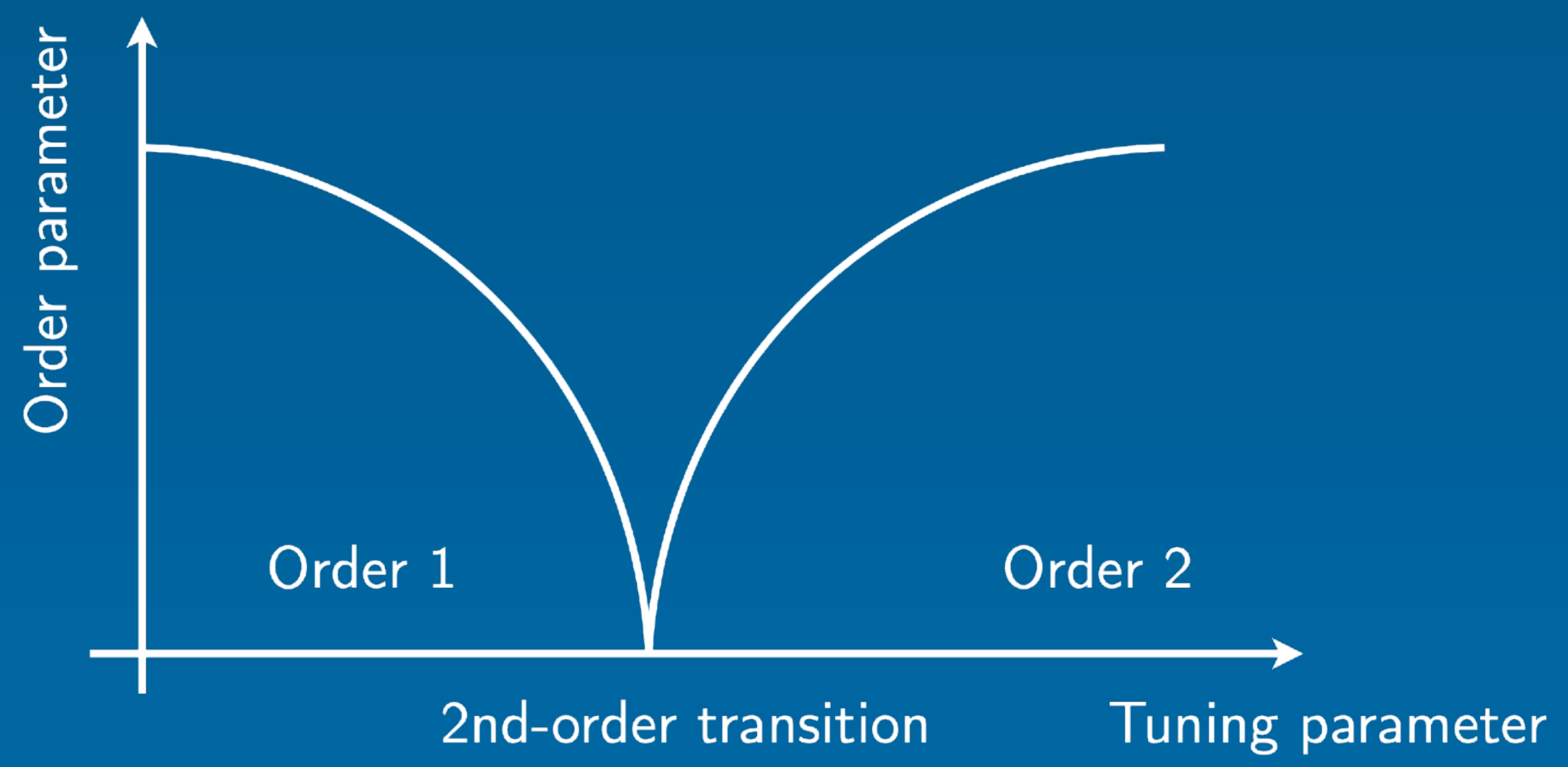
Continuous order-to-order transition from fixed-point annihilation

Lukas Janssen

David Moser



Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft
ct.qmat Complexity and Topology in Quantum Matter

Outline

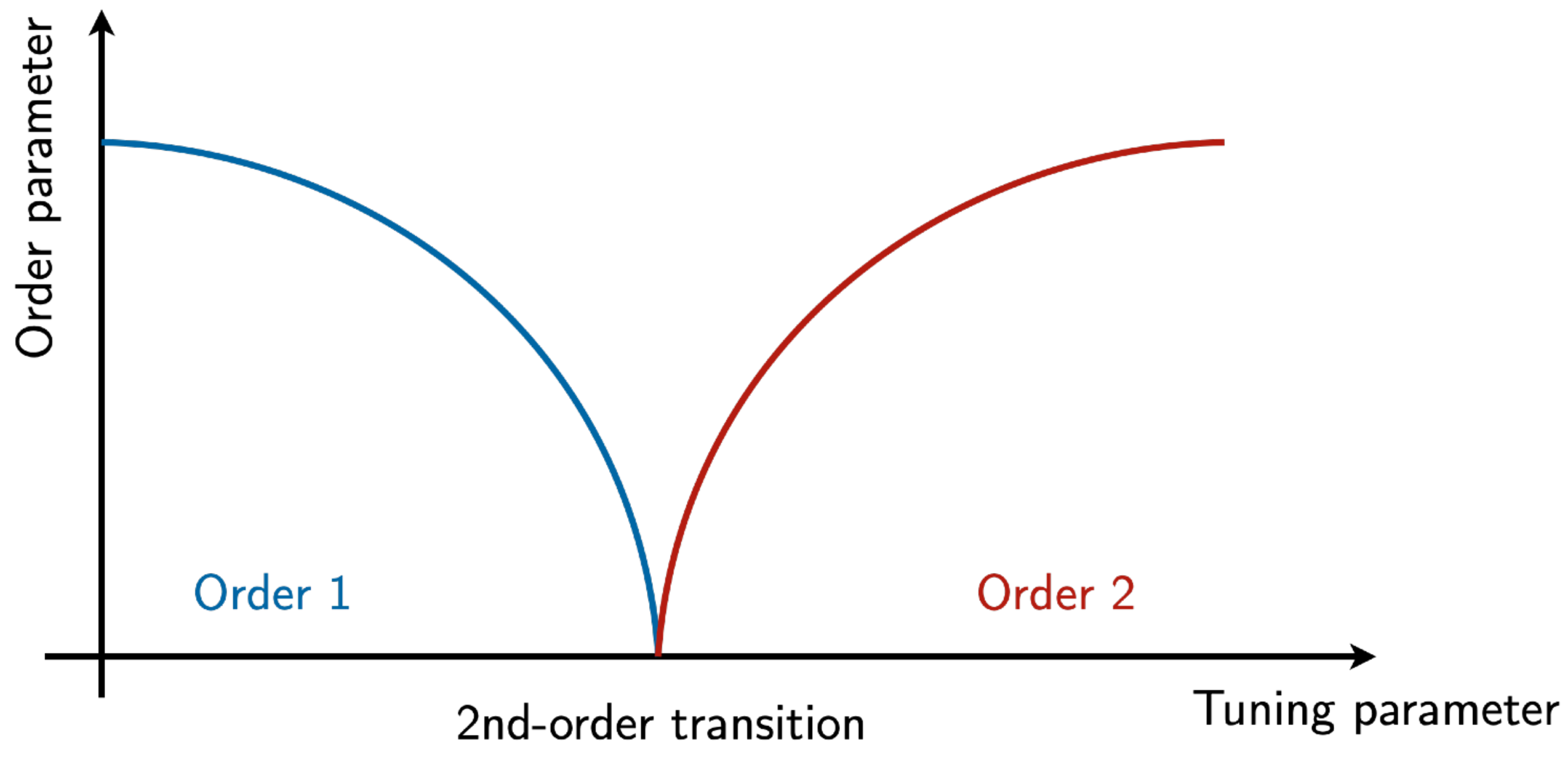
(1) Introduction

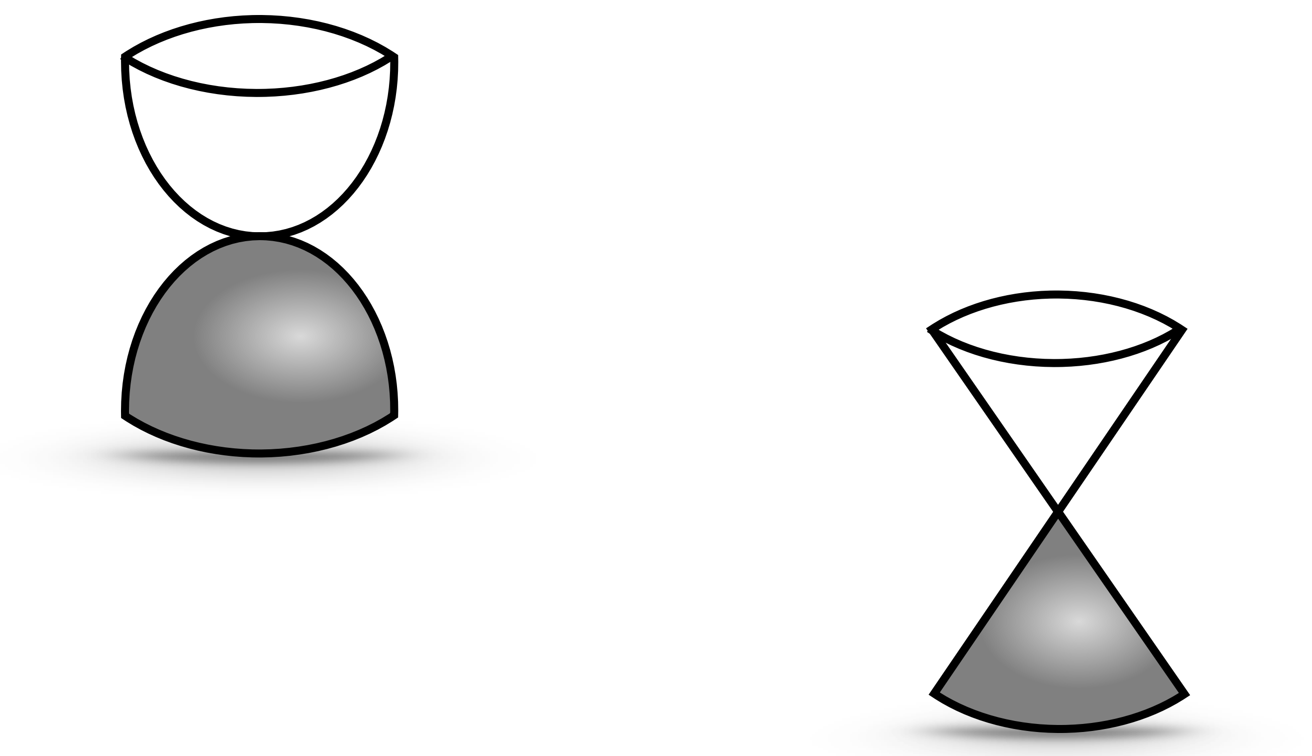
(2) Continuous order-to-order transition from fixed-point annihilation

(3) Examples

- Luttinger semimetals
- Dirac spin liquids

(4) Conclusions





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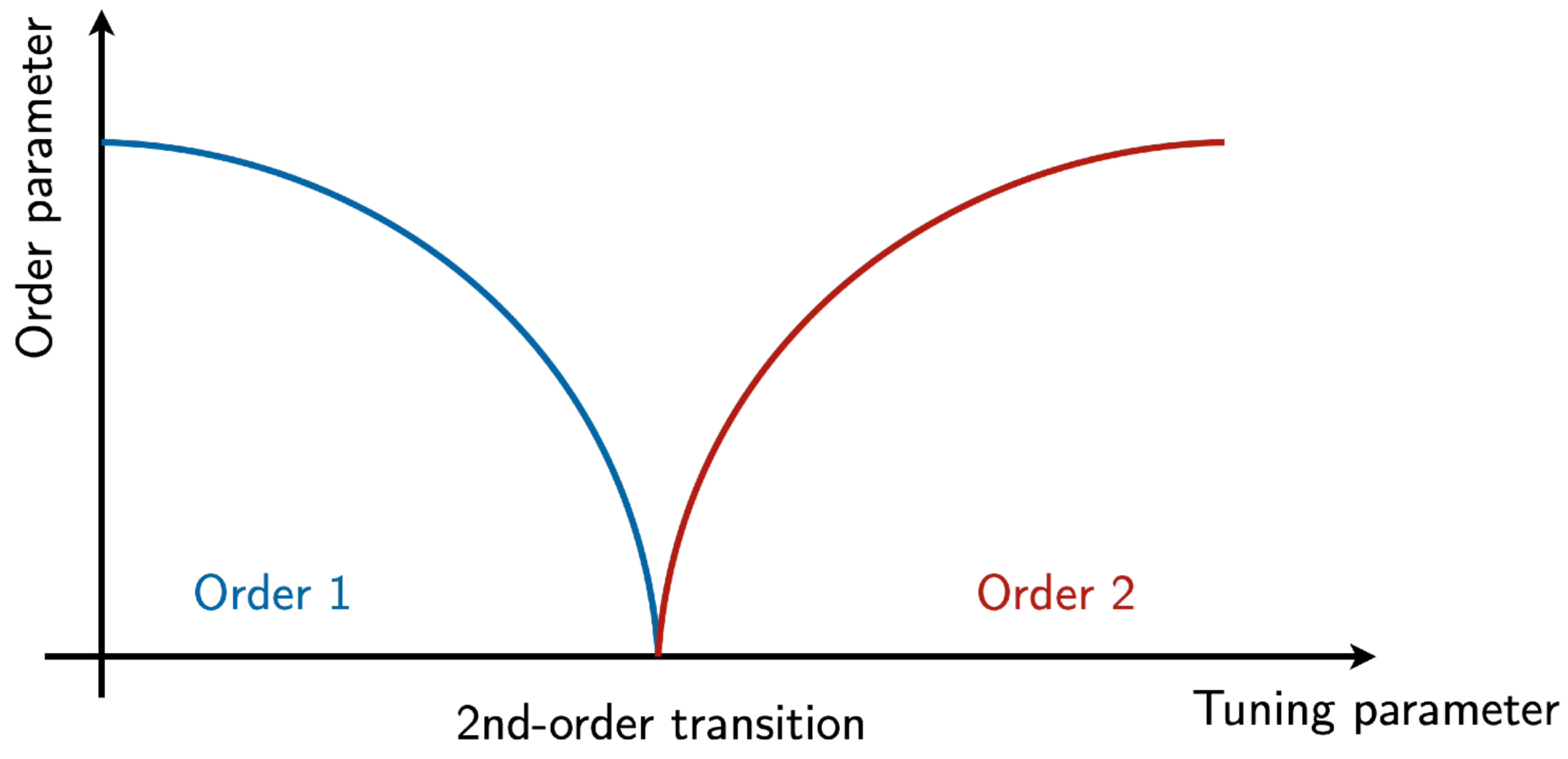
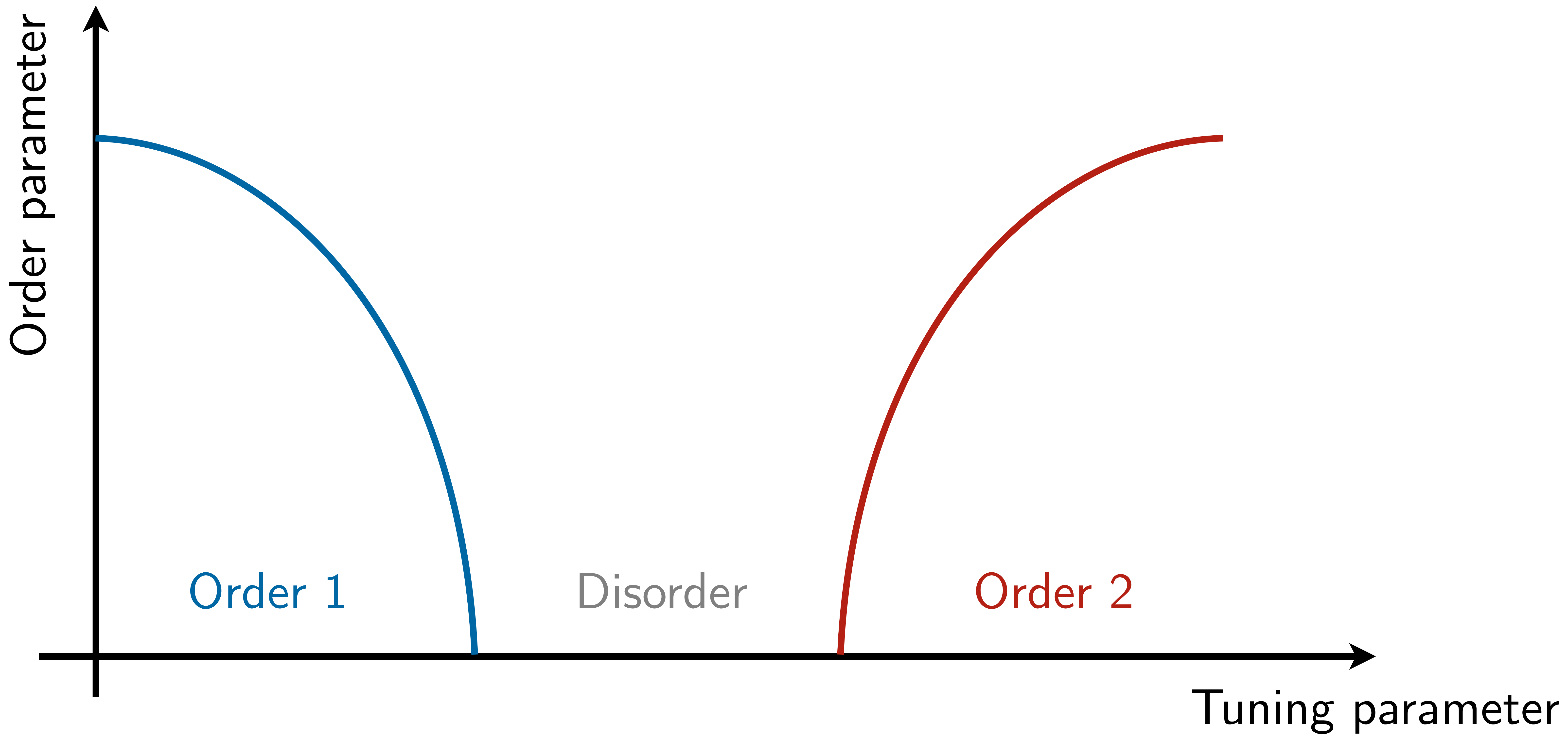


Competing orders



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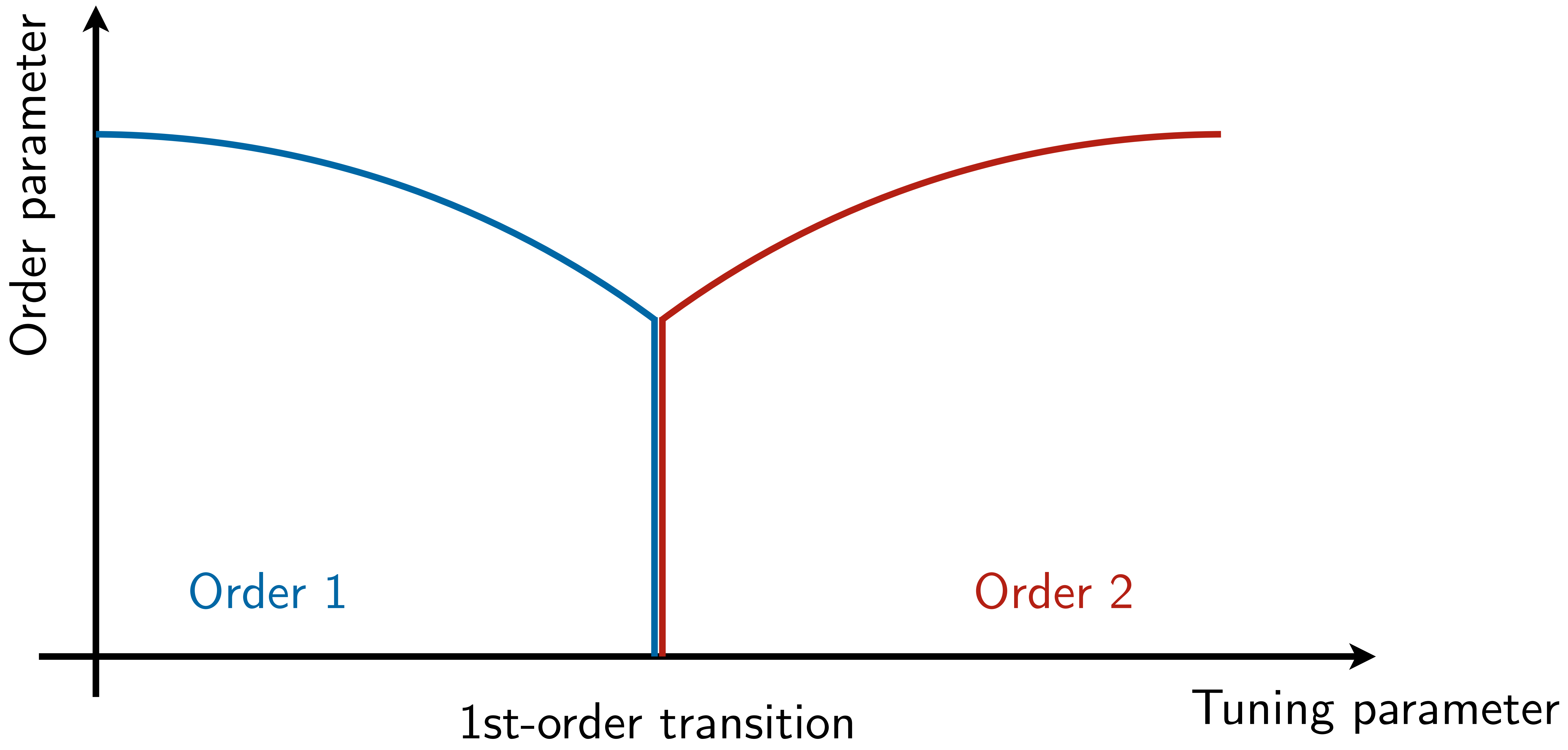

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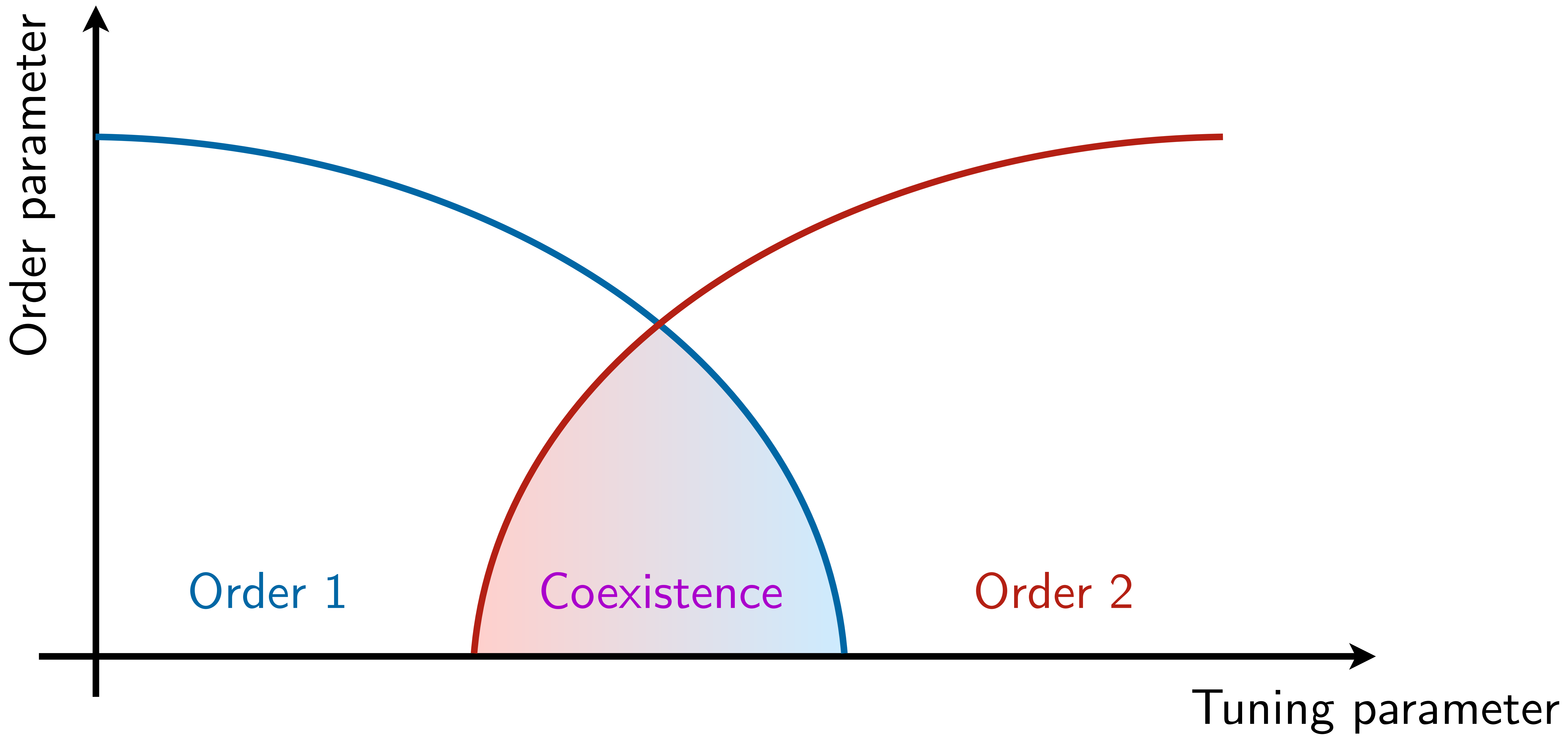

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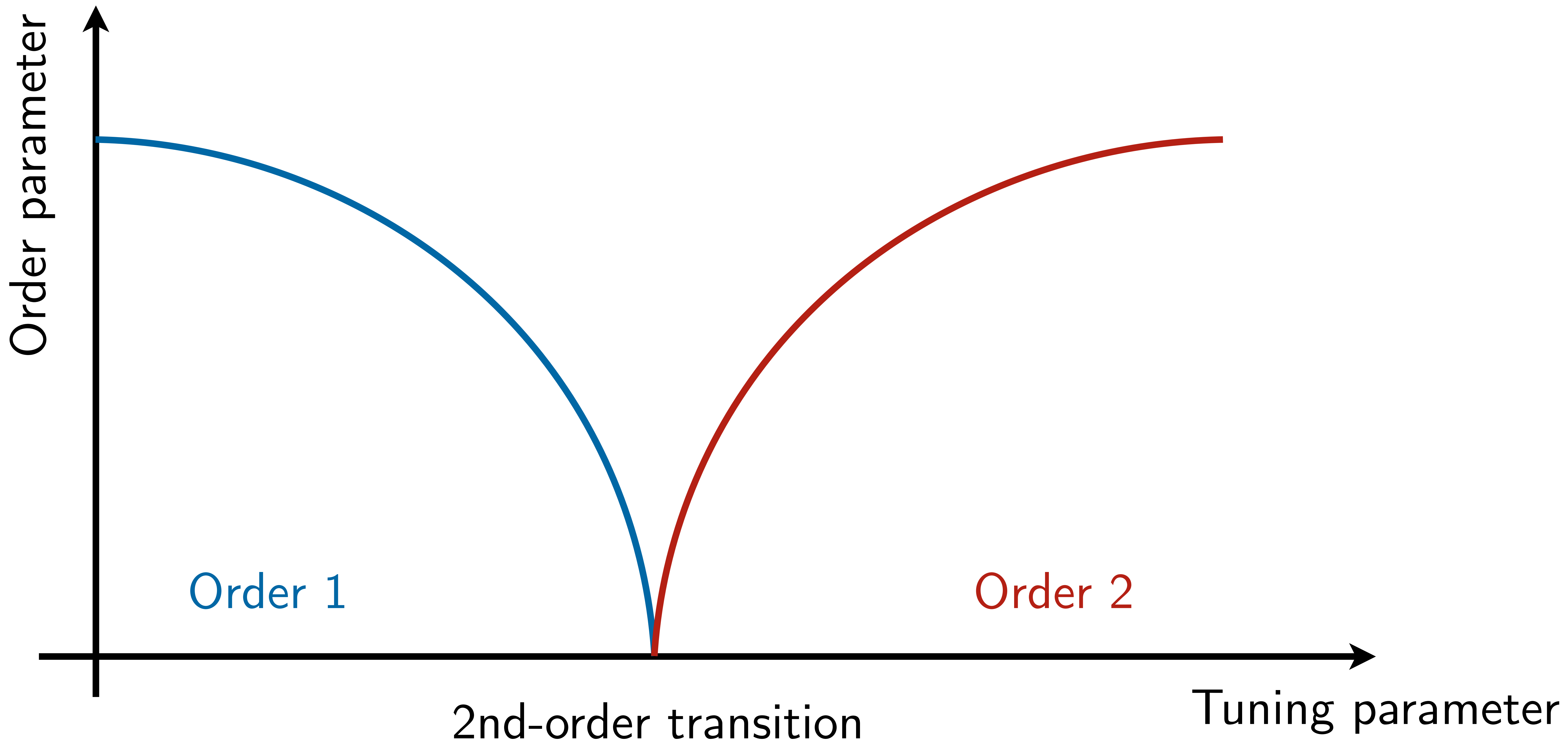

Landau

Landau theory

Landau functional: $$F(\phi, \varphi) = \frac{r_1}{2}\phi^2 + \frac{r_2}{2}\varphi^2 + \lambda_1\phi^4 + \lambda_2\varphi^4 + 2\lambda_{12}\phi^2\varphi^2 + \mathcal{O}((\phi, \varphi)^6)$$

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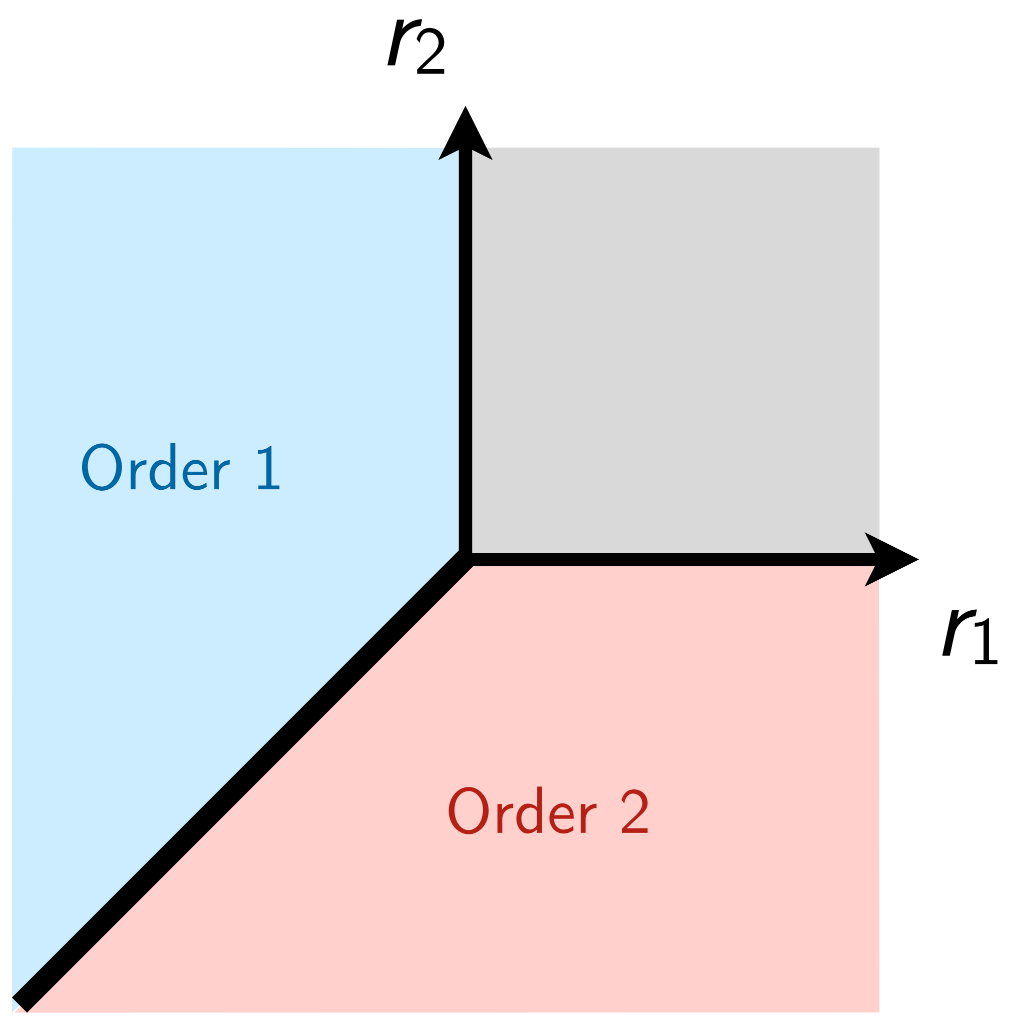
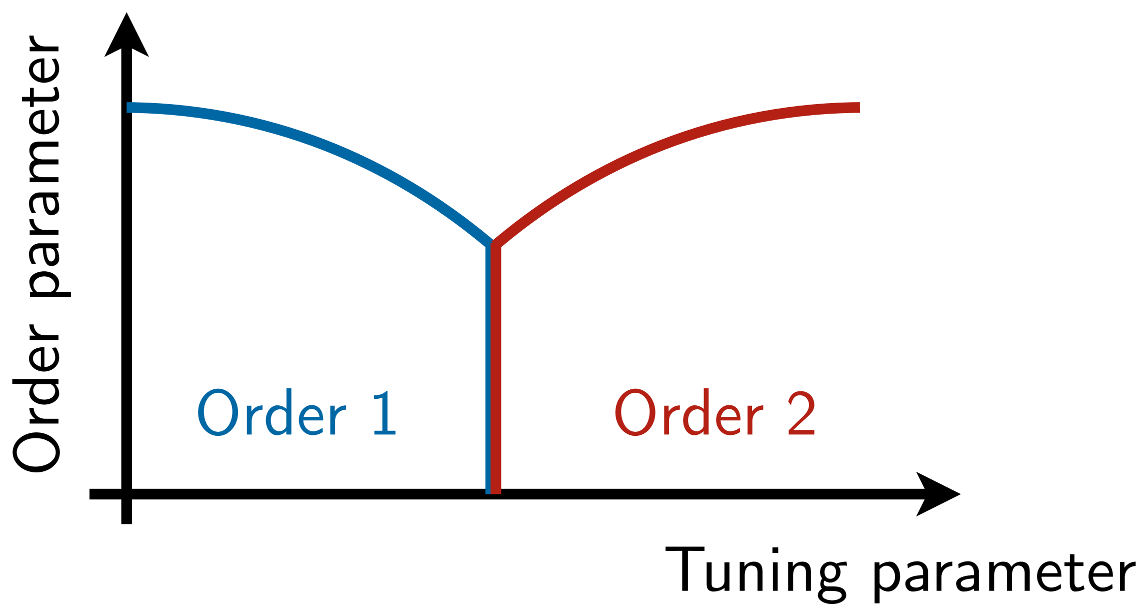
$$\lambda_1\lambda_2 - \lambda_{12}^2 \leq 0$$





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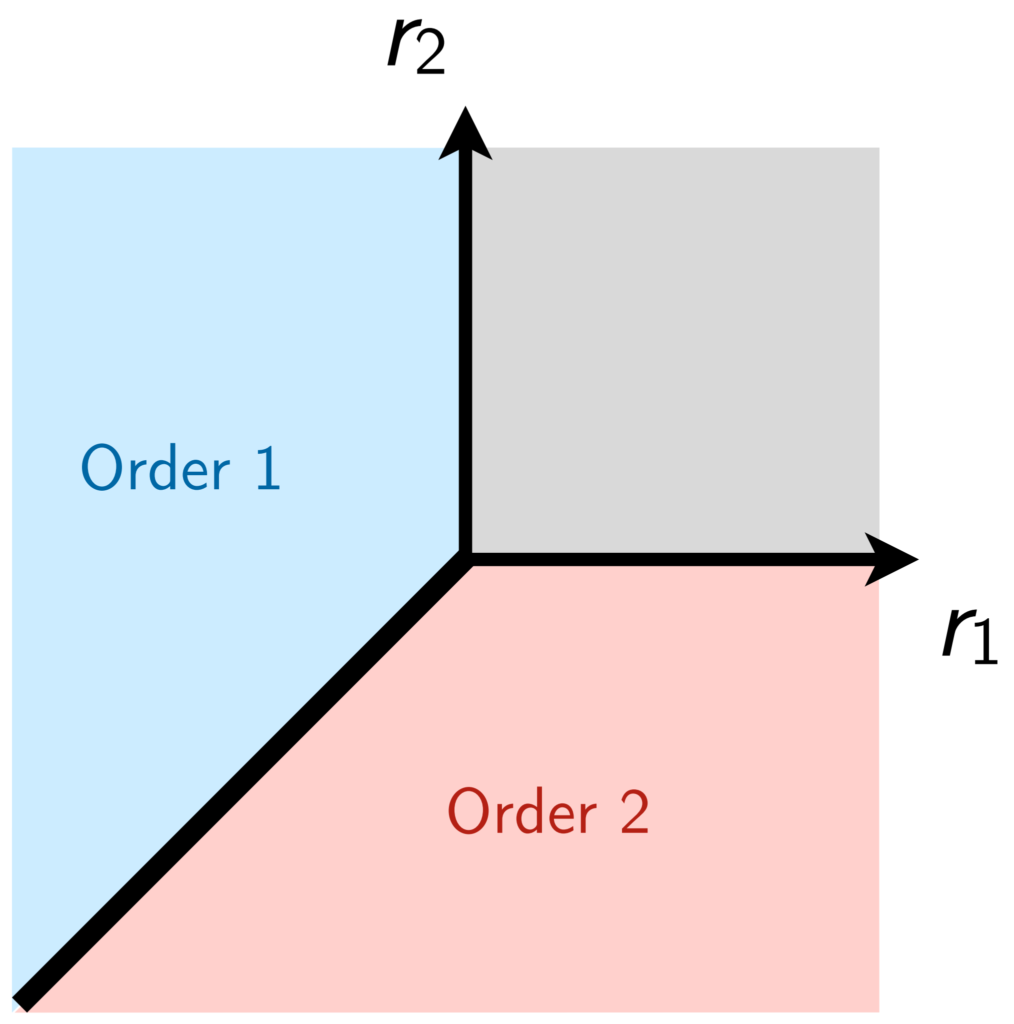
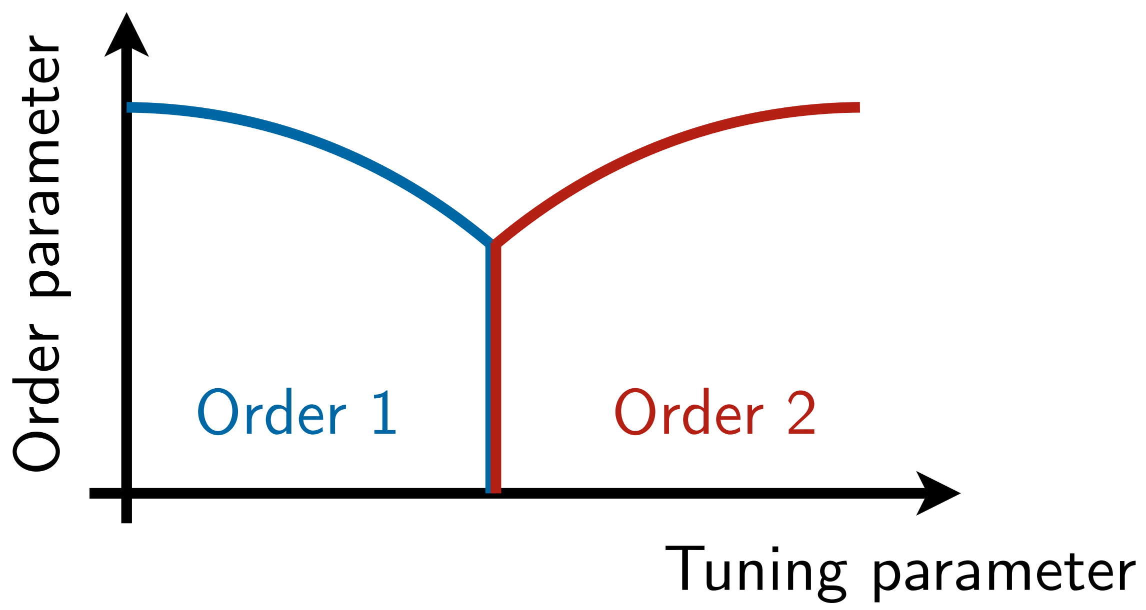
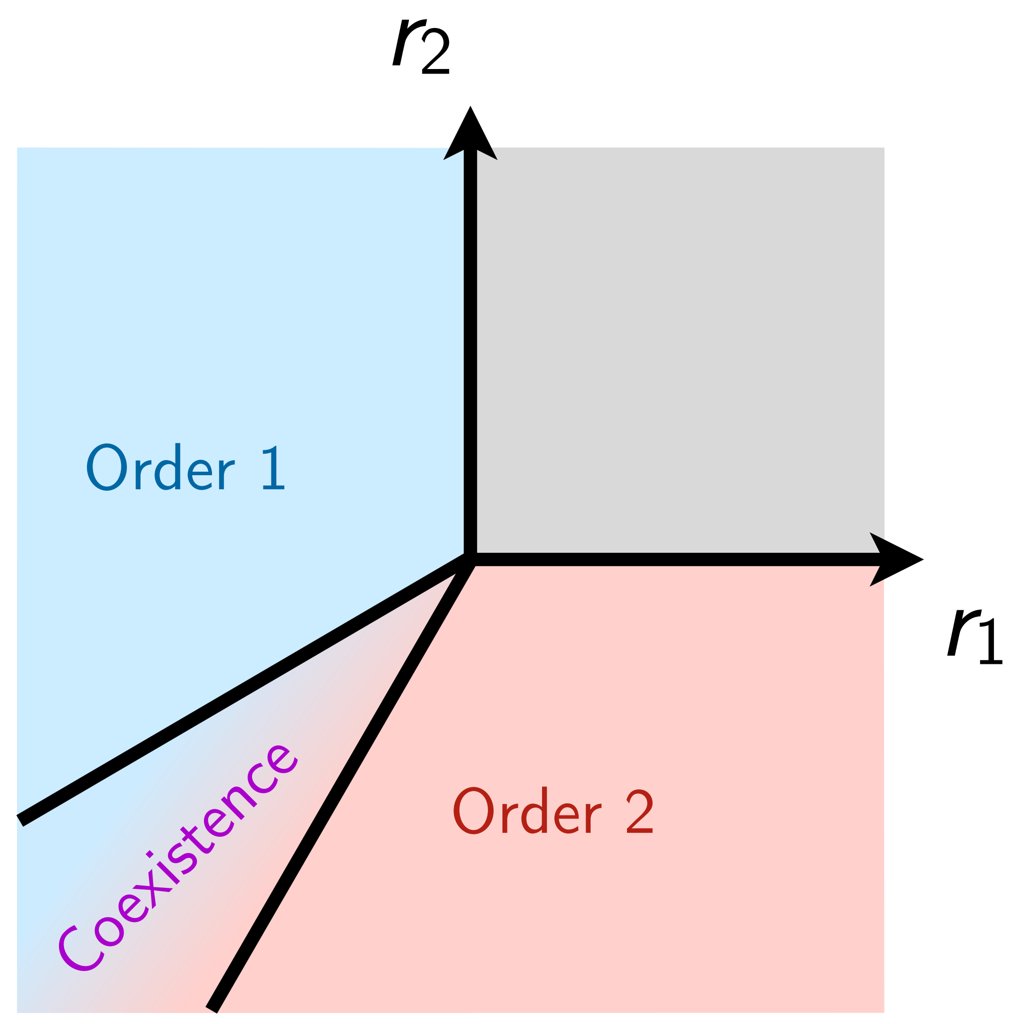
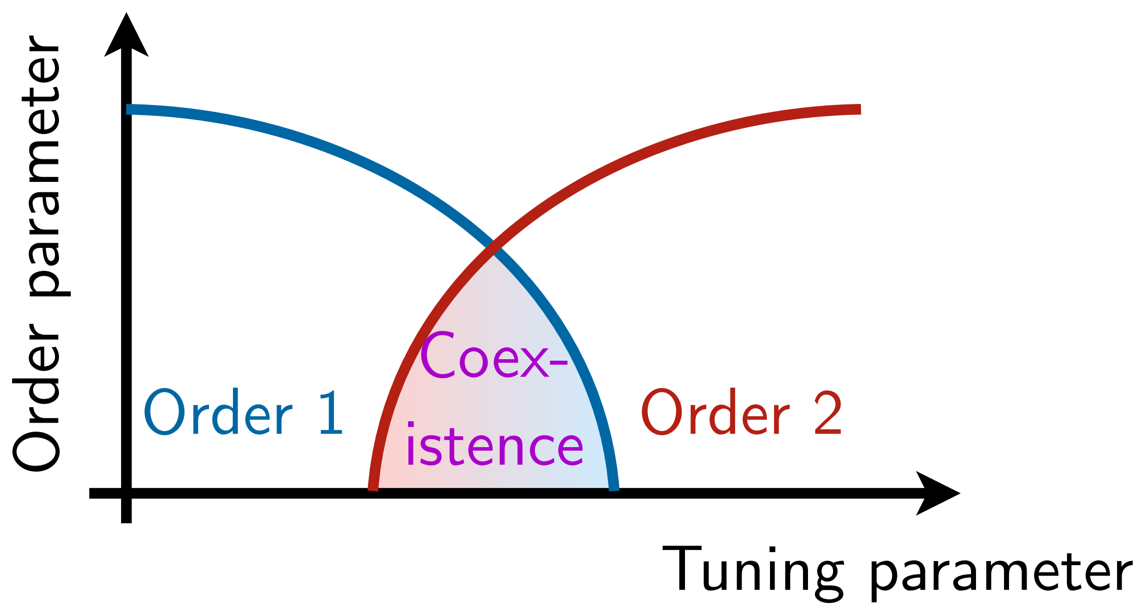




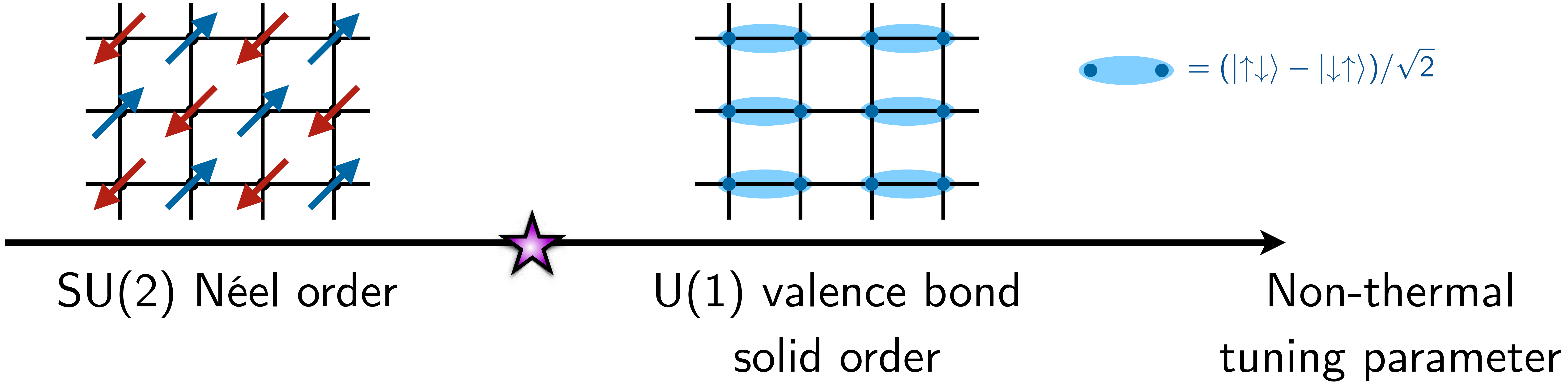
$\lambda_1\lambda_2 - \lambda_{12}^2 > 0$





Deconfined criticality

SU(2)-to-U(1) transition:



... driven by defects

[Senthil *et al.*, Science '04]

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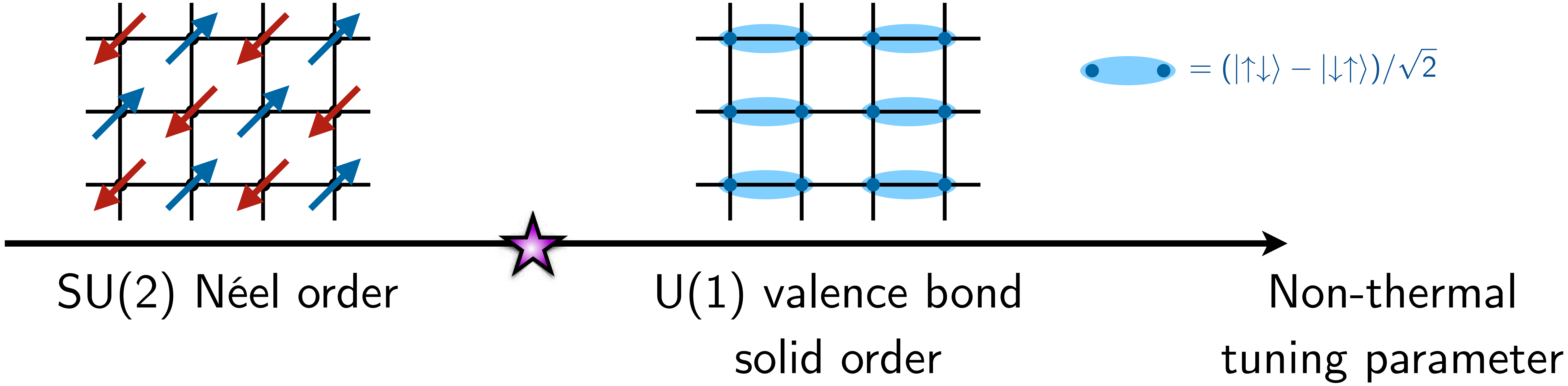

... driven by defects

[Senthil *et al.*, Science '04]

Effective field theory:

$$\mathcal{L} = (D_\mu z)^\dagger D_\mu z \quad \text{with} \quad z^\dagger z = 1 \qquad \text{“}\mathbb{C}P^1 \text{ model”}$$

where $D_\mu = \partial_\mu - i a_\mu$

Deconfined criticality

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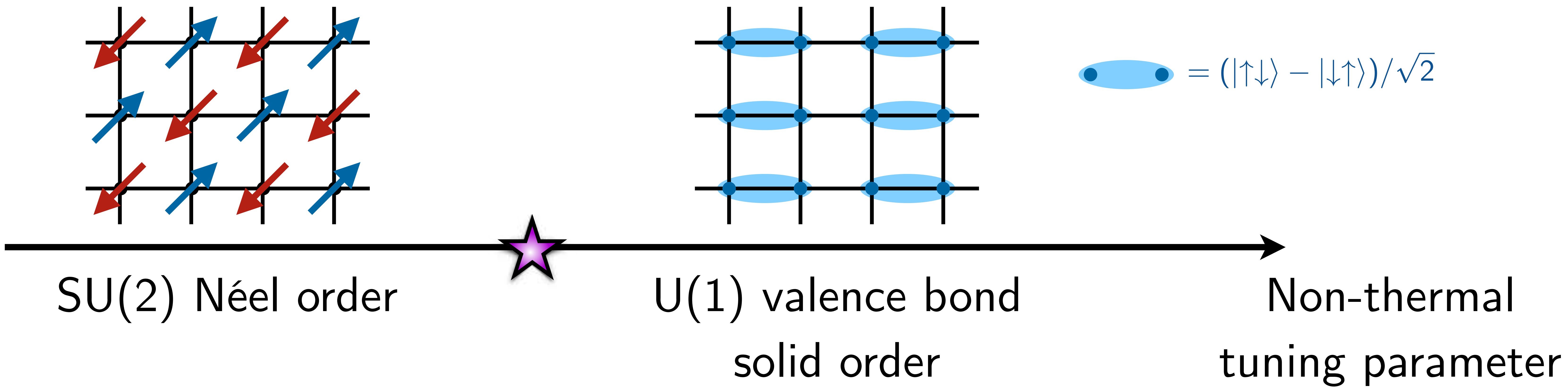

... driven by defects

[Senthil *et al.*, Science '04]

Effective field theory:

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where $D_\mu = \partial_\mu - i a_\mu$

Scenario very beautiful, but ...

