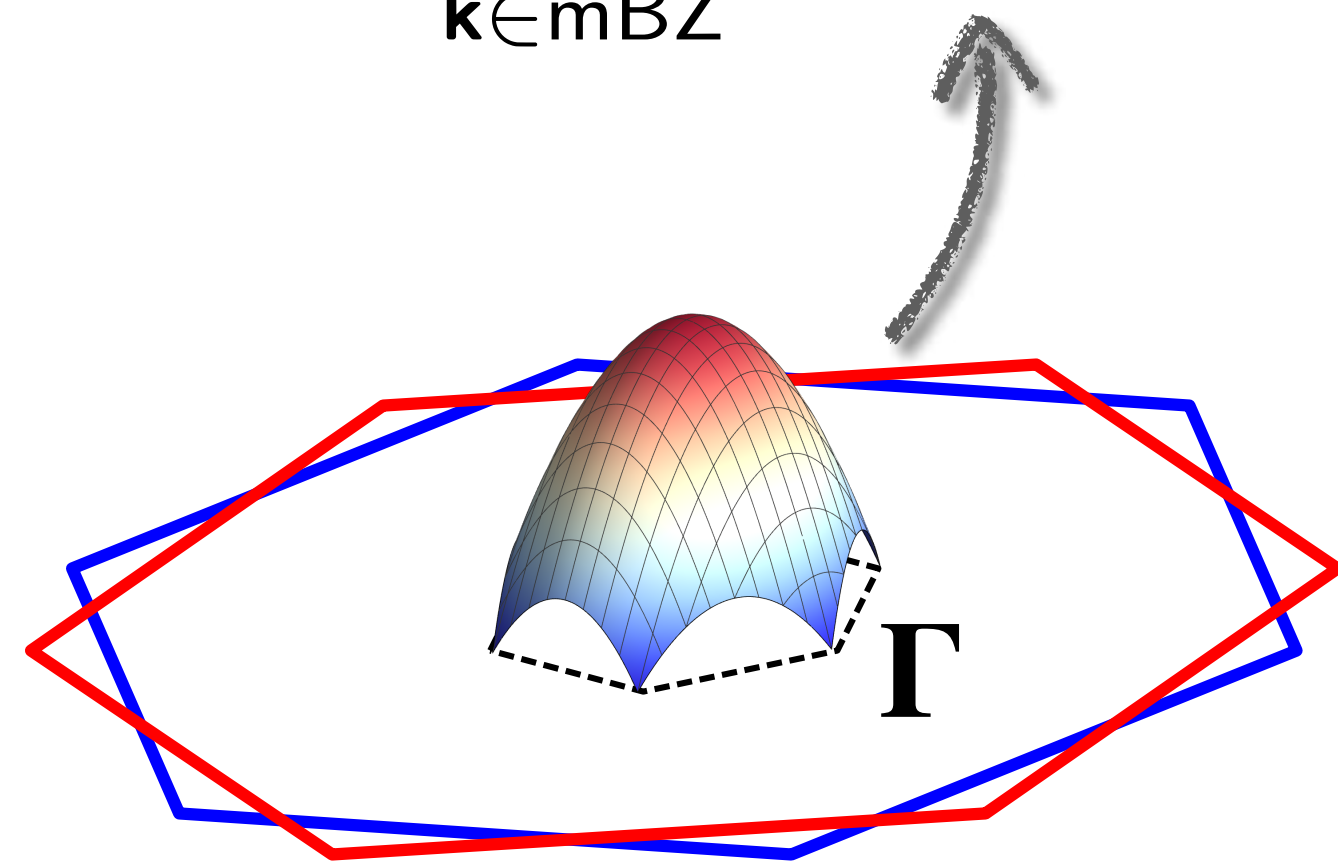
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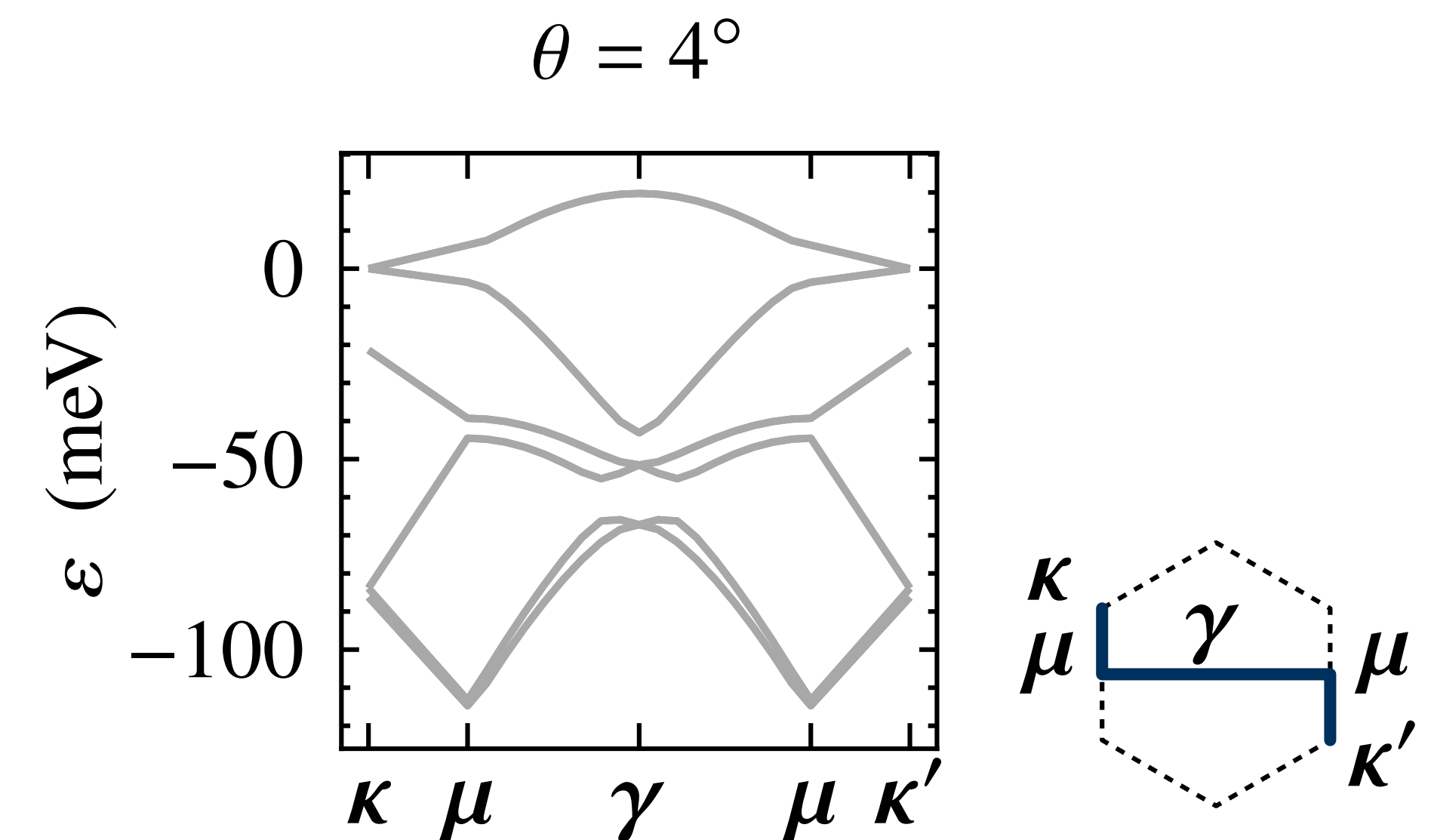
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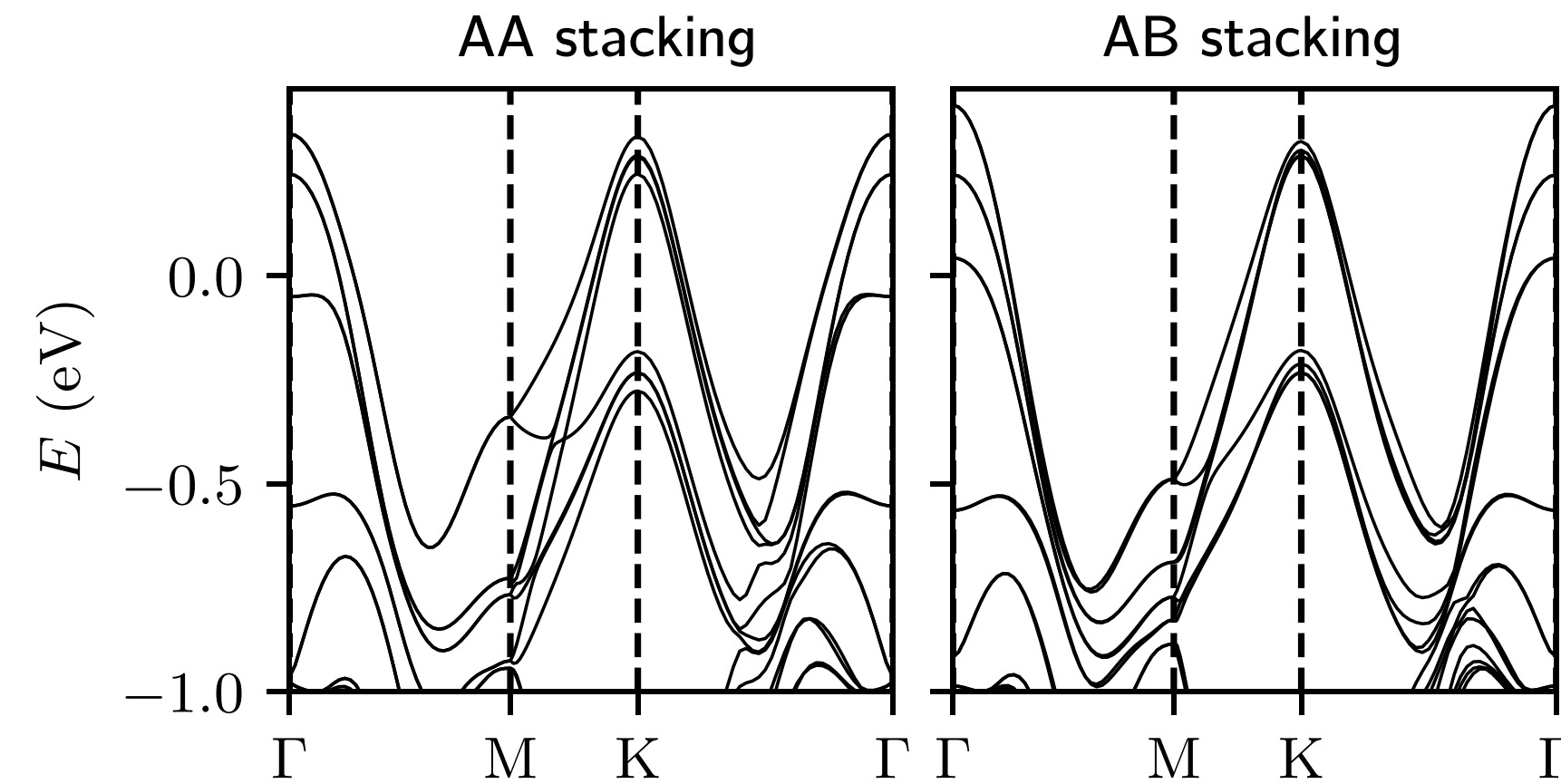
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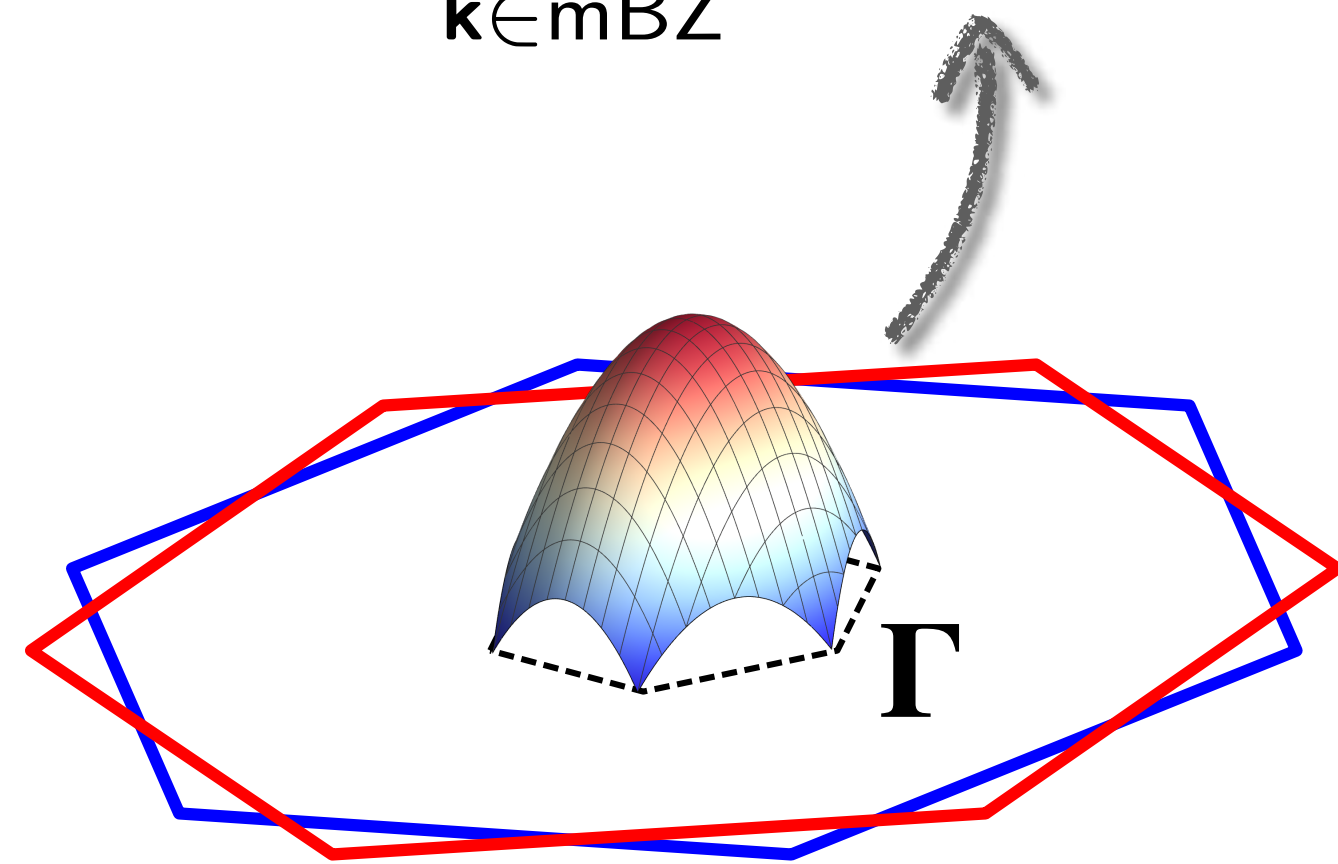
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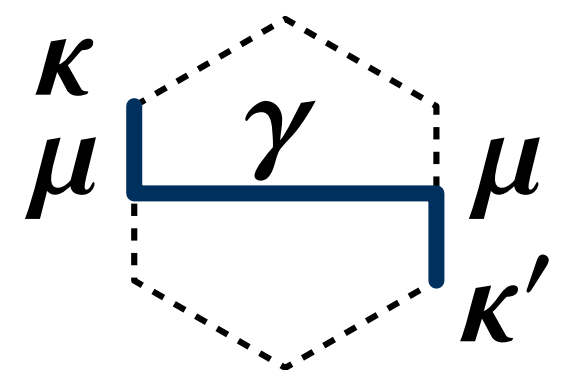
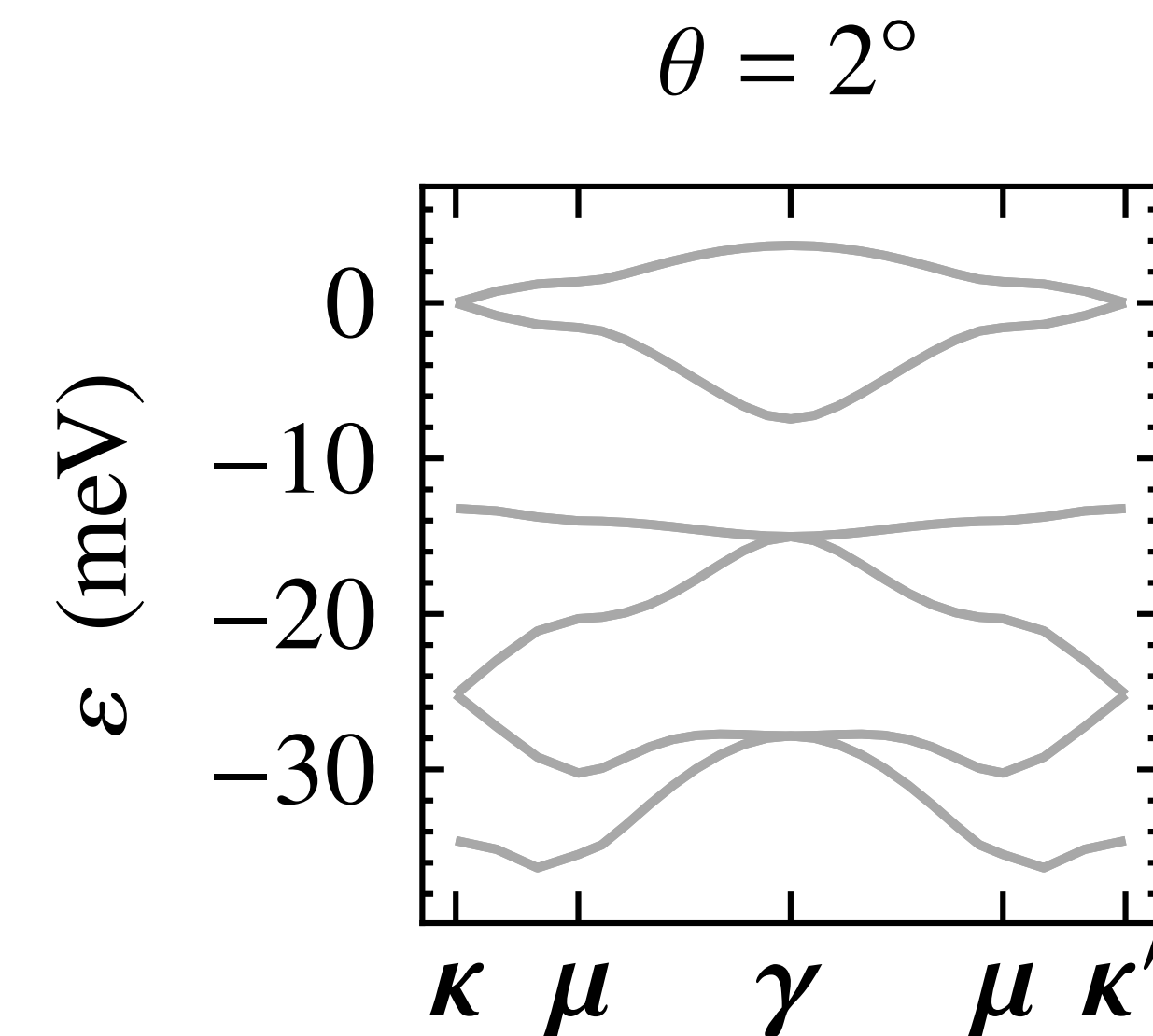
