

Emergent phenomena in quantum materials

Lukas Janssen

Emmy
Noether-
Programm



DFG Deutsche
Forschungsgemeinschaft



ct.qmat
Complexity and Topology
in Quantum Matter

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*How to realize coherent
quantum waves?*

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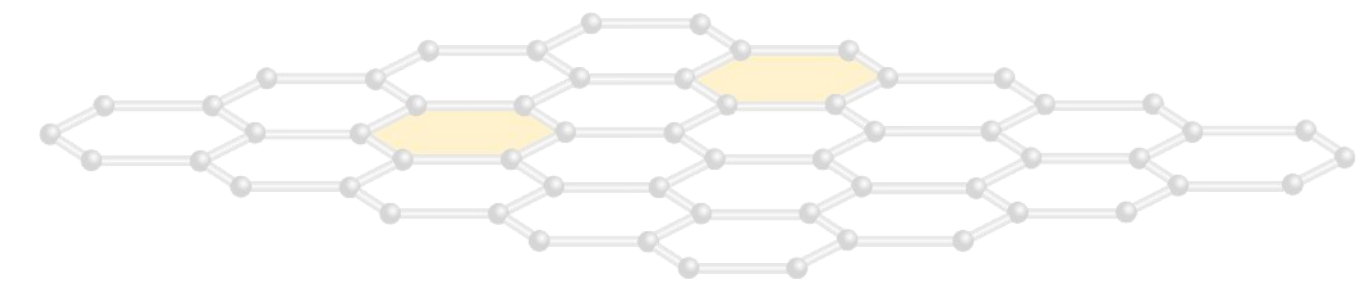
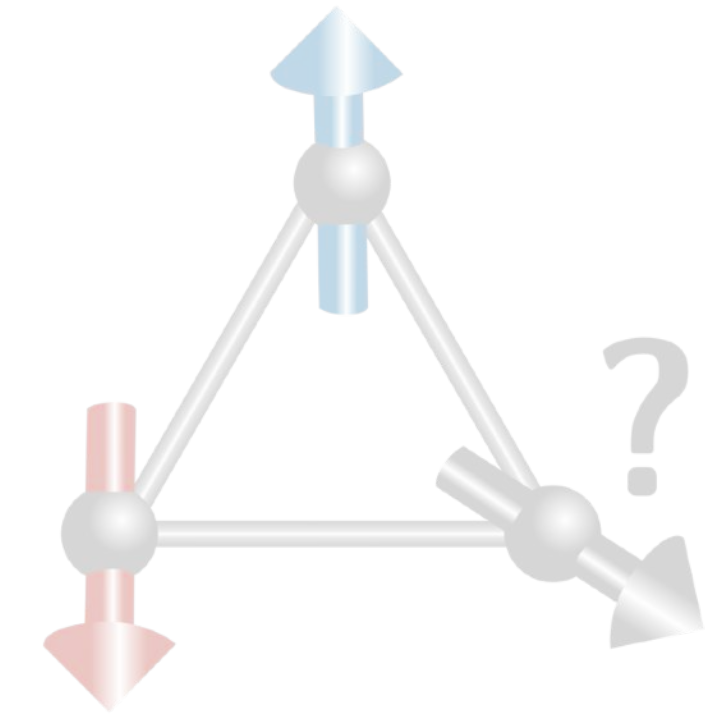
Outline

(1) Introduction: Emergence

(2) Quantum materials

(3) Emergent phenomena for Quantum Saxony

(4) Summary



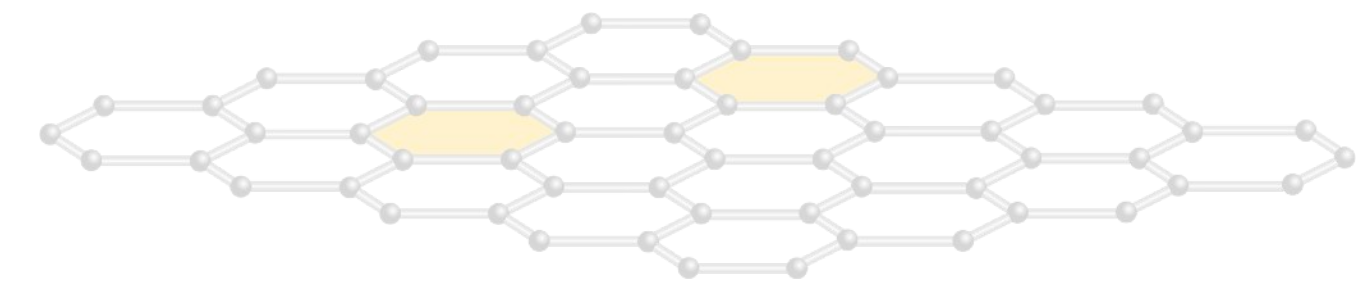
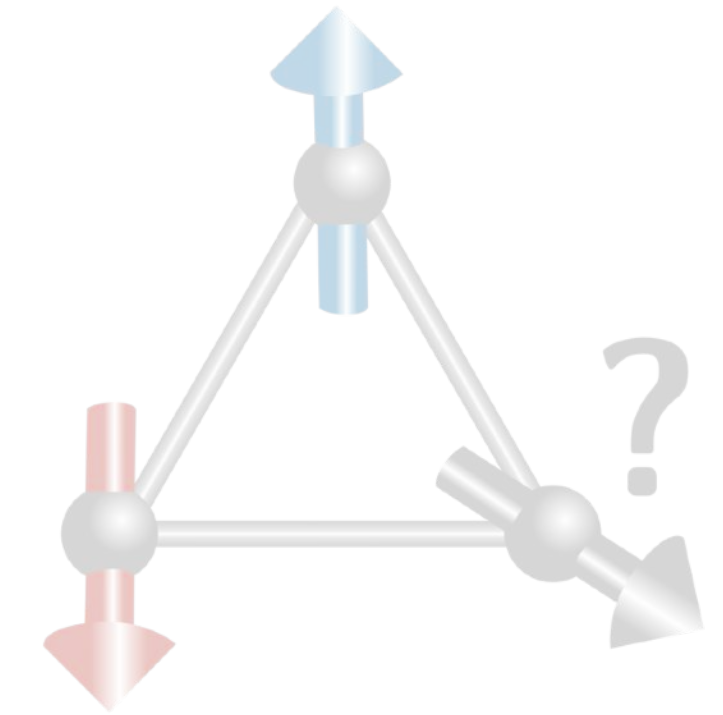
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Reductionism



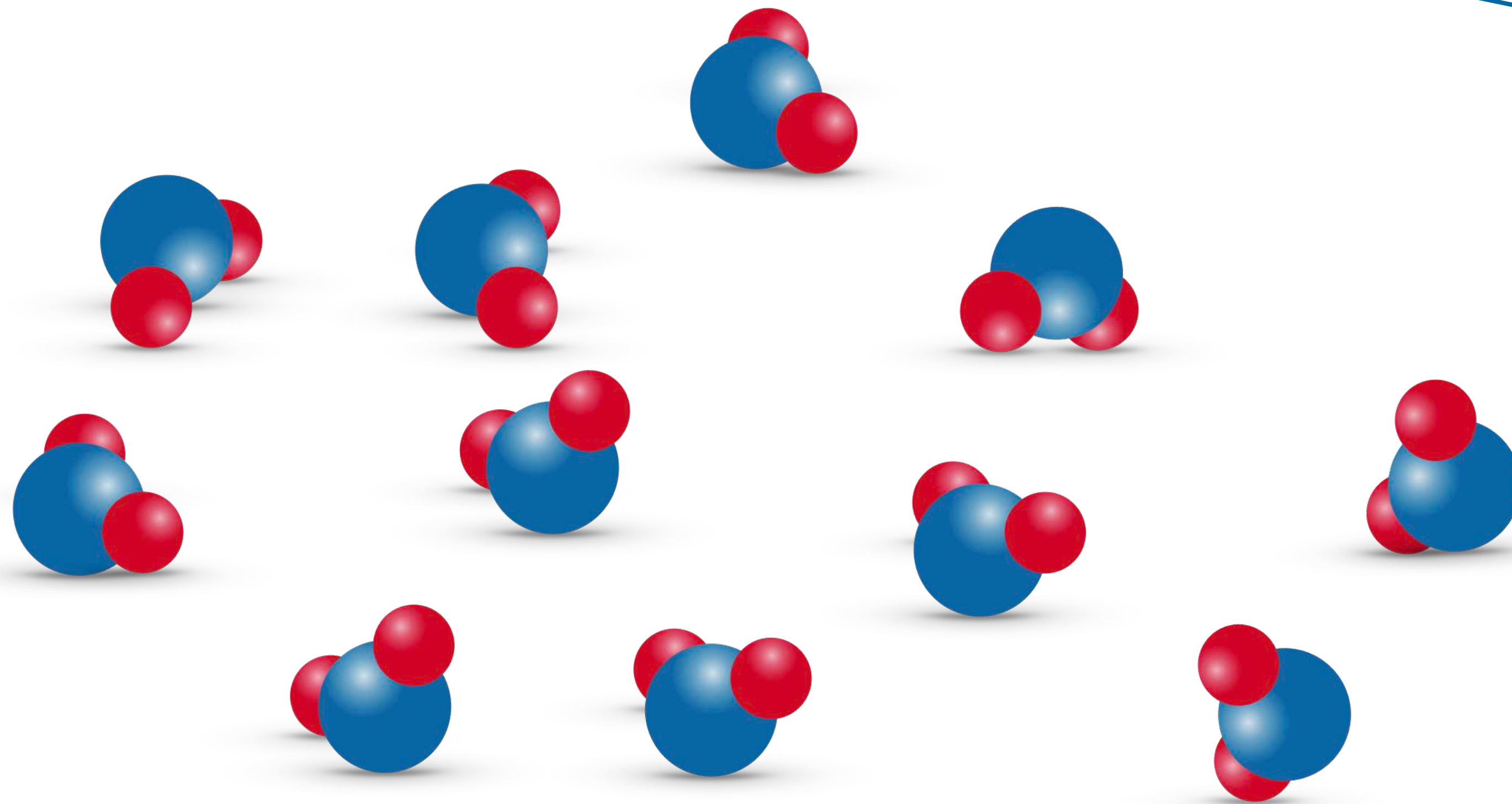
Matter

Reductionism

Matter



Molecules



Reductionism

Matter

Molecules

Atoms

1 H Hydrogen 1.008																	2 He Helium 4.002602
3 Li Lithium 6.94	4 Be Beryllium 9.0121831											13 B Boron 10.81	14 C Carbon 12.011	15 N Nitrogen 14.007	16 O Oxygen 15.999	17 F Fluorine 18.99840363	18 Ne Neon 20.1797
11 Na Sodium 22.98976928	12 Mg Magnesium 24.305											13 Al Aluminium 26.9815385	14 Si Silicon 28.085	15 P Phosphorus 30.973761998	16 S Sulfur 32.06	17 Cl Chlorine 35.45	18 Ar Argon 39.948
19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.955908	22 Ti Titanium 47.867	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938044	26 Fe Iron 55.845	27 Co Cobalt 58.933194	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.630	33 As Arsenic 74.921595	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 83.798
37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90584	40 Zr Zirconium 91.224	41 Nb Niobium 92.90637	42 Mo Molybdenum 95.95	43 Tc Technetium (98)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.90550	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.760	52 Te Tellurium 127.60	53 I Iodine 126.90447	54 Xe Xenon 131.293
55 Cs Caesium 132.90545196	56 Ba Barium 137.327	57 - 71 Lanthanoids	72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.084	79 Au Gold 196.966569	80 Hg Mercury 200.592	81 Tl Thallium 204.38	82 Pb Lead 207.2	83 Bi Bismuth 208.98040	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)
87 Fr Francium (223)	88 Ra Radium (226)	89 - 103 Actinoids	104 Rf Rutherfordium (261)	105 Db Dubnium (268)	106 Sg Seaborgium (269)	107 Bh Bohrium (270)	108 Hs Hassium (285)	109 Mt Meitnerium (278)	110 Ds Darmstadtium (281)	111 Rg Roentgenium (282)	112 Cn Copernicium (285)	113 Nh Nihonium (286)	114 Fl Flerovium (289)	115 Mc Moscovium (289)	116 Lv Livermorium (293)	117 Ts Tennessine (294)	118 Og Oganesson (294)

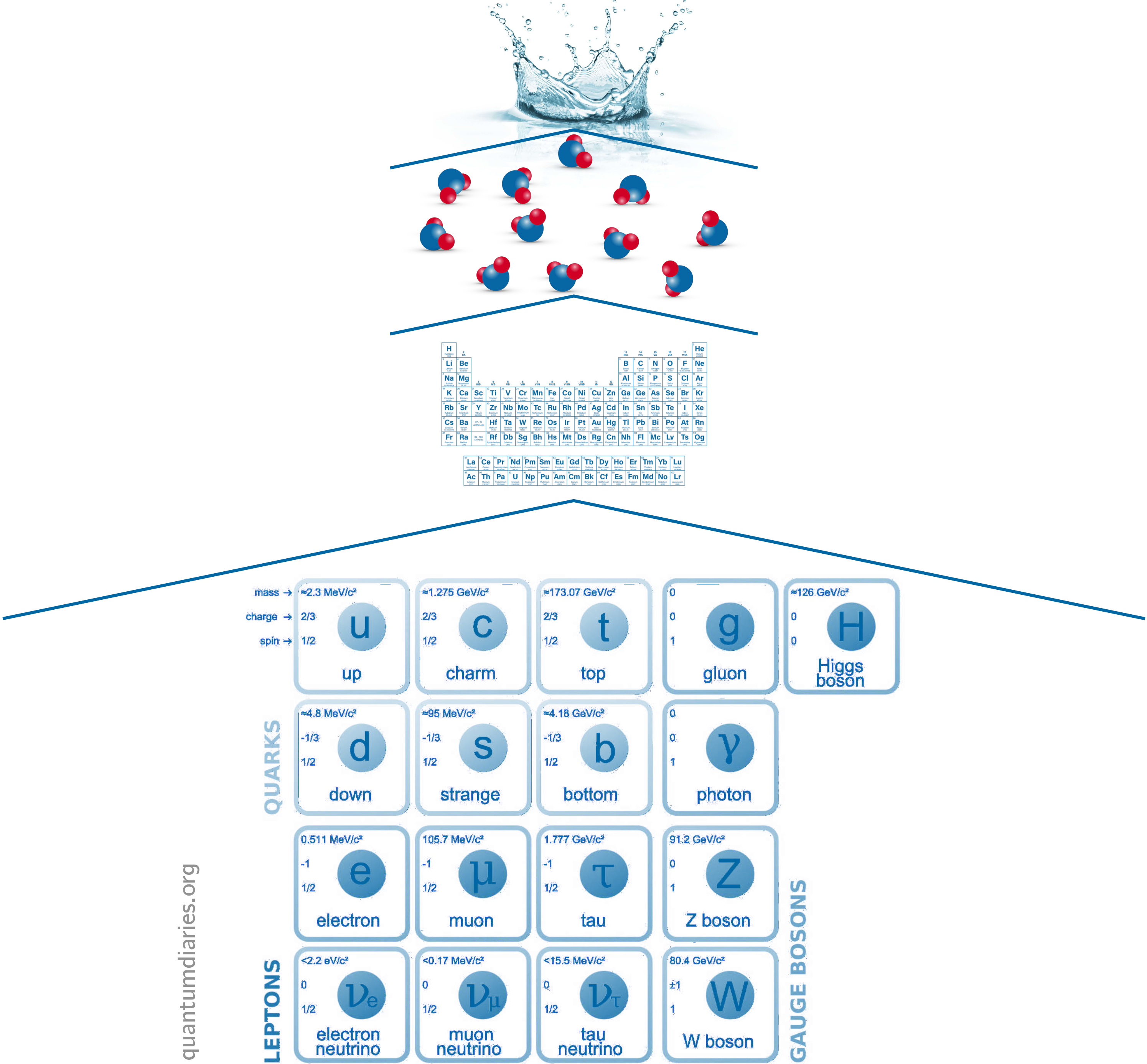
57 La Lanthanum 138.90547	58 Ce Cerium 140.16	59 Pr Praseodymium 140.90766	60 Nd Neodymium 144.242	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.500	67 Ho Holmium 164.93033	68 Er Erbium 167.259	69 Tm Thulium 168.93422	70 Yb Ytterbium 173.045	71 Lu Lutetium 174.9668
89 Ac Actinium (227)	90 Th Thorium 232.0377	91 Pa Protactinium 231.03688	92 U Uranium 238.02891	93 Np Neptunium (237)	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (260)

Reductionism

Matter

Molecules

Atoms



Elementary
particles

FIELDS ARRANGED BY PURITY

→
MORE PURE

SOCIOLOGY IS
JUST APPLIED
PSYCHOLOGY

PSYCHOLOGY IS
JUST APPLIED
BIOLOGY

BIOLOGY IS
JUST APPLIED
CHEMISTRY

WHICH IS JUST
APPLIED PHYSICS.
IT'S NICE TO
BE ON TOP.

OH, HEY, I DIDN'T
SEE YOU GUYS ALL
THE WAY OVER THERE.

...



SOCIOLOGISTS

PSYCHOLOGISTS

BIOLOGISTS

CHEMISTS

PHYSICISTS

MATHEMATICIANS

Complexity



Emergence

The whole is
greater than the sum of
its parts!



Aristotle, 385-322 BC

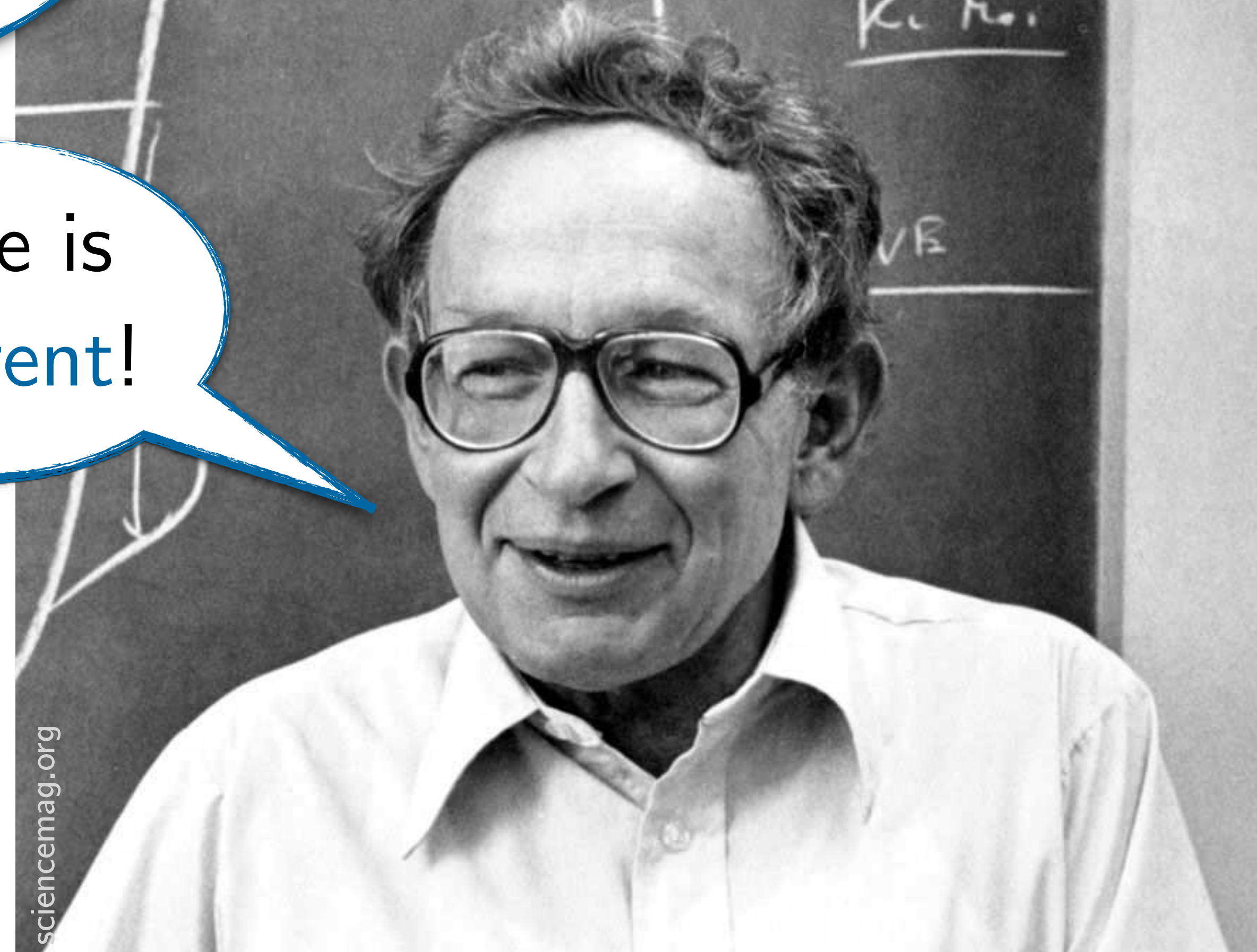
Emergence

The whole is
greater than the sum of
its parts!

More is
different!



Aristotle, 385-322 BC



Anderson, 1923-2020 AD

[Anderson, Science '72]

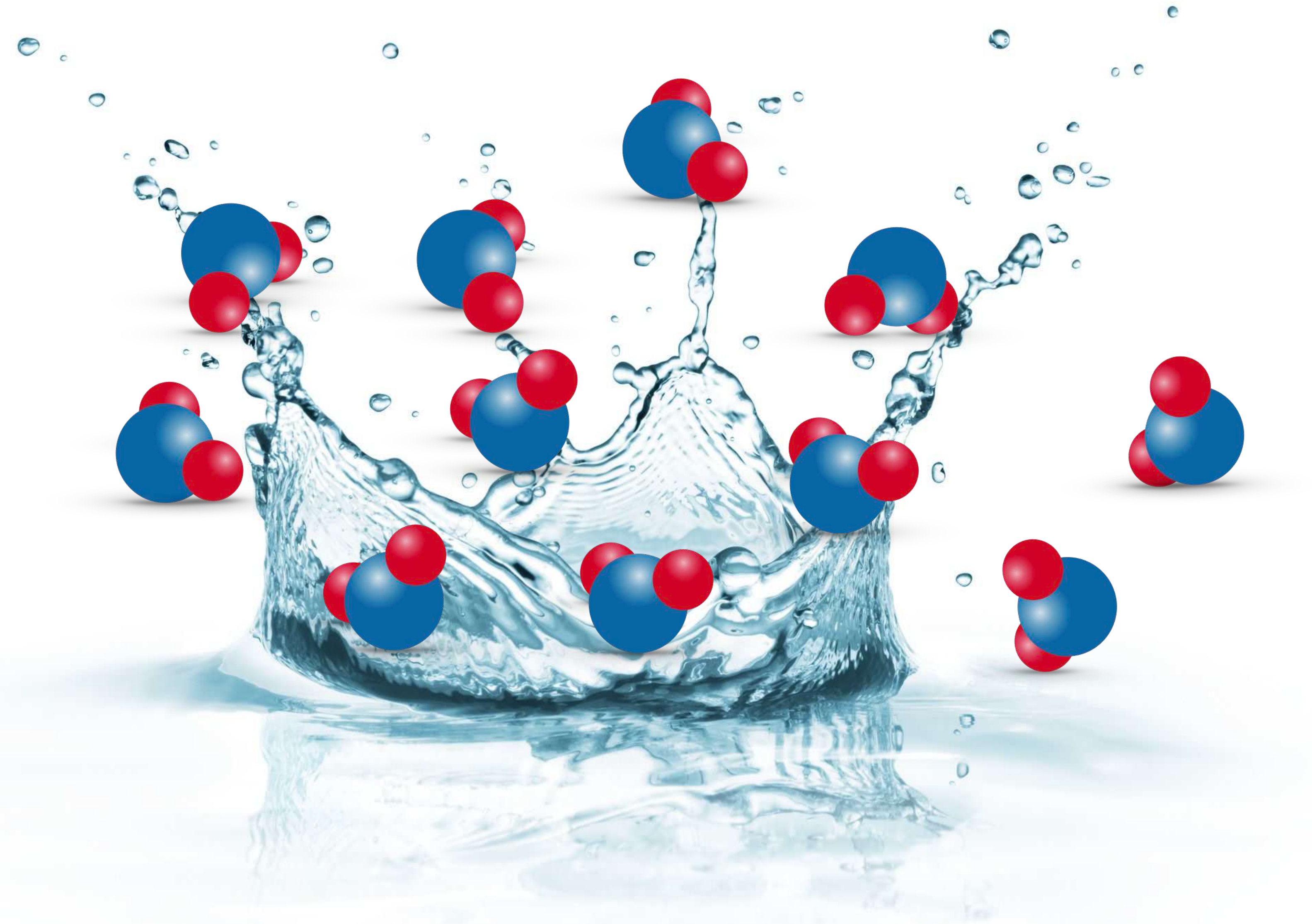
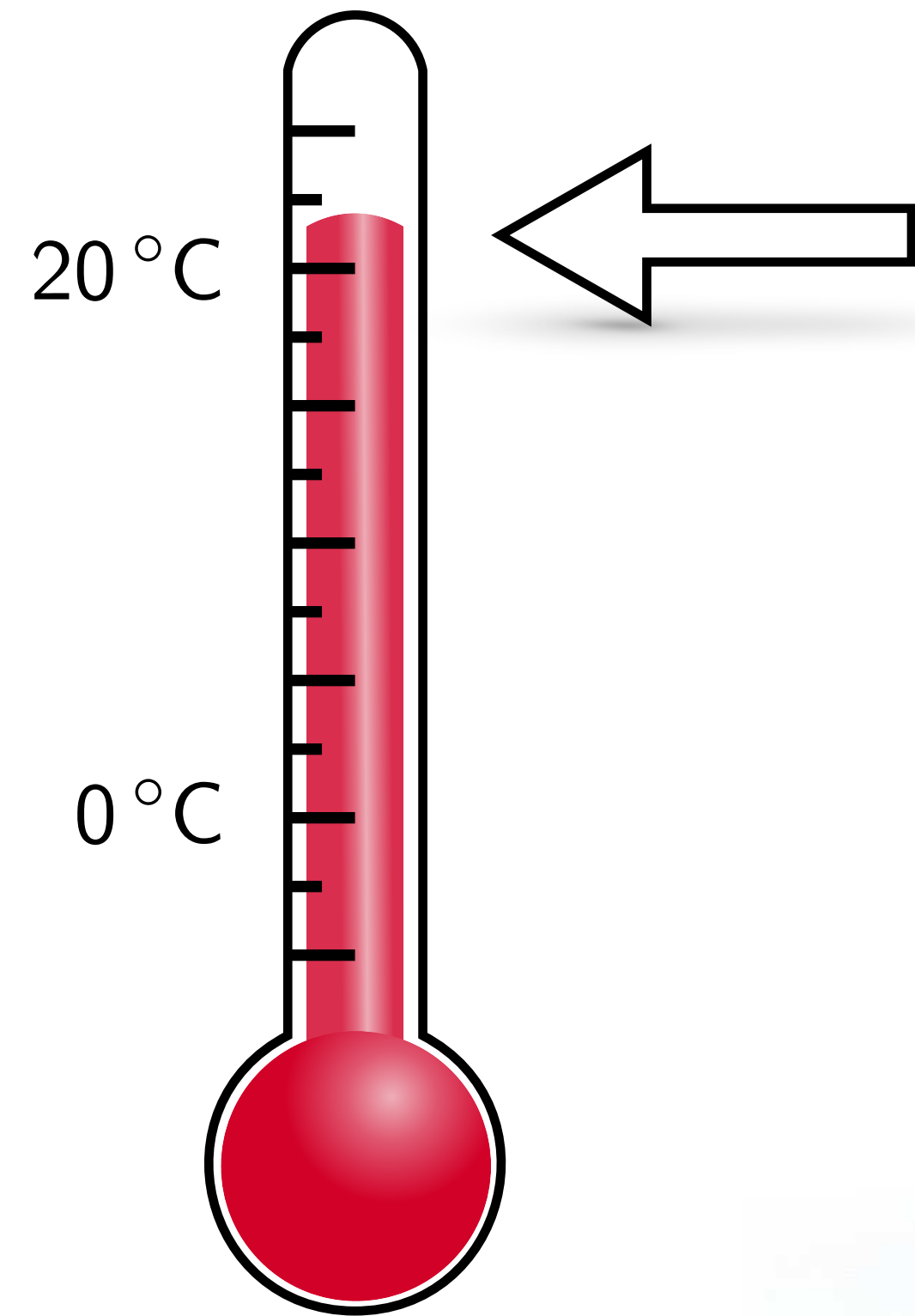
Living bridges