... quite complex

... very specific

... probably not realized

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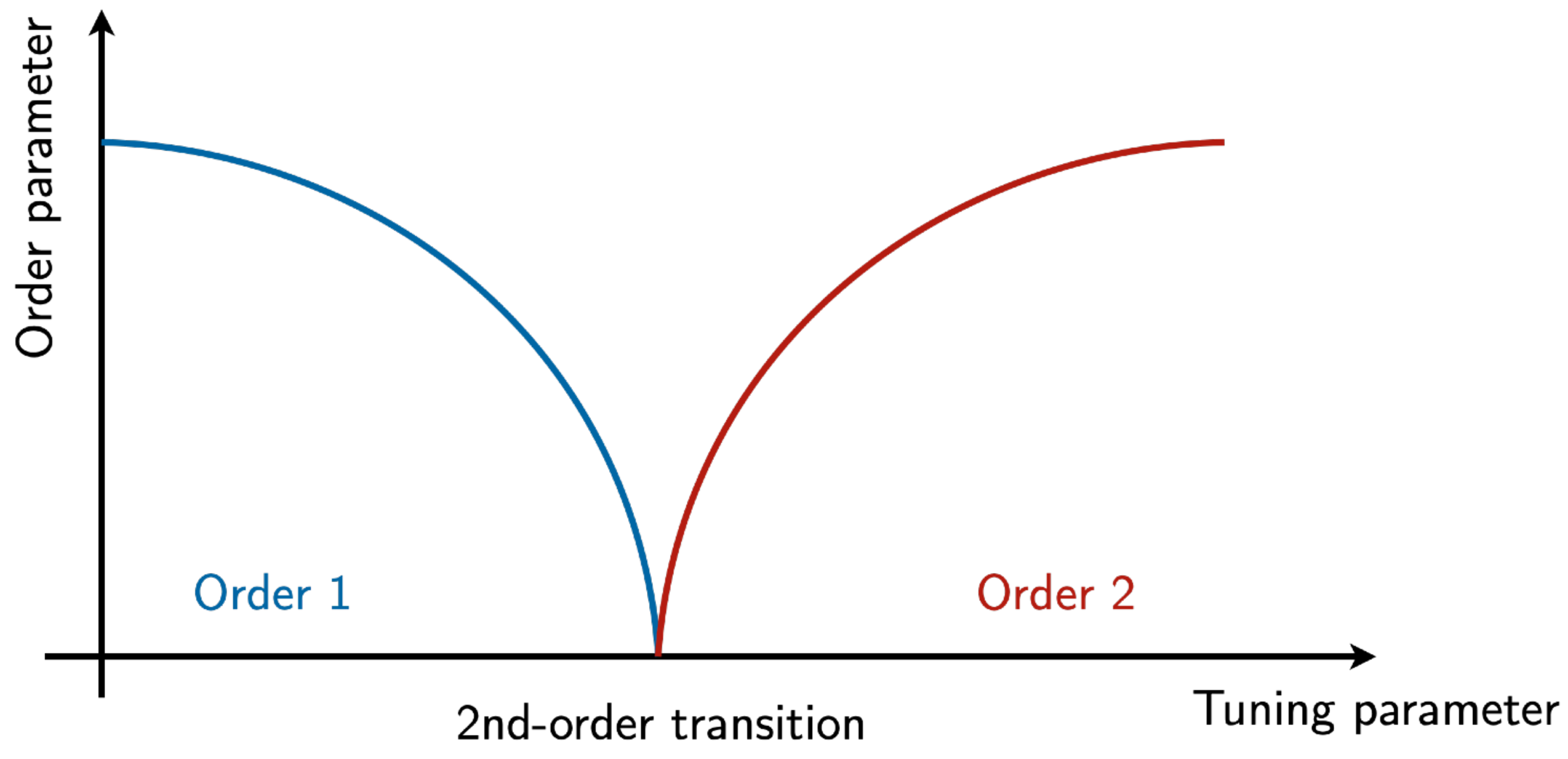
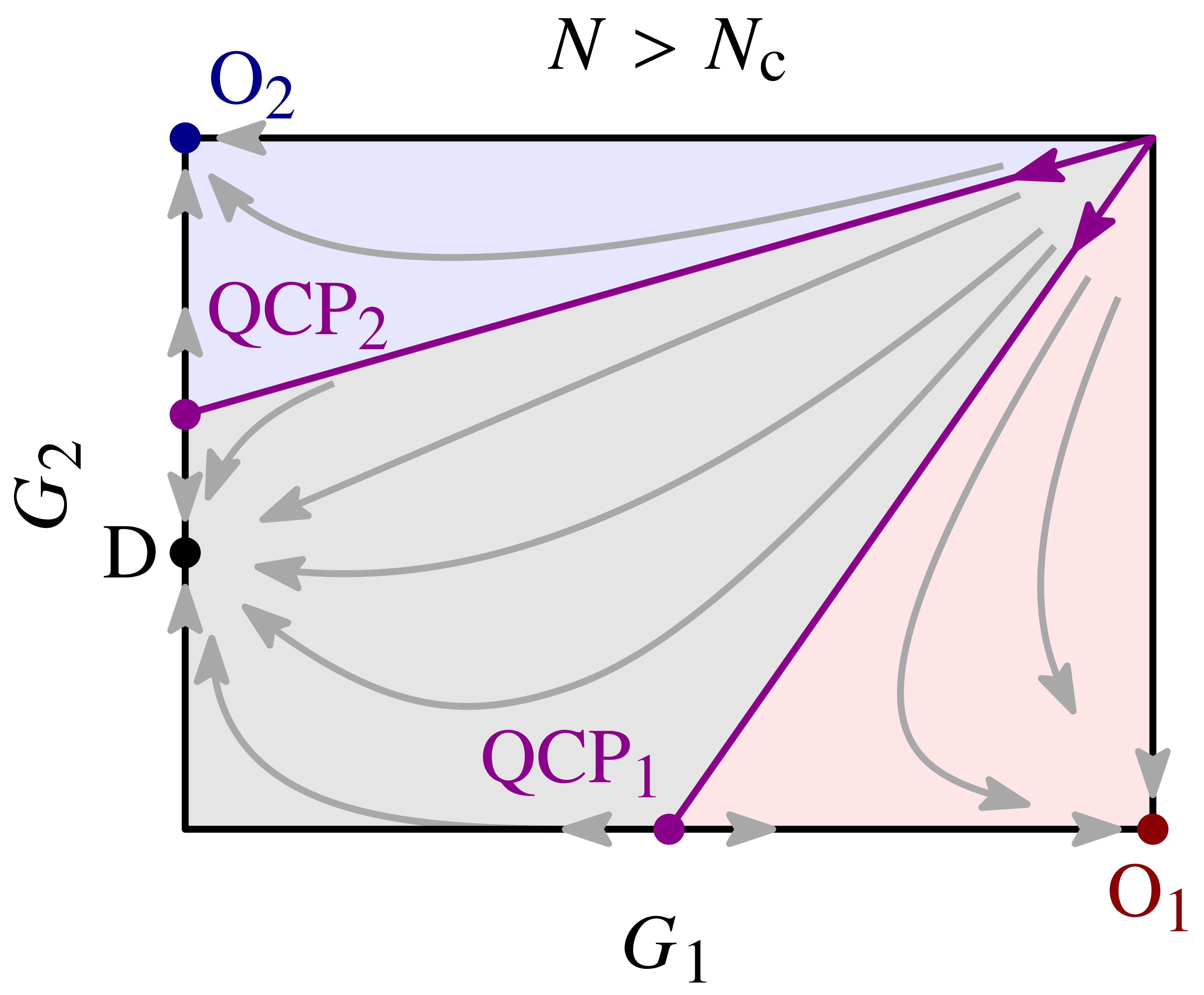


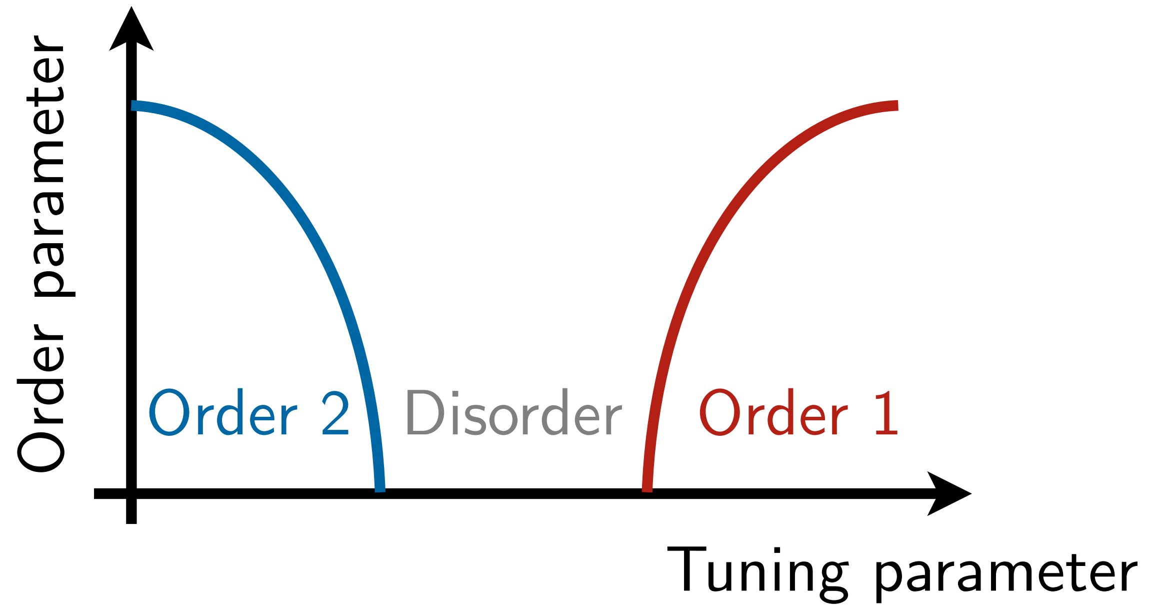
(3) Examples

- Luttinger semimetals
- Dirac spin liquids

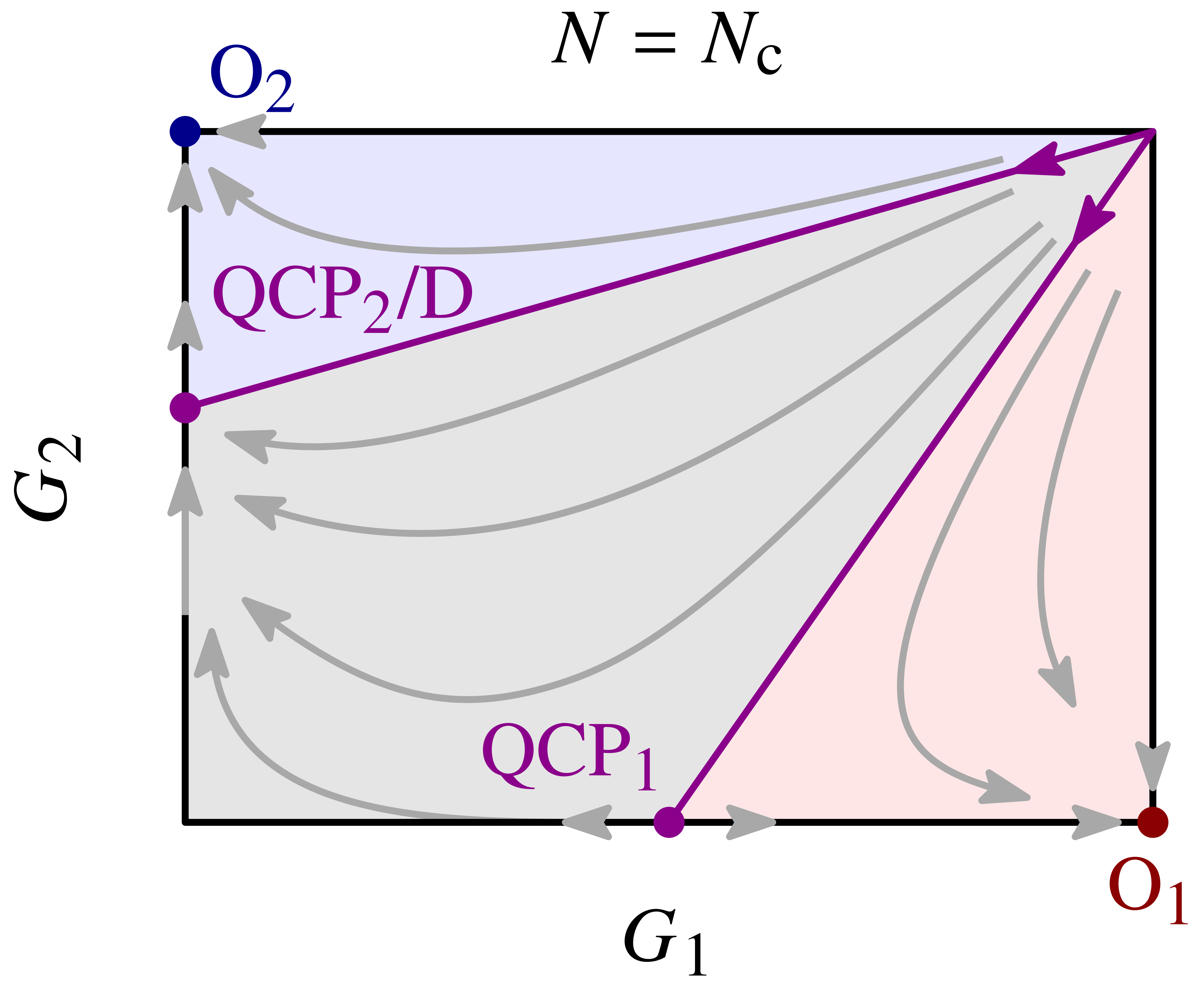
(4) Conclusions

Mechanism

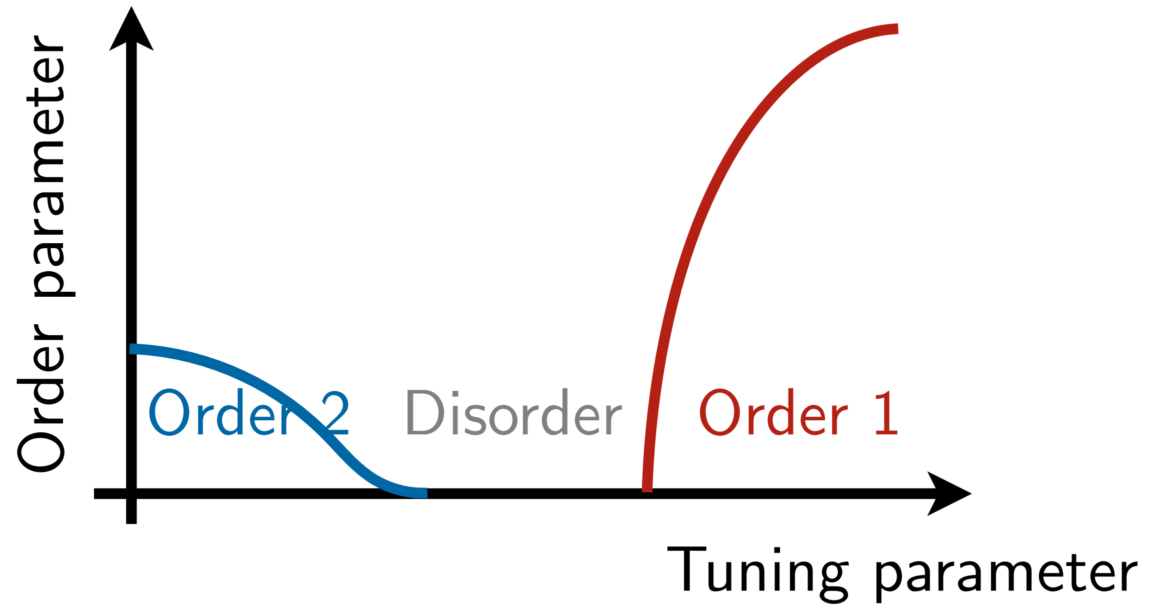

$N > N_c$
O_2
QCP_2
G_2
D
QCP_1
O_1
G_1


Order parameter
Order 2
Disorder
Order 1
Tuning parameter

Mechanism



$N = N_c$
O_2
QCP$_2$/D
G_2
QCP$_1$
O_1
G_1



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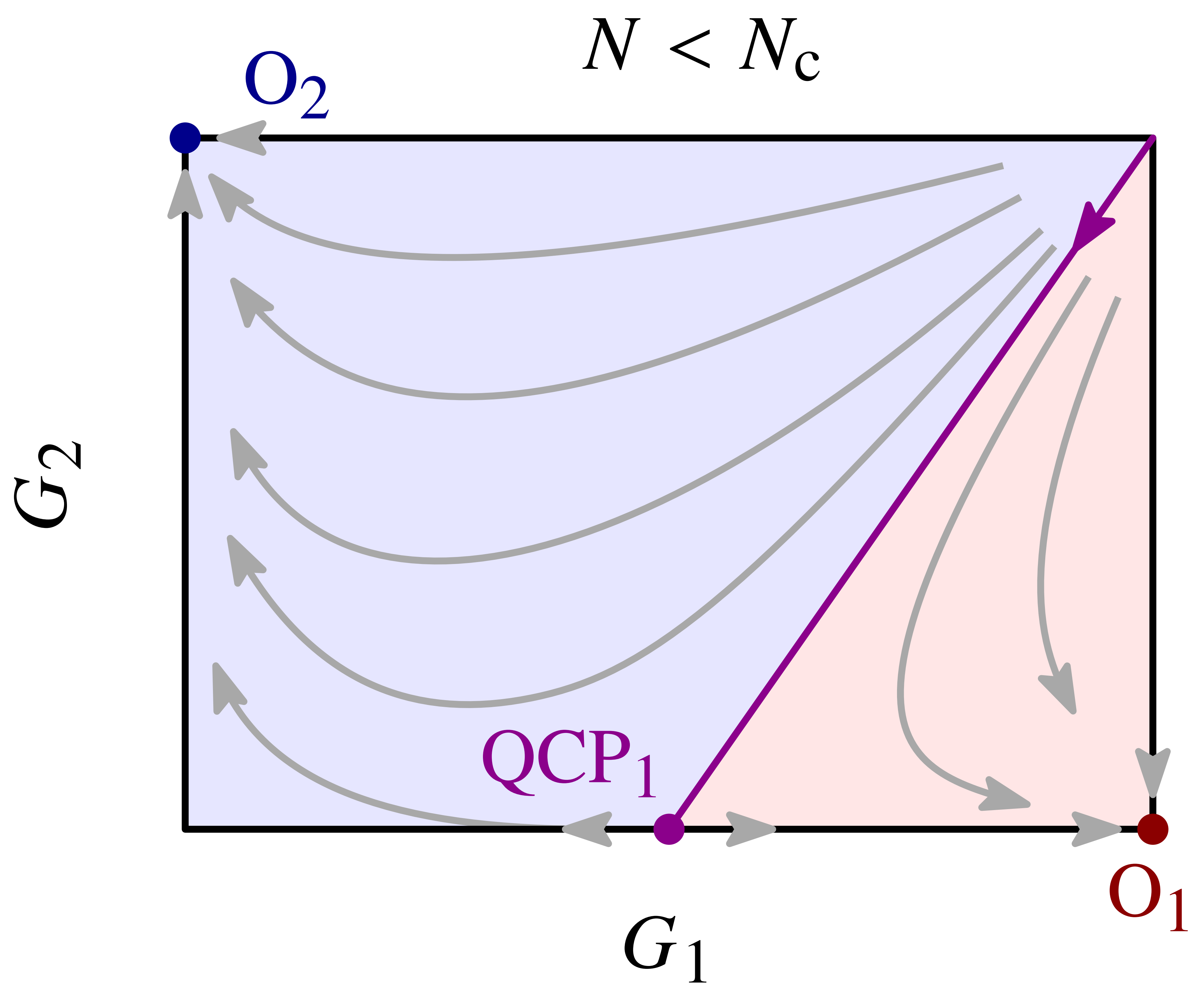
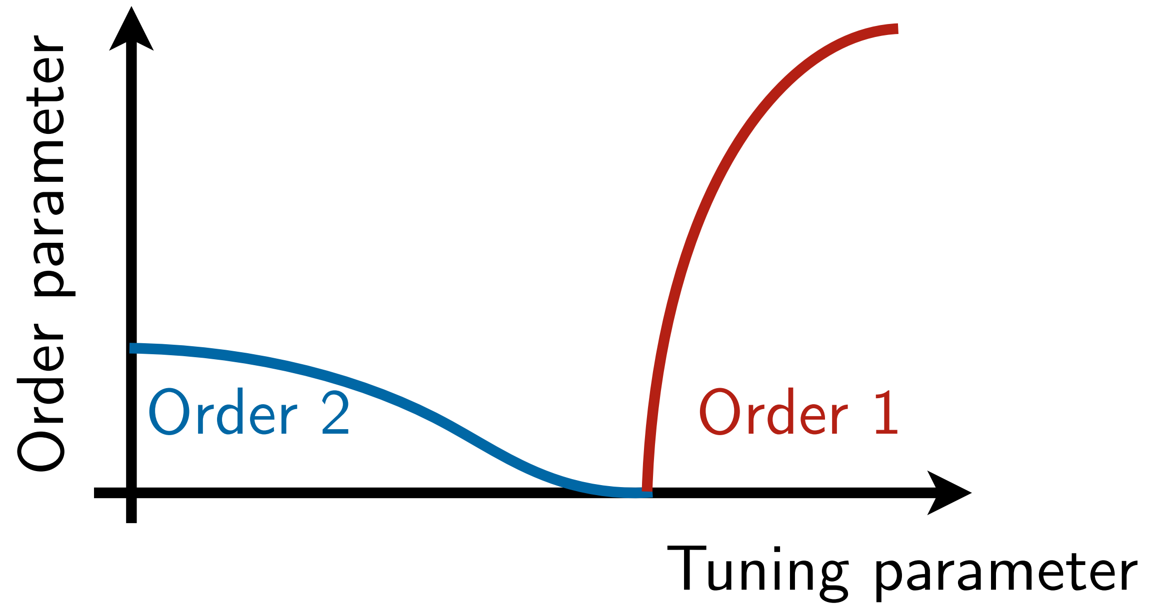


$N < N_c$
O_2
G_2
QCP_1
G_1
O_1



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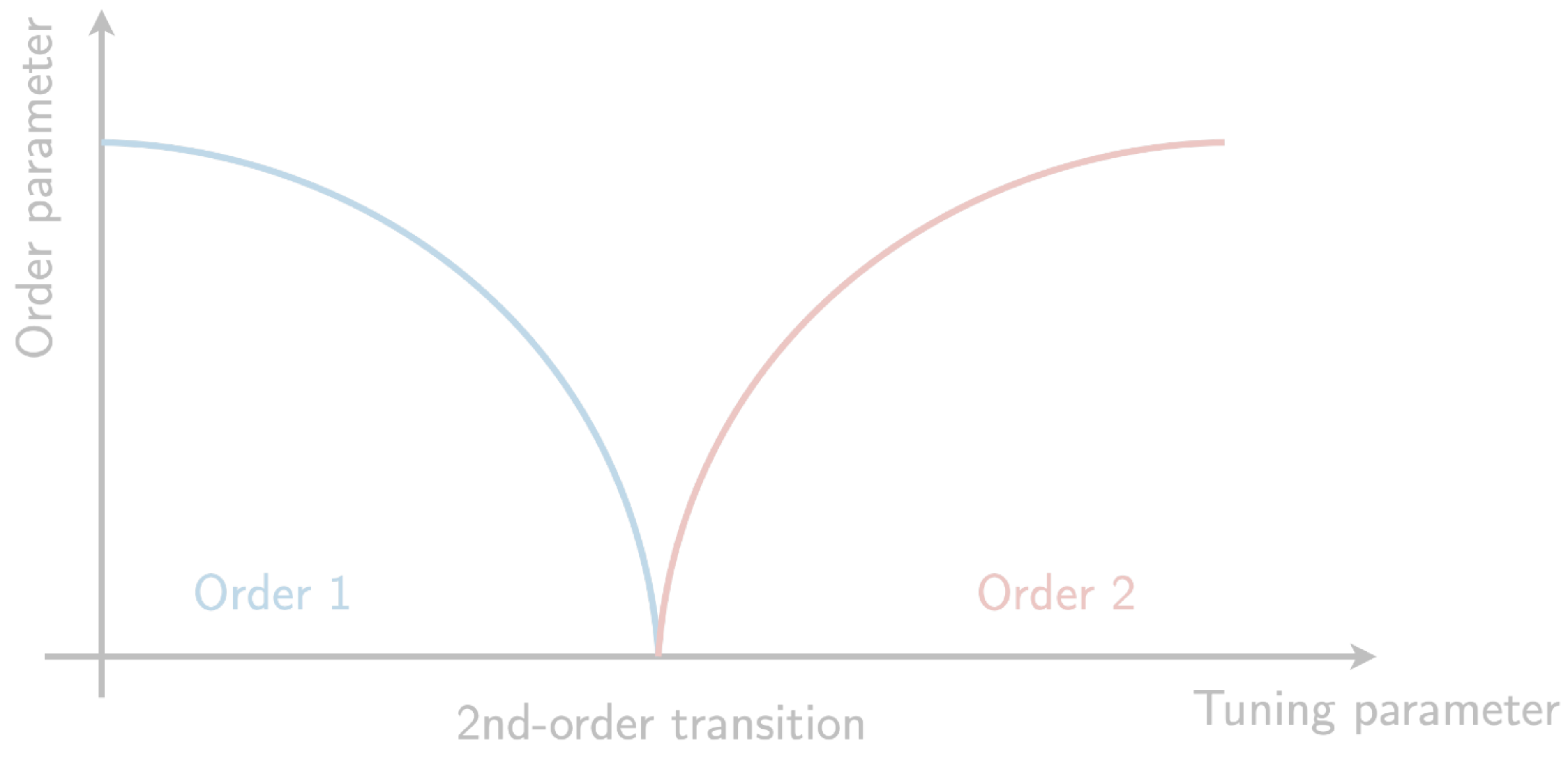

(3) Examples

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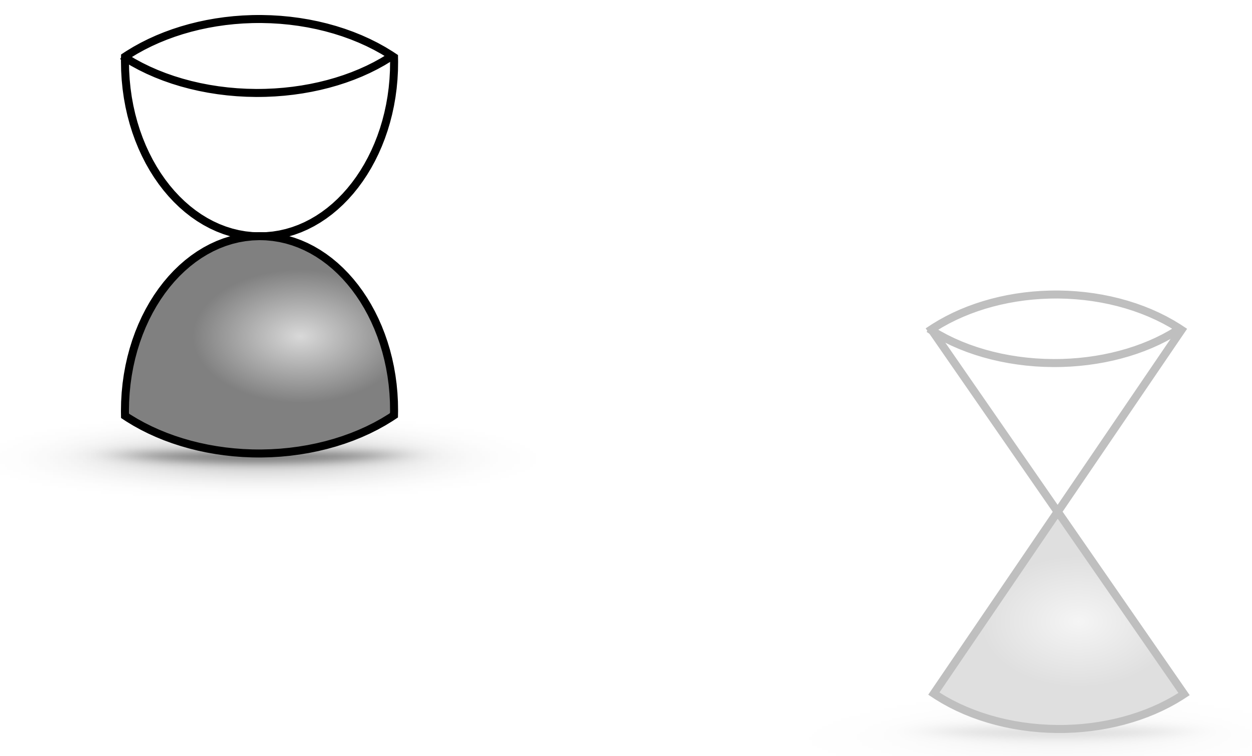
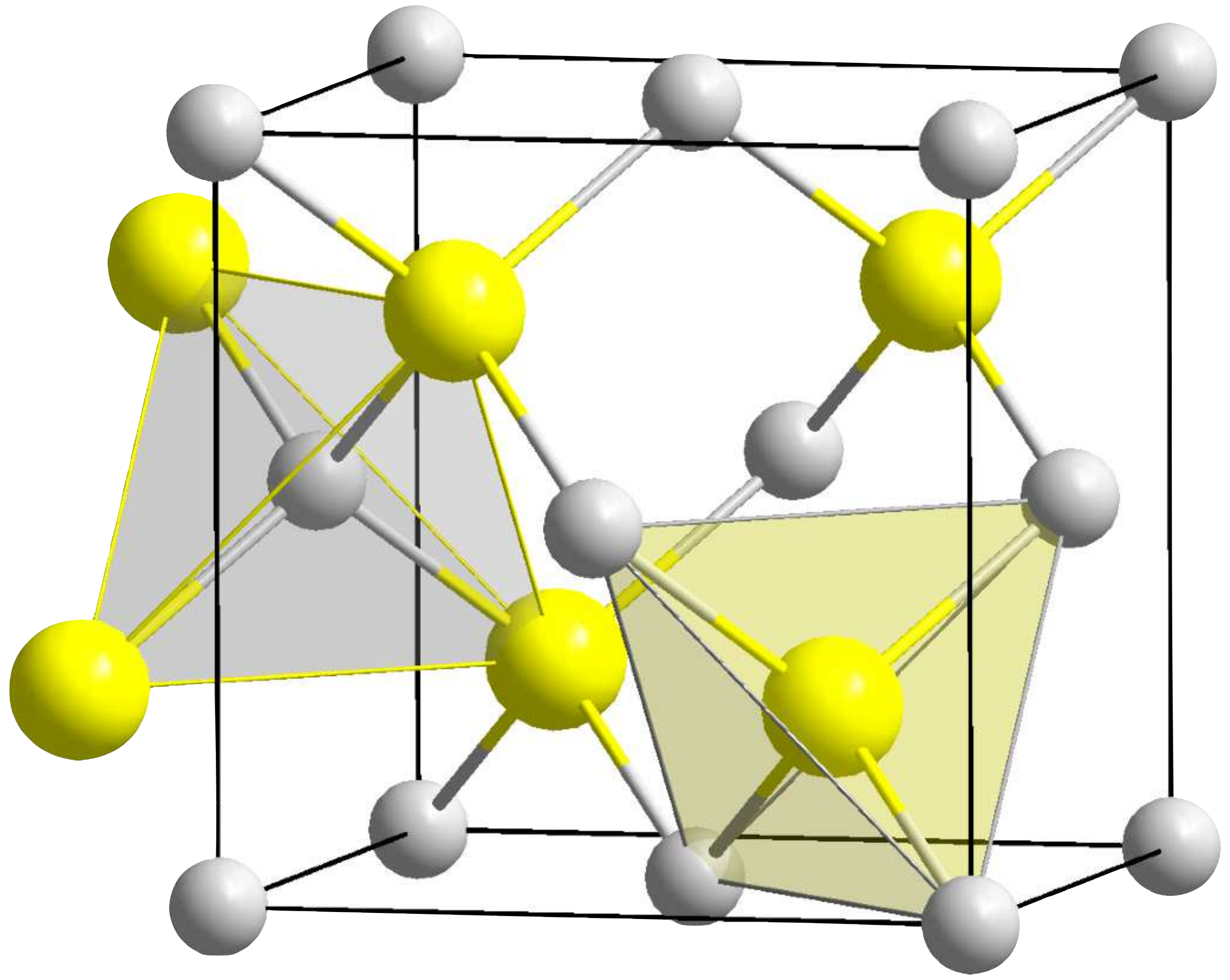


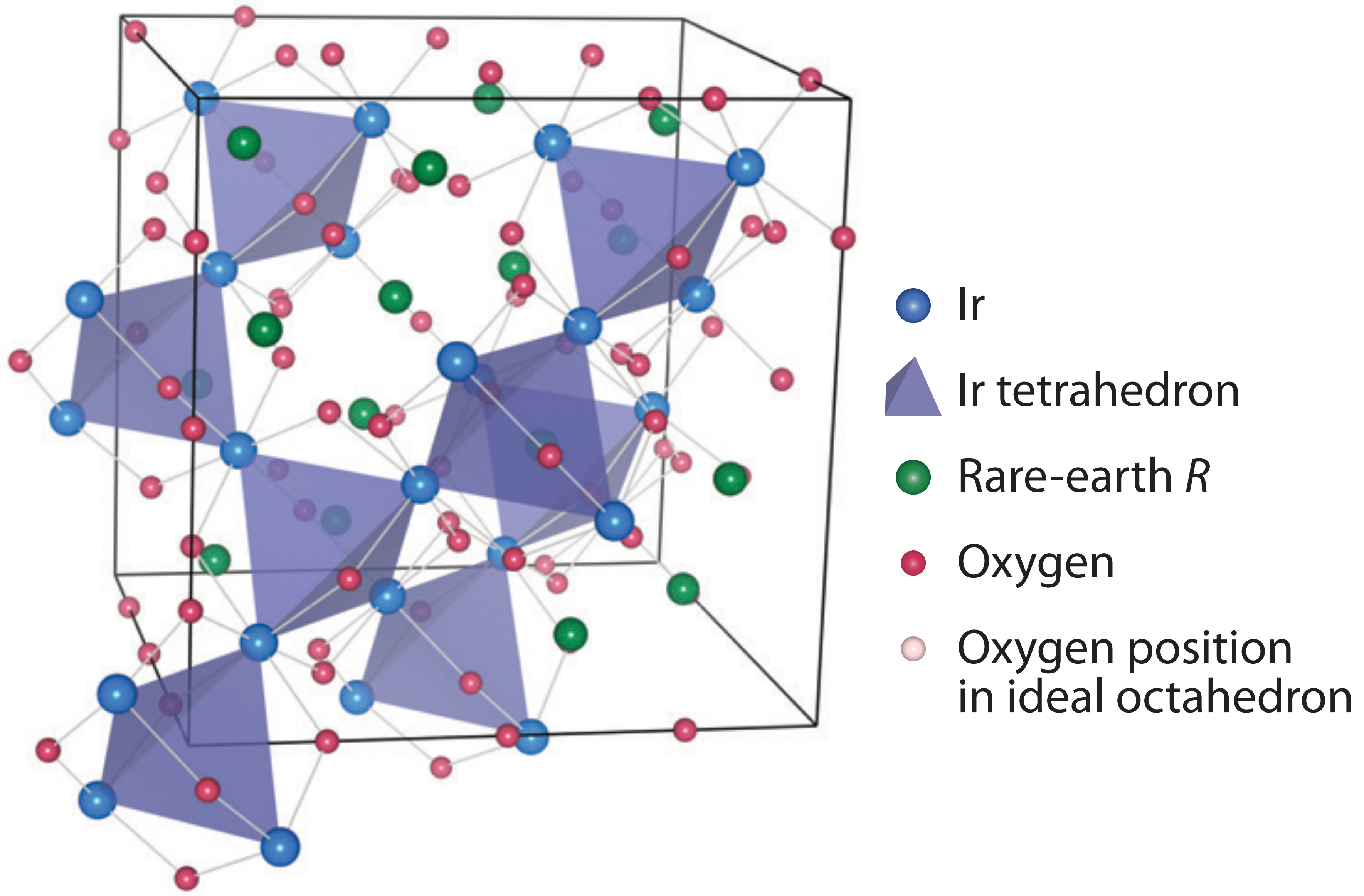
(4) Conclusions

Luttinger semimetals

Material realizations:



α-Sn, HgTe



$R_2Ir_2O_7$ (R = Pr, Nd)

[Witczak-Krempa *et al.*, Annu. Rev. Cond. Mat. Phys. '14]
[Armitage, Mele, Vishwanath, RMP '18]

Luttinger semimetals

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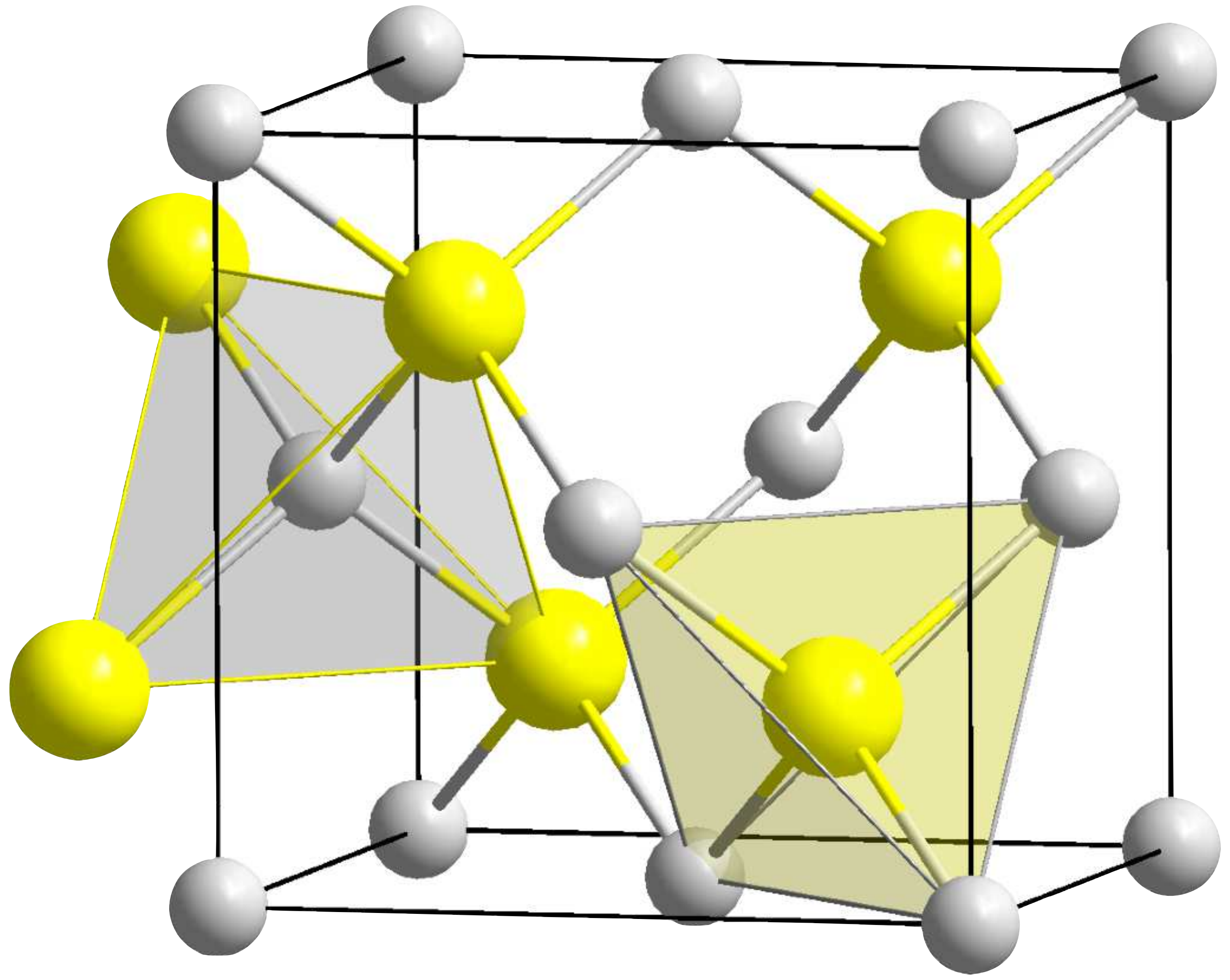

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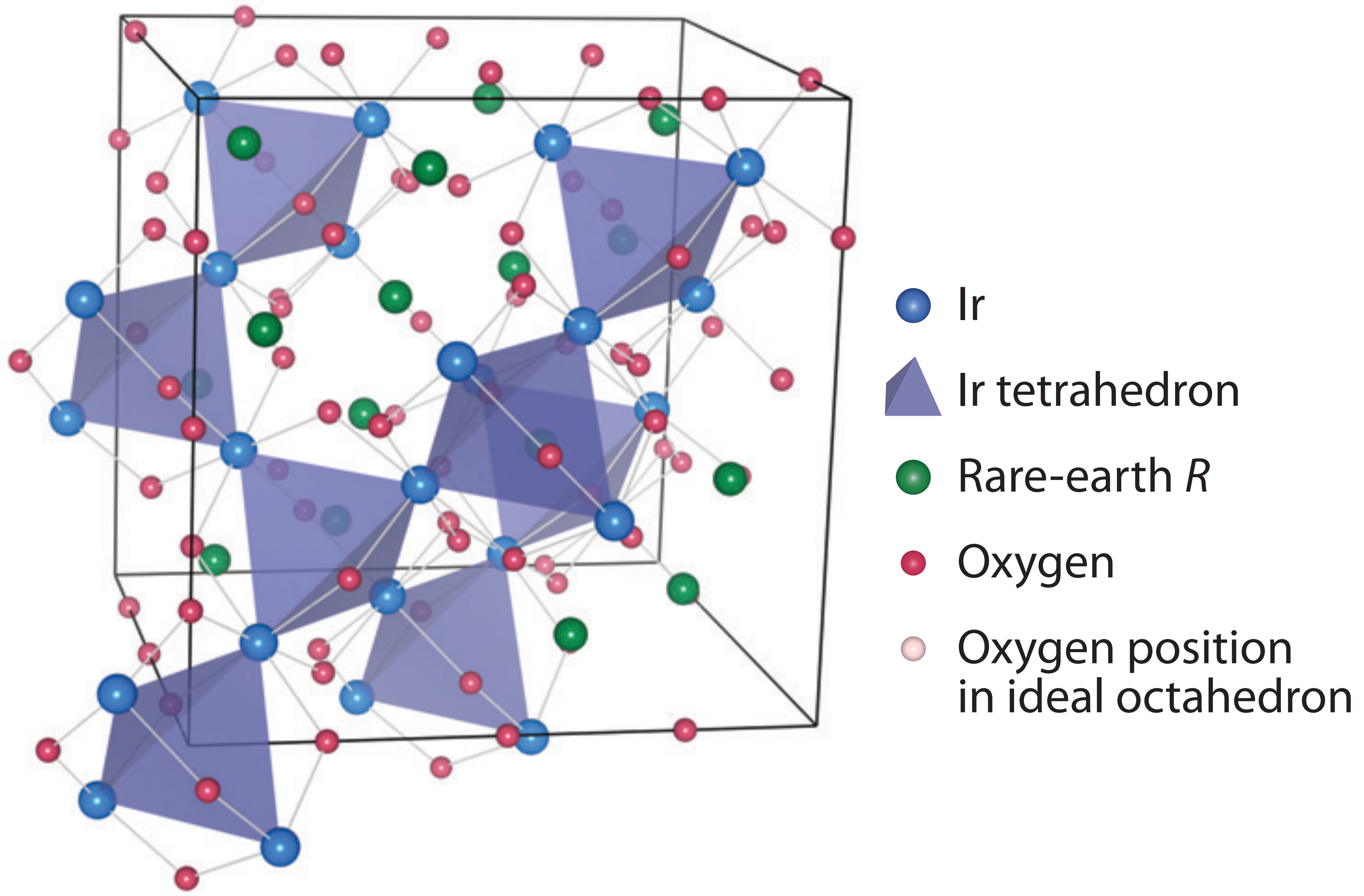

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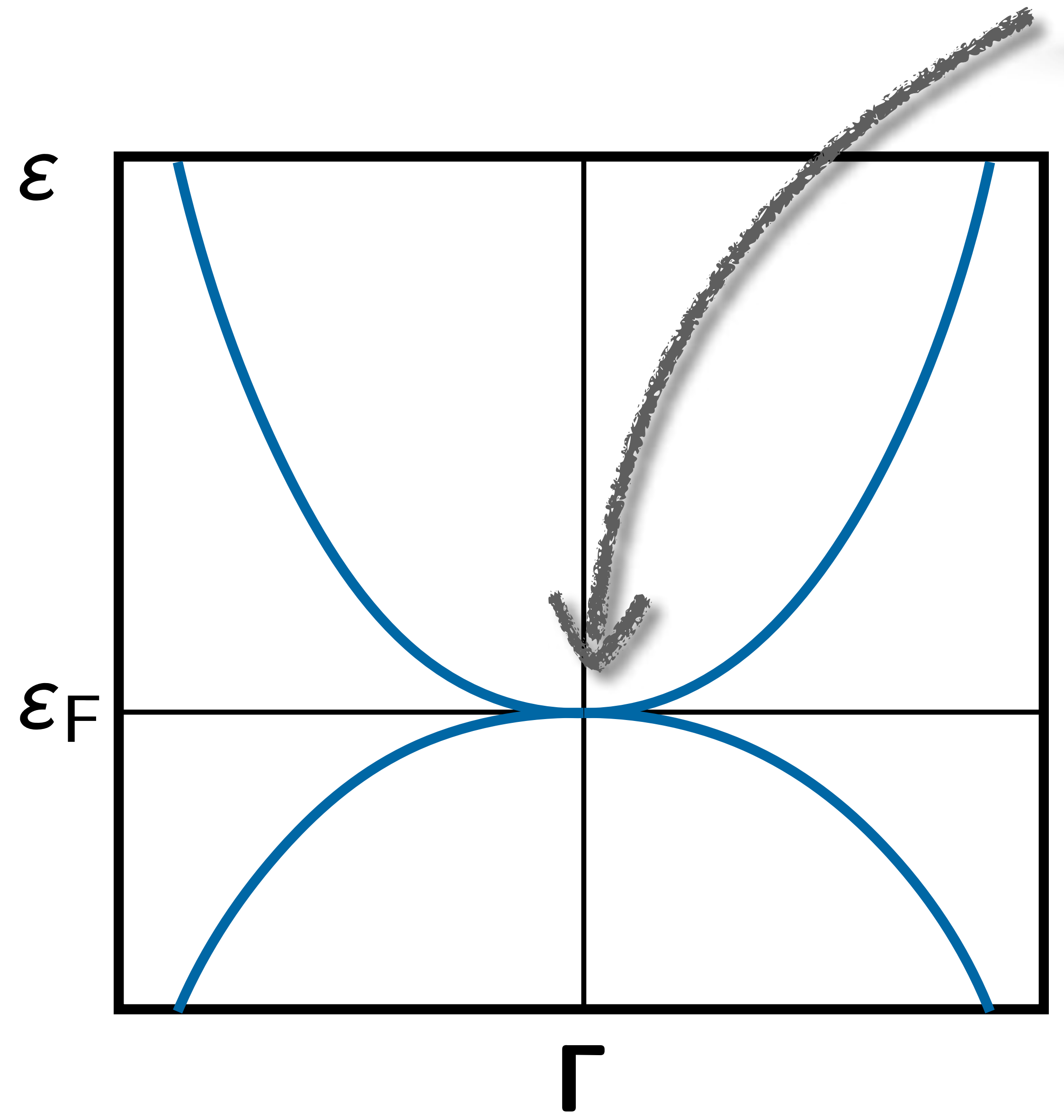
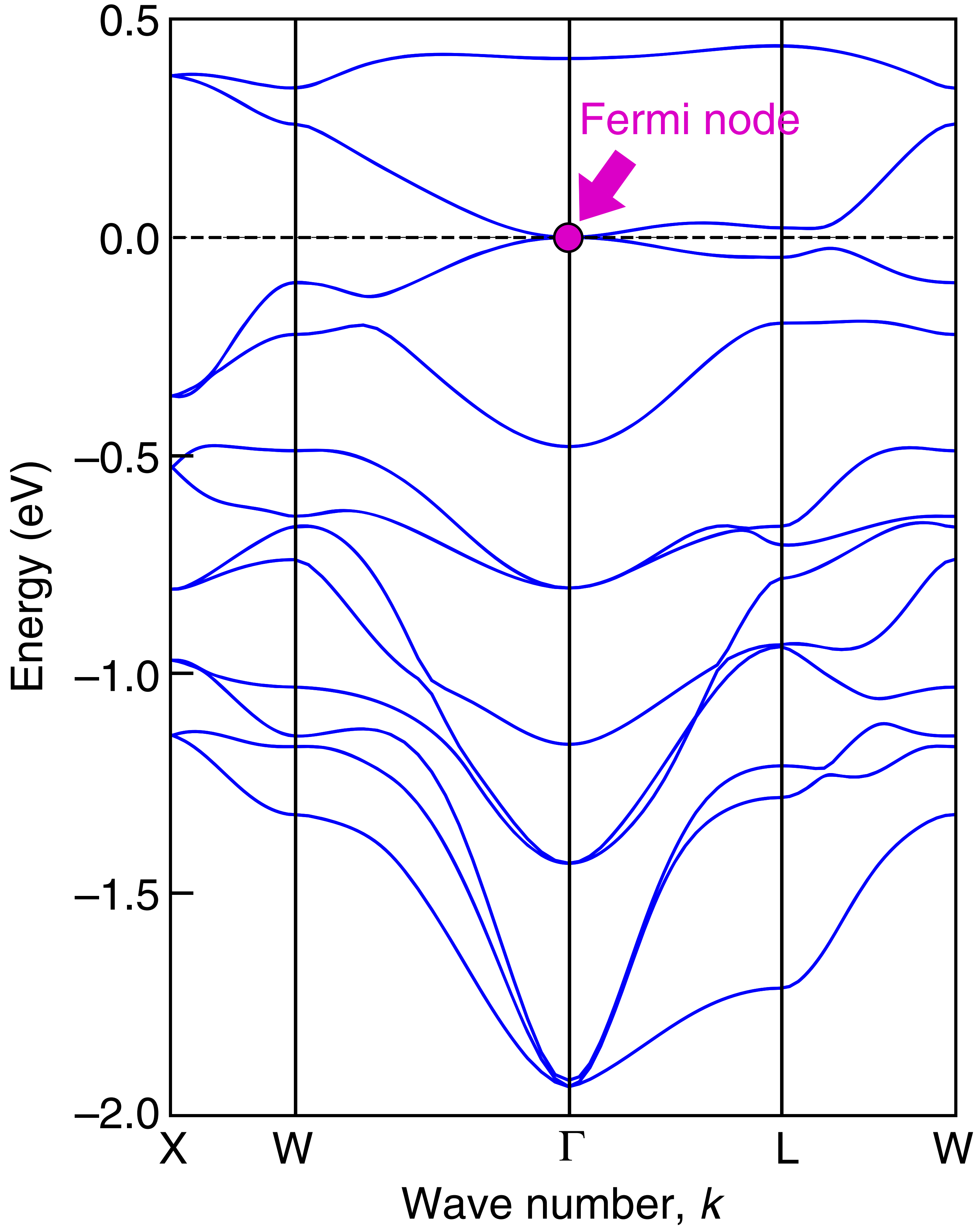
Low-energy spectrum:



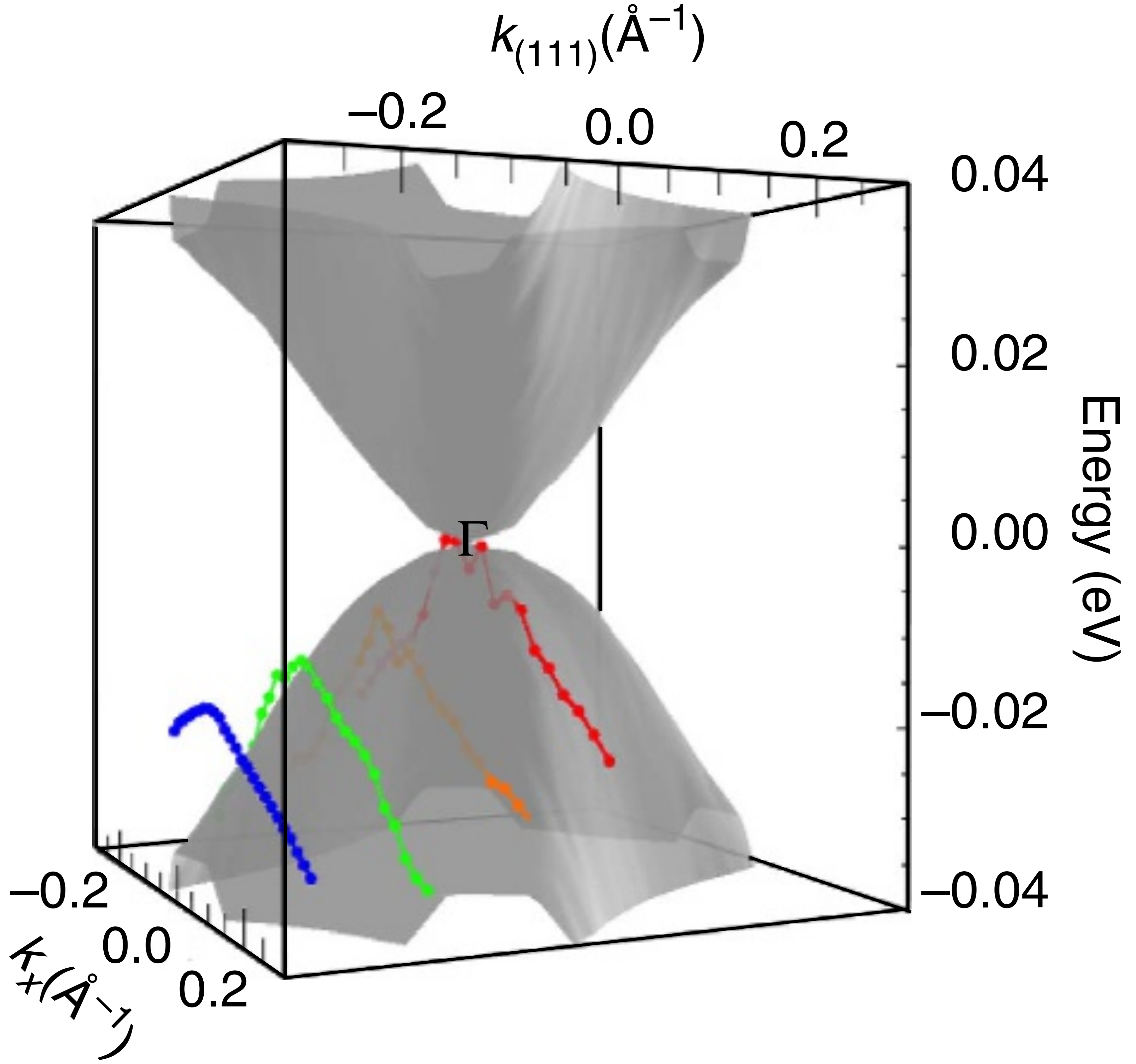
[Witczak-Krempa *et al.*, Annu. Rev. Cond. Mat. Phys. ’14]
[Armitage, Mele, Vishwanath, RMP ’18]

$Pr_2Ir_2O_7$

Band structure (DFT):



Band structure (ARPES):

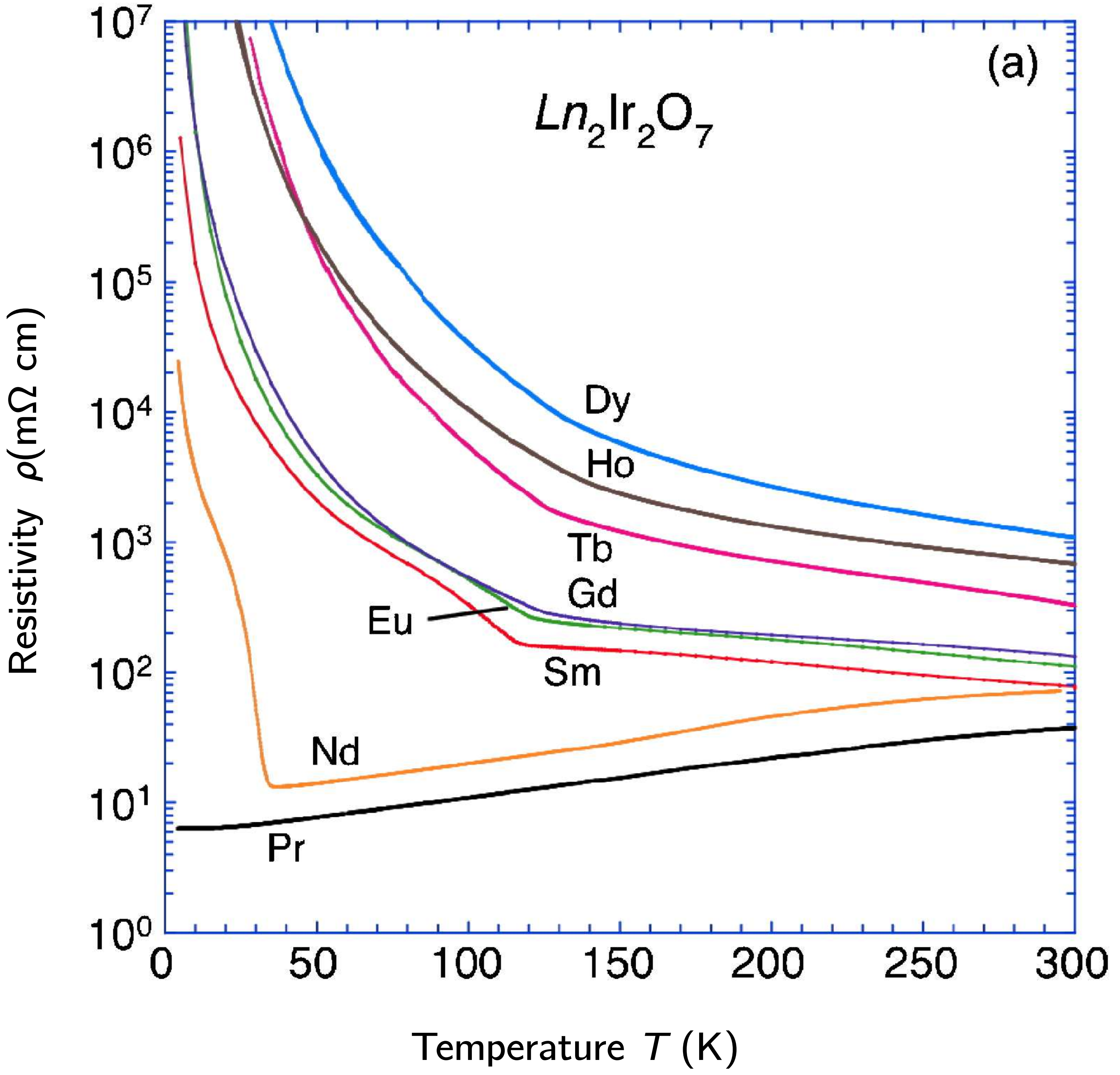


$$\varepsilon \propto \pm \frac{k^2}{2m_{\text{eff}}} \qquad \text{with } m_{\text{eff}}^{\text{hole}} \simeq 6m_0$$

[Kondo *et al.*, Nat. Commun. '15]

Pyrochlore iridates $R_2Ir_2O_7$

Electrical transport:



[Matsuhiro *et al.*, JPSJ '11]

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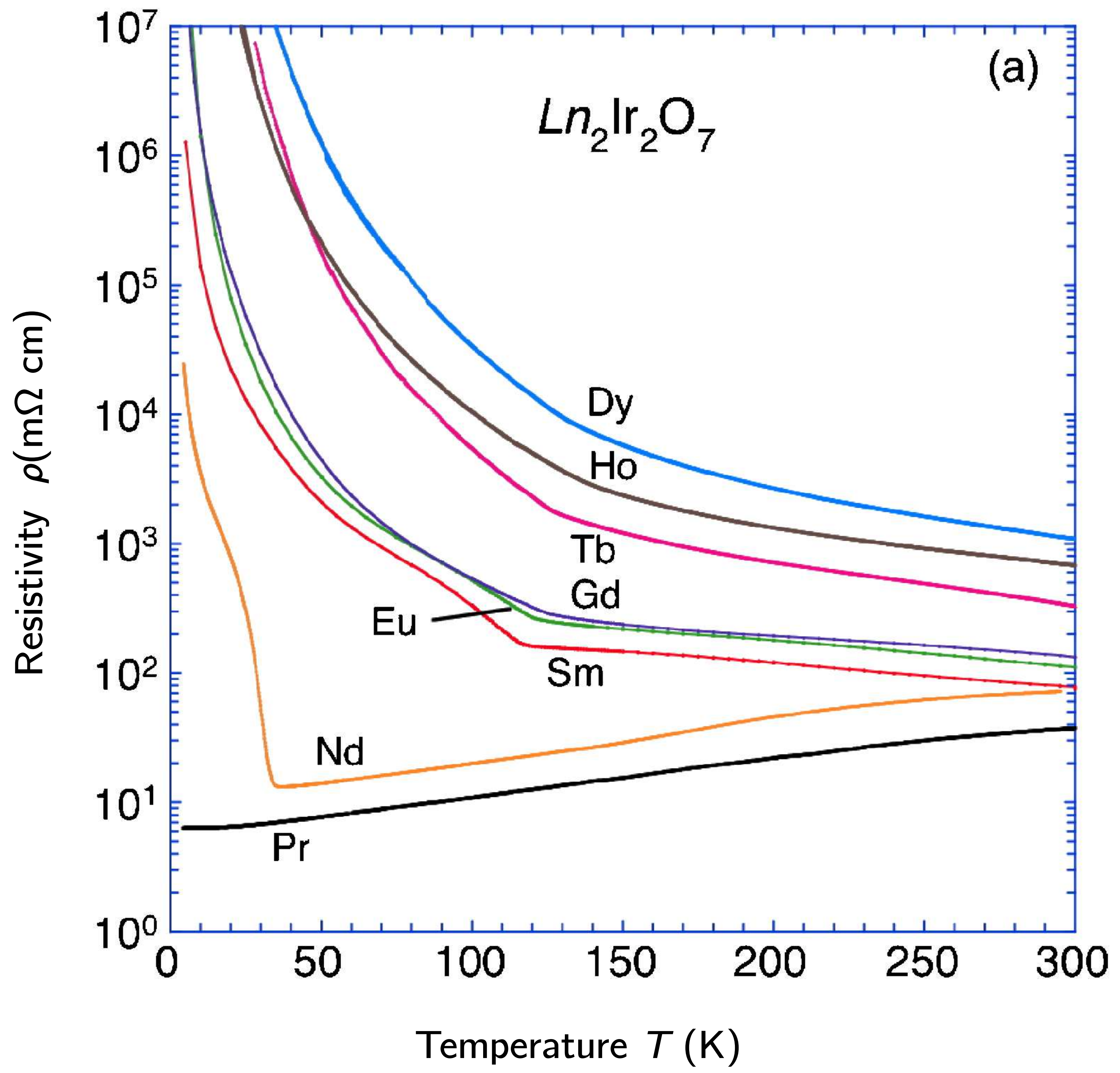

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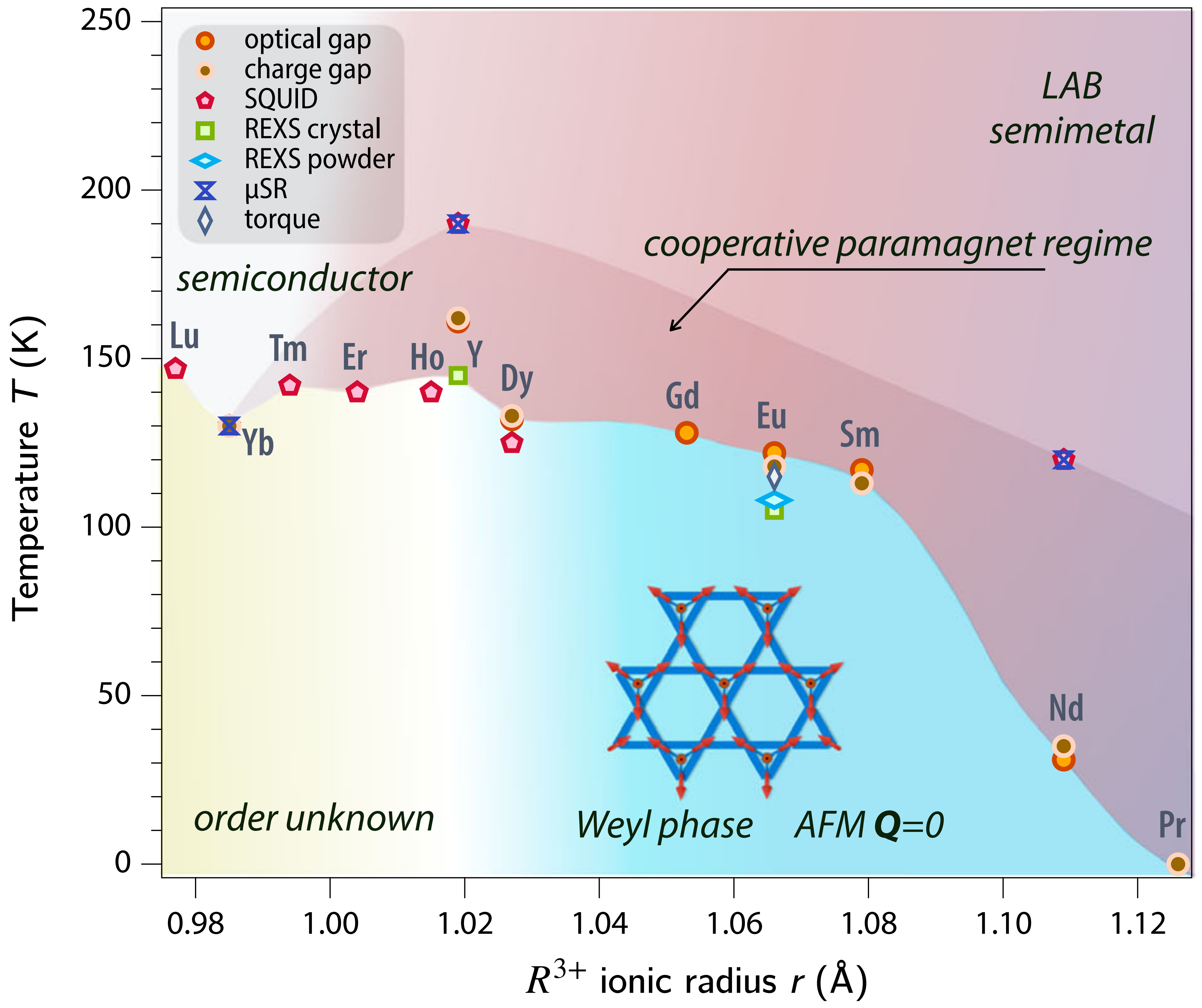
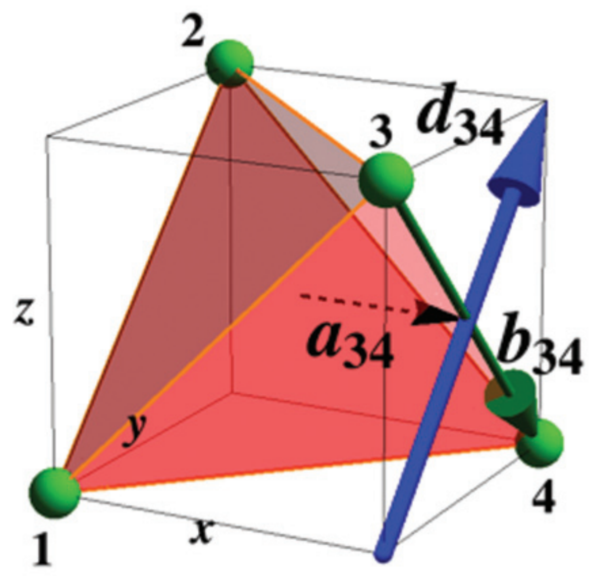
Phase diagram:



[Terilli *et al.*, Nat. Commun. '25]

Lattice model

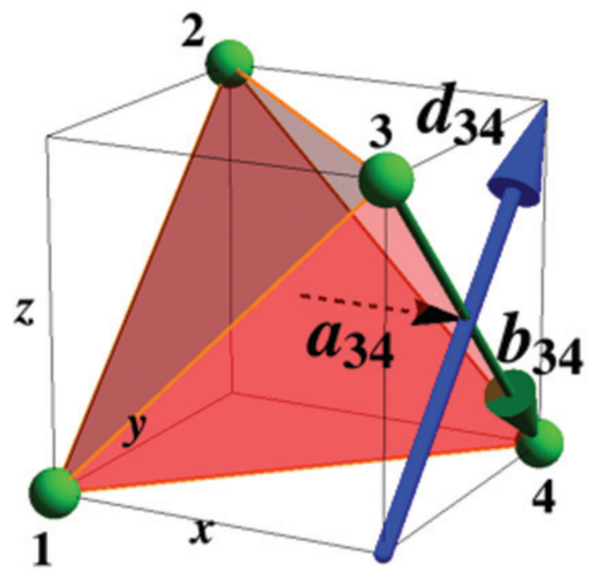


Hamiltonian:

$$\mathcal{H} = \sum_{\langle ij \rangle} c_i^\dagger (t_1 + \mathrm{i} t_2 \vec{d}_{ij} \cdot \vec{\sigma}) c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

[Witczak-Krempa *et al.*, PRB ’13]
[Witczak-Krempa *et al.*, Annu. Rev. Cond. Mat. Phys. ’14]

Lattice model



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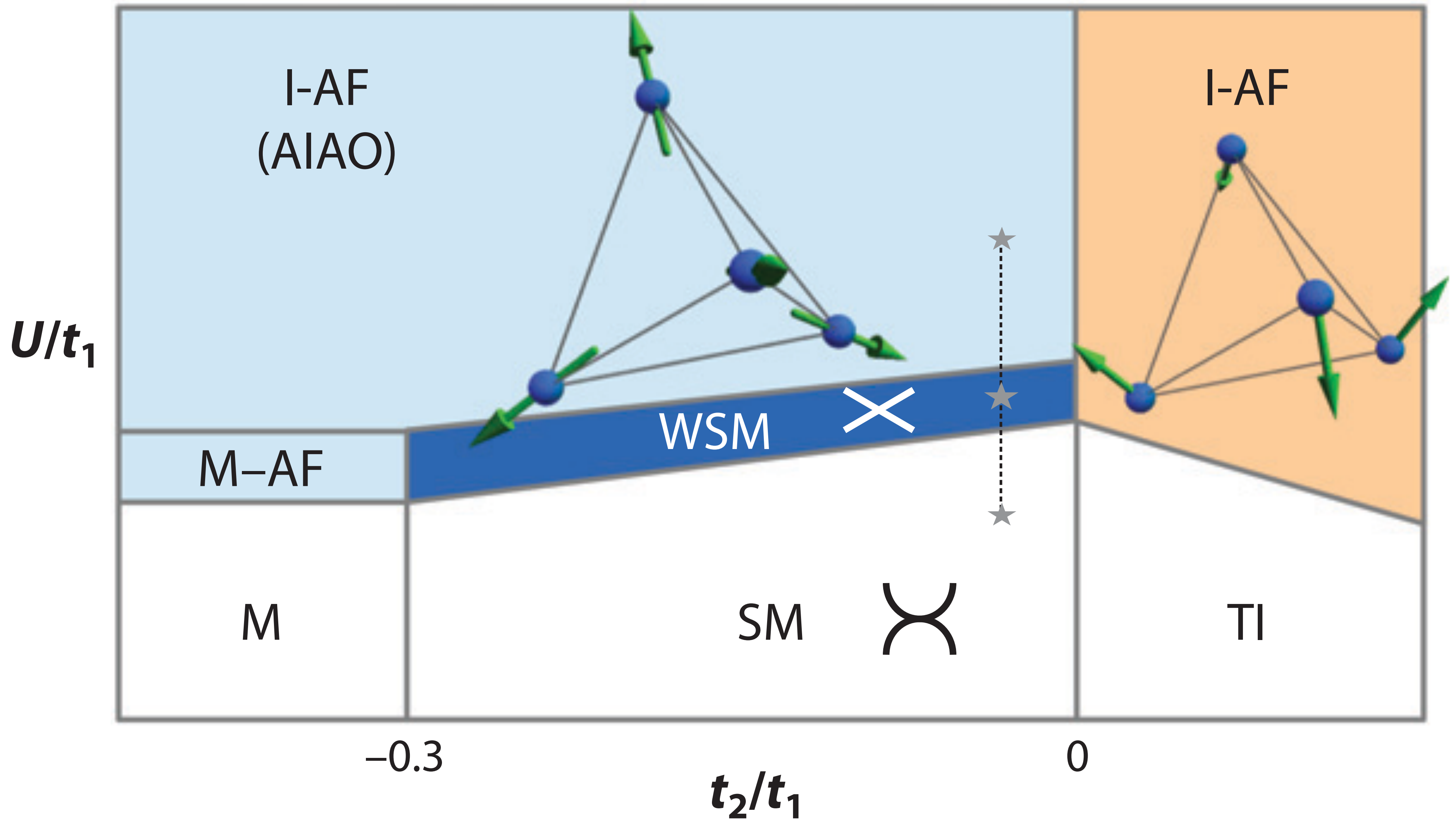
Mean-field phase diagram:



[Witczak-Krempa *et al.*, PRB '13]
[Witczak-Krempa *et al.*, Annu. Rev. Cond. Mat. Phys. '14]

Effective field theory

spherical harmonics $(d_a) = (\sqrt{3}p_y p_z, \sqrt{3}p_x p_z, \sqrt{3}p_x p_y, \frac{\sqrt{3}}{2}(p_x^2 - p_y^2), \frac{1}{2}(2p_z^2 - p_x^2 - p_y^2))$

$(s_a) = (+, +, +, -, -)$

cubic anisotropy $\delta \in [-1, 1]$

Luttinger Lagrangian:

$$\mathcal{L}_0 = \psi^\dagger \left(\partial_\tau + \sum_{a=1}^{5} (1 + s_a \delta) d_a(-\mathrm{i}\nabla)\gamma_a \right) \psi$$

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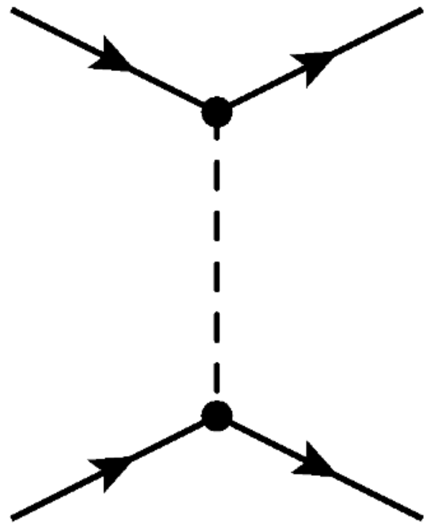
Contact interaction:

$$\mathcal{L}_\psi = G(\psi^\dagger \gamma_{45} \psi)^2$$



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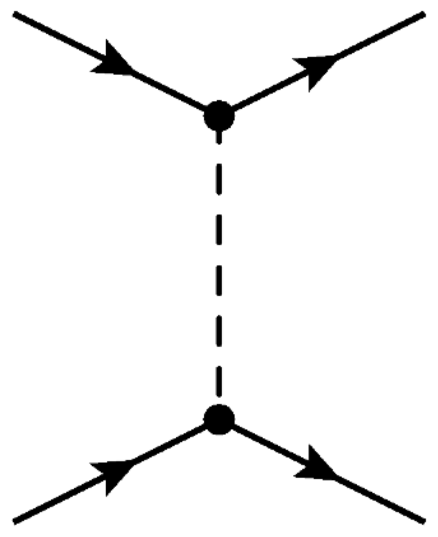
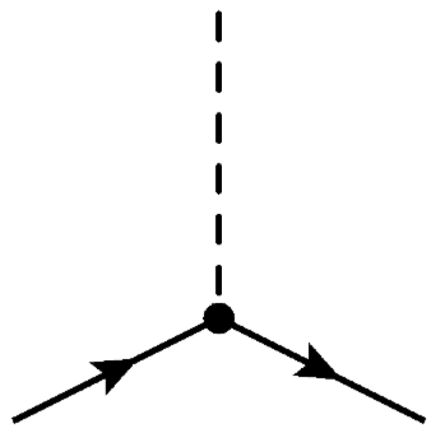


Hubbard-Stratonovich transformation:

$$\mathcal{L}_\phi = \frac{r}{2}\phi^2 + g\phi\psi^\dagger\gamma_{45}\psi$$



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$(s_a) = (+, +, +, -, -)$

cubic anisotropy $\delta \in [-1, 1]$

Luttinger Lagrangian:

$[\delta] = 0$

$$\mathcal{L}_0 = \psi^\dagger \left(\partial_\tau + \sum_{a=1}^{5} (1 + s_a \delta) d_a(-\mathrm{i}\nabla)\gamma_a \right) \psi$$

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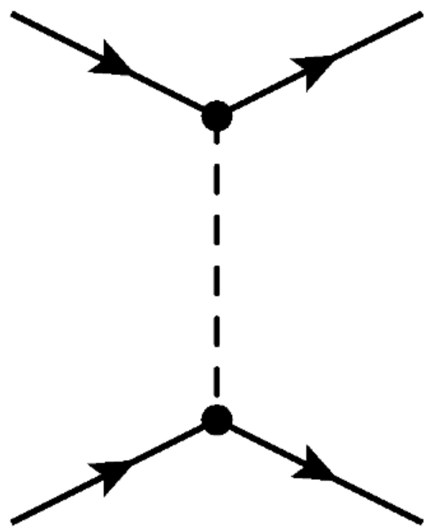
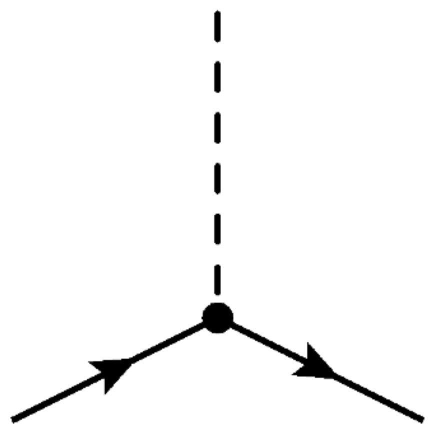


Hubbard-Stratonovich transformation:

$[r] = 2$

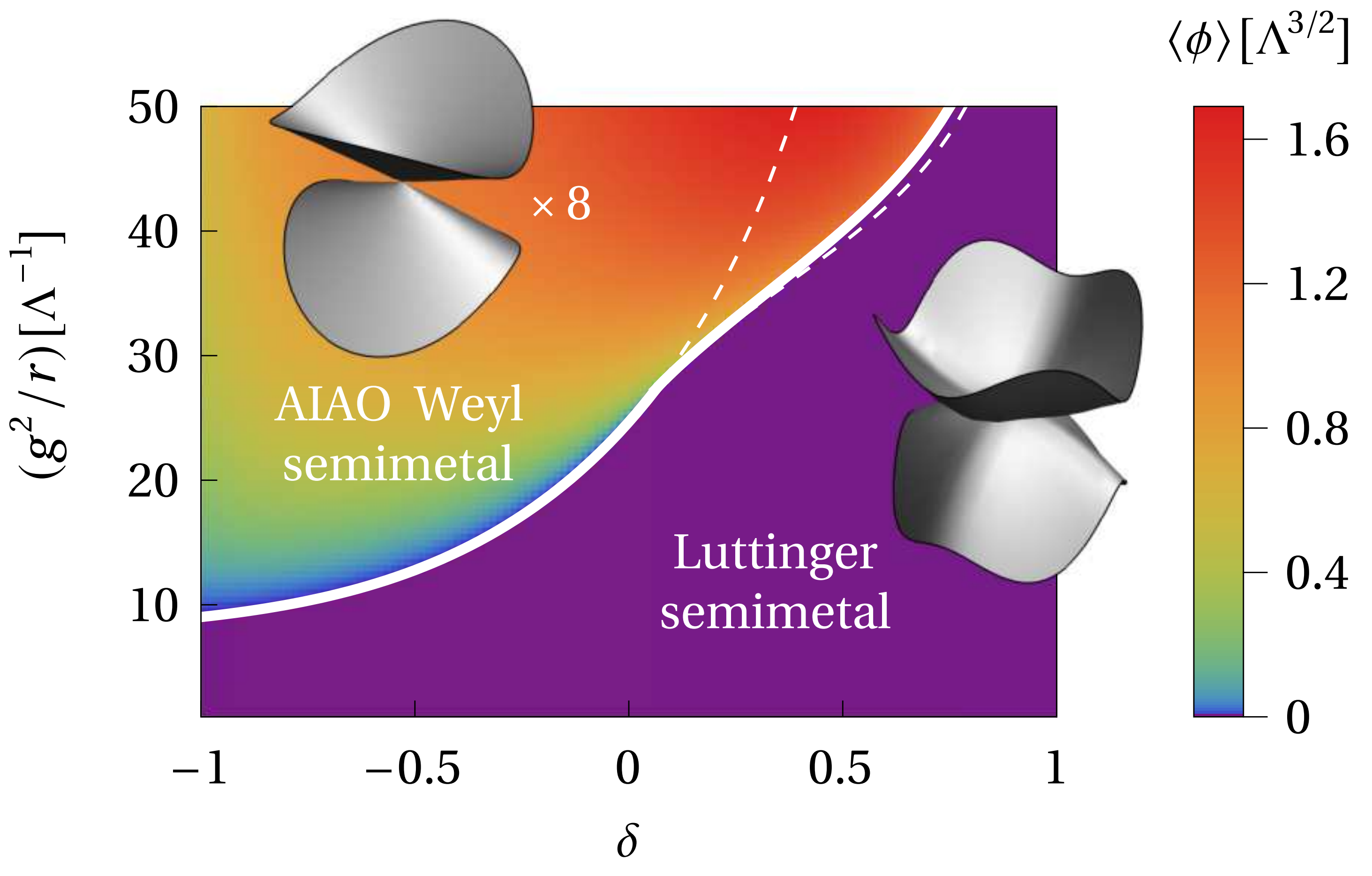
$$\mathcal{L}_\phi = \frac{r}{2}\phi^2 + g\phi\psi^\dagger\gamma_{45}\psi$$

$[g] = \frac{4-d}{2}$



Luttinger-to-Weyl transition

Mean-field phase diagram:



[Moser & LJ, PRB '24]

Luttinger-to-Weyl transition

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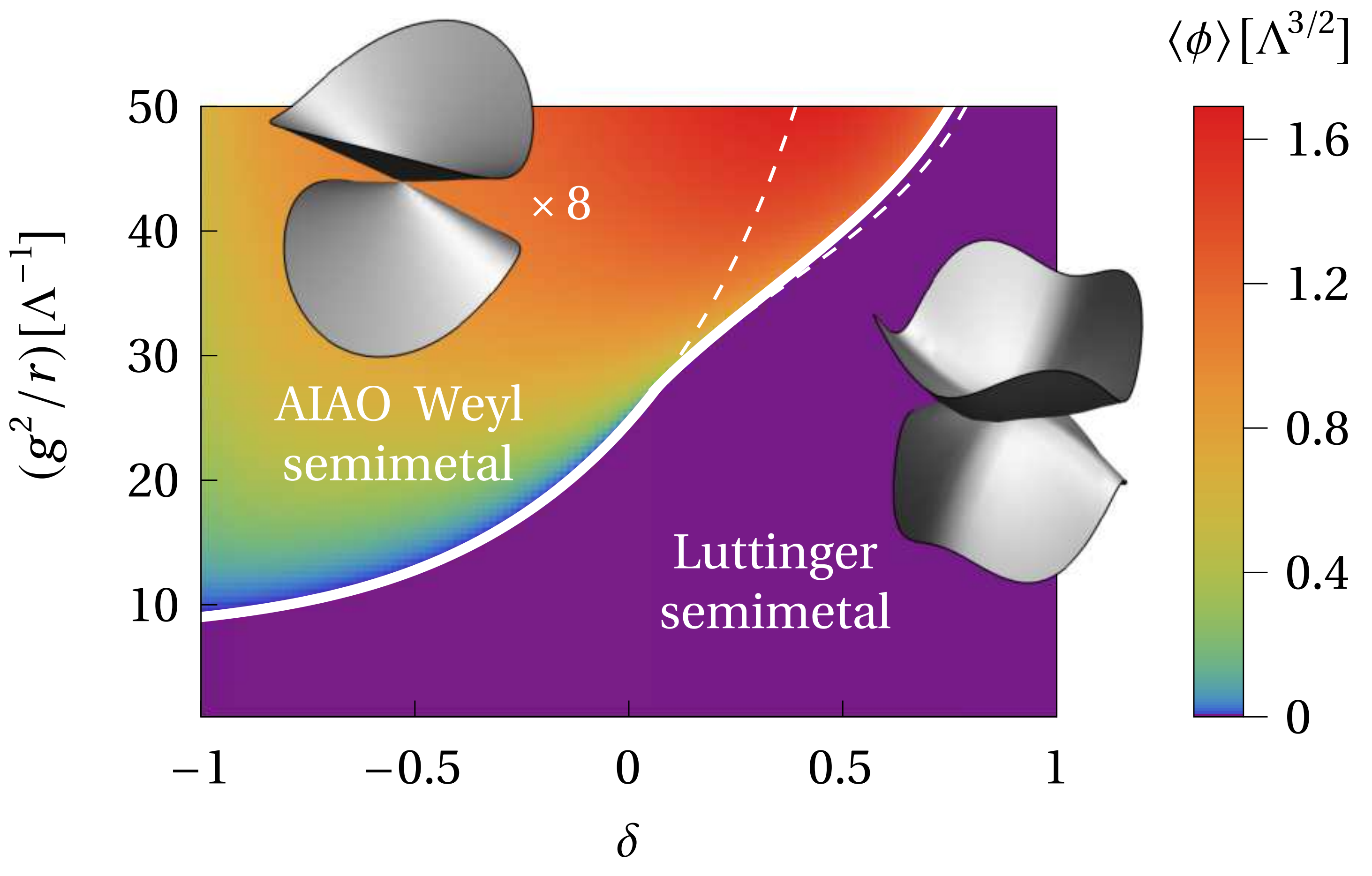
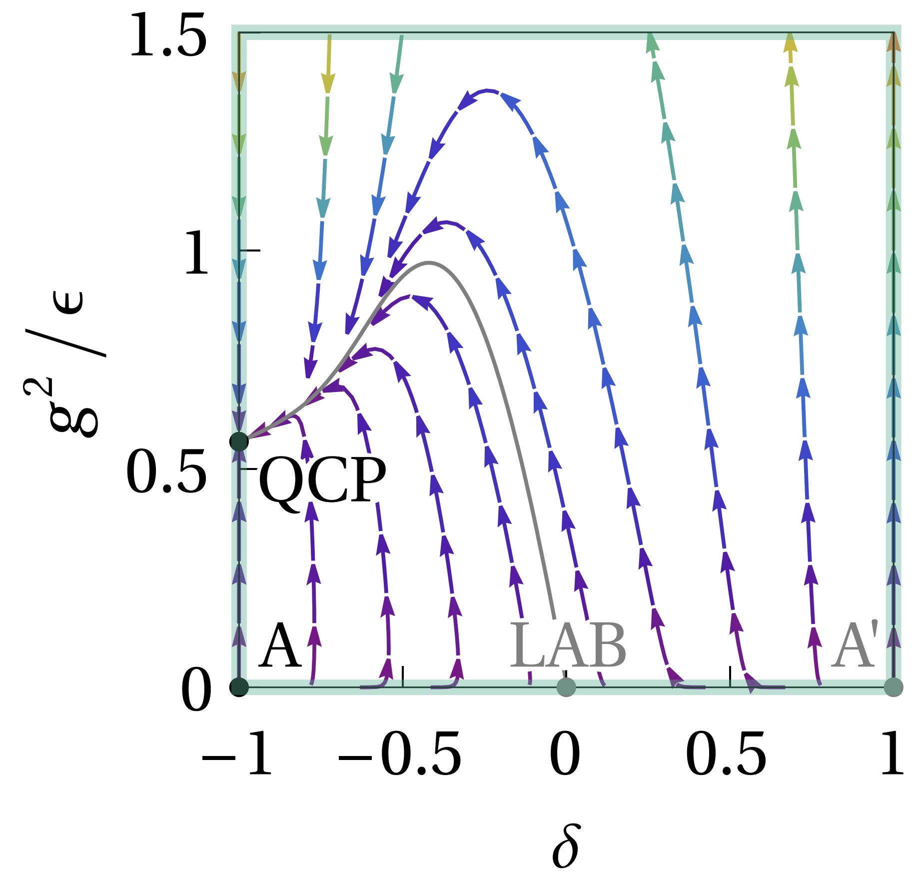


RG flow ($r = 0$):



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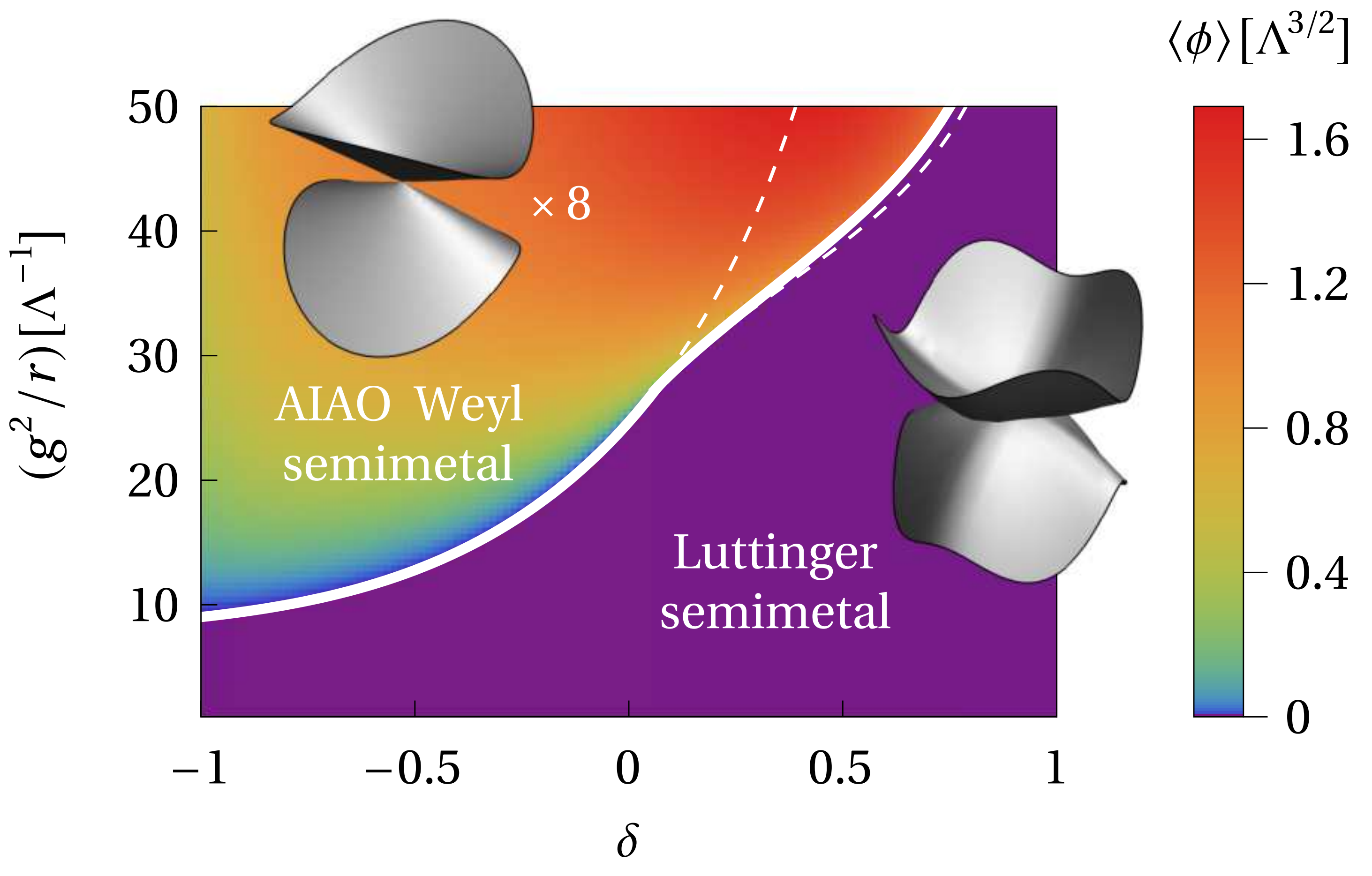

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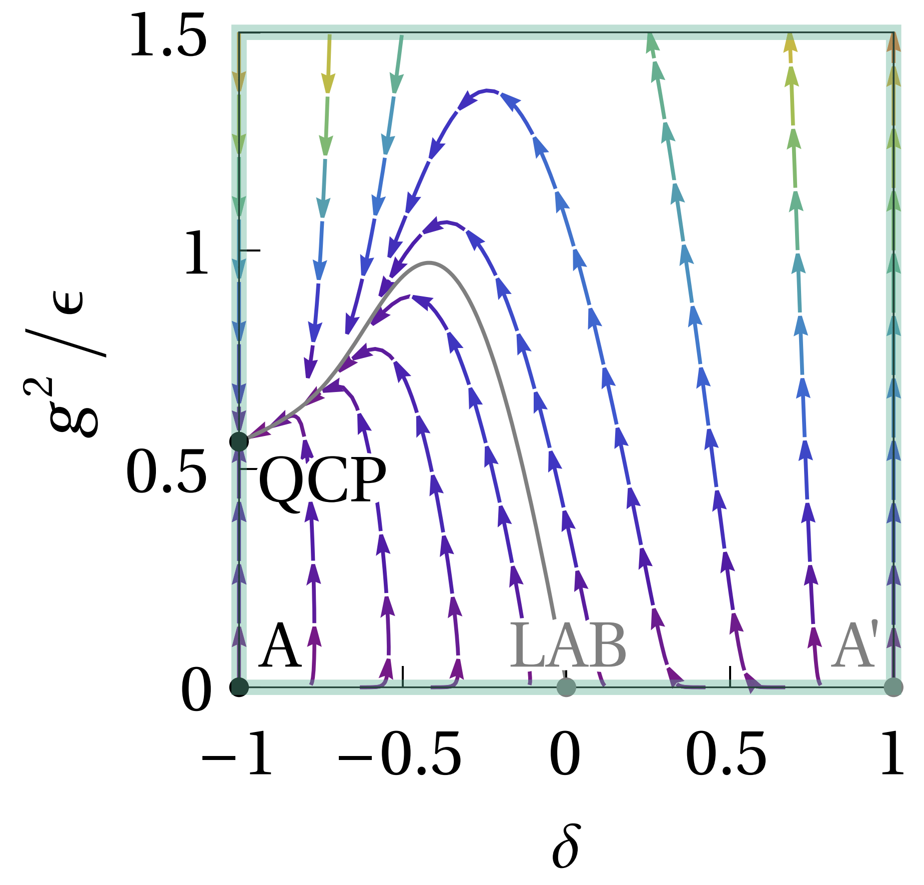
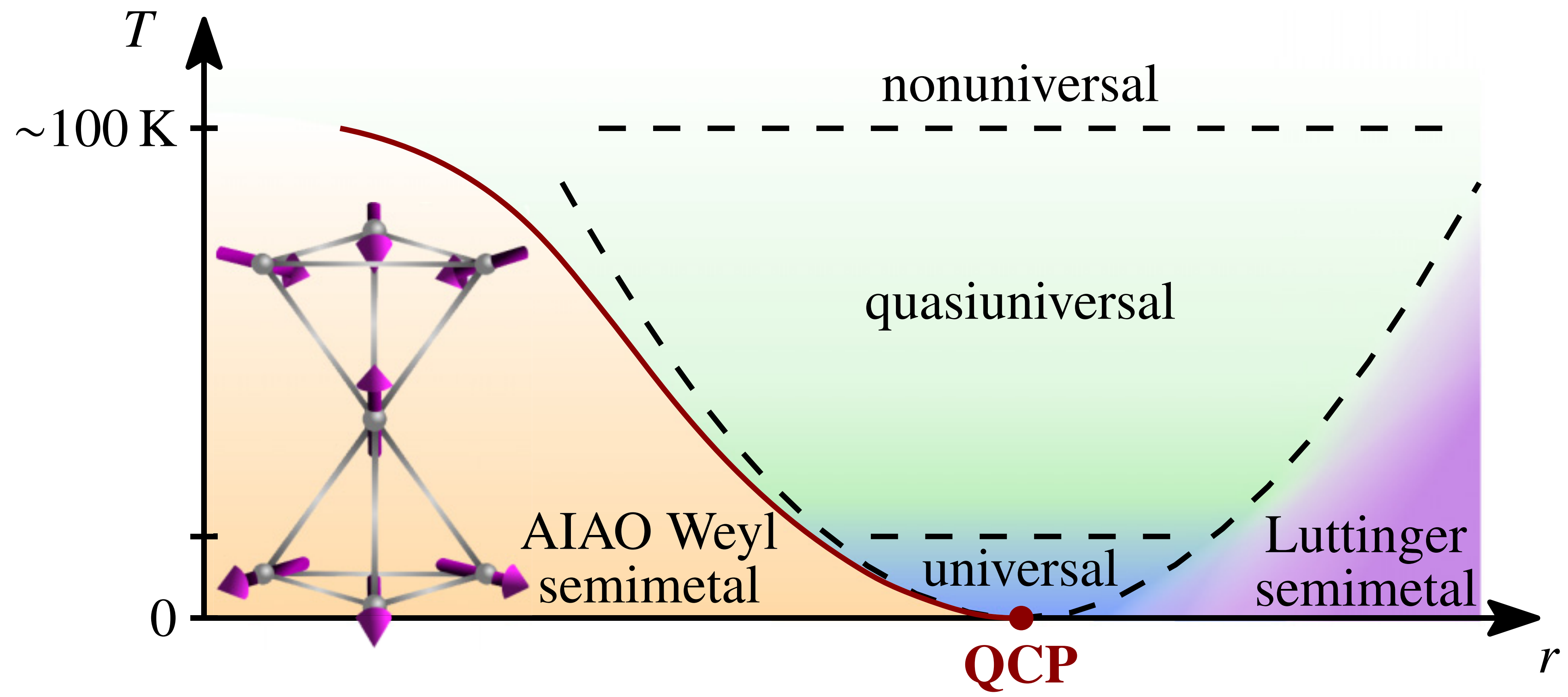