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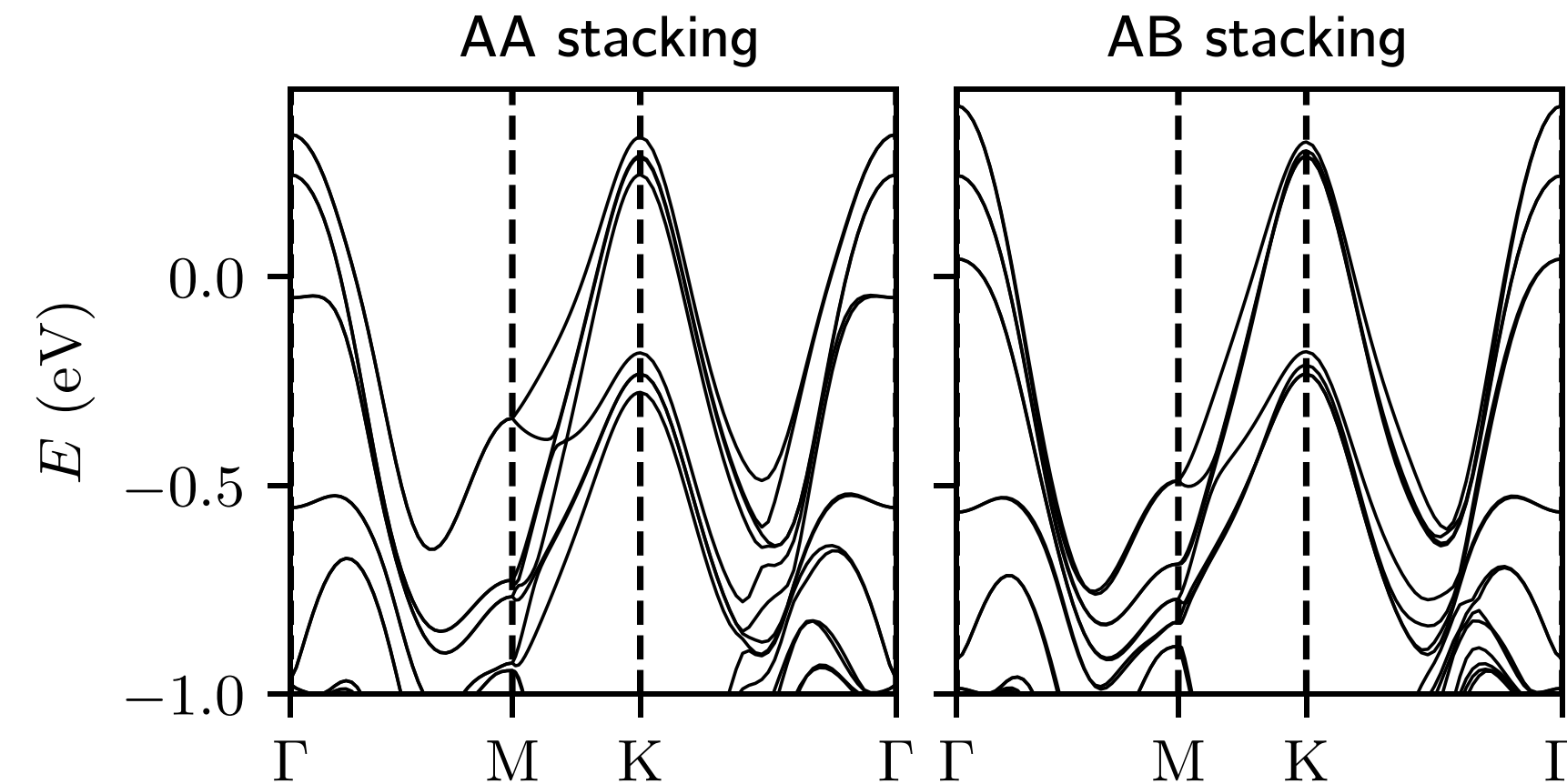
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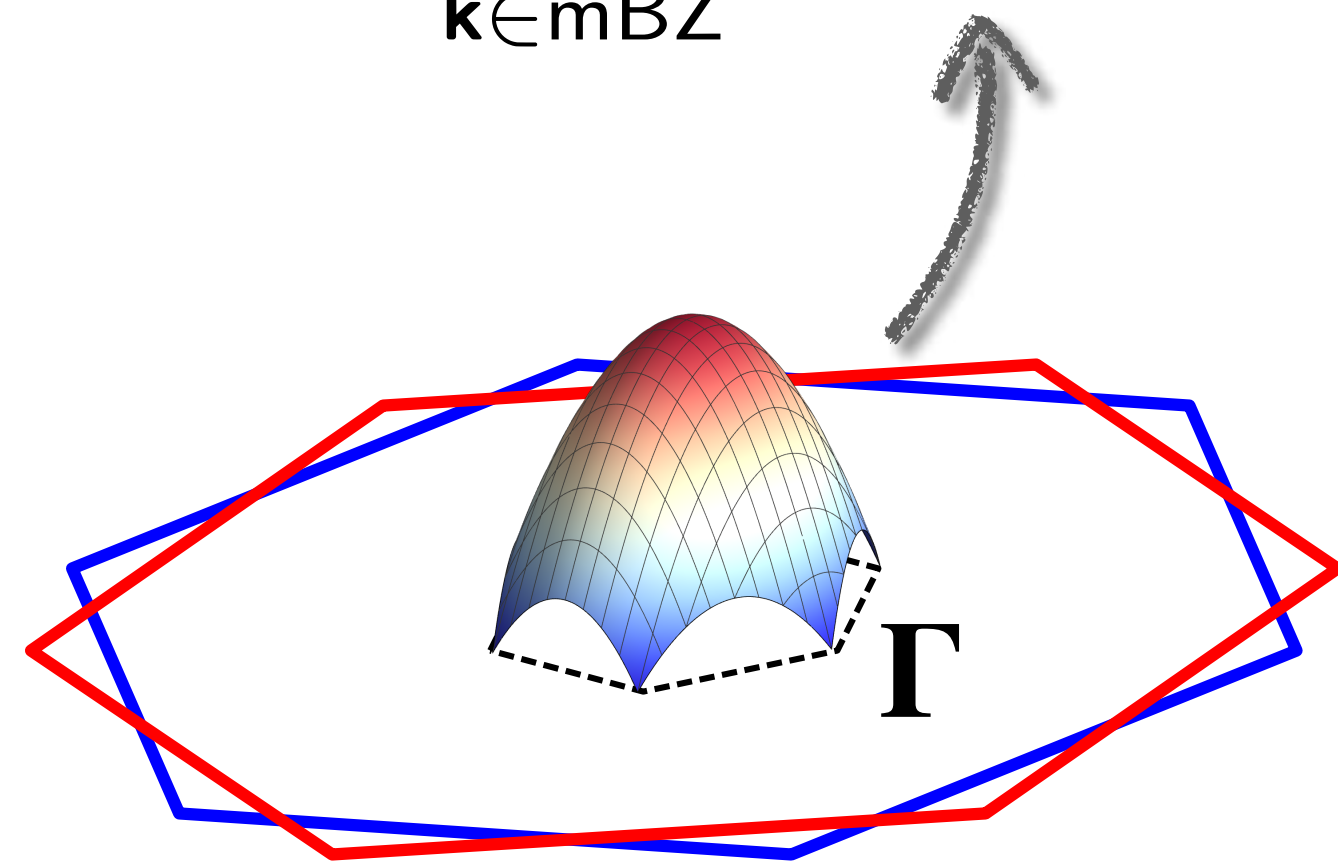
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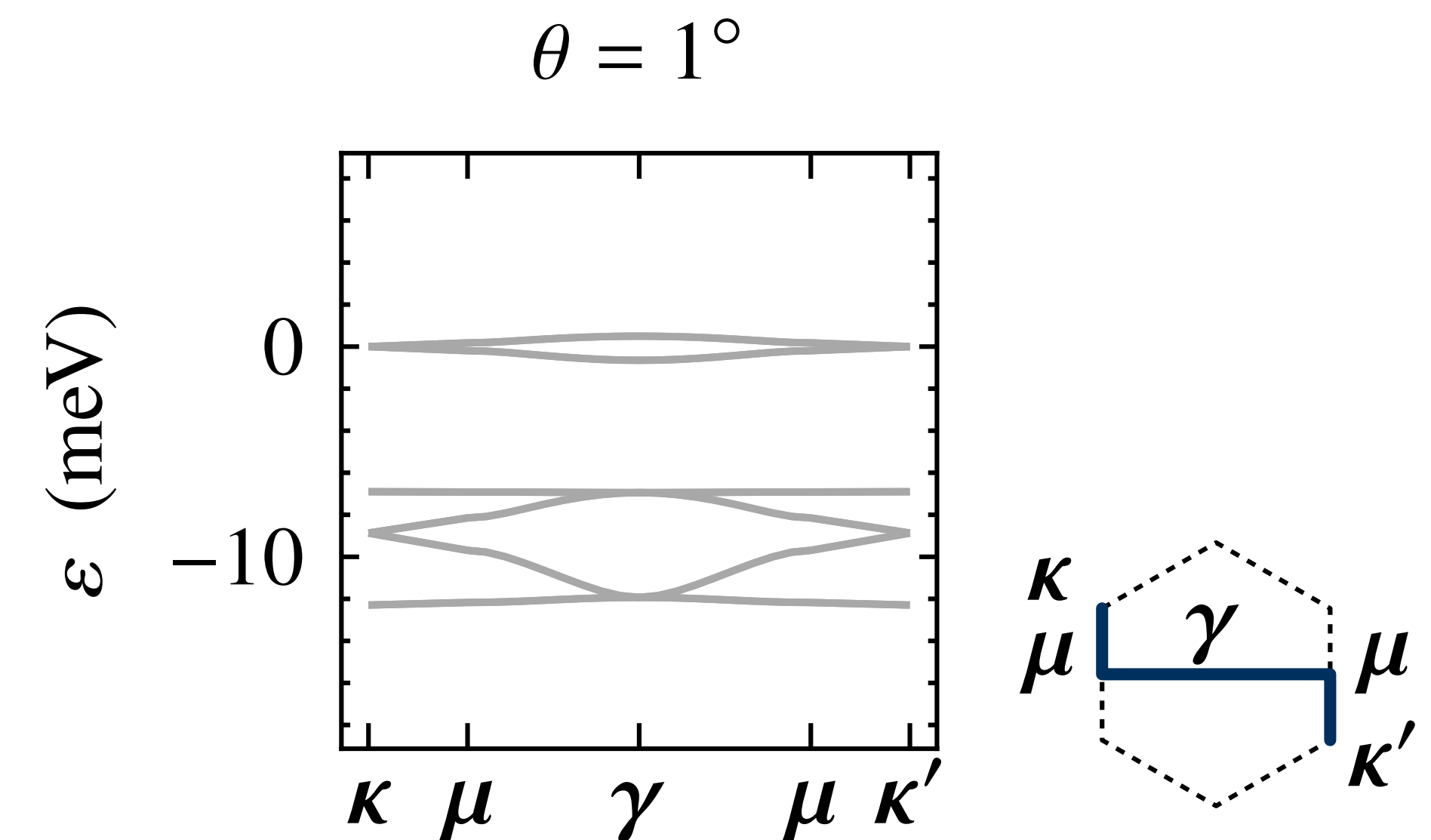
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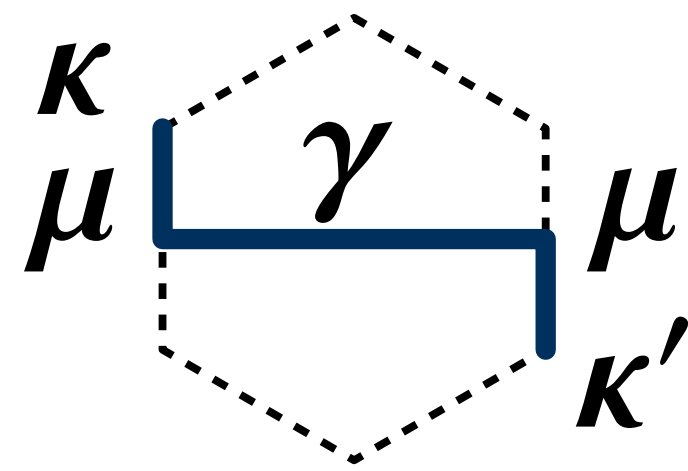
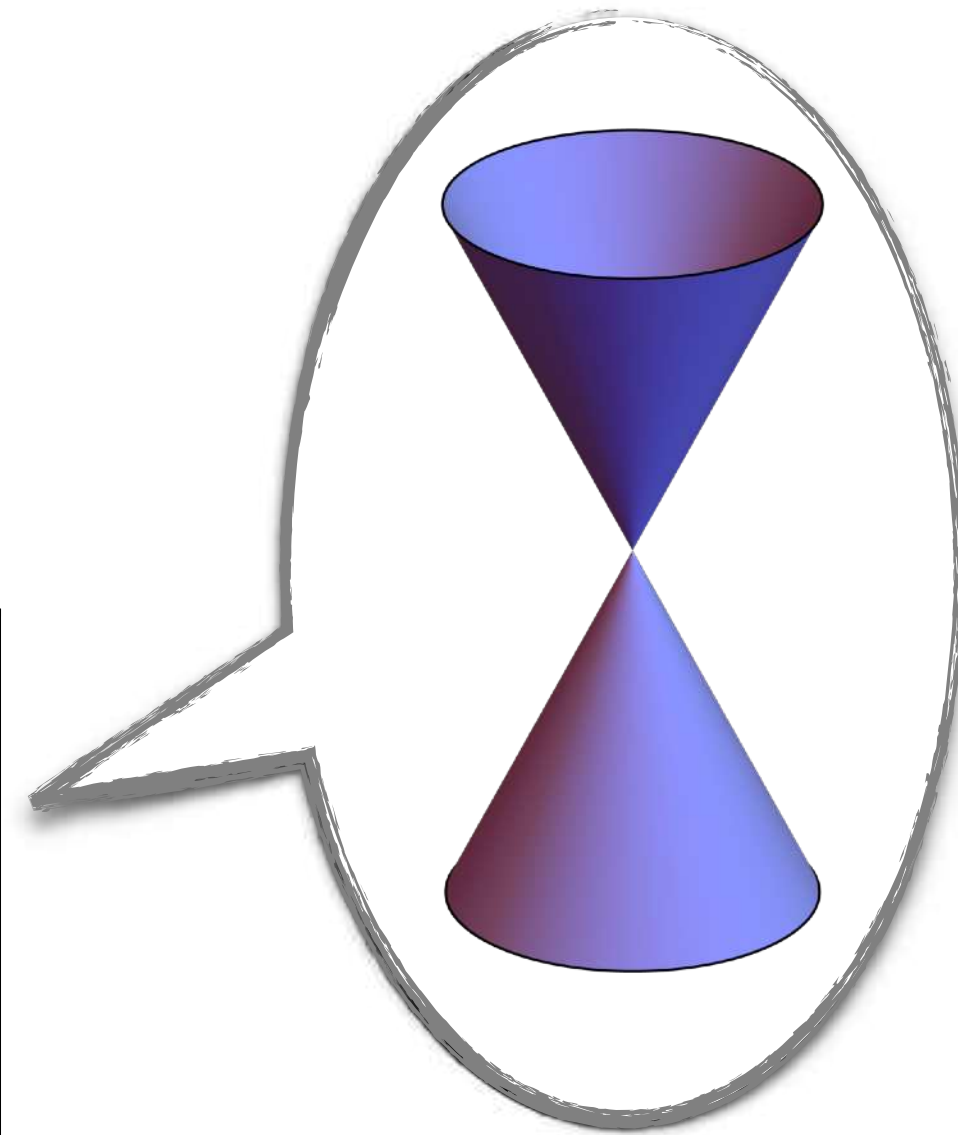
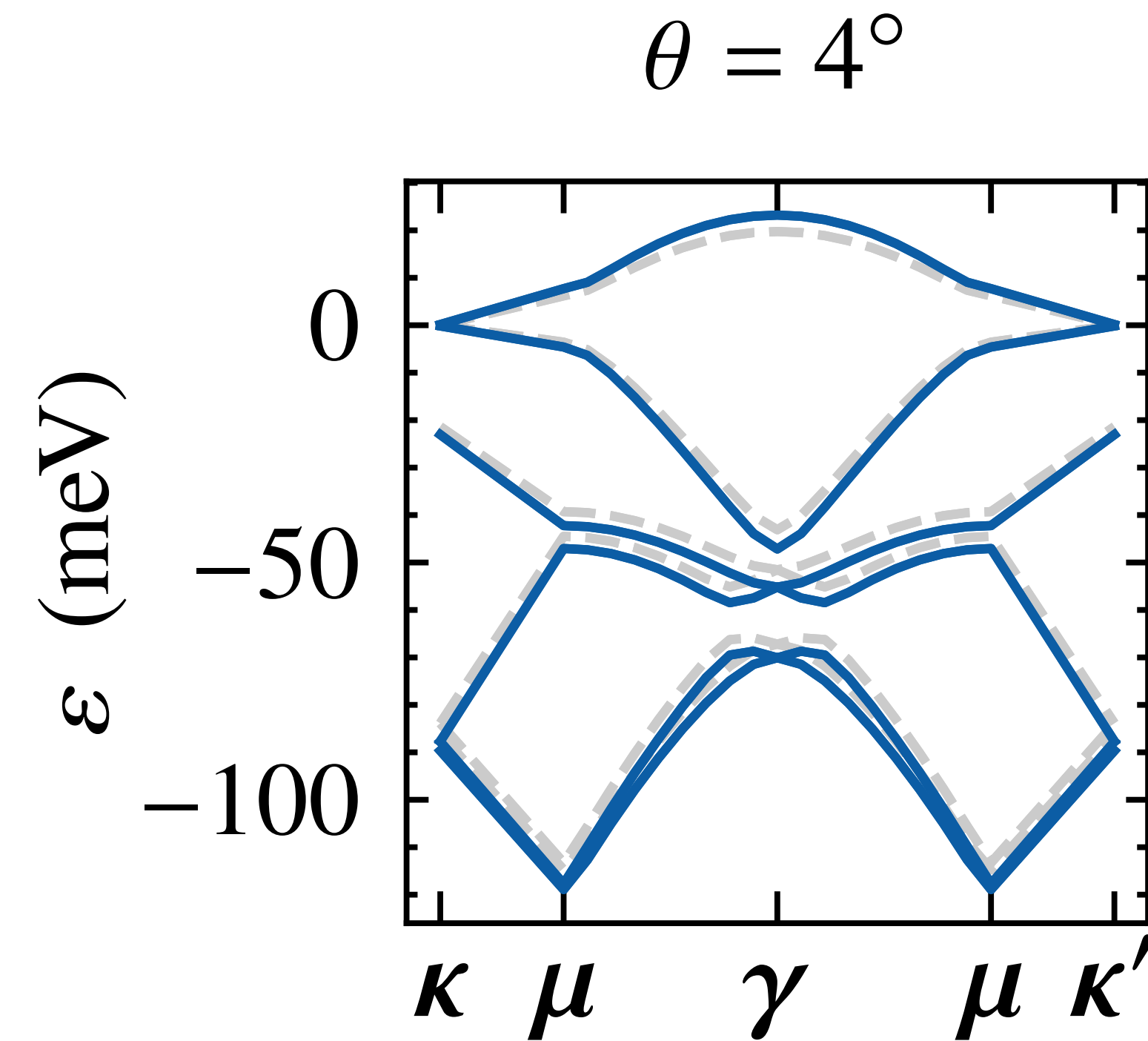
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Twisted noninteracting spectrum:



Twisted double bilayer TMDs: Dirac semimetal

Interacting spectrum:



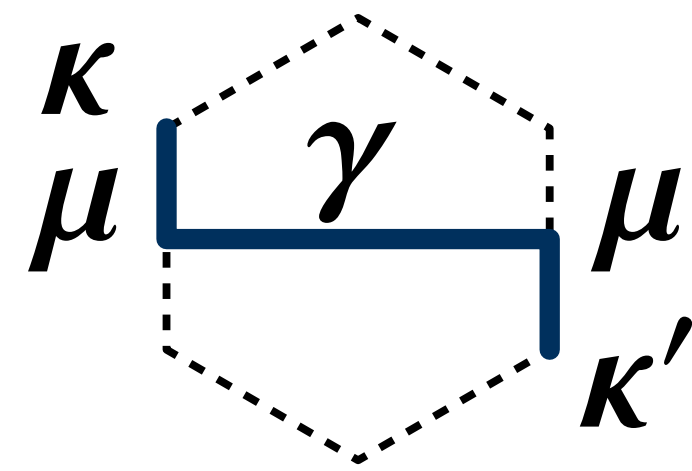
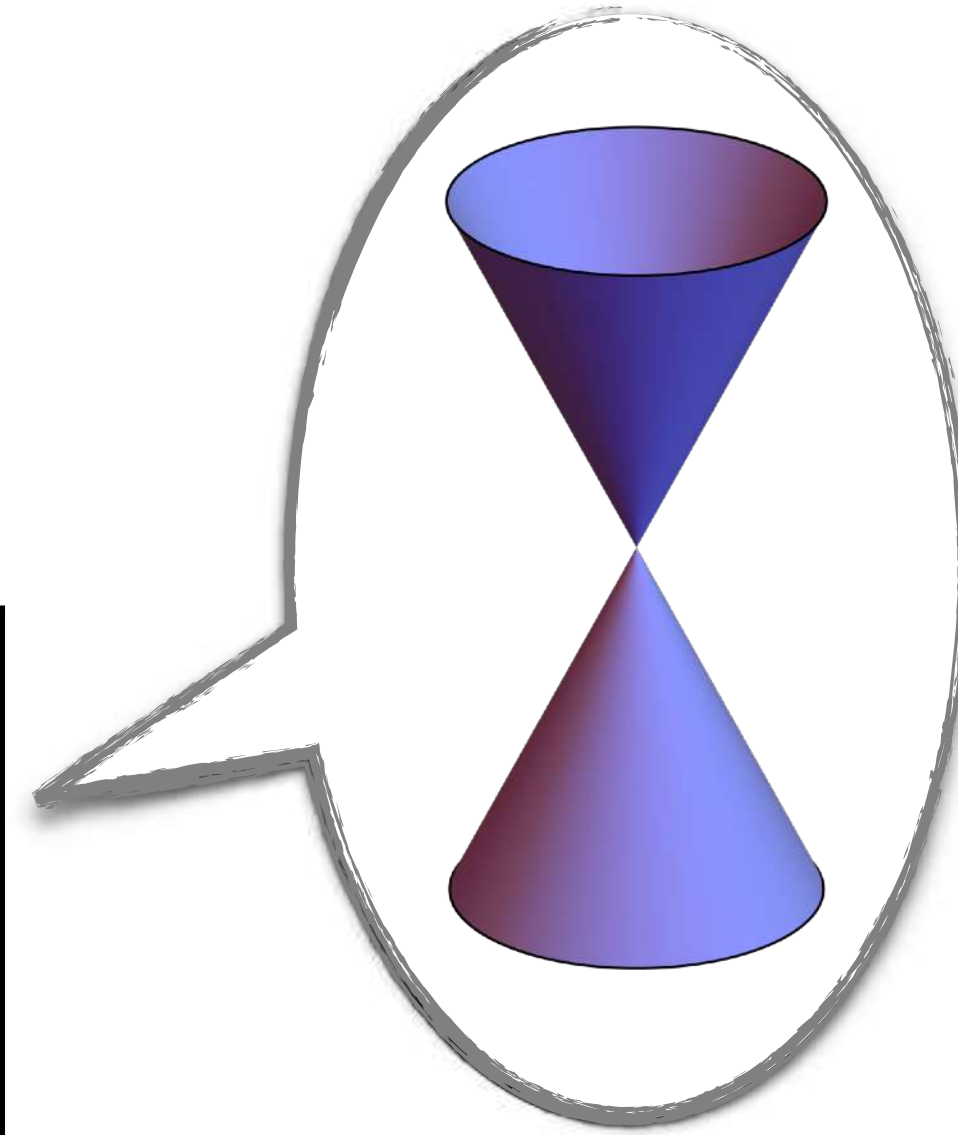
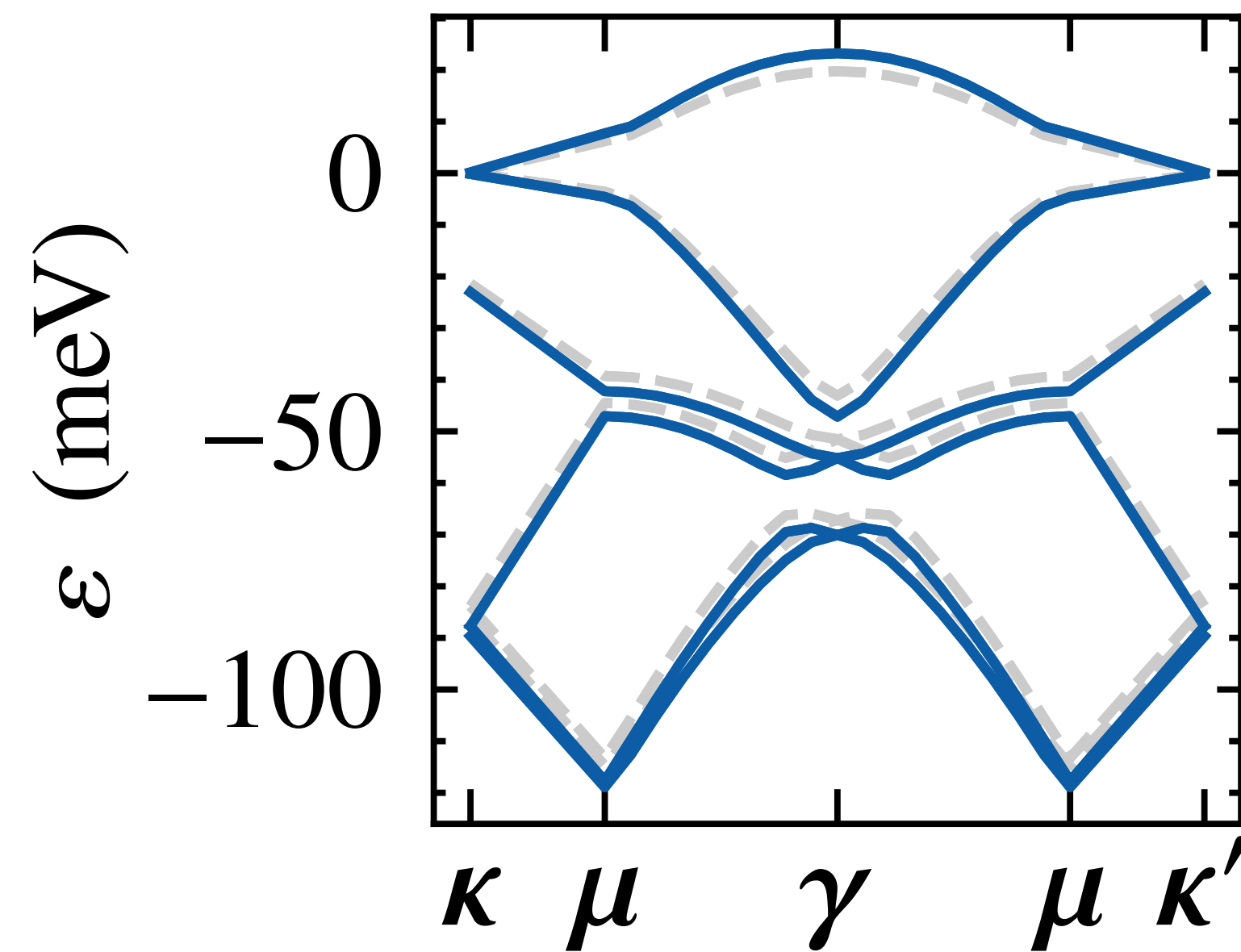
Dirac semimetal

θ

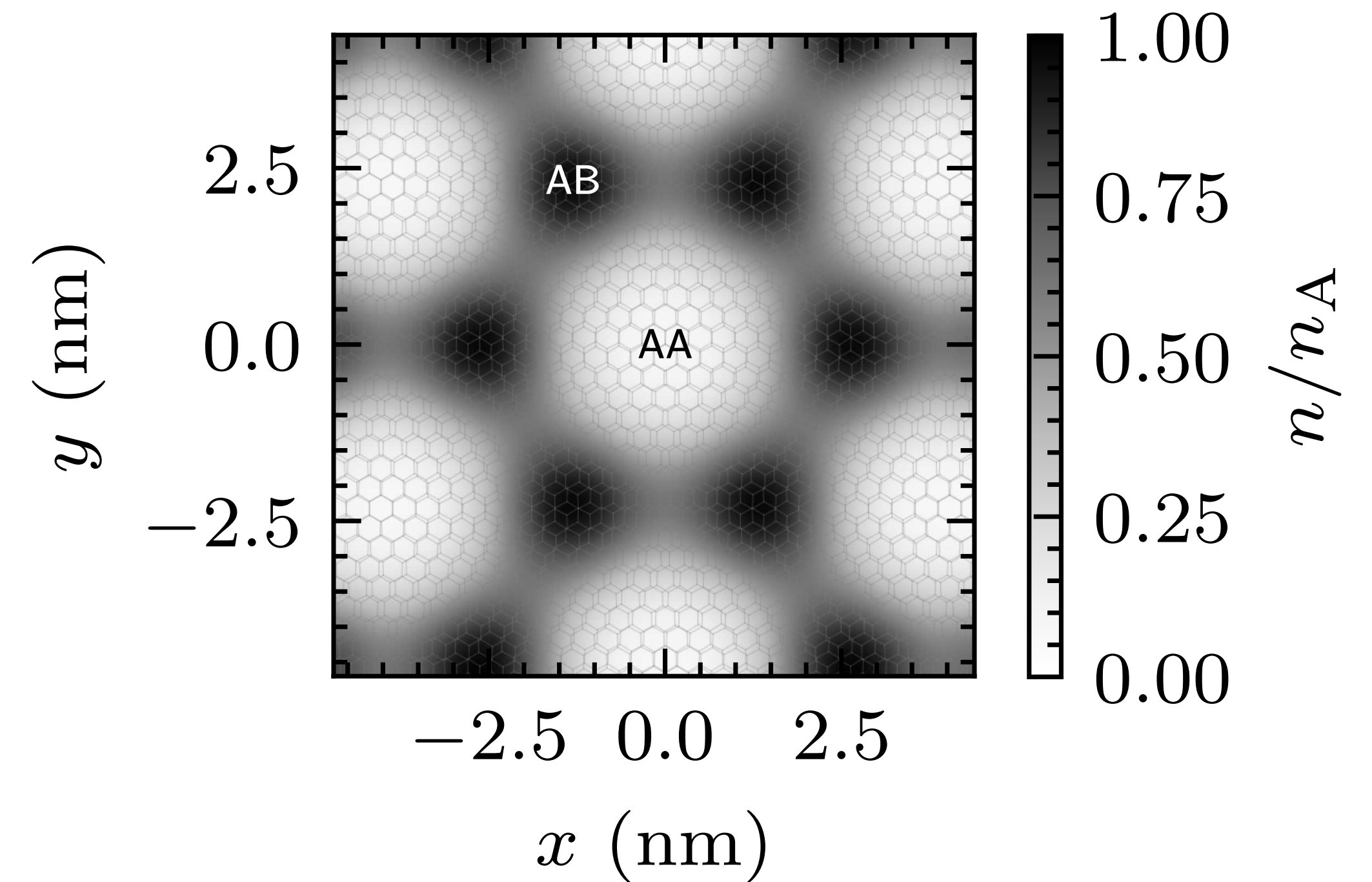
Twisted double bilayer TMDs: Dirac semimetal

