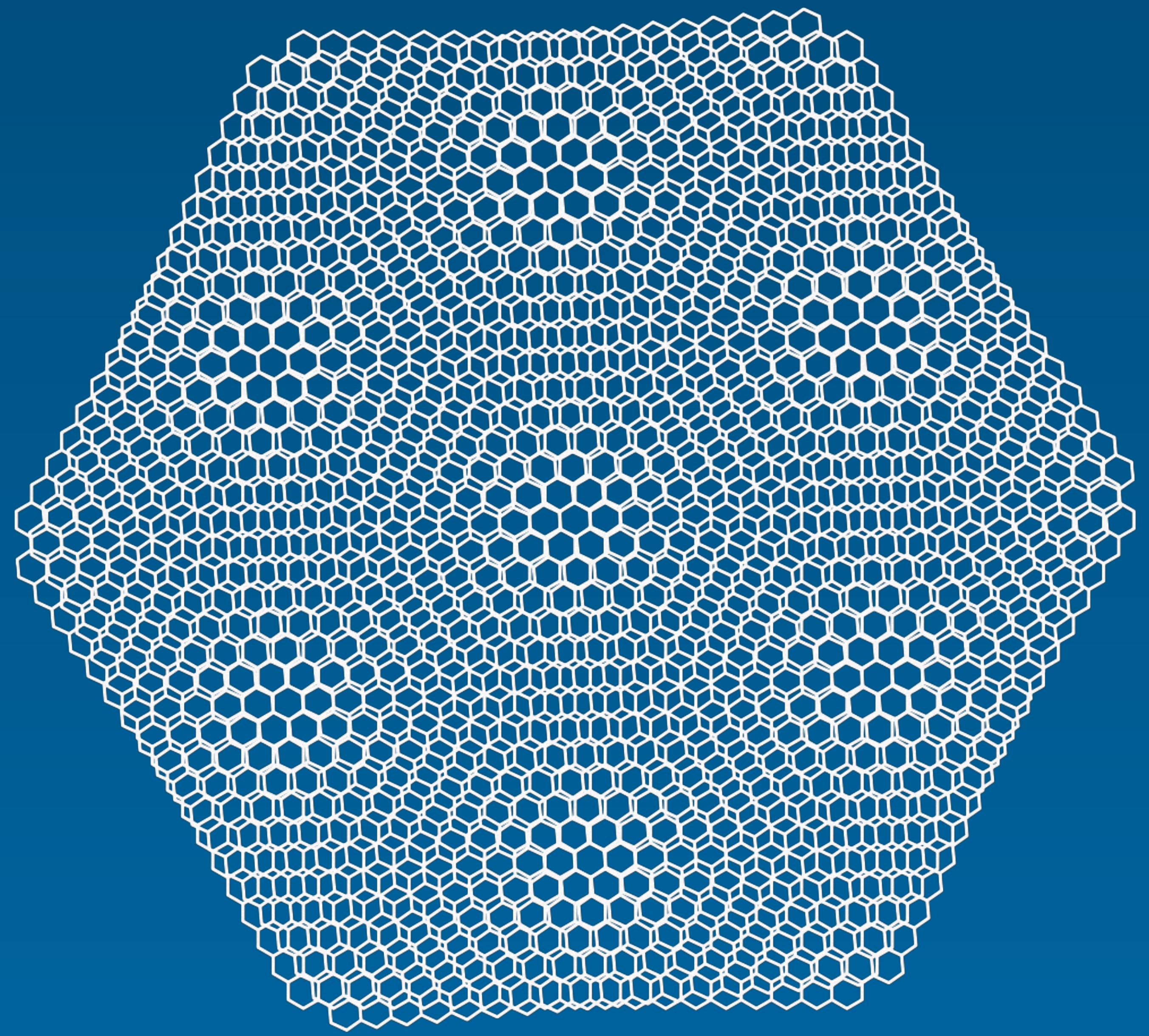
Dresden University of Technology

DRESDEN concept

Dirac quantum criticality in moiré systems



Lukas Janssen

Jan Biedermann

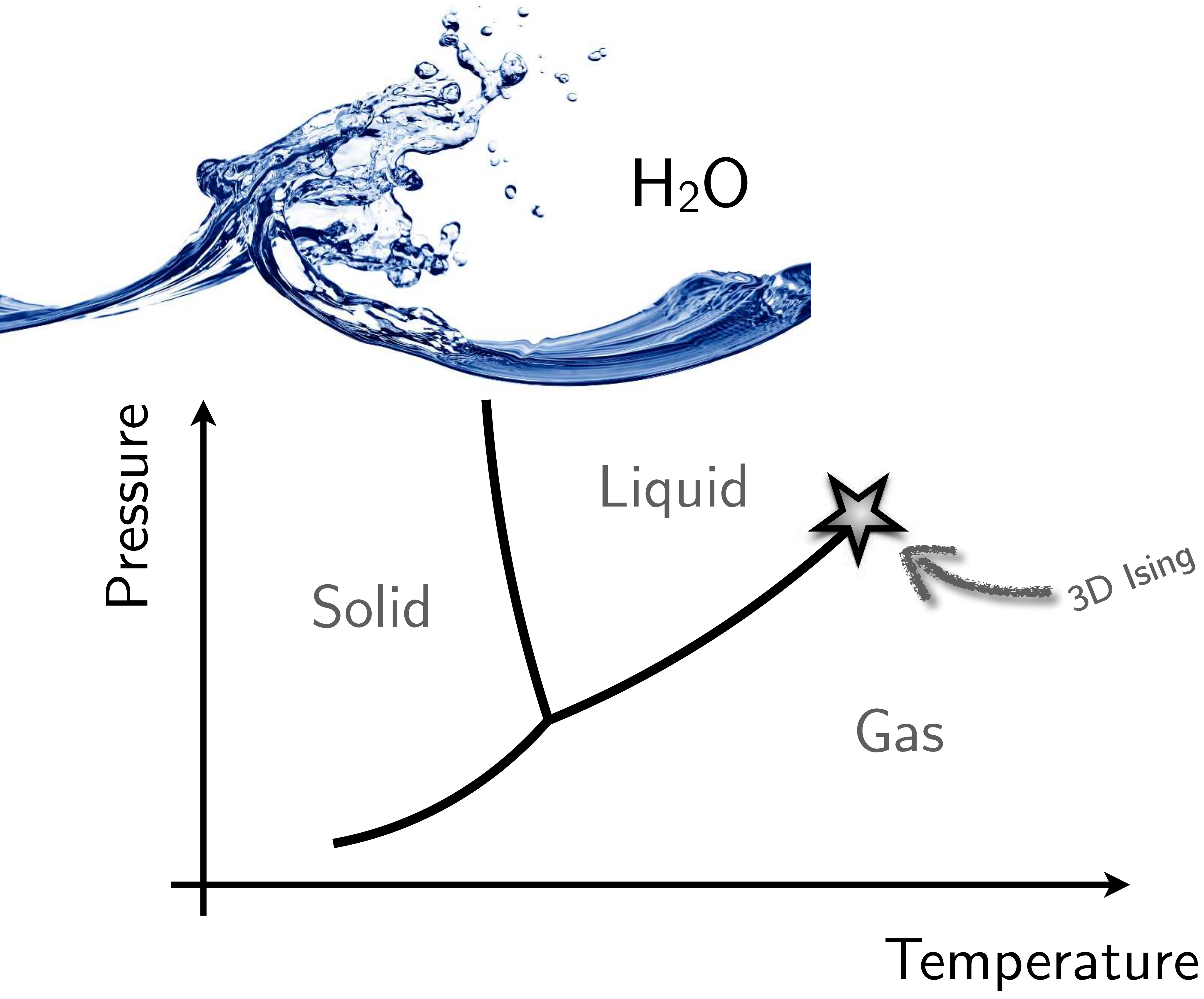