Quantum phase diagram:



Critical temperature: $T_c \propto (r_c - r)^{-\nu z}$

Dynamical structure factor:

$$\mathcal{S}(\vec{k}, \omega) = \frac{1}{|\vec{k}|^{2-\eta}} \mathcal{F}(\omega/|\vec{k}|^z)$$

with $\nu = 1$, $\eta = 1$, $z = 2$ exactly

[Moser & LJ, PRB '24]

Coulomb interaction

Lagrangian:

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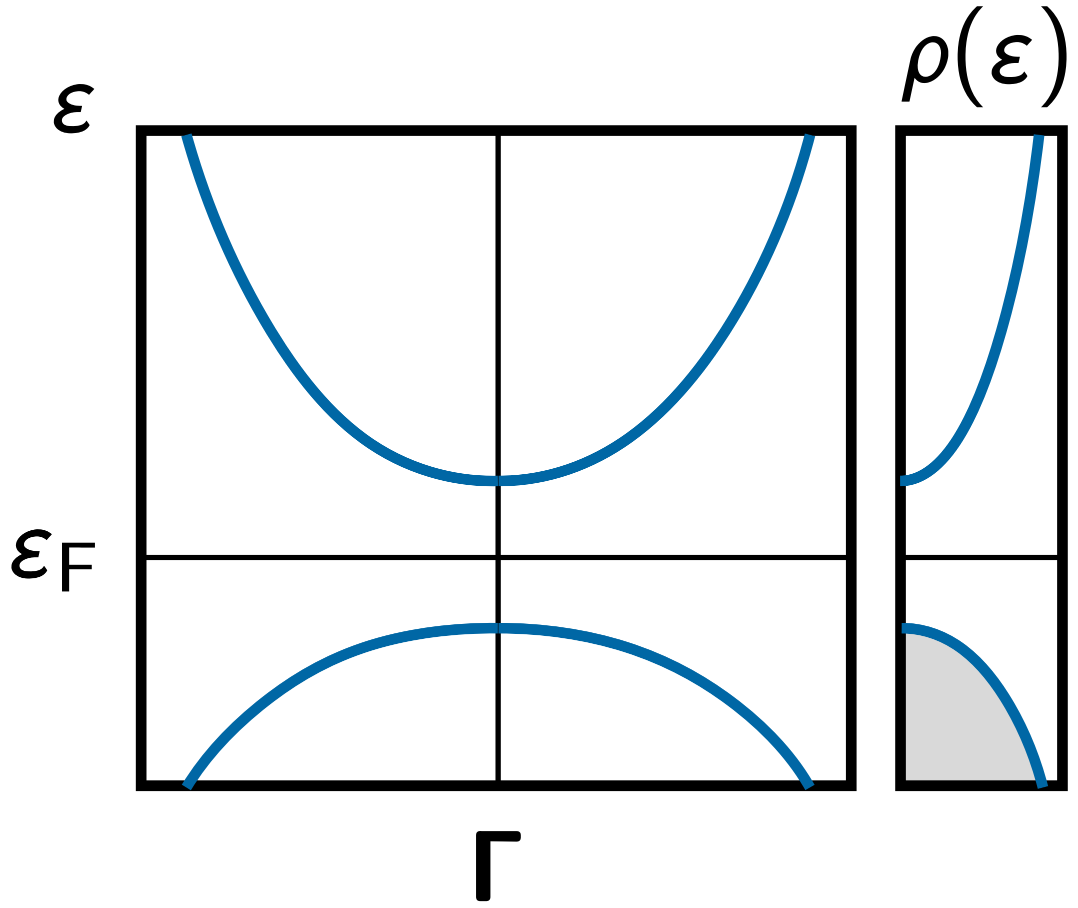
Small-gap semiconductor



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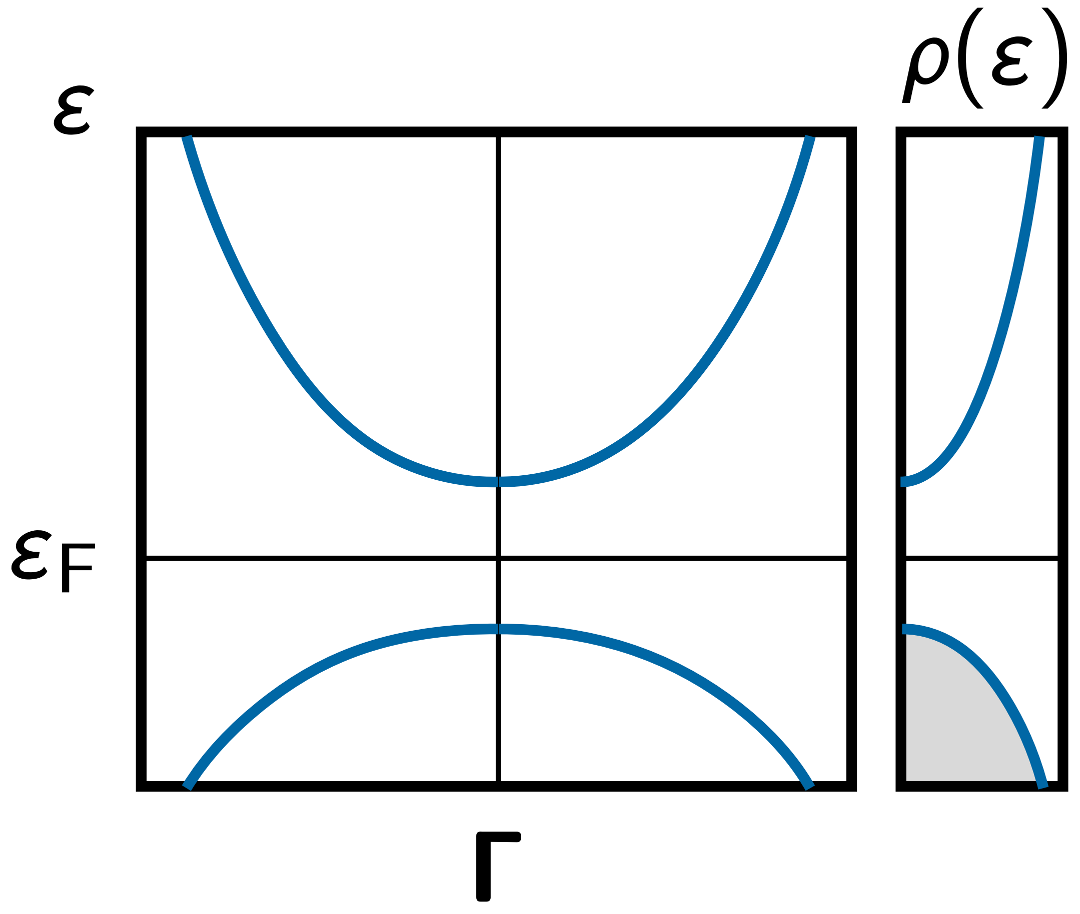

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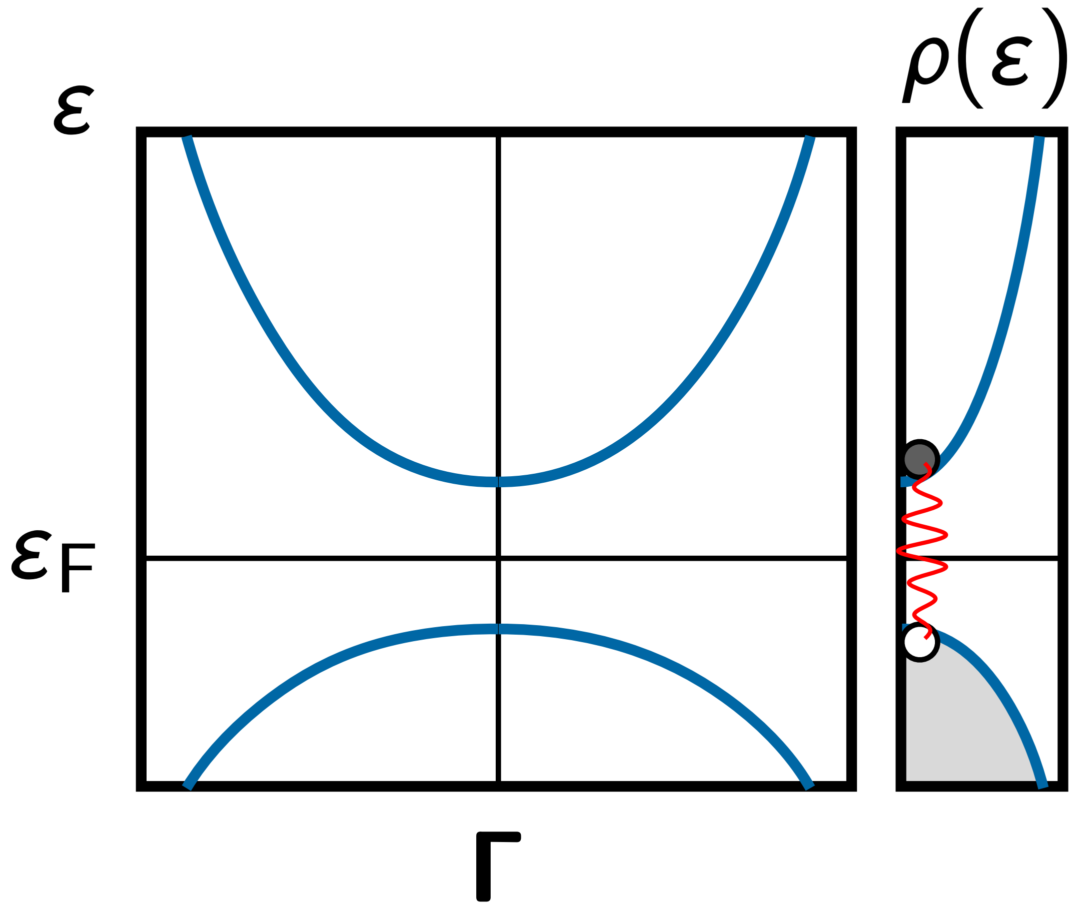

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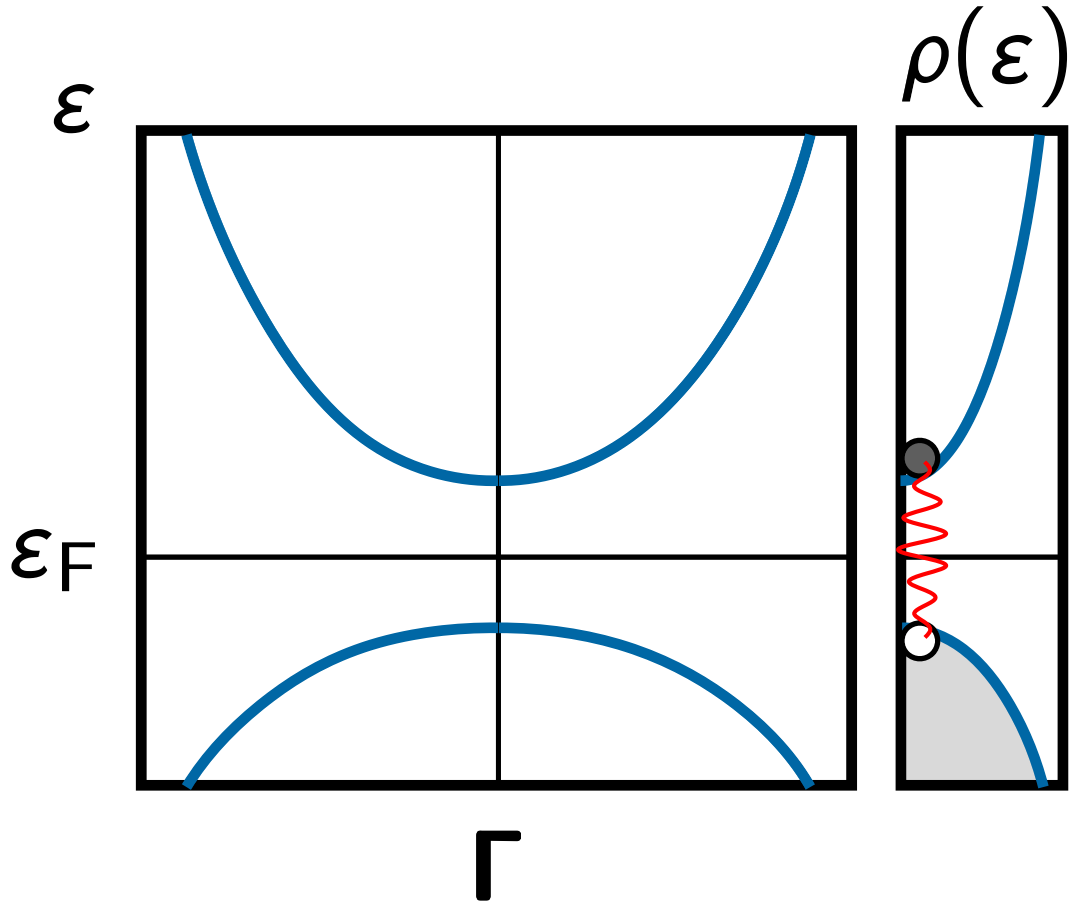

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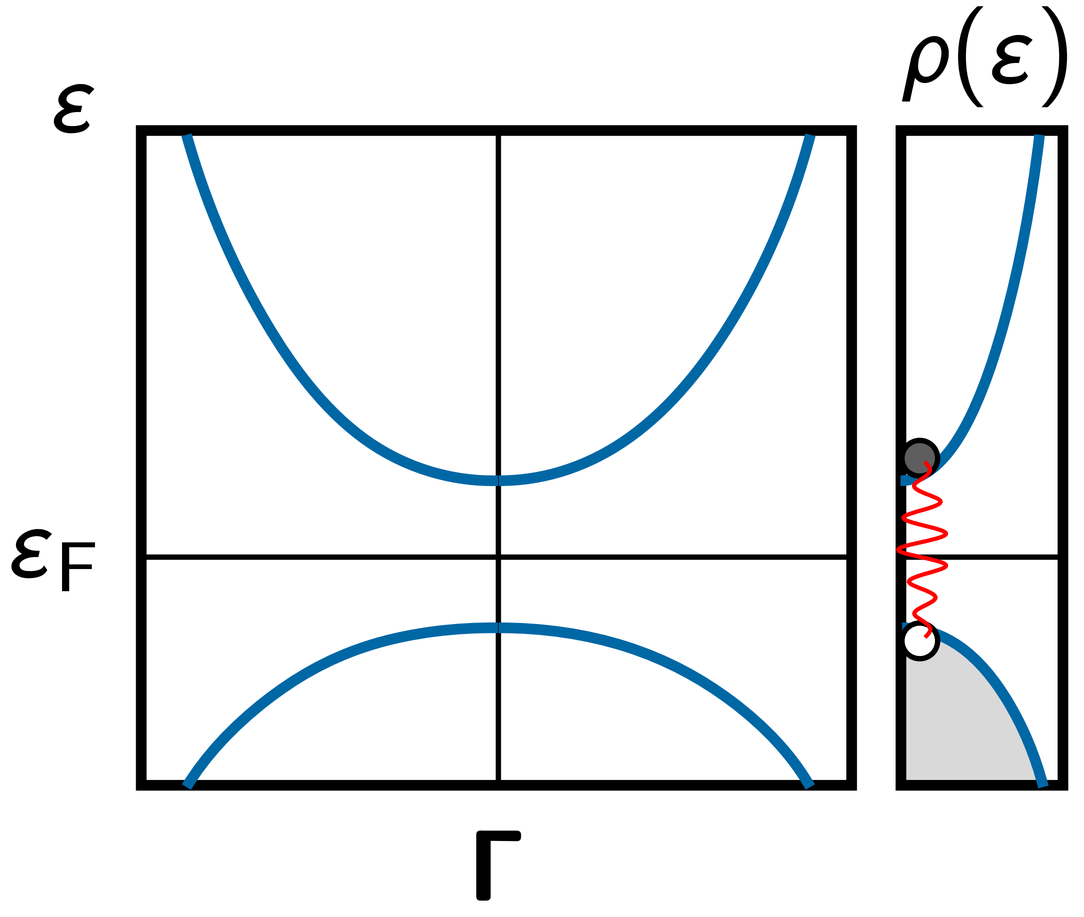
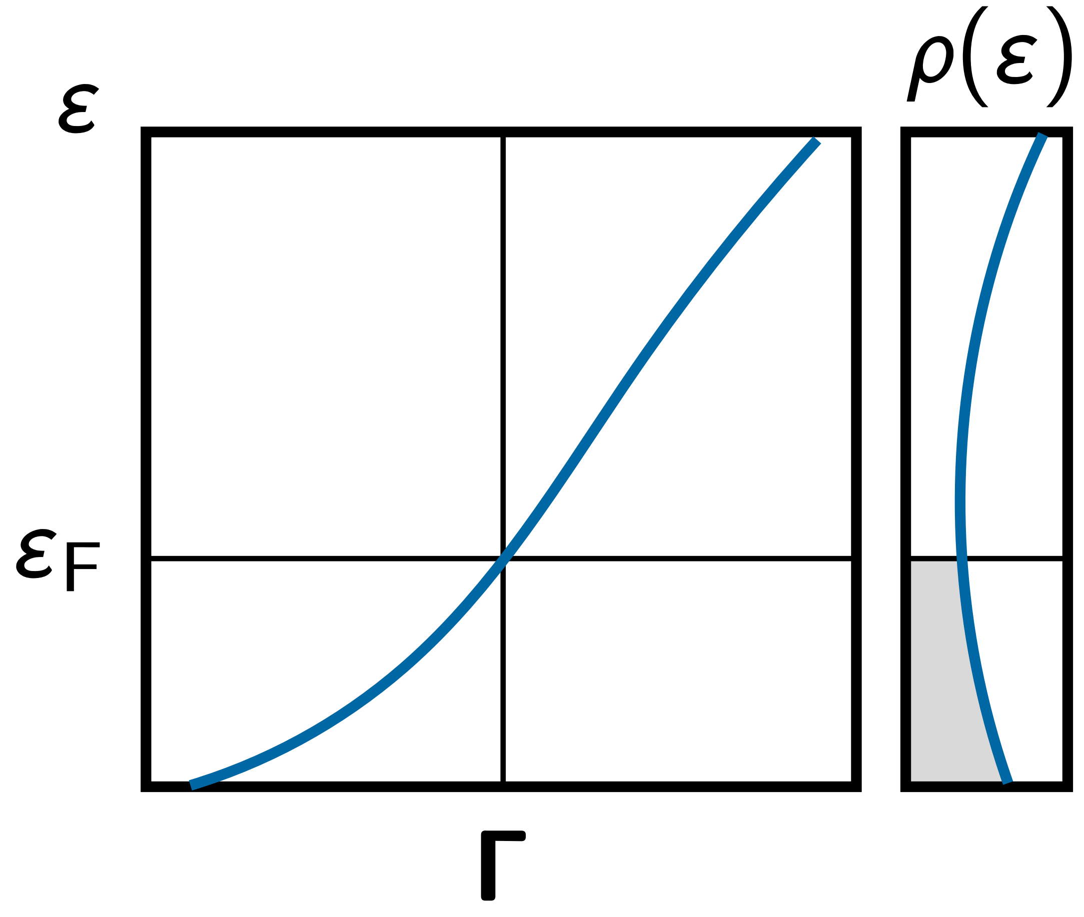


Metal



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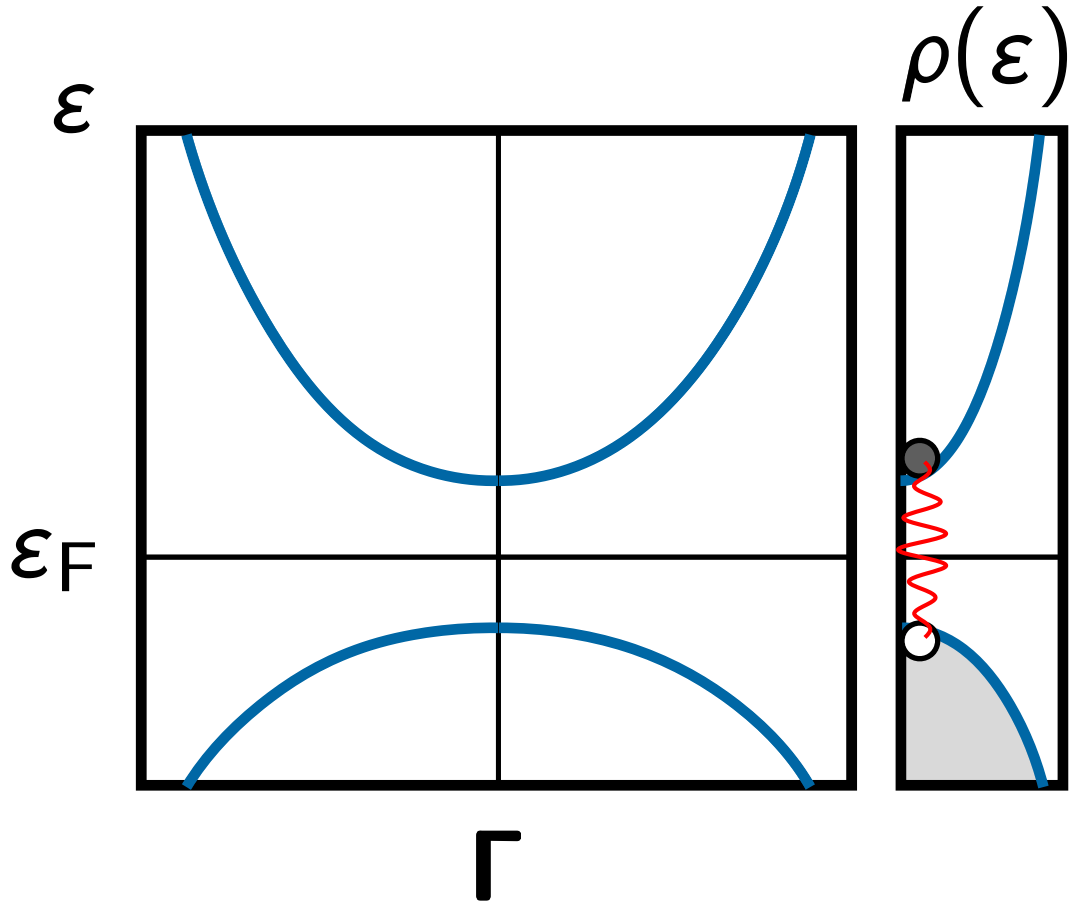

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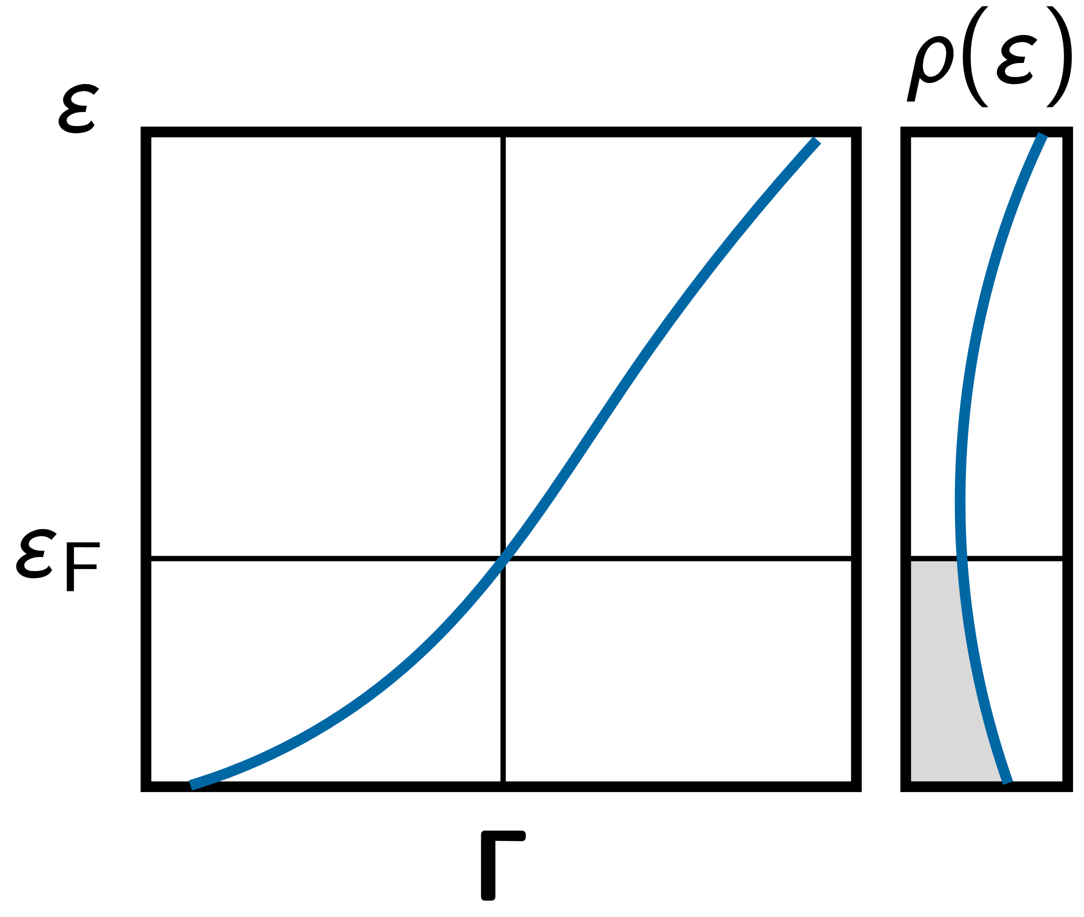

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Effect of interaction: Excitonic instability for $\Delta < E_{\mathrm{exc}} = \frac{m_{\mathrm{eff}}}{\epsilon^2 m_0} 13.6\,\mathrm{eV}$ (semiconductor); Screening (metal)

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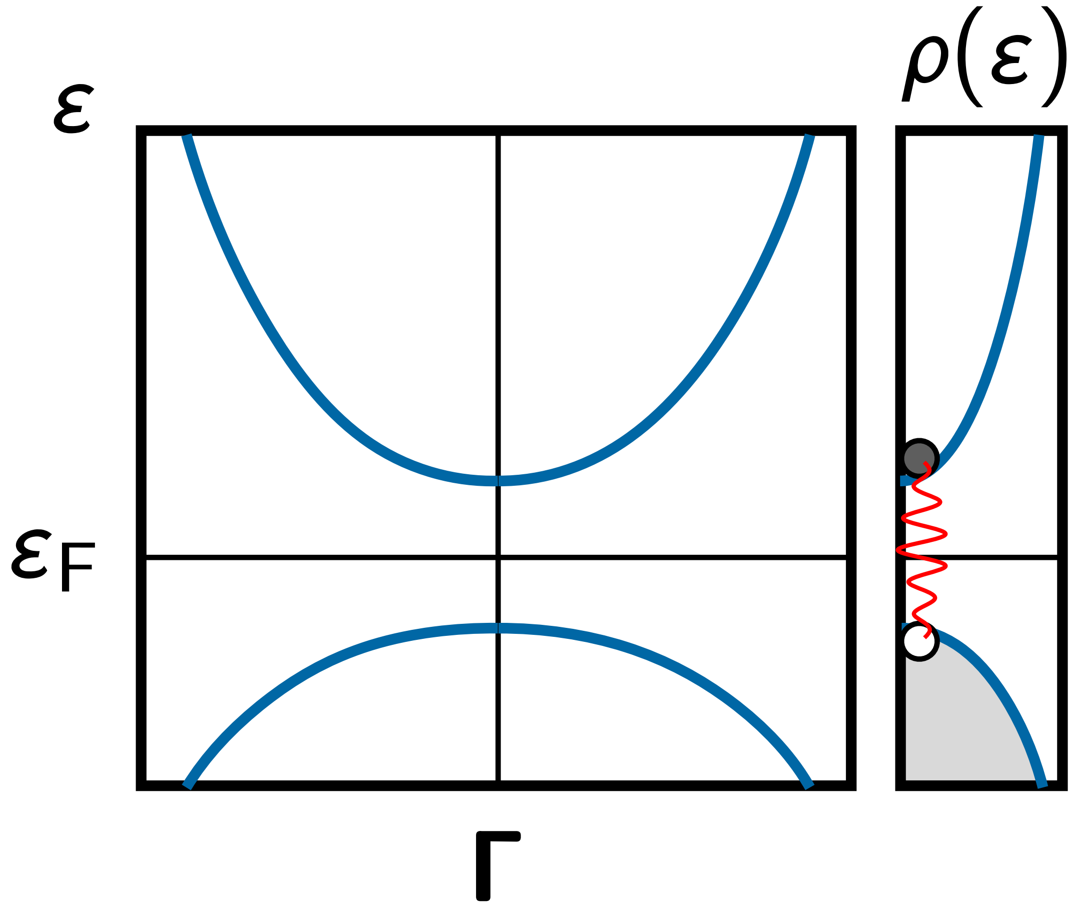
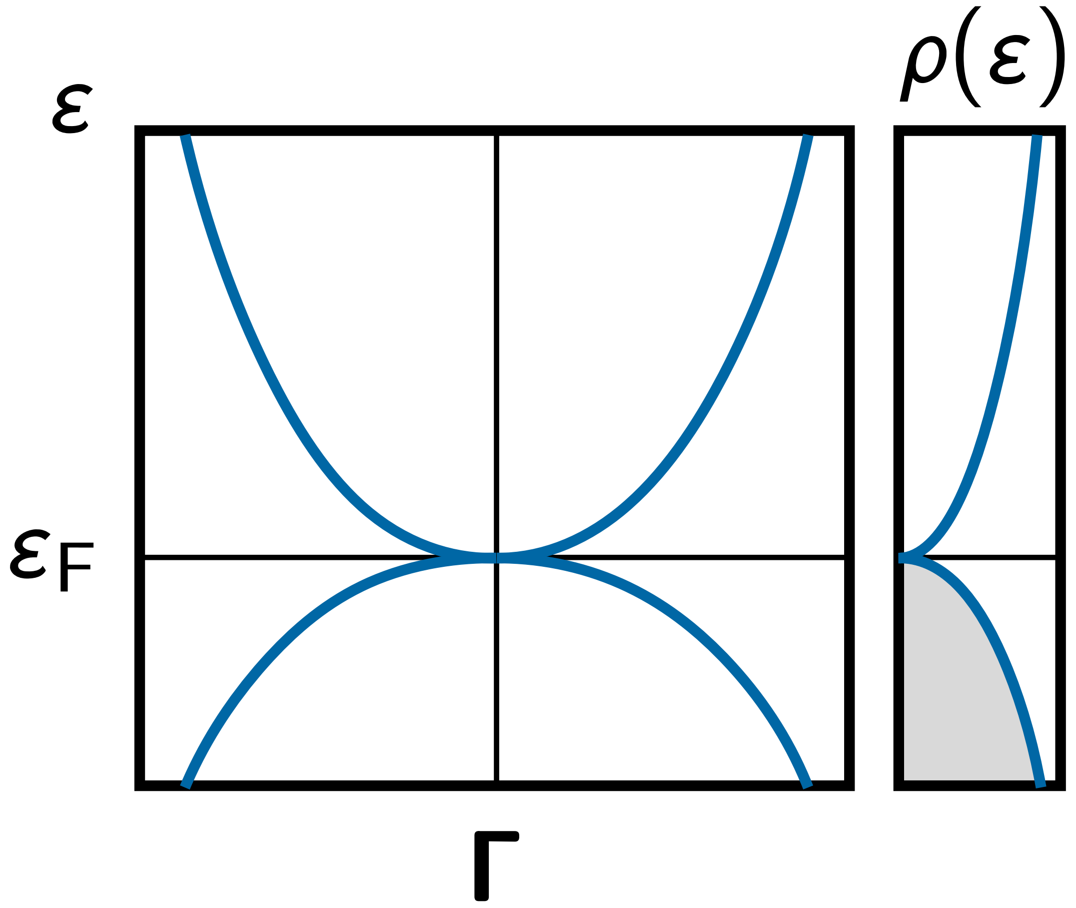
Small-gap semiconductor



Zero-gap semiconductor



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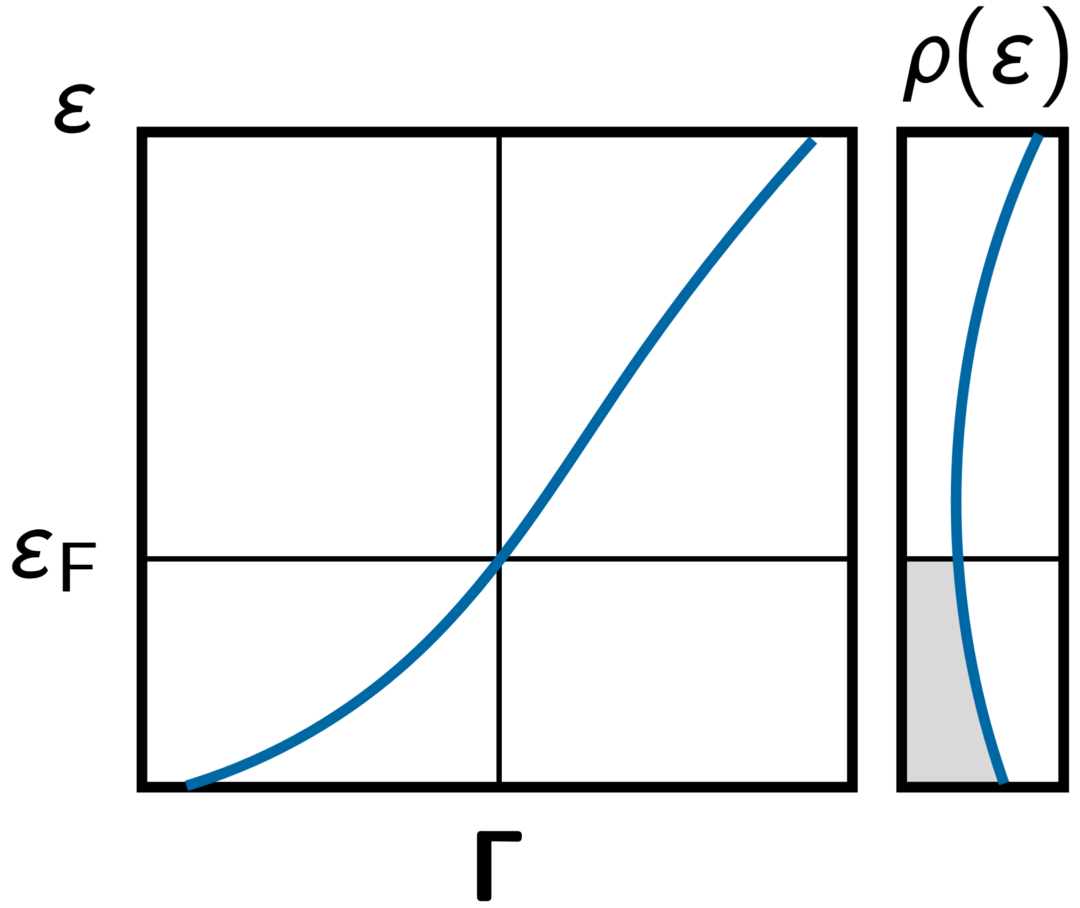

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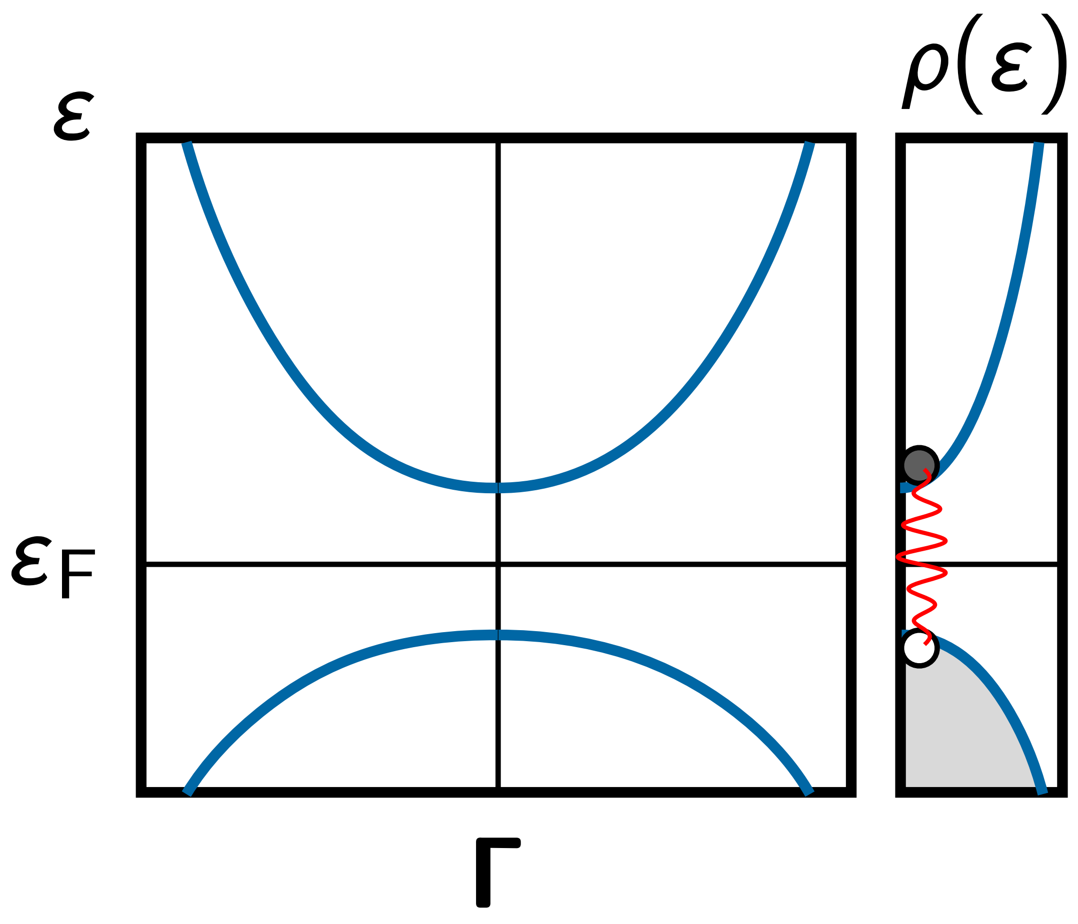

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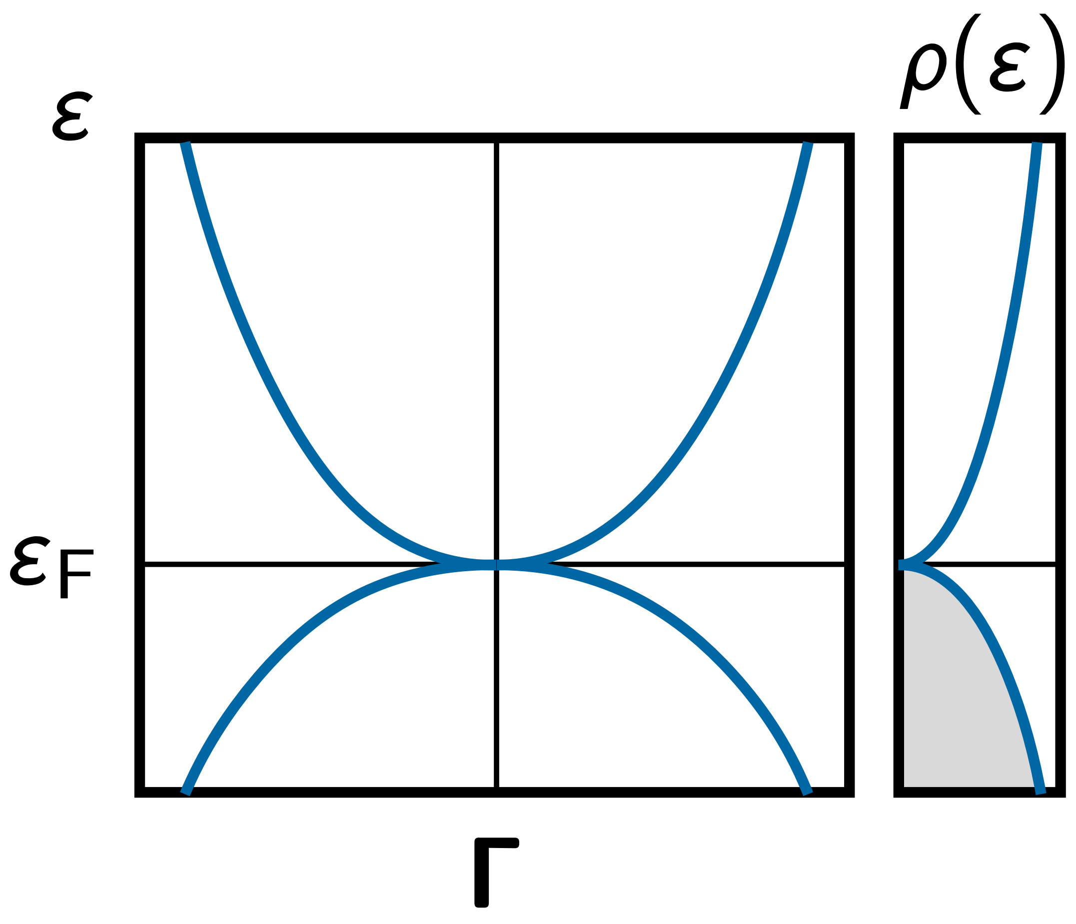

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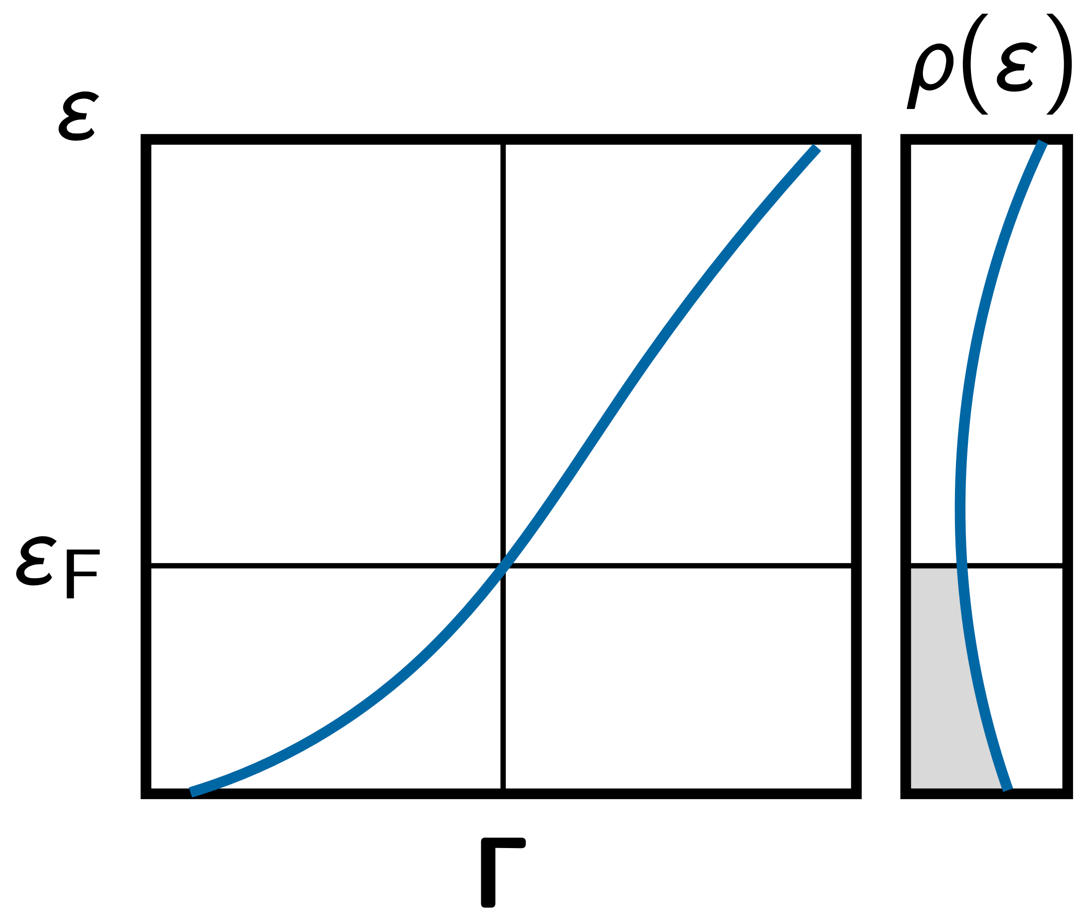

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[Landau & Lifshitz vol. 3, '77]

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$\Rightarrow$ N-dependent excitonic instability

[Landau & Lifshitz vol. 3, '77]

[LJ & Herbut, PRB '16]

Luttinger-Abrikosov-Beneslavskii state

RG flow ($\delta = 0$):

$$\frac{\mathrm{d}e^2}{\mathrm{d}\ln b} = (4 - d)e^2 - Ne^2$$

[Abrikosov, JETP '74]

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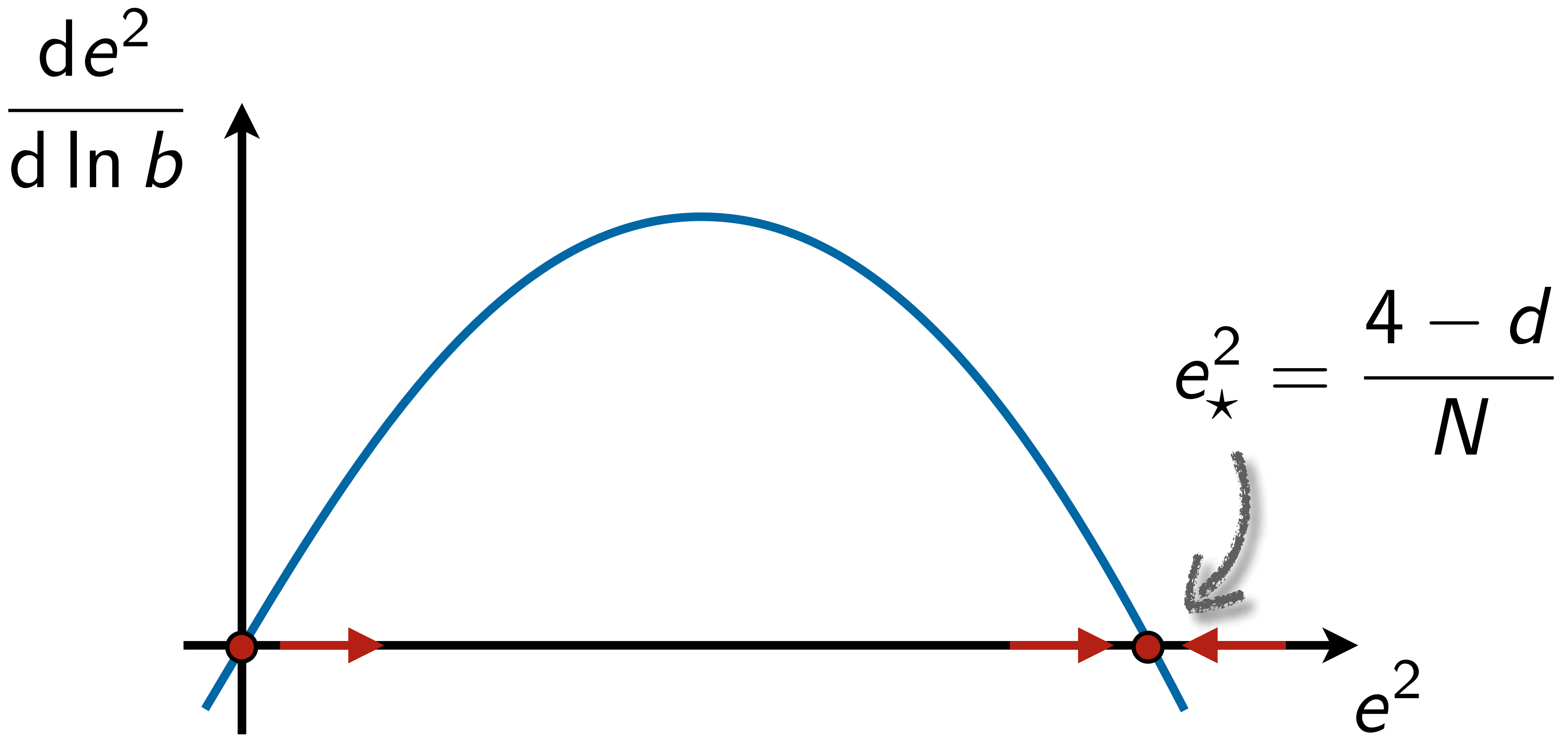