Interacting spectrum:

$$\theta = 4^\circ$$



Charge density:

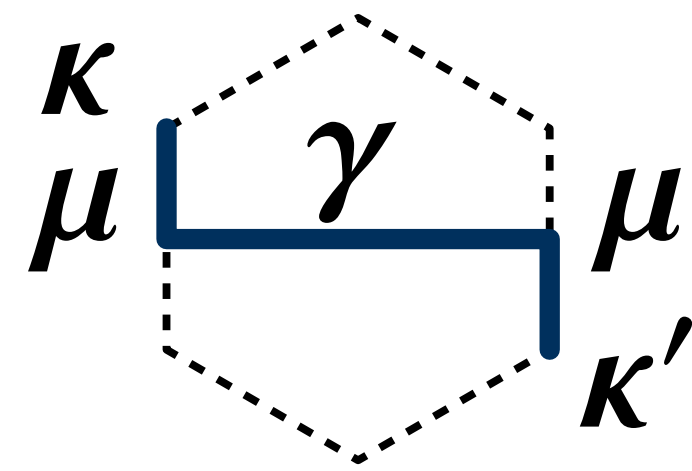
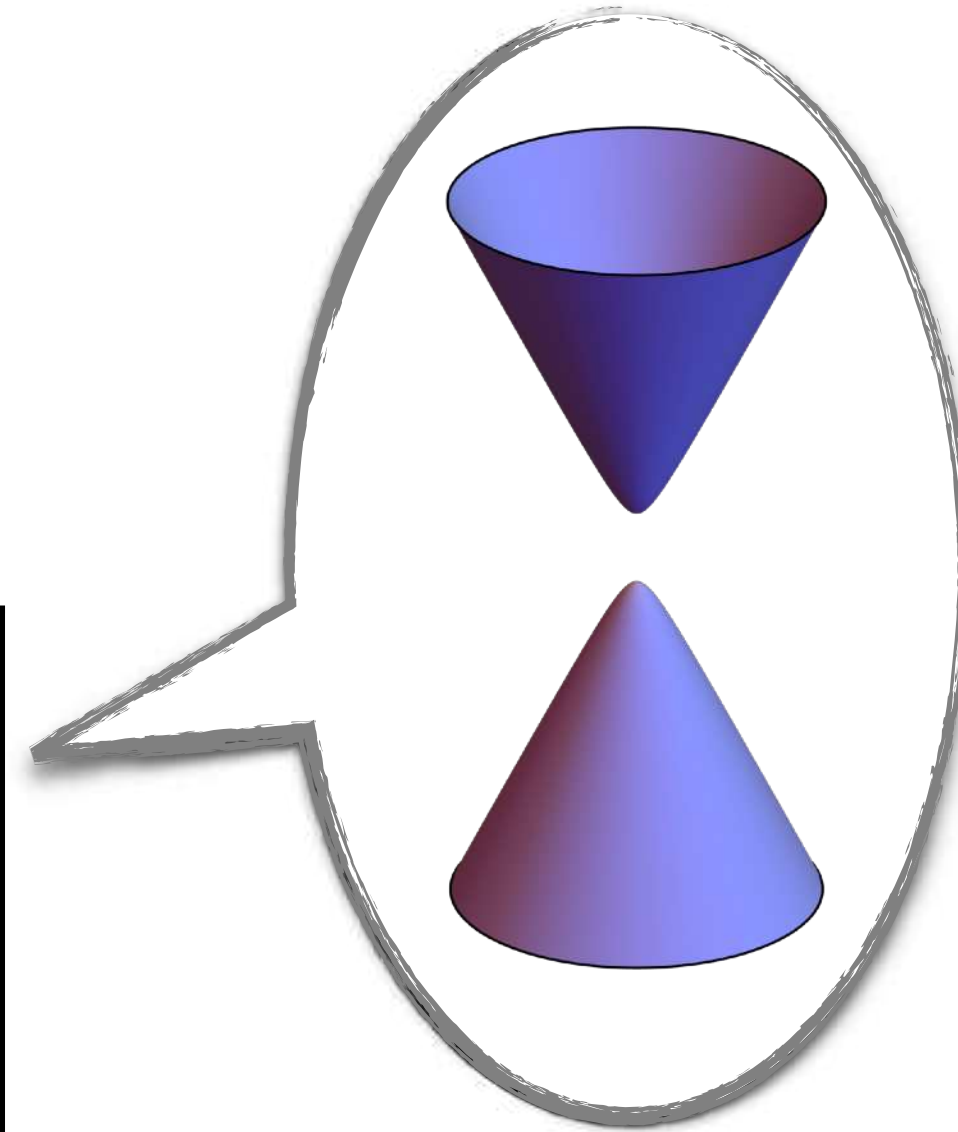
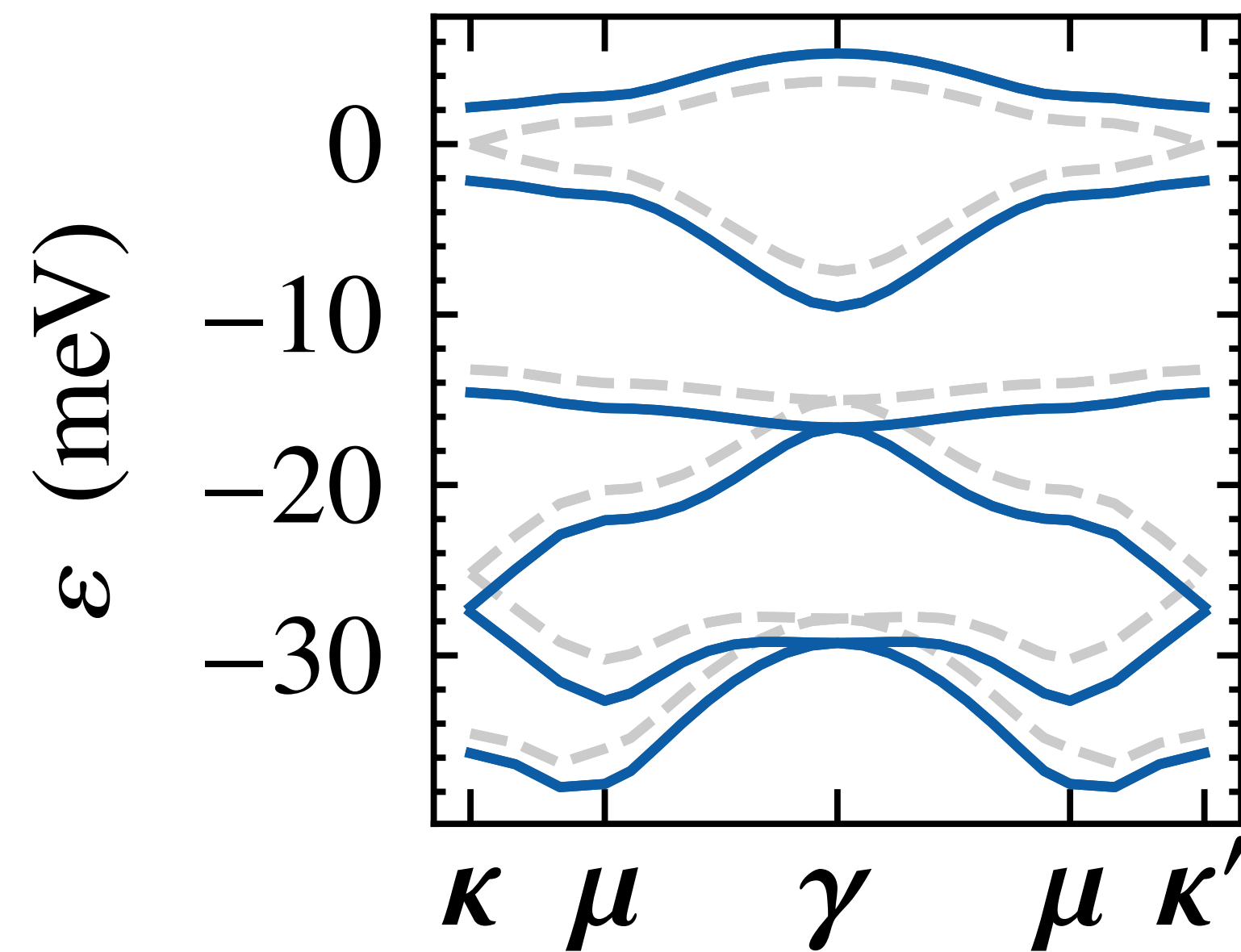


Dirac semimetal θ

Twisted double bilayer TMDs: Antiferromagnetic insulator

Interacting spectrum:

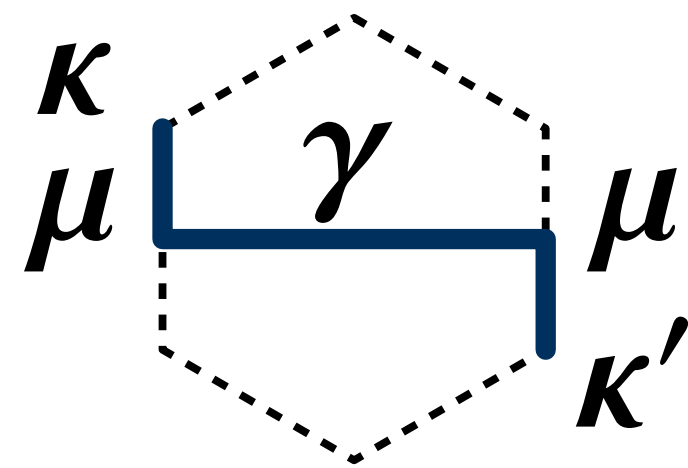
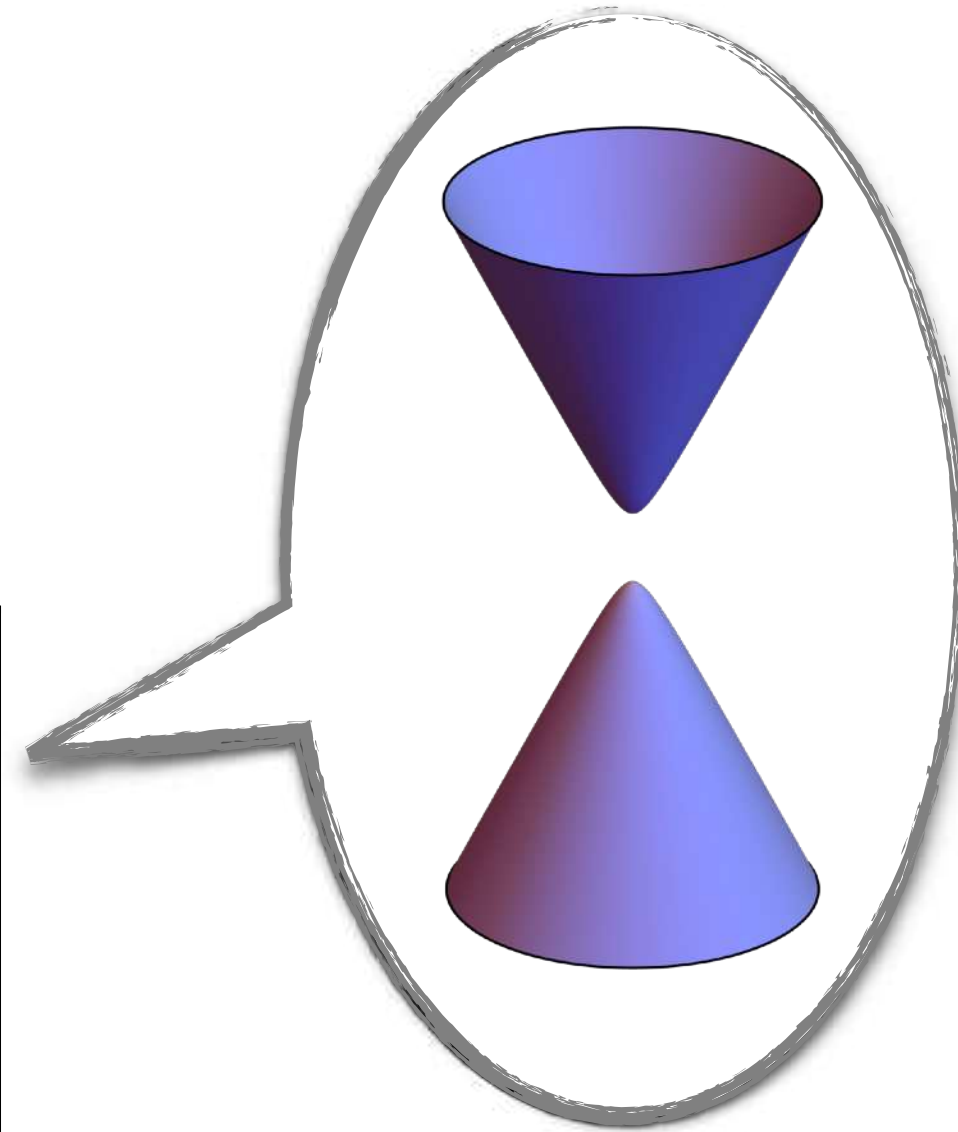
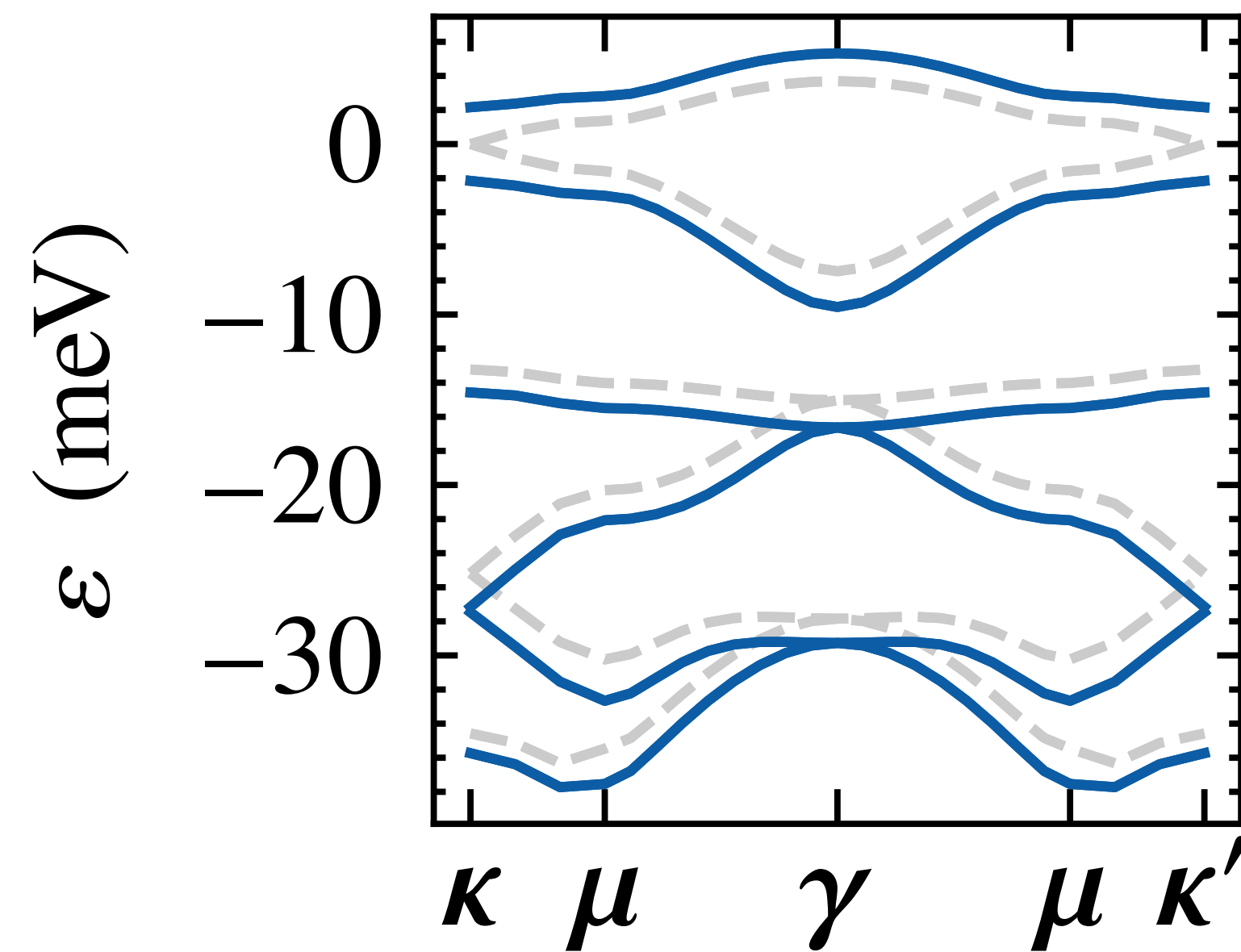
$$\theta = 2^\circ$$