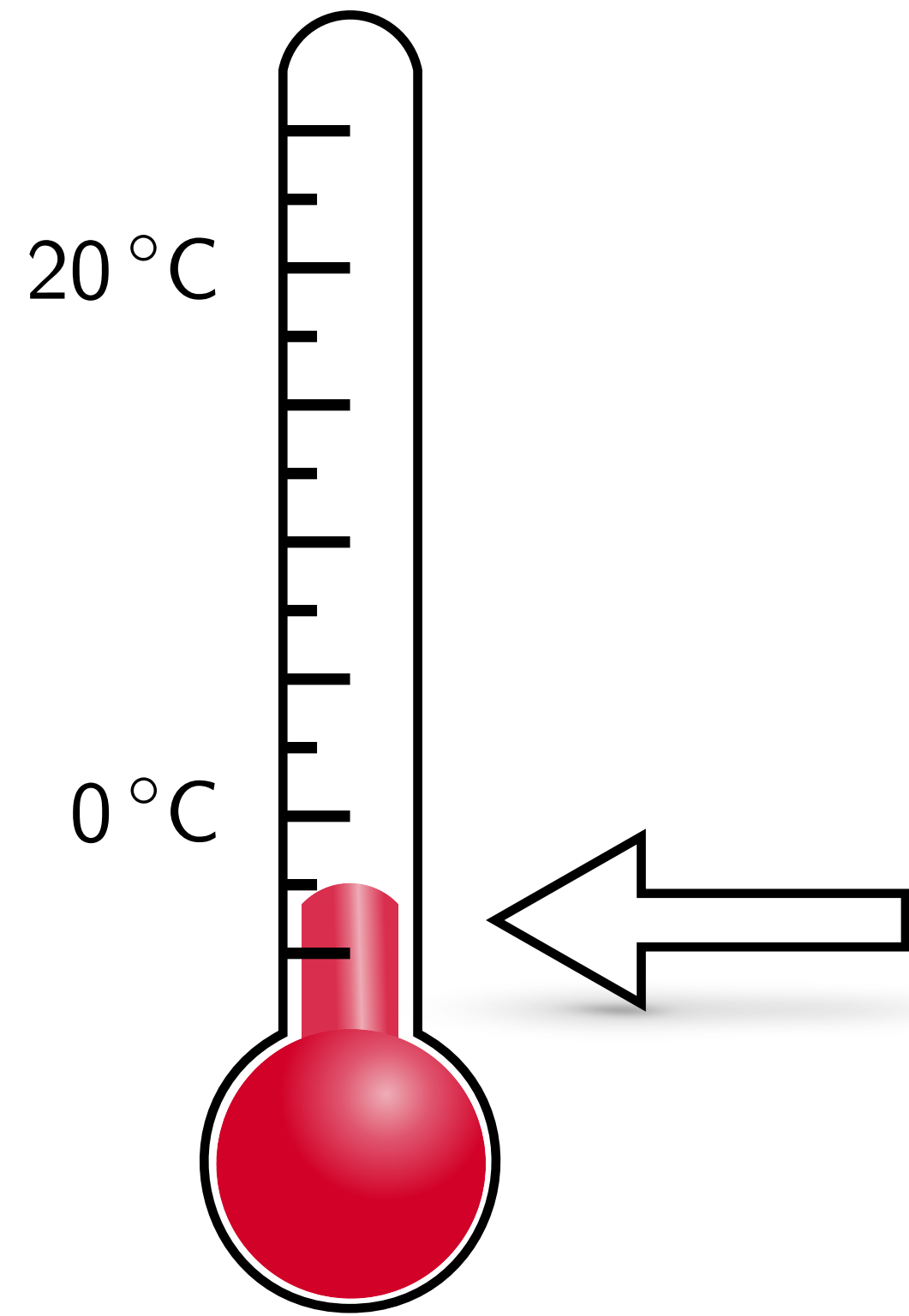
Living bridges



Snowflakes

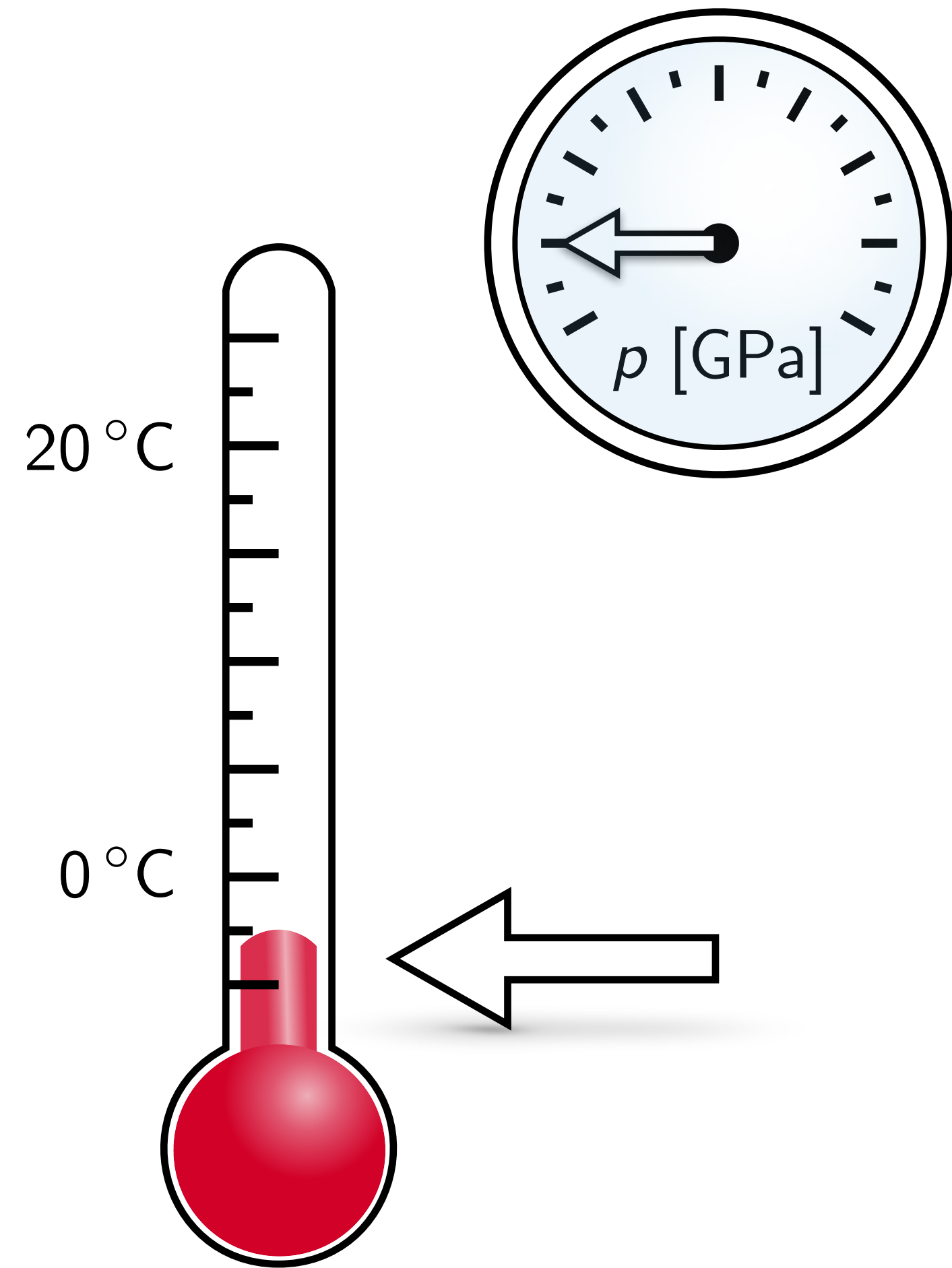


Snowflakes

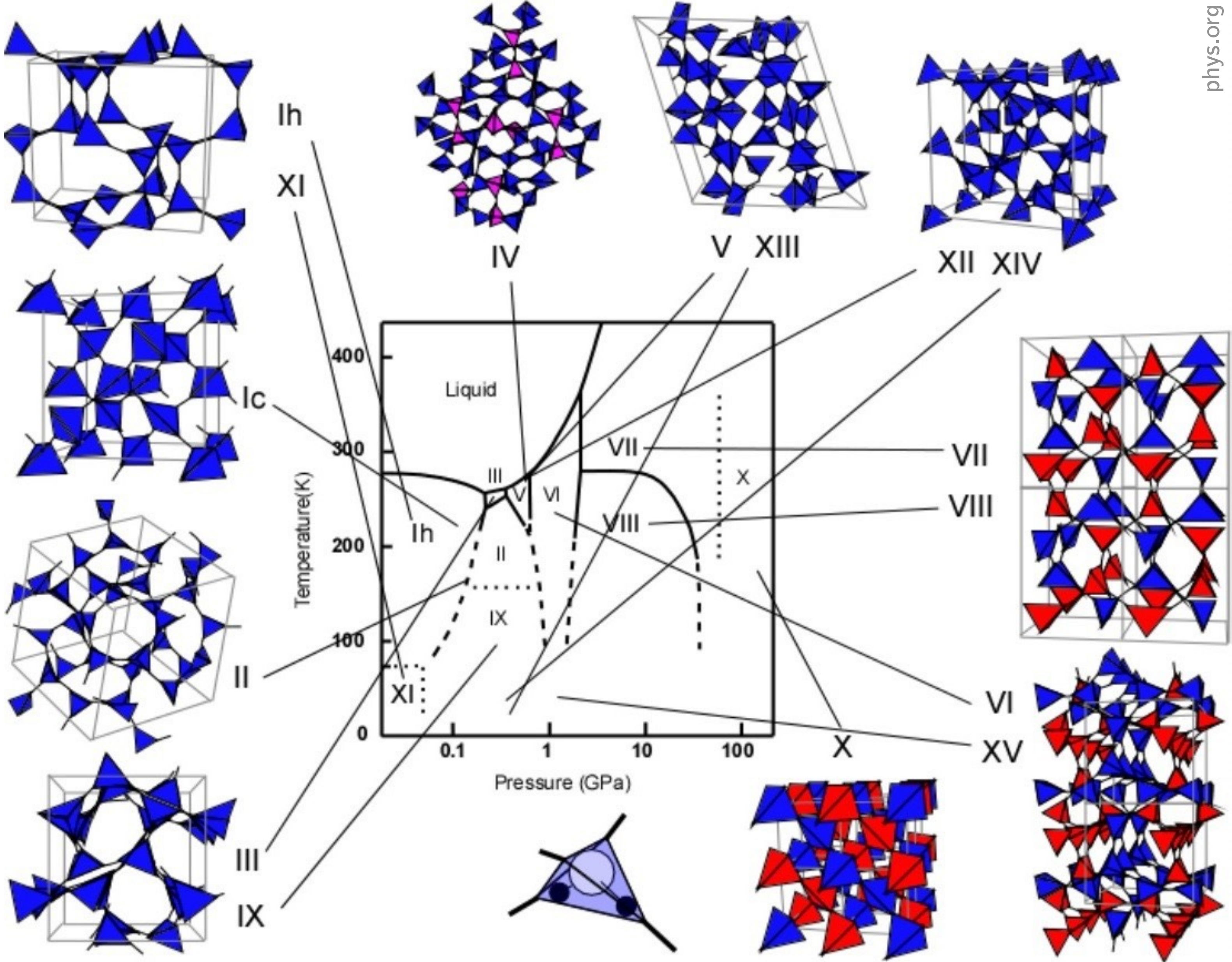
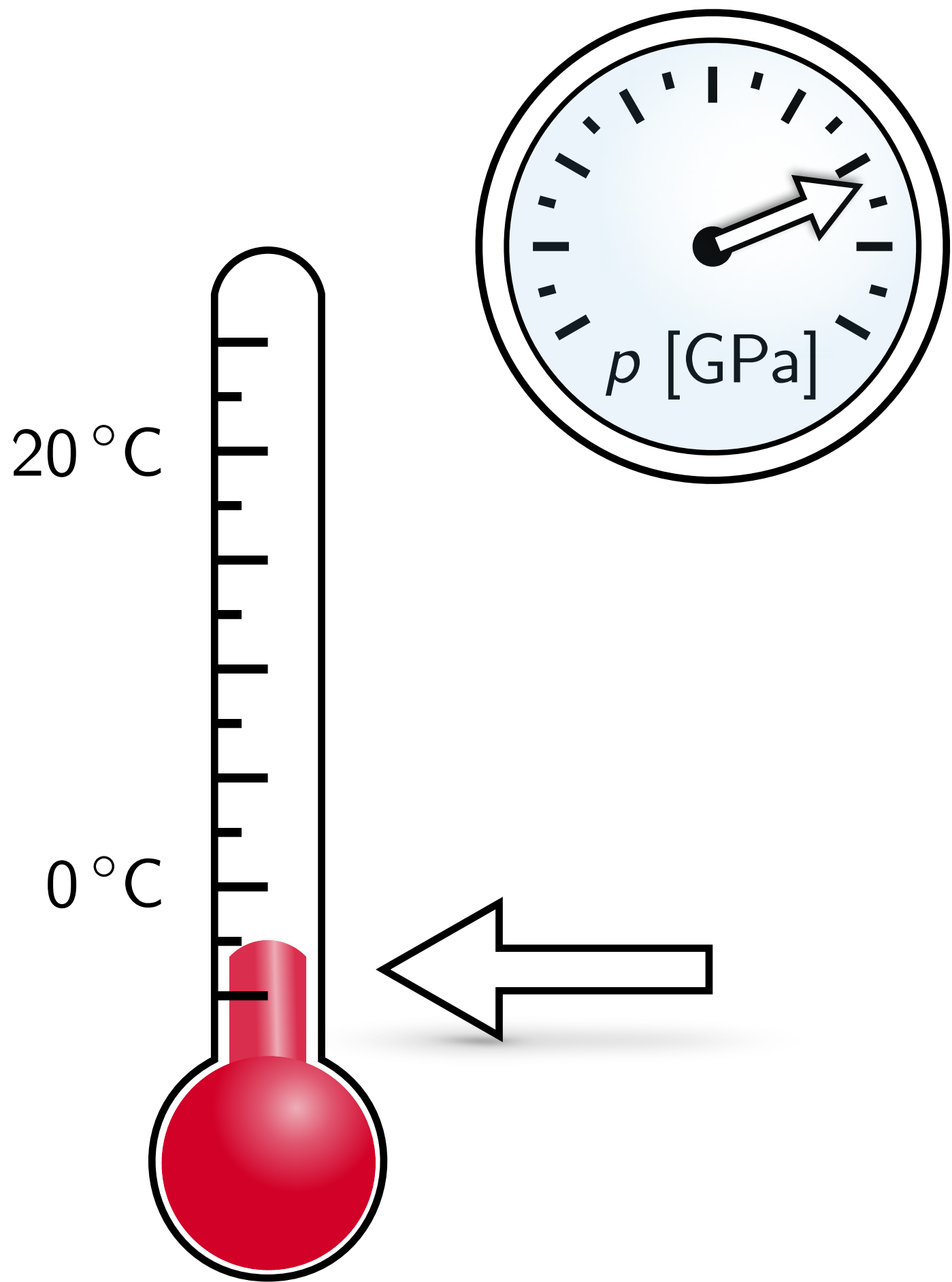


American Chemical Society

Phases of ice



Phases of ice

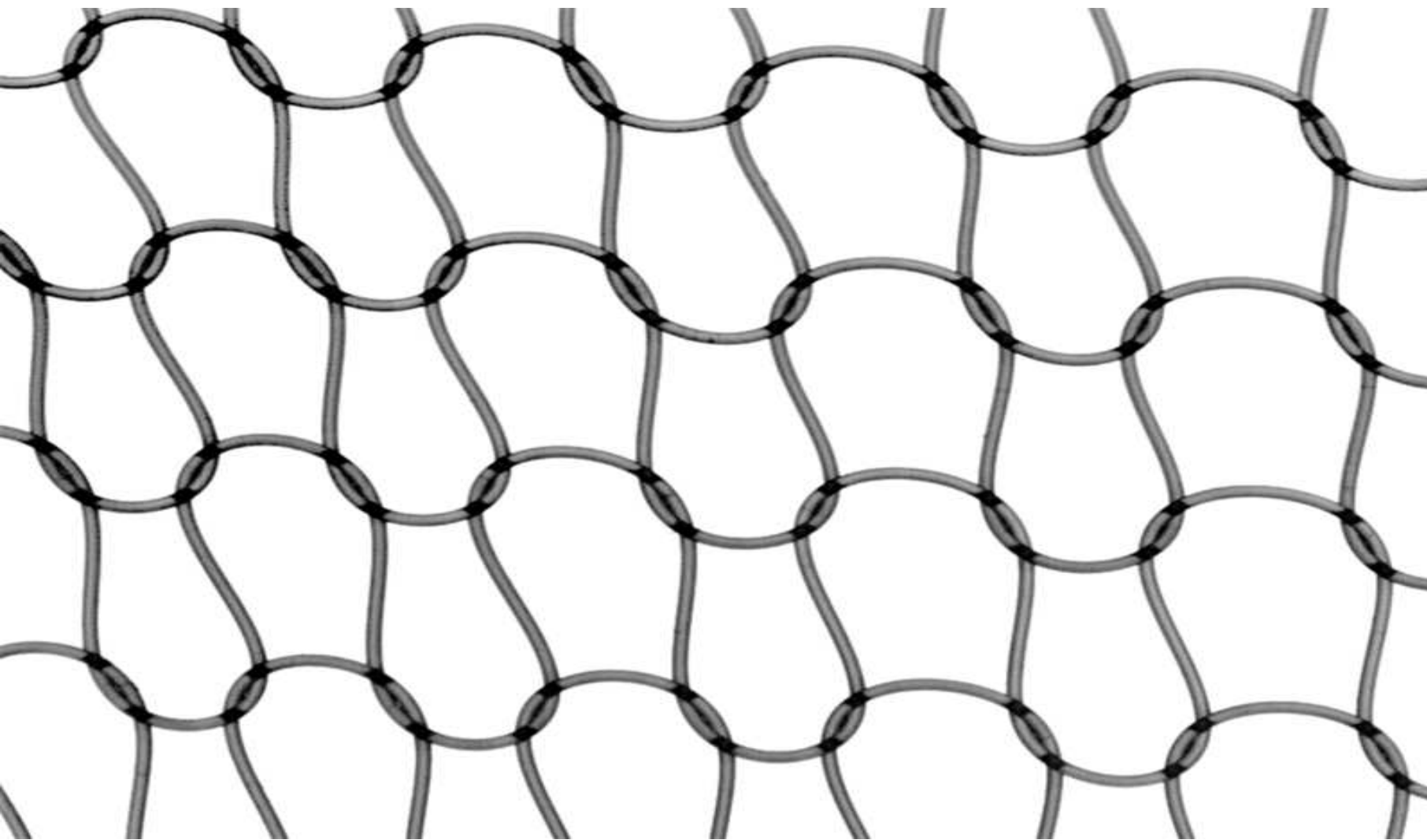


phys.org

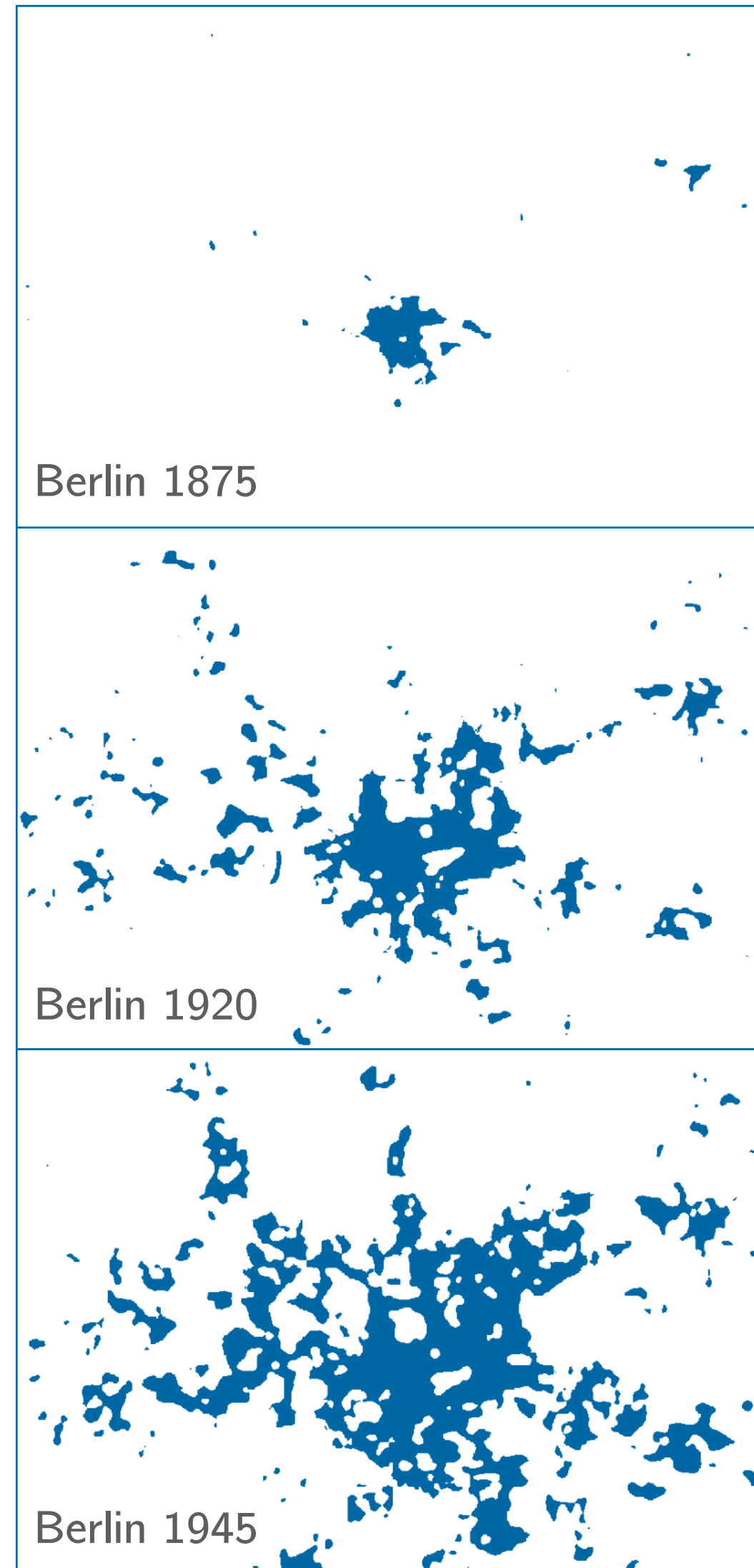
Emergent phenomena



[Portugal *et al.*, Nature '14]



[Poincloux *et al.*, PRX '18]

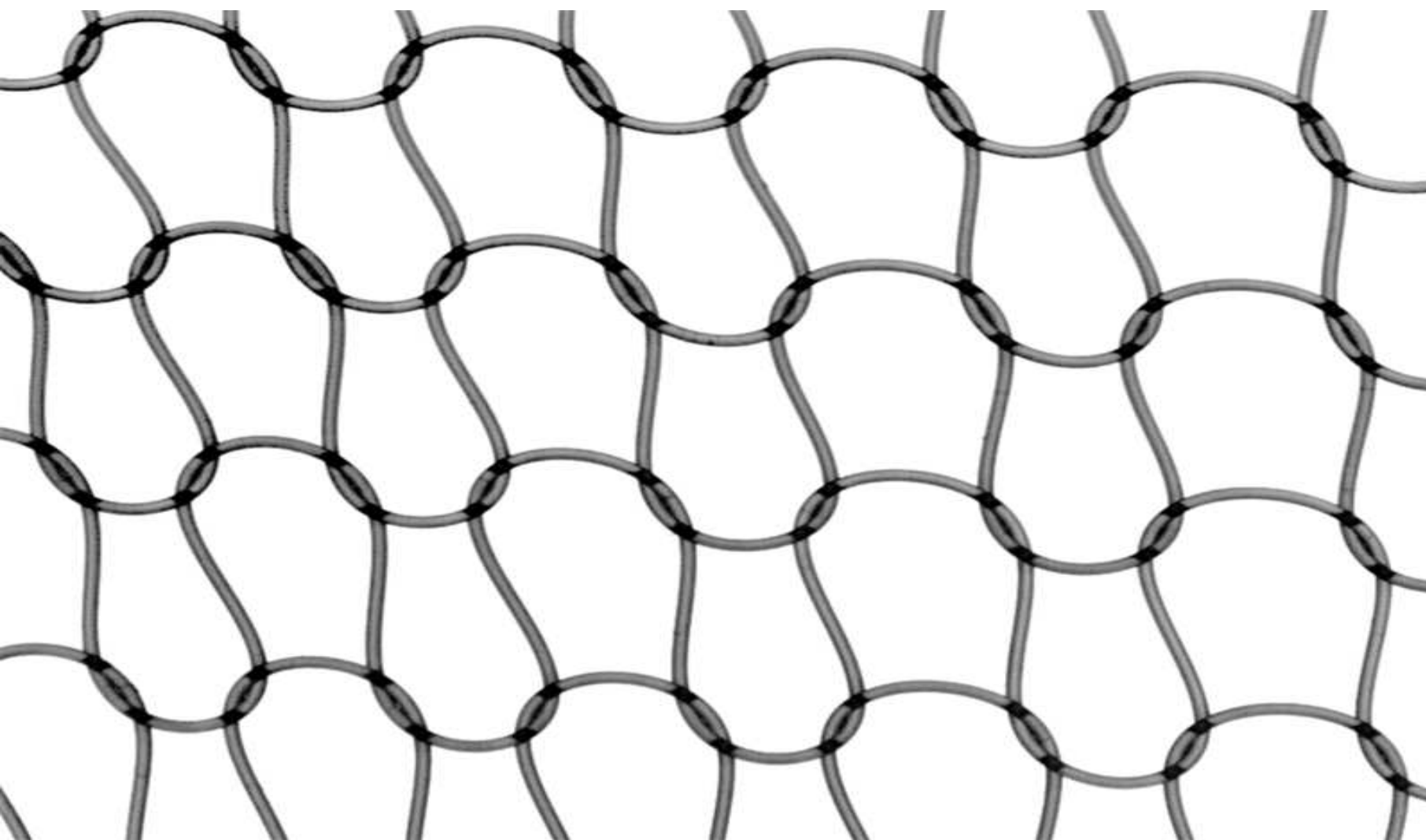


[Makse *et al.*, Nature '95]

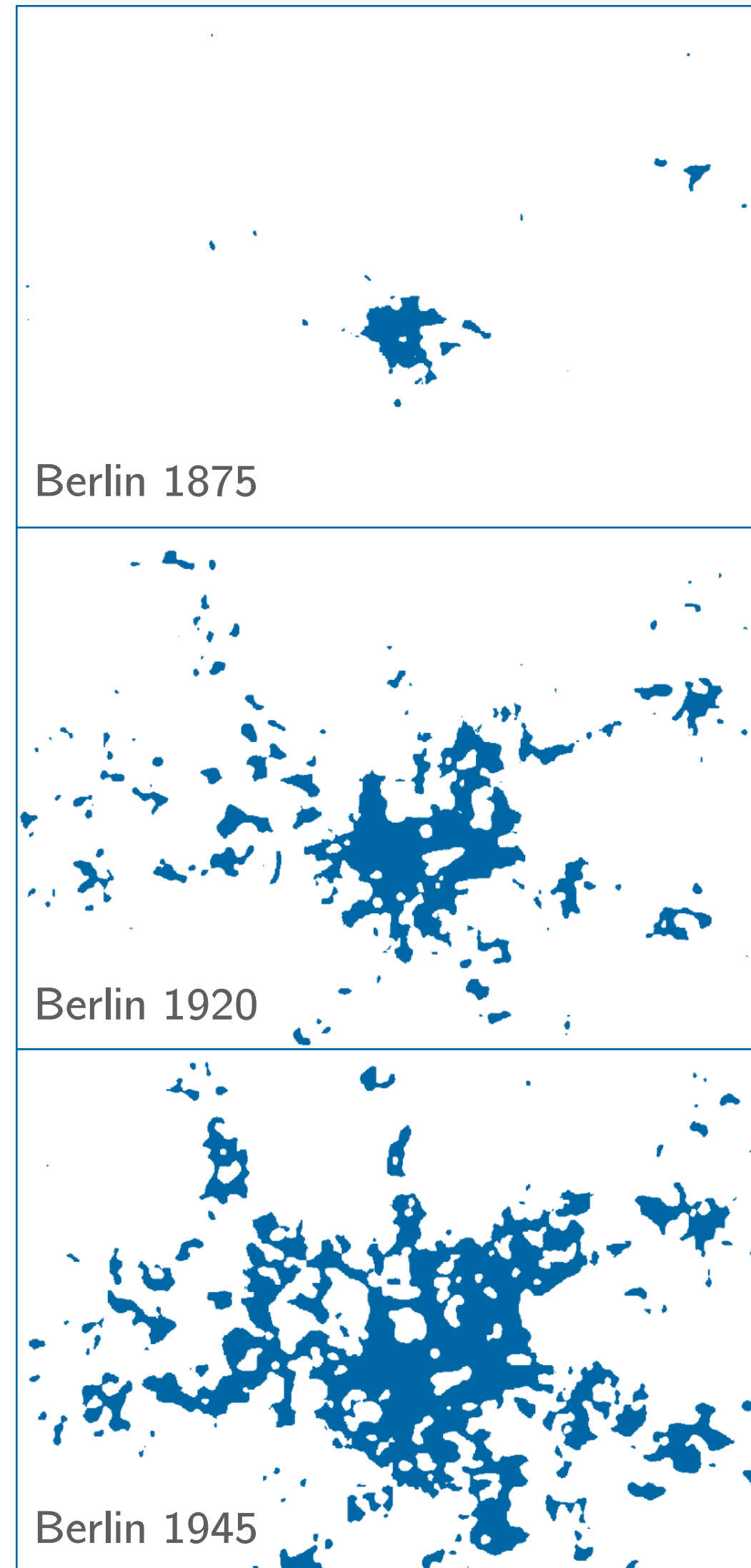
Emergent phenomena



[Portugal *et al.*, Nature '14]

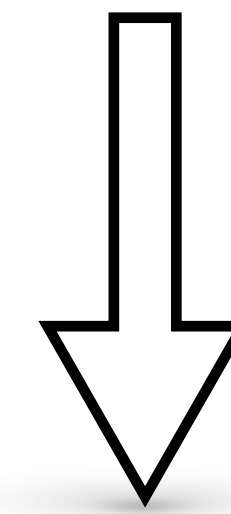


[Poincloux *et al.*, PRX '18]



[Makse *et al.*, Nature '95]

Many constituents
with simple rules



Complex structures
with new properties

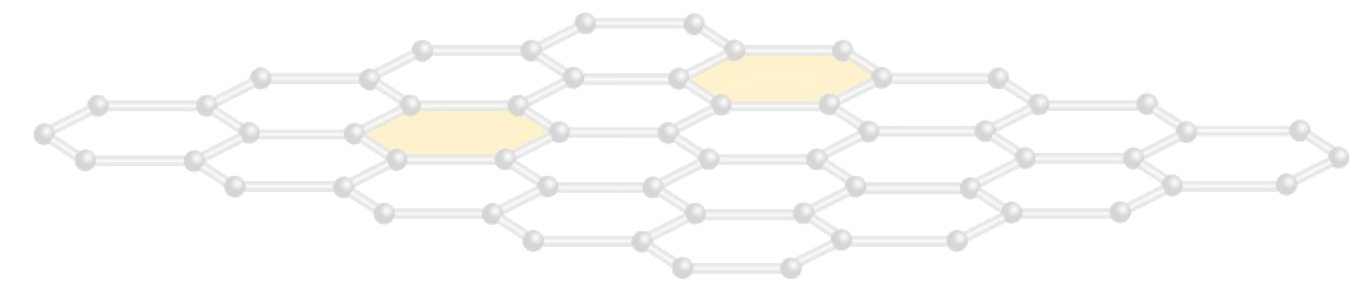
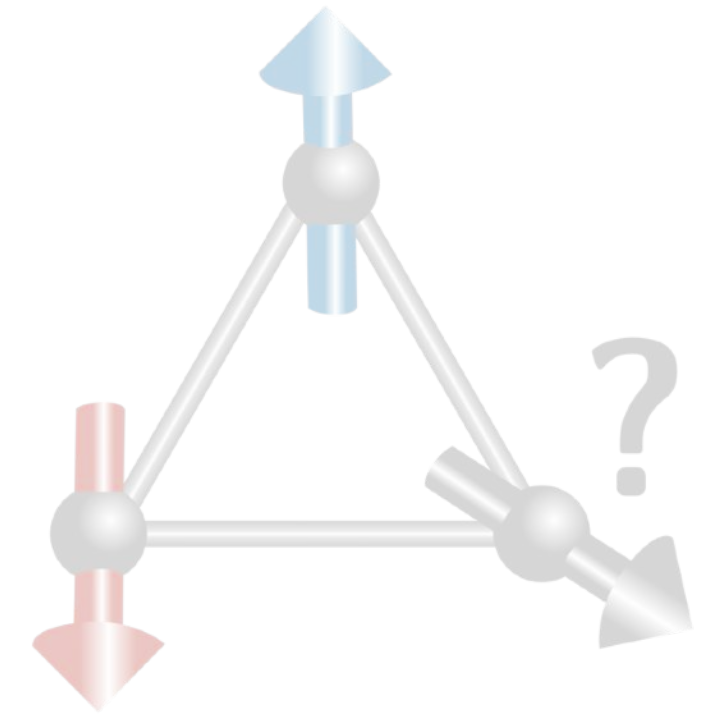
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Quantum physics: From microscopics ...

atomicarchive.com



Atom

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atomicarchive.com



Atom

Schrödinger equation:

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$

Quantum physics: From microscopics ...

atomicarchive.com

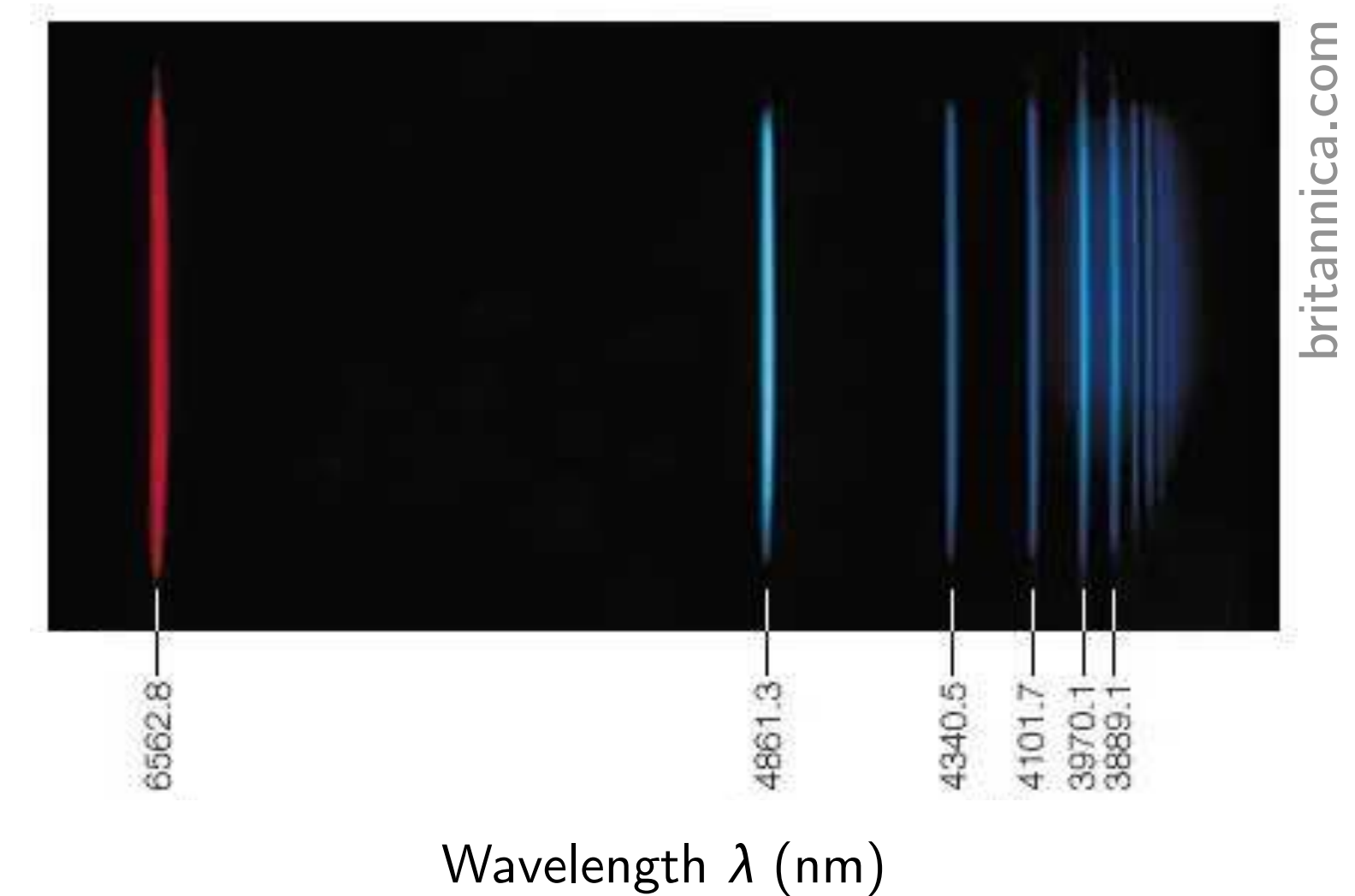


Atom

Schrödinger equation:

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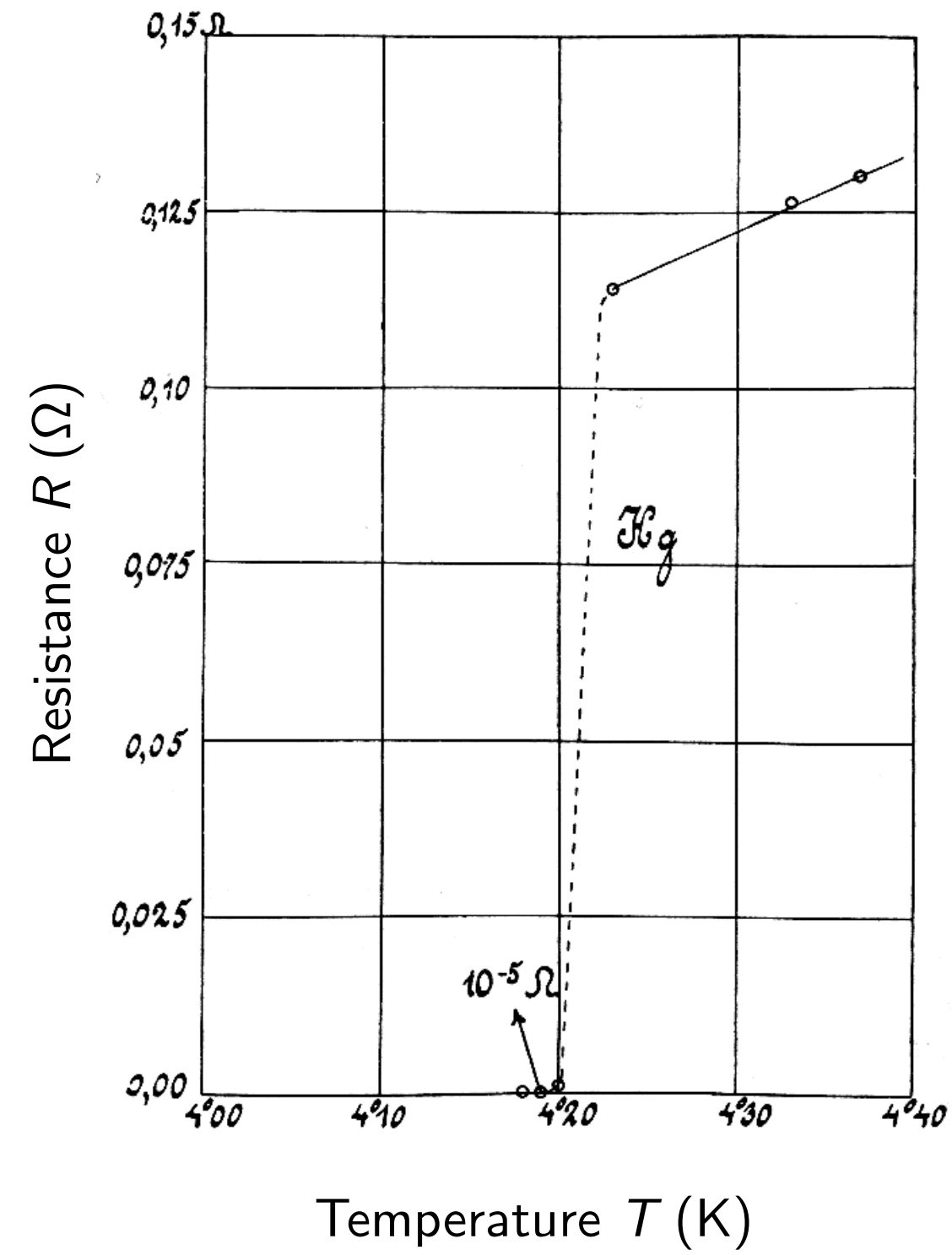
Emission spectra:



britannica.com

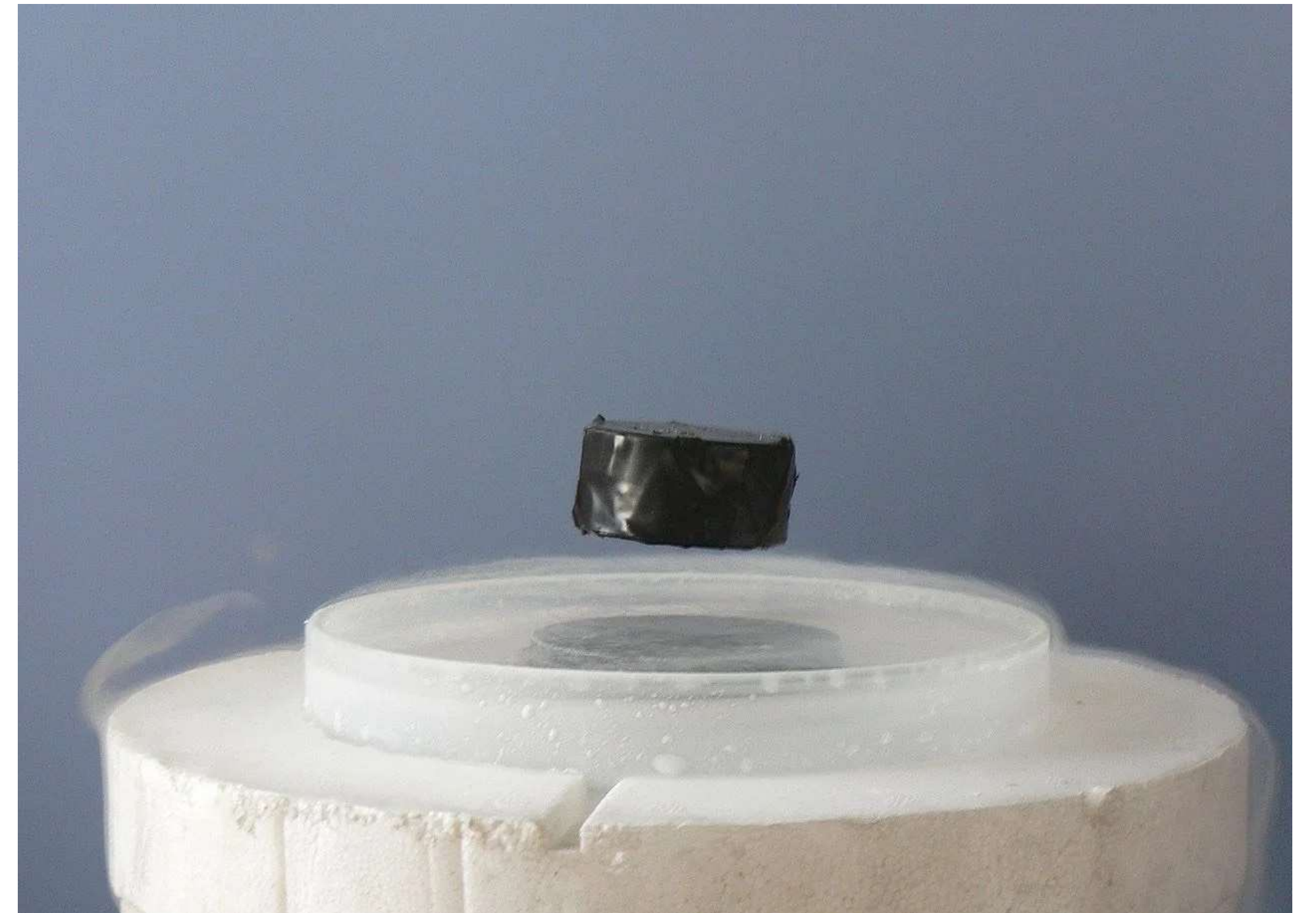
... to macrosopics ...