Emmy Noether-Programm

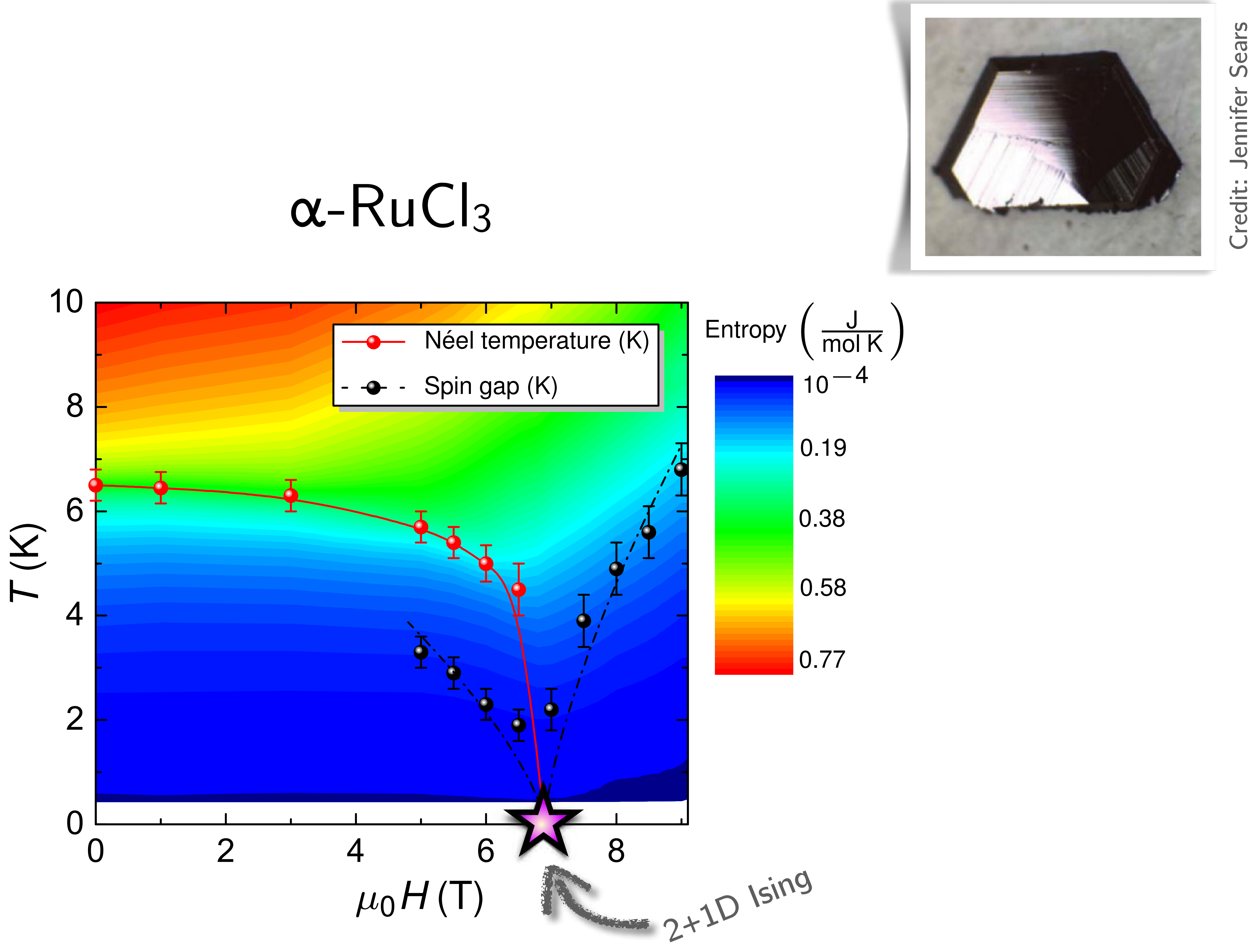
DFG Deutsche Forschungsgemeinschaft

ctd.qmat Complexity, Topology and Dynamics in Quantum Matter

Quantum SAX-QT

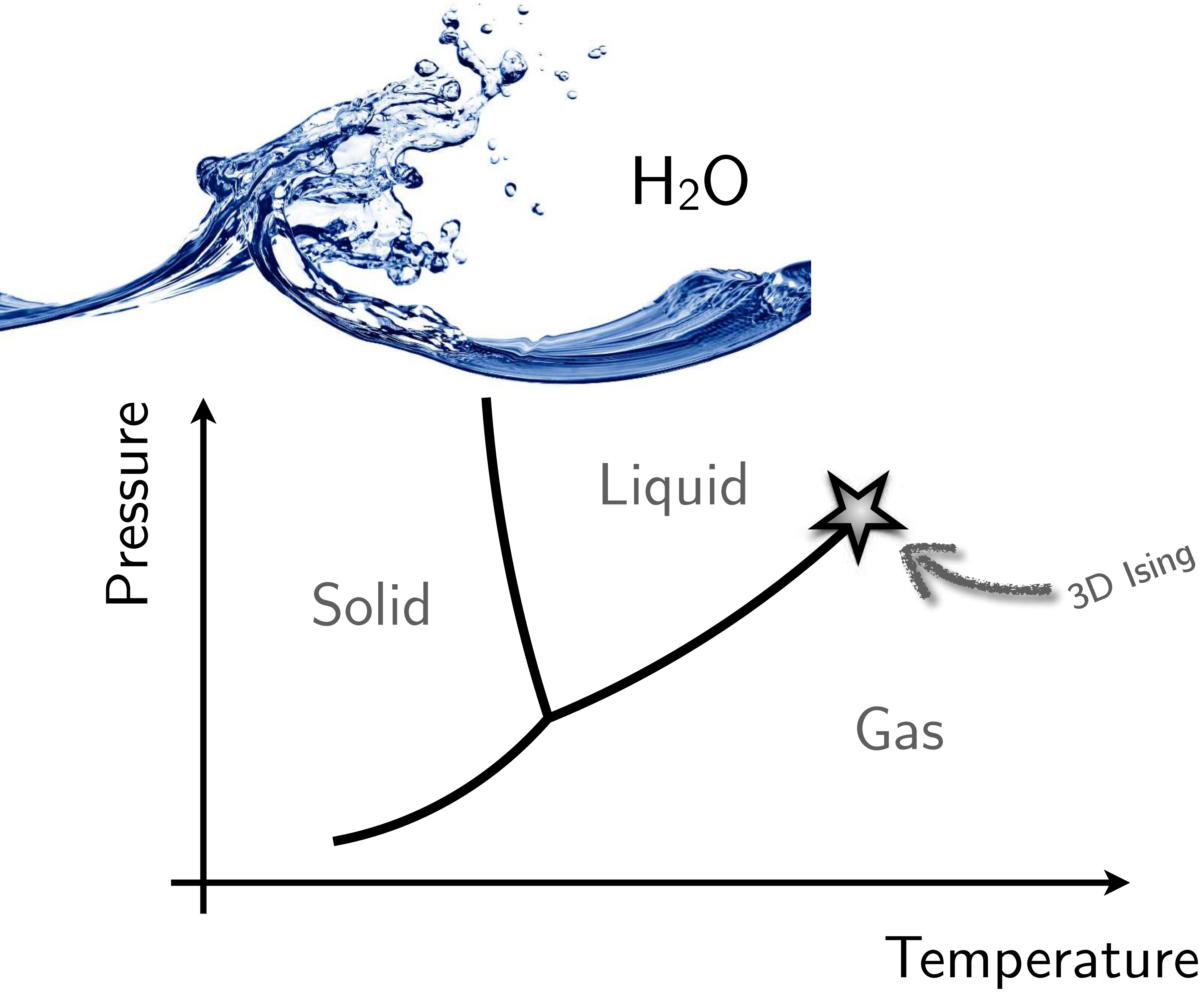