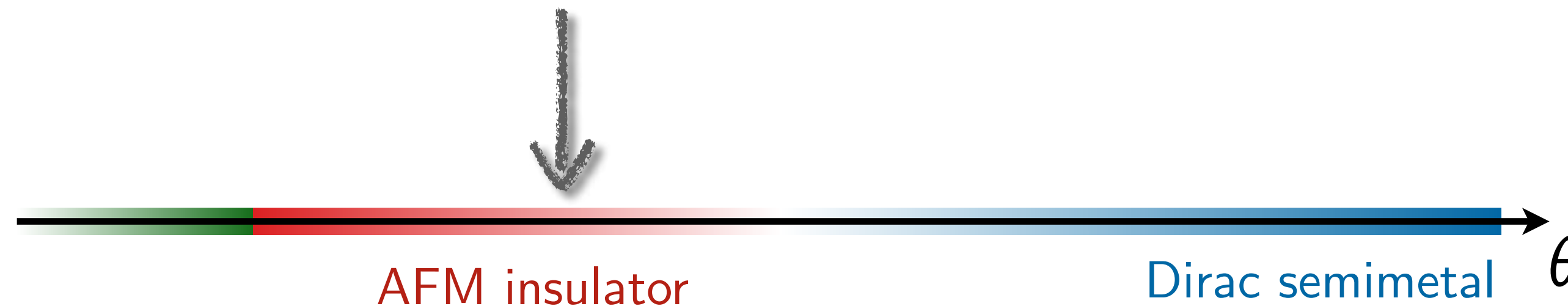
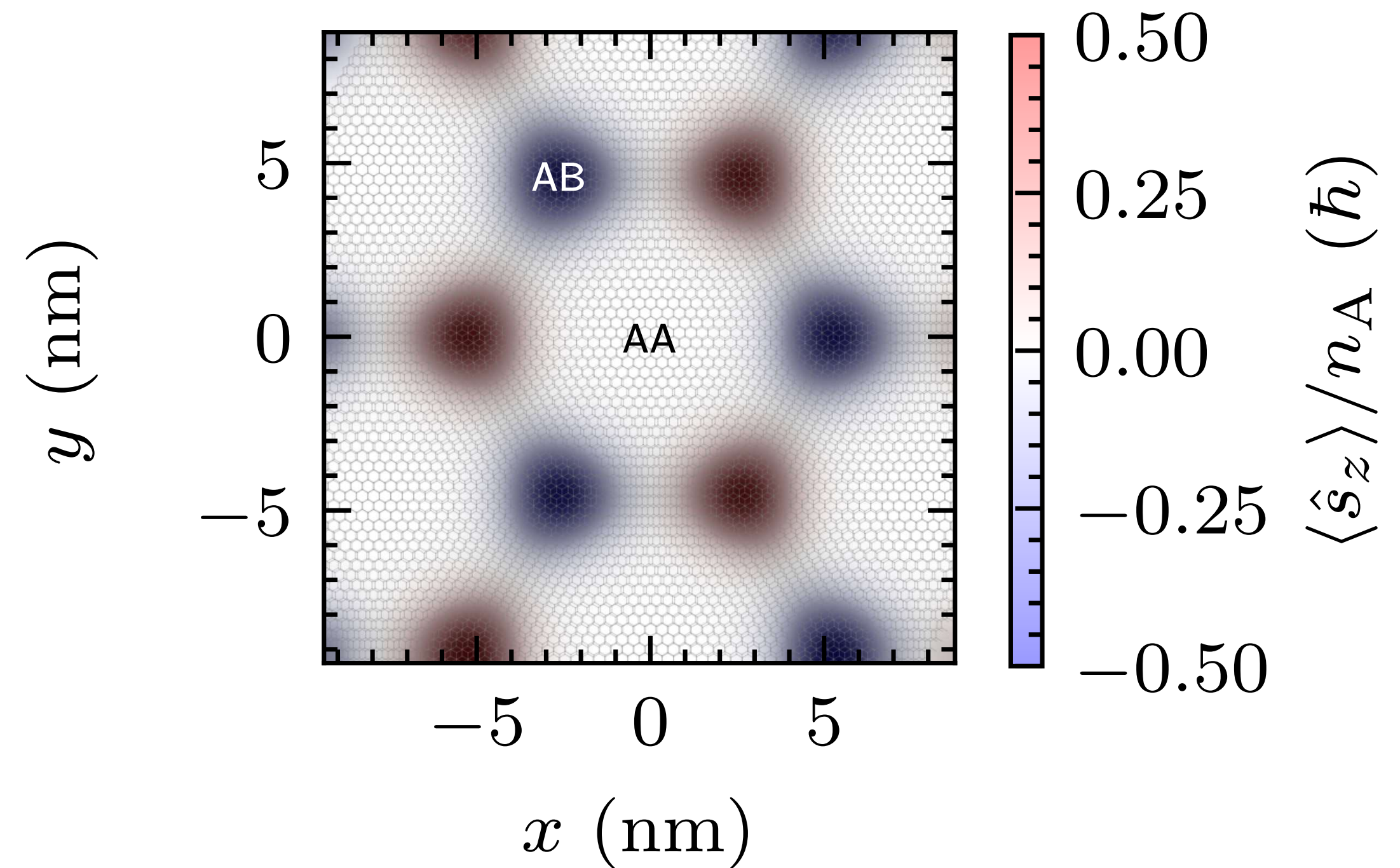
Twisted double bilayer TMDs: Antiferromagnetic insulator

Interacting spectrum:

$$\theta = 2^\circ$$



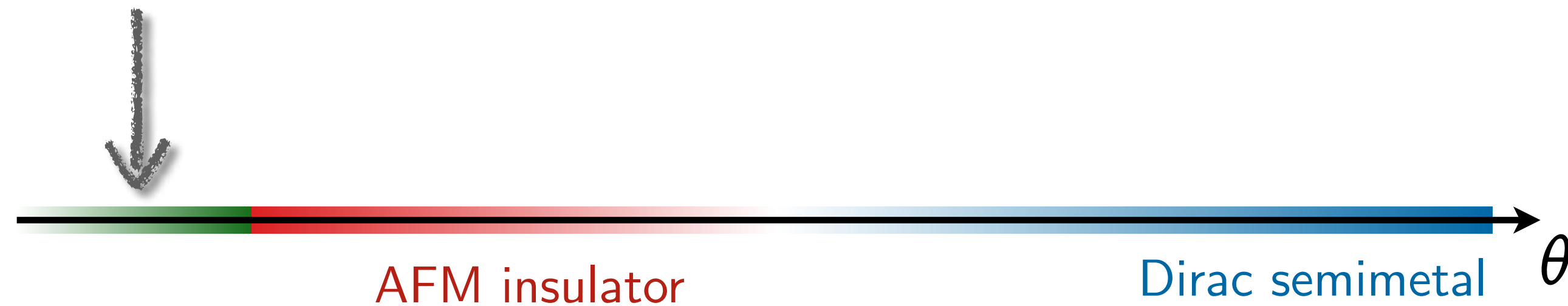
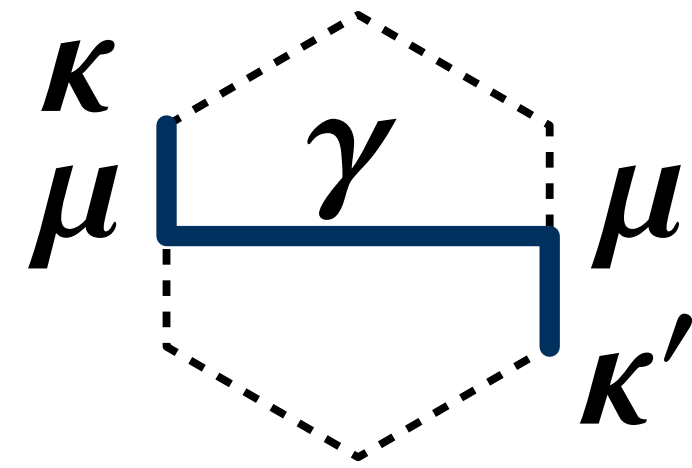
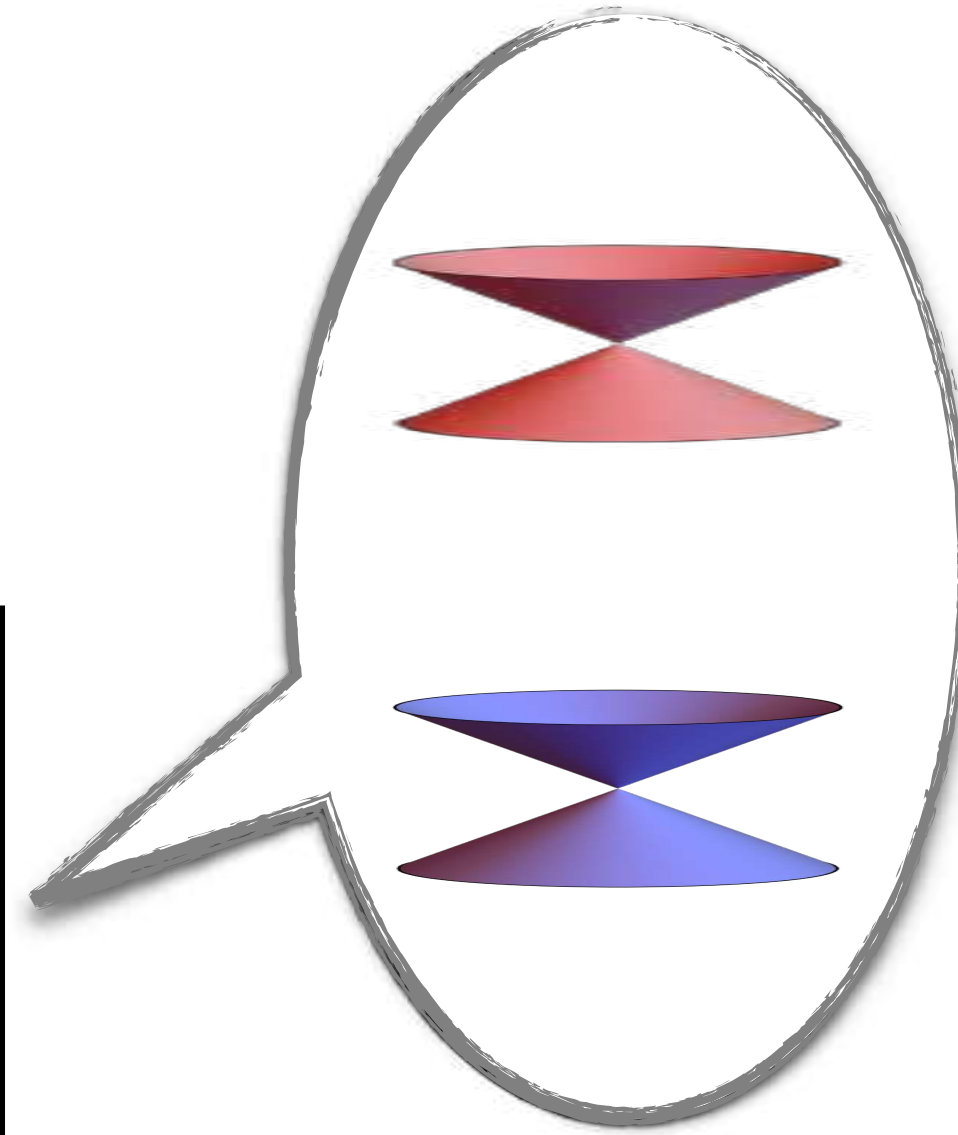
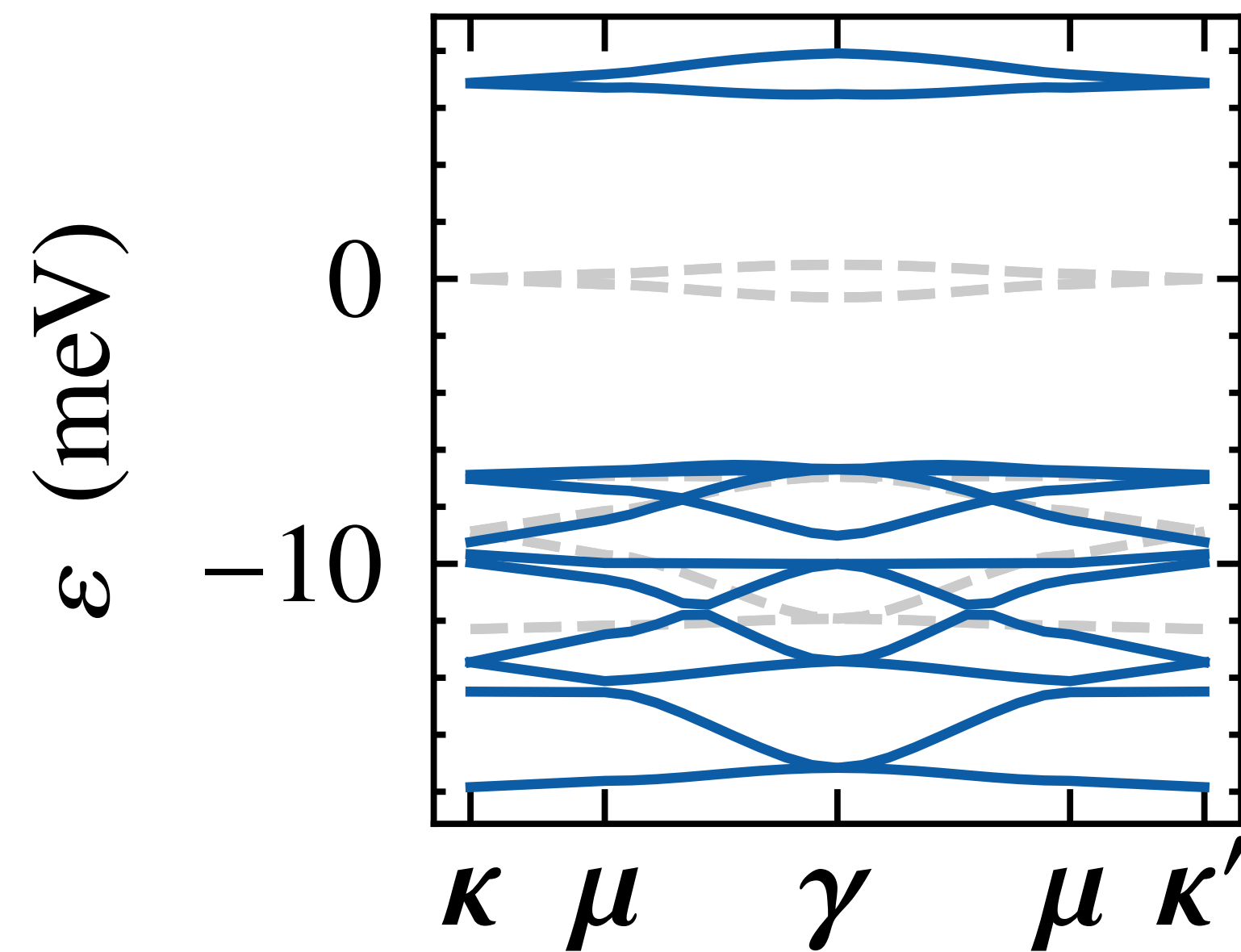
Spin density:



Twisted double bilayer TMDs: Antiferromagnetic insulator

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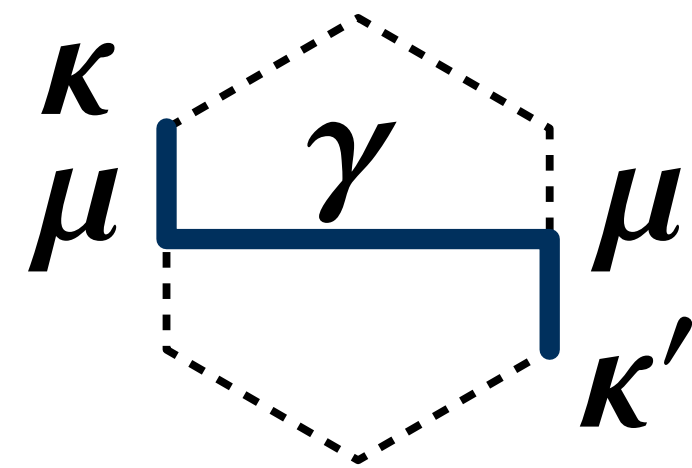
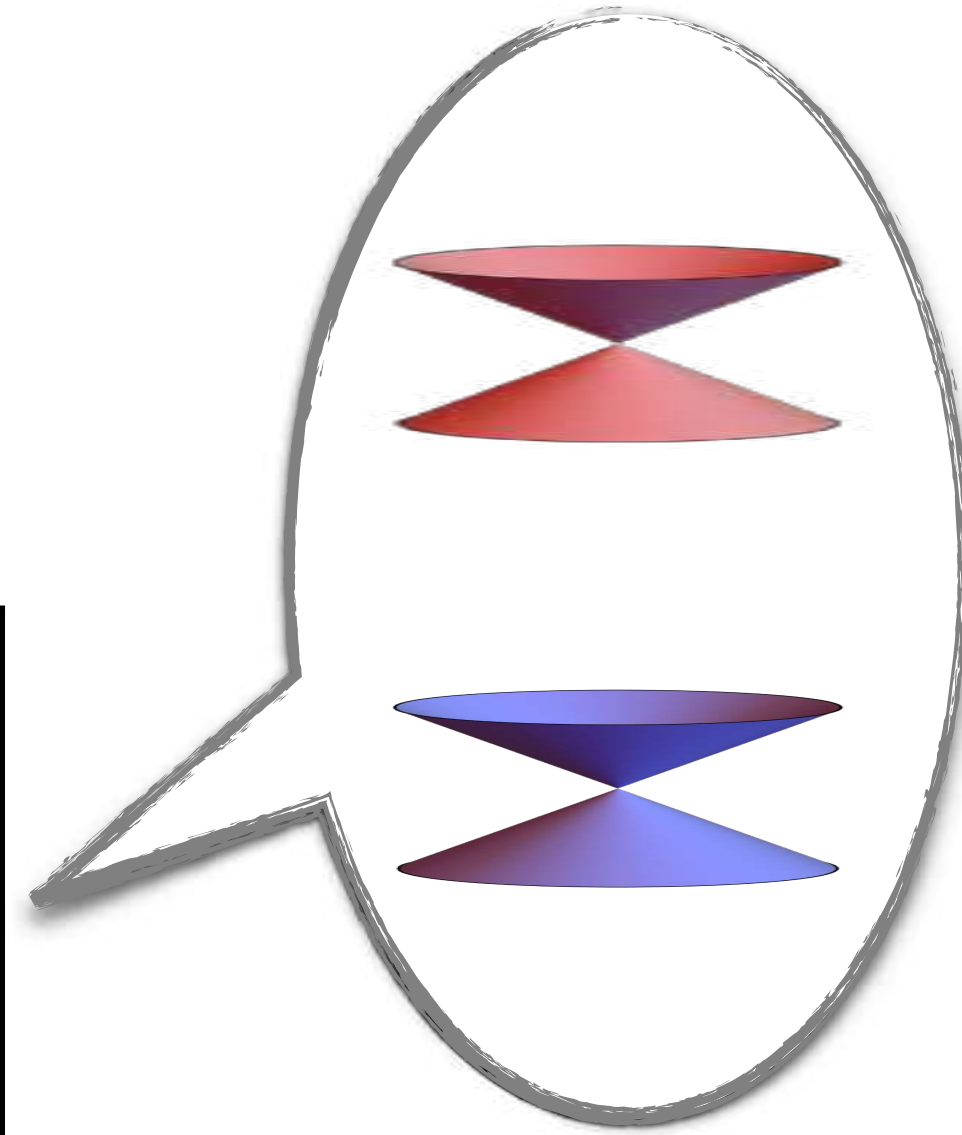
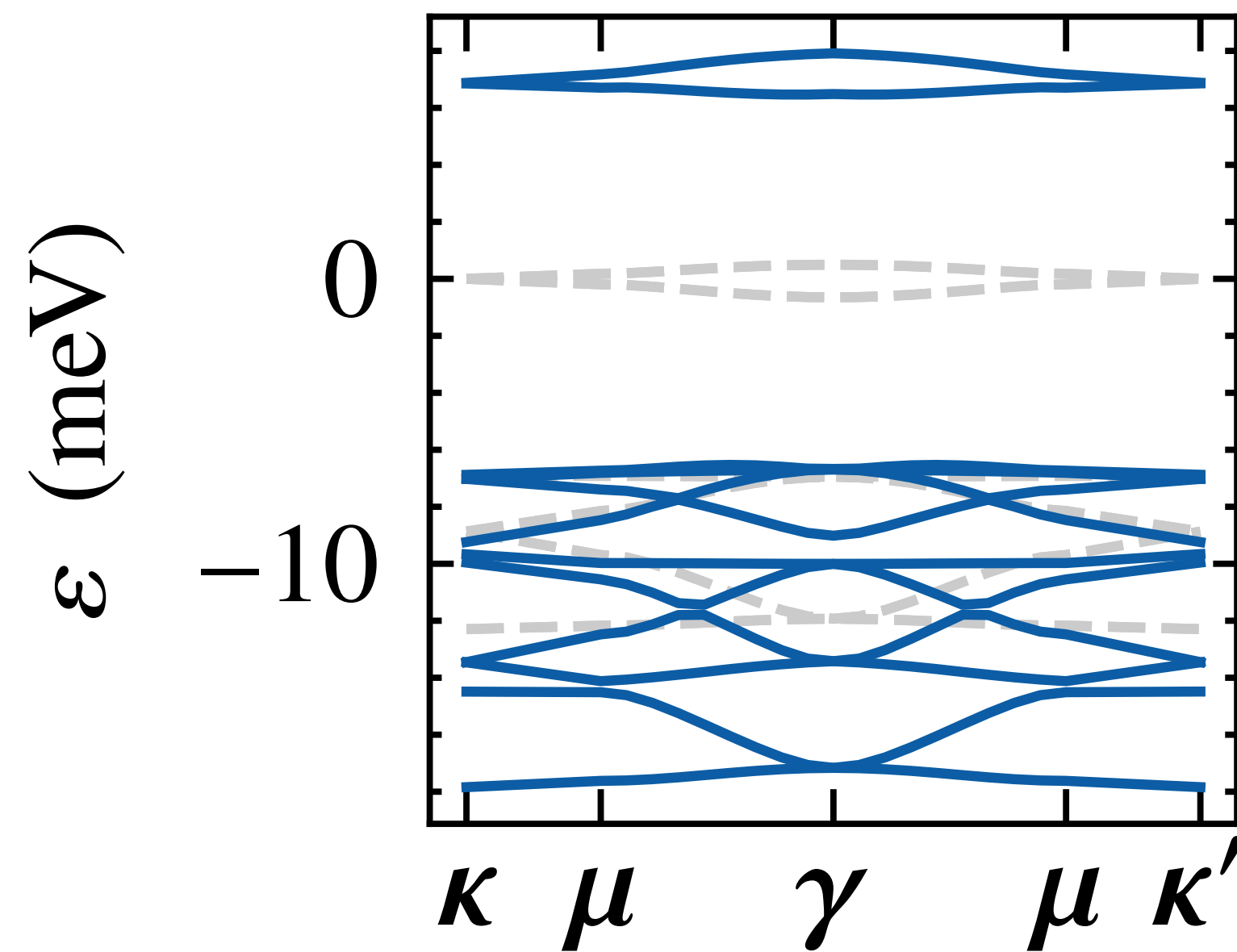
$$\theta = 1^\circ$$



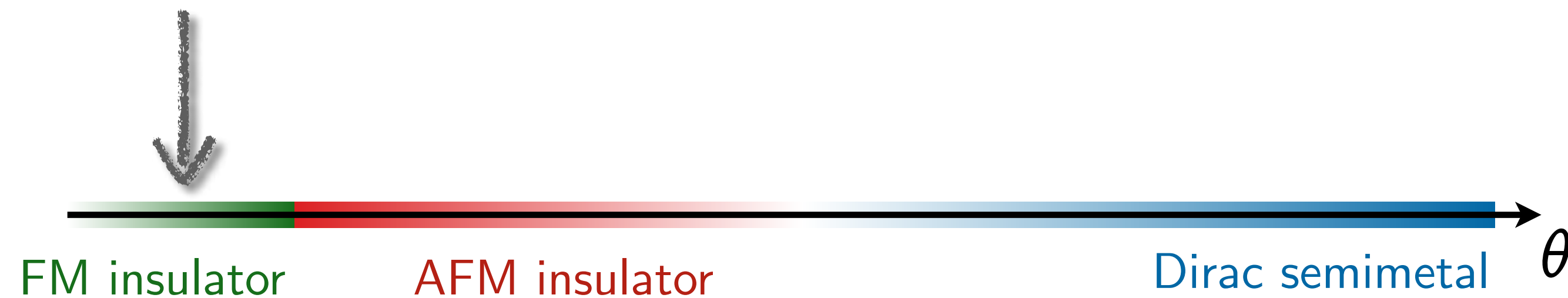
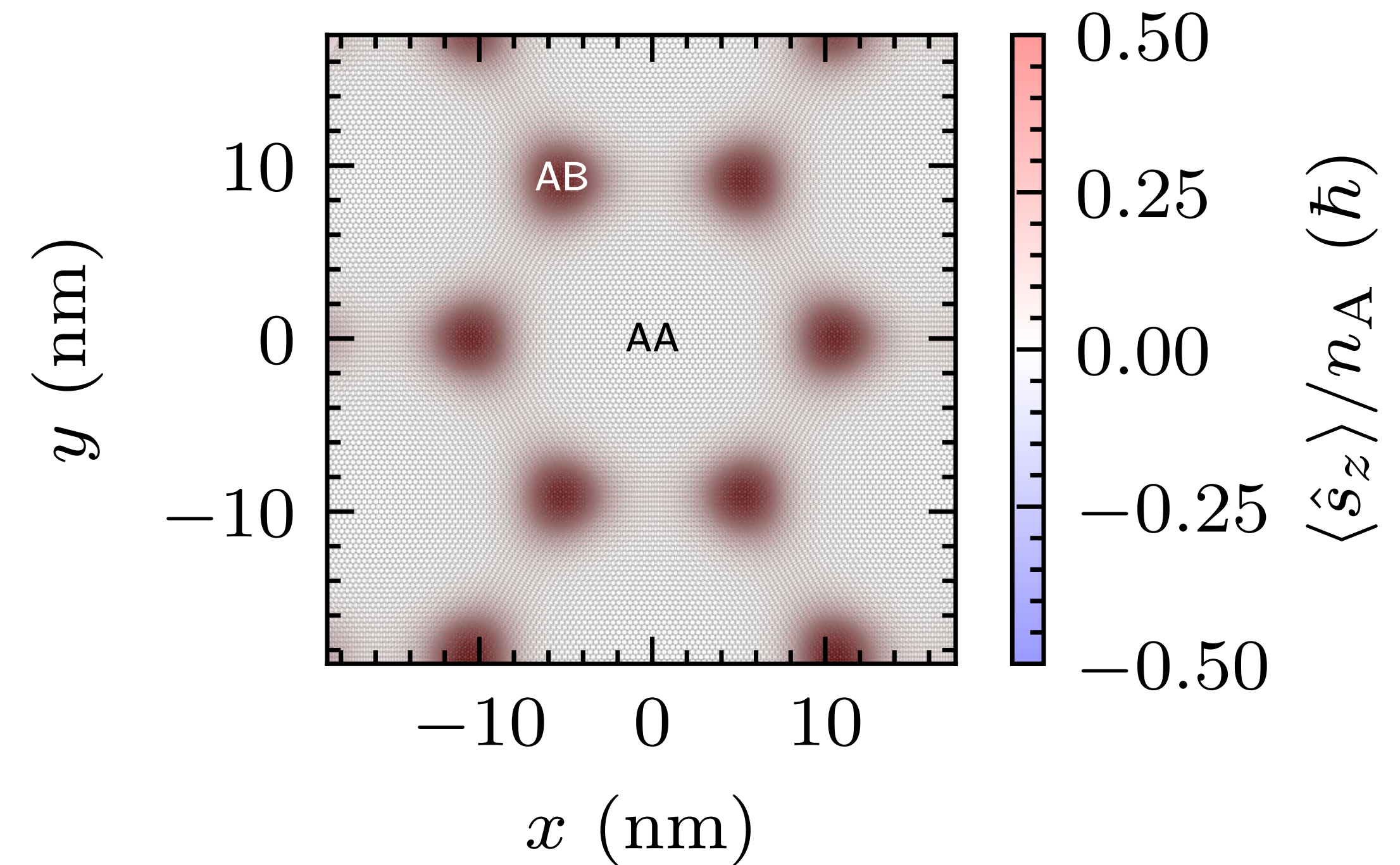
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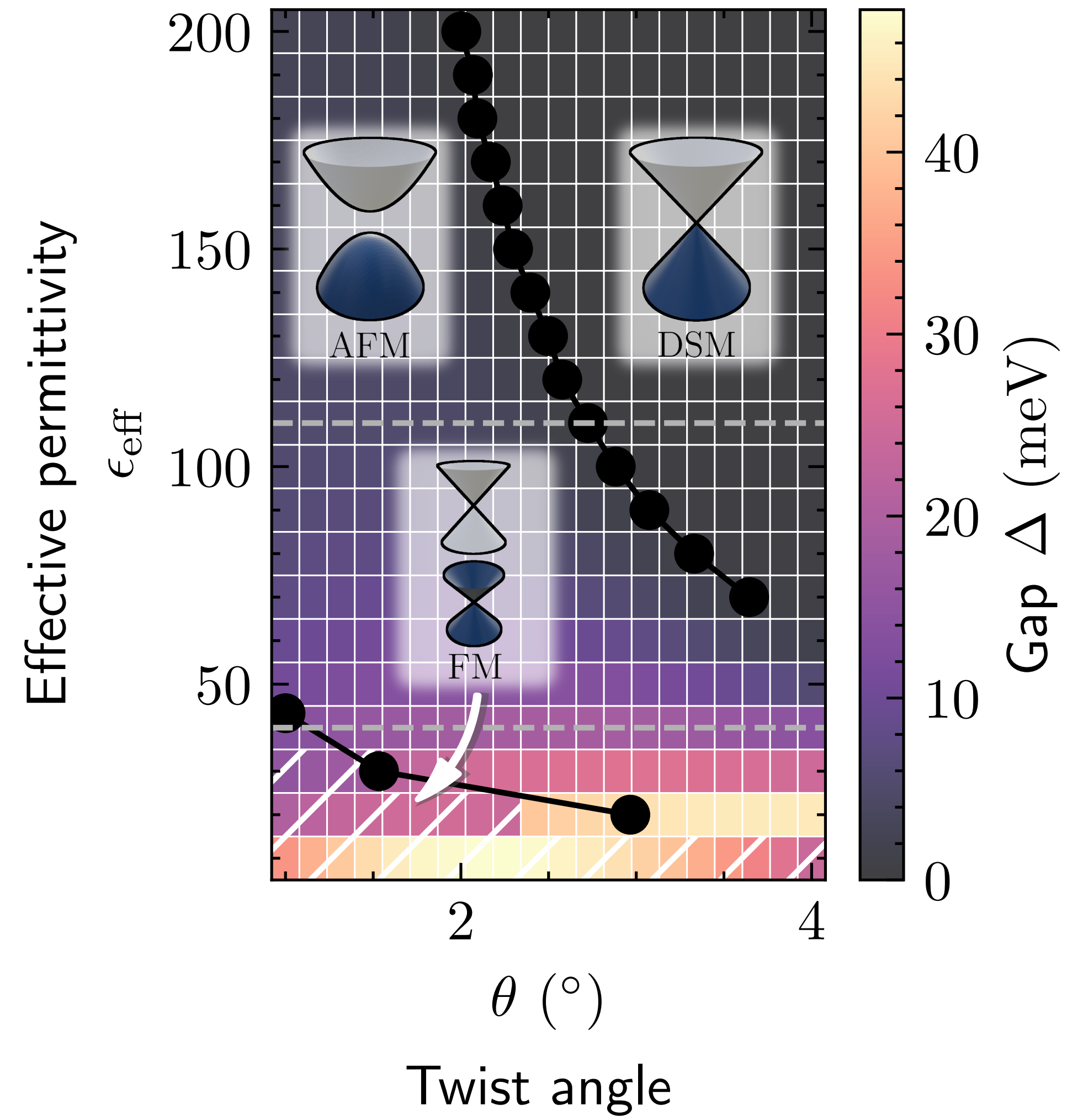
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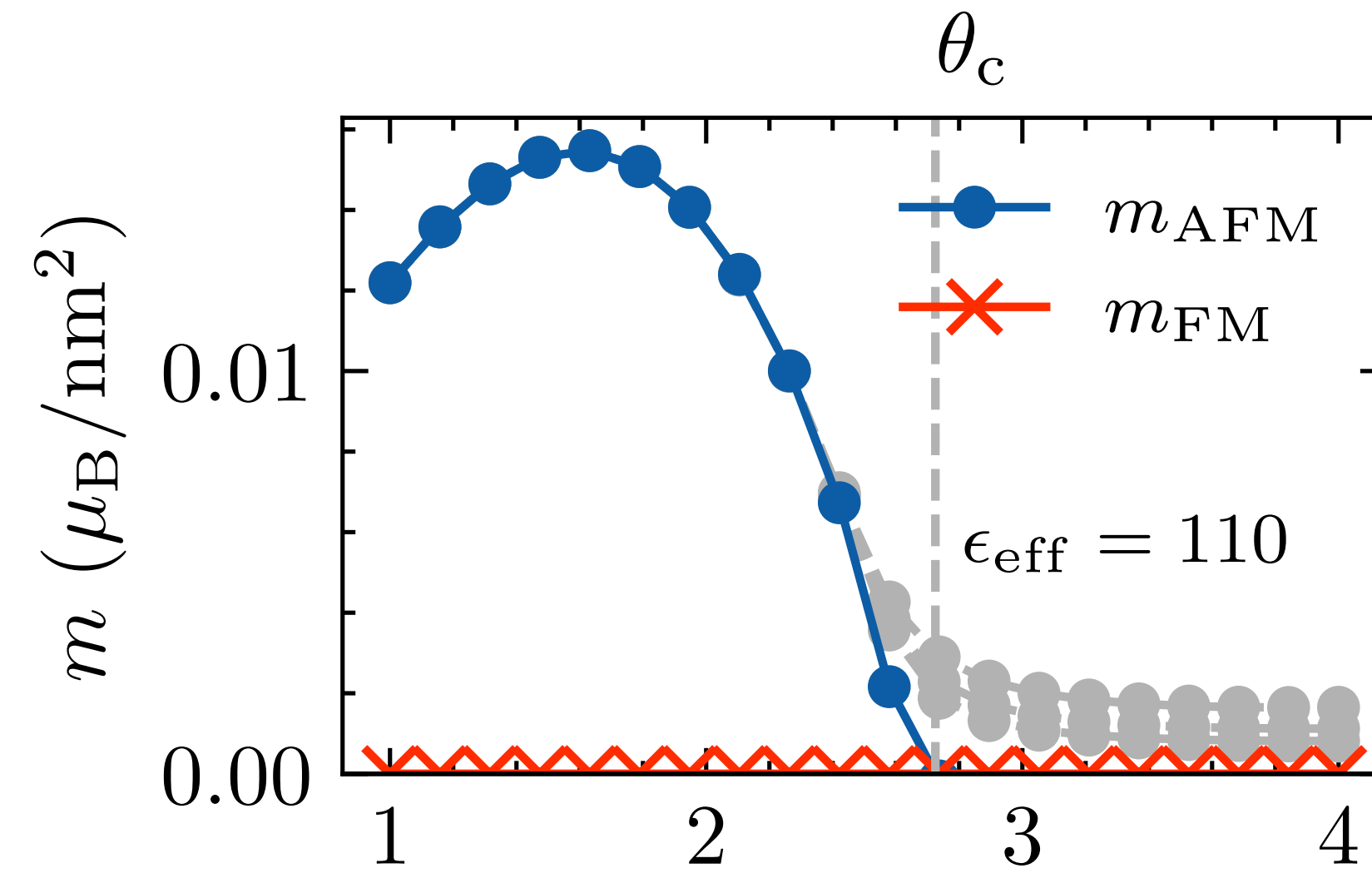