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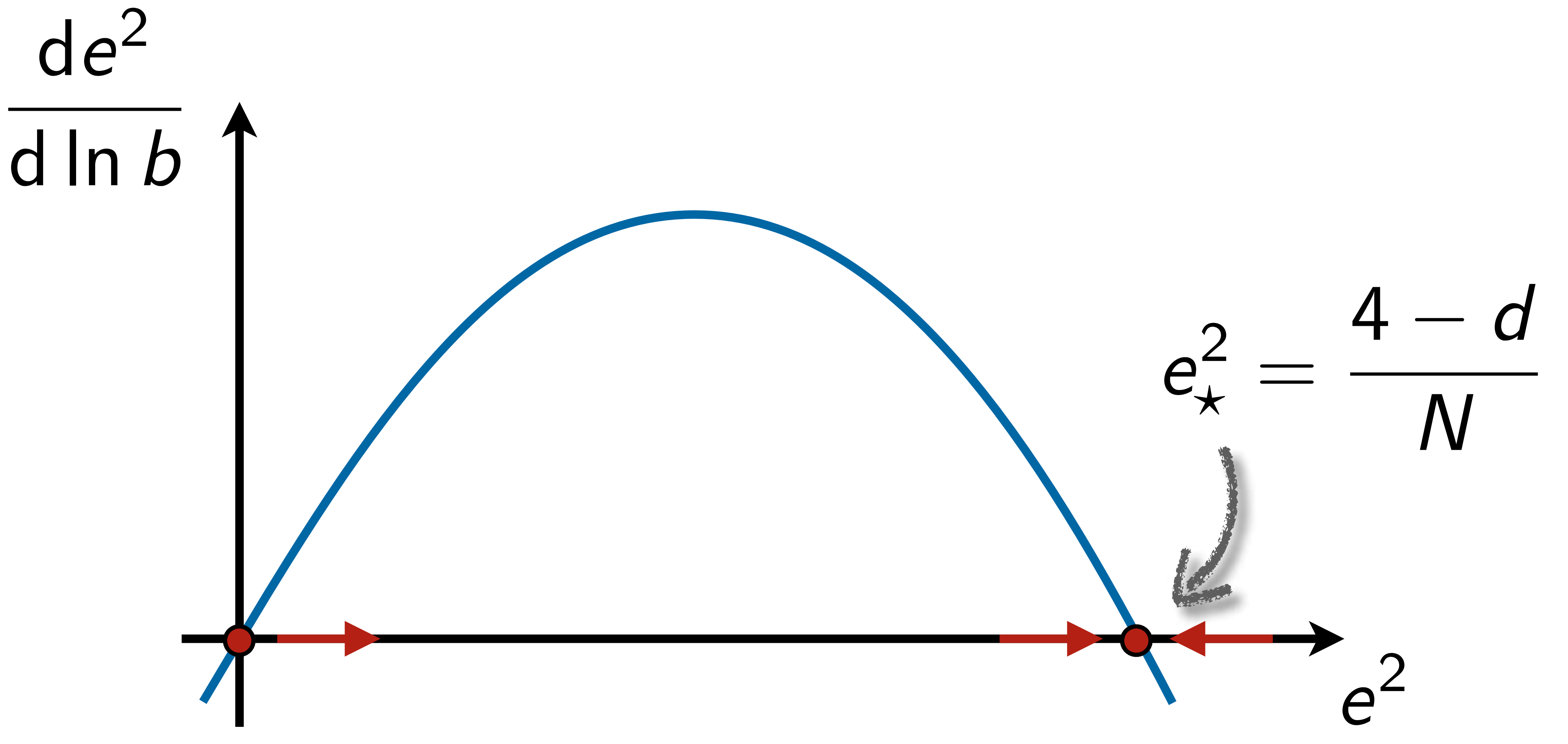


[Abrikosov, JETP ’74]

Non-Fermi liquid:

$$C_V \propto T^{d/z}$$

with $d = 3$ and $z \approx 1.8$

$$\sigma(\omega, T) \propto T^{1/z}\mathcal{F}(\omega/T)$$

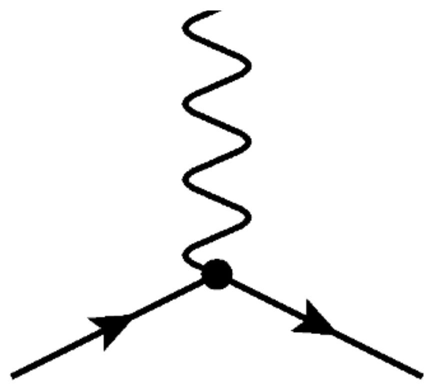
[Moon, Xu, Kim, Balents, PRL ’13]

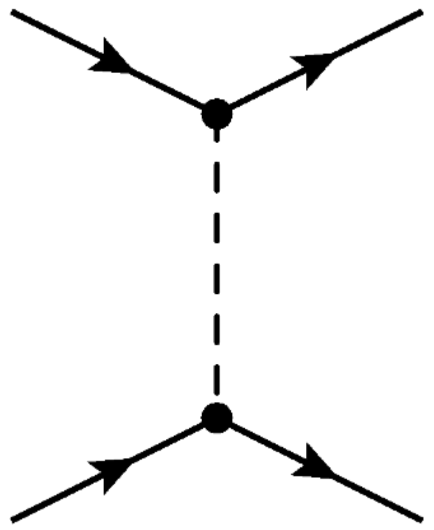
Excitonic instability

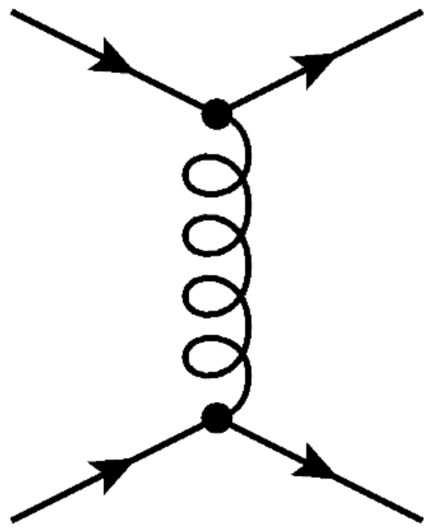
Lagrangian:

generated by RG

$$\mathcal{L} = \psi^\dagger \left(\partial_\tau + \sum_{a=1}^{5} (1 + s_a \delta) d_a(-\mathrm{i}\nabla)\gamma_a + \mathrm{i}ea \right) \psi + \frac{1}{2}(\nabla a)^2 + \sum_i G_i (\psi^\dagger M_i \psi)^2$$







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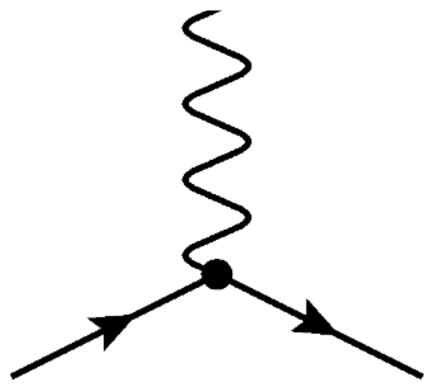

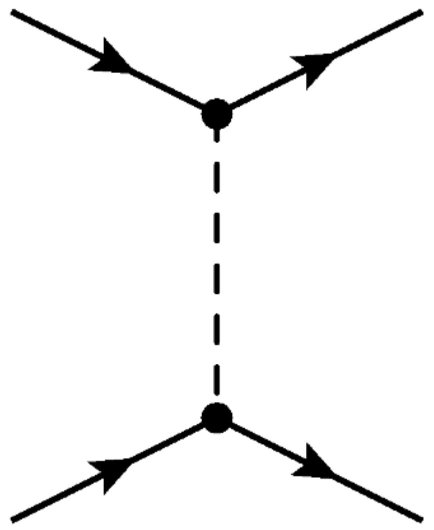

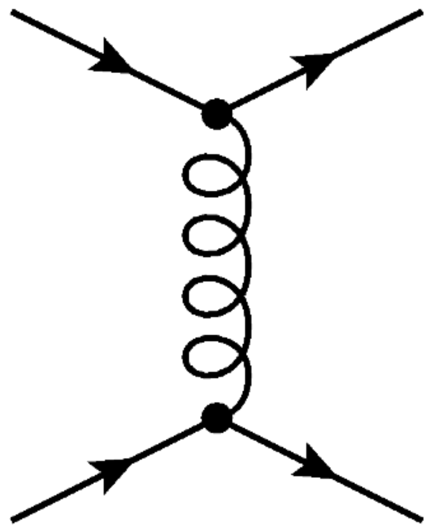

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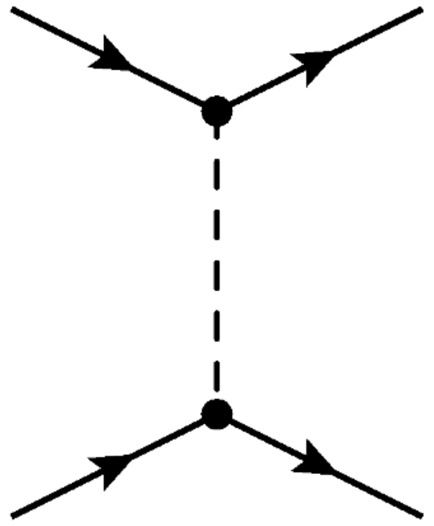
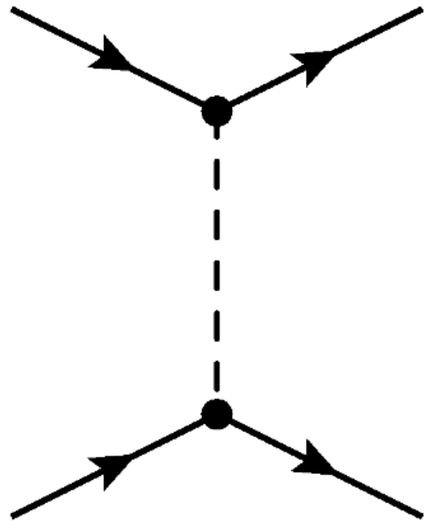
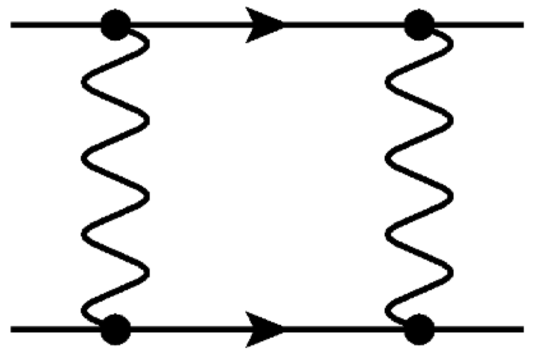
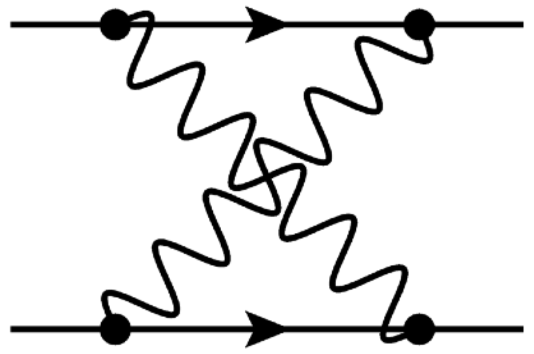
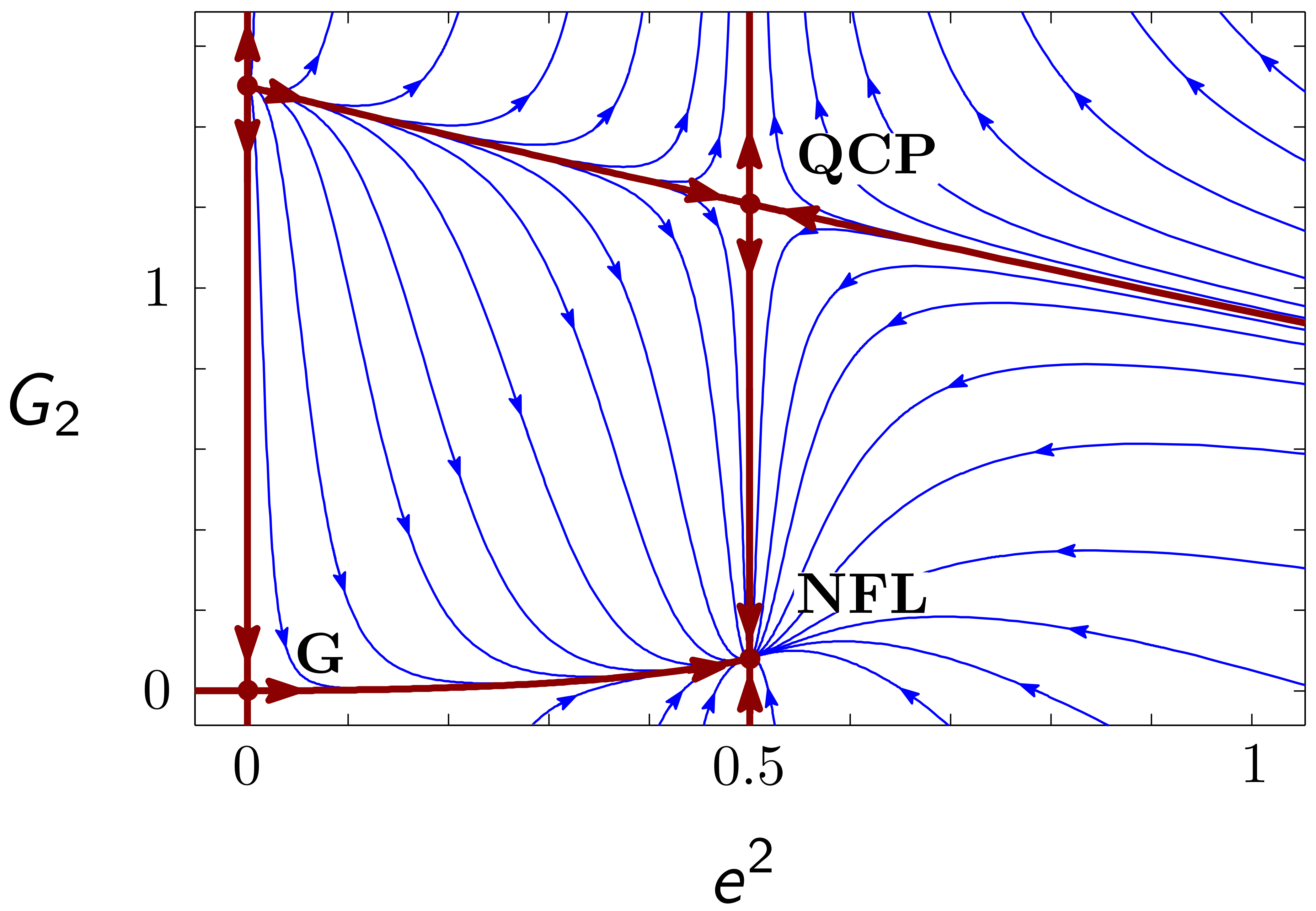
$\frac{\mathrm{d}}{\mathrm{d}\ln b}$  $= (2-d)$  $+$  $+$  $+ \ldots$

G_i G_i e^4 e^4

Flow diagram

$N > N_c \approx 2$



[LJ & Herbut, PRB '17]

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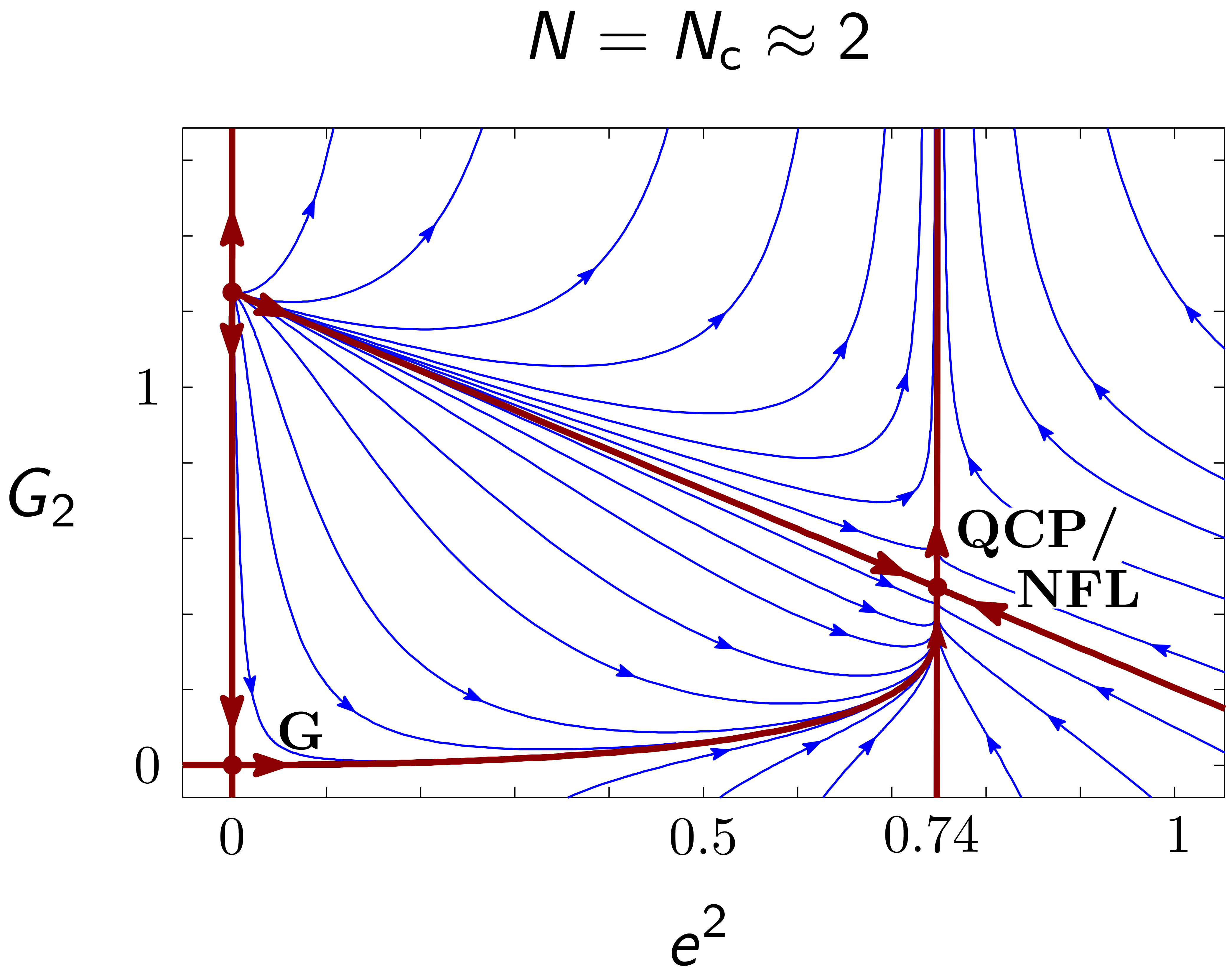

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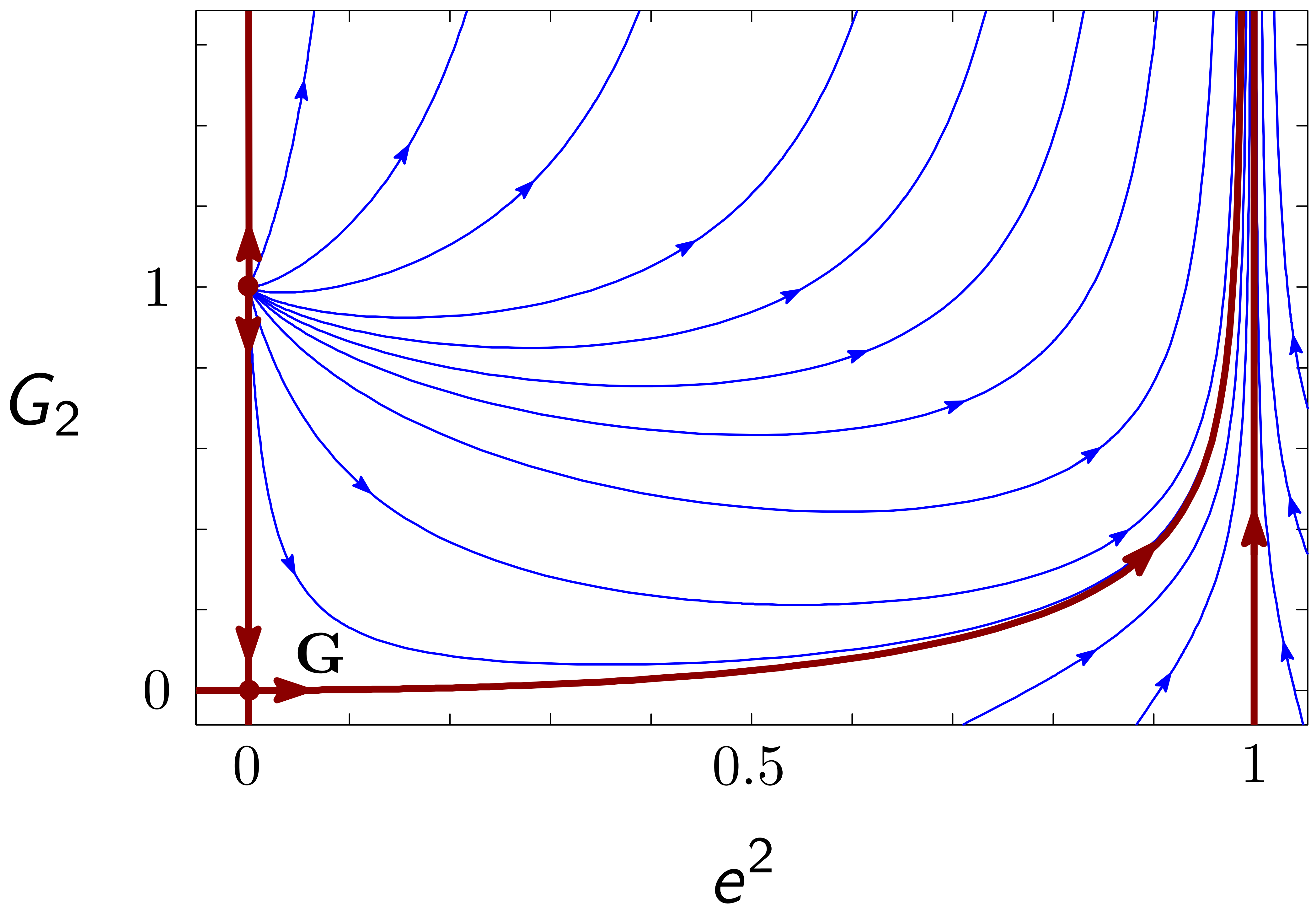

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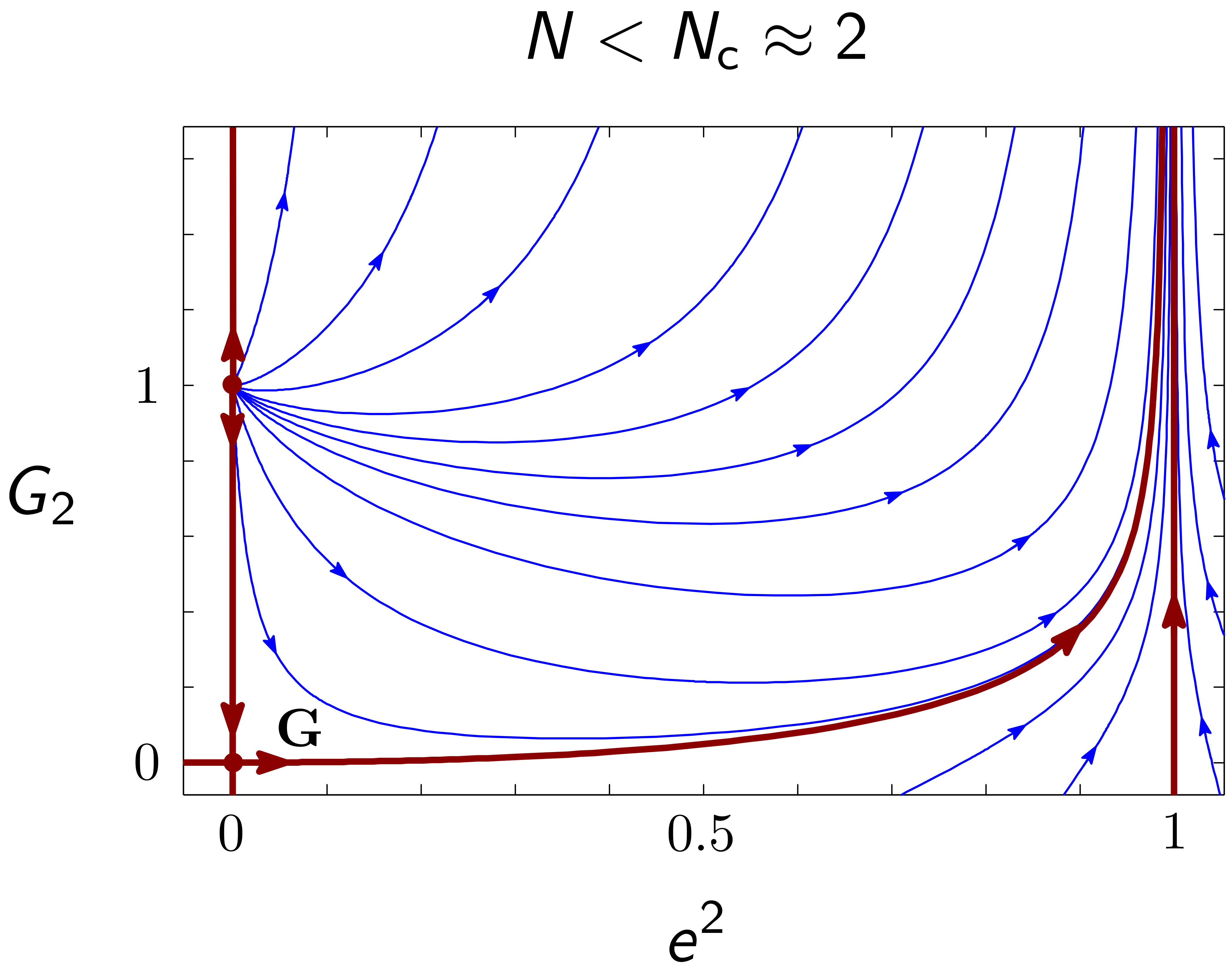
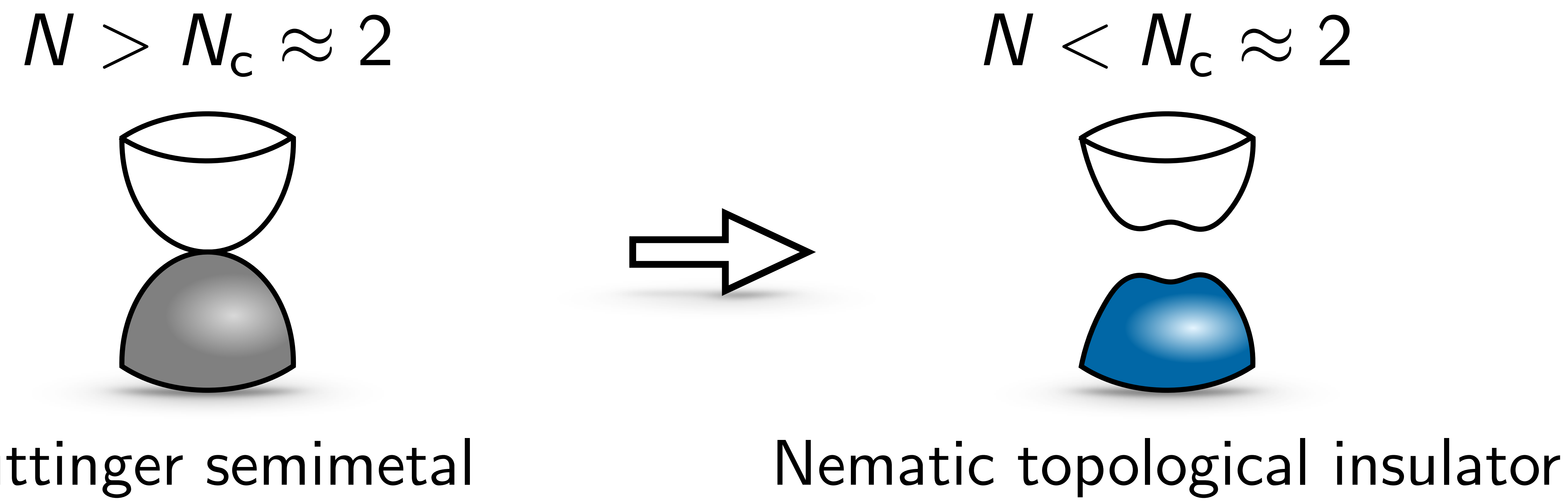


Ground state:



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Pyrochlore iridates

Full model:

$$\mathcal{L} = \psi^\dagger \left(\partial_\tau + \sum_{a=1}^{5} (1 + s_a \delta) d_a(-\mathrm{i}\nabla)\gamma_a + \mathrm{i}ea \right) \psi + \frac{1}{2}(\nabla a)^2 + \frac{r_1}{2}\phi^2 + g_1 \phi \psi^\dagger \gamma_{45} \psi + \frac{r_2}{2} \sum_{a=1}^{5} \varphi_a^2 + g_2 \sum_{a=1}^{5} \varphi_a \psi^\dagger \gamma_a \psi$$

[Moser & LJ, Rep. Progr. Phys. '25]

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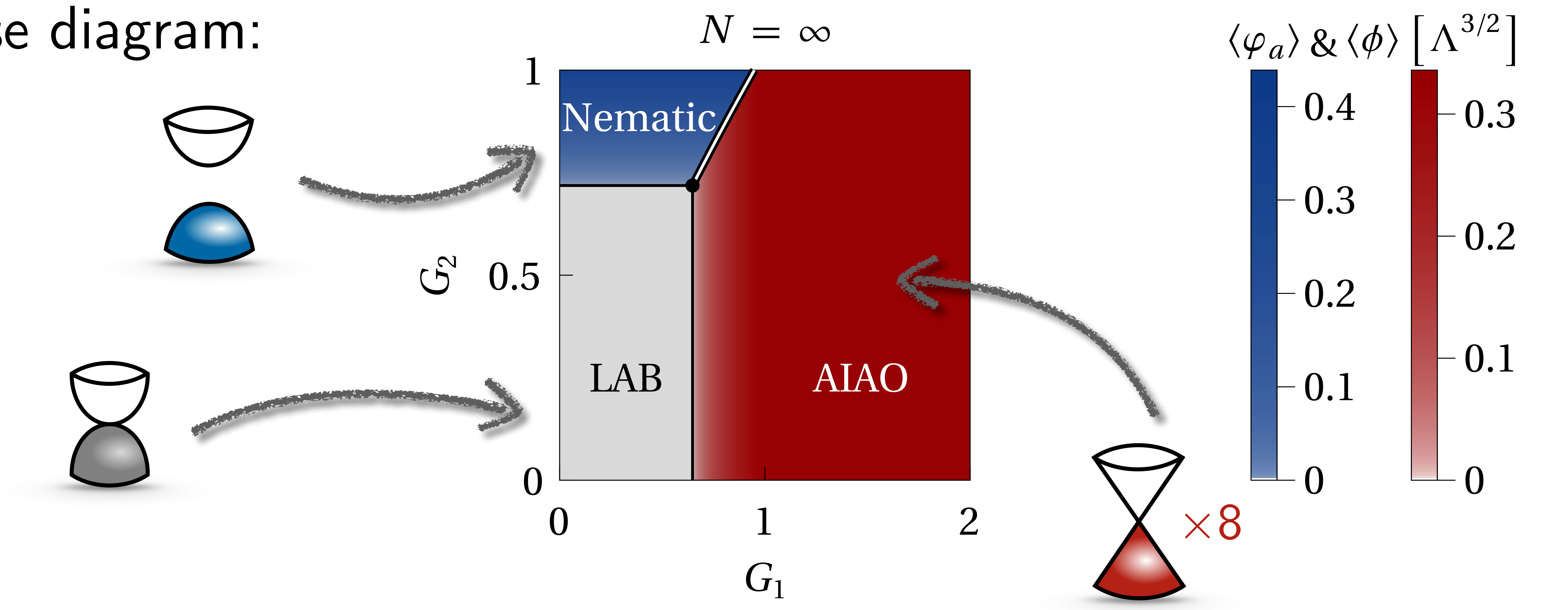
Mean-field phase diagram:



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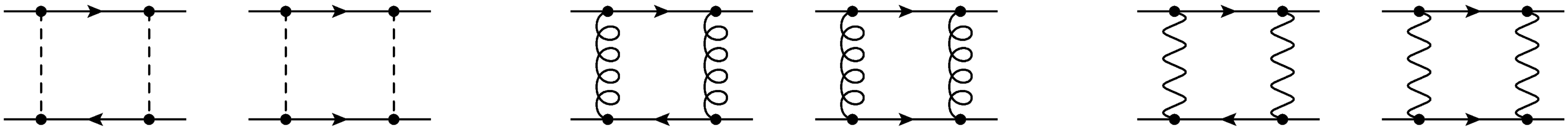
Dynamical bosonization

Fermion box diagrams:



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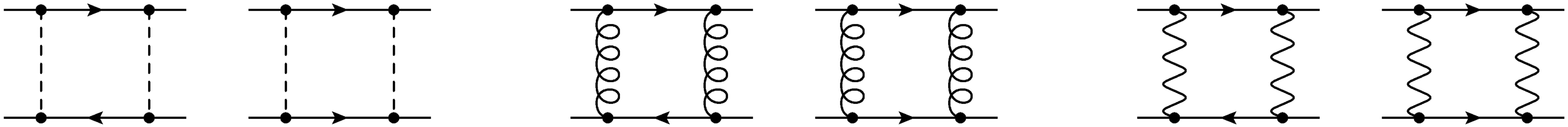

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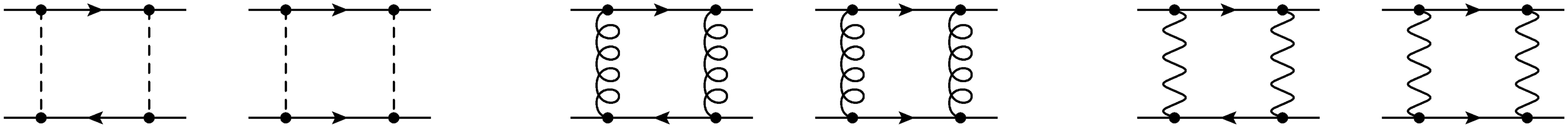

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New terms!

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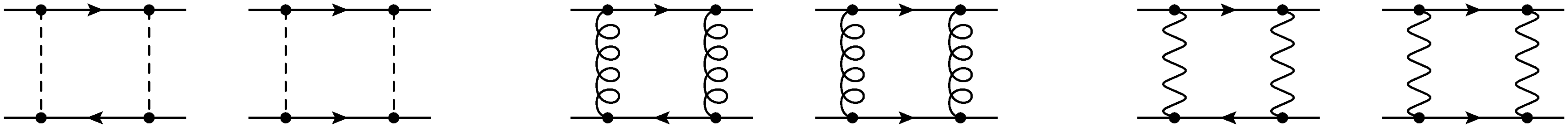

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Modified Yukawa-coupling flow: $\left.\frac{\mathrm{d} g_2}{\mathrm{d}\ln b}\right|_{\text{dyn. bos.}} = -r_2 \frac{\partial \delta G_2}{\partial \ln b}$

[Gies, Wetterich, PRD '02]
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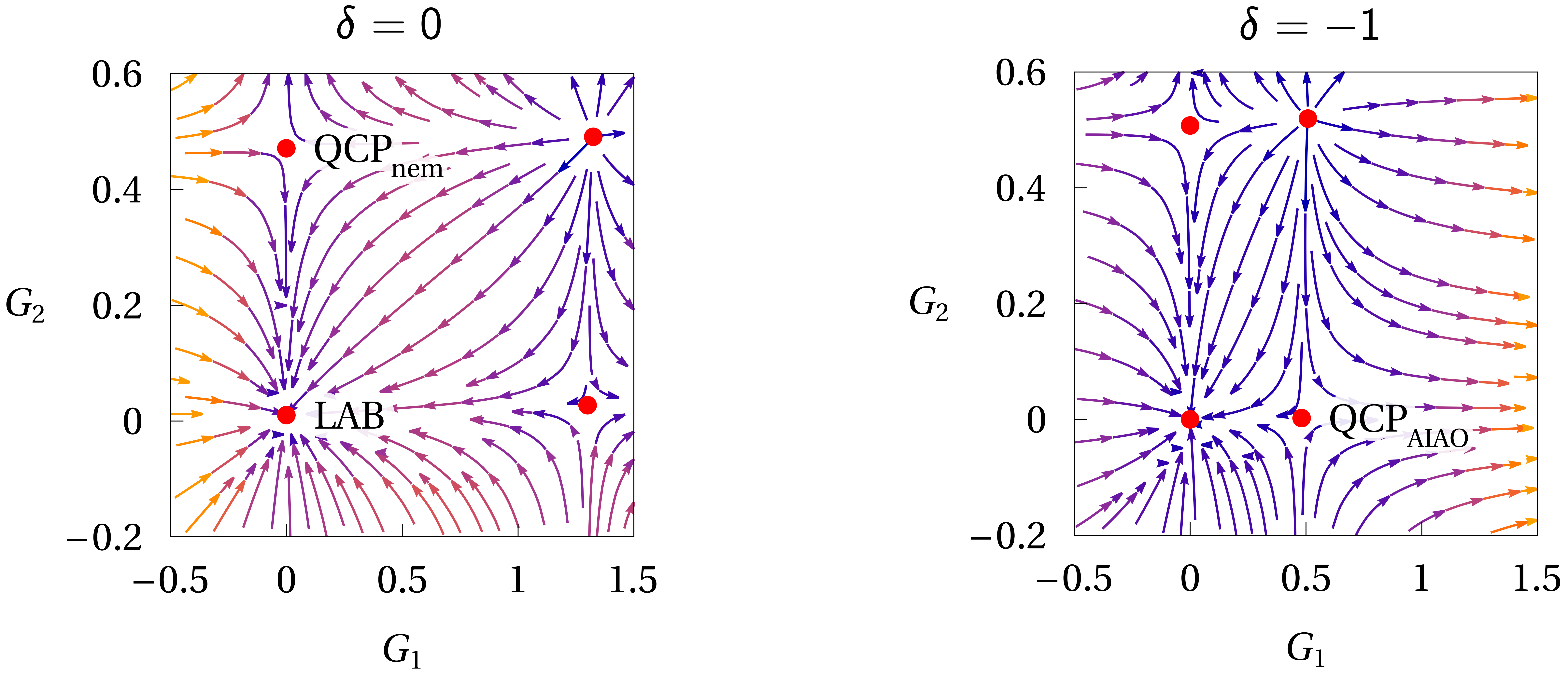
...

Point-like limit $\left(r_i \to \infty \text{ with } G_i = g_i^2/r_i \text{ fixed}\right)$

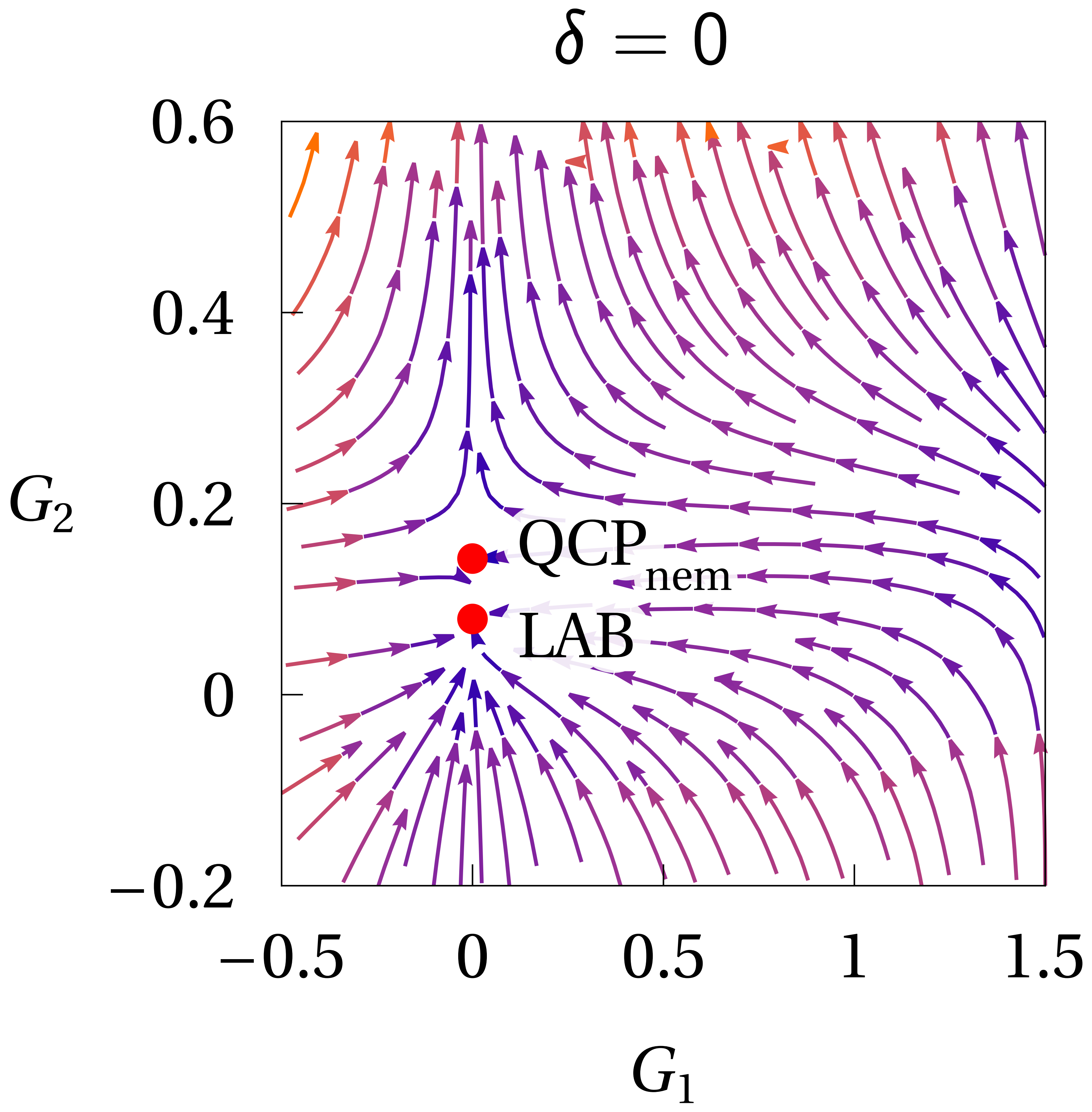
$N = 10$

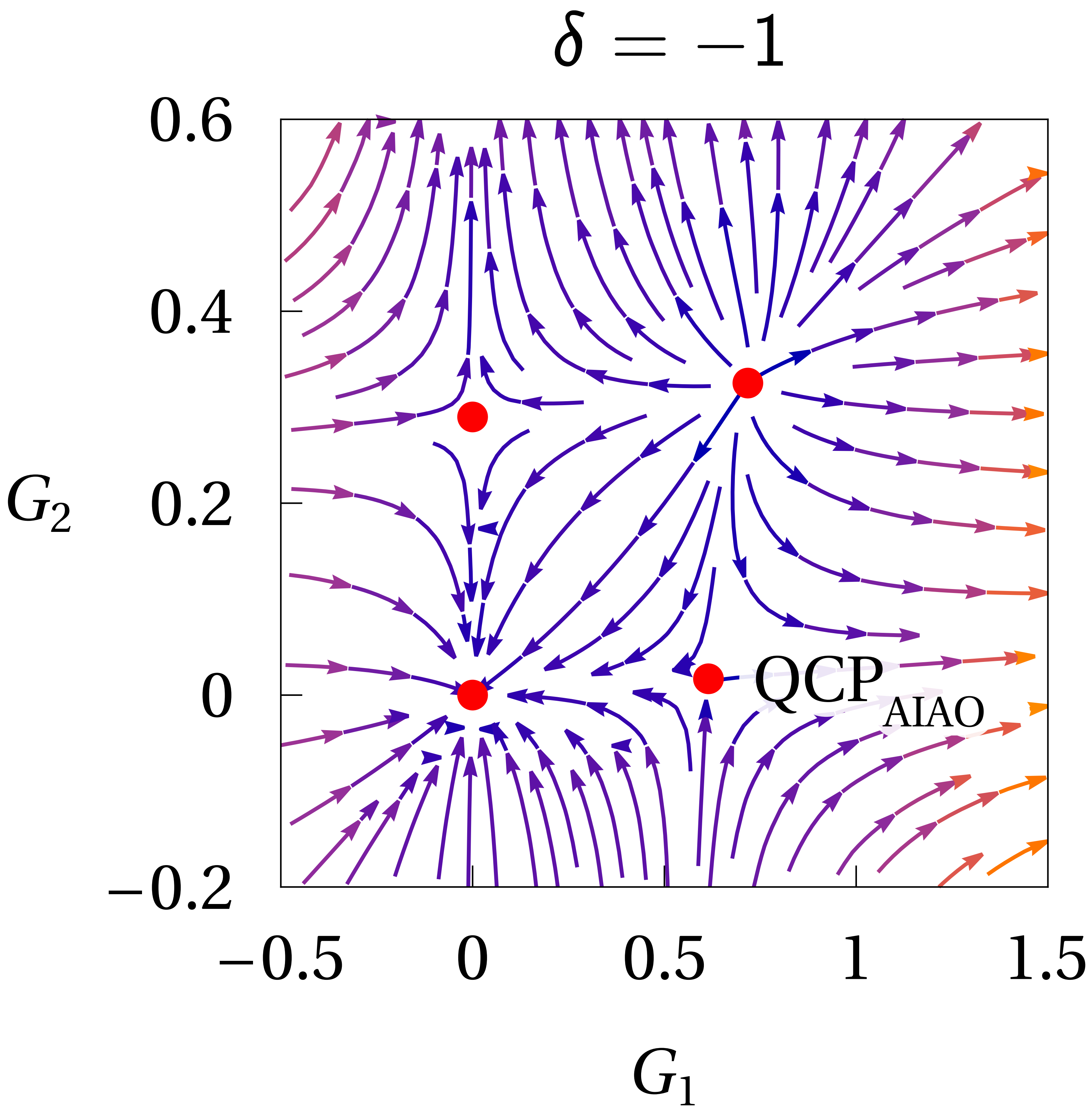


… for charge $e^2 = e_\star^2 > 0$

Point-like limit ($r_i \to \infty$ with $G_i = g_i^2/r_i$ fixed)

$N = 2$

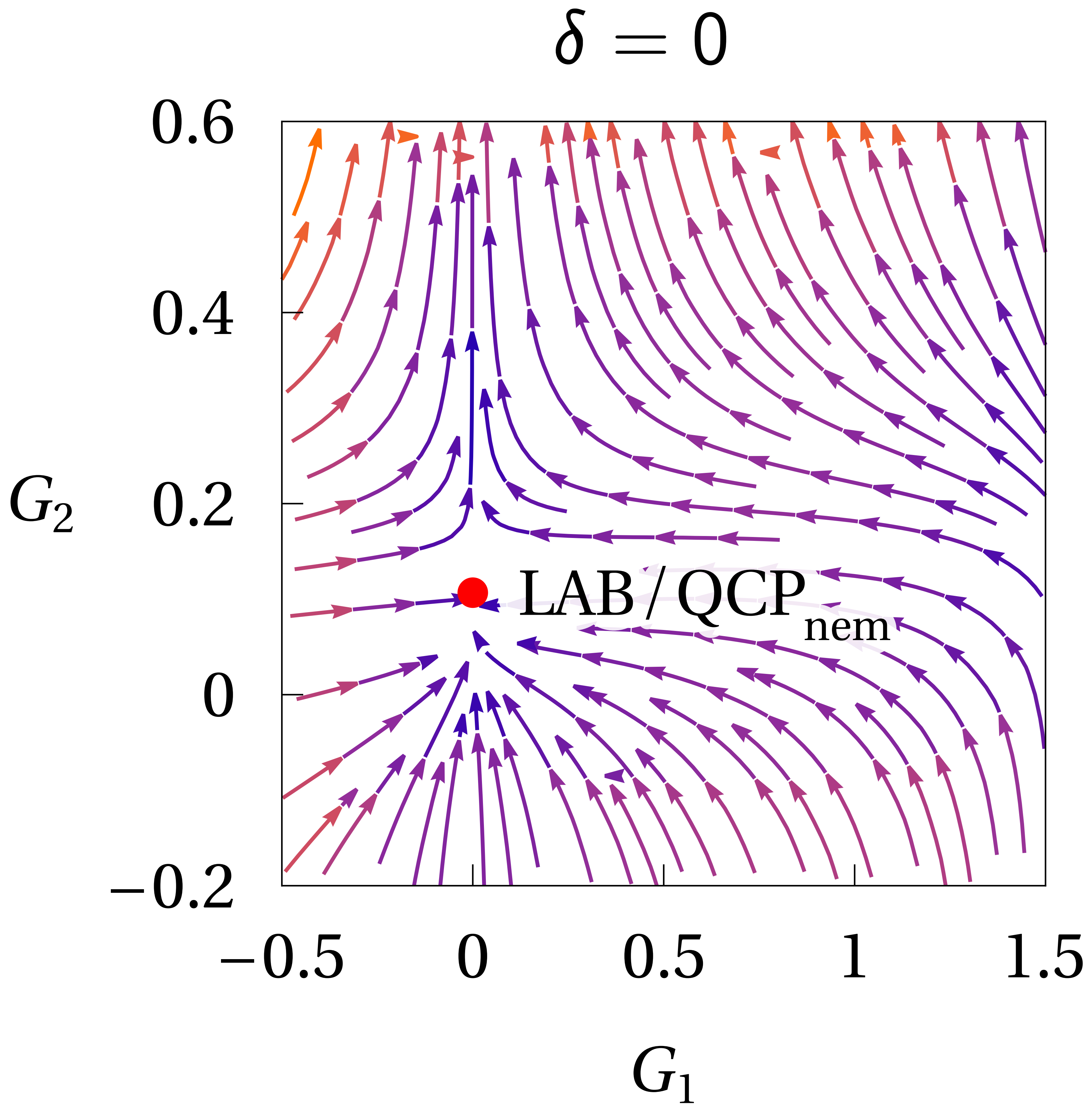


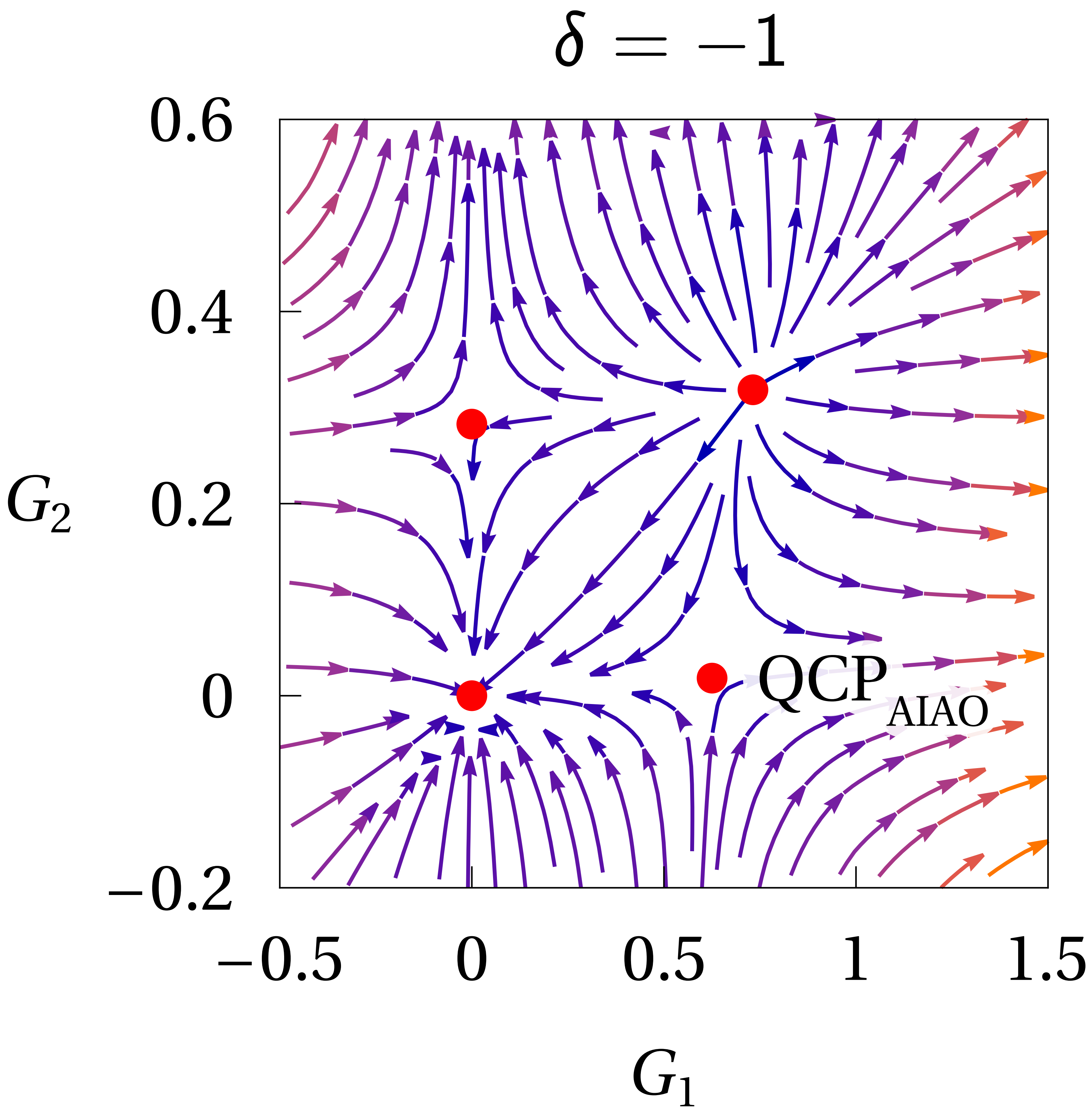


... for charge $e^2 = e_\star^2 > 0$

Point-like limit ($r_i \to \infty$ with $G_i = g_i^2/r_i$ fixed)