Example: Superconductivity



Perfect conductor

[H. Kamerlingh Onnes, KNAW Proc. 1911]



Perfect diamagnet

[Meissner & Ochsenfeld, Sci. Nat. 1933]

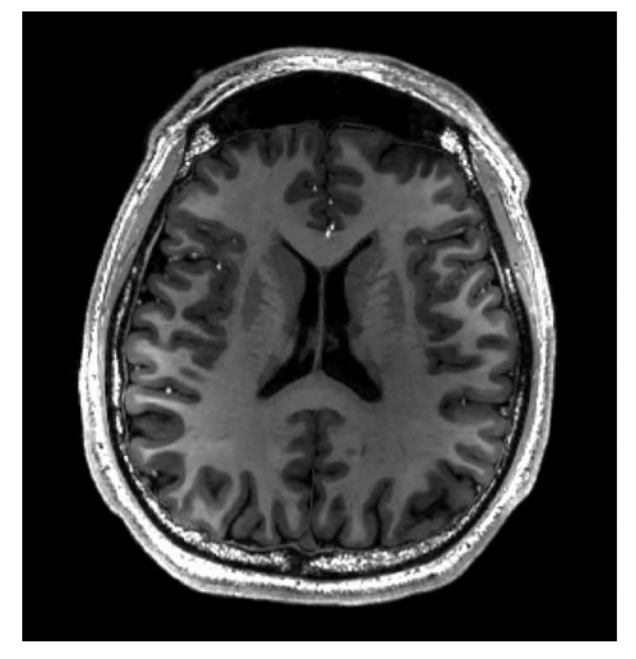
... to applications

Magnetic resonance imaging

onestophospital.co.uk



wikimedia.org



Human brain

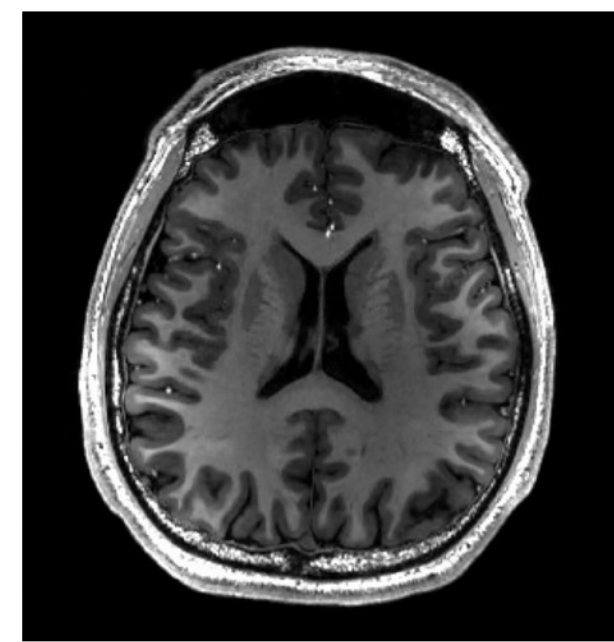
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wikimedia.org



Human brain

Superconducting power grid

nexans.com



Chicago's resilient electric grid

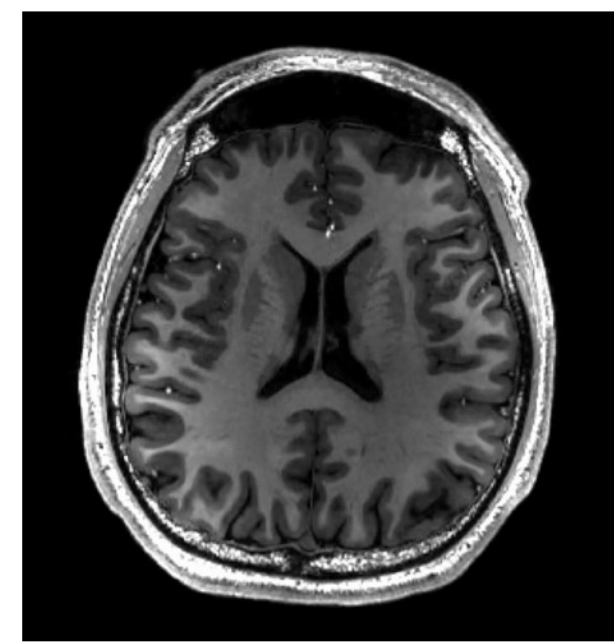
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Magnetic resonance imaging

onestophospital.co.uk



wikimedia.org



Human brain

Superconducting power grid

nexans.com



Chicago's resilient electric grid

Superconducting maglev train

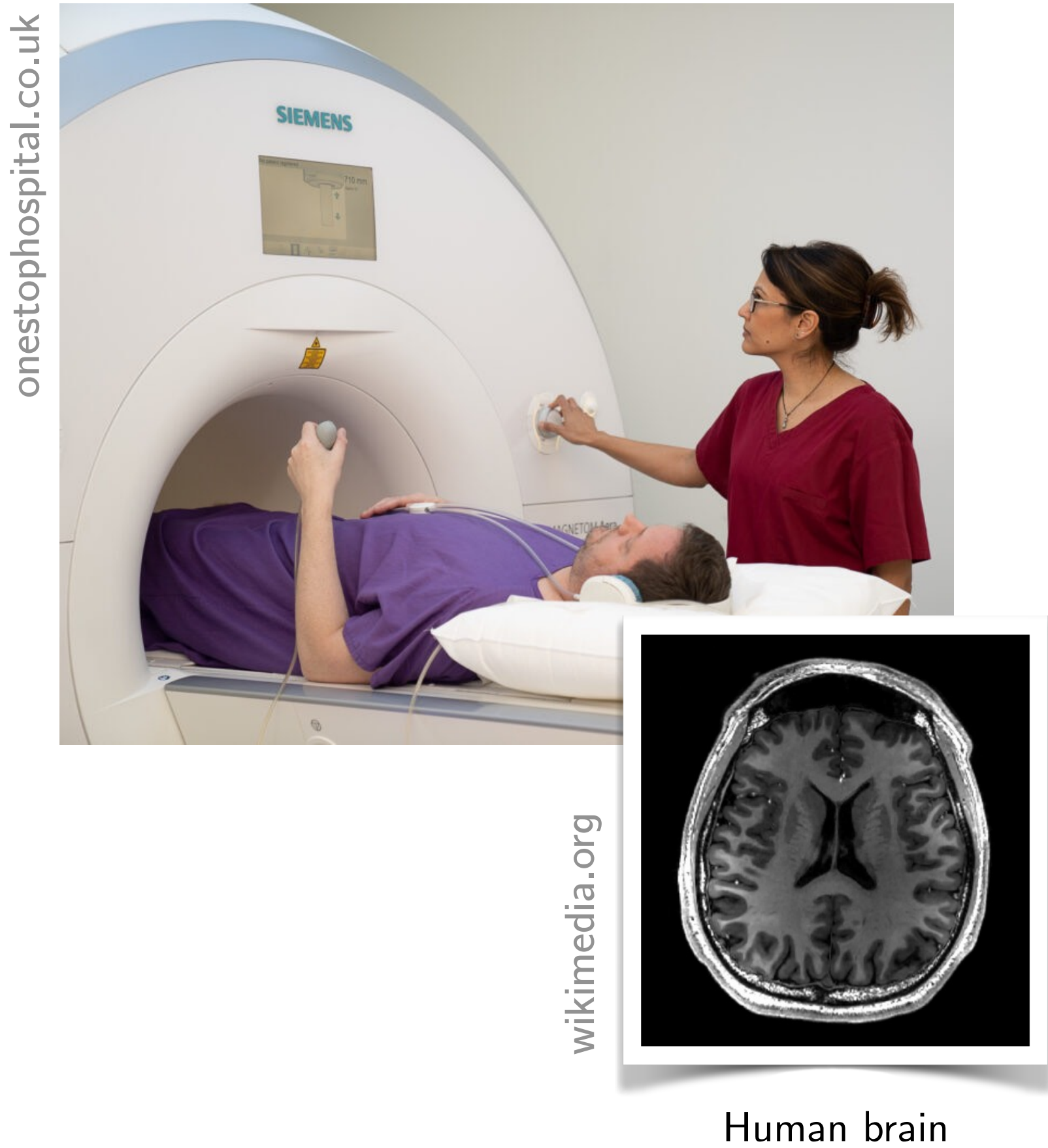
IFW Dresden



IFW's superconducting train

... to applications

Magnetic resonance imaging



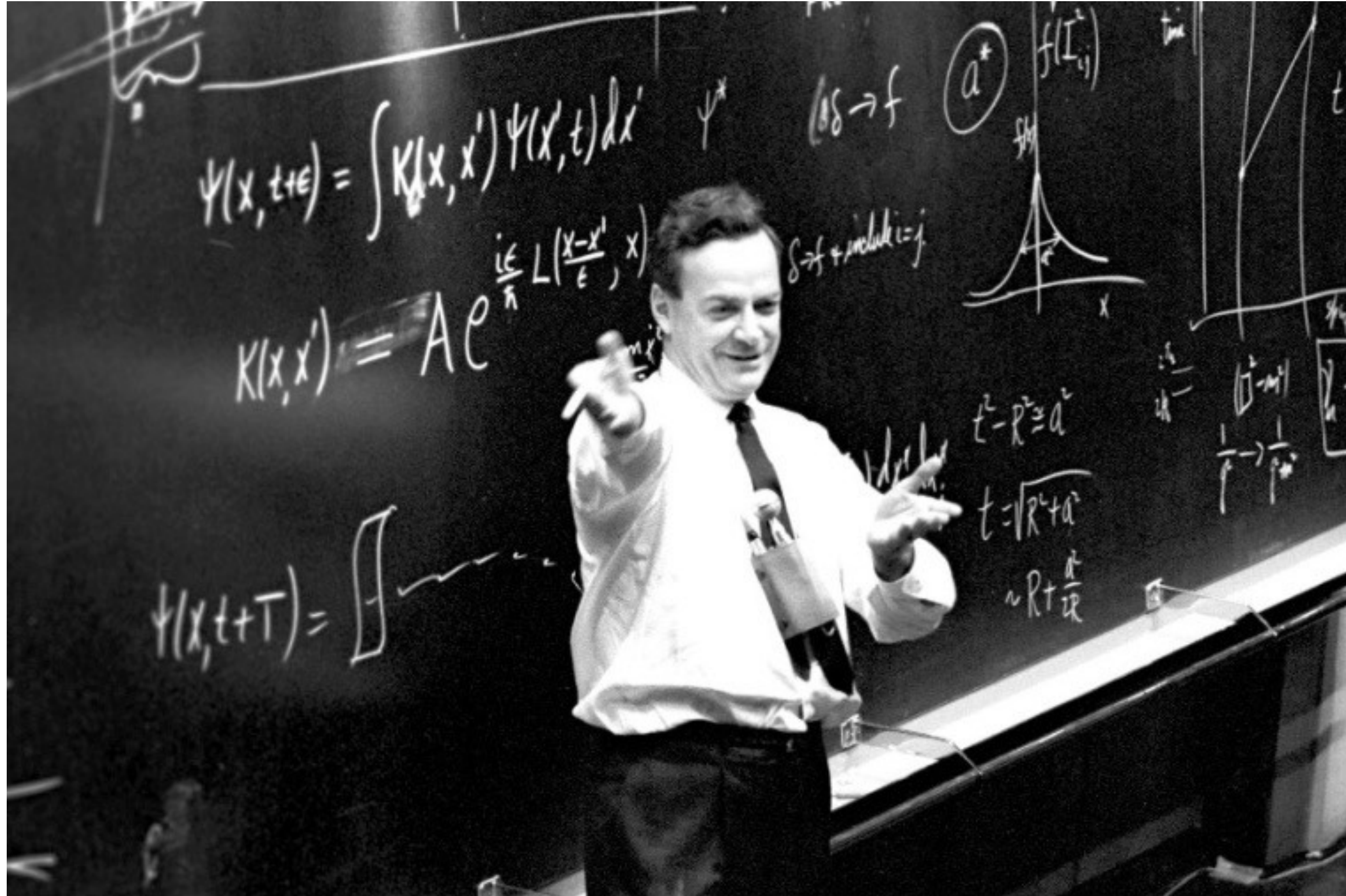
Superconducting power grid



Superconducting maglev train

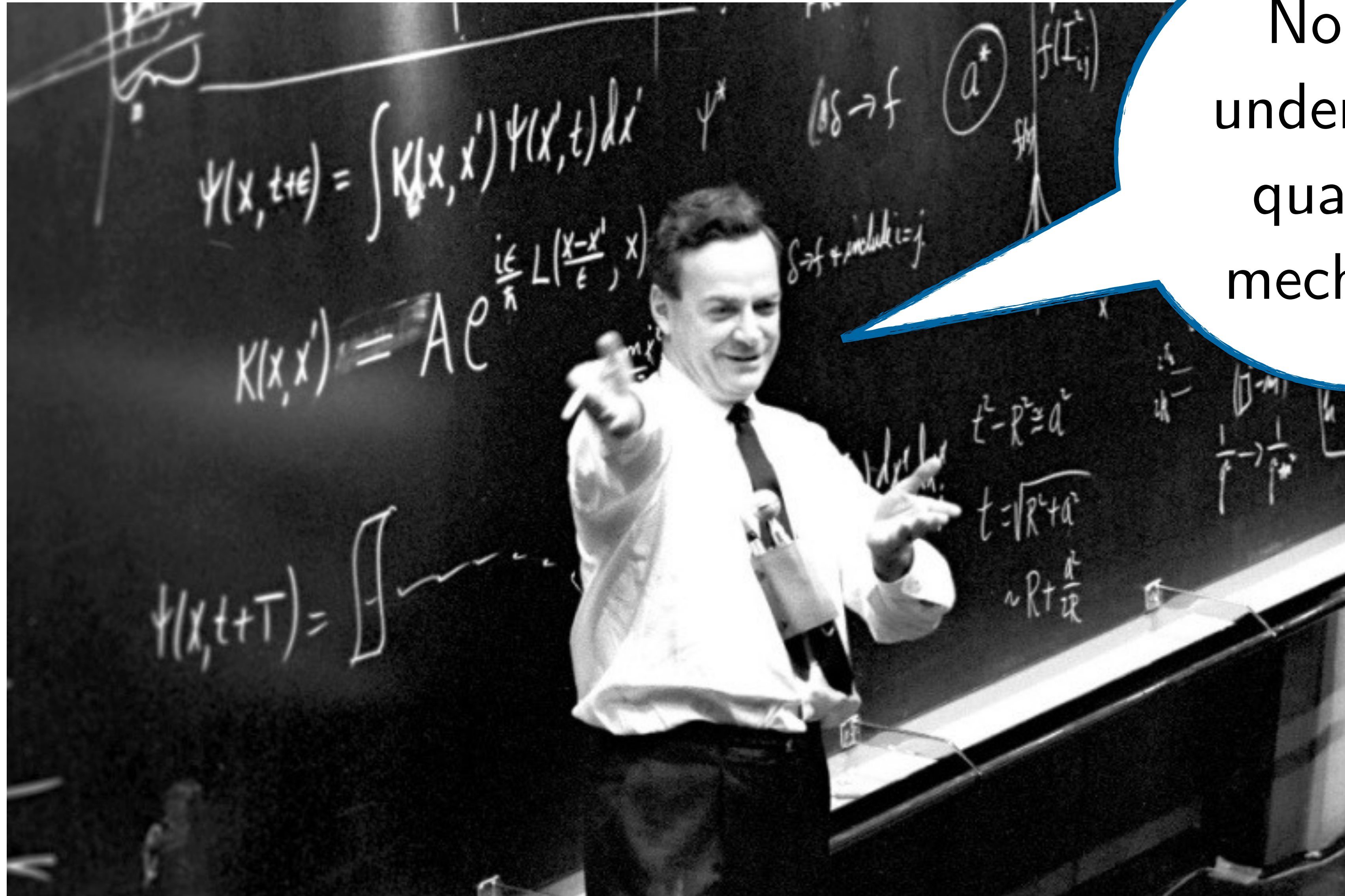


Macroscopic quantum phenomena



Richard Feynman

Macroscopic quantum phenomena

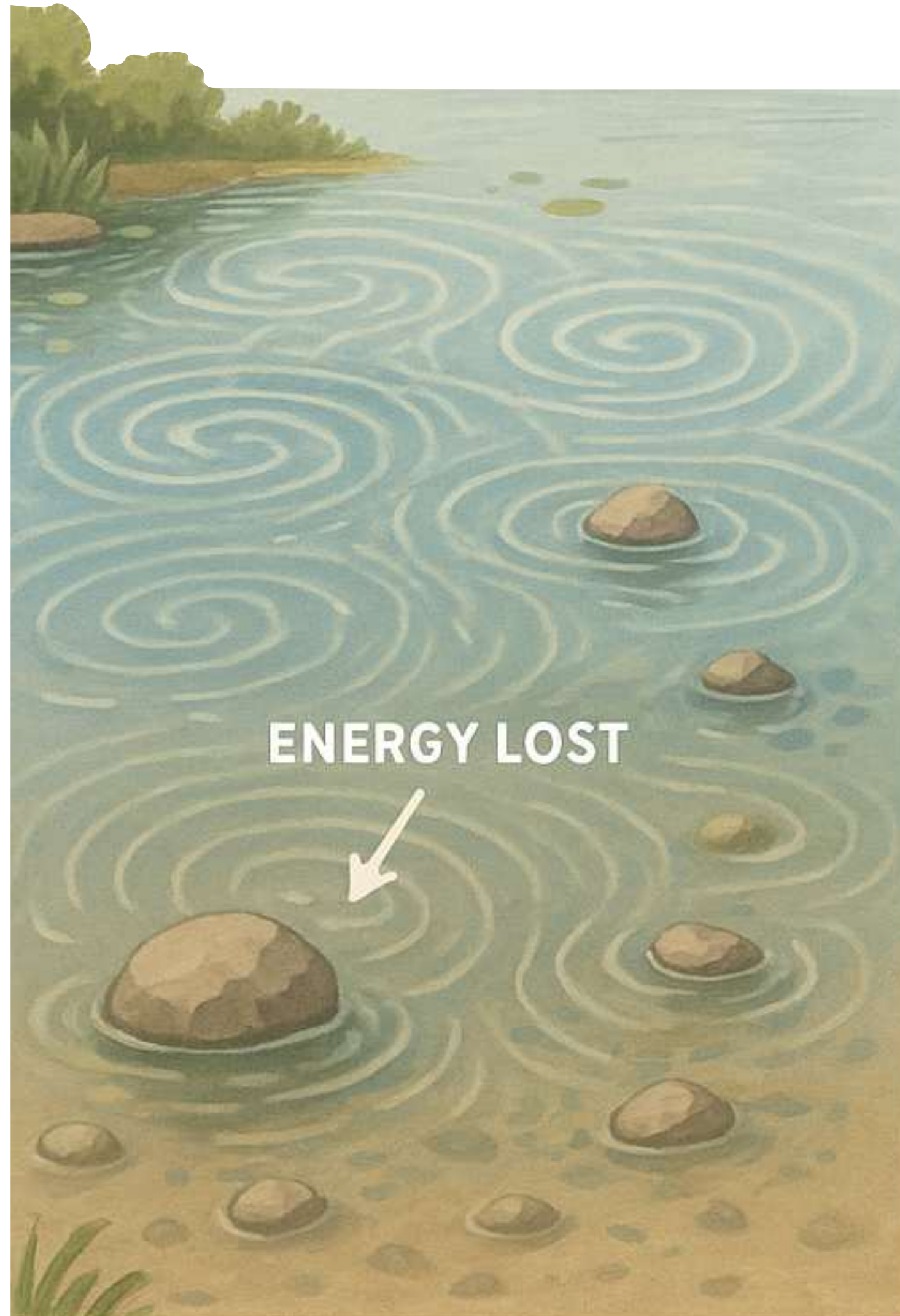


Nobody
understands
quantum
mechanics!

Richard Feynman

Macroscopic quantum phenomena

Normal metal

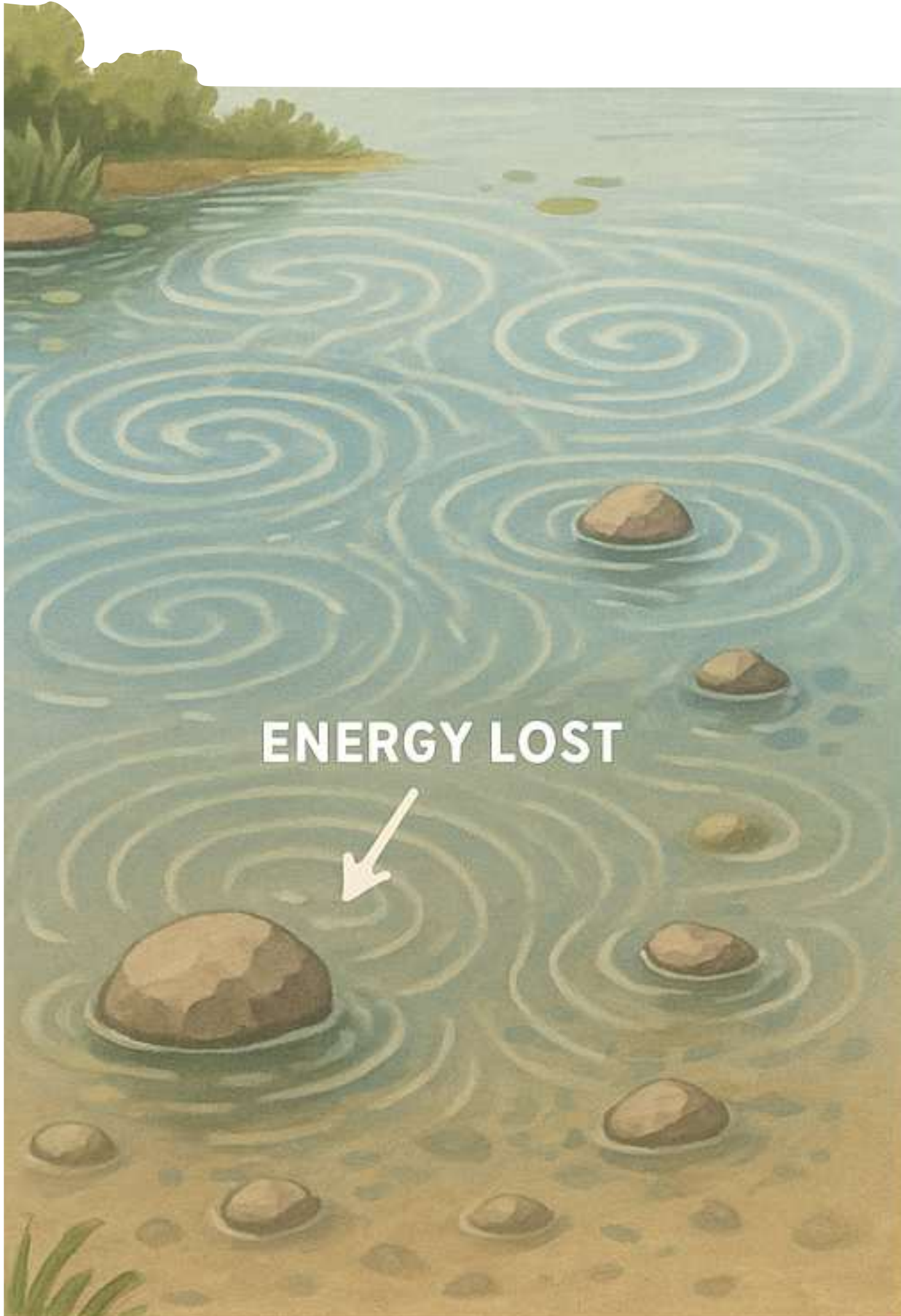


AI generated

Normal electrons

Macroscopic quantum phenomena

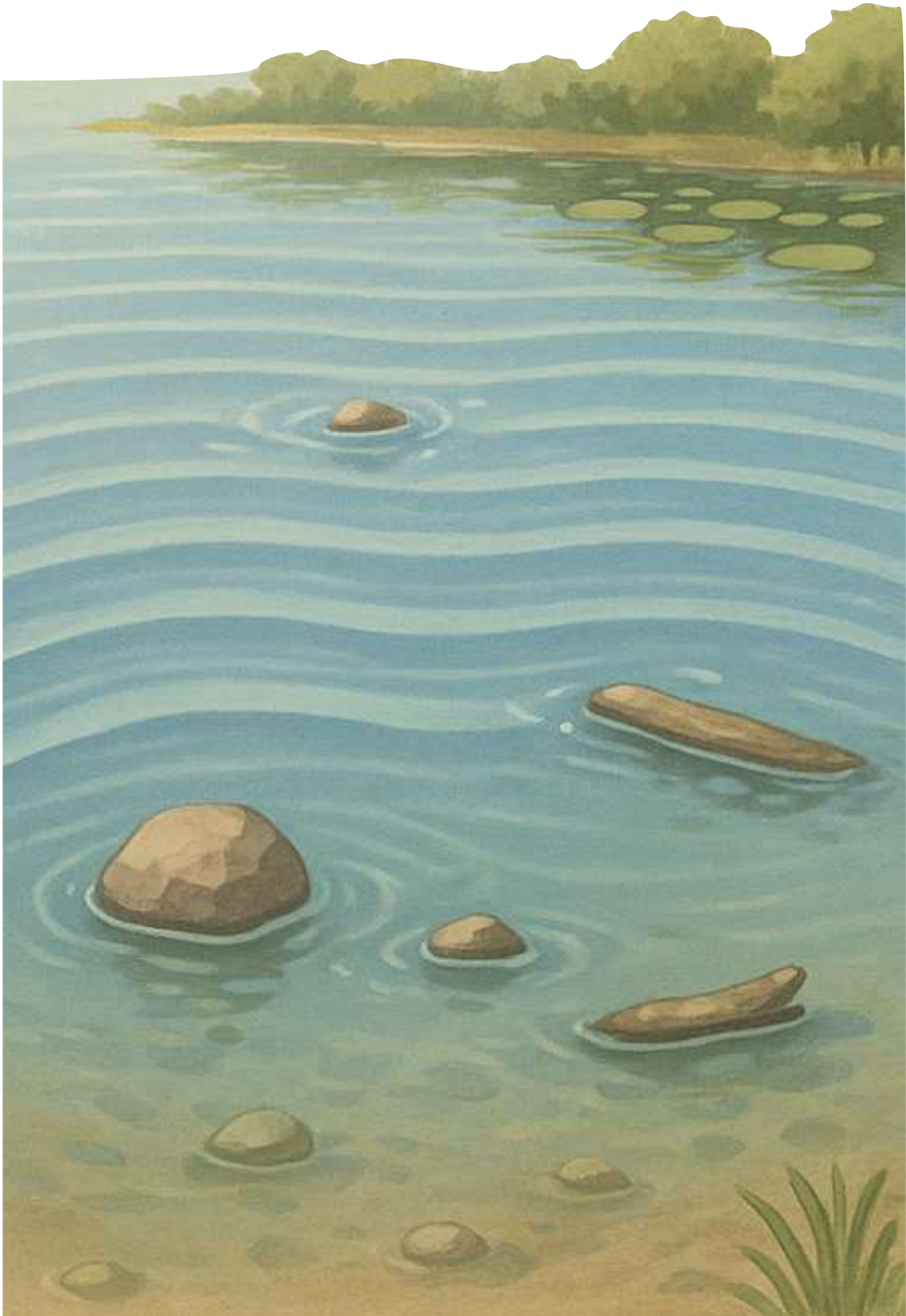
Normal metal



AI generated

Normal electrons

Superconductor



AI generated

Cooper pairs

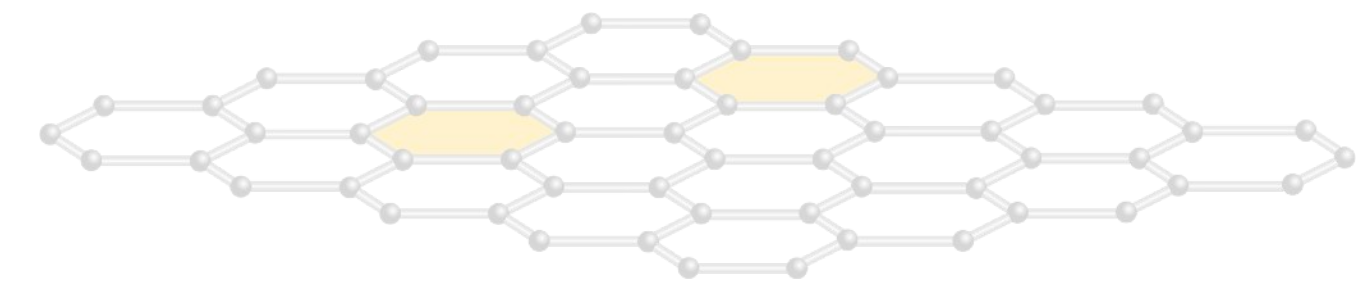
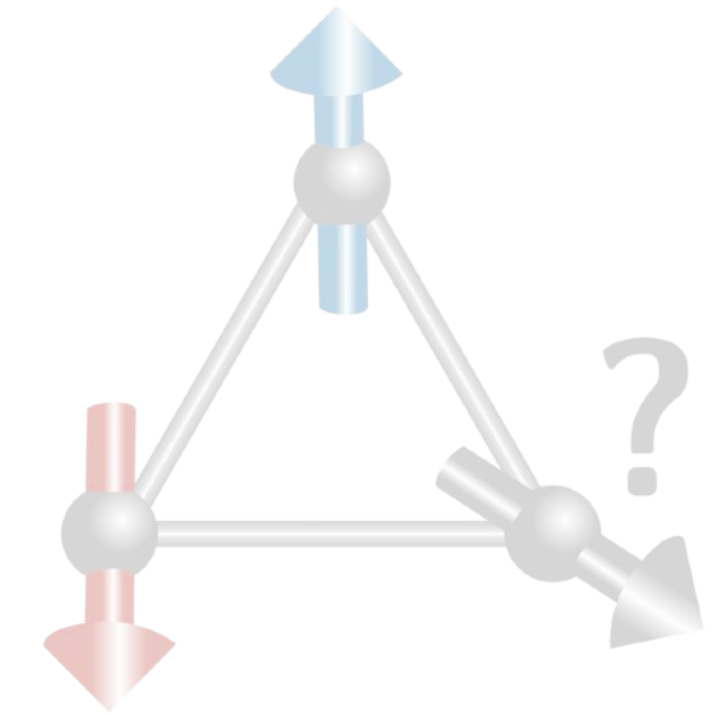
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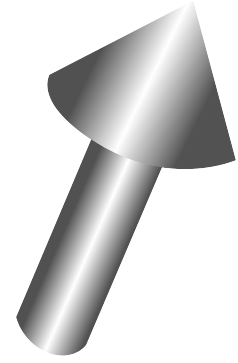
(4) Summary



Qubits

Spin 1/2:

$$|\psi\rangle = |\uparrow\rangle$$

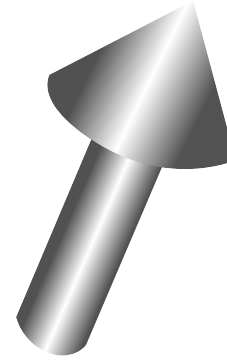


Qubits

Spin 1/2:

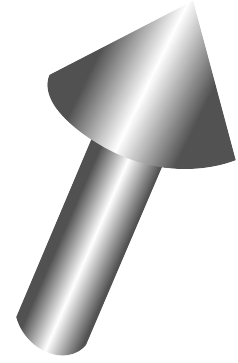
$$|\psi\rangle =$$

$$|\downarrow\rangle$$



Qubits

Spin 1/2:

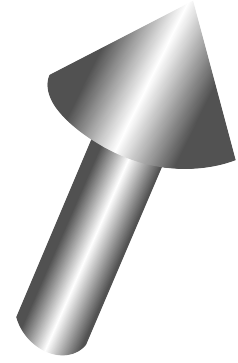


$$|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$$

Qubits

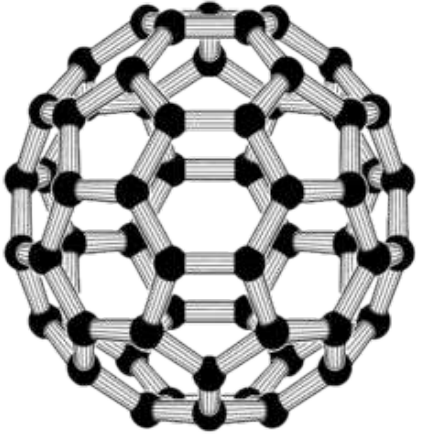
Spin 1/2:

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Spin-split quantum dot:

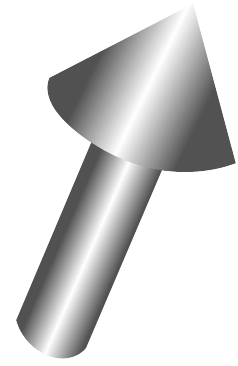
$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$



Qubits

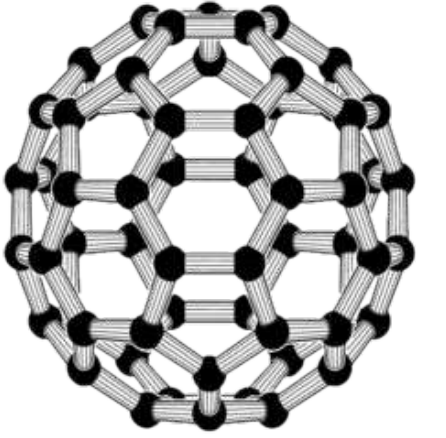
Spin 1/2:

$$\begin{aligned} |\psi\rangle &= \alpha |\uparrow\rangle + \beta |\downarrow\rangle \\ &\hat{=} \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$



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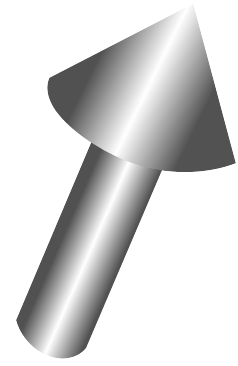


Qubits

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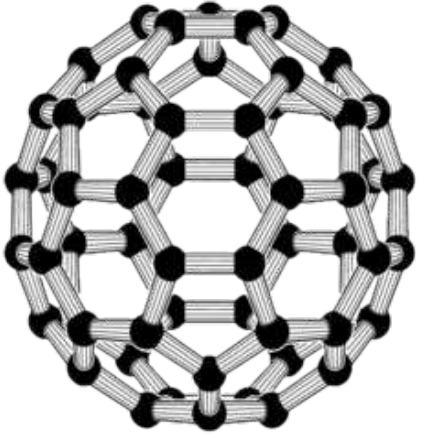
Complex numbers



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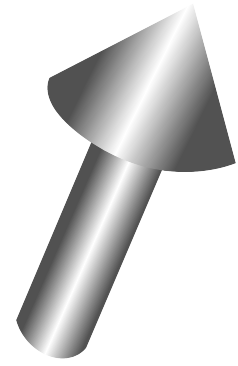
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Complex numbers



Qubits

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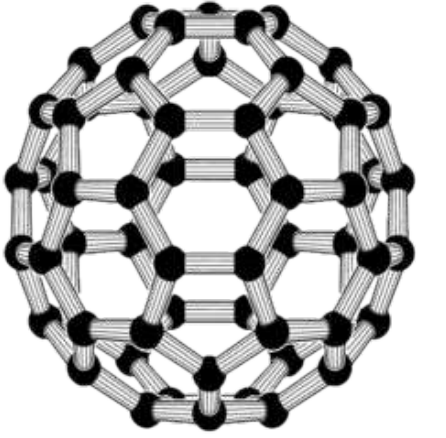
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Complex numbers

Density matrix:

$$\rho = |\psi\rangle\langle\psi| \hat{=} \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

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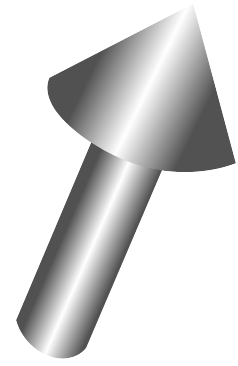


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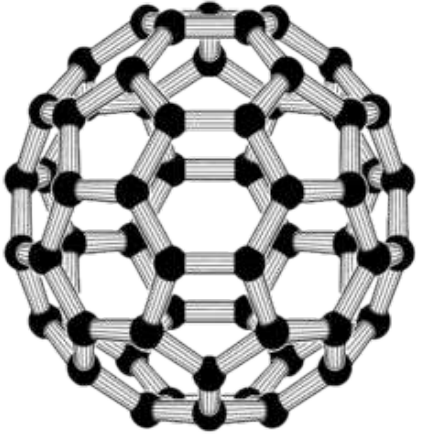
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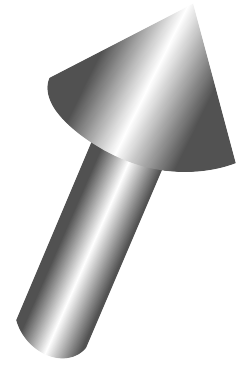
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"Coherences"

Qubits

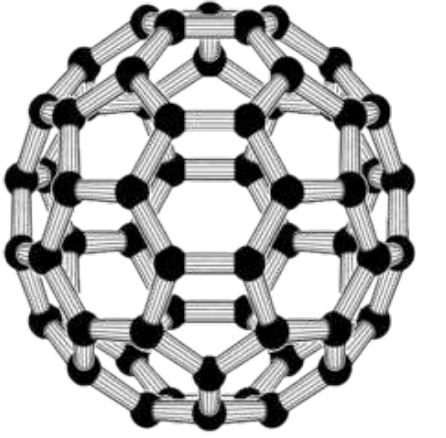
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Complex numbers

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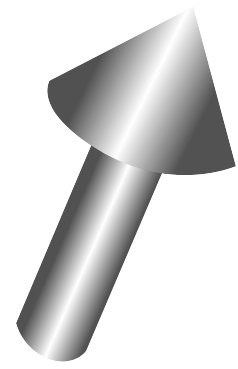
"Coherences"

Decoherence:

$$\rho \rightarrow \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

Qubits

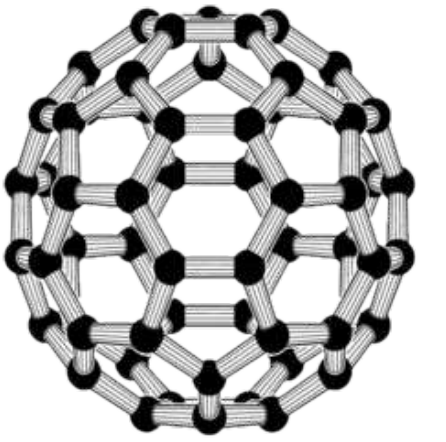
Spin 1/2:



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Complex numbers

Spin-split quantum dot:



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"Coherences"

Decoherence:

$$\rho \rightarrow \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$



Classical bit

Qubits

Spin 1/2:

$$|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$$

$$\hat{=} \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Density matrix:

$$\rho \rightarrow \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Decoherence:

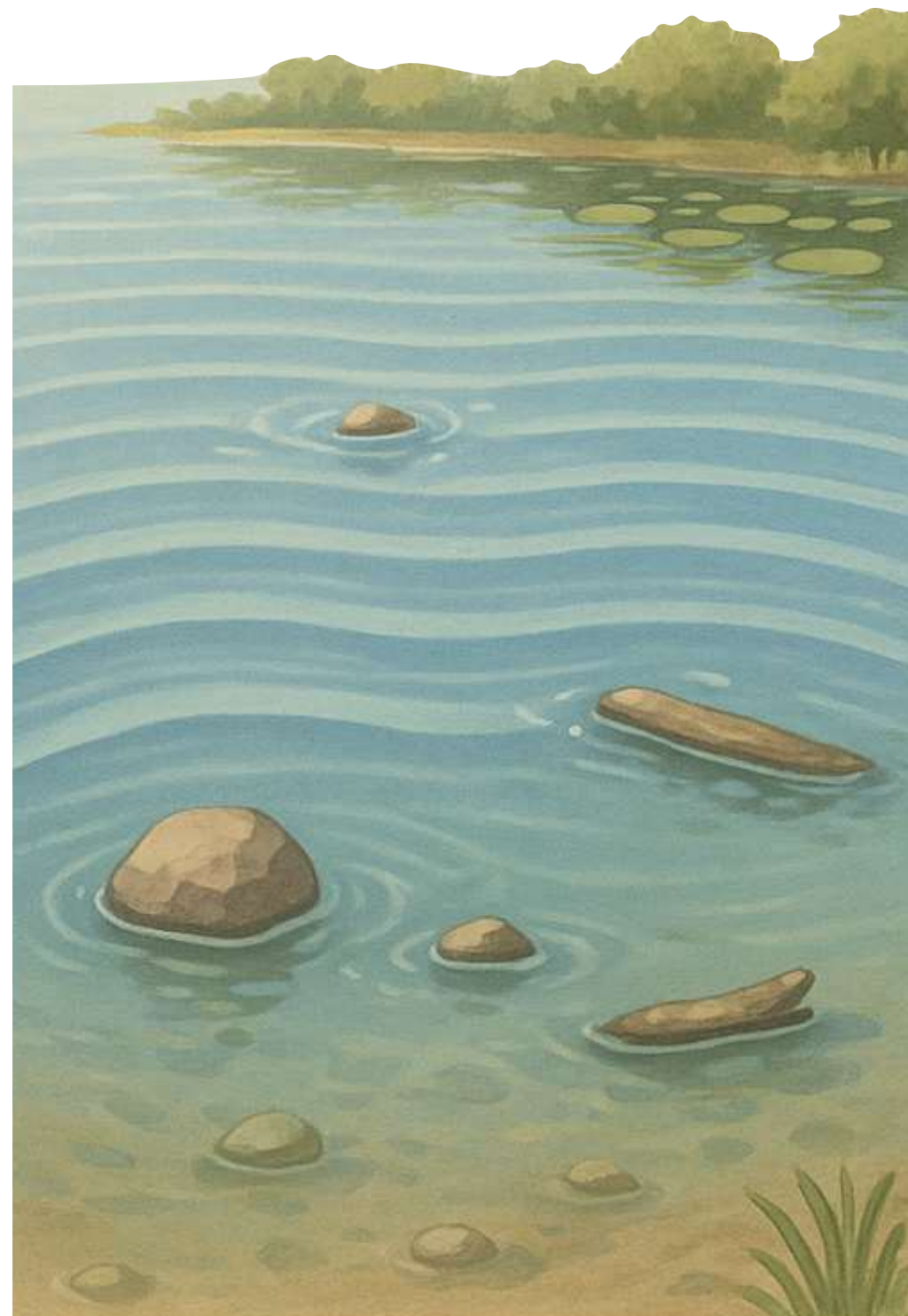
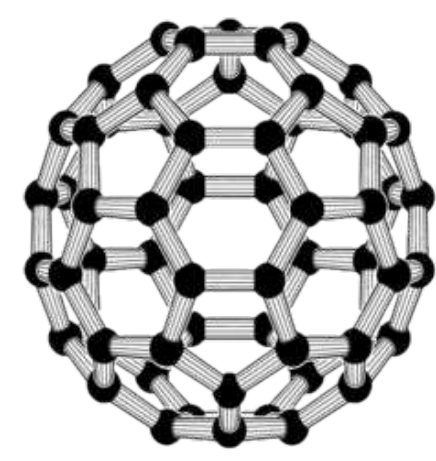
Classical bit

Coherent quantum wave?

quantum dot:

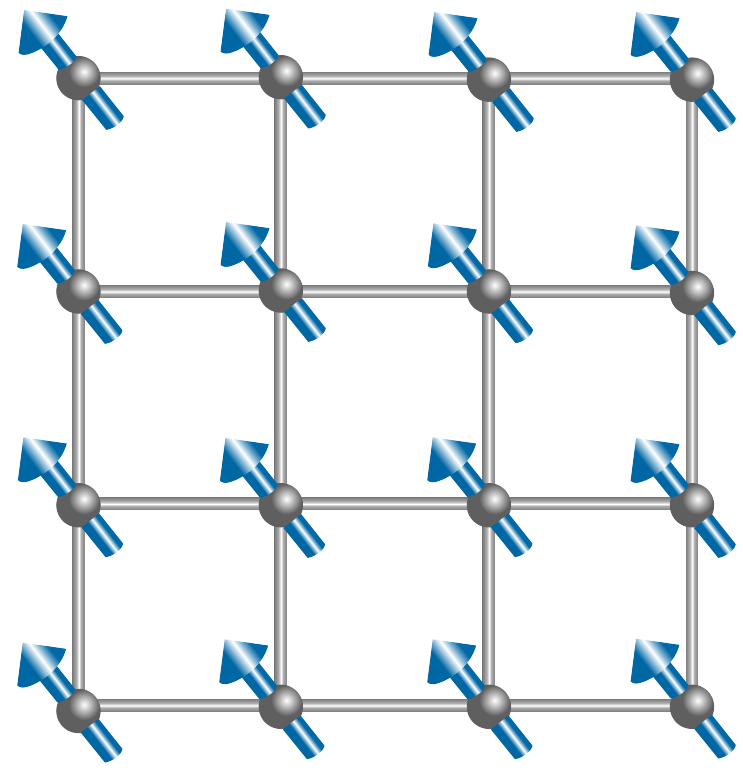
$$= \alpha |0\rangle + \beta |1\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



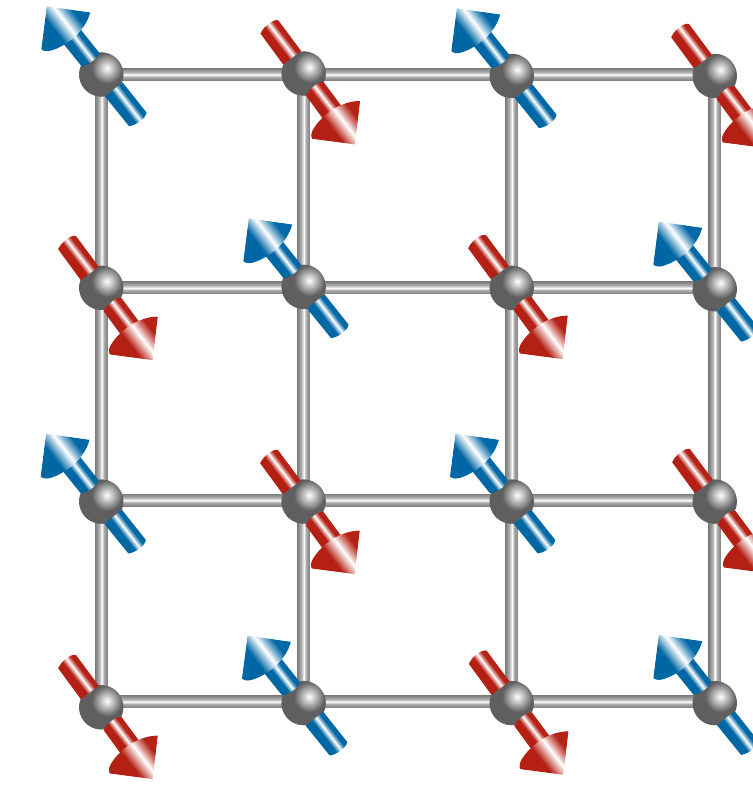
Possible states of matter?

Symmetry classification:



Ferromagnet

Spin: $SU(2) \rightarrow U(1)$



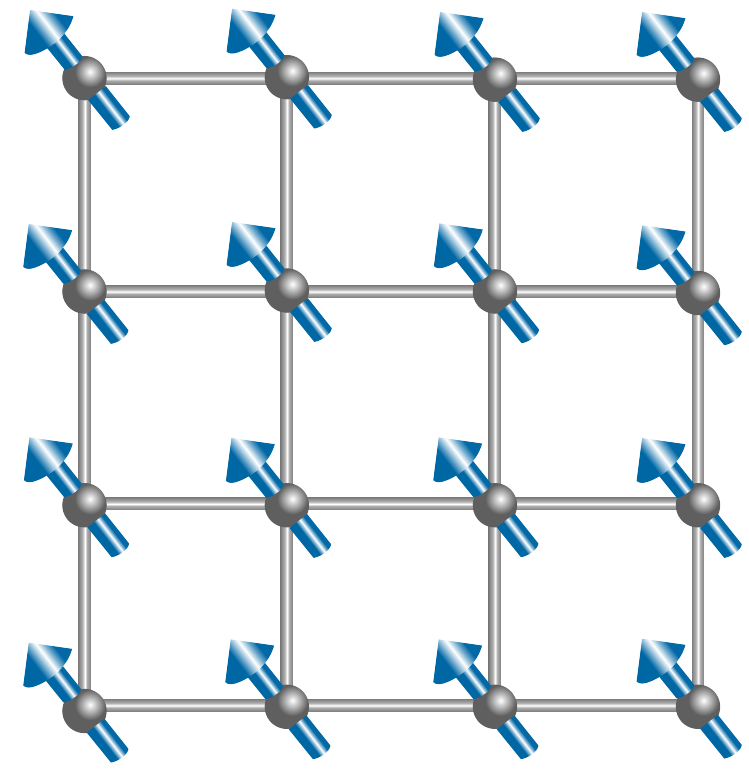
Antiferromagnet

Spin: $SU(2) \rightarrow U(1)$

Lattice: $T_a \rightarrow T_{2a}$

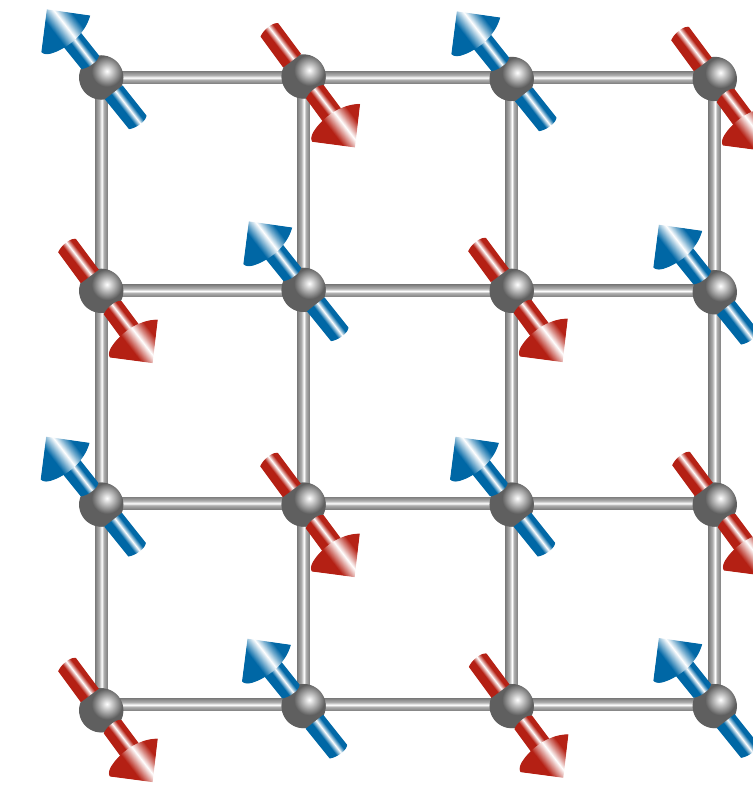
Possible states of matter?

Symmetry classification:



Ferromagnet

Spin: $SU(2) \rightarrow U(1)$

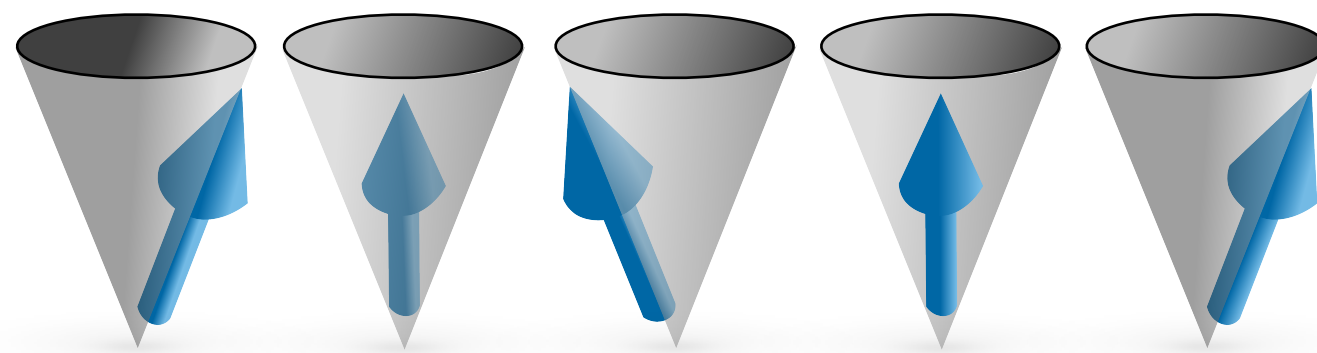


Antiferromagnet

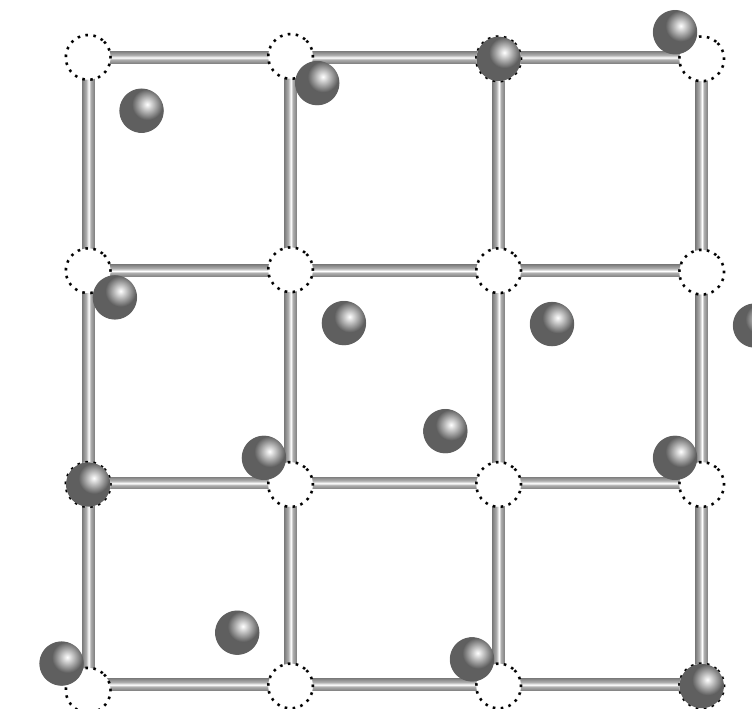
Spin: $SU(2) \rightarrow U(1)$

Lattice: $T_a \rightarrow T_{2a}$

Effective excitations: Quasiparticles



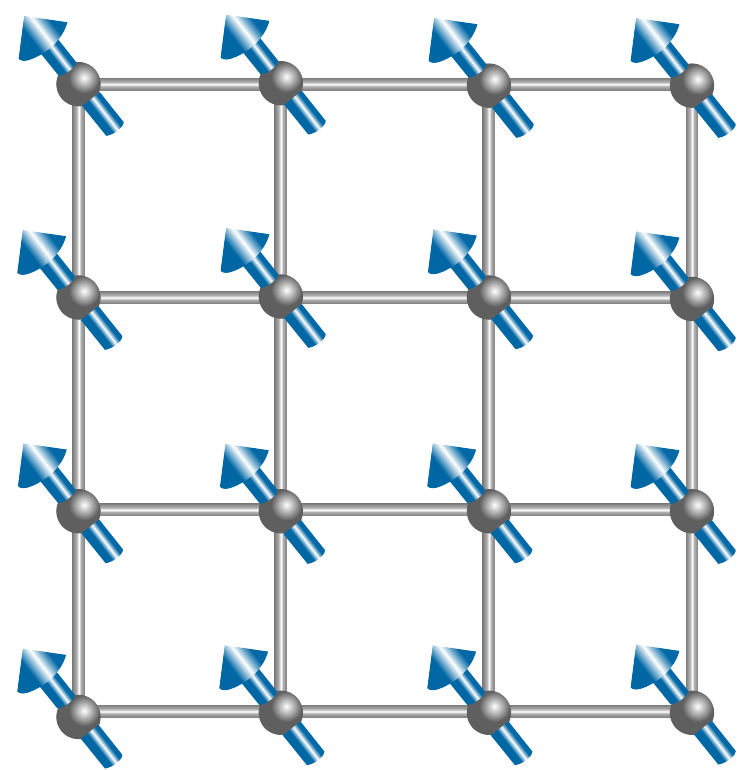
Spin: Magnons



Lattice: Phonons

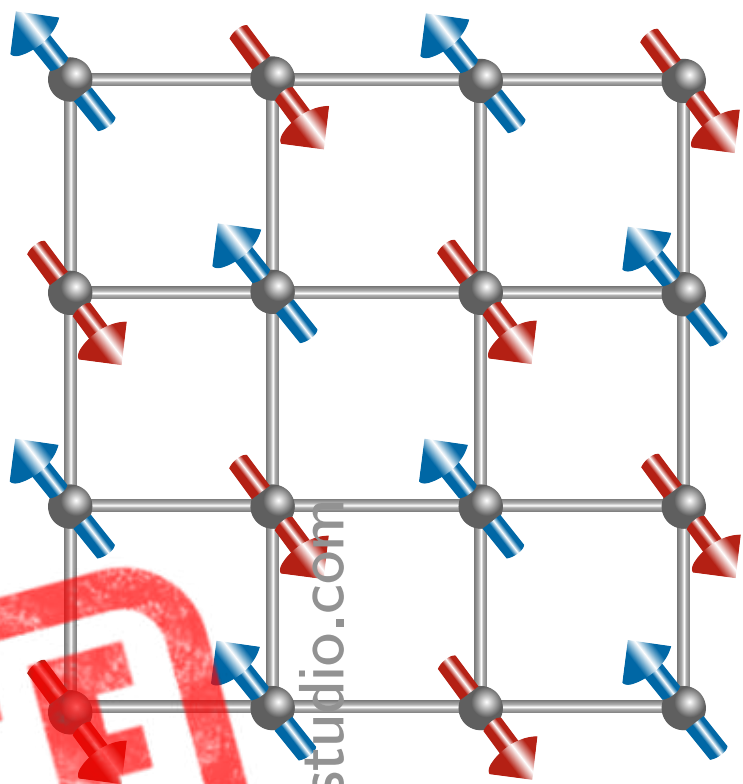
Possible states of matter?

Symmetry classification:



Ferromagnet

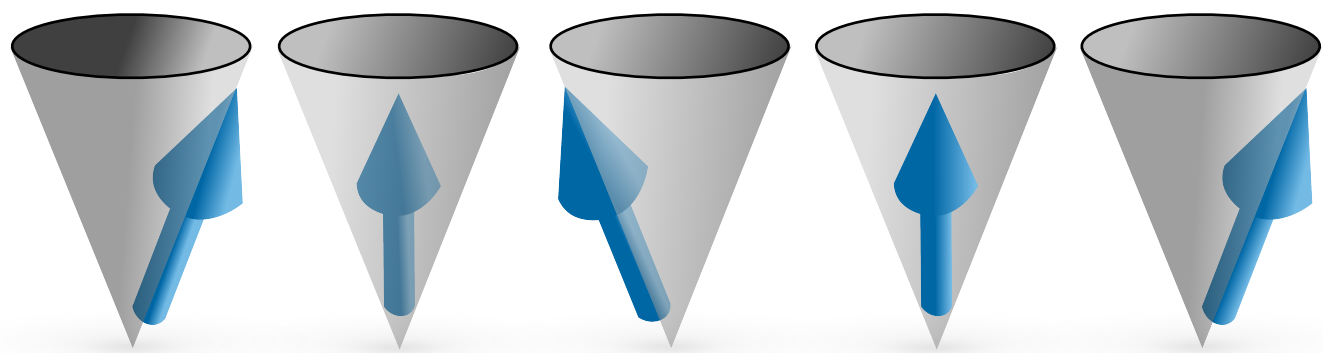
Spin: $SU(2) \rightarrow U(1)$



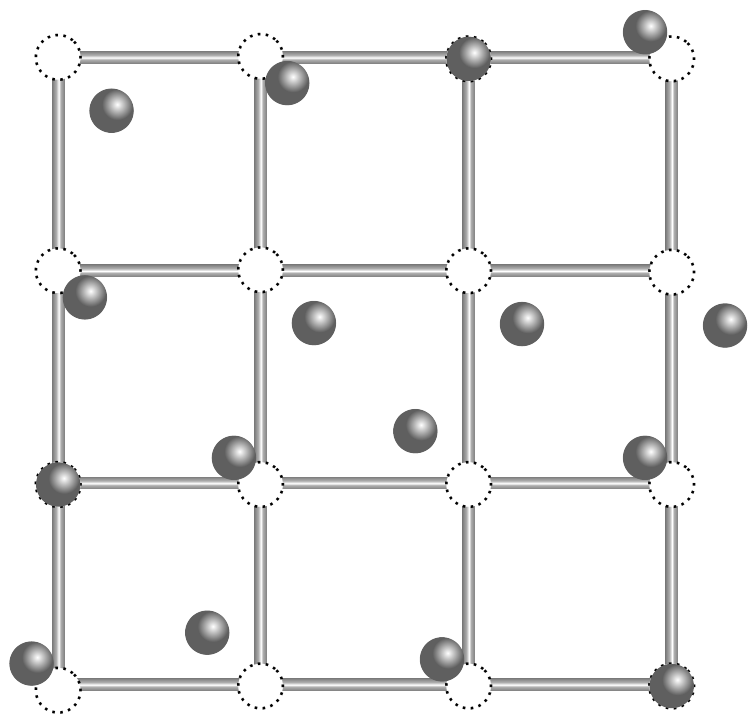
Antiferromagnet

Spin: $SU(2) \rightarrow U(1)$
Lattice: $T_a \rightarrow T_{2a}$

Effective excitations: Quasiparticles



Spin: Magnons



Lattice: Phonons

INCOMPLETE

Ingredient #1: Topology

Topological insulator:

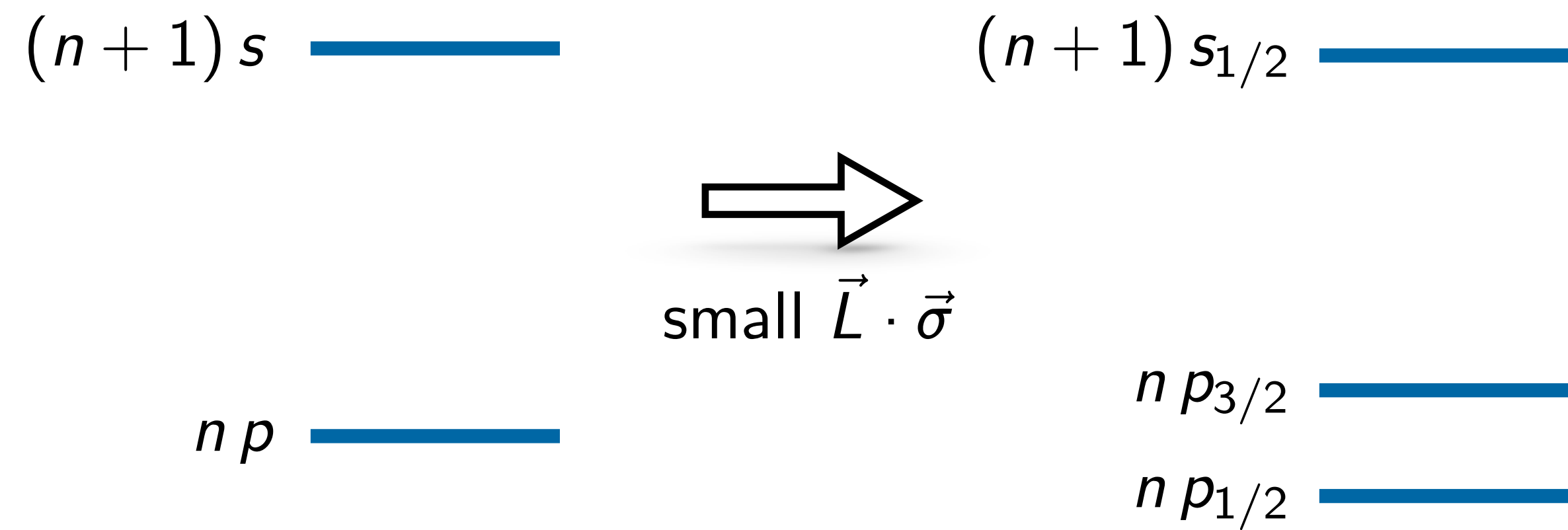


Ingredient #1: Topology

Topological insulator:



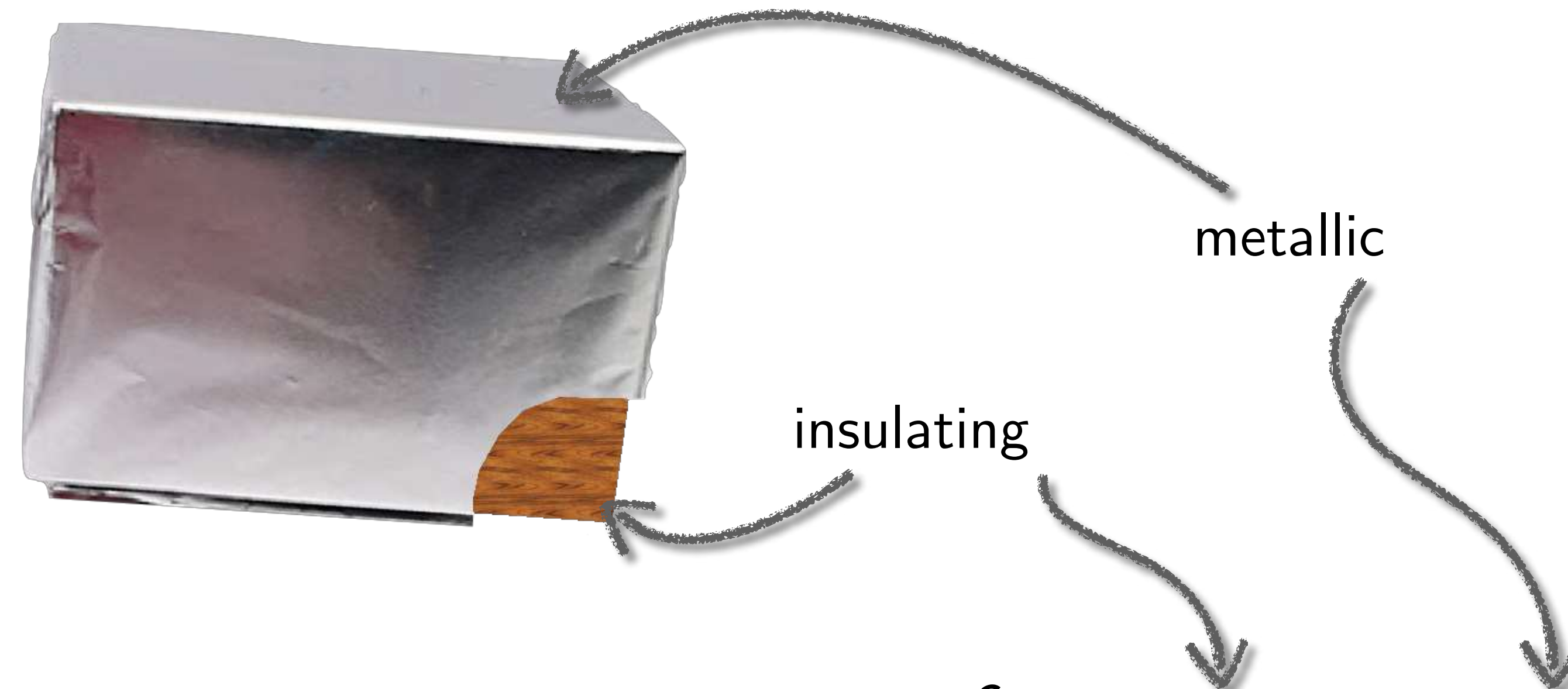
Band inversion:



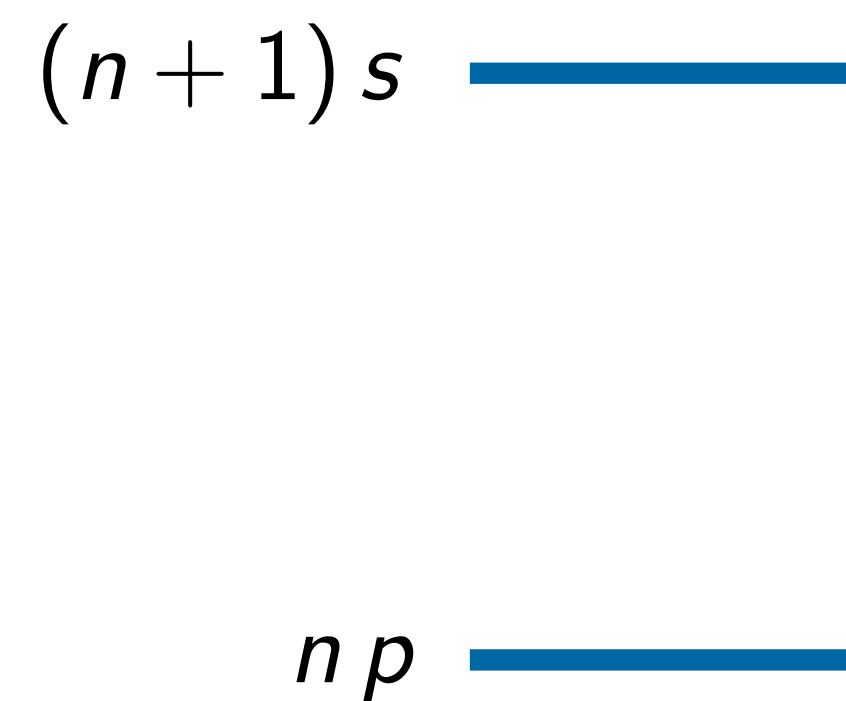
[Kane, Mele, PRL '05]
[König *et al.*, Science '07]

Ingredient #1: Topology

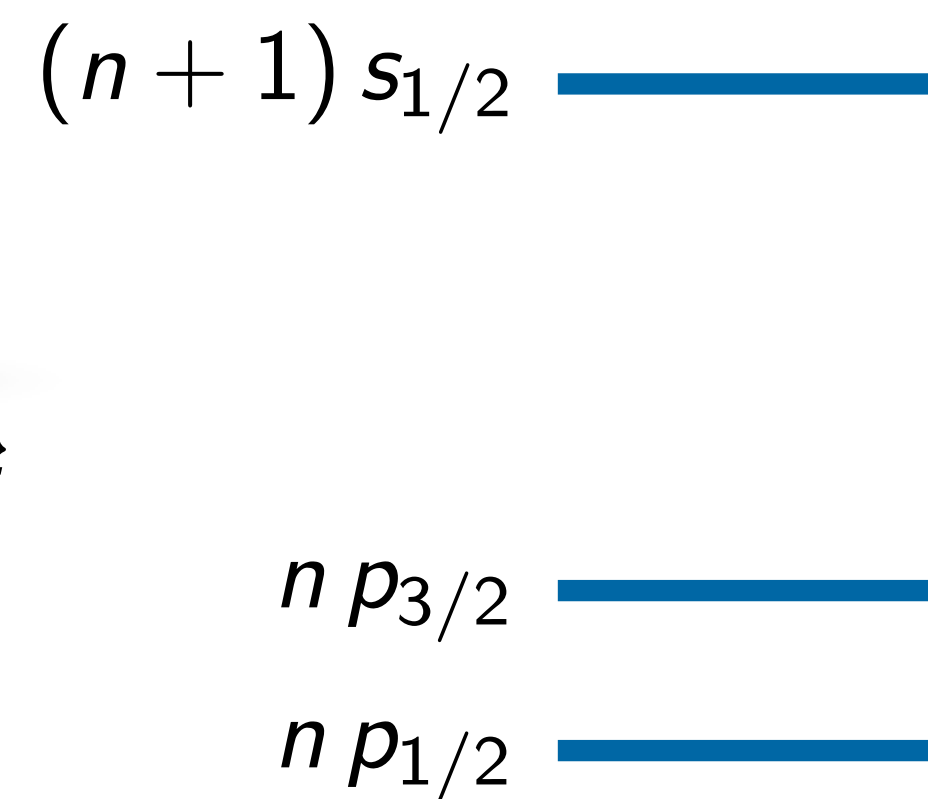
Topological insulator:



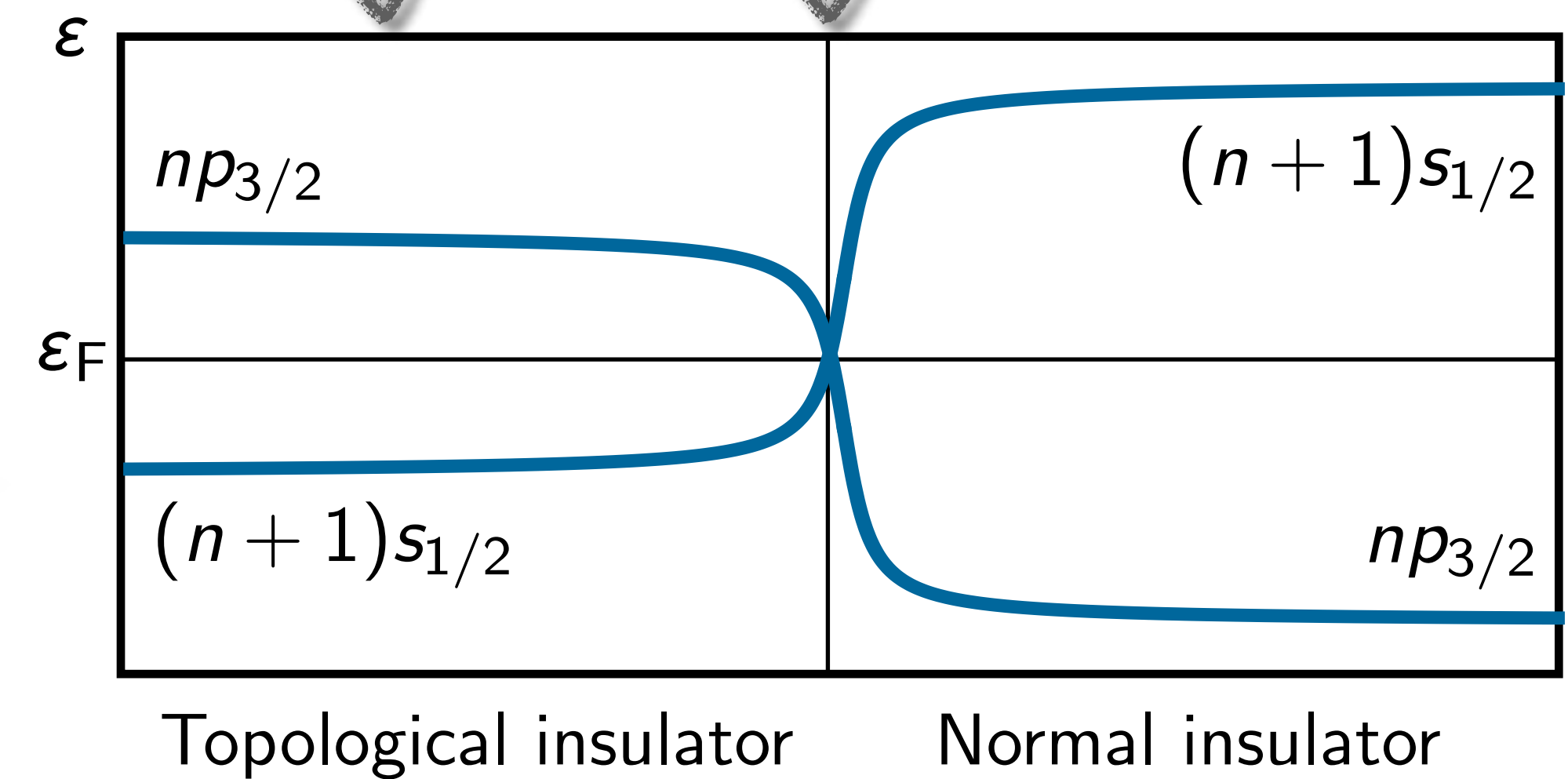
Band inversion:



small $\vec{L} \cdot \vec{\sigma}$



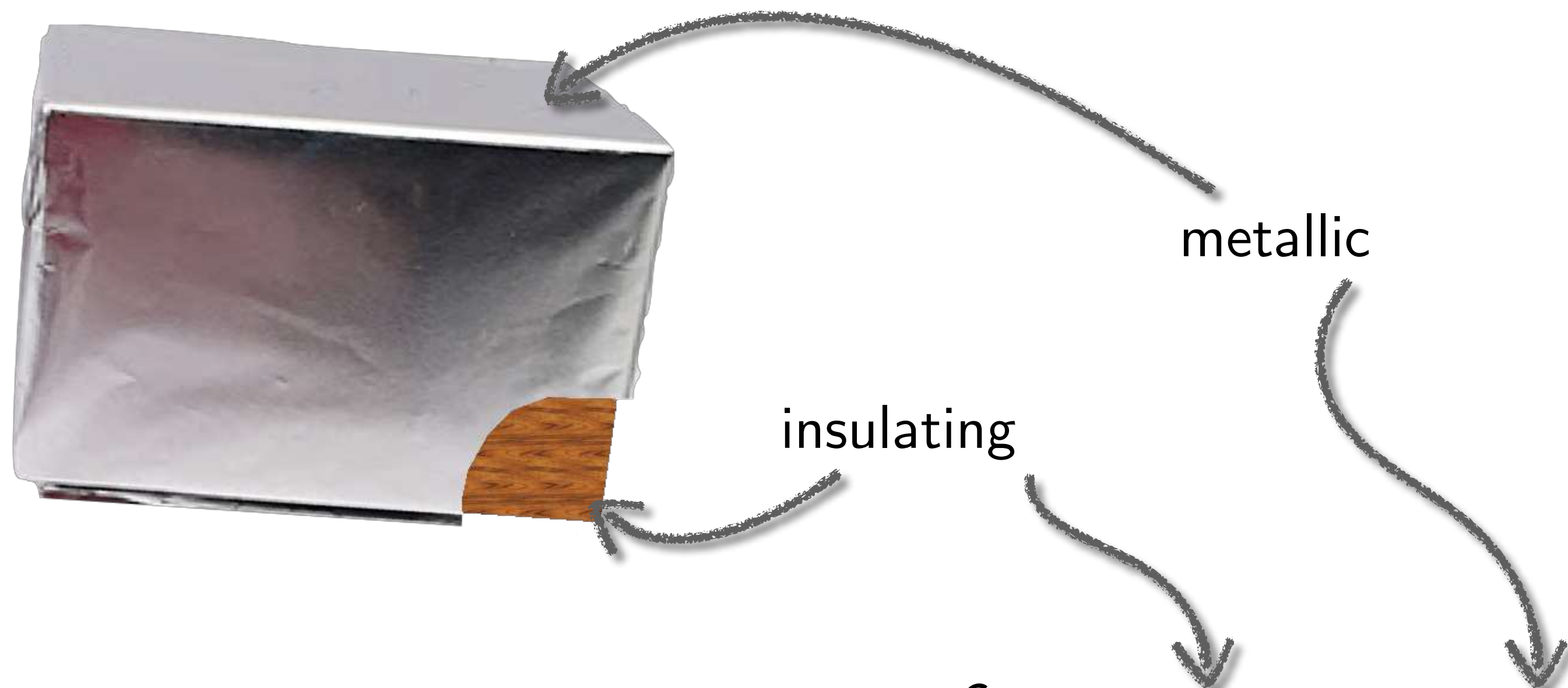
large $\vec{L} \cdot \vec{\sigma}$



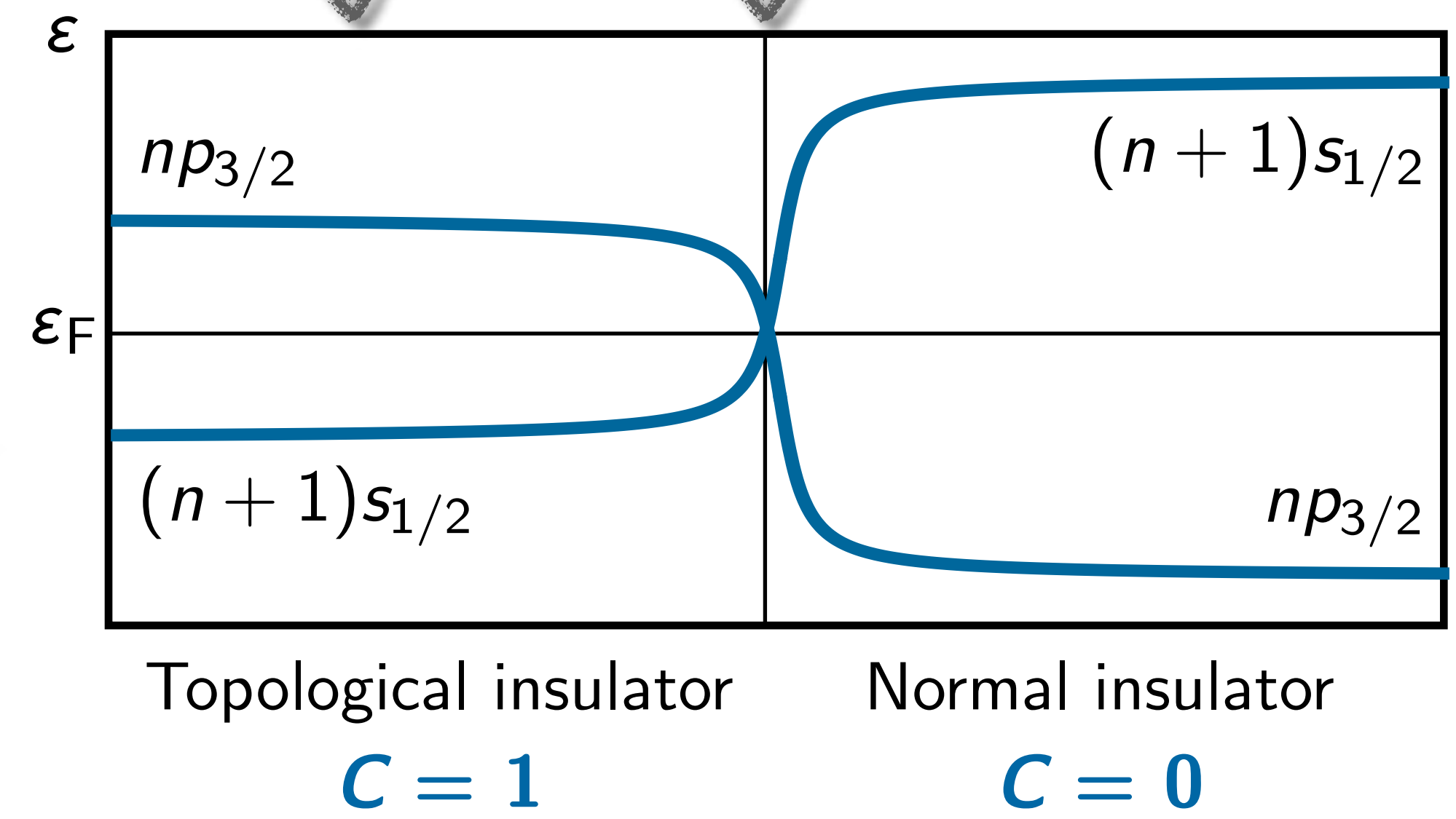
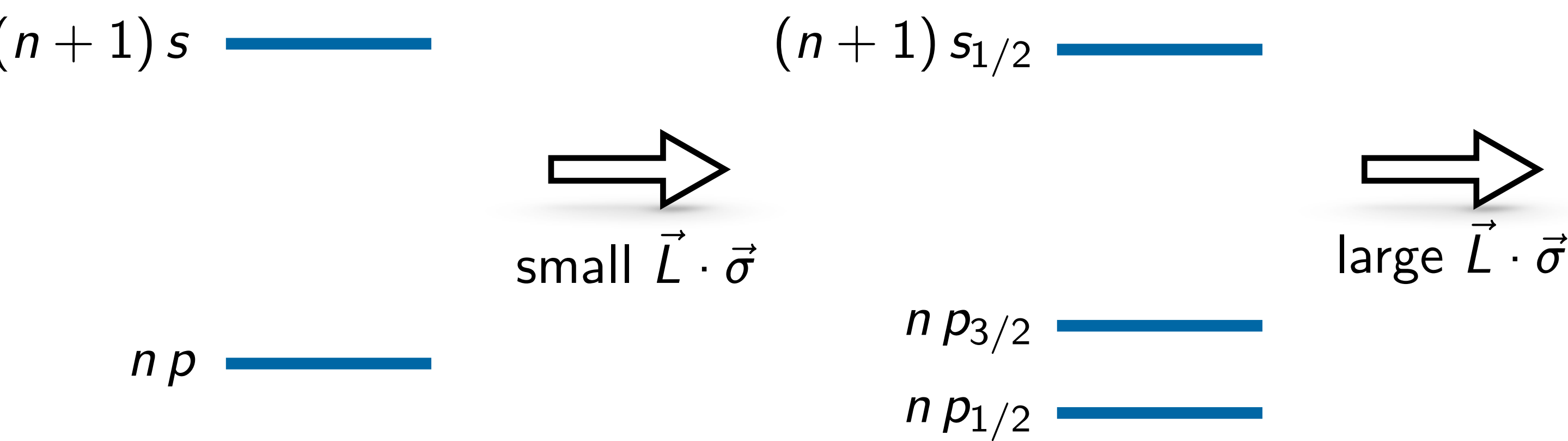
[Kane, Mele, PRL '05]
[König *et al.*, Science '07]

Ingredient #1: Topology

Topological insulator:



Band inversion:

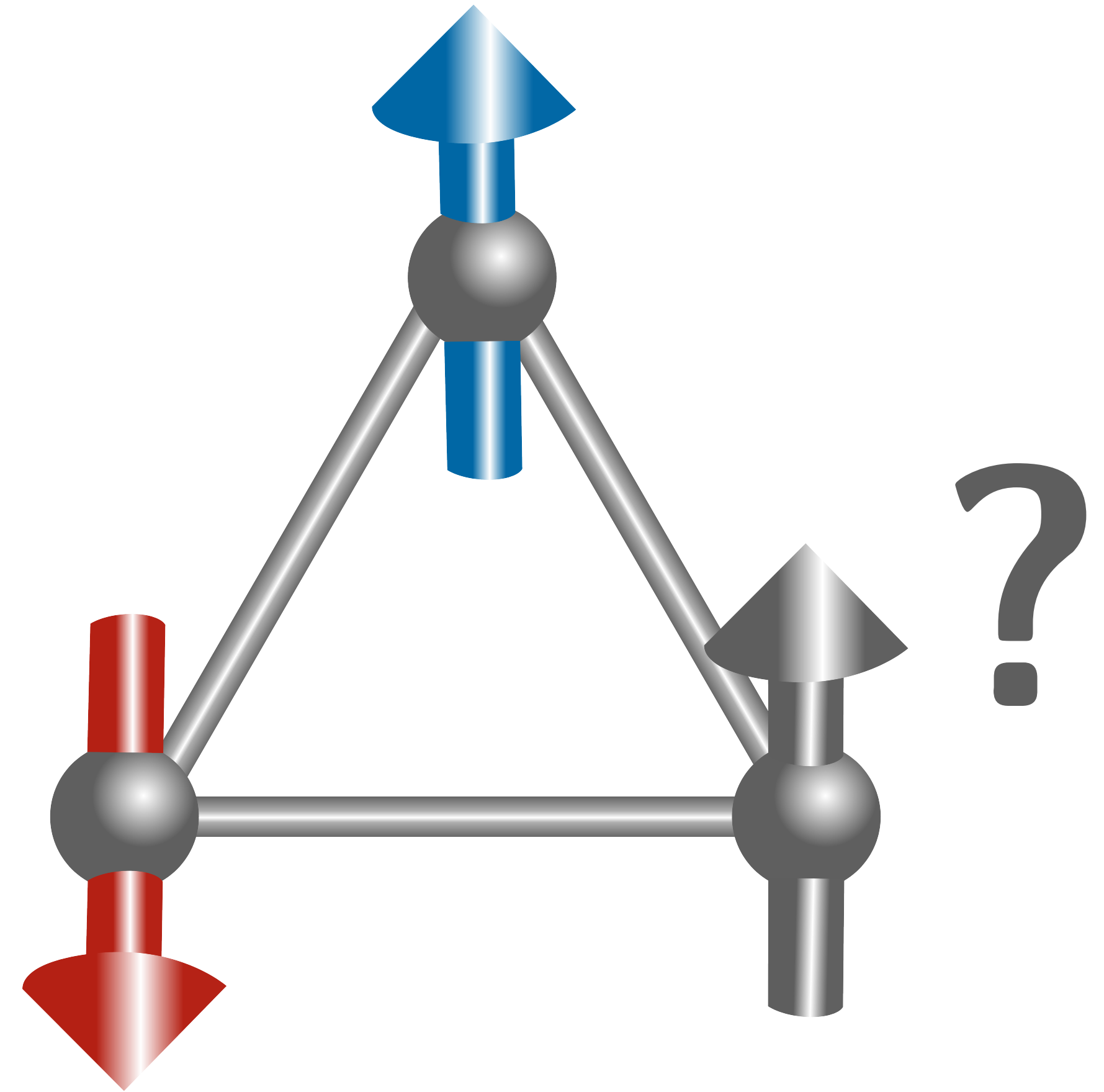


[Kane, Mele, PRL '05]
[König et al., Science '07]

Ingredient #2: Frustration

Antiferromagnetic interaction:

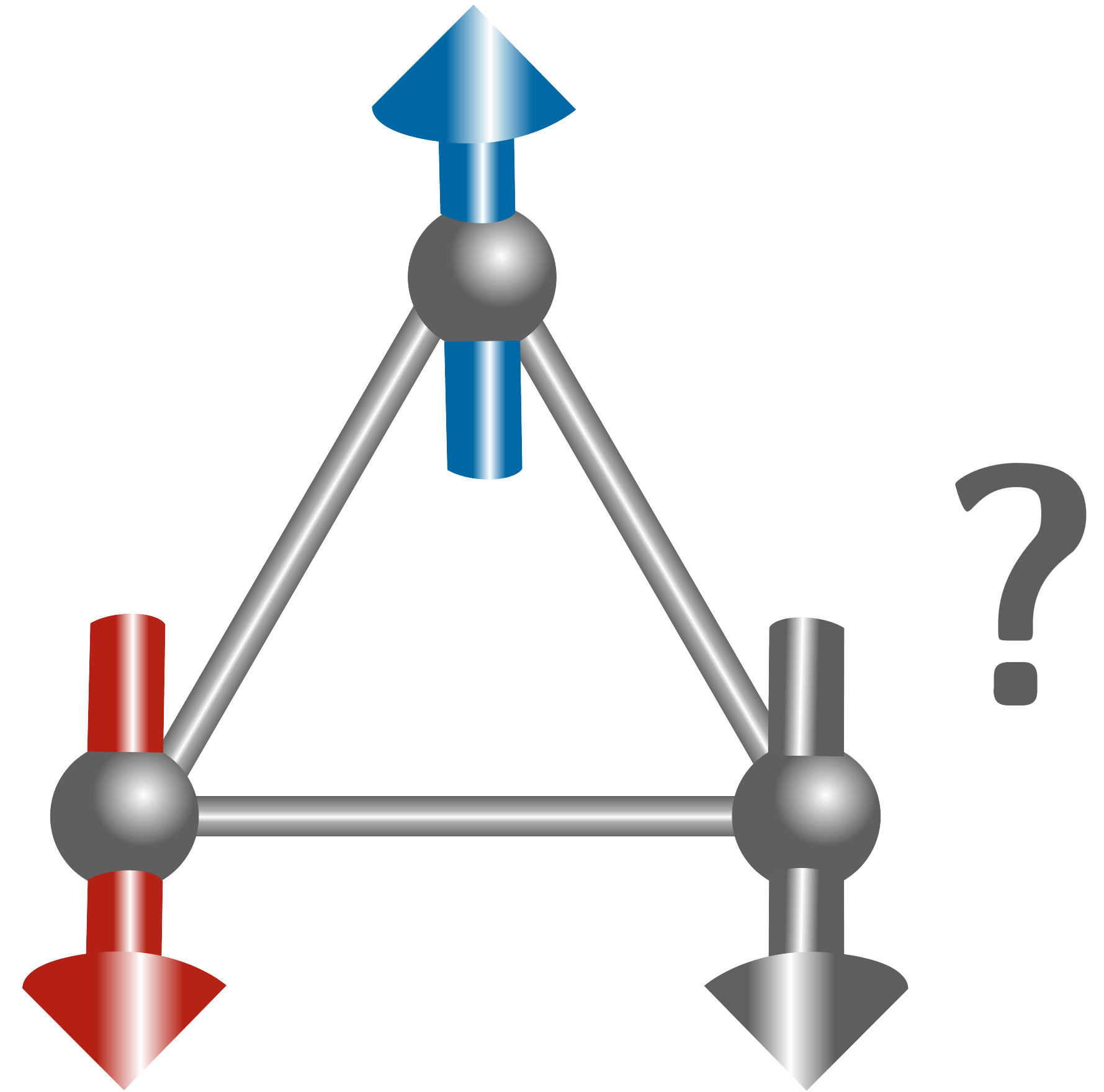
$$\mathcal{H}_{ij} = JS_i^z S_j^z \quad J > 0$$



Ingredient #2: Frustration

Antiferromagnetic interaction:

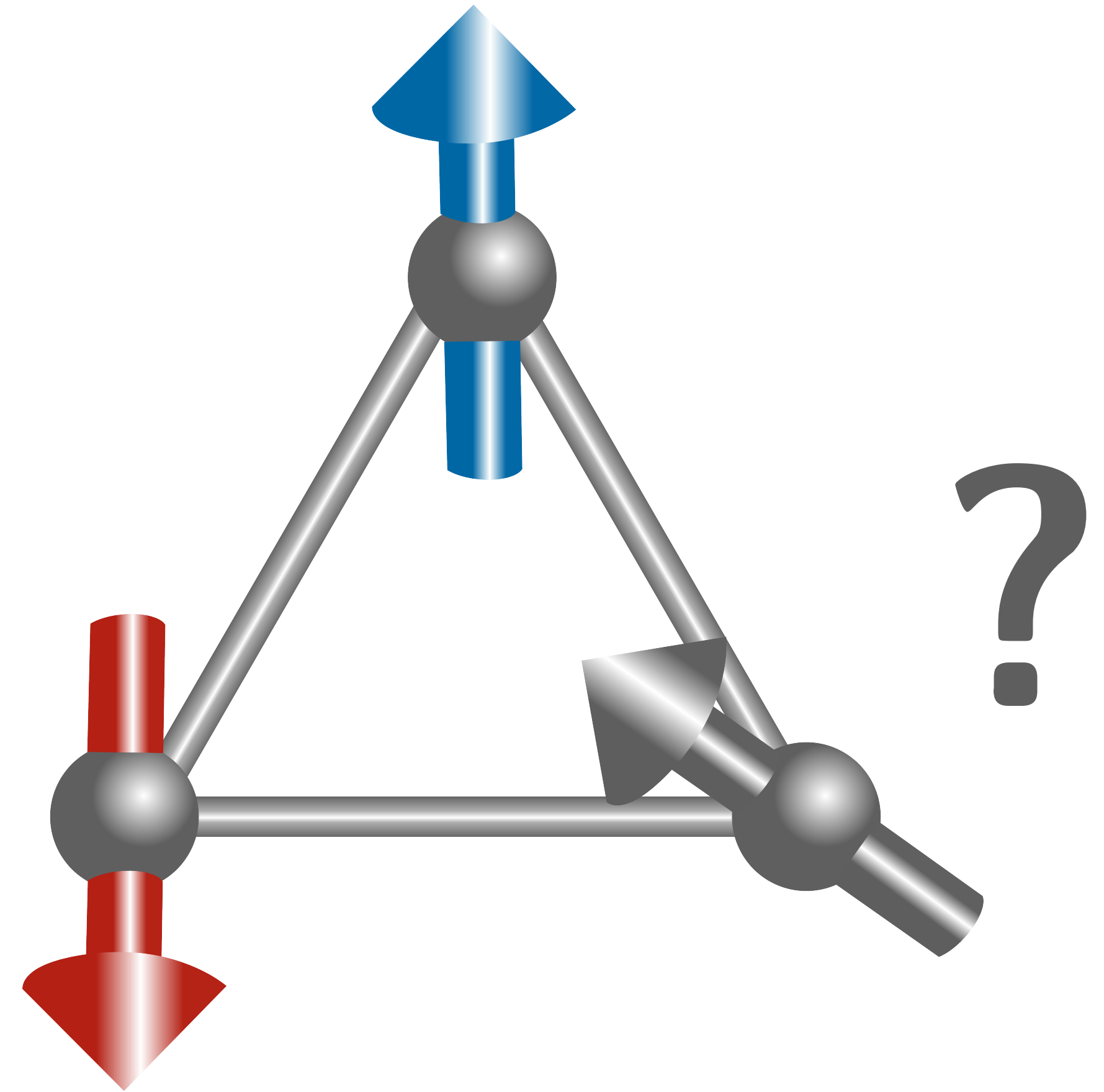
$$\mathcal{H}_{ij} = JS_i^z S_j^z \quad J > 0$$



Ingredient #2: Frustration

Antiferromagnetic interaction:

$$\mathcal{H}_{ij} = JS_i^z S_j^z \quad J > 0$$



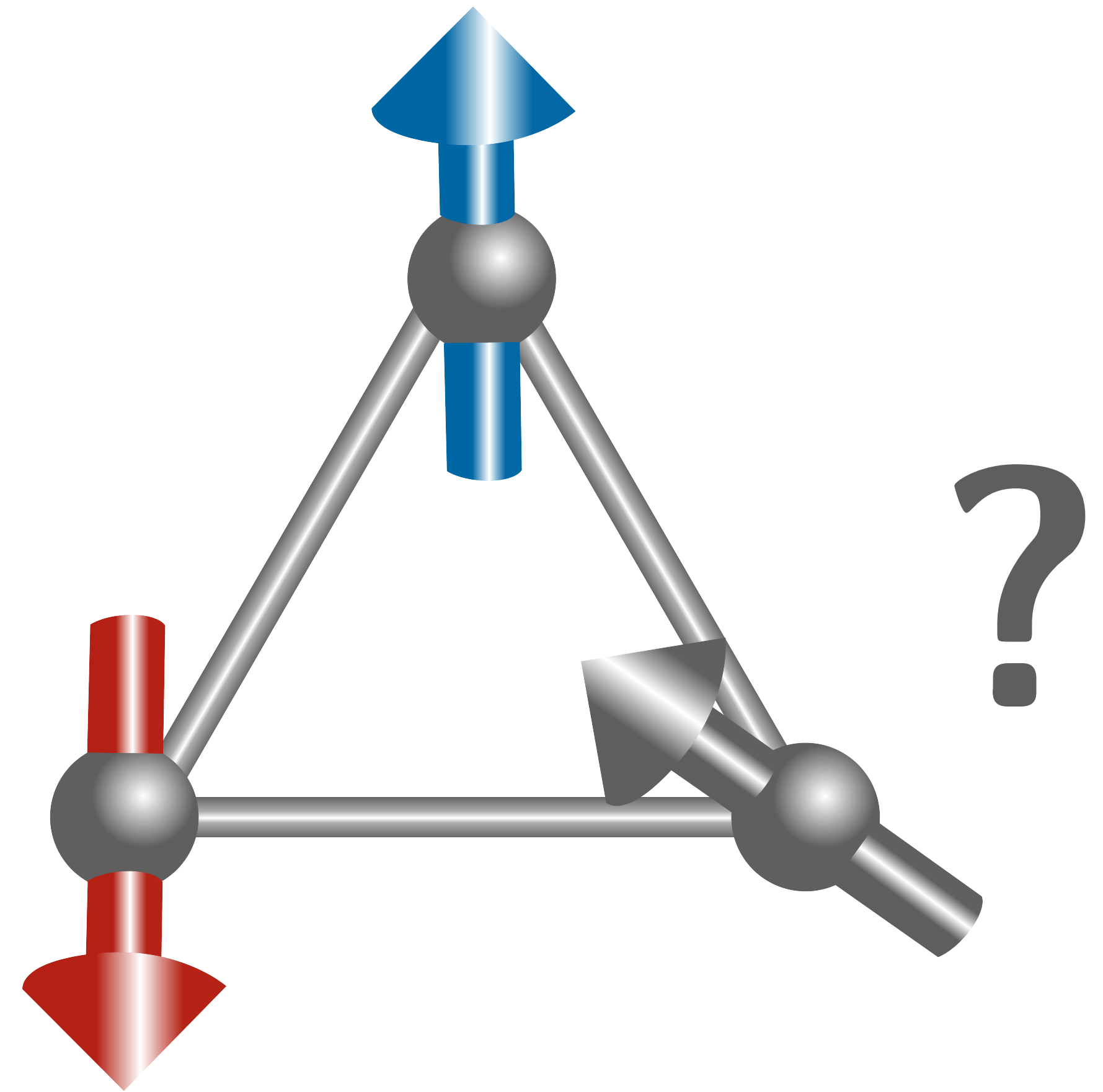
Ingredient #2: Frustration

Antiferromagnetic interaction:

$$\mathcal{H}_{ij} = JS_i^z S_j^z \quad J > 0$$

Frustration:

Incompatible interactions



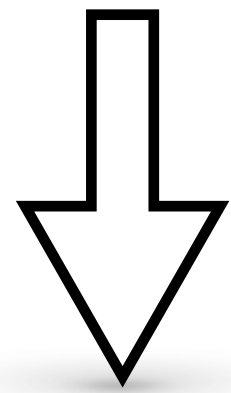
Ingredient #2: Frustration

Antiferromagnetic interaction:

$$\mathcal{H}_{ij} = JS_i^z S_j^z \quad J > 0$$

Frustration:

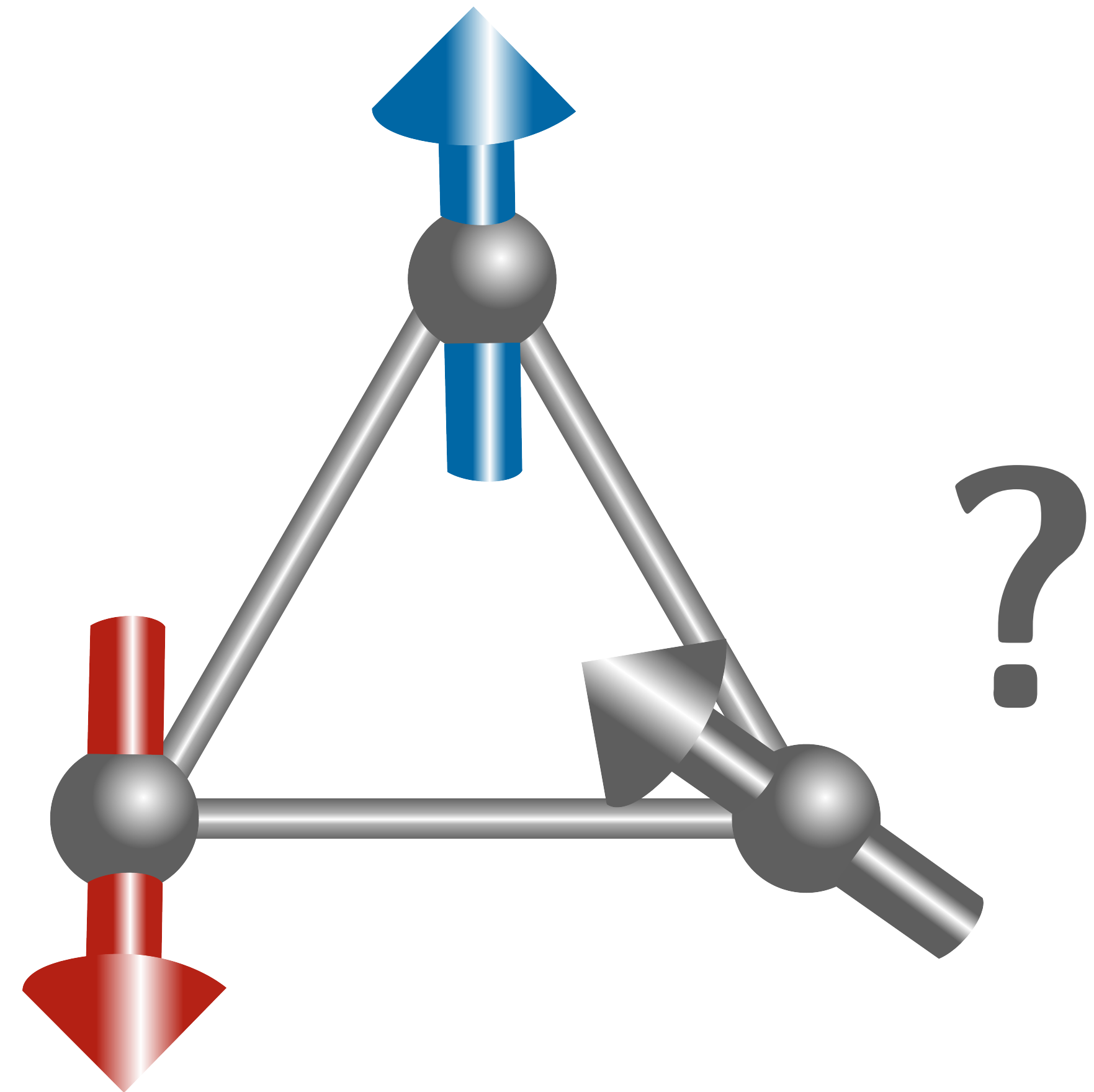
Incompatible interactions



New states of matter
with **exotic excitations**



illustrationsource.com

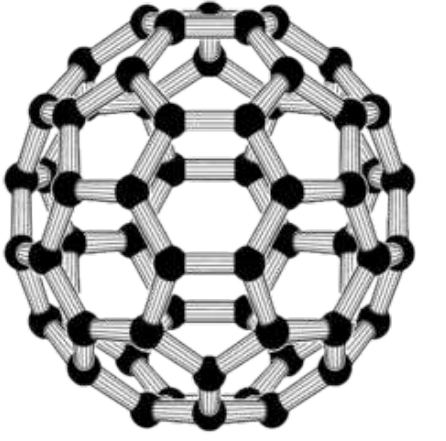


Majorana fermions

Conventional fermions:

f

$f^\dagger$



Majorana fermions

Conventional fermions:

 f $f^\dagger$

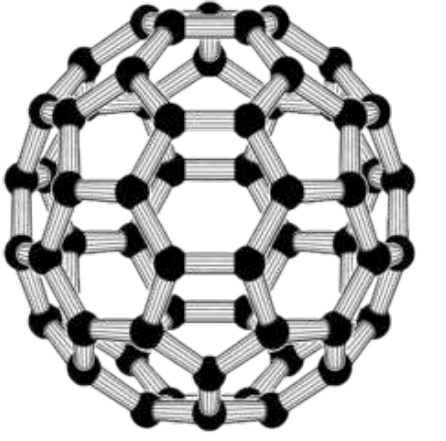
with

$$f|1\rangle = |0\rangle$$

$$f^\dagger|1\rangle = 0$$

$$f|0\rangle = 0$$

$$f^\dagger|0\rangle = |1\rangle$$



Majorana fermions

Conventional fermions:

$$\begin{array}{c} f \\ f^\dagger \end{array}$$

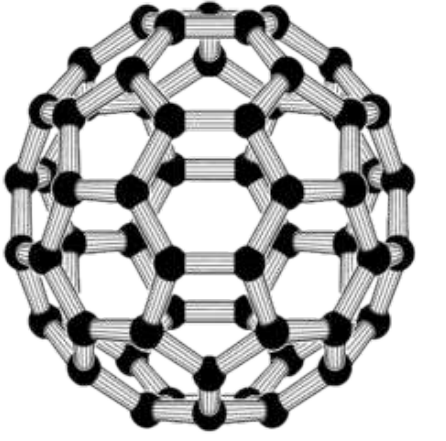
with

$$\begin{array}{l} f|1\rangle = |0\rangle \\ f^\dagger|1\rangle = 0 \end{array}$$

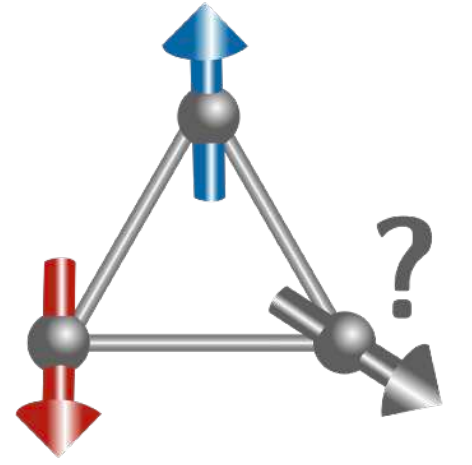
$$\begin{array}{l} f|0\rangle = 0 \\ f^\dagger|0\rangle = |1\rangle \end{array}$$

Particle number:

$$n = f^\dagger f = \begin{cases} 1 \\ 0 \end{cases}$$



Majorana fermions



$$f = \frac{1}{2}(c_1 + ic_2)$$
$$f^\dagger = \frac{1}{2}(c_1 - ic_2)$$

with

$$f|1\rangle = |0\rangle$$
$$f^\dagger|1\rangle = 0$$

$$f|0\rangle = 0$$
$$f^\dagger|0\rangle = |1\rangle$$

Particle number:

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Majorana fermions



$$f = \frac{1}{2}(c_1 + ic_2)$$

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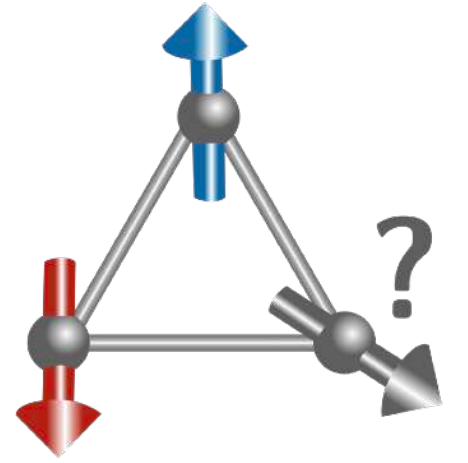
with

$$f|1\rangle = |0\rangle$$

$$f^\dagger|1\rangle = 0$$

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Particle number:

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Majorana fermions



$$f = \frac{1}{2}(c_1 + ic_2)$$

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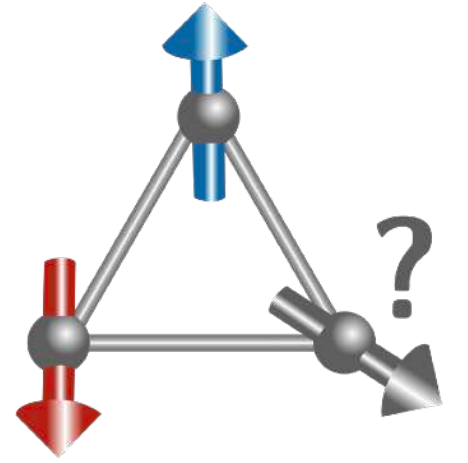
with

$$f|1\rangle = |0\rangle$$

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$$f|0\rangle = 0$$

$$f^\dagger|0\rangle = |1\rangle$$



Particle number:

$$n = f^\dagger f = \frac{1}{2}(1 + ic_1 c_2) = \begin{cases} 1 & \text{if } ic_1 c_2 = 1 \\ 0 & \text{if } ic_1 c_2 = -1 \end{cases}$$

Majorana fermions



$$f = \frac{1}{2}(c_1 + ic_2)$$

$$f^\dagger = \frac{1}{2}(c_1 - ic_2)$$

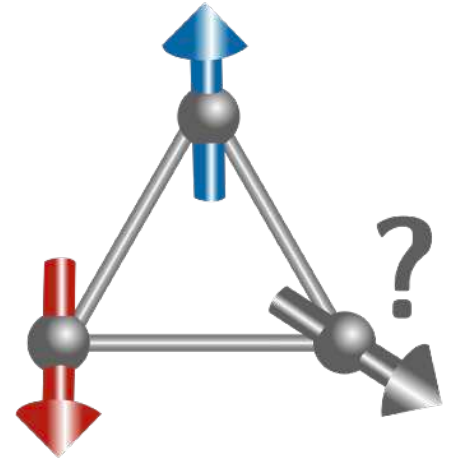
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Particle number:

$$n = f^\dagger f = \frac{1}{2}(1 + ic_1 c_2) = \begin{cases} 1 & \text{if } ic_1 c_2 = 1 \\ 0 & \text{if } ic_1 c_2 = -1 \end{cases}$$

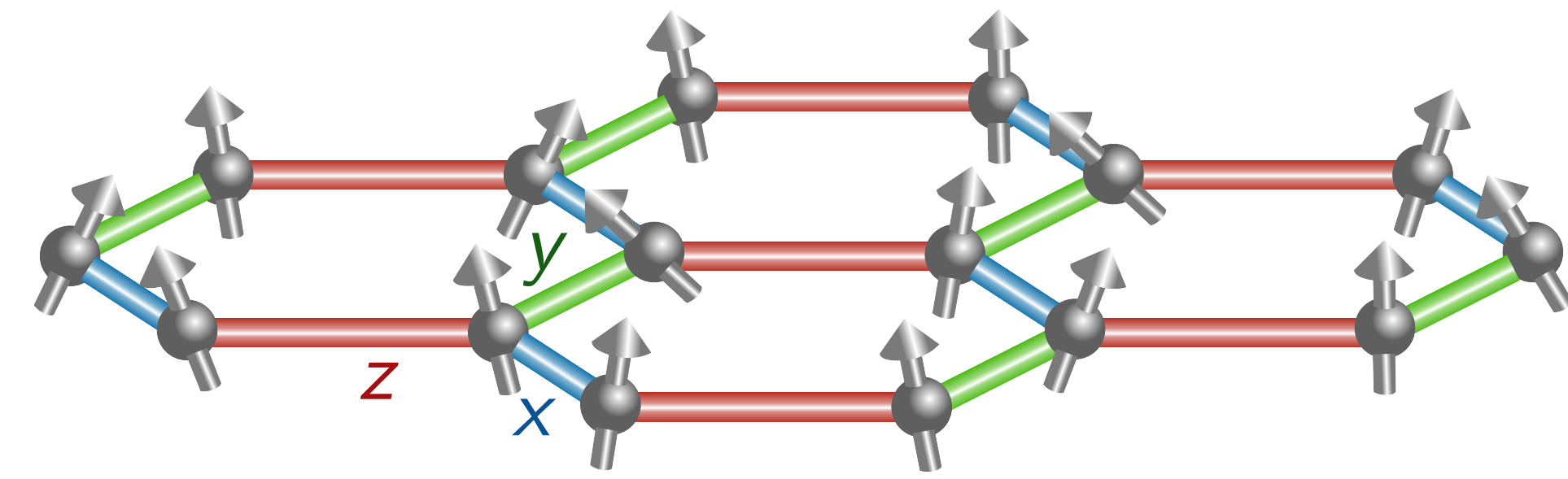
Potential delocalized
information storage



Kitaev quantum magnets

Hamiltonian:

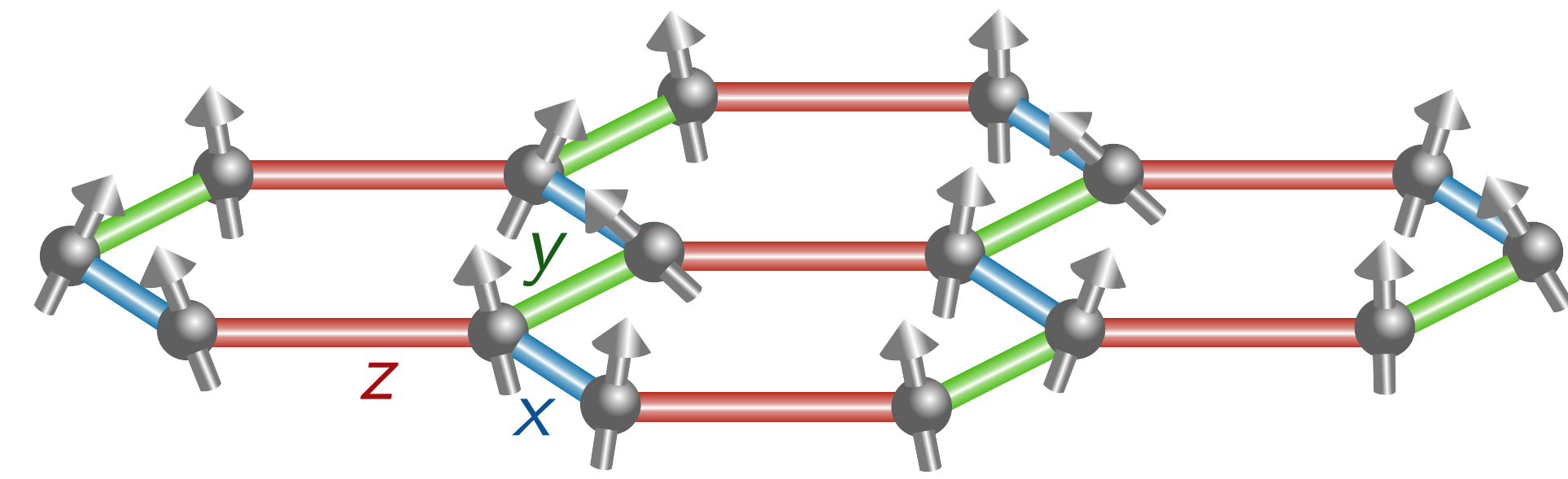
$$\mathcal{H} = -K \left(\sum_{\langle ij \rangle_x} S_i^x S_j^x + \sum_{\langle ij \rangle_y} S_i^y S_j^y + \sum_{\langle ij \rangle_z} S_i^z S_j^z \right)$$



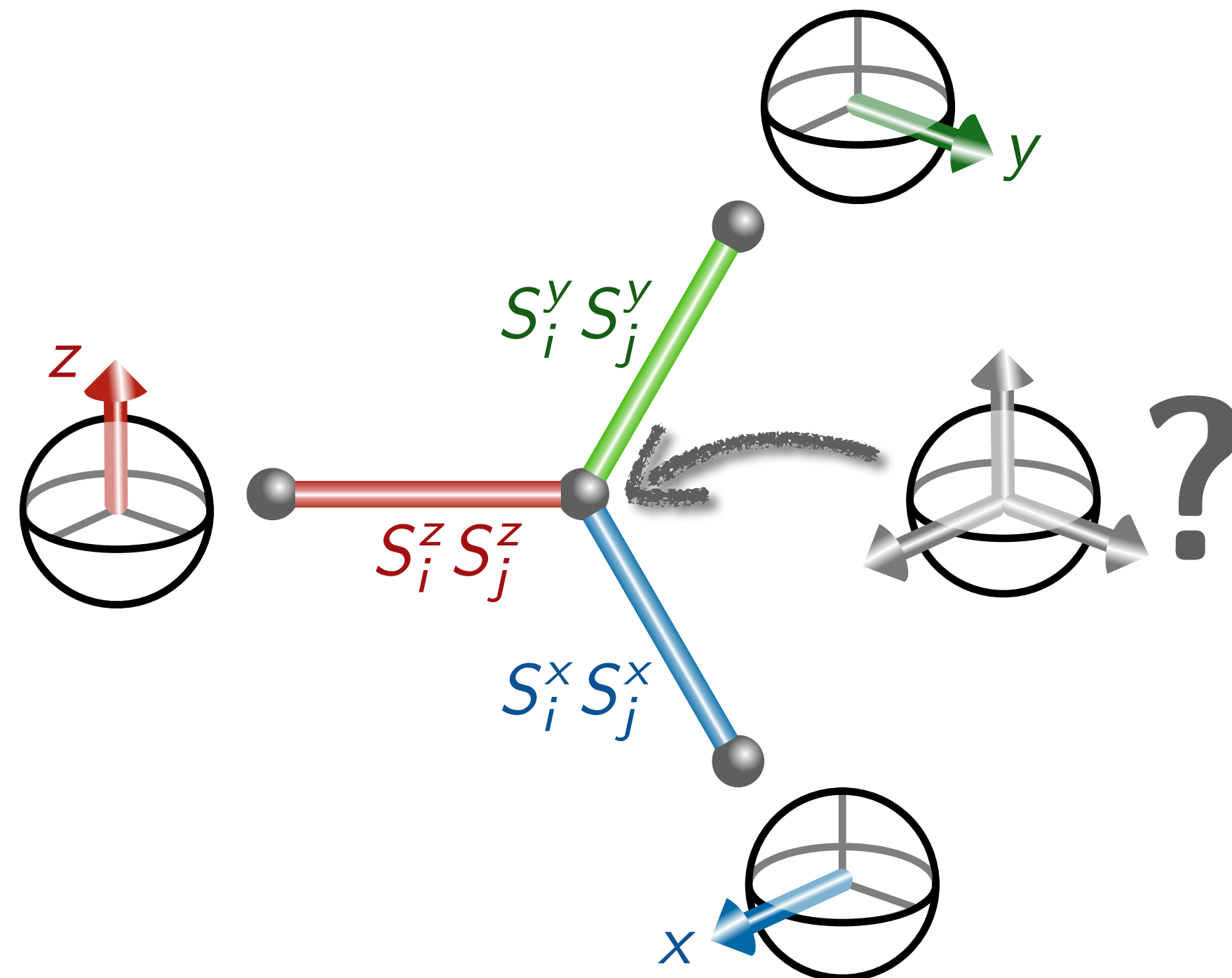
Kitaev quantum magnets

Hamiltonian:

$$\mathcal{H} = -K \left(\sum_{\langle ij \rangle_x} S_i^x S_j^x + \sum_{\langle ij \rangle_y} S_i^y S_j^y + \sum_{\langle ij \rangle_z} S_i^z S_j^z \right)$$



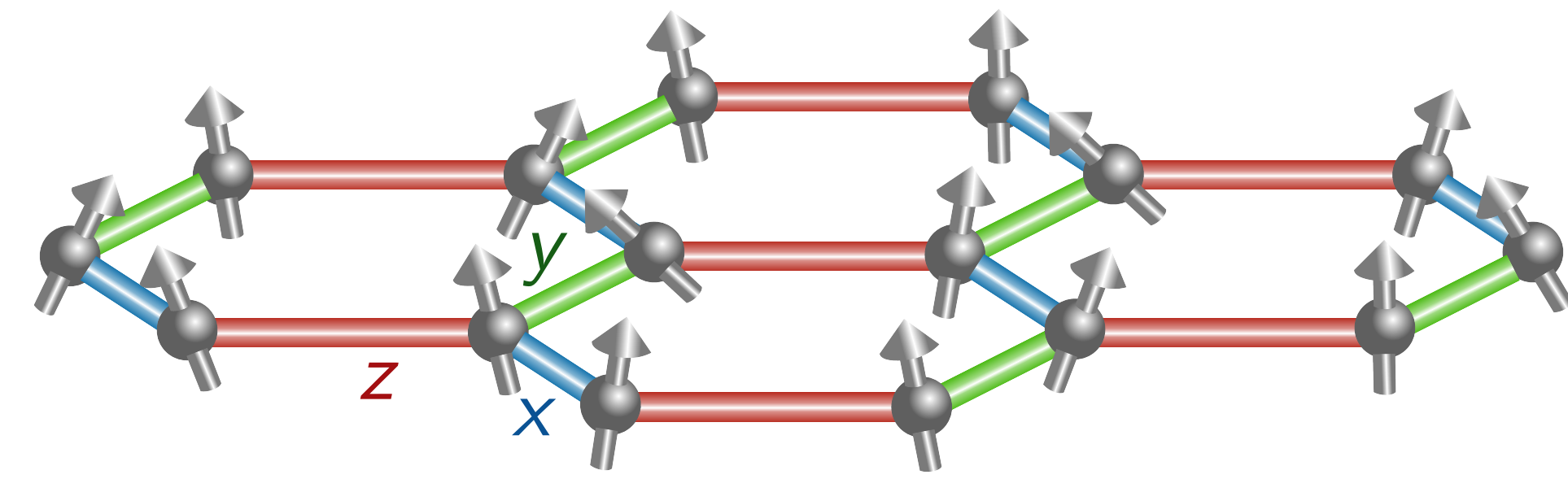
Frustration:



Kitaev quantum magnets

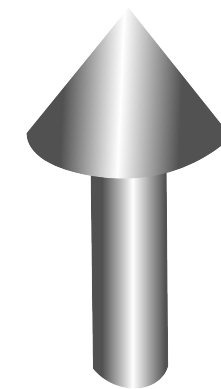
Hamiltonian:

$$\mathcal{H} = -K \left(\sum_{\langle ij \rangle_x} S_i^x S_j^x + \sum_{\langle ij \rangle_y} S_i^y S_j^y + \sum_{\langle ij \rangle_z} S_i^z S_j^z \right)$$



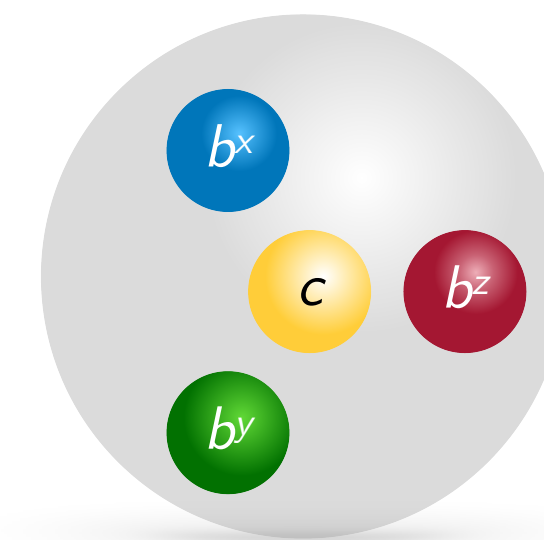
Majorana representation:

$$\begin{aligned} S_i^x &= \frac{i\hbar}{2} b_i^x c_i \\ S_i^y &= \frac{i\hbar}{2} b_i^y c_i \\ S_i^z &= \frac{i\hbar}{2} b_i^z c_i \end{aligned}$$



Spin

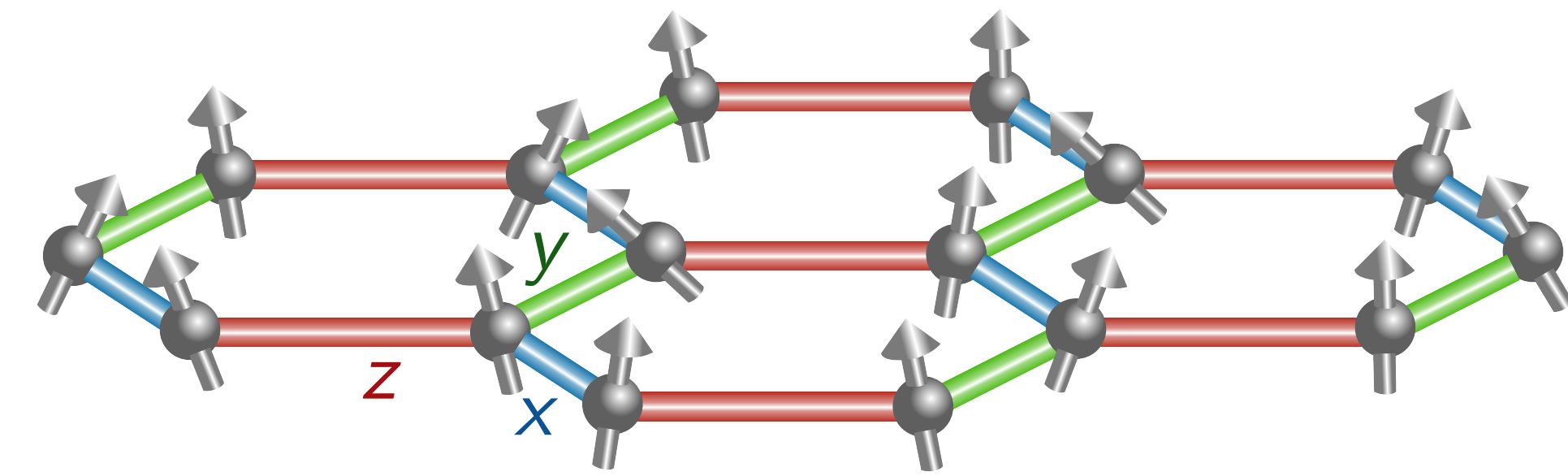
=



4 Majoranas
with gauge constraint

Kitaev quantum magnets

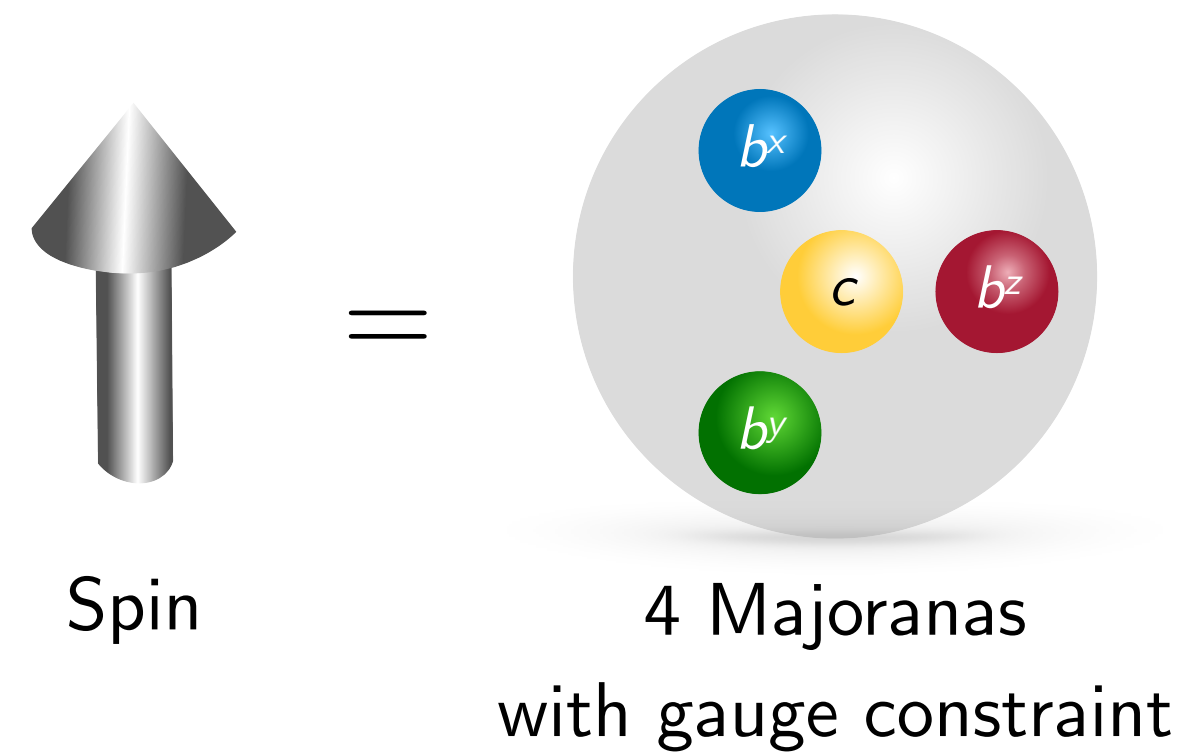
Hamiltonian:



$$\mathcal{H} = -K \left(\sum_{\langle ij \rangle_x} S_i^x S_j^x + \sum_{\langle ij \rangle_y} S_i^y S_j^y + \sum_{\langle ij \rangle_z} S_i^z S_j^z \right) = -\frac{i\hbar^2 K}{4} \sum_{\langle ij \rangle_\alpha} (i b_i^\alpha b_j^\alpha) c_i c_j$$

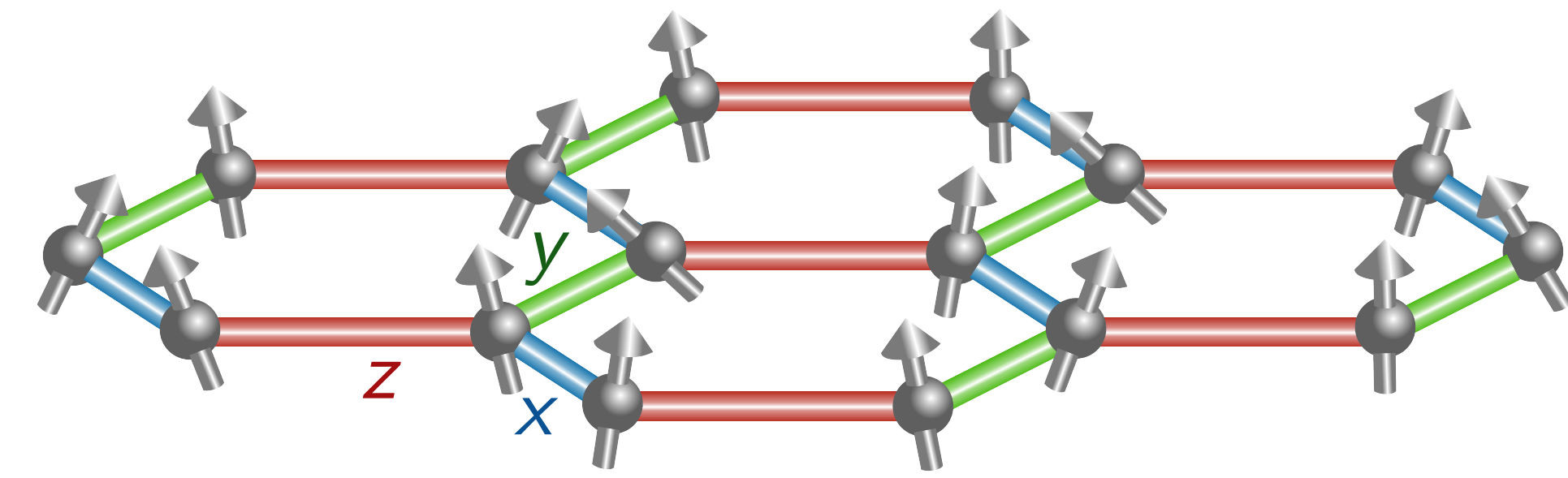
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Kitaev quantum magnets

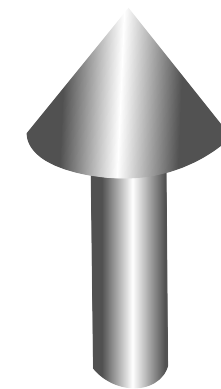
Hamiltonian:



$$\mathcal{H} = -K \left(\sum_{\langle ij \rangle_x} S_i^x S_j^x + \sum_{\langle ij \rangle_y} S_i^y S_j^y + \sum_{\langle ij \rangle_z} S_i^z S_j^z \right) = -\frac{i\hbar^2 K}{4} \sum_{\langle ij \rangle_\alpha} \underbrace{(i b_i^\alpha b_j^\alpha)}_{\equiv \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j \quad \text{with } [\hat{u}_{ij}, \tilde{\mathcal{H}}] = 0$$