Classical vs quantum criticality



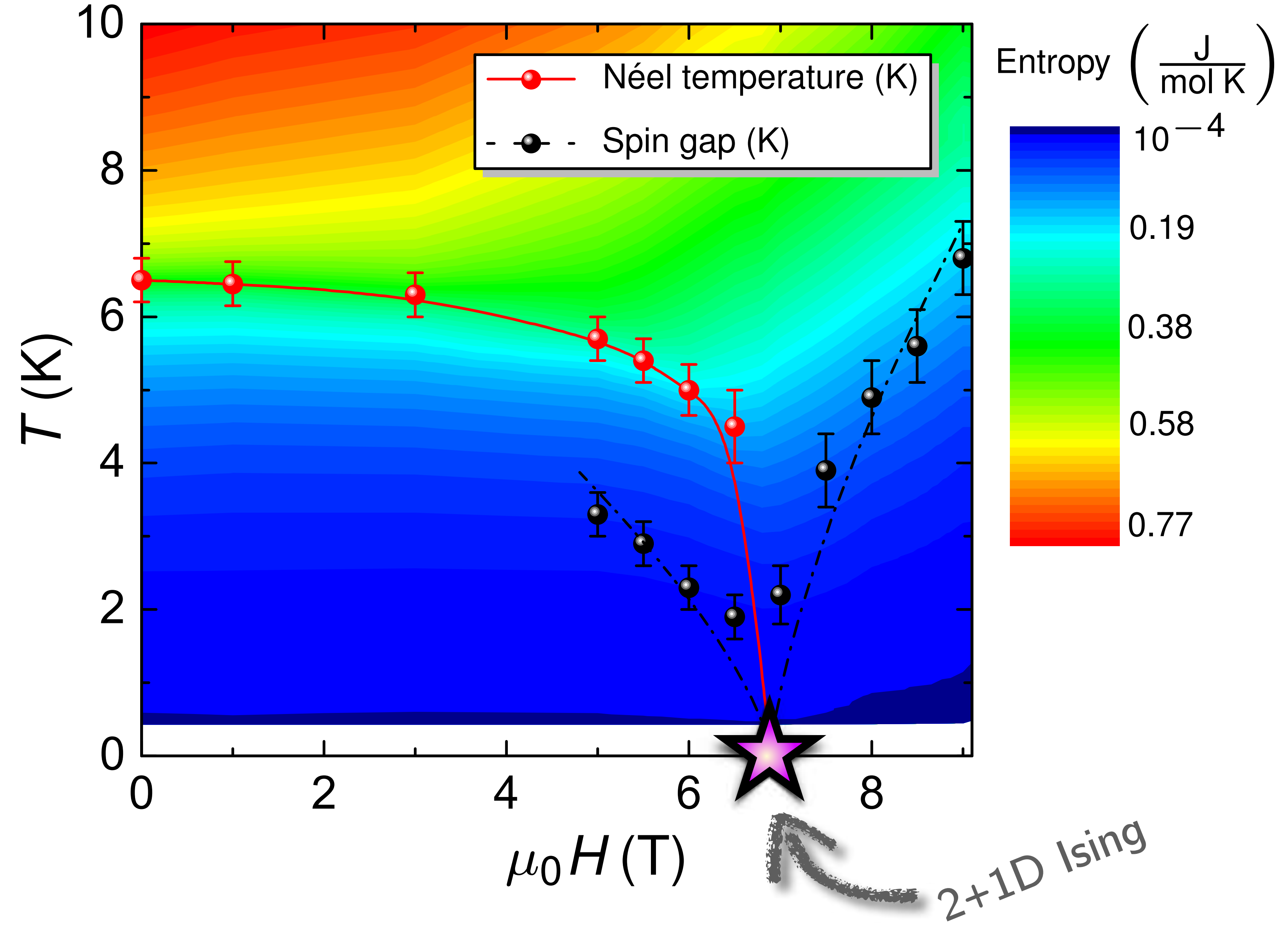


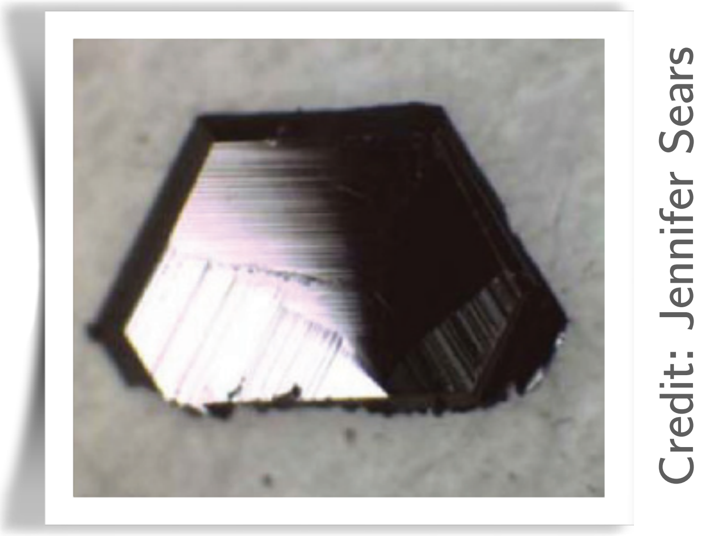
[Wolter, Corredor, LJ, *et al.*, PRB '17]

Classical vs quantum criticality



α-$RuCl_3$





[Wolter, Corredor, LJ, *et al.*, PRB ’17]

Universal field theory:

$$\mathcal{L} = \frac{1}{2}\varphi(-\partial_\tau^2 - v_B^2\vec{\nabla}^2)\varphi + \lambda\varphi^4 + \ldots$$

Genuine *quantum* criticality?