$N = 1.91$

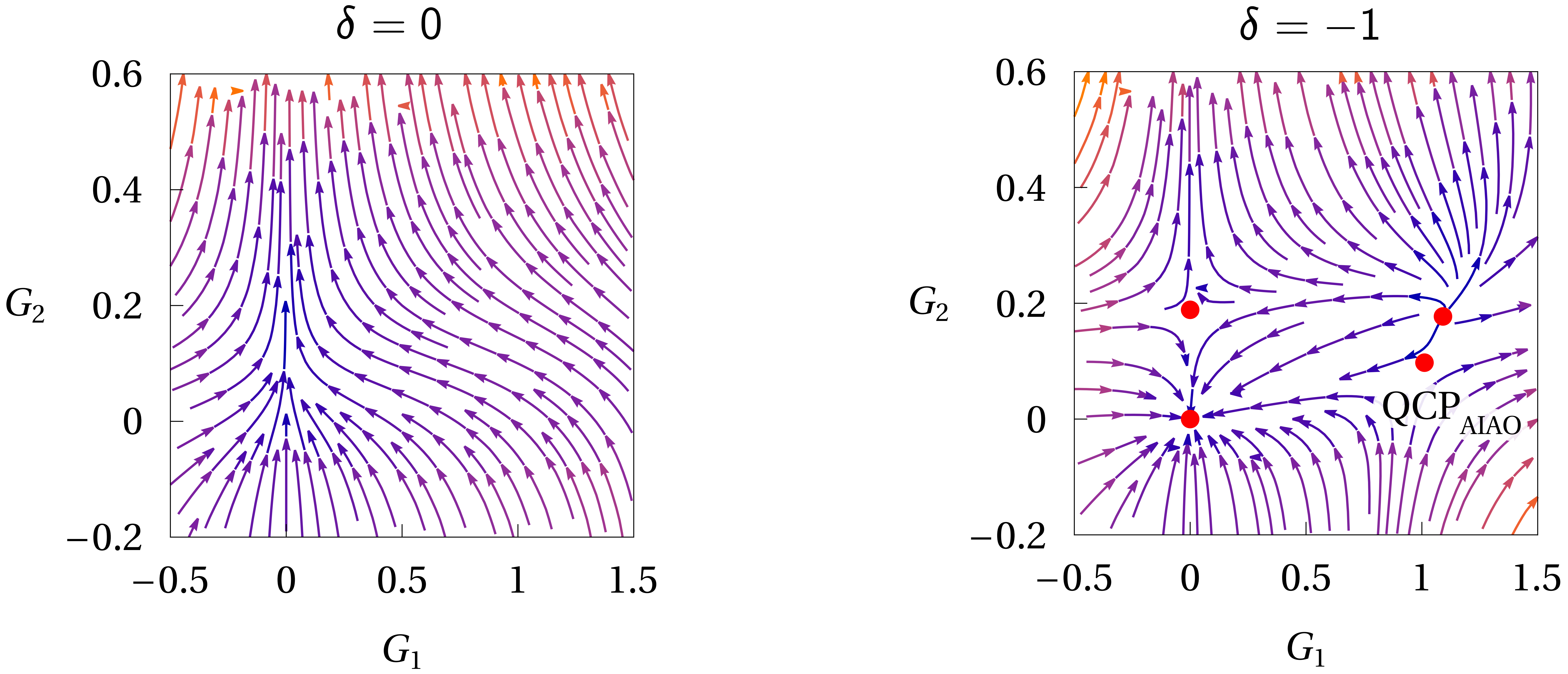




... for charge $e^2 = e_\star^2 > 0$

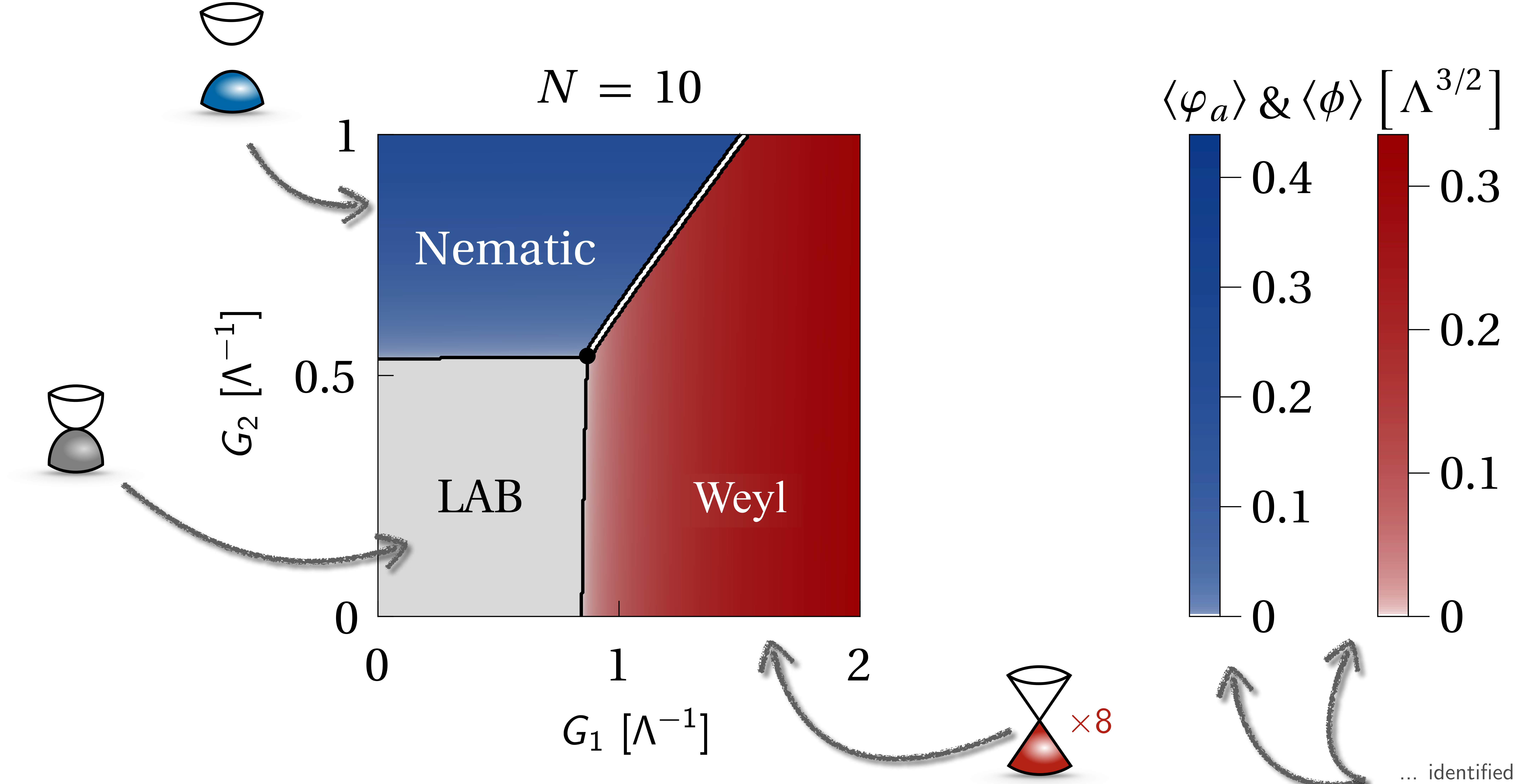
Point-like limit $(r_i \to \infty$ with $G_i = g_i^2/r_i$ fixed$)$

$N = 1$



... for charge $e^2 = e_\star^2 > 0$

Dynamically bosonized RG flow

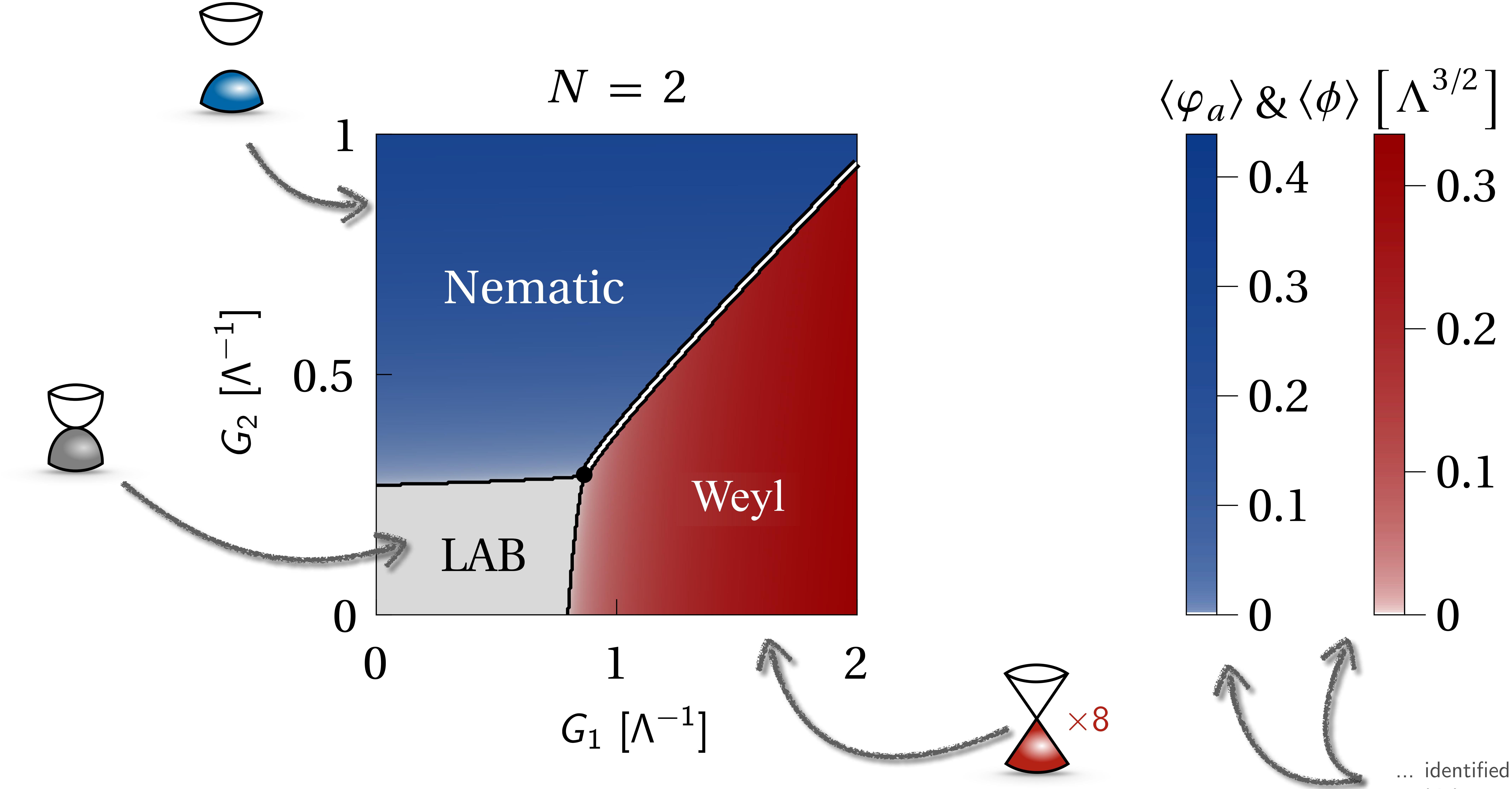


... with $G_{1,2} = g_{1,2}^2 / r_{1,2}$ in units of Λ^{-1}

... identified by scale t_{IR}, at which r_1 or r_2 vanishes, $\langle \phi \rangle \sim \exp(-2 t_{\mathrm{IR}})$

[Moser & LJ, Rep. Progr. Phys. '25]

Dynamically bosonized RG flow



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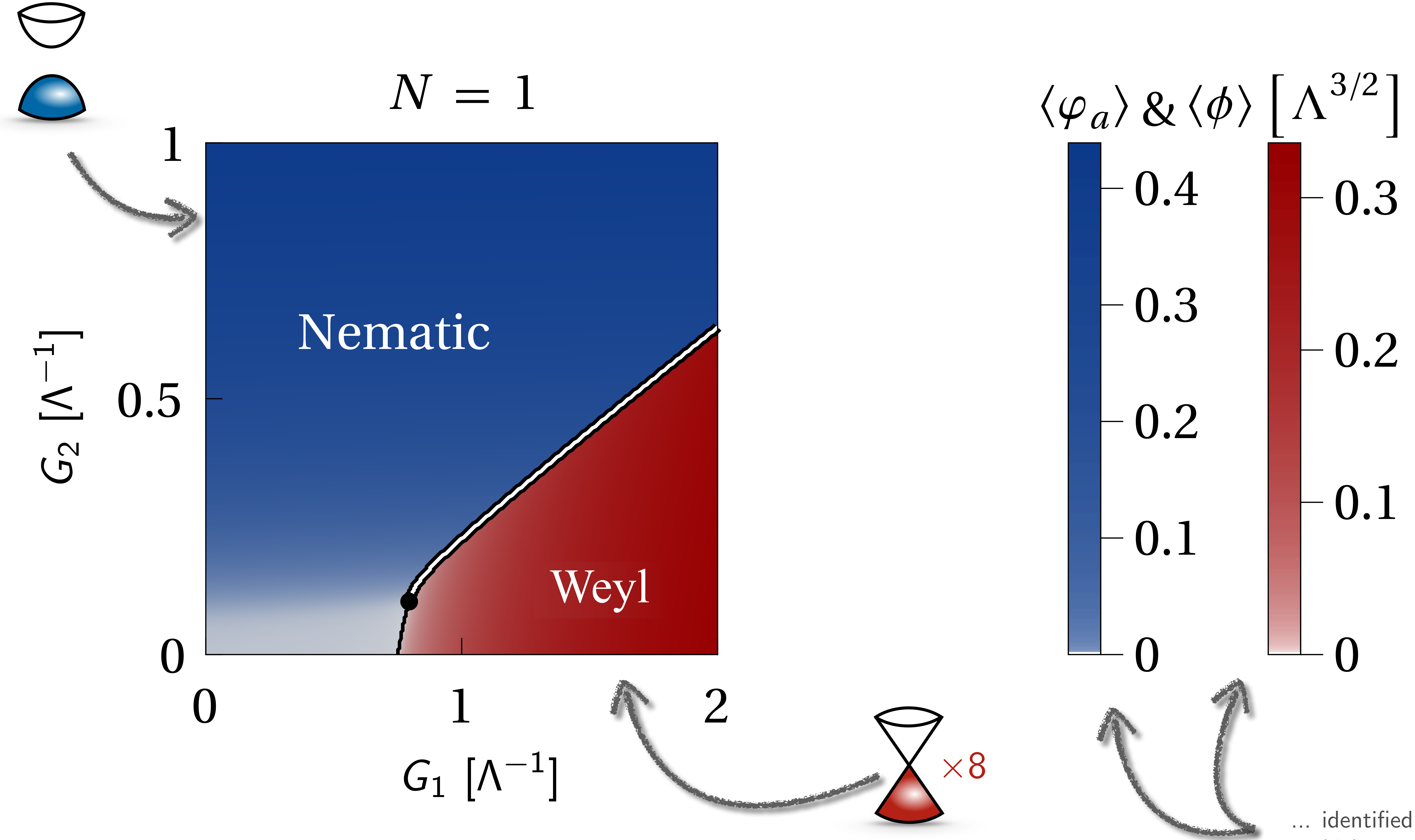

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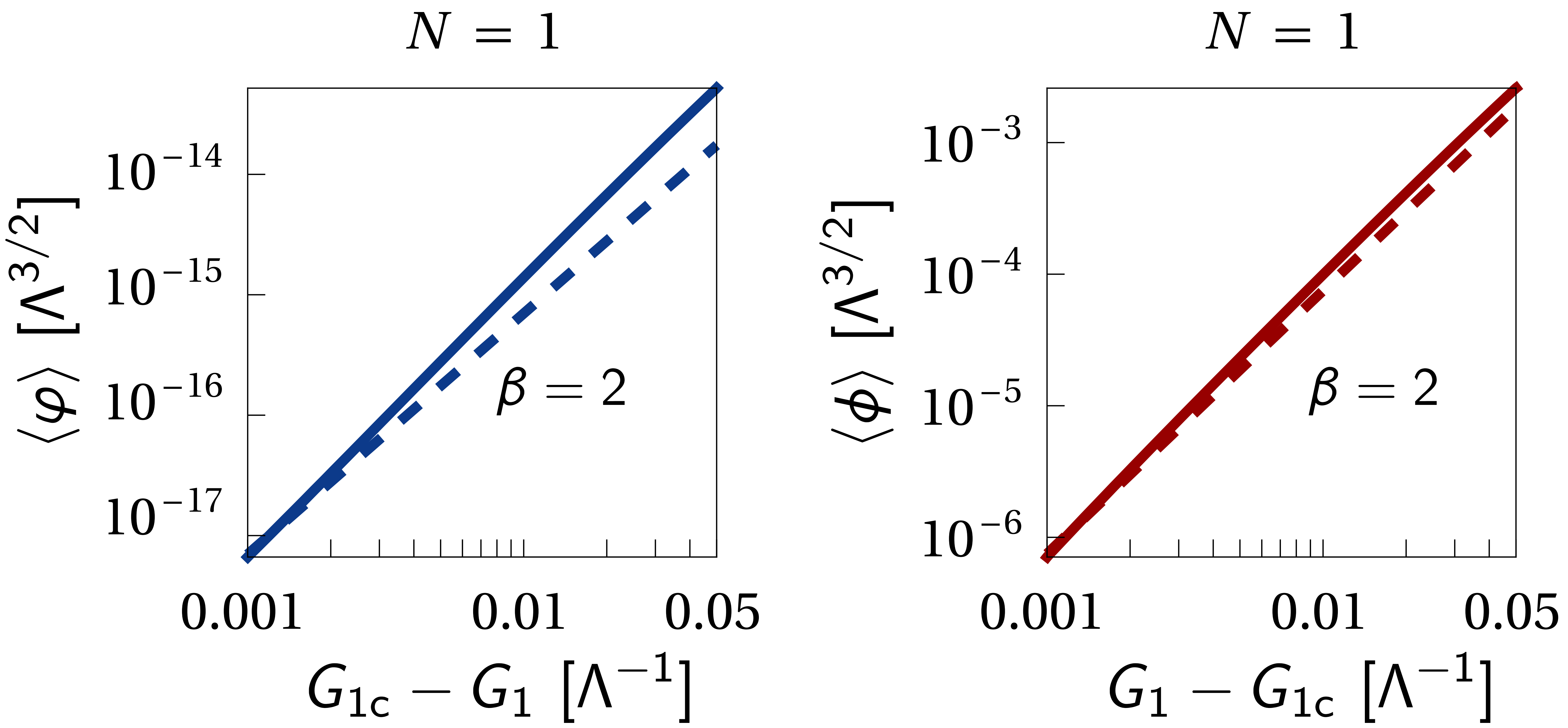
... identified by scale t_{IR}, at which r_1 or r_2 vanishes, $\langle \phi \rangle \sim \exp(-2t_{\mathrm{IR}})$

[Moser & LJ, Rep. Progr. Phys. '25]

Critical behavior

Order parameters:



[Moser & LJ, Rep. Progr. Phys. ’25]

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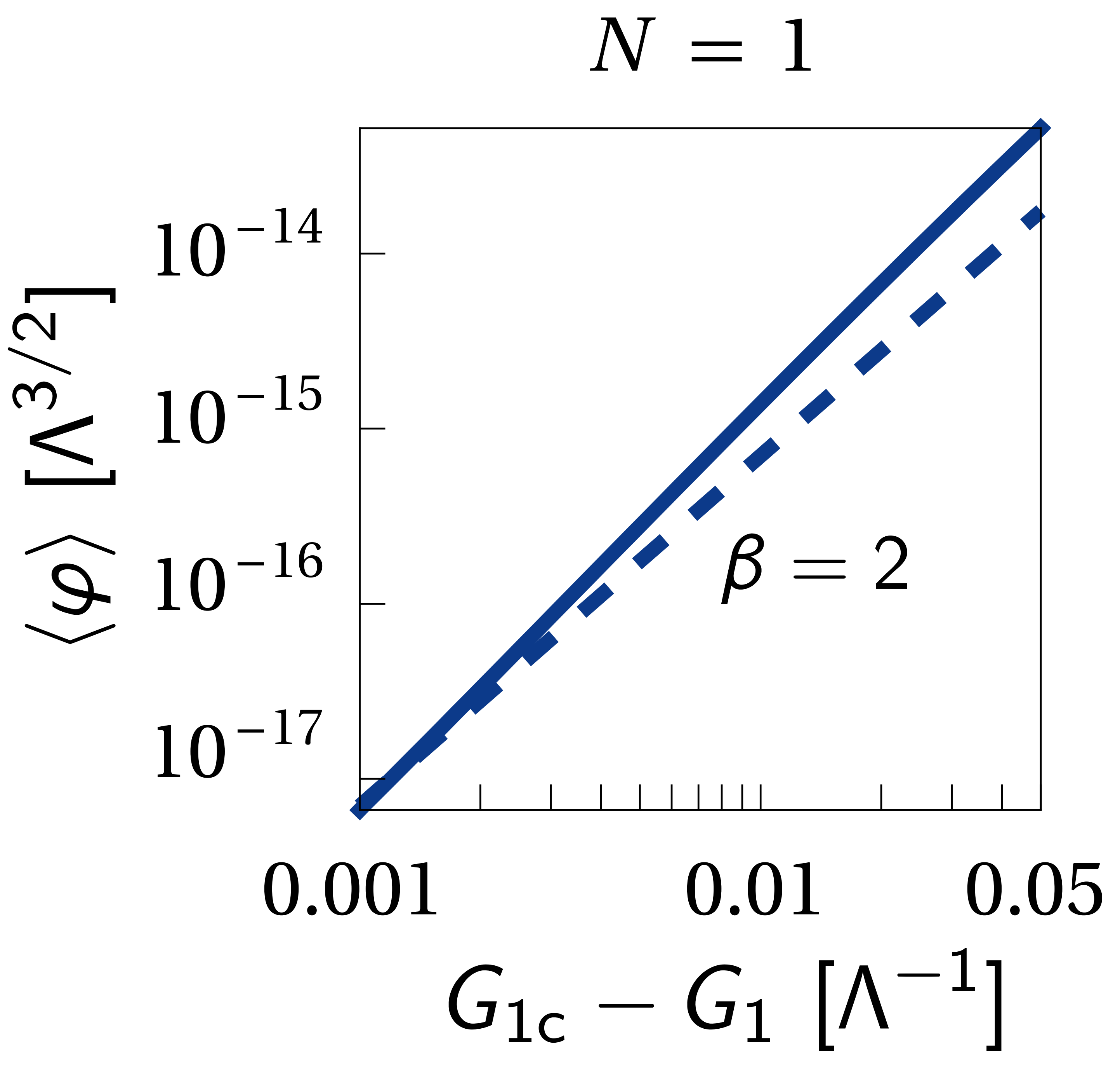
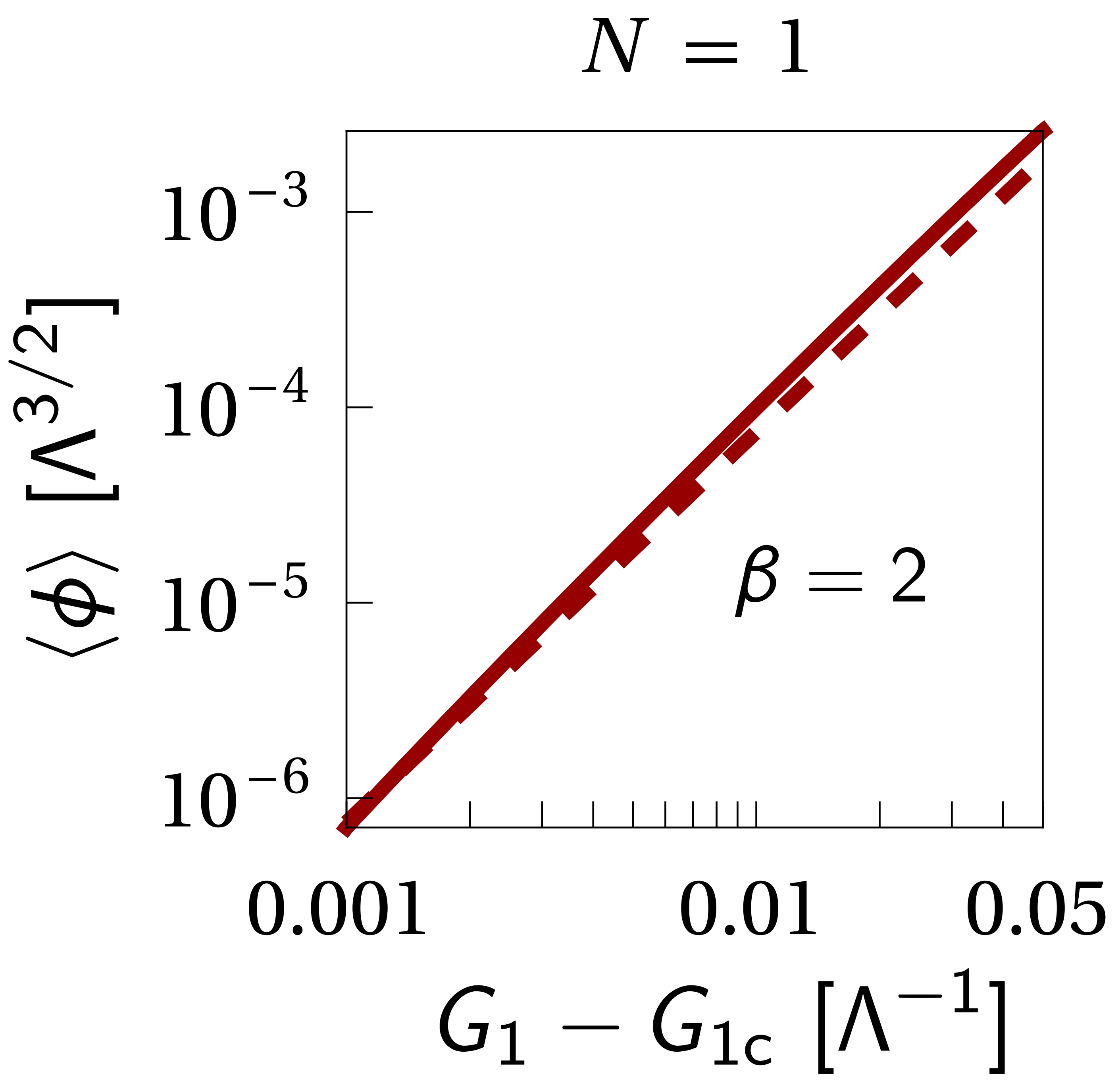
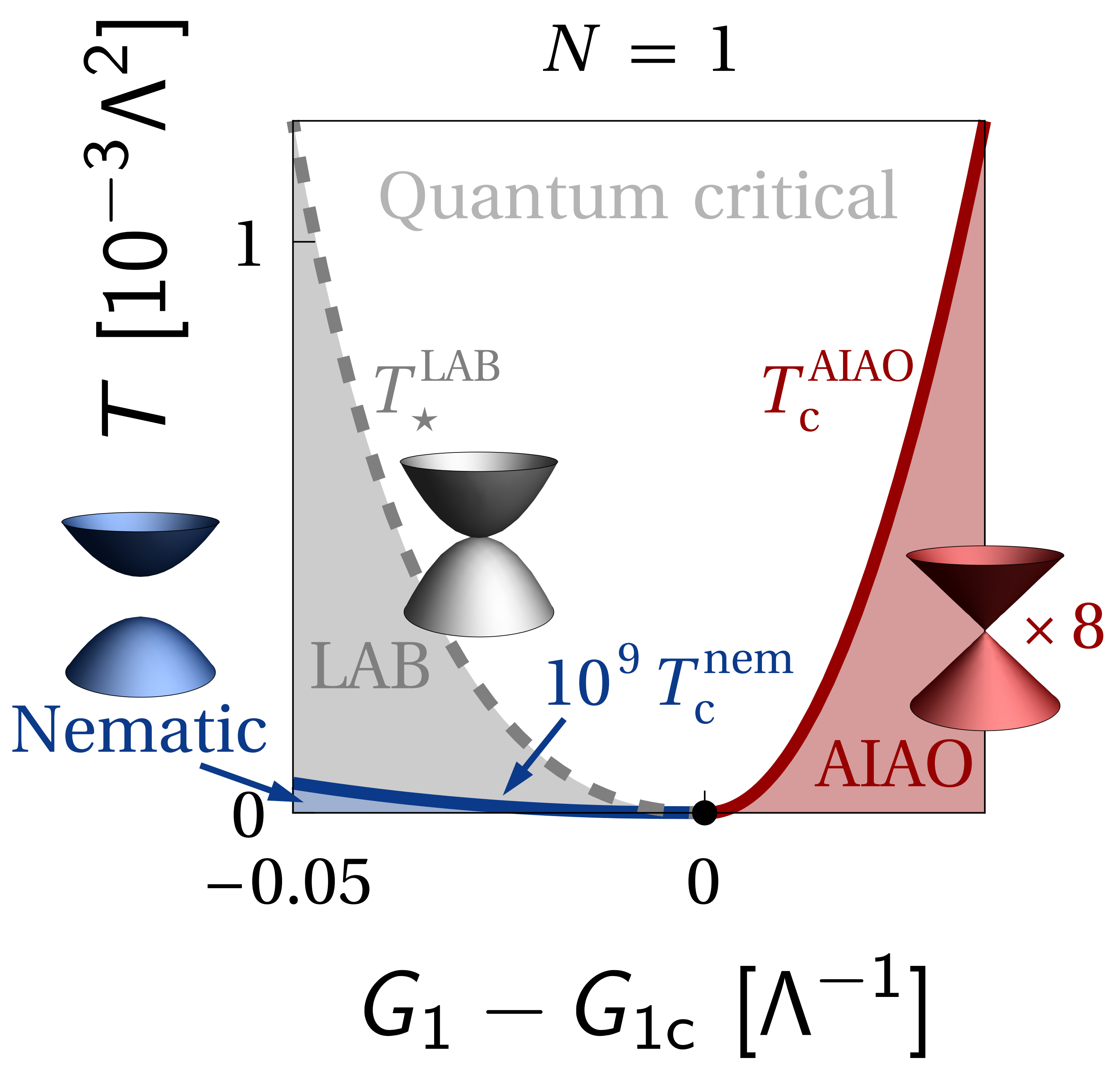




Finite-temperature phase diagram:



... asymmetry in energy scales!

[Moser & LJ, Rep. Progr. Phys. '25]

Outline

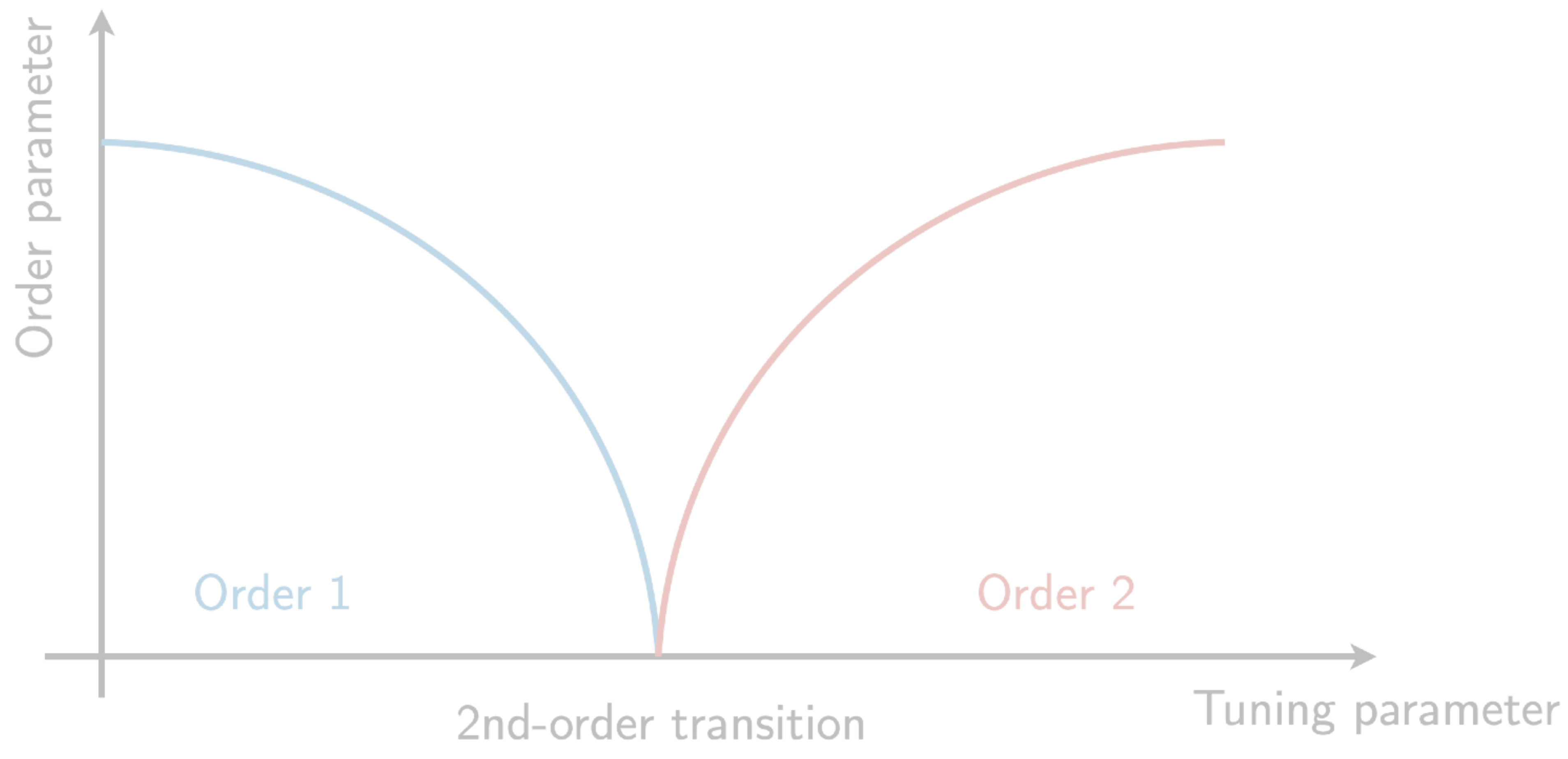
(1) Introduction

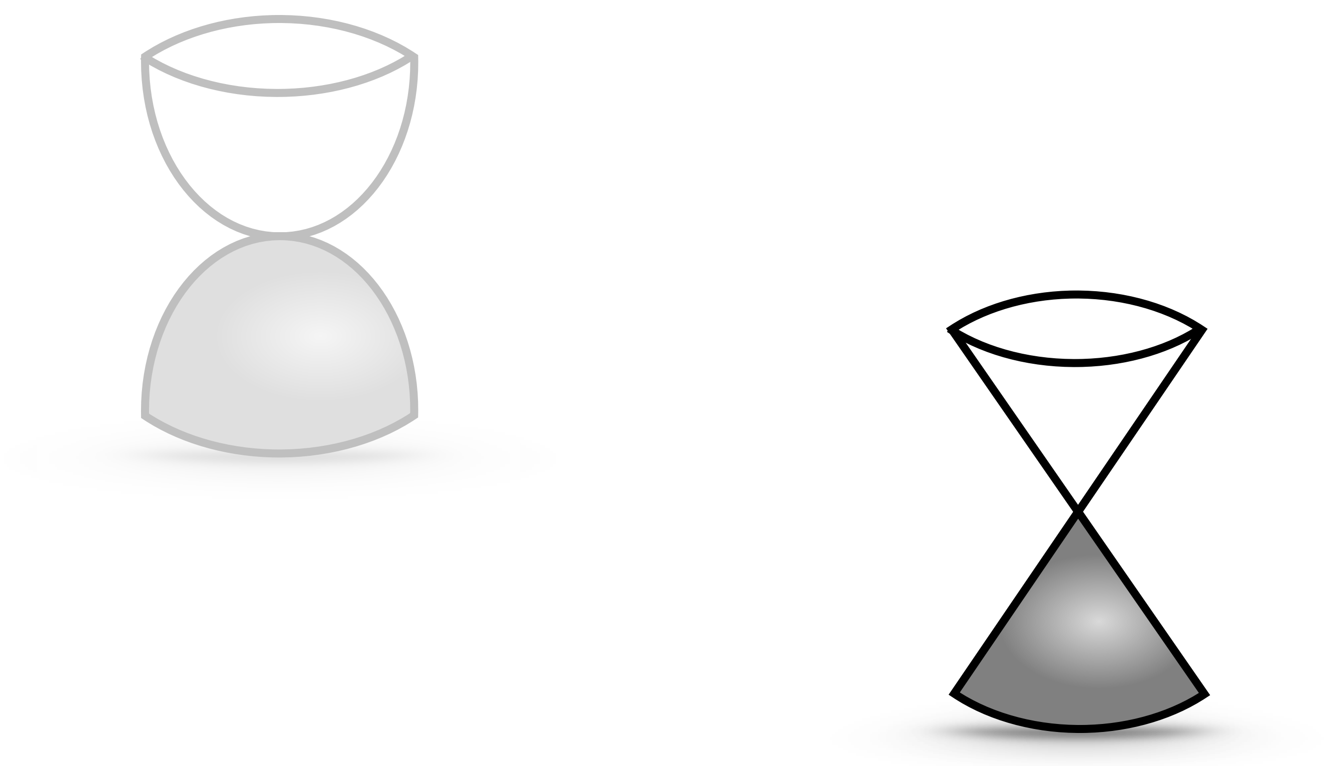
(2) Continuous order-to-order transition from fixed-point annihilation

(3) Examples

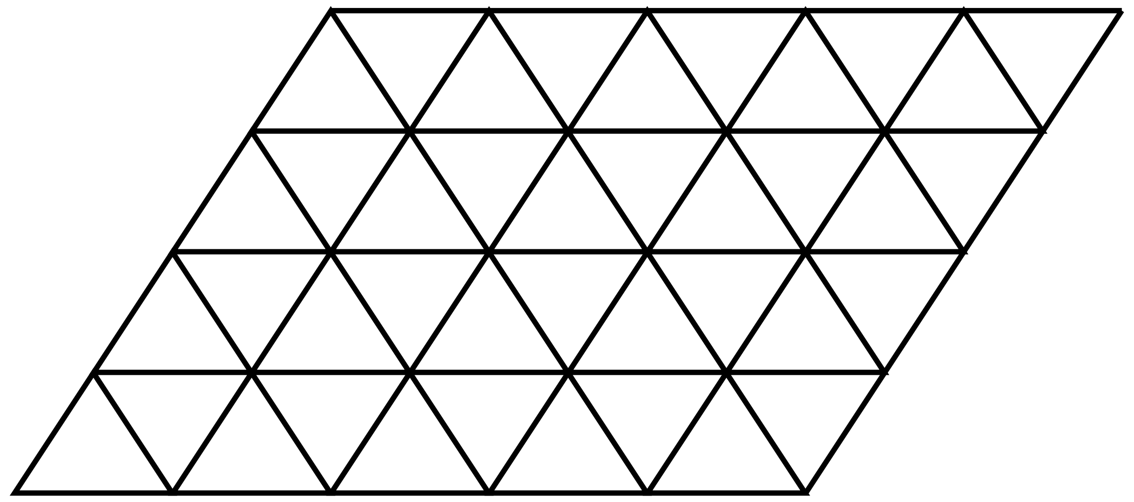
- Luttinger semimetals
- Dirac spin liquids

(4) Conclusions





Triangular-lattice antiferromagnet



Hamiltonian:

$$\mathcal{H} = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

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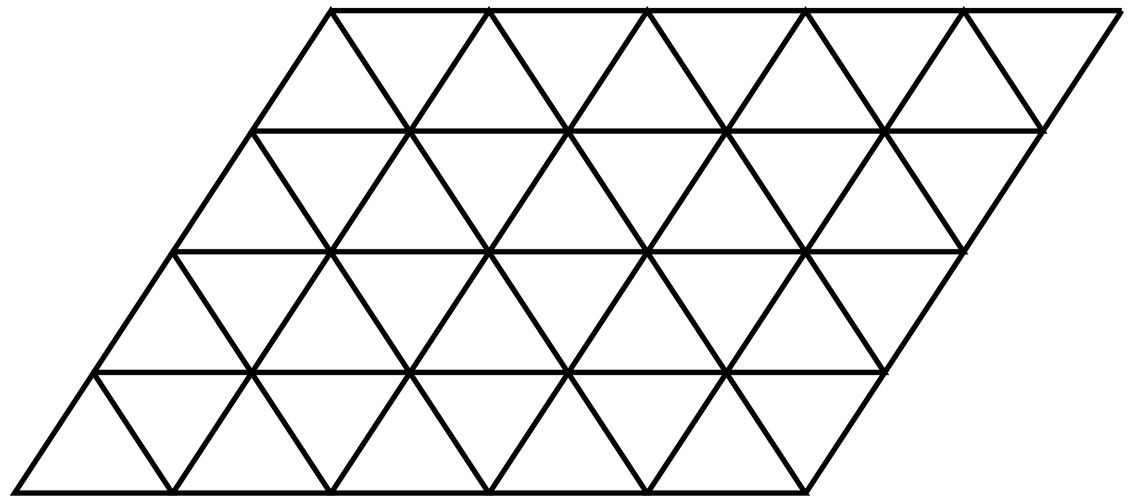

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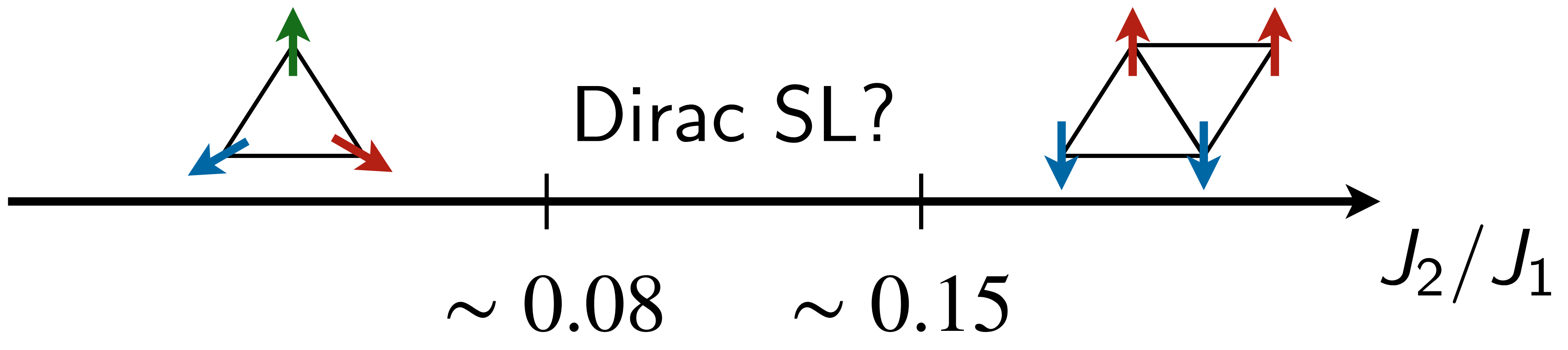
Phase diagram:



[Wietek *et al.*, PRX '24]

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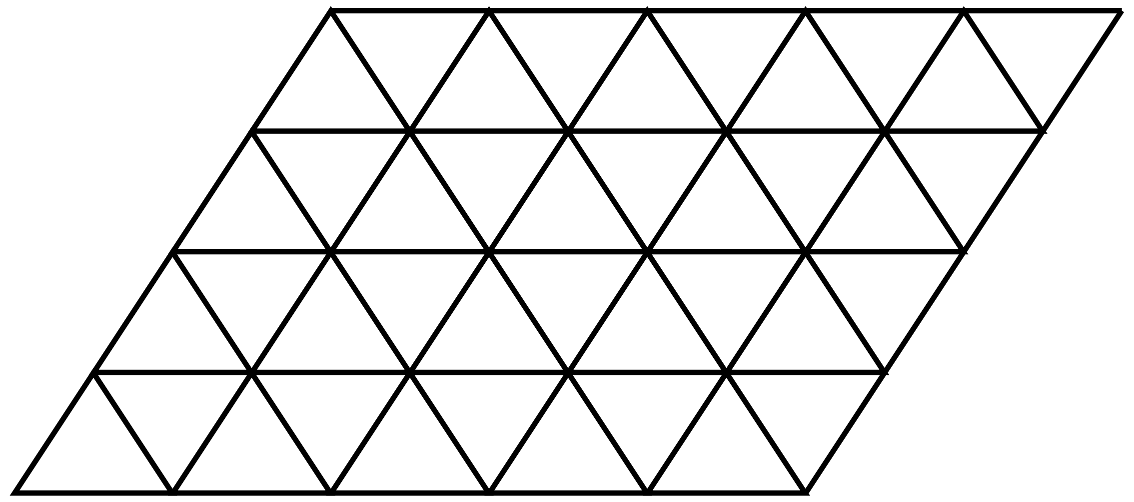

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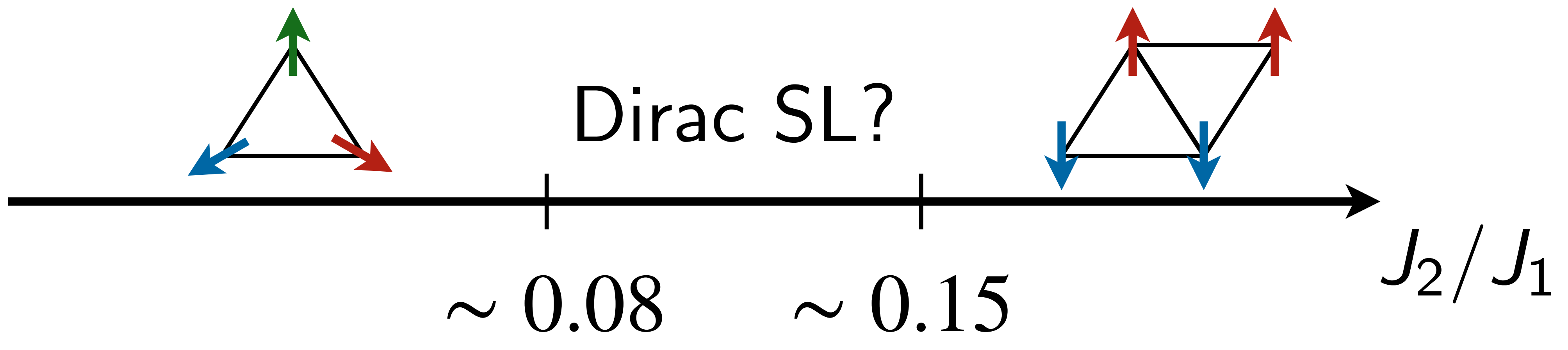