Twisted double bilayer TMDs: Phase diagram



Twisted double bilayer TMDs: Quantum criticality

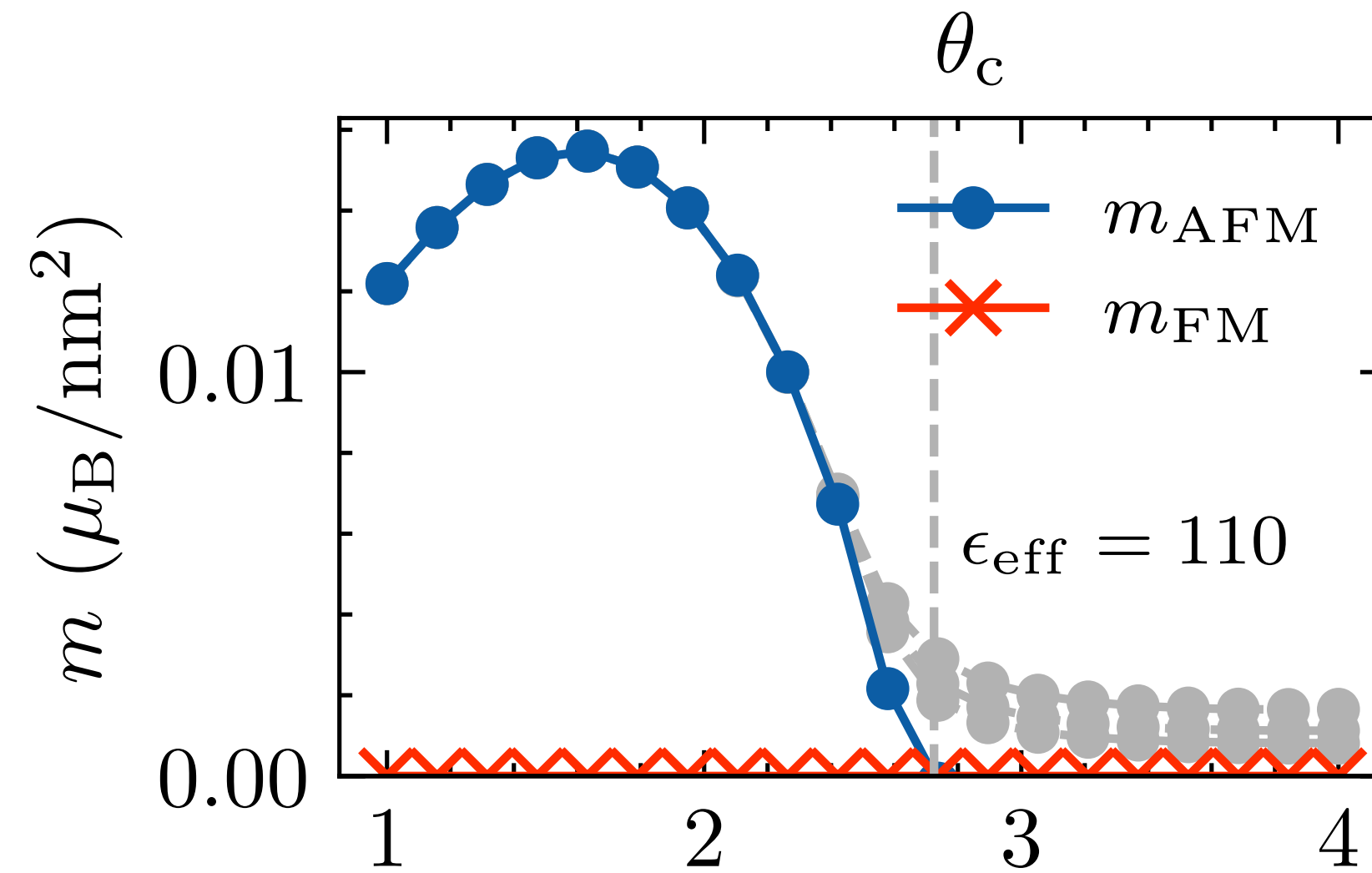
Staggered magnetization:



$$m_{\text{AFM}} \propto (\theta_c - \theta)^\beta$$

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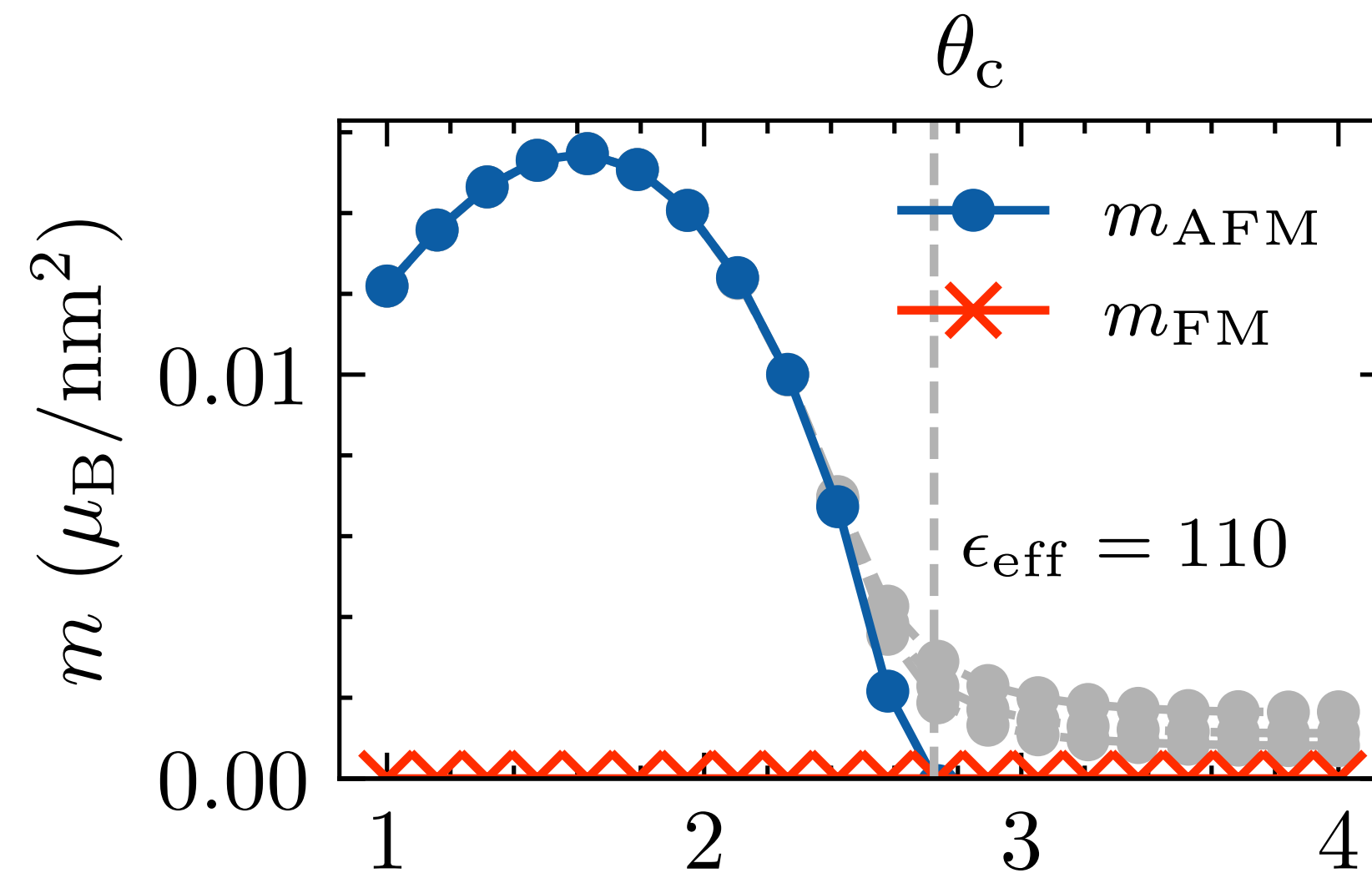
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[Biedermann & LJ, arXiv:2509.04561]

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[Ladovrechis, Ray, Meng, LJ, PRB '23]