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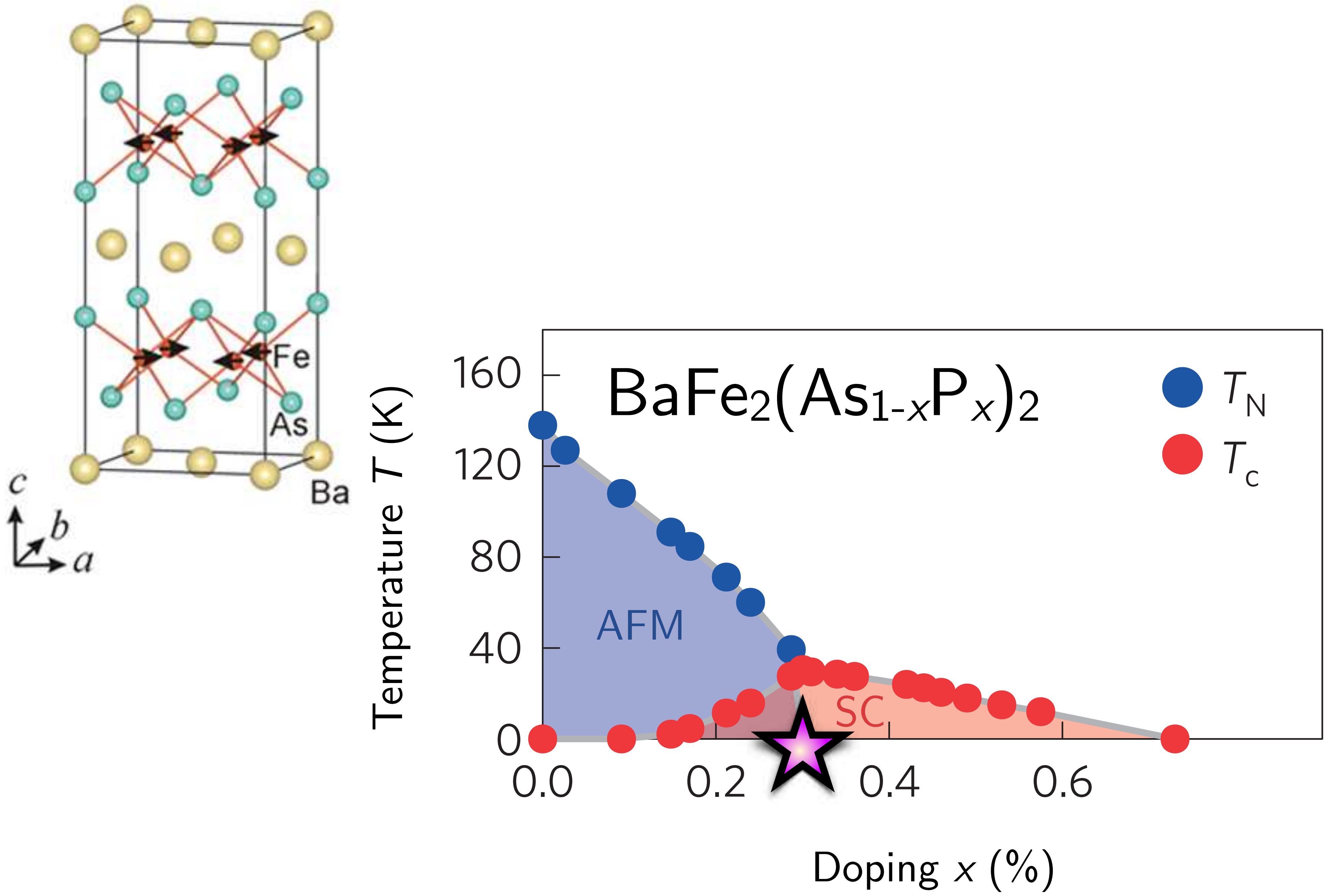