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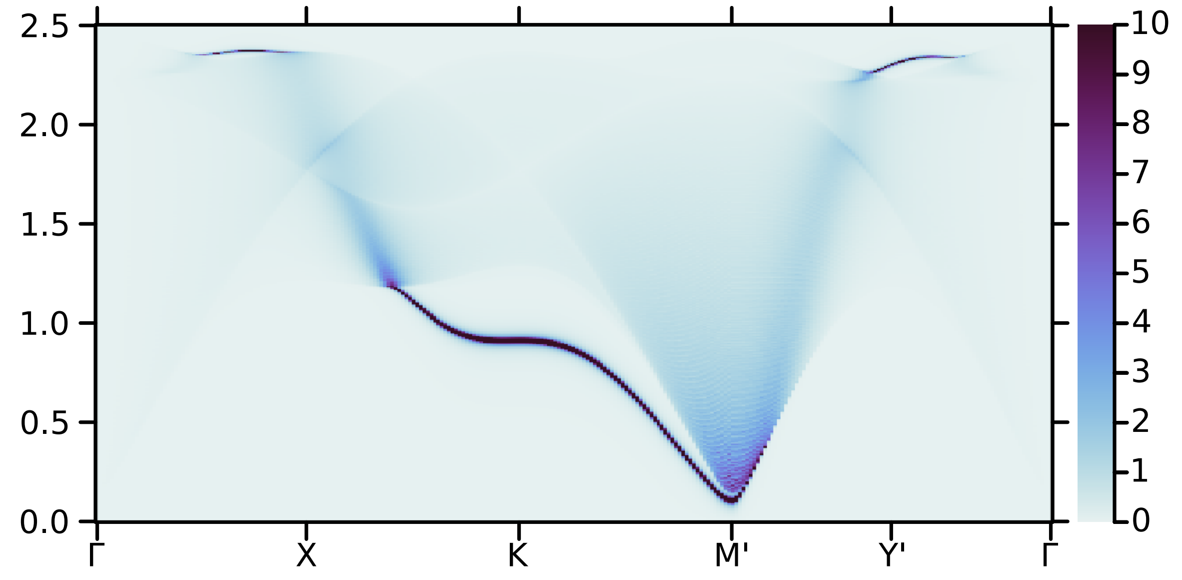
Effective field theory:

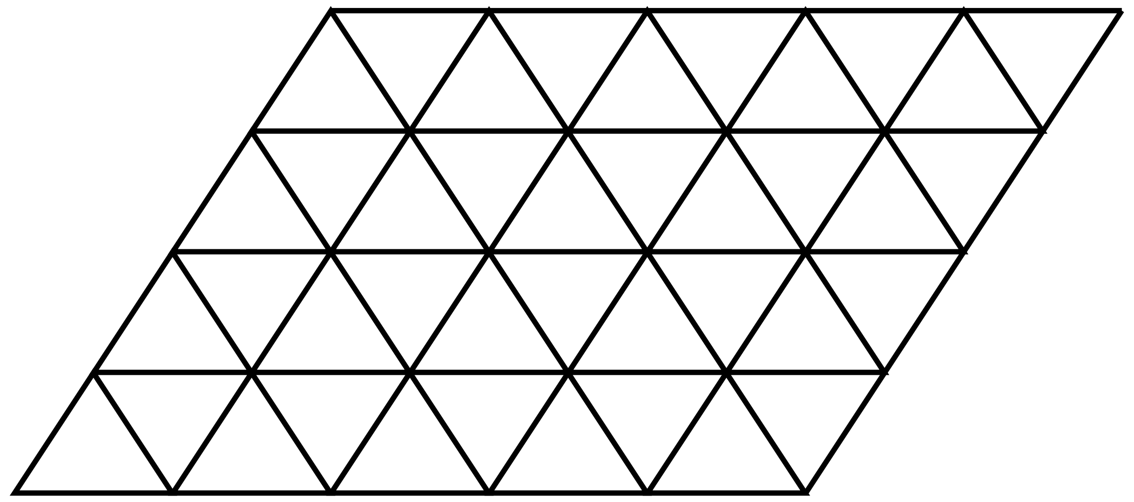
$$\mathcal{L} = \bar{\psi}\gamma^\mu(\partial_\mu - \mathrm{i}ea_\mu)\psi + \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \qquad \text{QED}_3$$

Dynamical structure factor



[Willsher & Knolle, arXiv:2503.13831]

Triangular-lattice antiferromagnet



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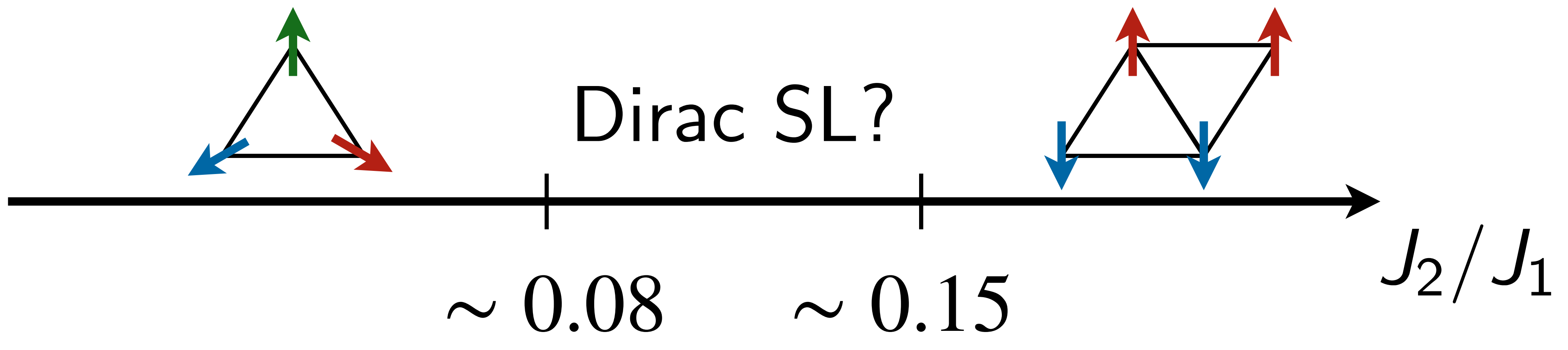
[Cookmeyer *et al.*, PRL ’21]

Phase diagram:



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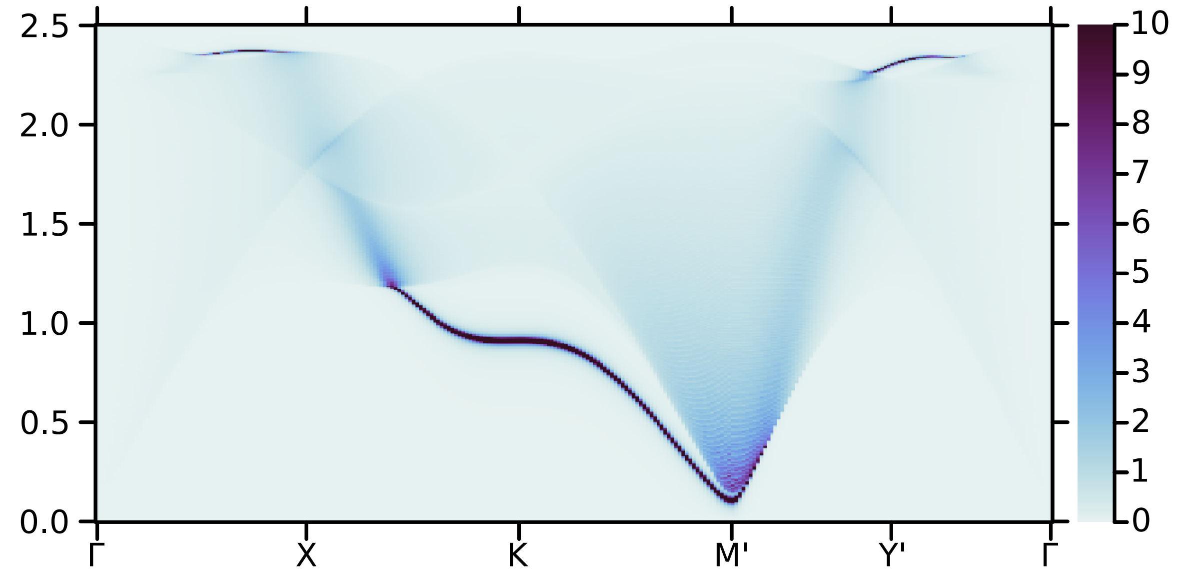

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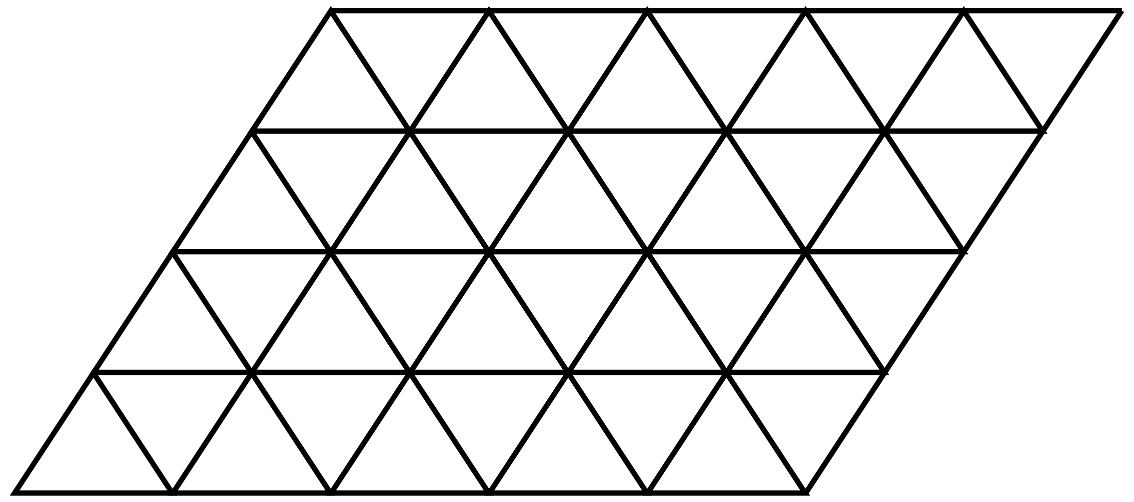

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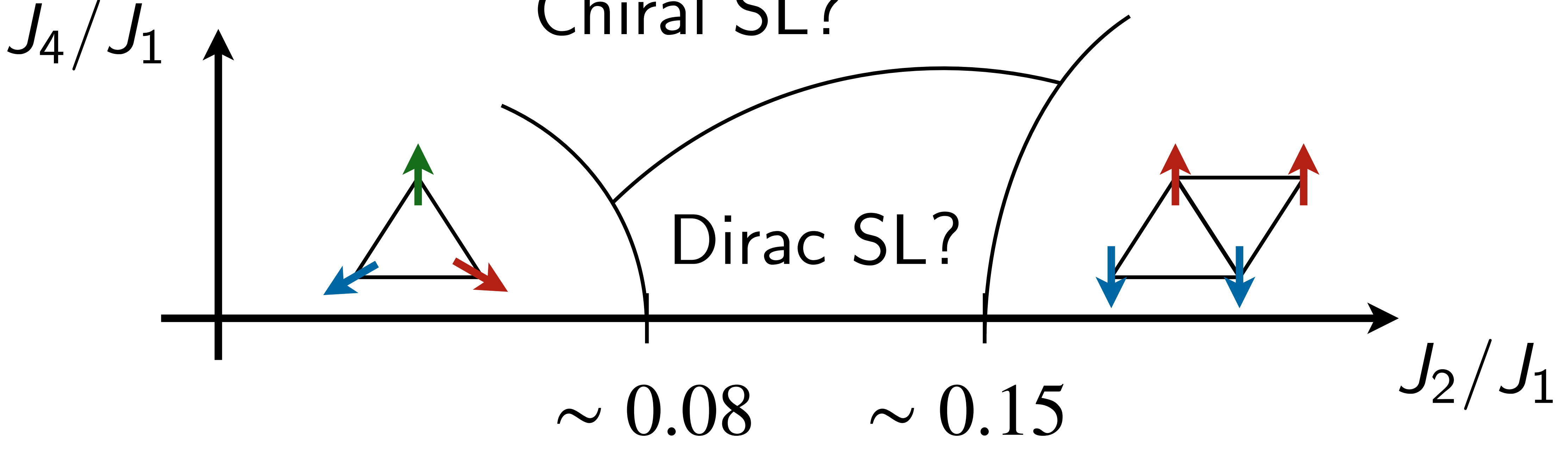

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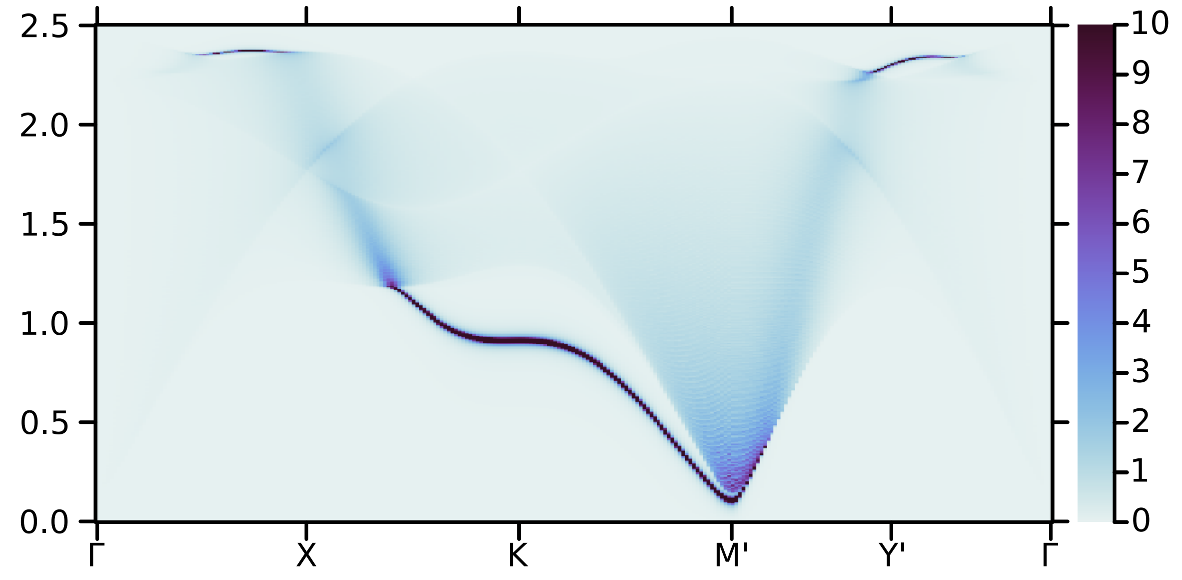

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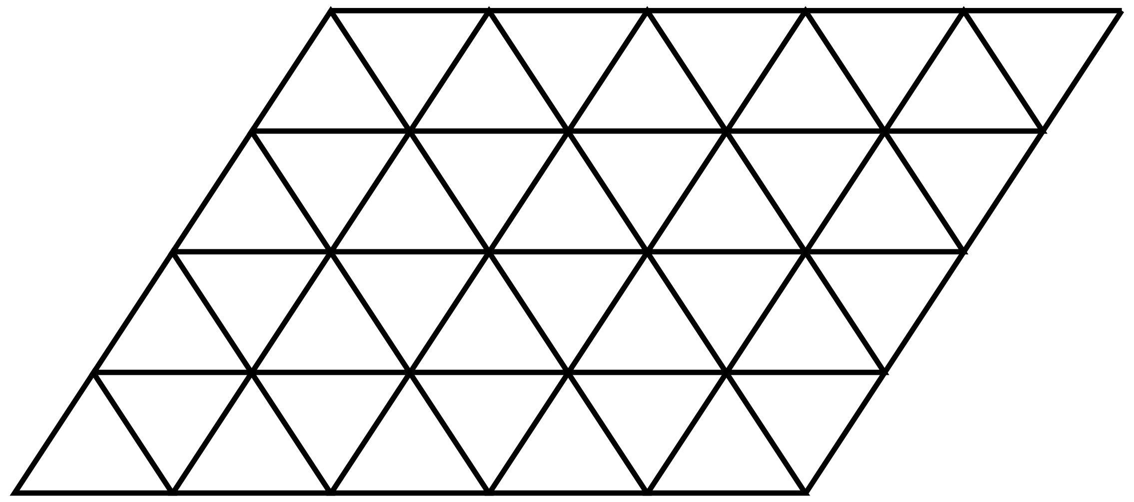

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$$+ J_4 \sum_{\langle ijkl \rangle} (\vec{S}_i \cdot \vec{S}_j)(\vec{S}_k \cdot \vec{S}_l) + (\vec{S}_i \cdot \vec{S}_l)(\vec{S}_j \cdot \vec{S}_k) - (\vec{S}_i \cdot \vec{S}_k)(\vec{S}_j \cdot \vec{S}_l)$$

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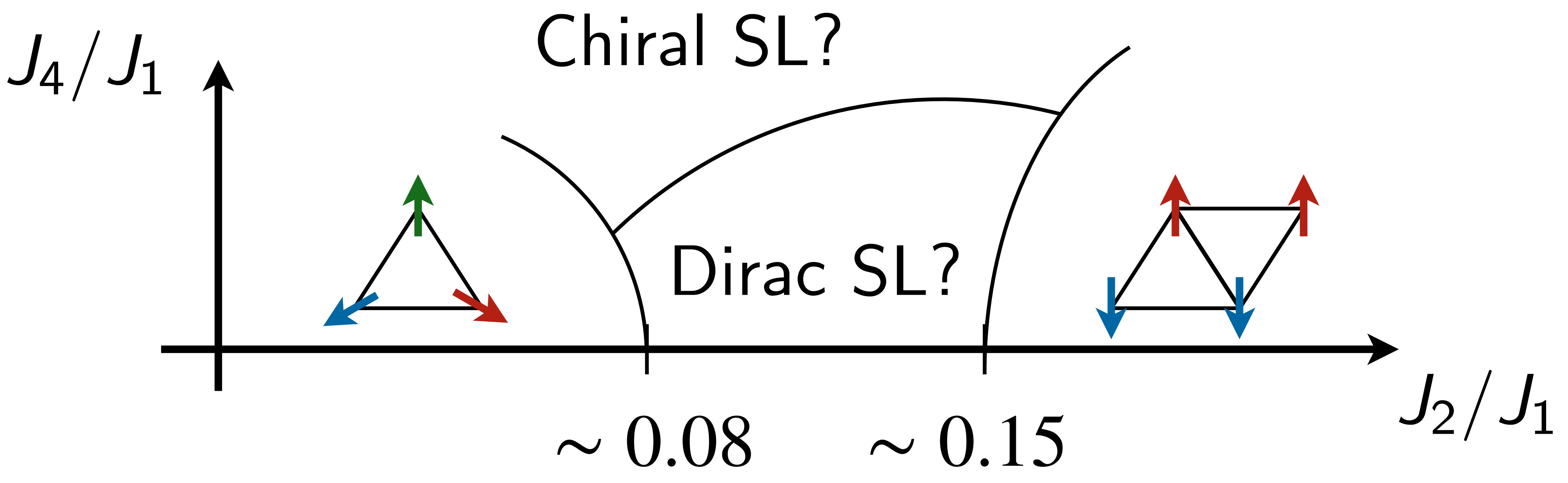
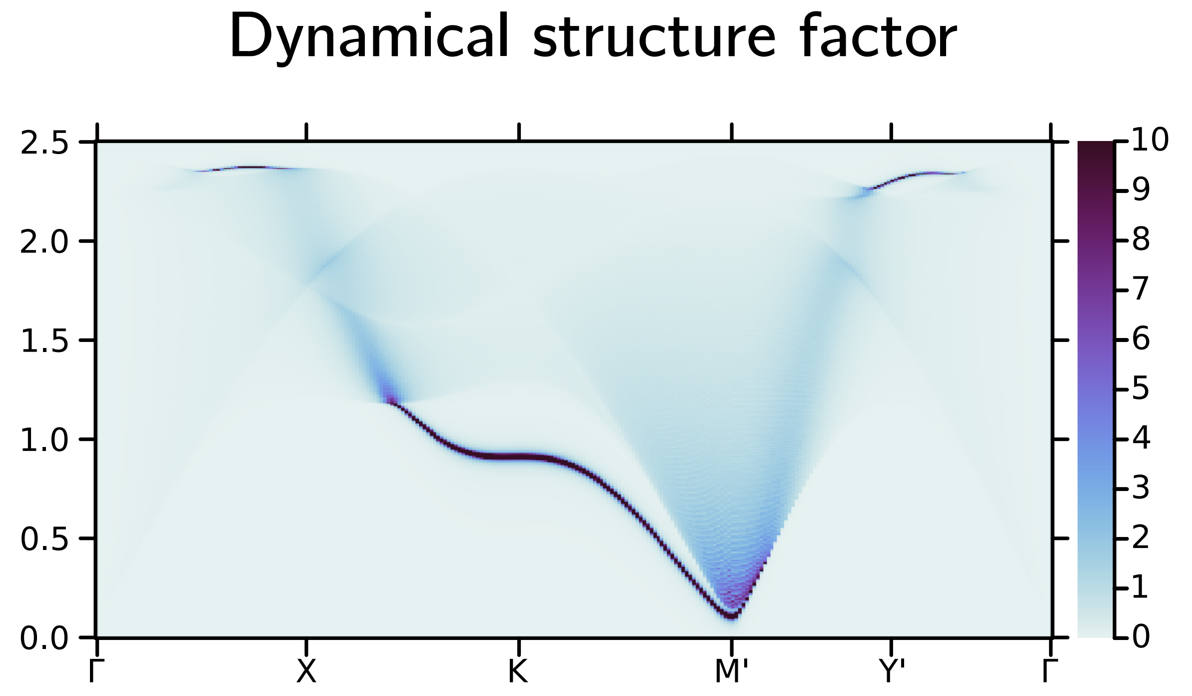


[Wietek *et al.*, PRX ’24]



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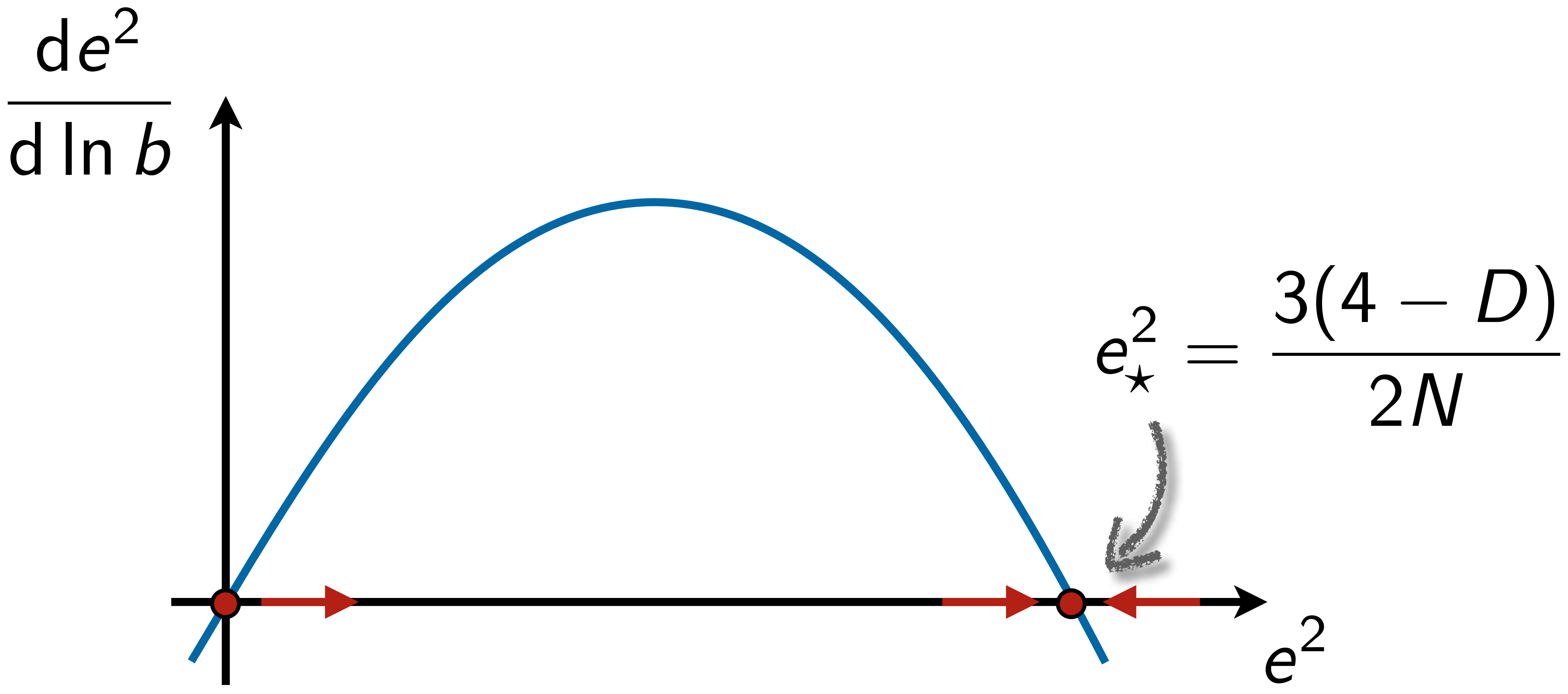
$$- G_1(\bar{\psi}\psi)^2$$

[Willsher & Knolle, arXiv:2503.13831]

RG flow

Charge:

$$\frac{\mathrm{d}e^2}{\mathrm{d}\ln b} = (4-D)e^2 - \frac{2N}{3}e^2$$



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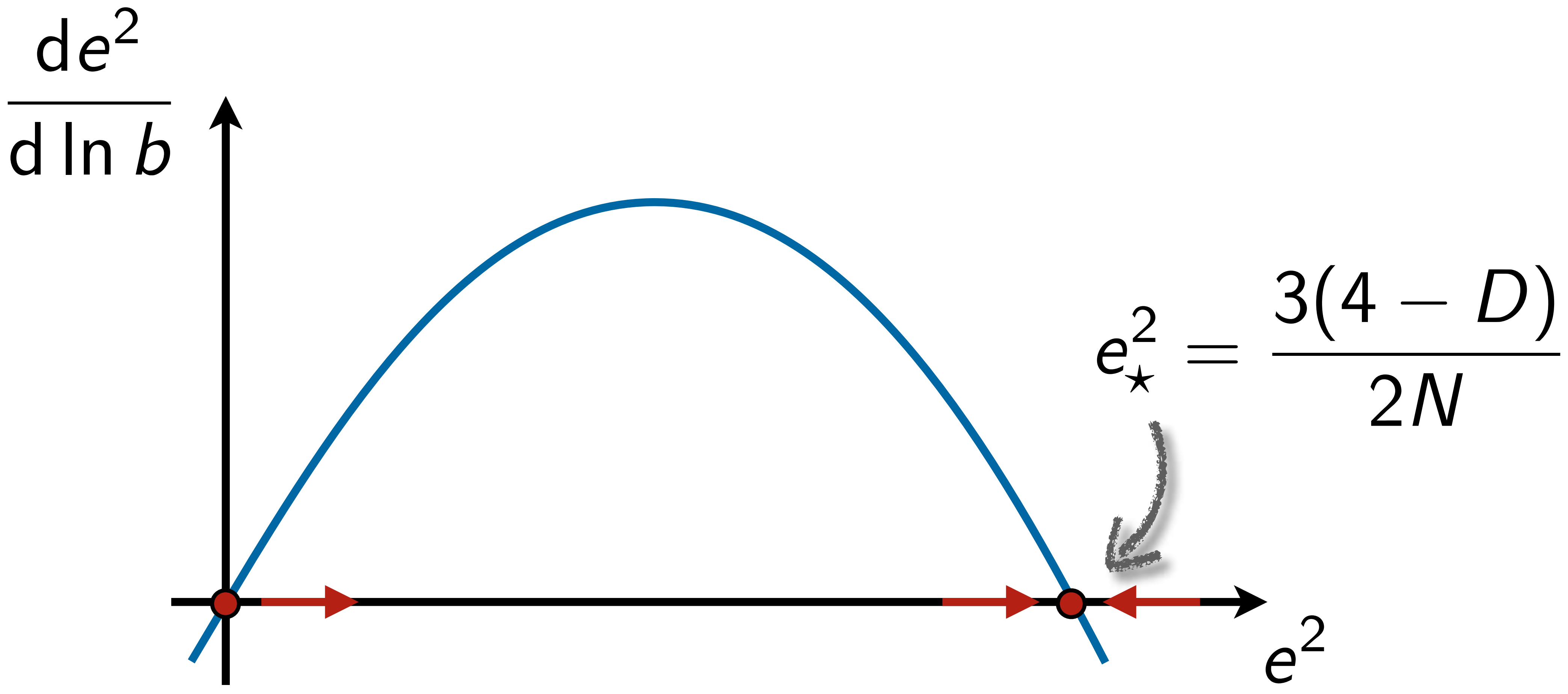
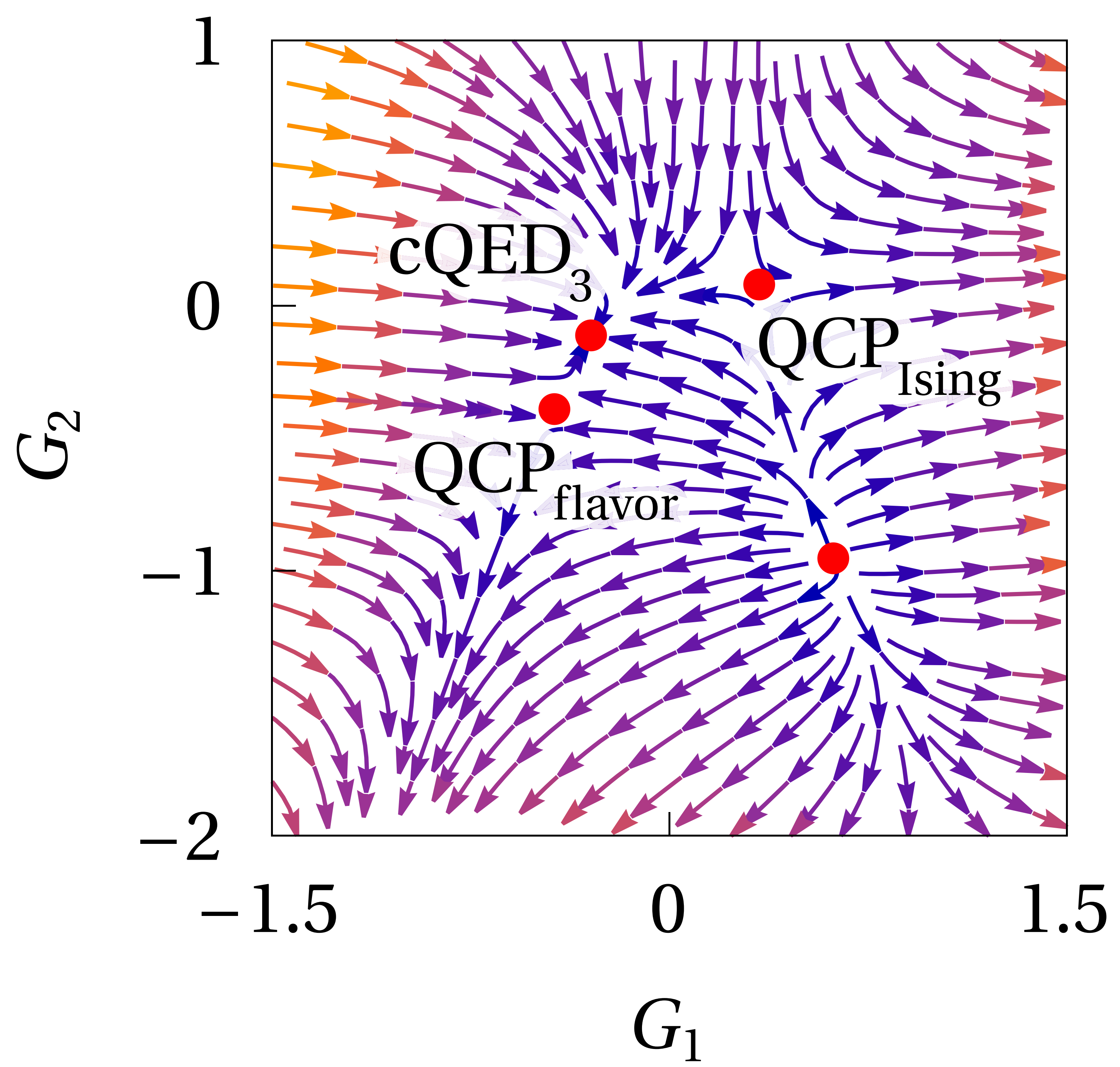


Four-fermion interactions:

$$\mathcal{L}_\psi = -G_1(\bar{\psi}\psi)^2 - G_2(\bar{\psi}\gamma_\mu\psi)^2$$

RG flow diagram

$N > N_\mathrm{c} \approx 12$



[Kubota & Terao, Progr. Ther. Phys. '01]
[Braun, Gies, LJ, Roscher, PRD '14]
[Herbut, PRD '16]
...

RG flow diagram

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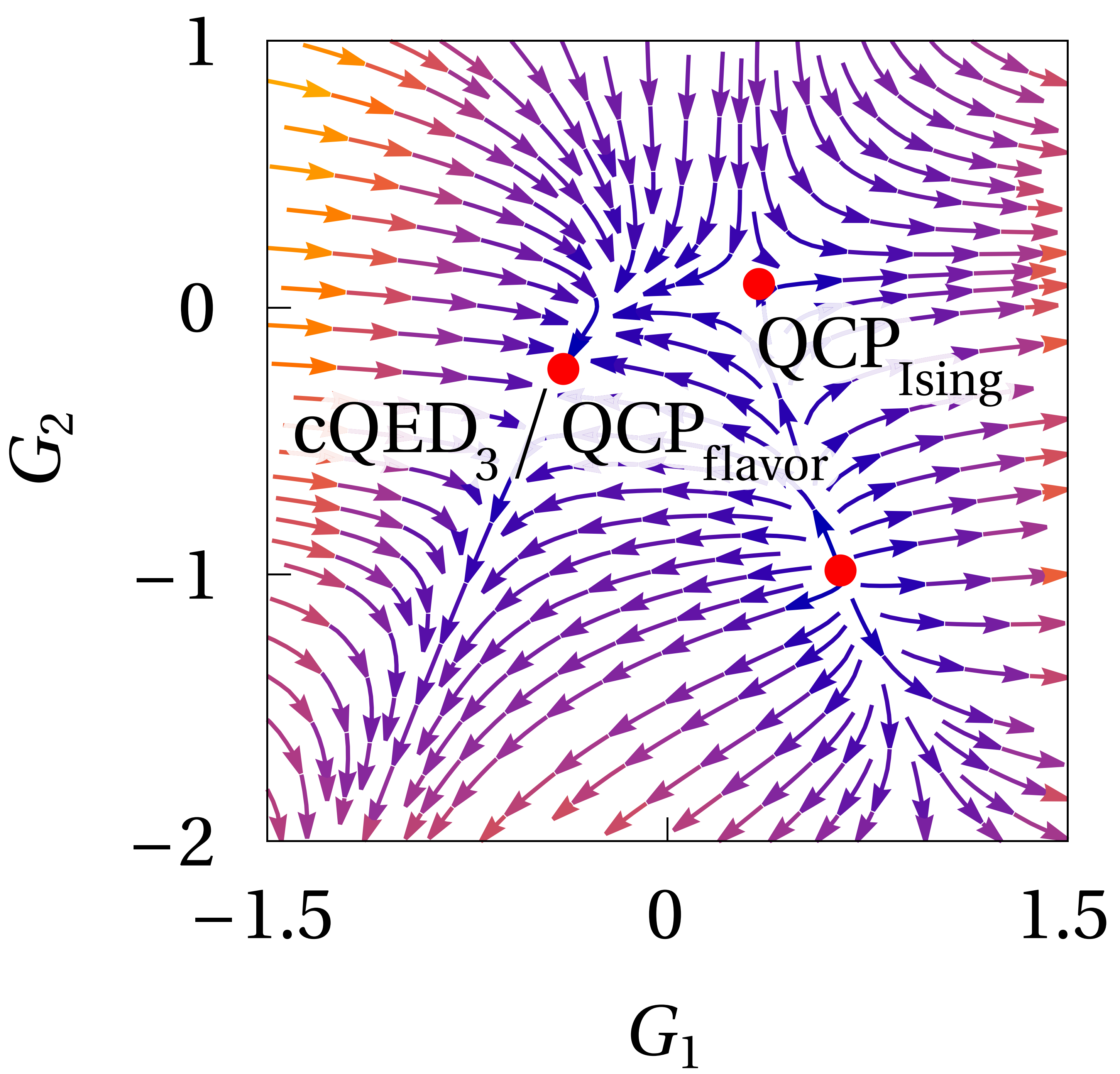

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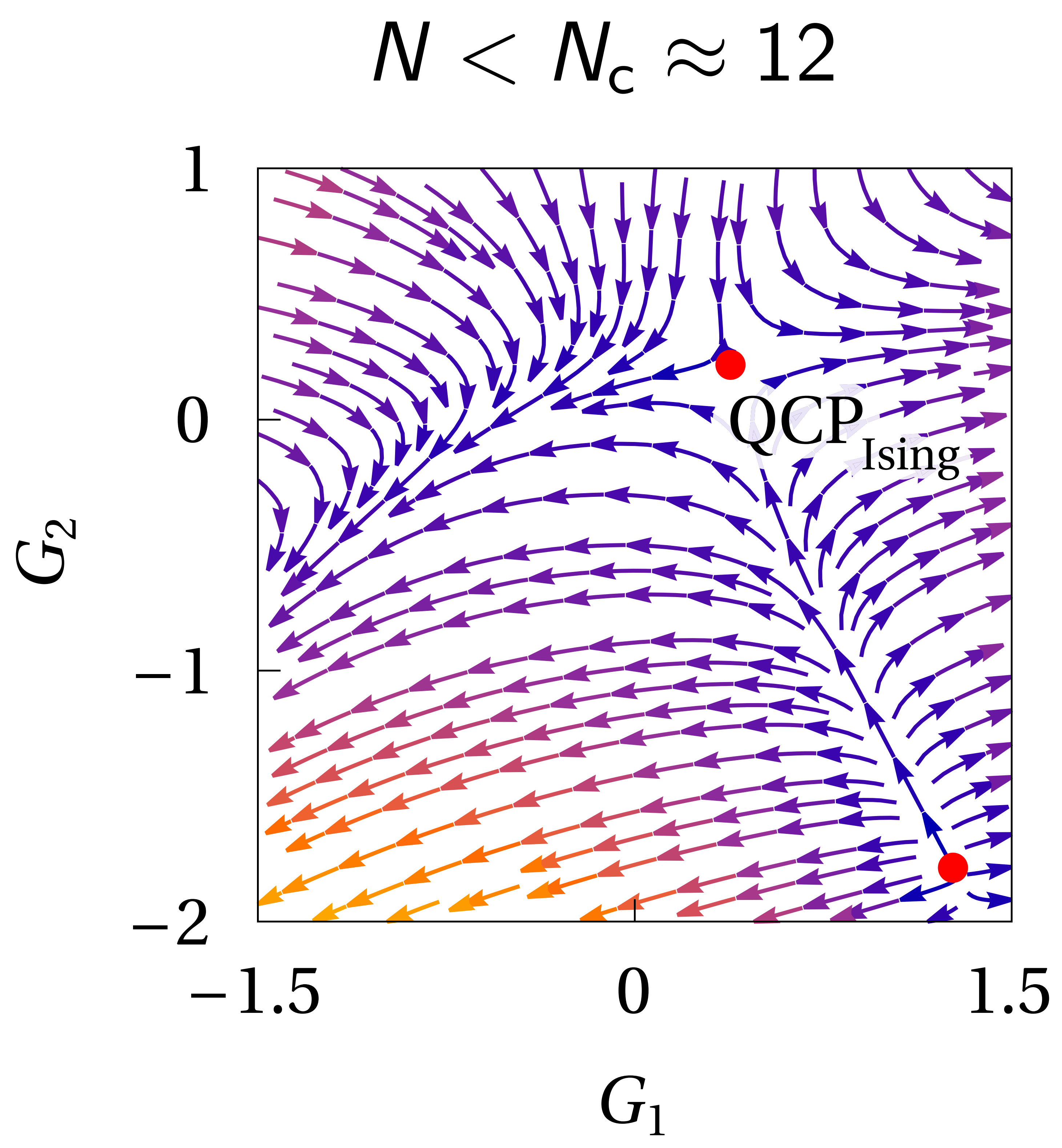

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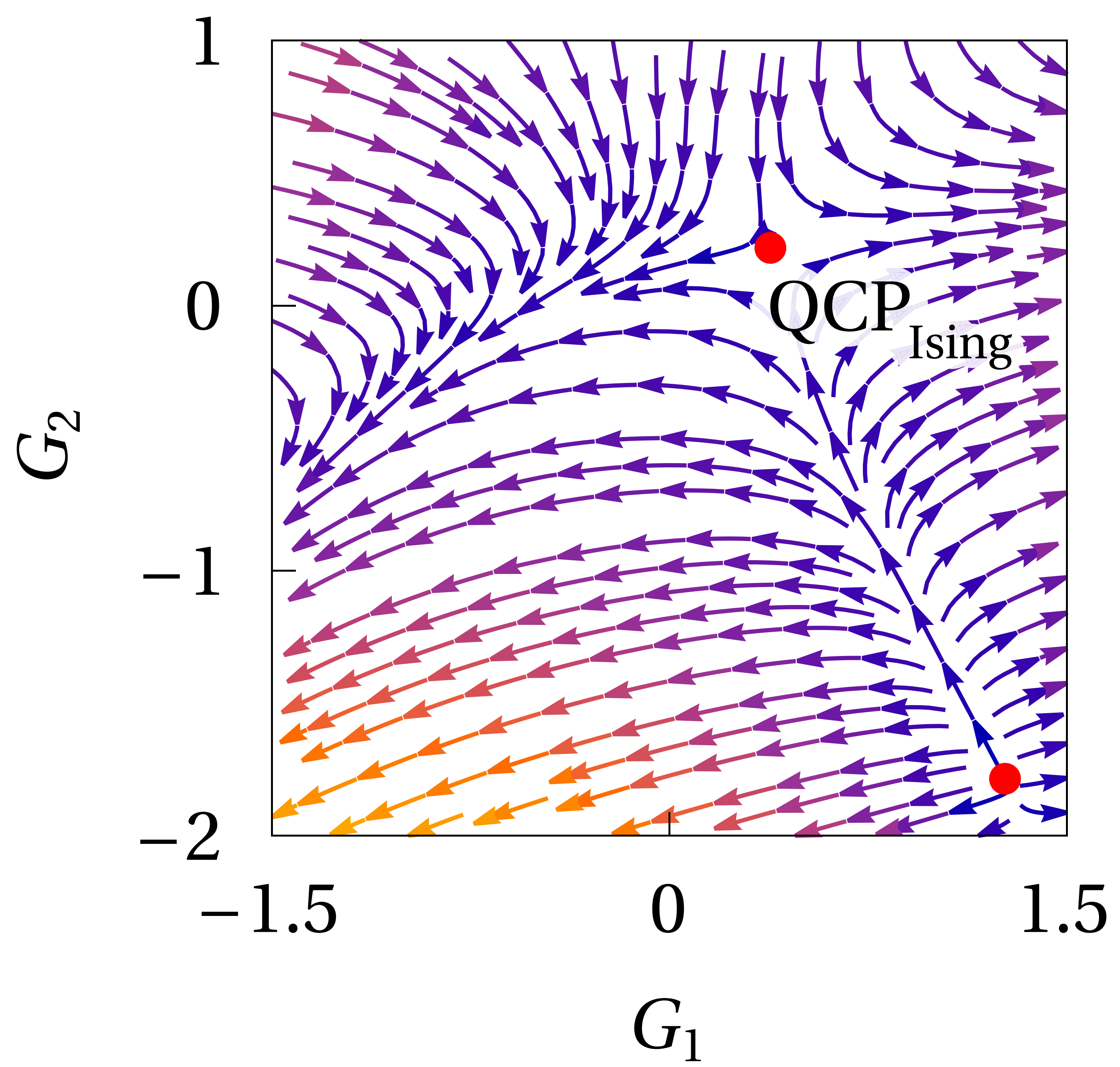
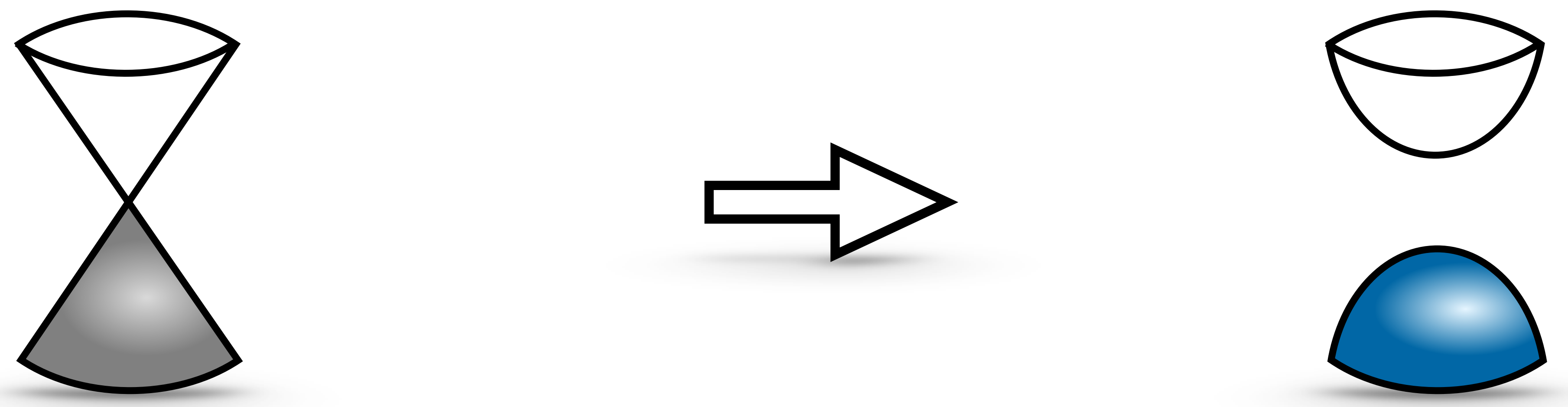


[Kubota & Terao, Progr. Ther. Phys. '01]
[Braun, Gies, LJ, Roscher, PRD '14]
[Herbut, PRD '16]
...