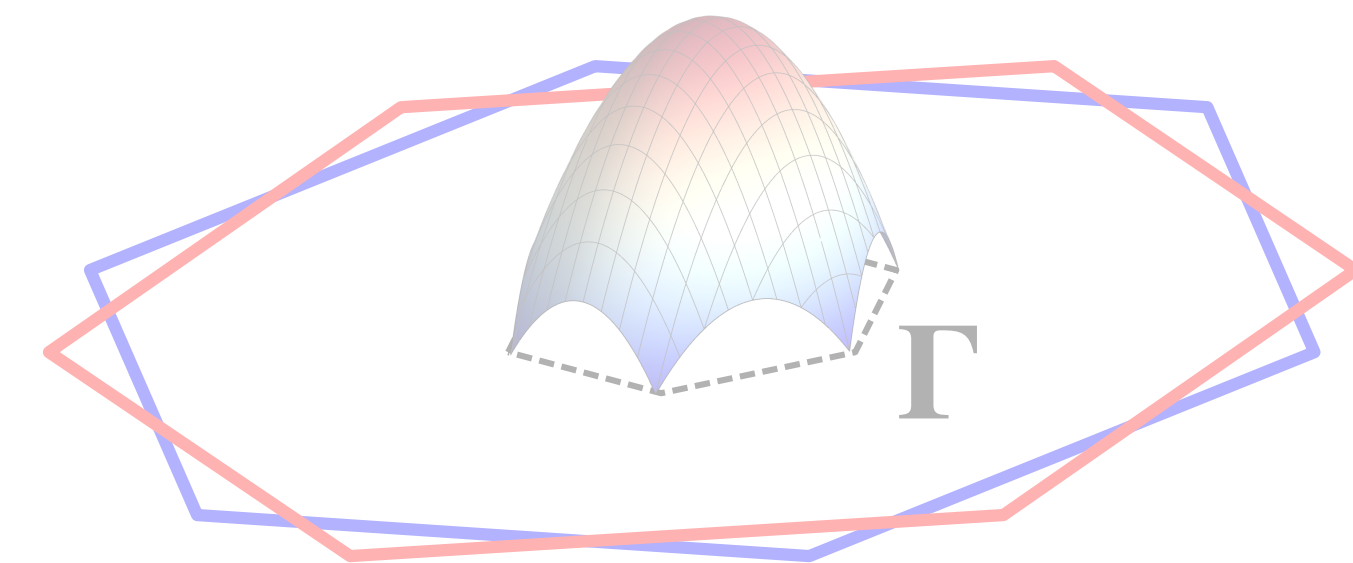
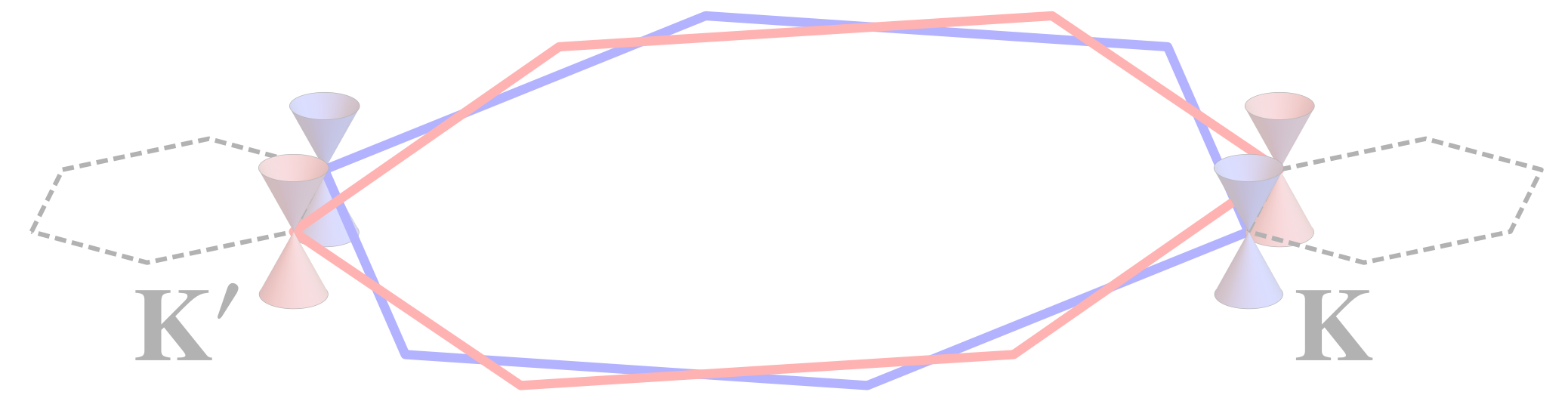
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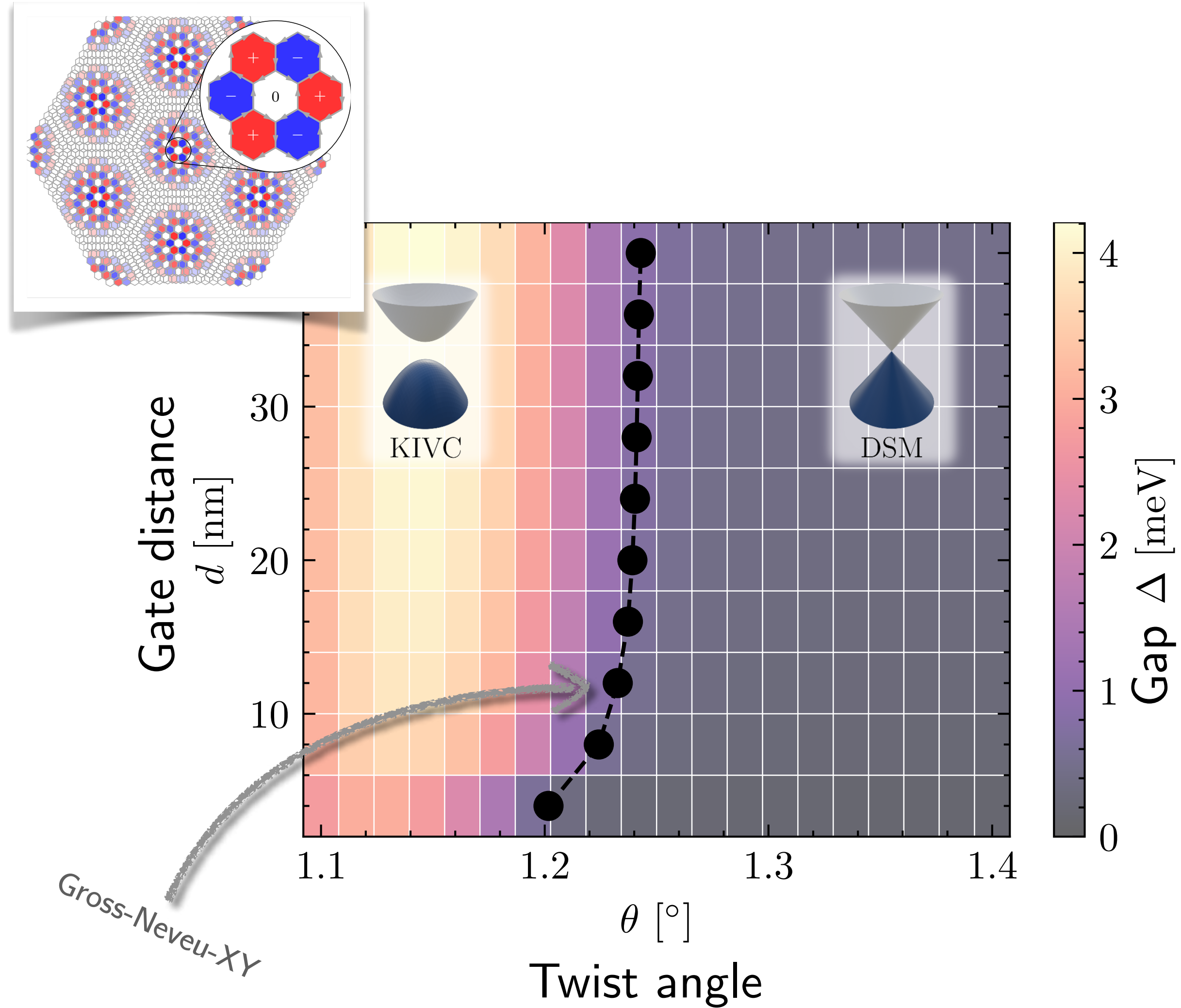
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(4) Conclusions



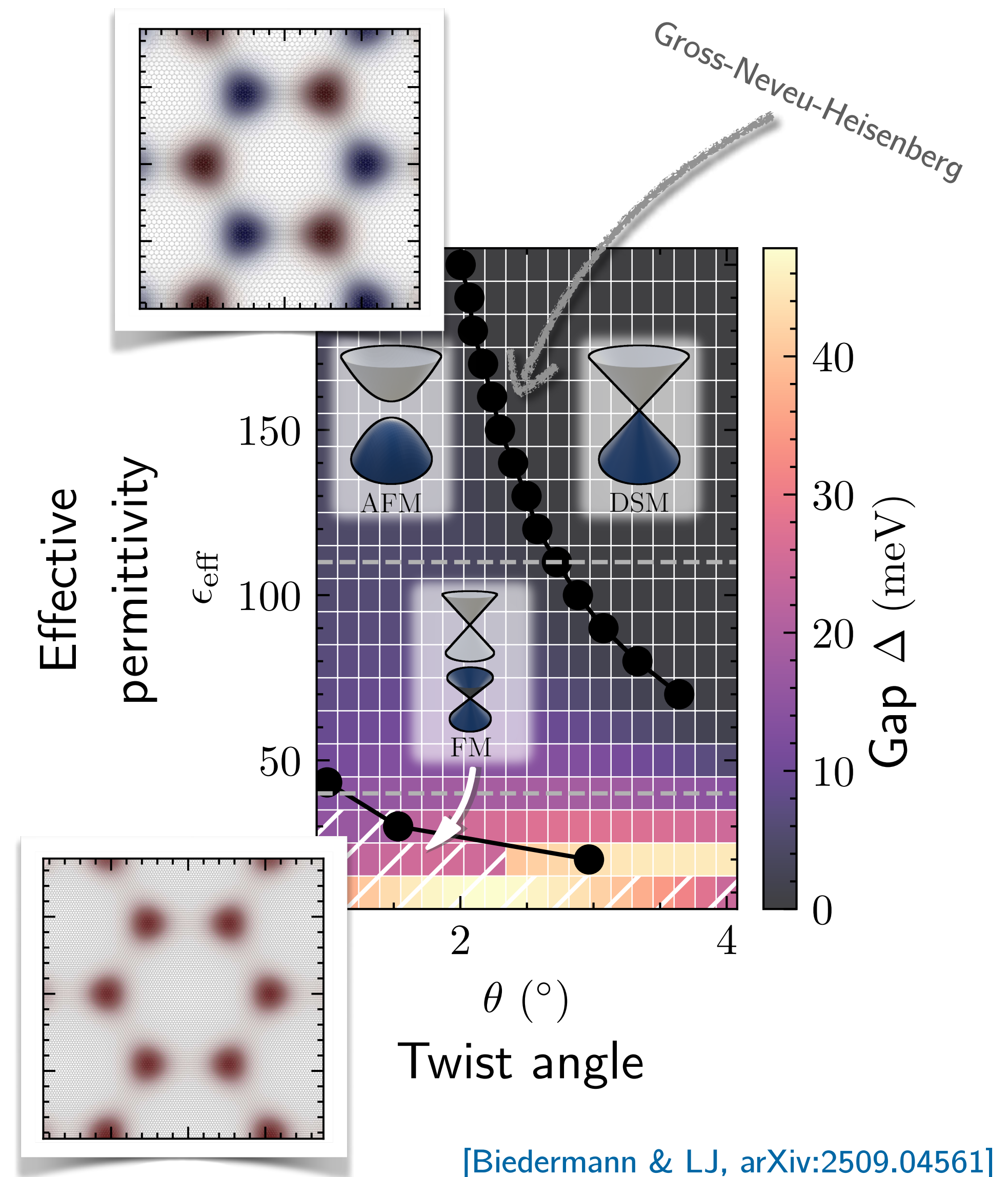
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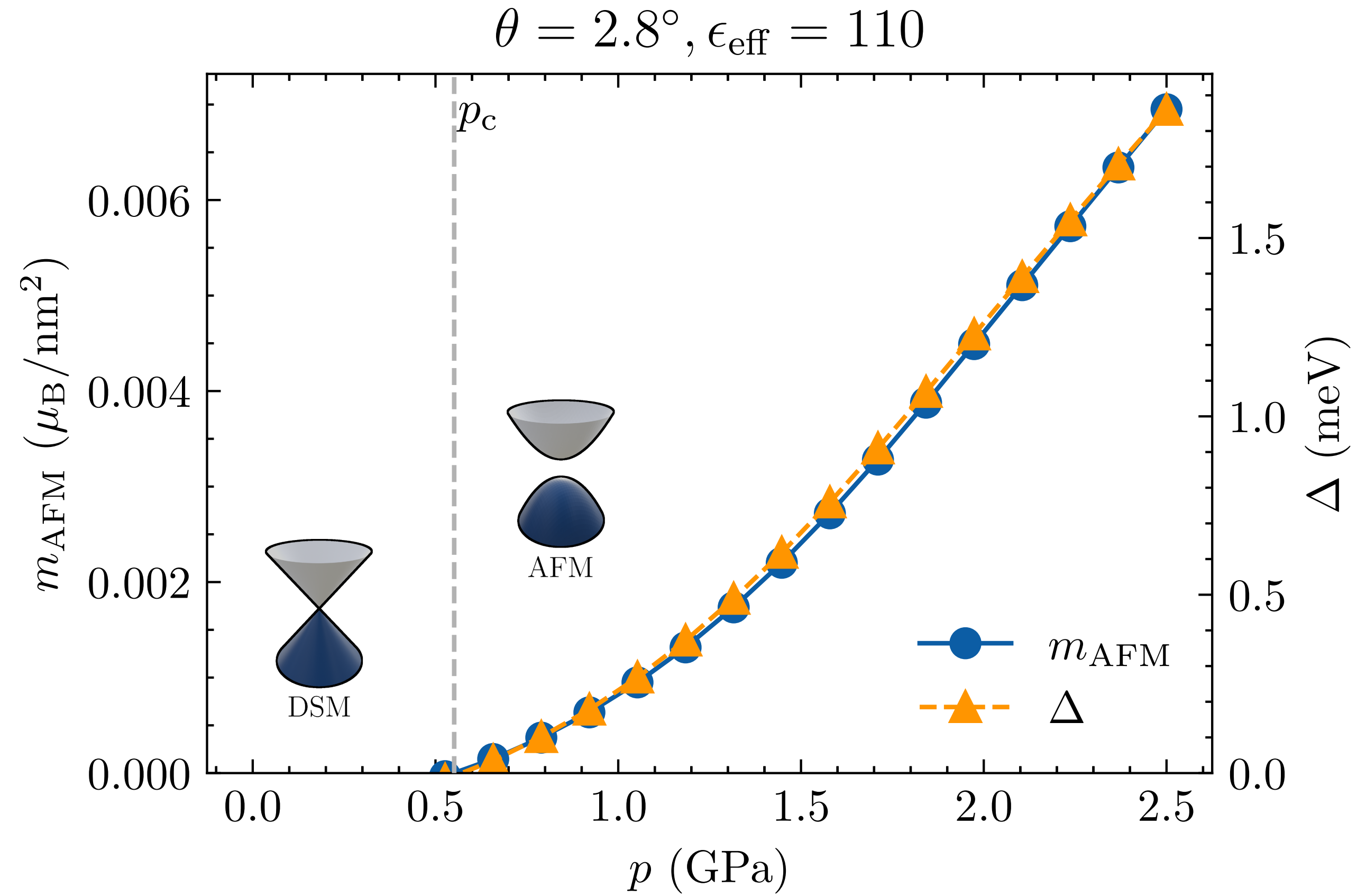
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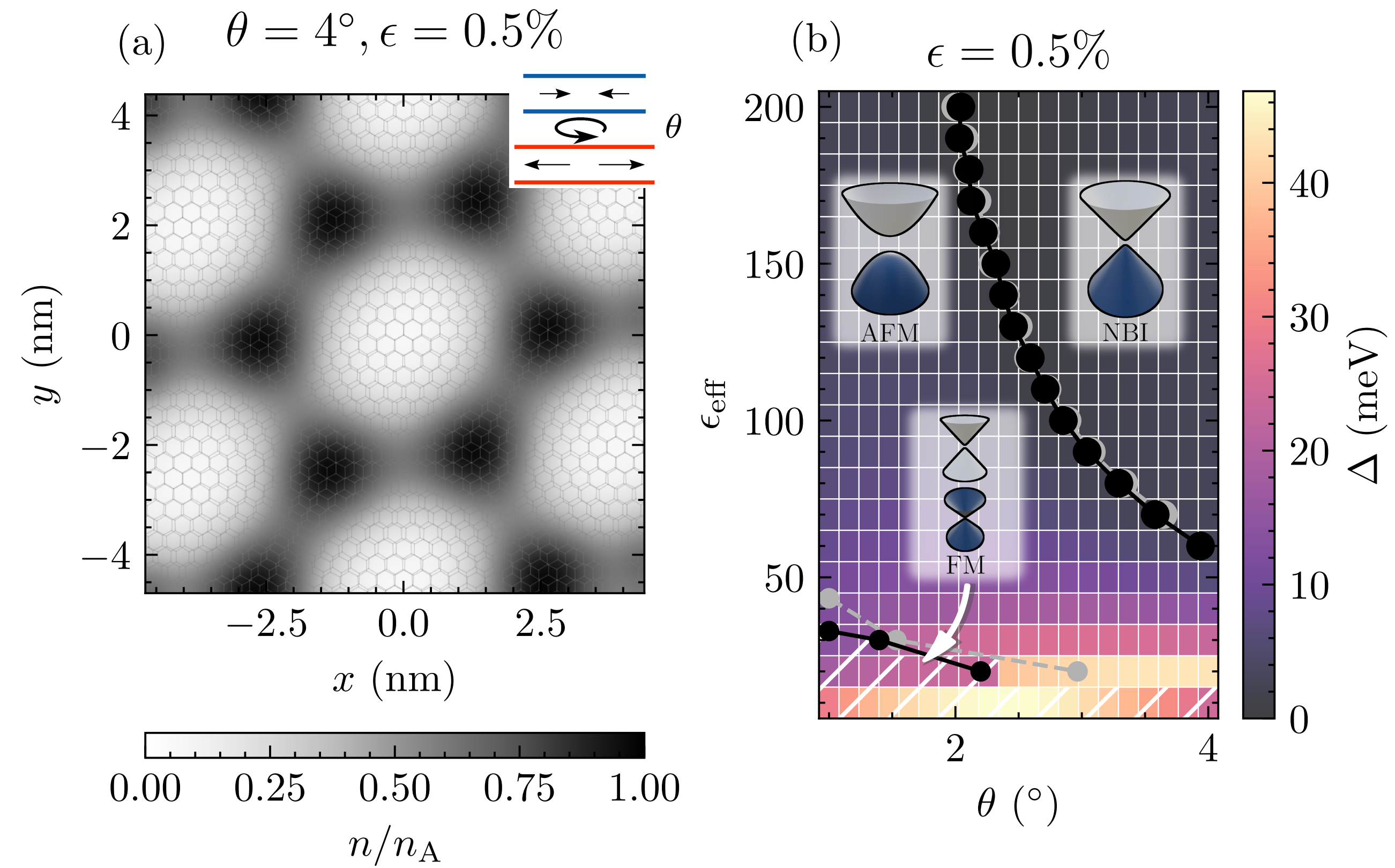


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Twisted double bilayer TMDs: Pressure-tuned transition



Twisted double bilayer TMDs: Heterostrain



Twisted double bilayer TMDs: Heterostrain