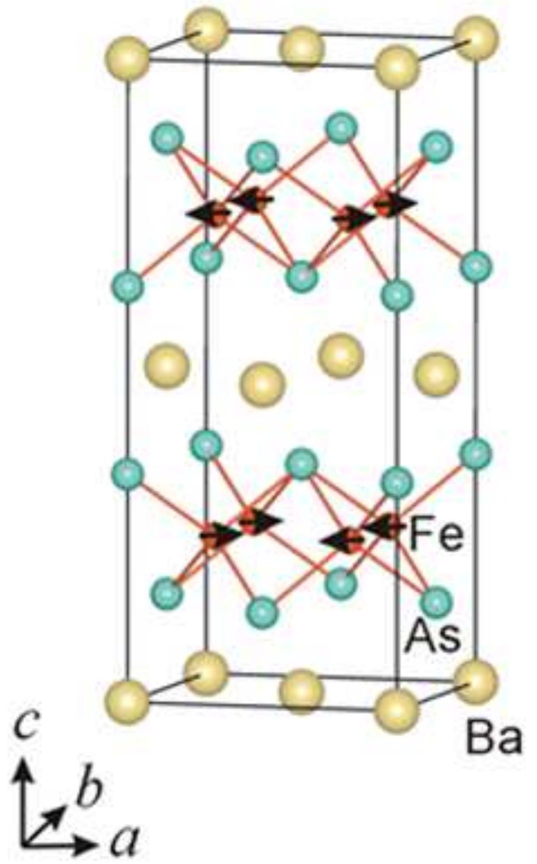
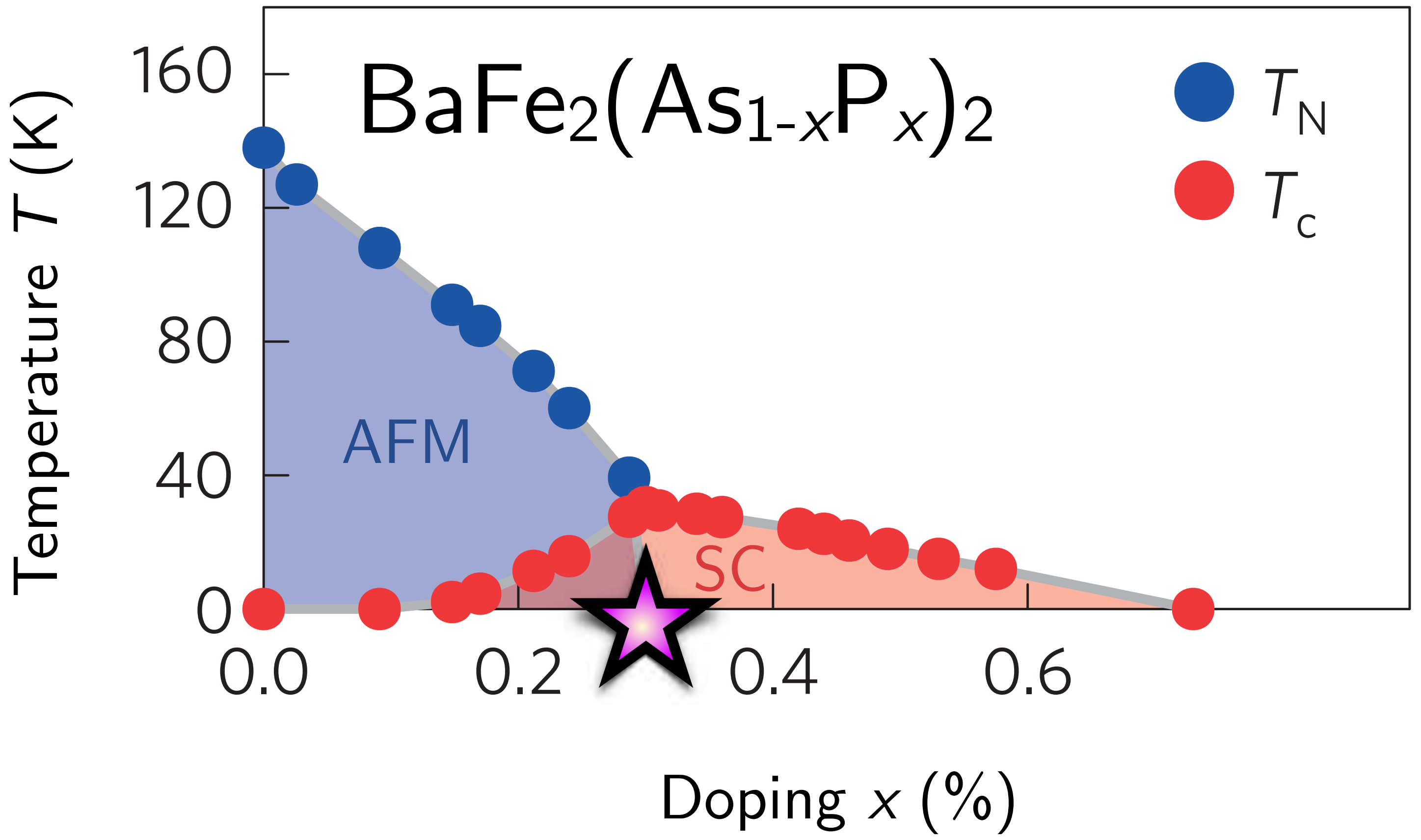