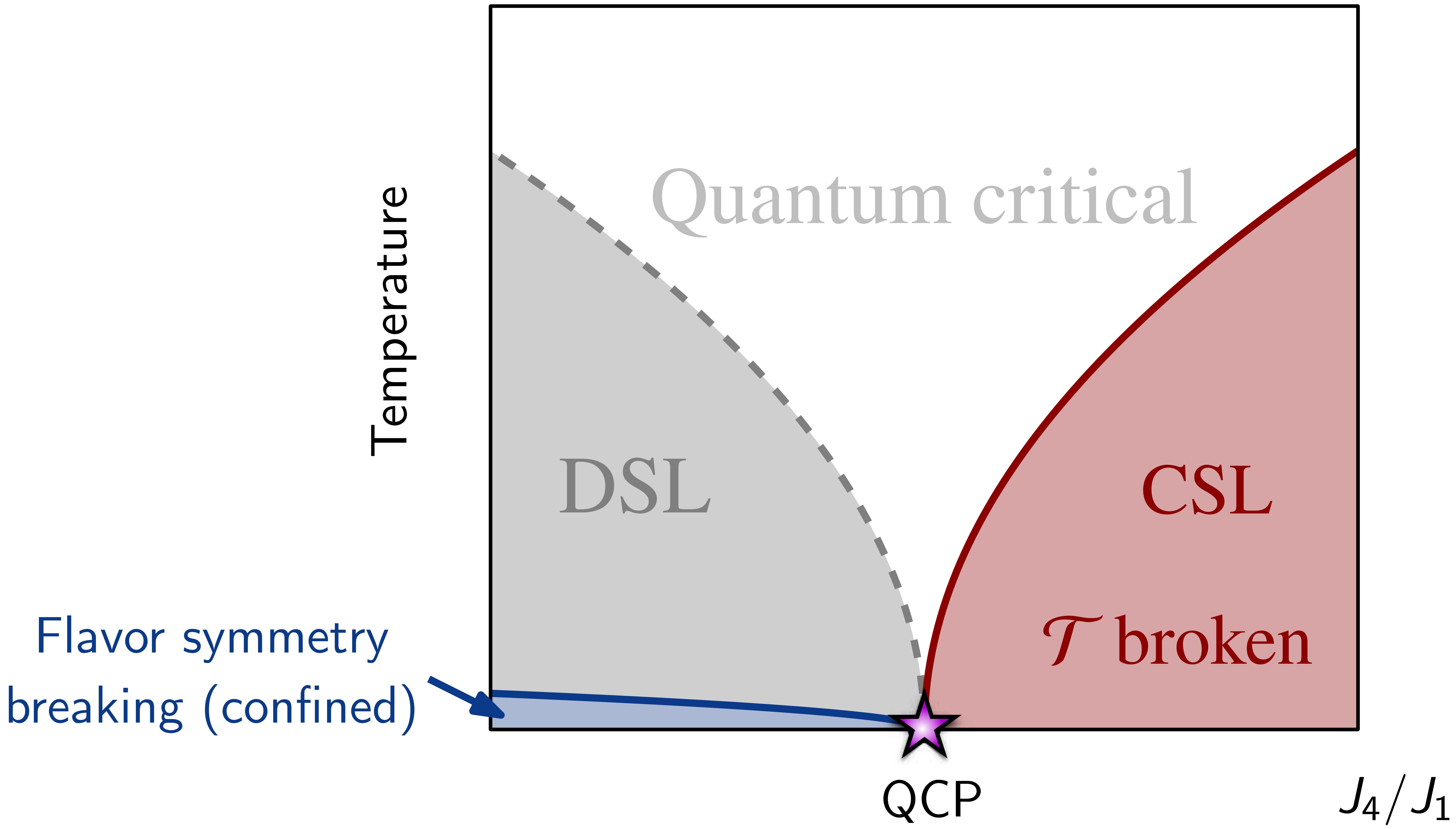
Ground state:

$N > N_c \approx 12$ $\qquad$ $N < N_c \approx 12$



cQED$_3$ $\qquad$ Flavor symmetry breaking (confinement)

Schematic phase diagram ($N < N_c$)



[Moser & LJ, Rep. Progr. Phys. ’25]

Outline

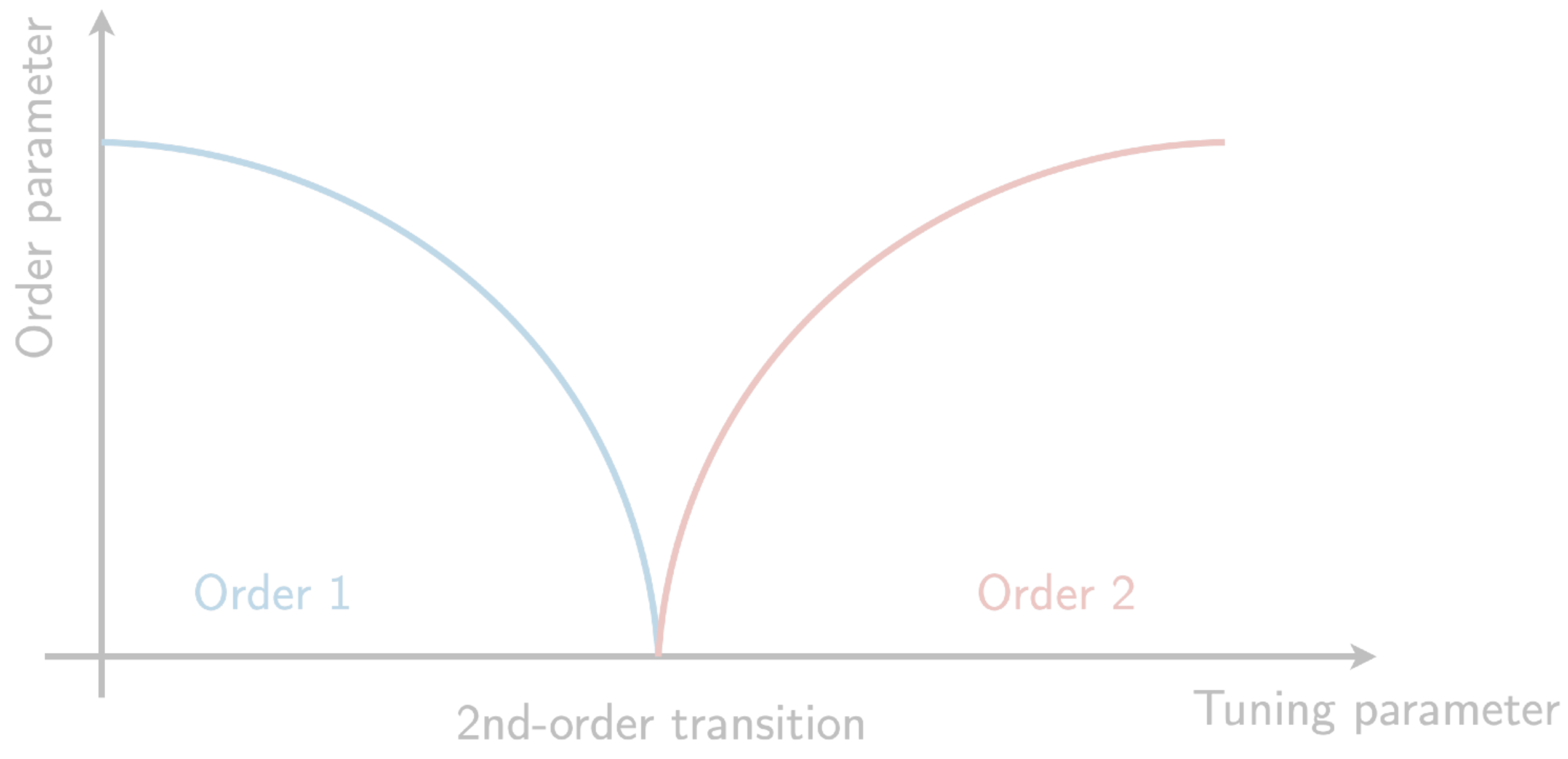
(1) Introduction

(2) Continuous order-to-order transition from fixed-point annihilation

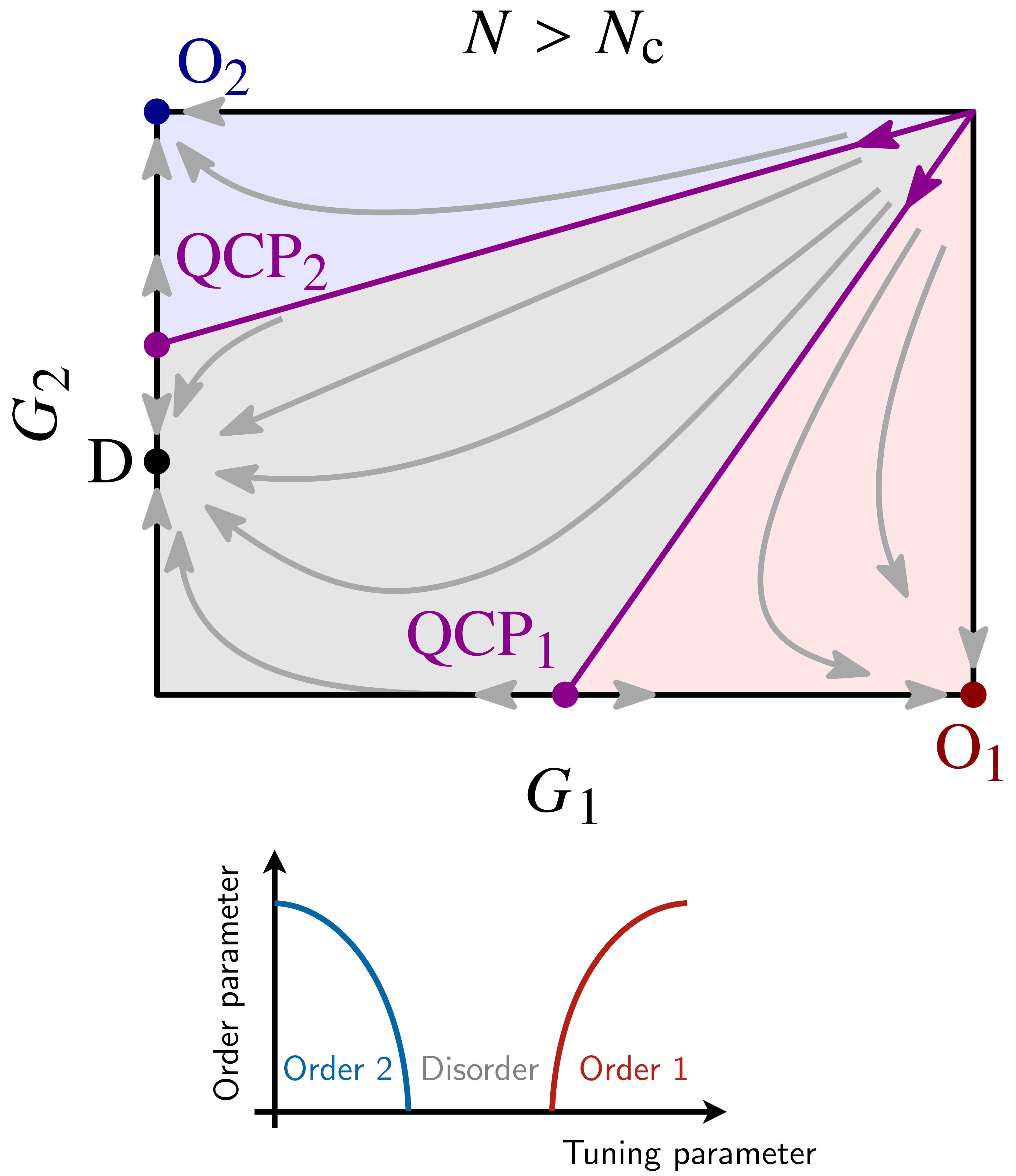
(3) Examples

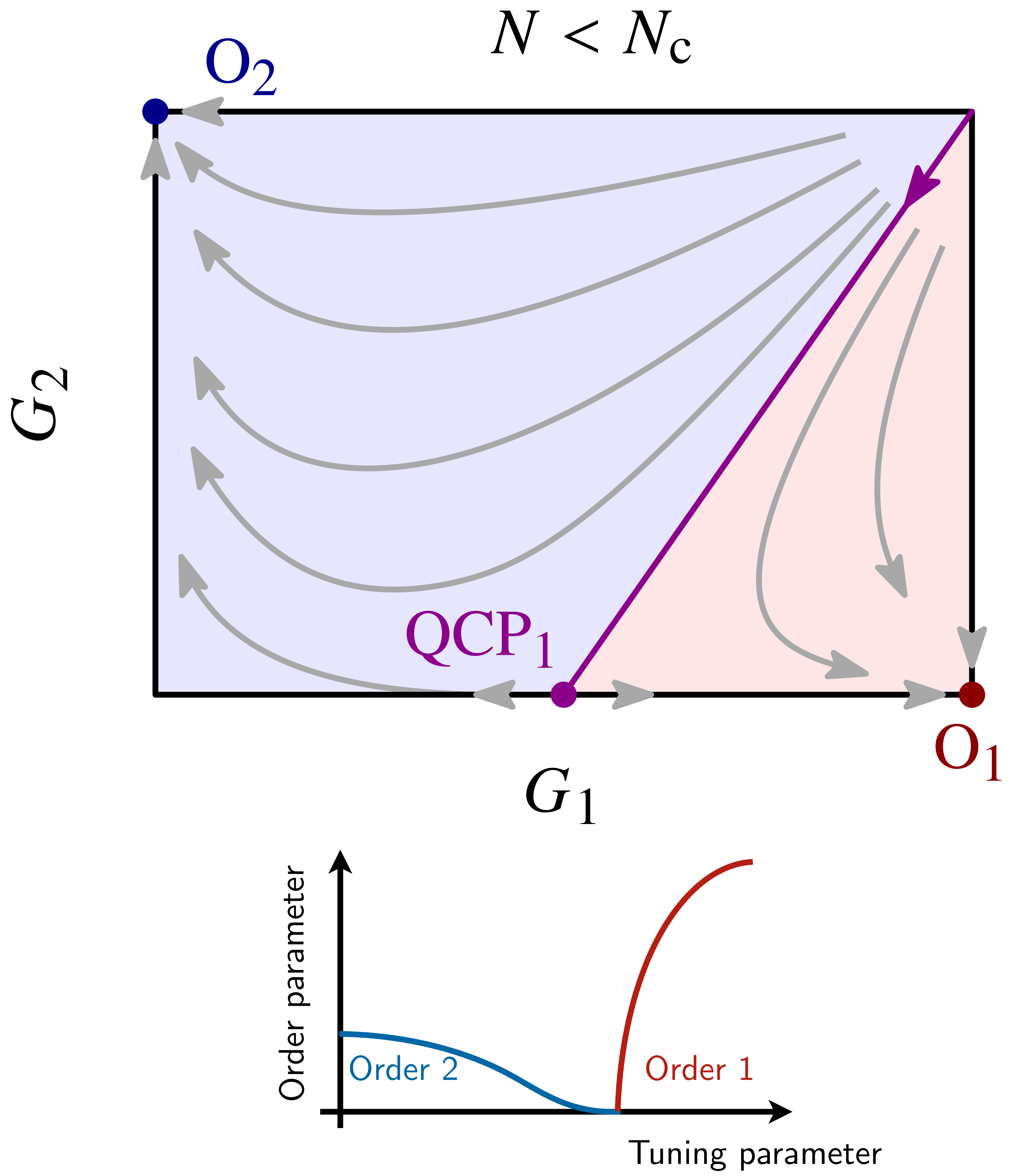
- Luttinger semimetals
- Dirac spin liquids

(4) Conclusions



Conclusions





[Moser, LJ, arXiv:2412.06890]

Spin-boson model

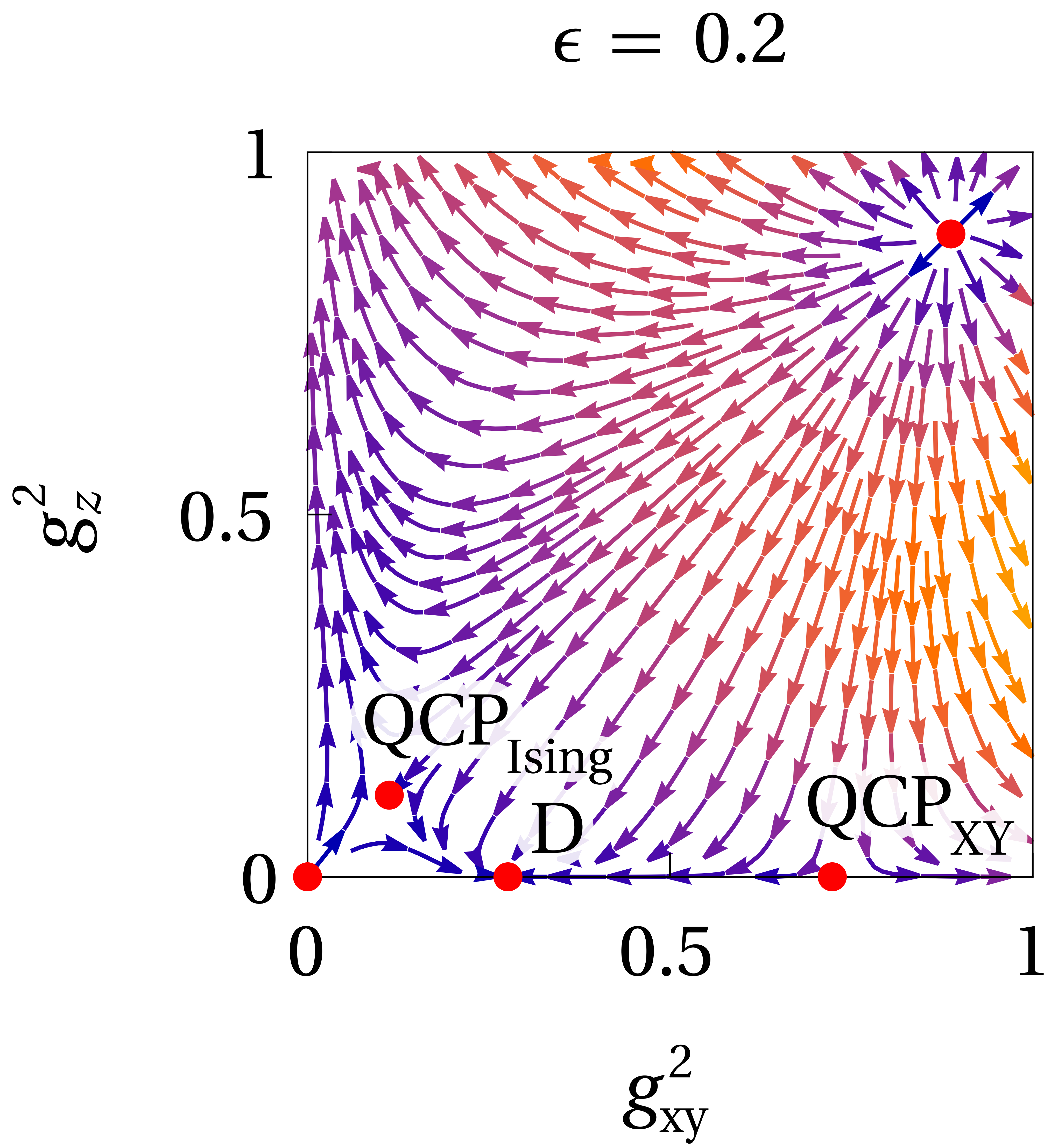
Spin-1/2 in fluctuating field:

$$\mathcal{H}_{\text{spin-boson}} = g_{xy}(h^x S^x + h^y S^y) + g_z h^z S^z + \mathcal{H}_{\text{bulk}}(\vec{h})$$

with $\langle \mathcal{T}_\tau h^a(\tau) h^b(0) \rangle \propto \dfrac{\delta^{ab}}{|\tau|^{2-\epsilon}}$

[Sengupta, PRB '00]
[Zhu, Si, PRB '02]
[Zaránd, Demler, PRB '02]
[Weber, Vojta, PRL '23]
[Weber, arXiv '24]
...

RG flow diagram



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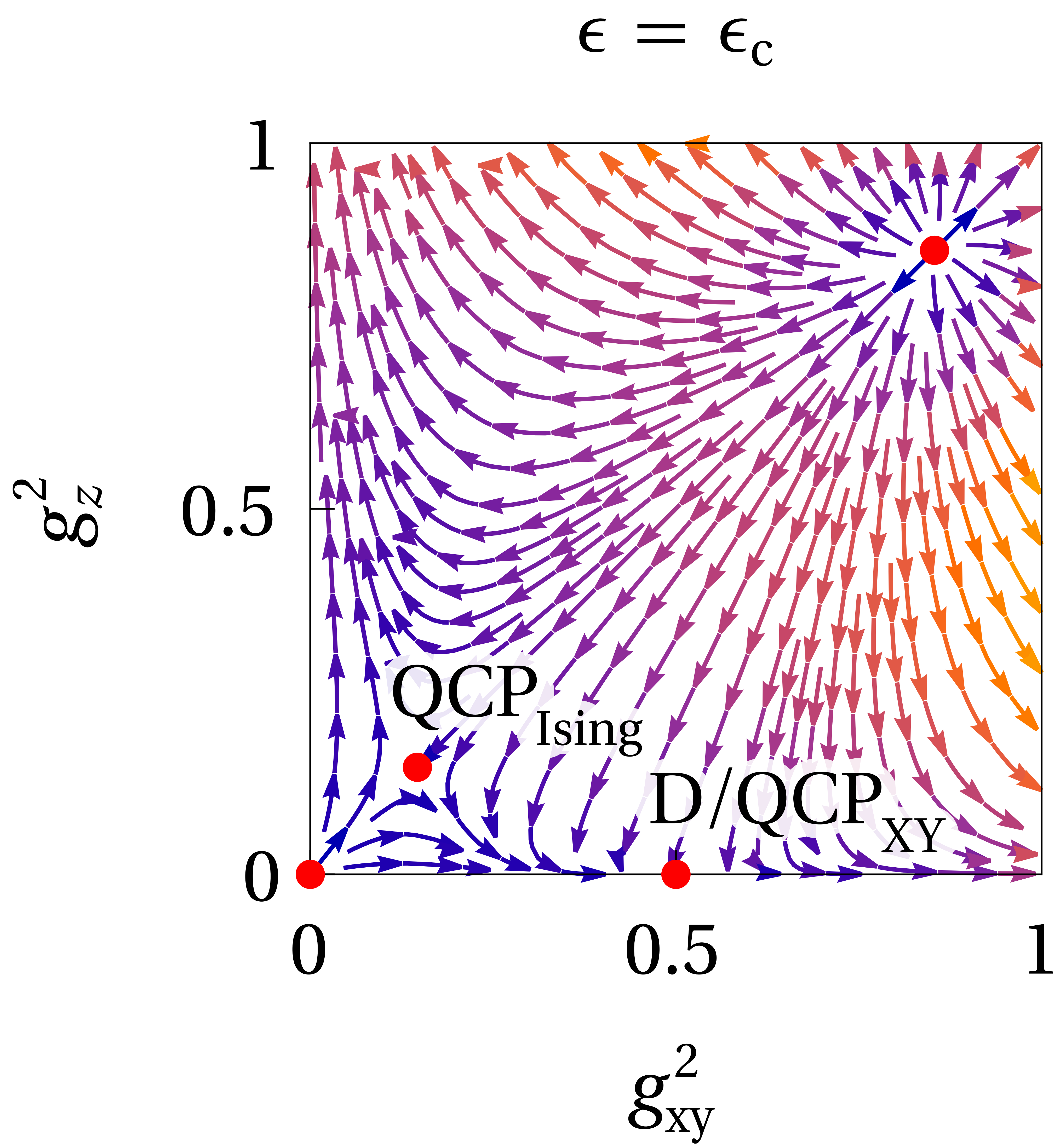

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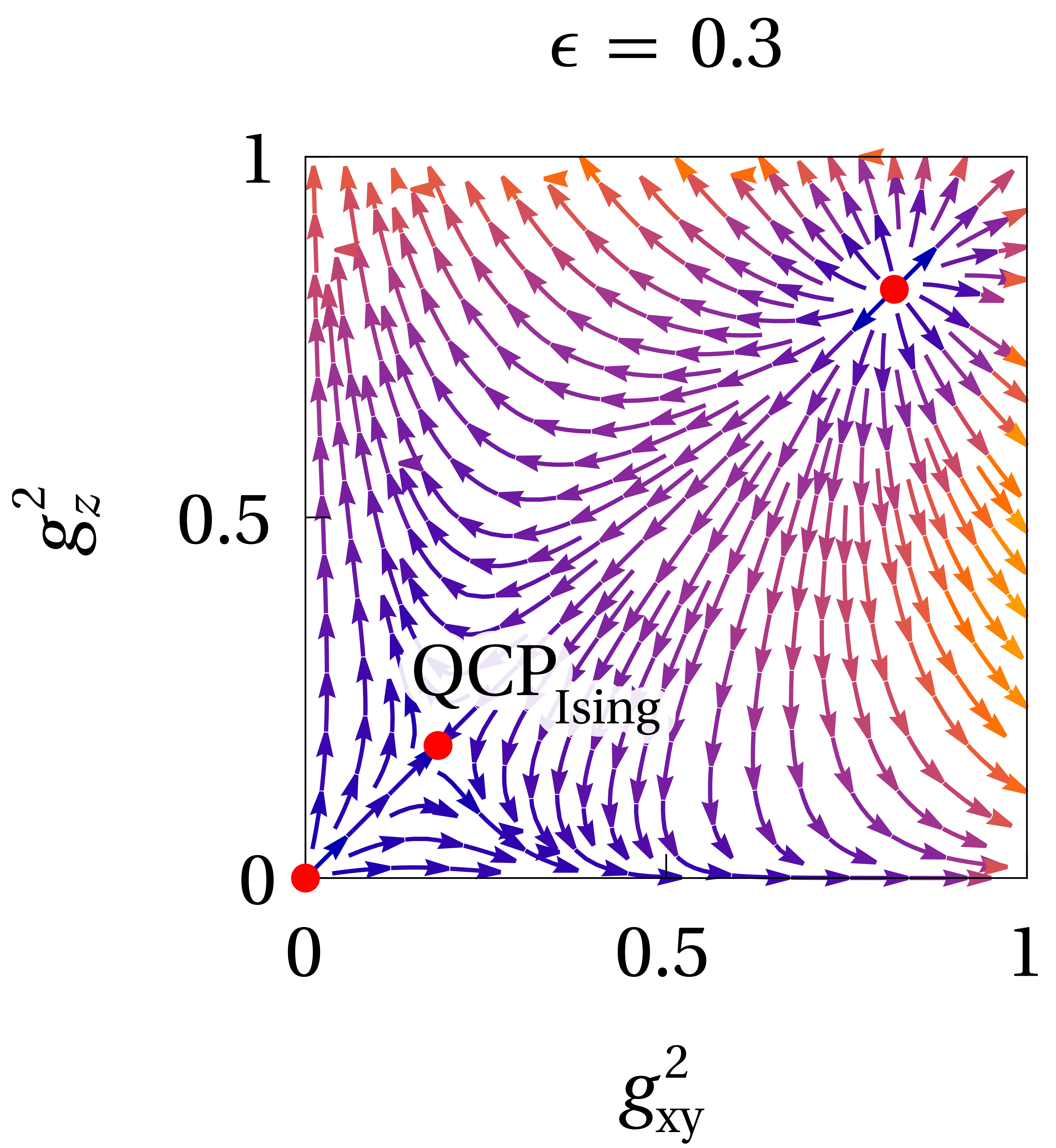
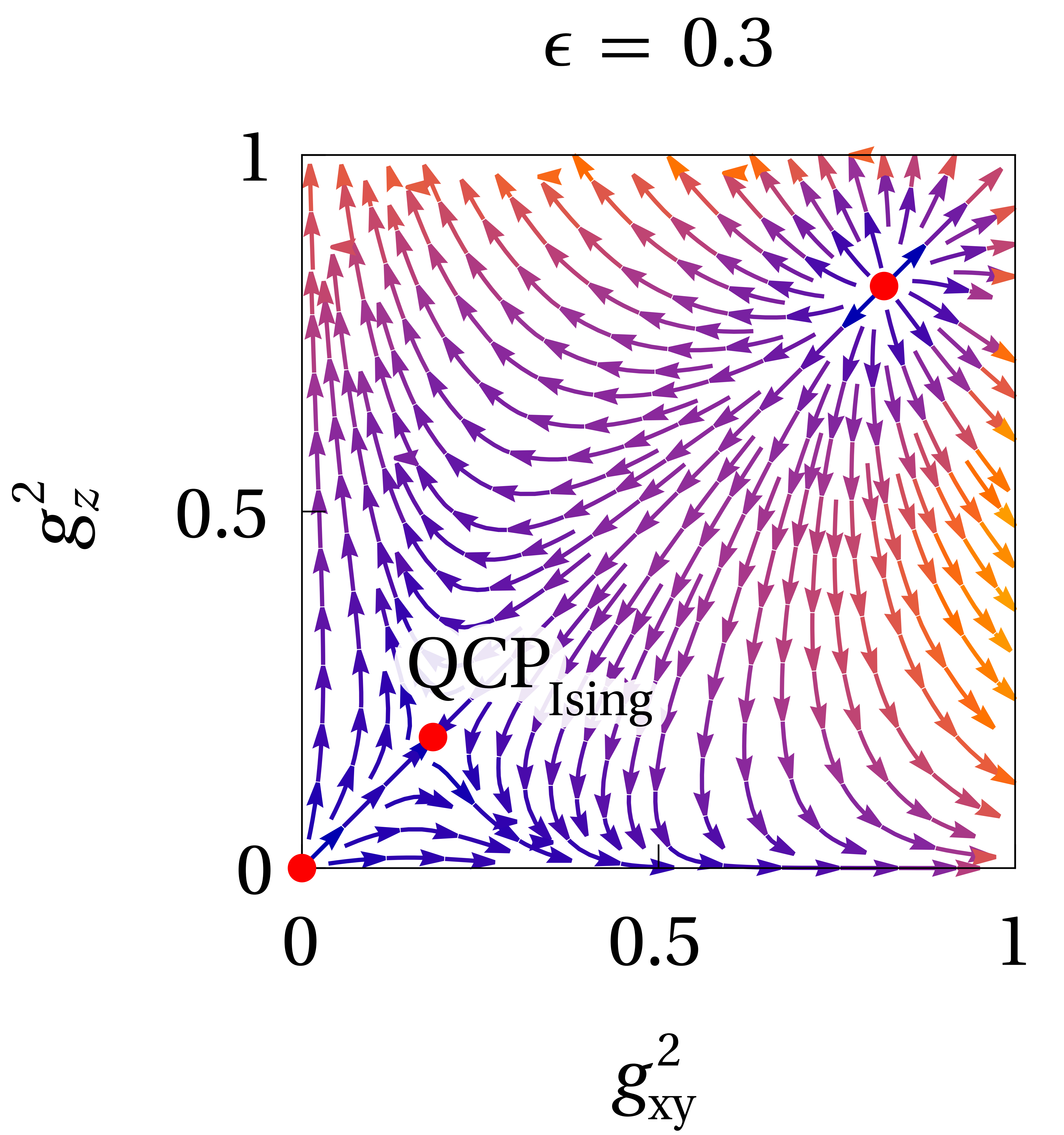
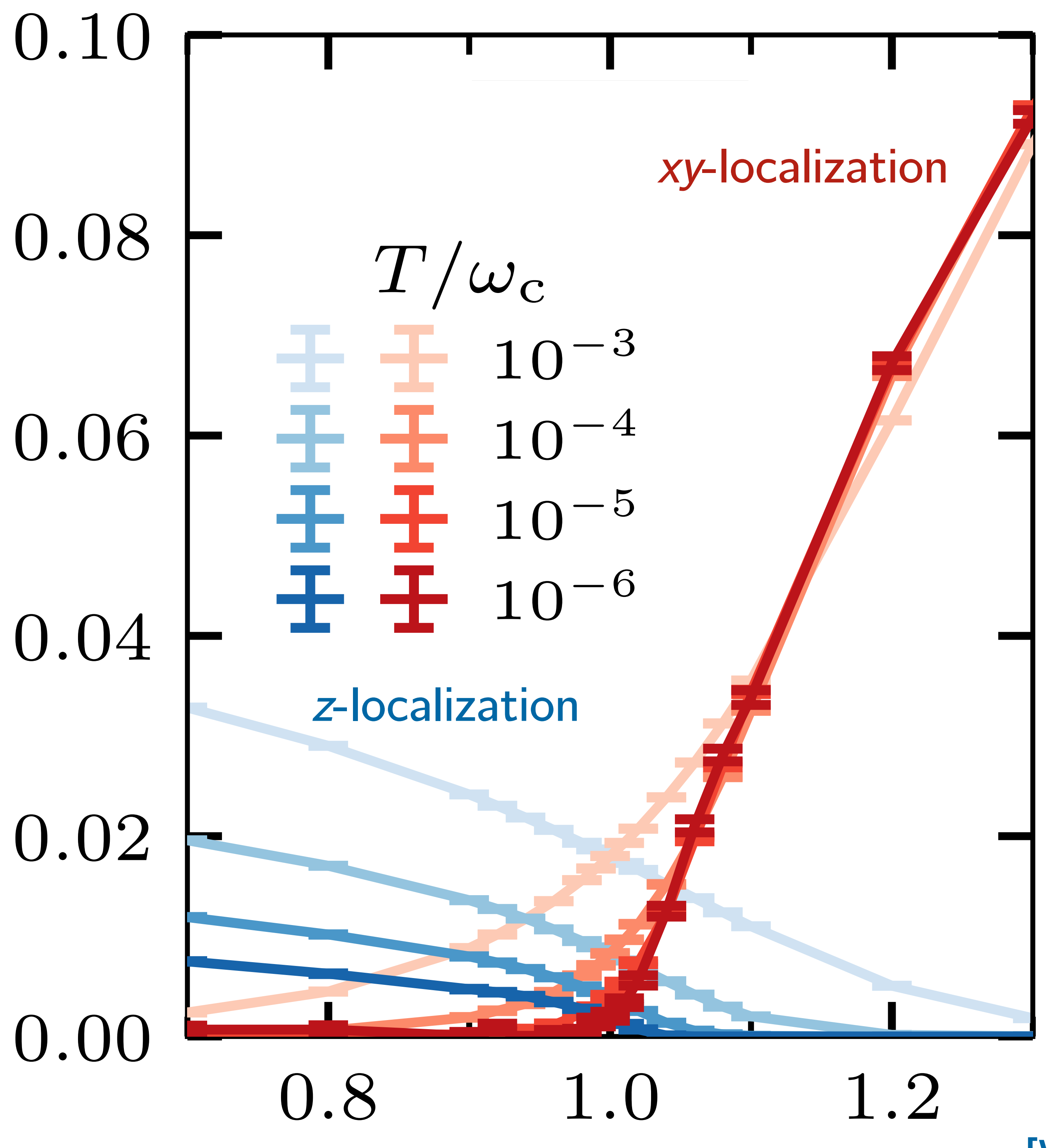


RG flow diagram



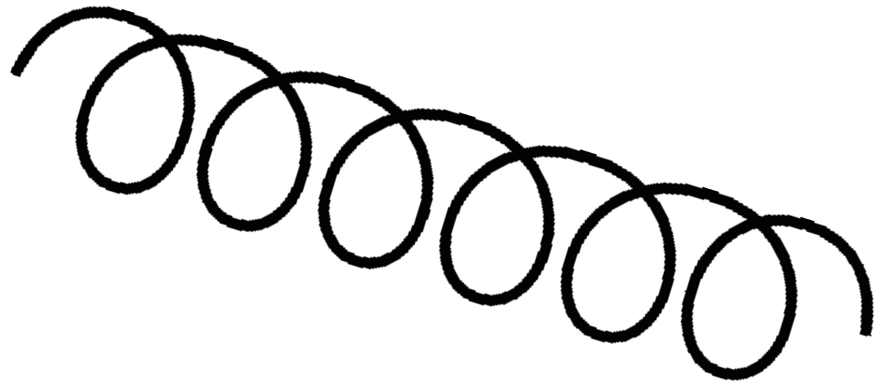
Quantum Monte Carlo:



[Weber, arXiv '24]

... consistent with predictions!

QCD$_4$ plus 4-fermion interactions



Lagrangian:

$$\mathrm{i}\bar{\psi}\gamma^{\mu} D_{\mu}\psi + \frac{1}{4} F_{z}^{\mu\nu} F^{z}_{\mu\nu} + \frac{1}{2}\sum_{\alpha=1}^{2} G_{\alpha}\mathcal{O}_{\alpha}$$

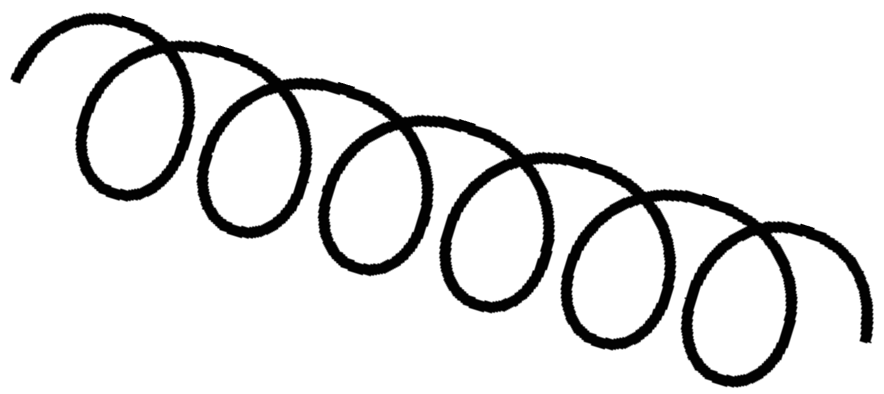
in Veneziano limit $N_{\text{color}}, N_{\text{flavor}} \to \infty$ with fixed $x = N_{\text{flavor}}/N_{\text{color}}$

$$\mathcal{O}_1 = \left(\bar{\psi}^a \gamma_{\mu}\psi^b\right)^2 + \left(\bar{\psi}^a \gamma_{\mu}\gamma_5\psi^b\right)^2$$

$$\mathcal{O}_2 = \left(\bar{\psi}^a \psi^b\right)^2 - \left(\bar{\psi}^a \gamma_5\psi^b\right)^2$$

[Gies, Jaeckel, Wetterich, PRD '04]
[Gies, Jaeckel, EPJC '06]
[Braun, JPG '12]
[Gukov, NPB '17]
...

QCD$_4$ plus 4-fermion interactions



Lagrangian:

$$\mathrm{i}\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \frac{1}{4}F_{z}^{\mu\nu}F^{z}_{\mu\nu} + \frac{1}{2}\sum_{\alpha=1}^{2} G_{\alpha}\mathcal{O}_{\alpha}$$

in Veneziano limit $N_{\text{color}}, N_{\text{flavor}} \to \infty$ with fixed $x = N_{\text{flavor}}/N_{\text{color}}$

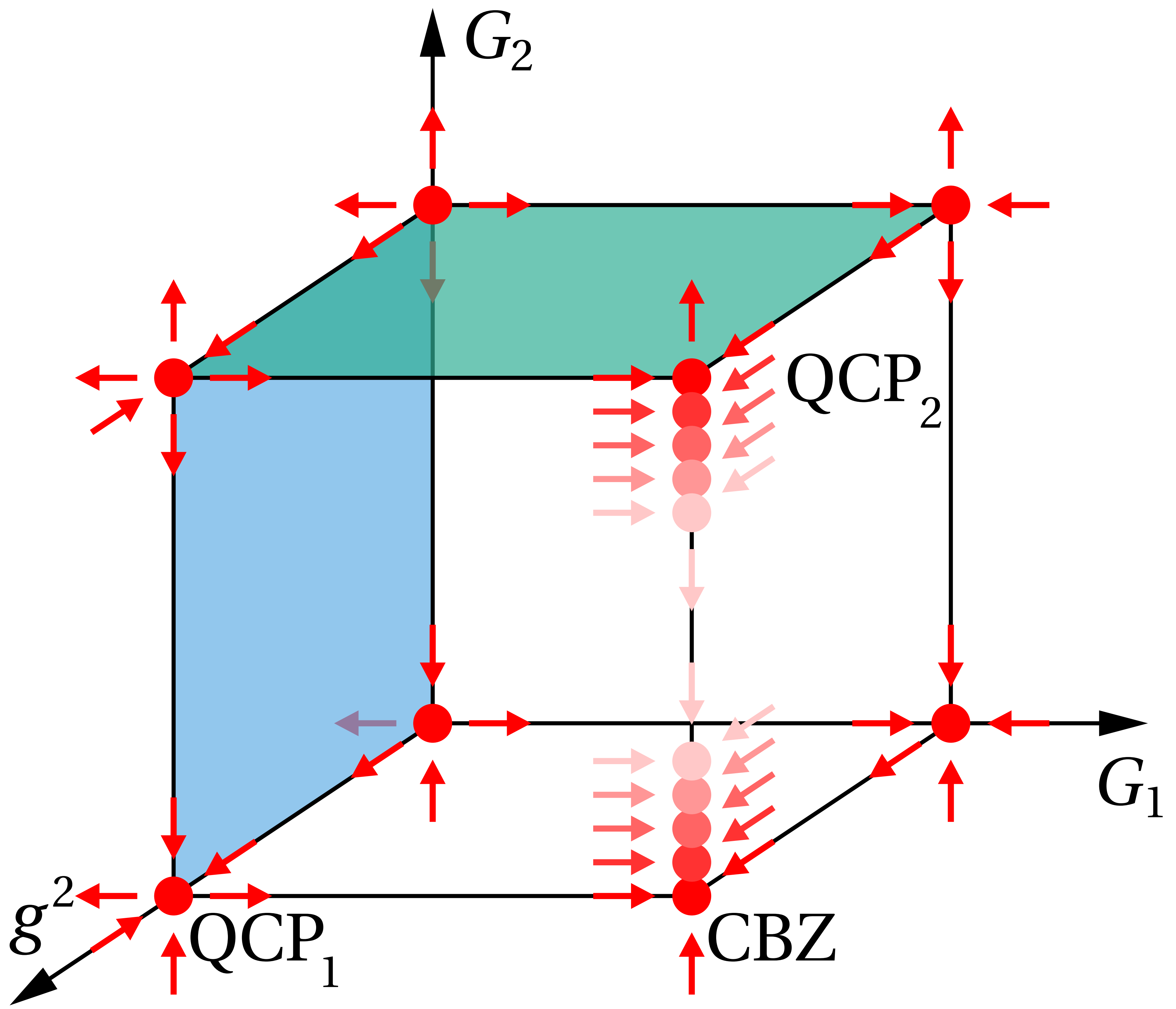
$$\mathcal{O}_1 = \left(\bar{\psi}^a\gamma_{\mu}\psi^b\right)^2 + \left(\bar{\psi}^a\gamma_{\mu}\gamma_5\psi^b\right)^2$$
$$\mathcal{O}_2 = \left(\bar{\psi}^a\psi^b\right)^2 - \left(\bar{\psi}^a\gamma_5\psi^b\right)^2$$

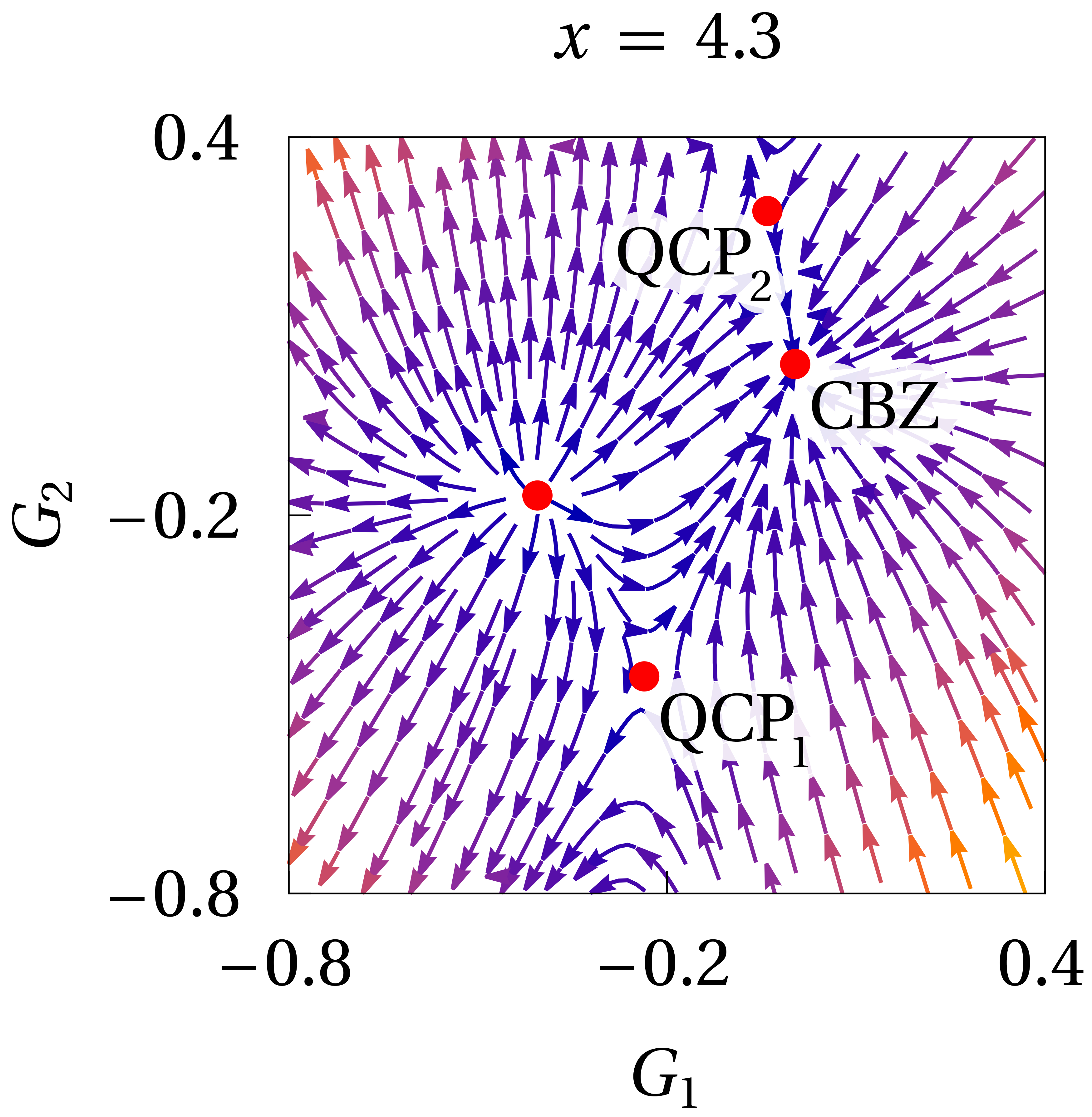
[Gies, Jaeckel, Wetterich, PRD '04]
[Gies, Jaeckel, EPJC '06]
[Braun, JPG '12]
[Gukov, NPB '17]
...

Tree-level scaling:

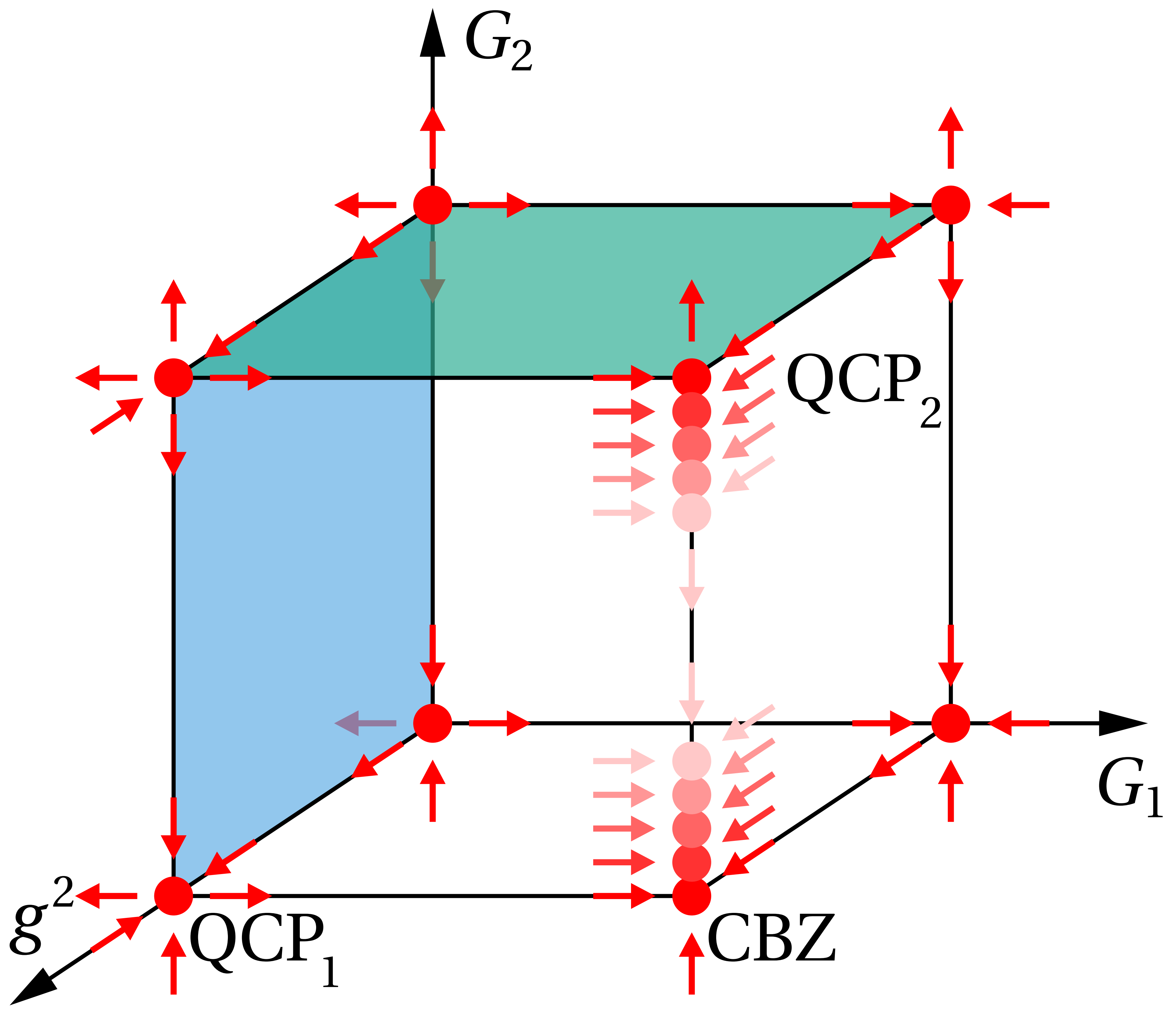
Gauge coupling	$[g] = 0$	marginal	... marginally relevant (asymptotic freedom)
4-fermion couplings	$[G_{1,2}] = -2$	irrelevant	

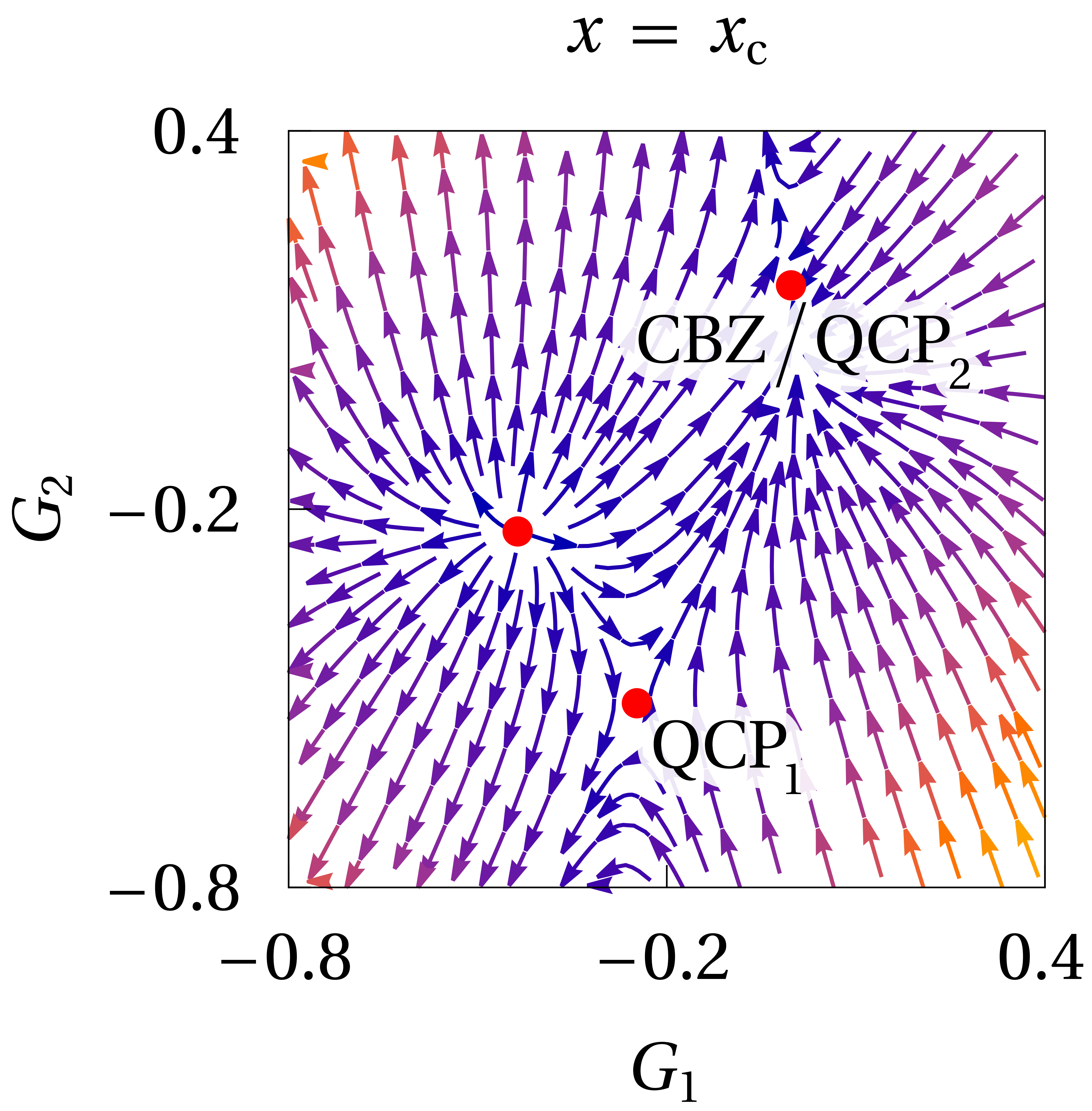
RG flow diagram





RG flow diagram





RG flow diagram