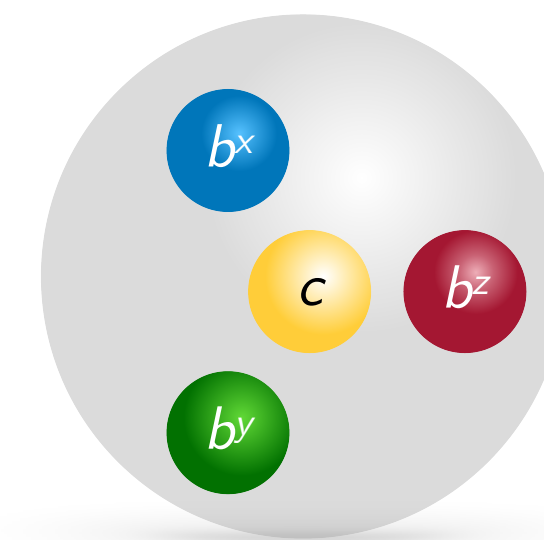
Majorana representation:

$$\begin{aligned} S_i^x &= \frac{i\hbar}{2} b_i^x c_i \\ S_i^y &= \frac{i\hbar}{2} b_i^y c_i \\ S_i^z &= \frac{i\hbar}{2} b_i^z c_i \end{aligned}$$



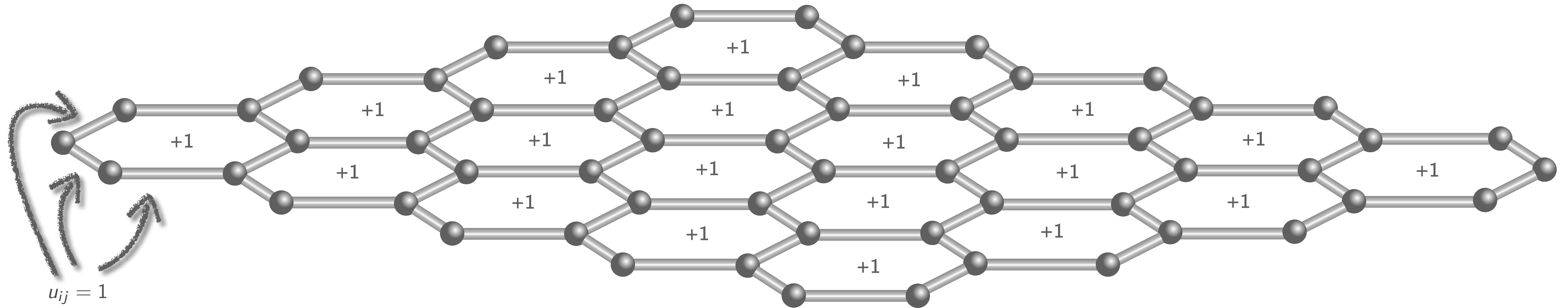
Spin

=

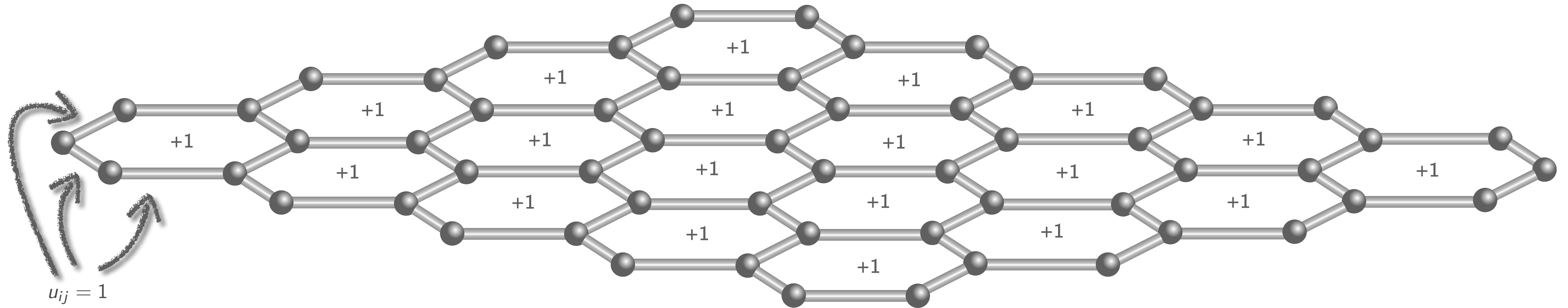


4 Majoranas
with gauge constraint

Kitaev quantum spin liquid: Ground state

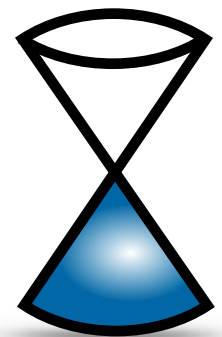


Kitaev quantum spin liquid: Ground state

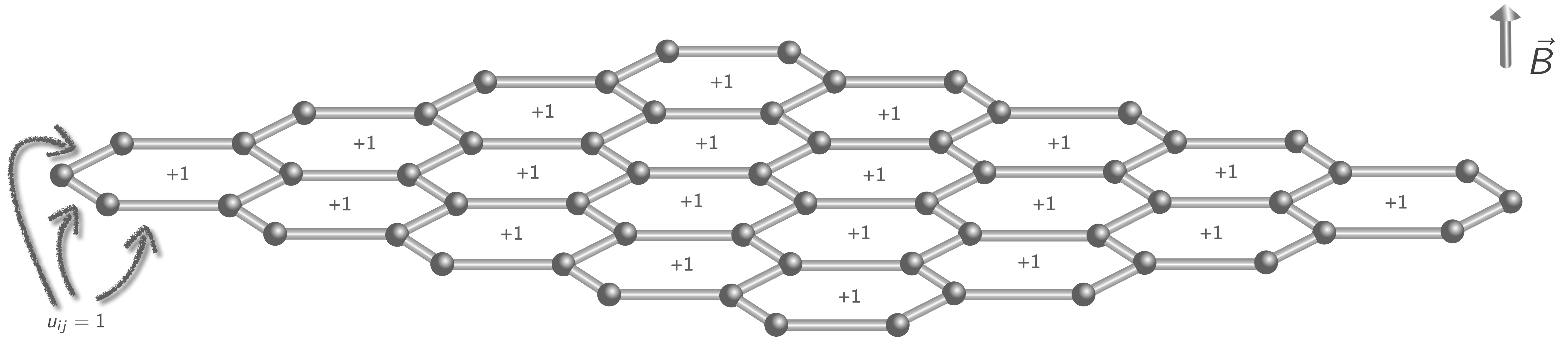


Majorana spectrum:

$$B = 0$$

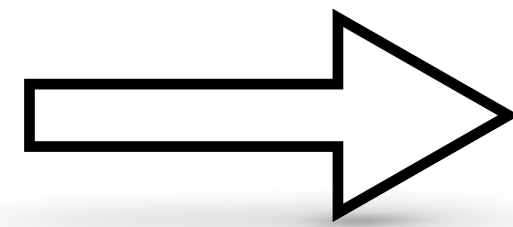
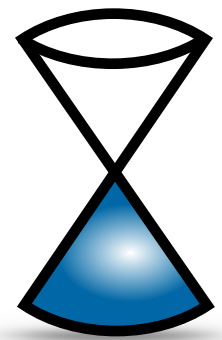


Kitaev quantum spin liquid: Ground state

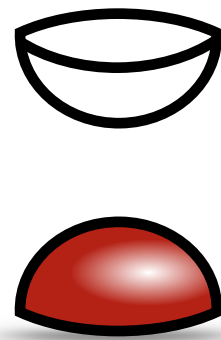


Majorana spectrum:

$$B = 0$$

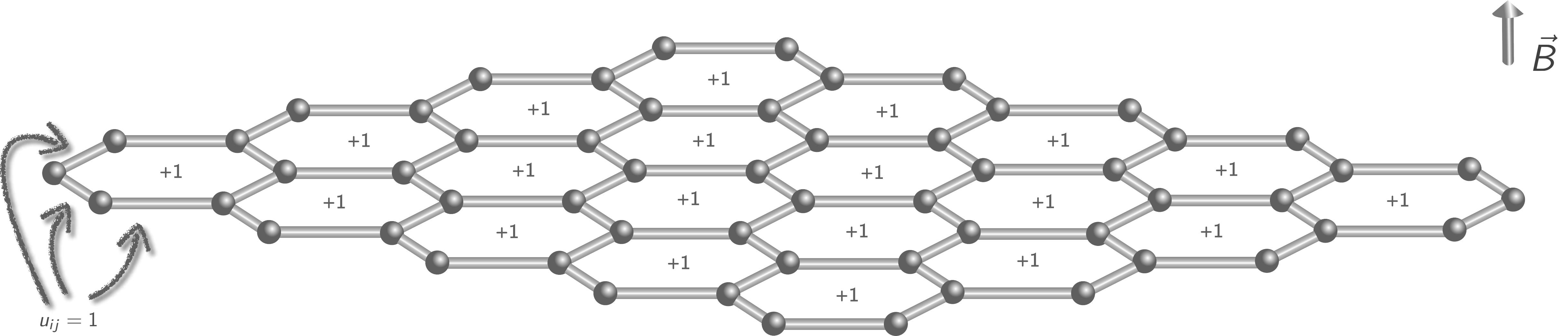


$$B > 0$$



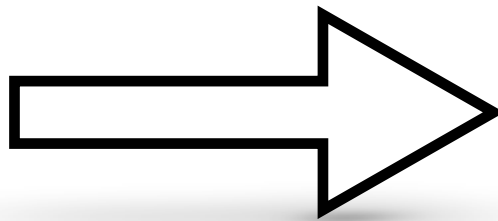
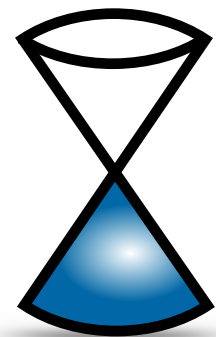
$$C = 1$$

Kitaev quantum spin liquid: Ground state

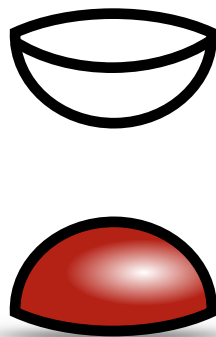


Majorana spectrum:

$B = 0$



$B > 0$

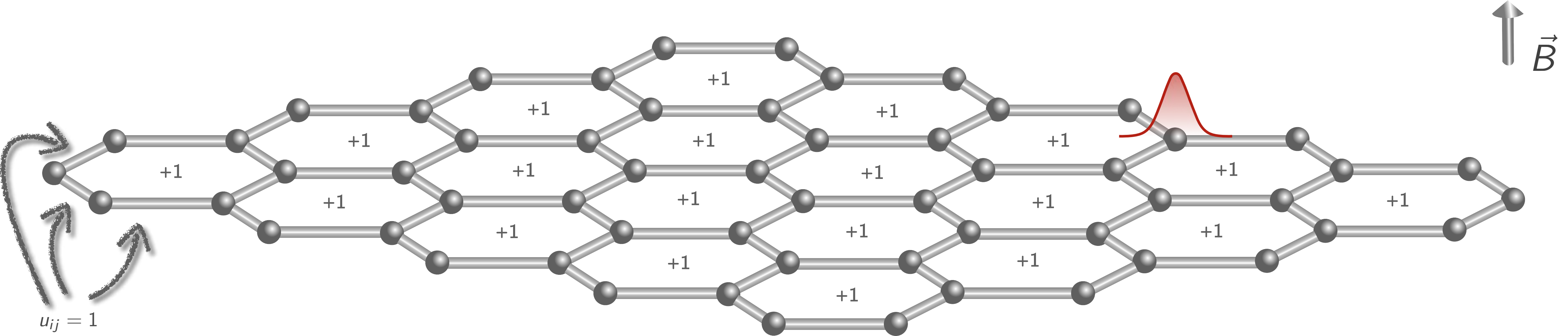


$C = 1$

Fractionalized version of topological insulator

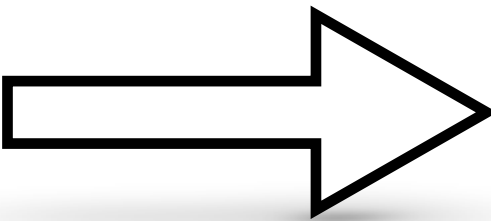
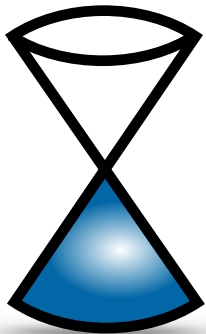
... half-quantized thermal Hall response

Kitaev quantum spin liquid: Excitations

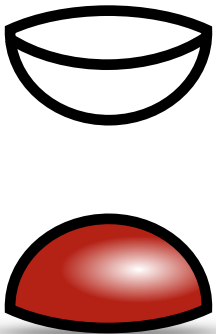


Majorana spectrum:

$B = 0$



$B > 0$

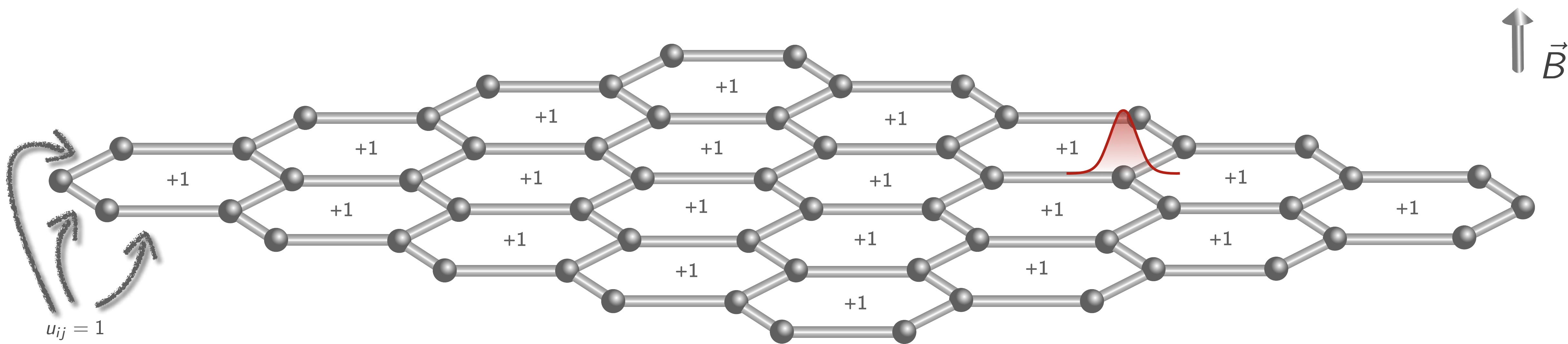


$C = 1$

Fractionalized version of topological insulator

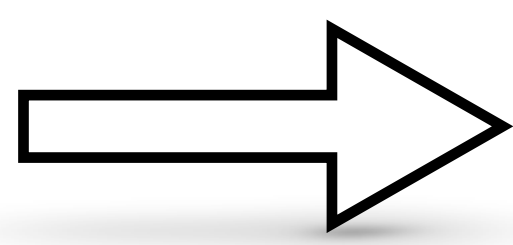
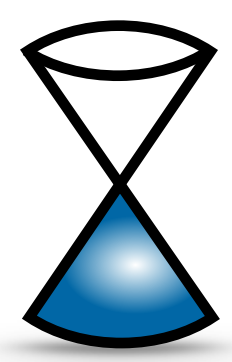
... half-quantized thermal Hall response

Kitaev quantum spin liquid: Excitations

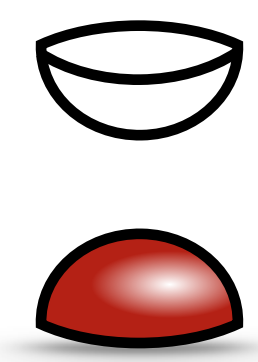


Majorana spectrum:

$B = 0$



$B > 0$

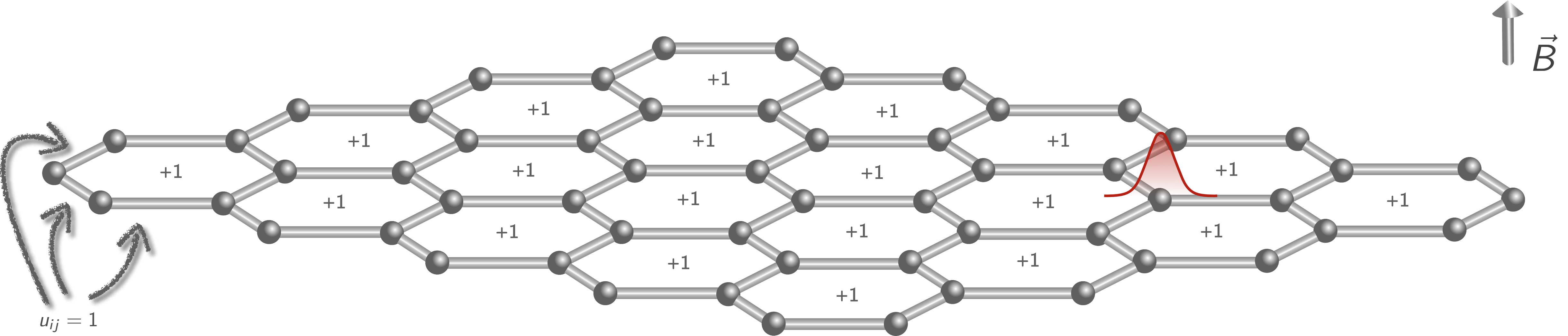


$C = 1$

Fractionalized version of topological insulator

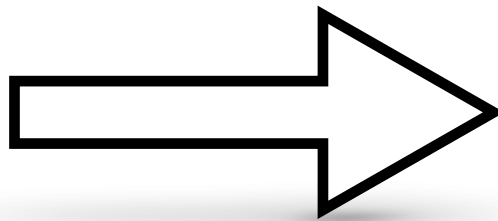
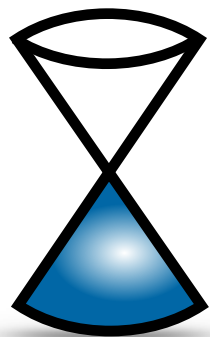
... half-quantized thermal Hall response

Kitaev quantum spin liquid: Excitations

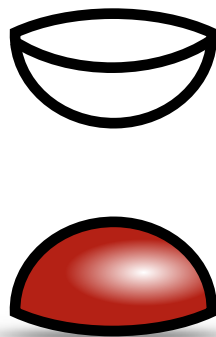


Majorana spectrum:

$B = 0$



$B > 0$

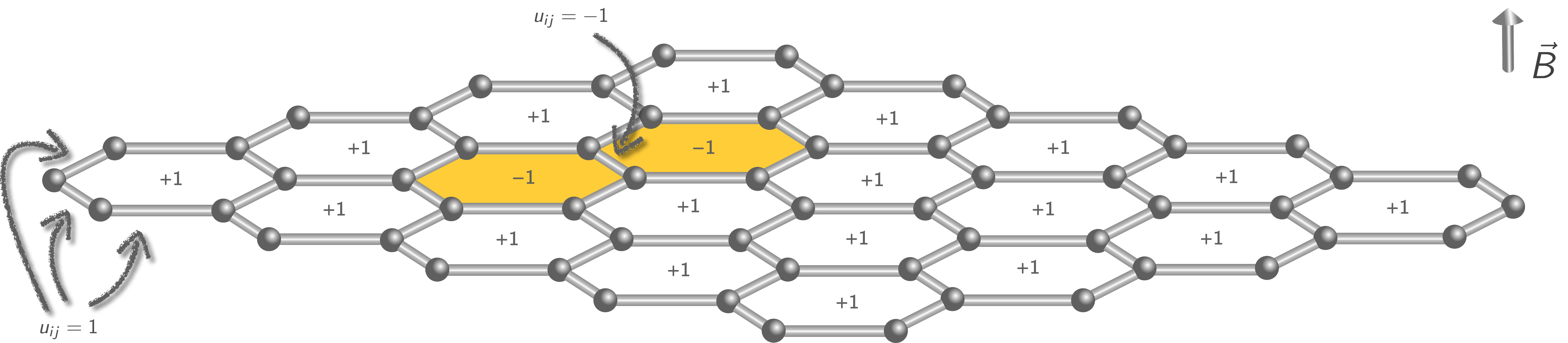


$C = 1$

Fractionalized version of topological insulator

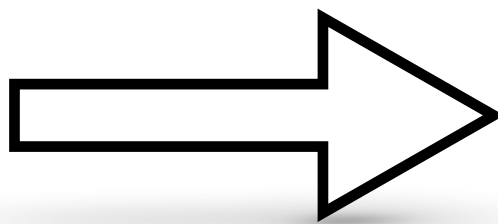
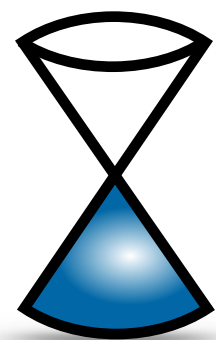
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Kitaev quantum spin liquid: Excitations

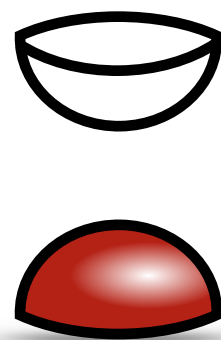


Majorana spectrum:

$B = 0$



$B > 0$

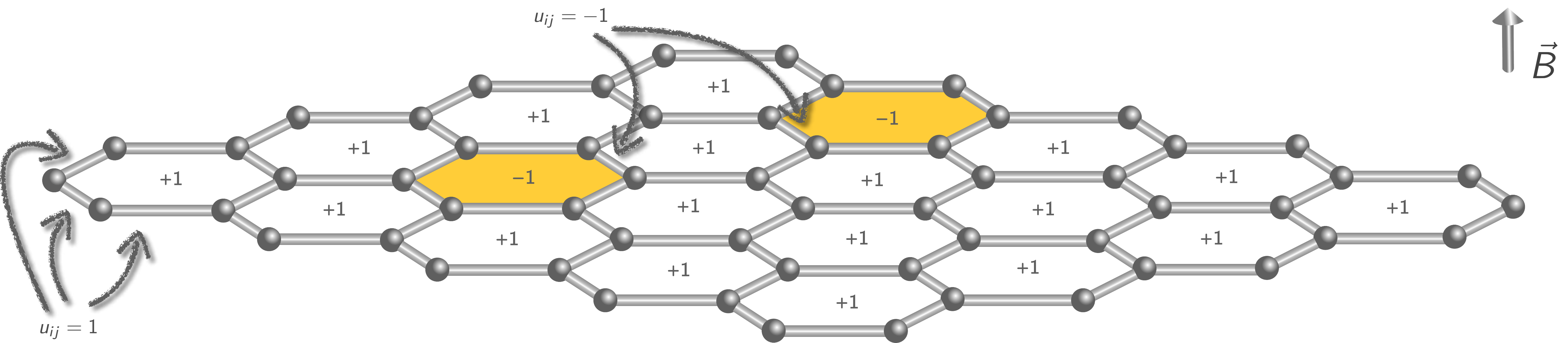


$C = 1$

Fractionalized version of topological insulator

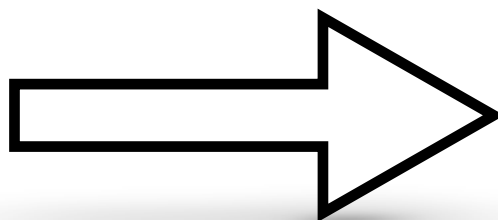
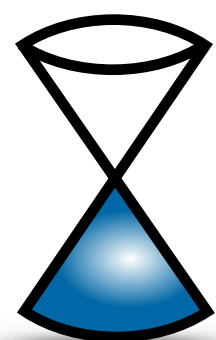
... half-quantized thermal Hall response

Kitaev quantum spin liquid: Excitations

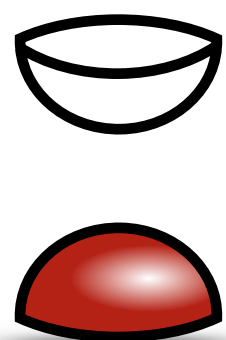


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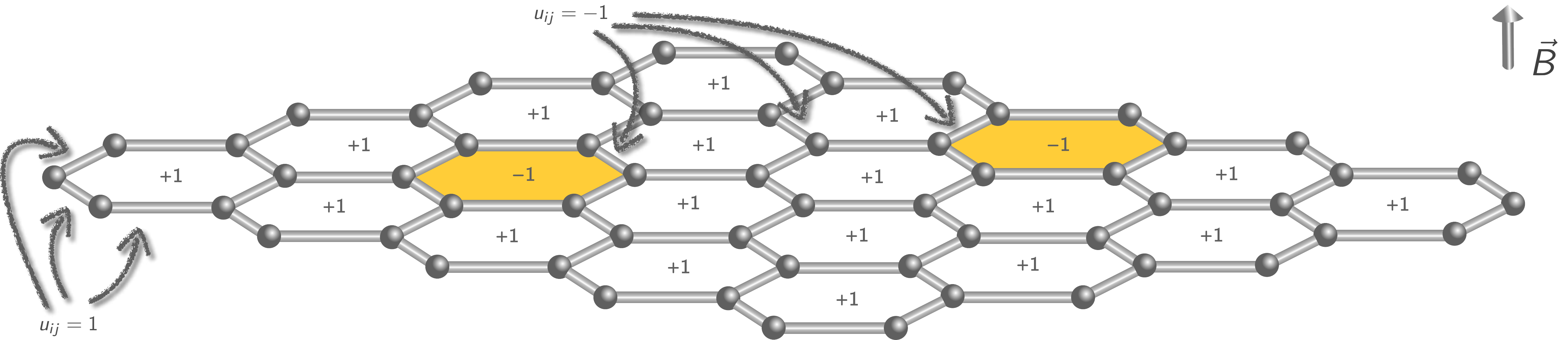


$C = 1$

Fractionalized version of topological insulator

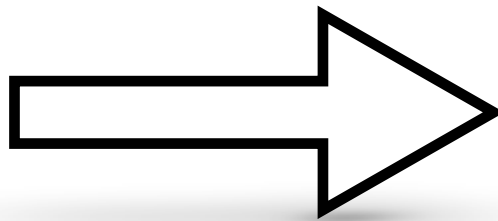
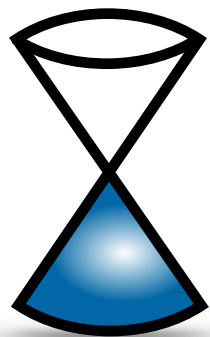
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Kitaev quantum spin liquid: Excitations

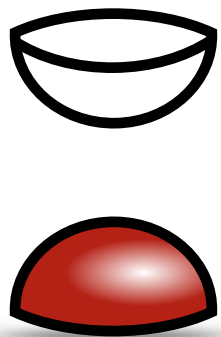


Majorana spectrum:

$B = 0$



$B > 0$

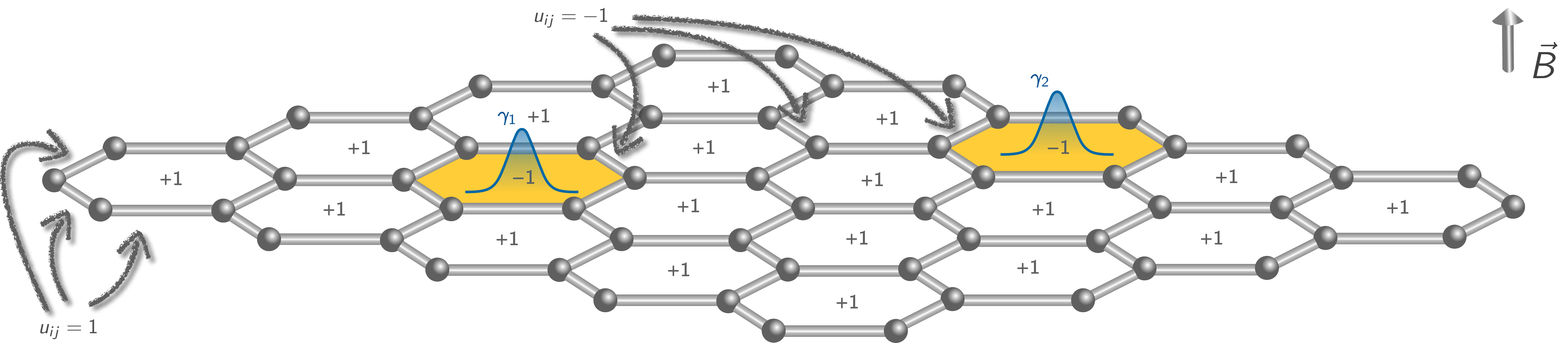


$C = 1$

Fractionalized version of topological insulator

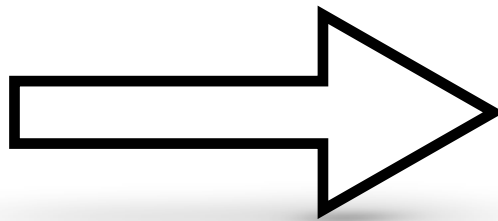
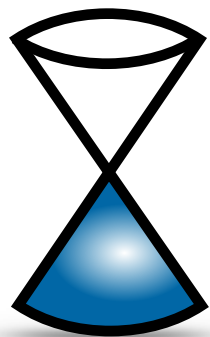
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Kitaev quantum spin liquid: Excitations

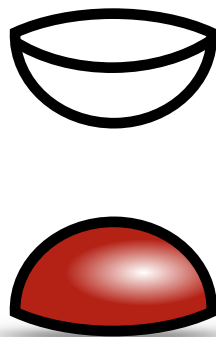


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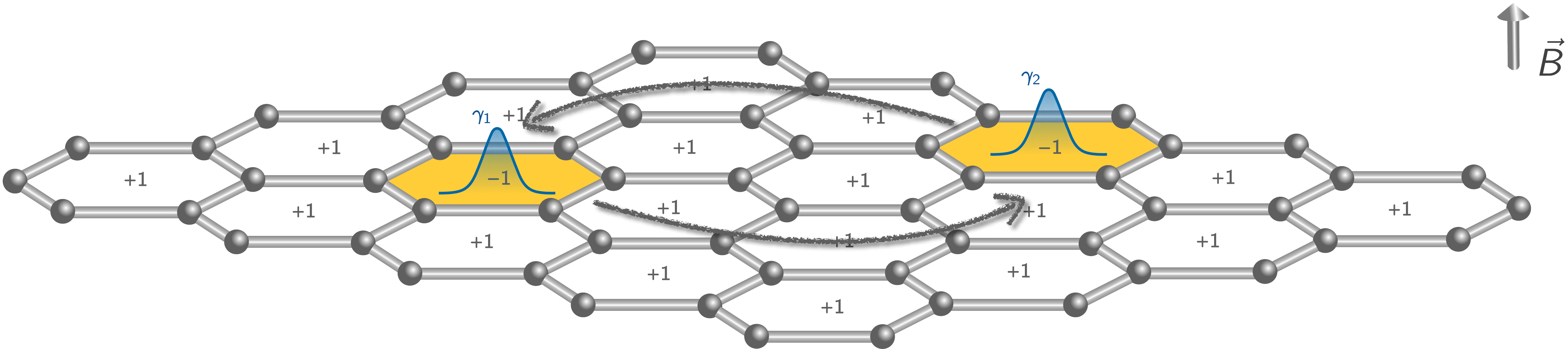


$C = 1$

Fractionalized version of topological insulator

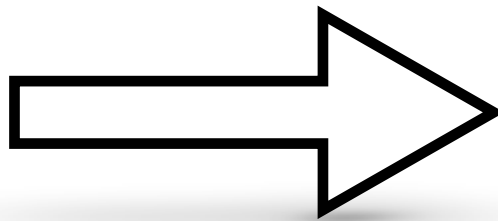
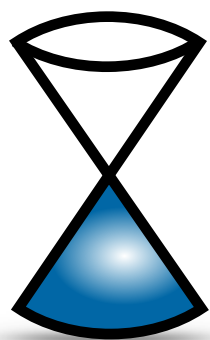
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Kitaev quantum spin liquid: Excitations

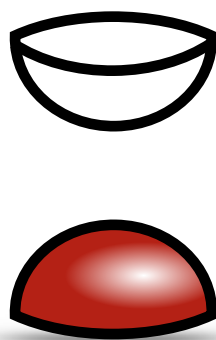


Majorana spectrum:

$$B = 0$$



$$B > 0$$



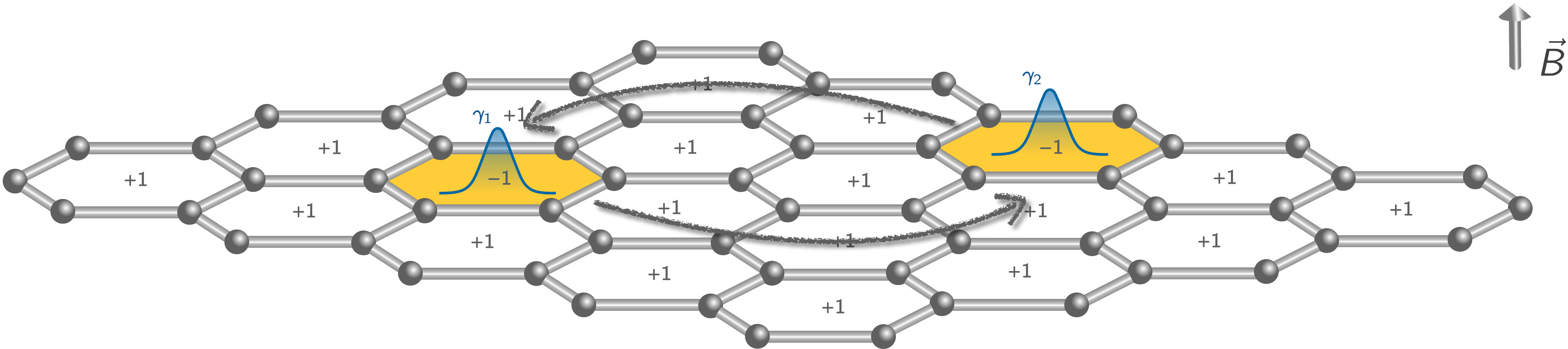
$$C = 1$$

Braiding:

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \mapsto \pm \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

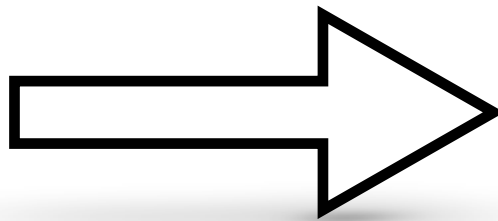
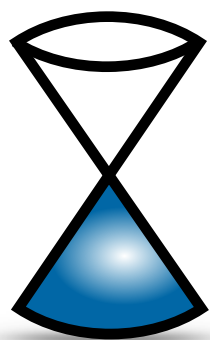
... non-Abelian anyons

Kitaev quantum spin liquid: Excitations

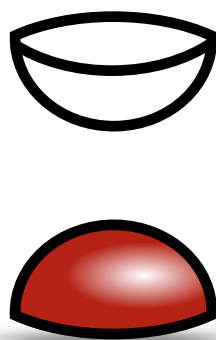


Majorana spectrum:

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... non-Abelian anyons

⇒ Fault-tolerant quantum computing

[Kitaev, Ann. Phys. '06]

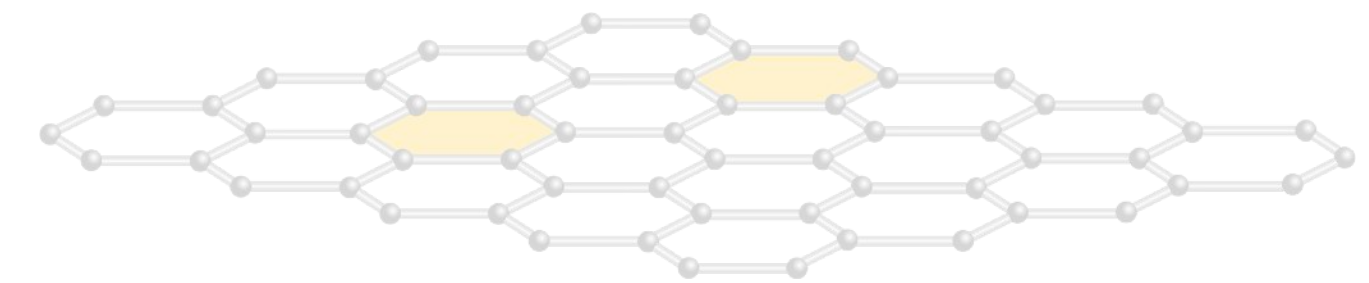
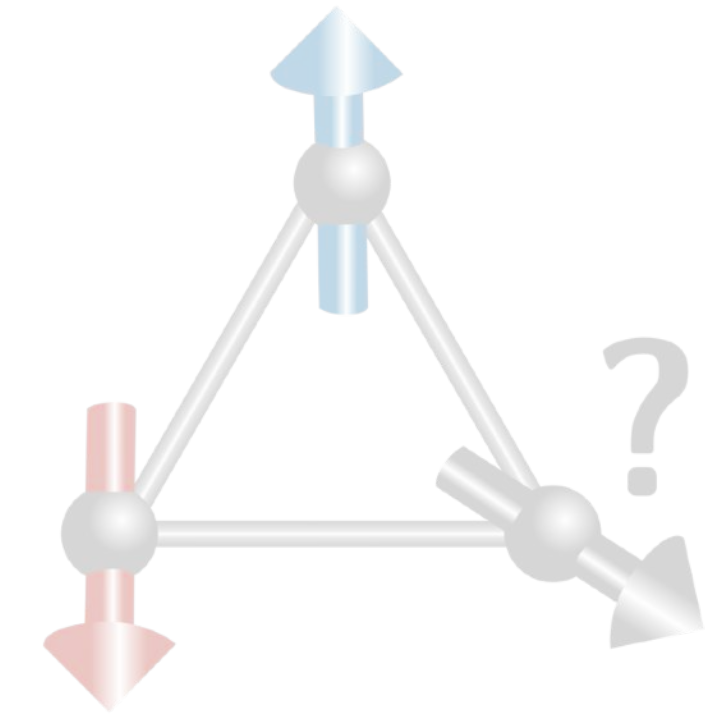
Outline

(1) Introduction: Emergence

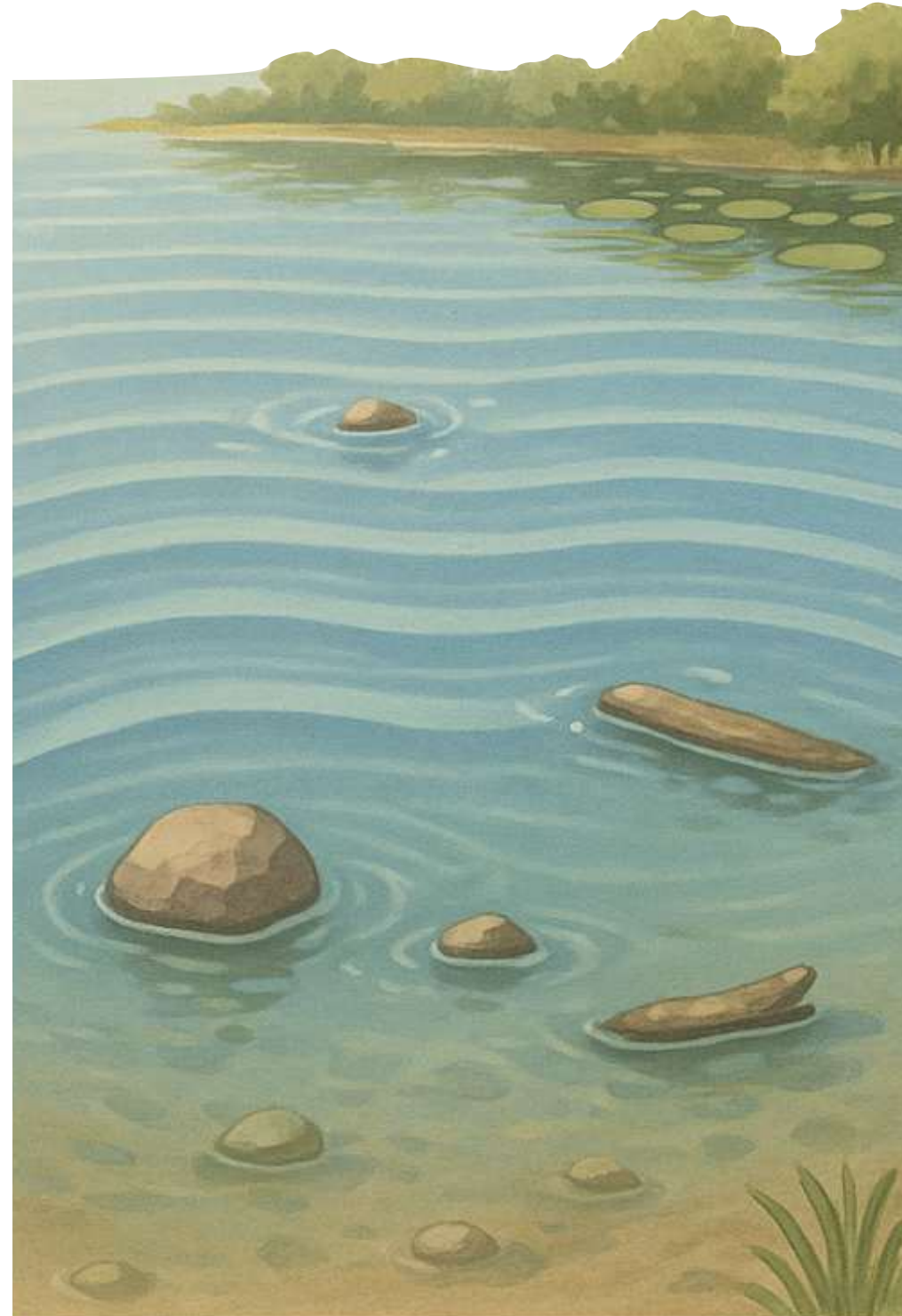
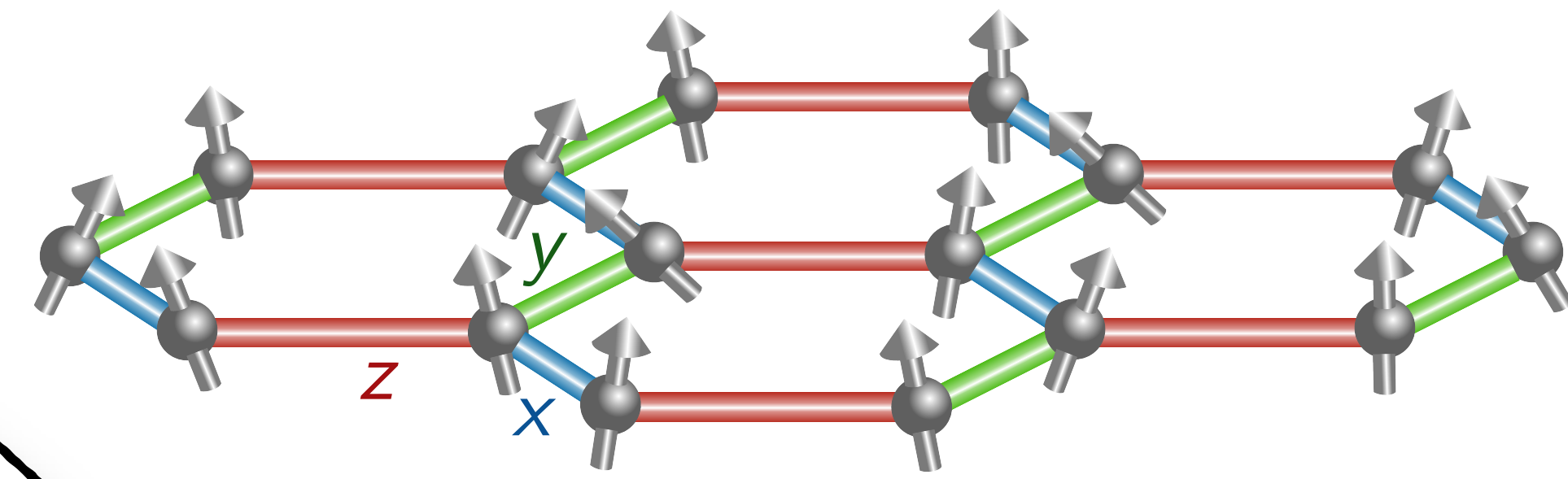
(2) Quantum materials

(3) Emergent phenomena for Quantum Saxony

(4) Summary



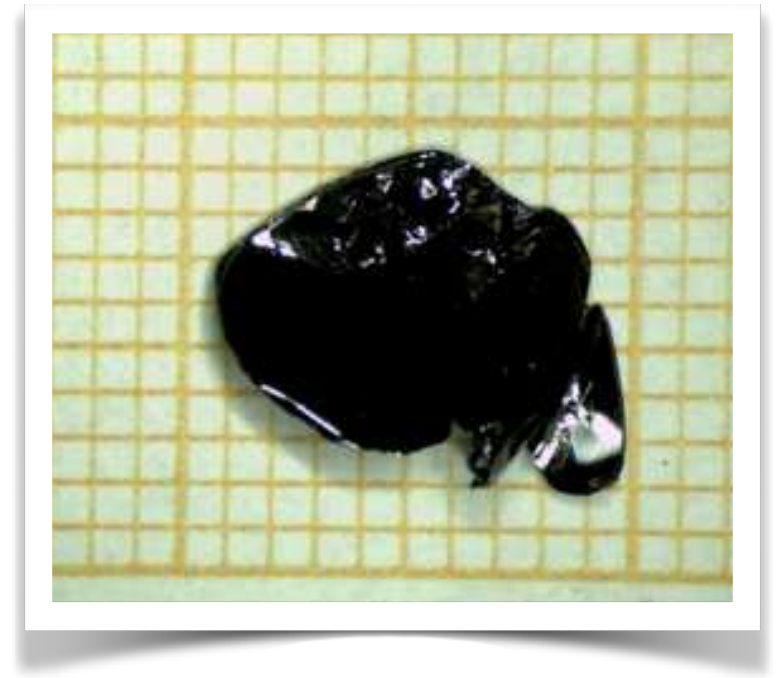
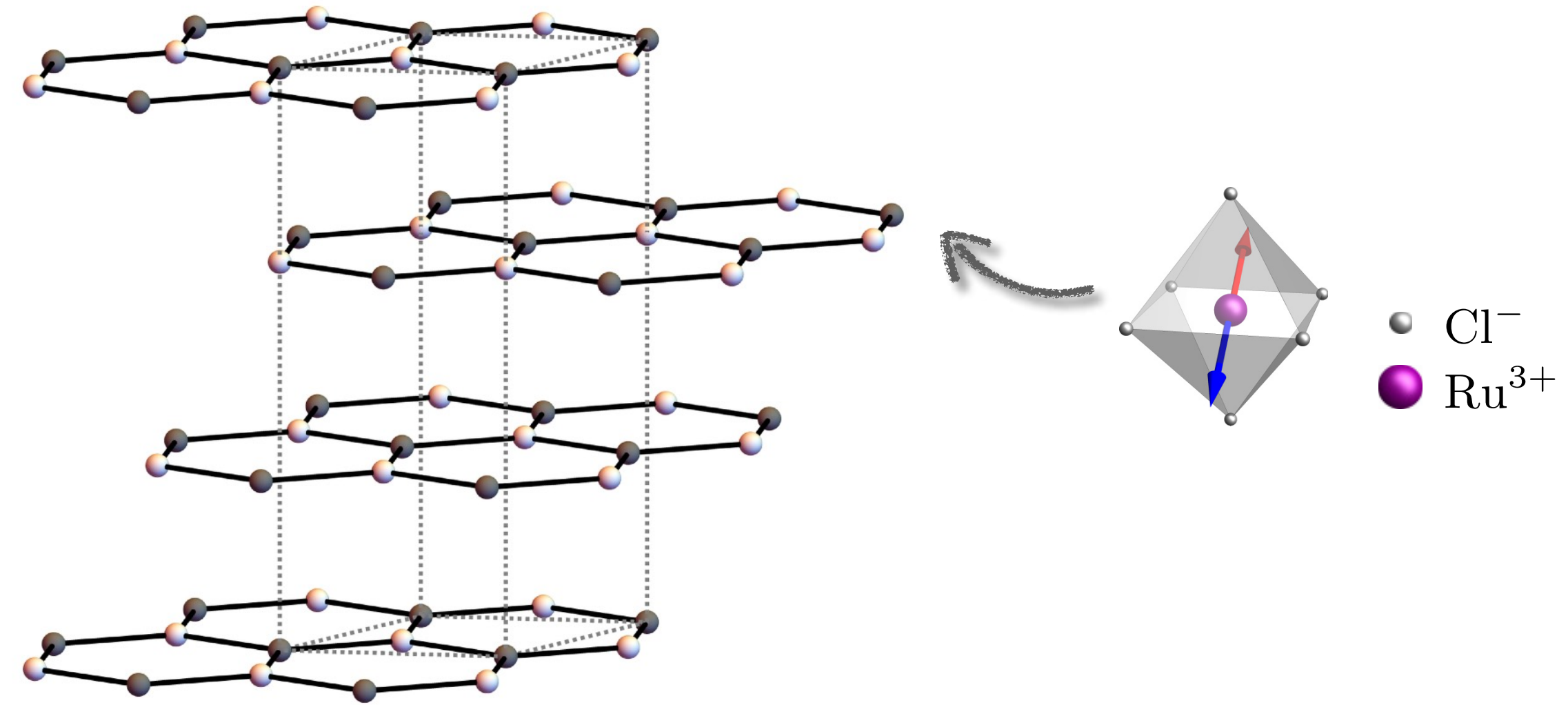
Emergent phenomena in quantum materials



Coherent quantum wave

α -RuCl₃

Crystal structure:

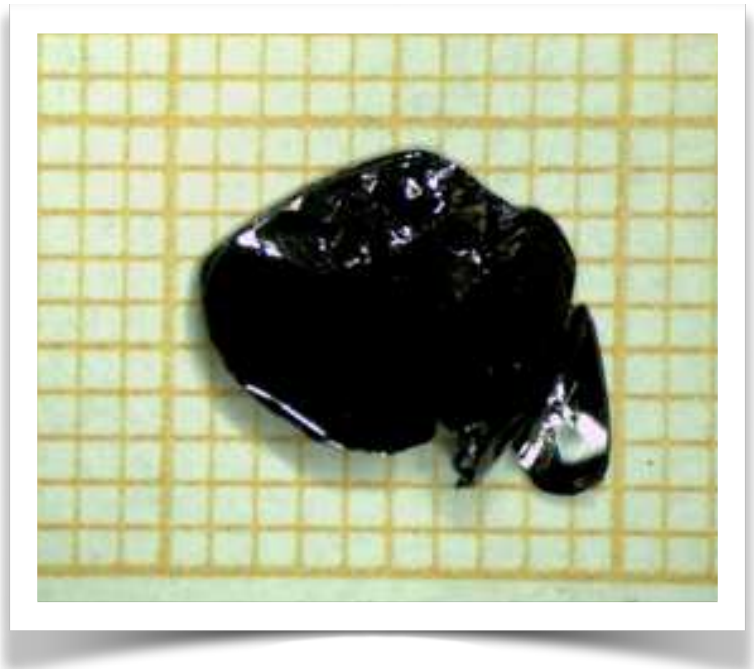
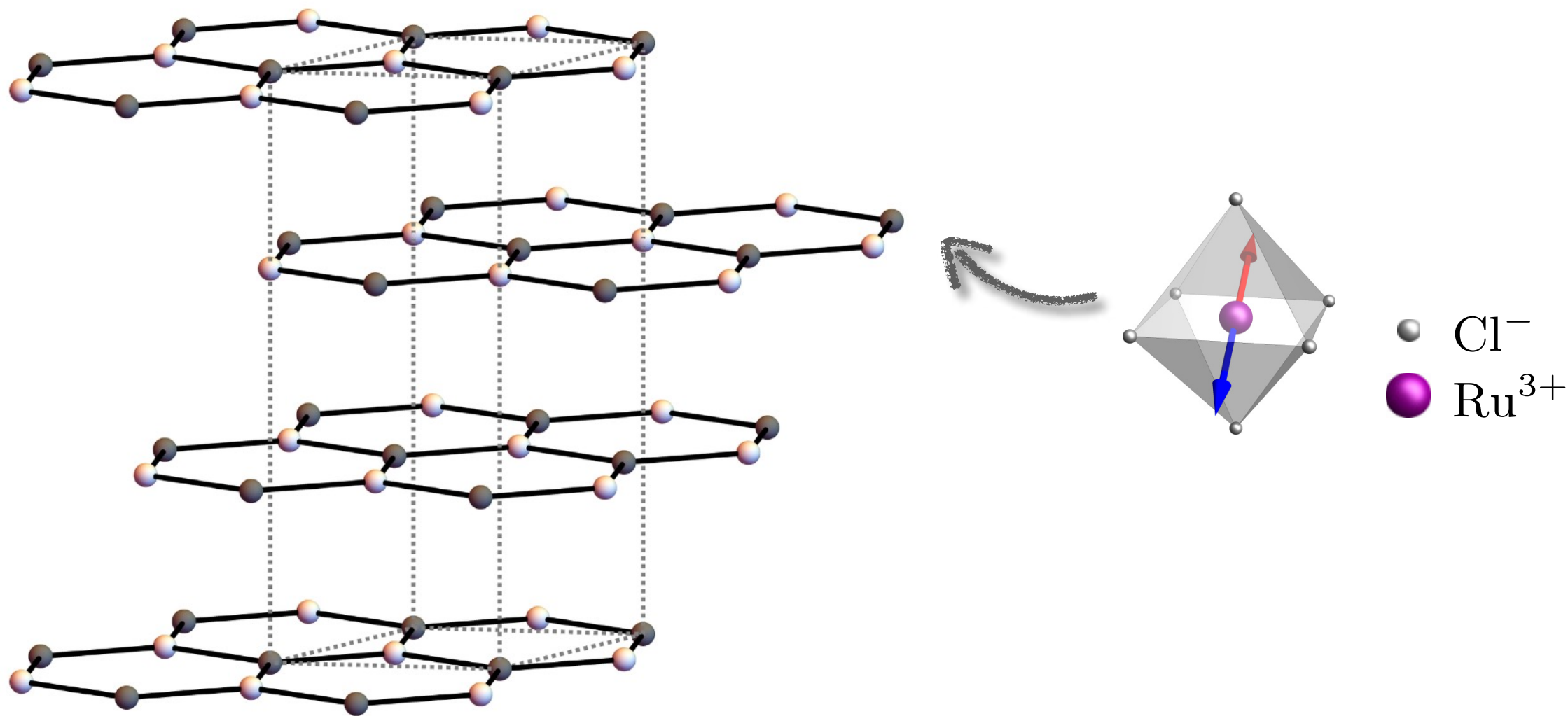


Credit: S. Wurmehl

[Balz *et al.*, PRB '21]

α -RuCl₃

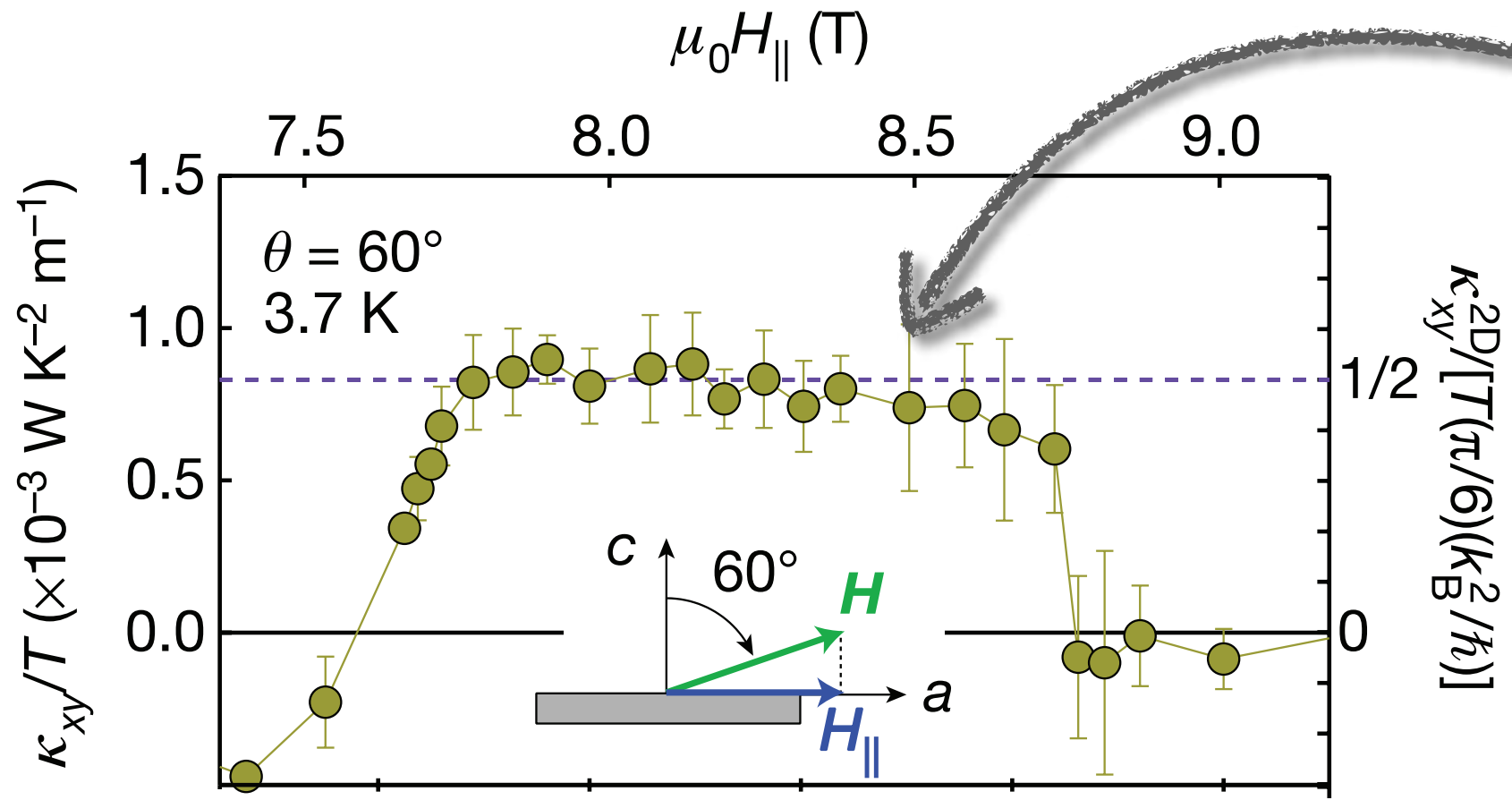
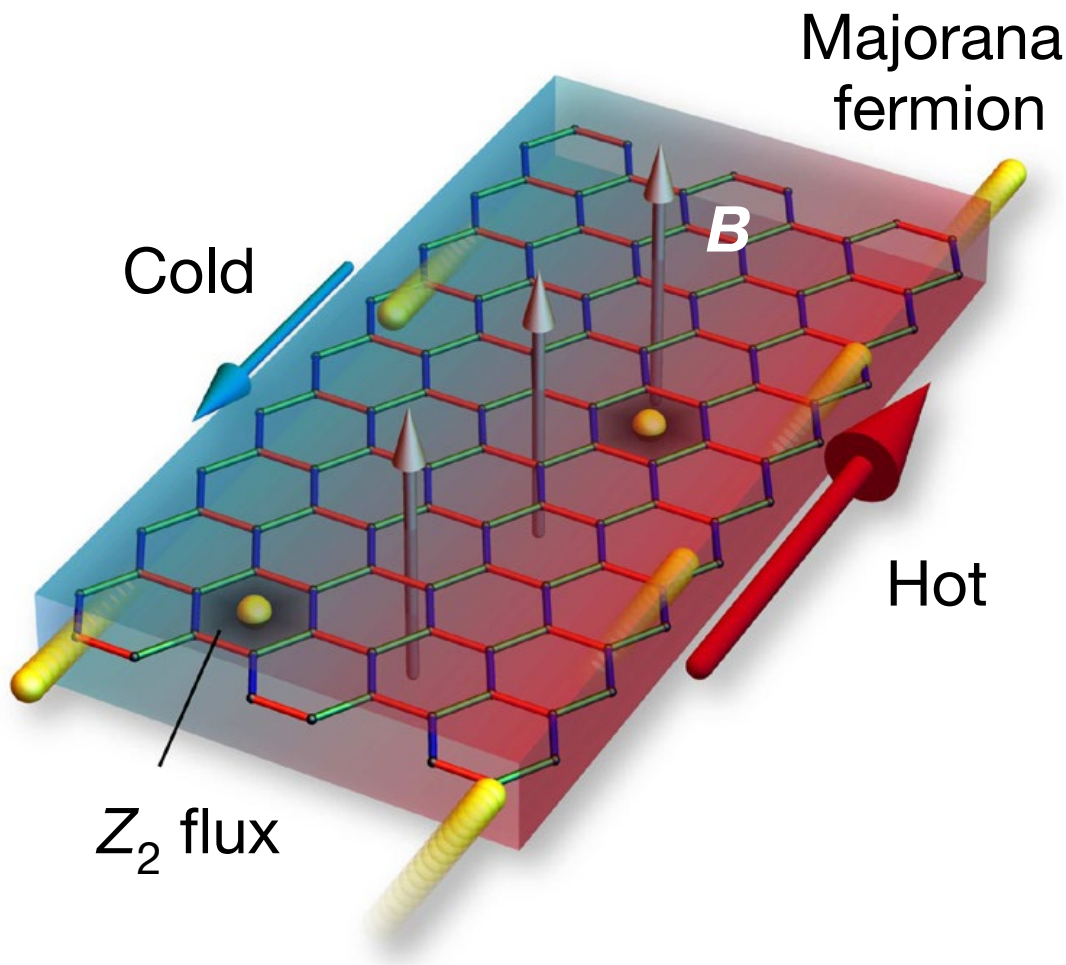
Crystal structure:



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[Balz *et al.*, PRB '21]

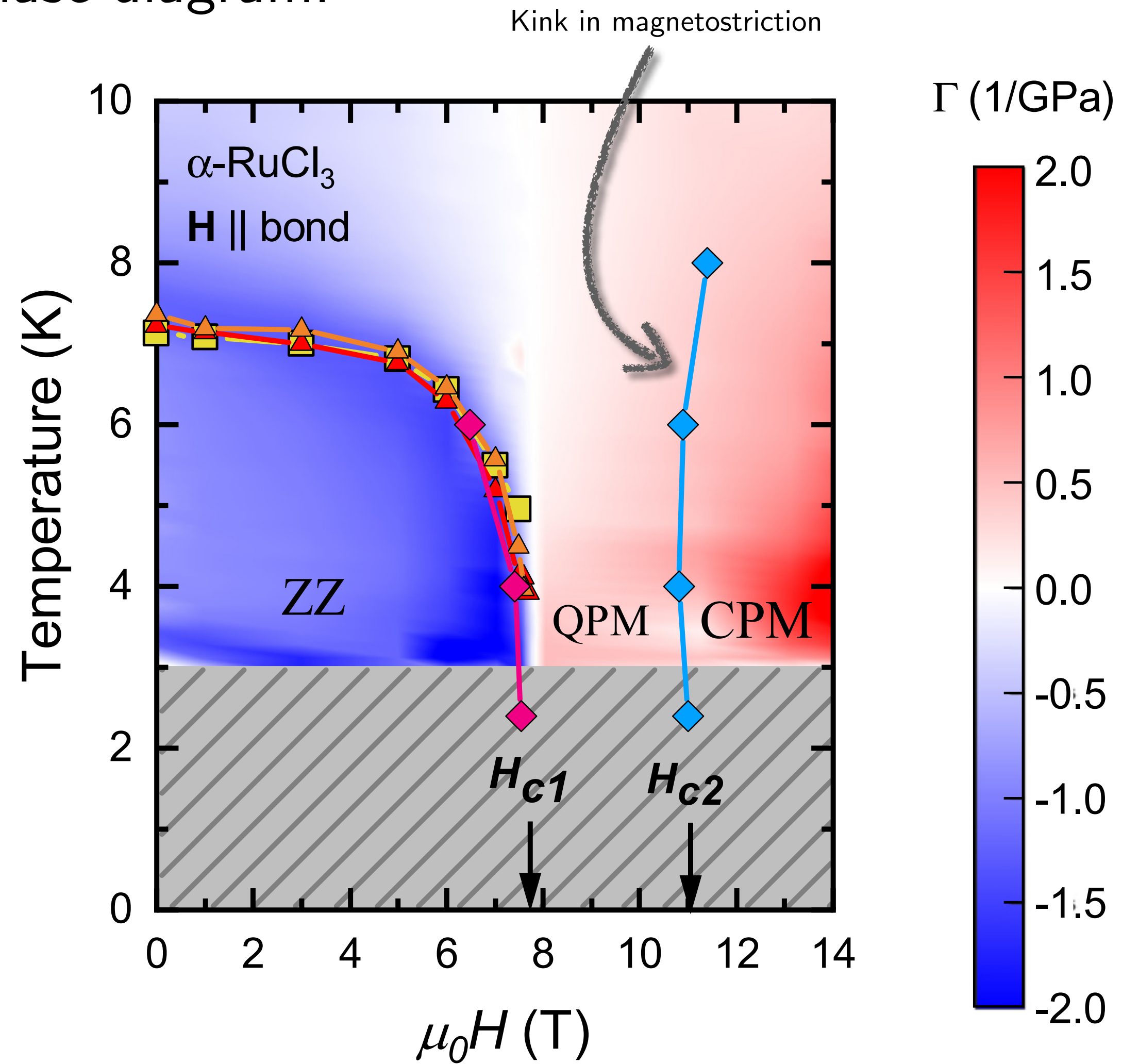
Half-integer quantum Hall effect:



[Kasahara *et al.*, Nature '18]
[Yokoi *et al.*, Science '21]

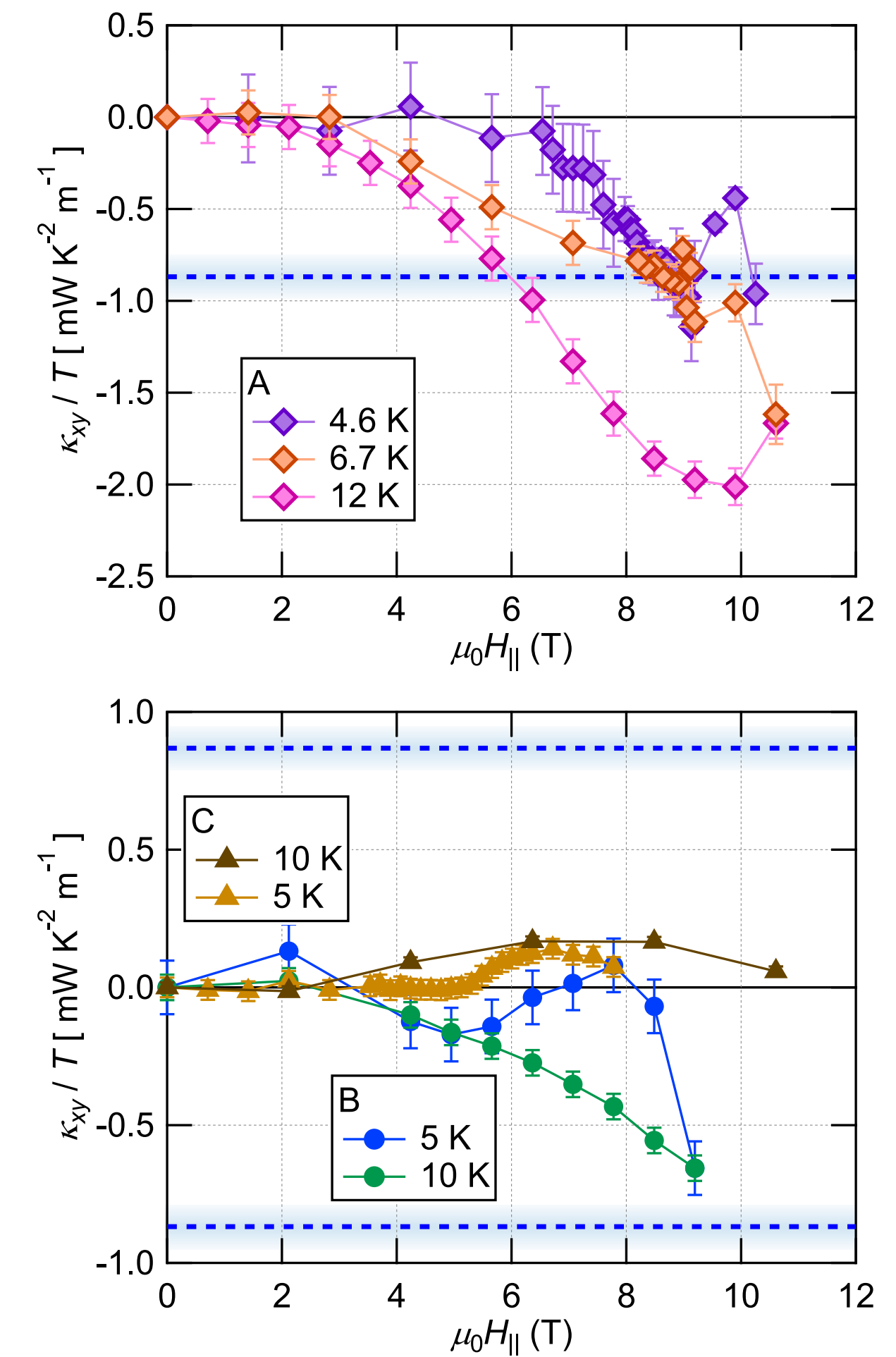
α -RuCl₃: Quantum phase diagram

Phase diagram:



[Gass *et al.*, PRB '20]

Sample dependence:



[Yamashita *et al.*, PRB '20]