[Huang *et al.*, PRL '08]
[Hashimoto *et al.*, Science '12]
[Hayes *et al.*, Nat. Phys. '16]

Genuine *quantum* criticality?





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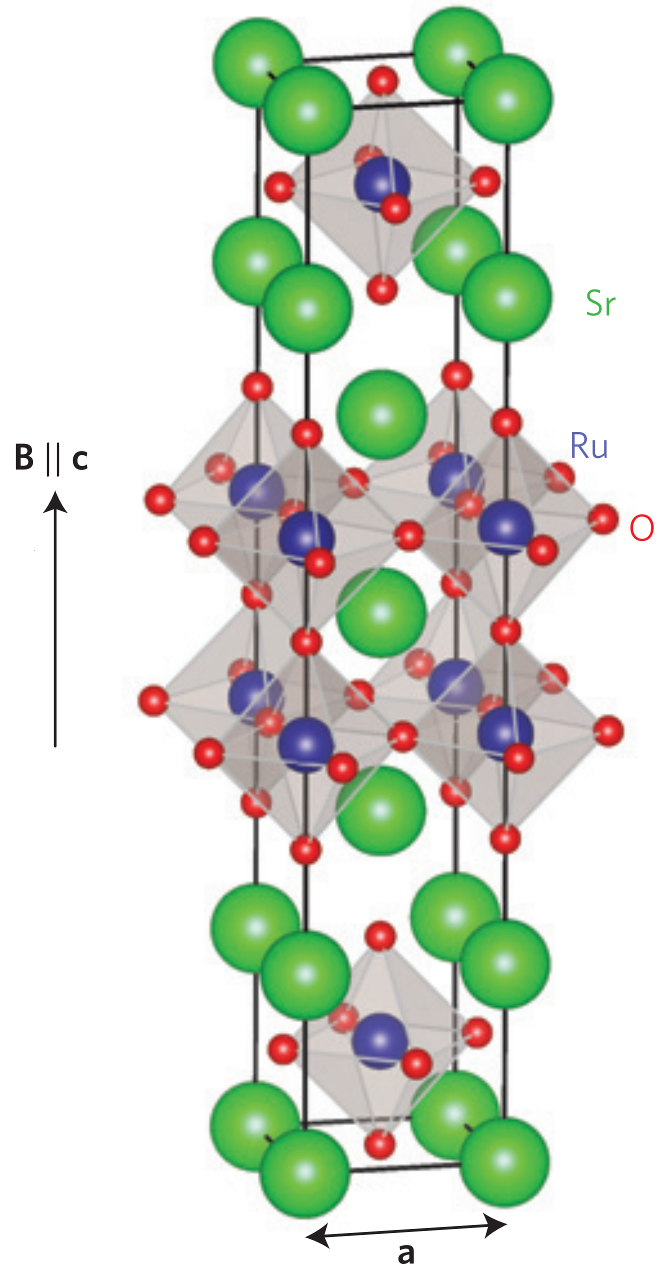

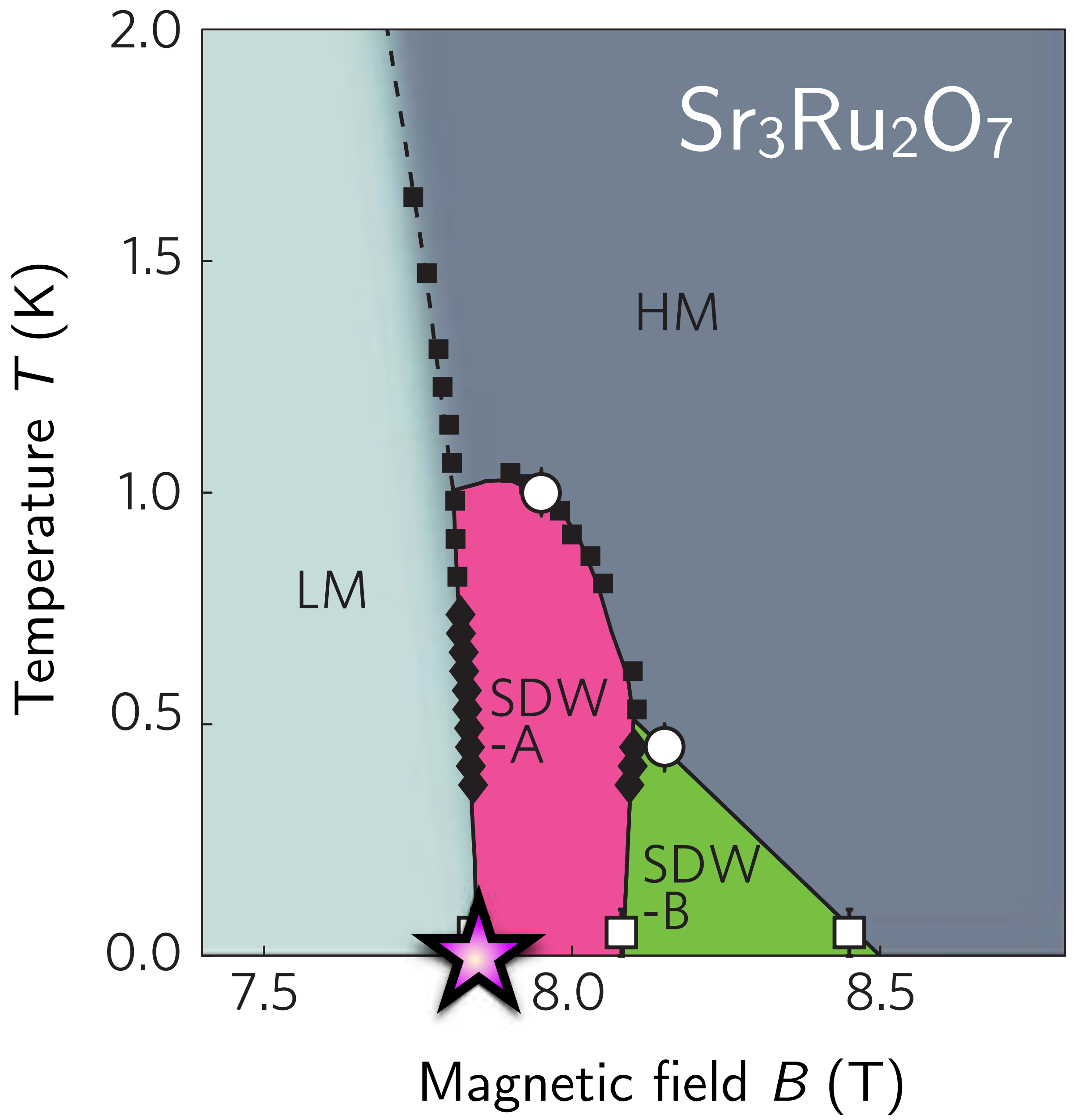

[Lester *et al.*, Nat. Mat. '15]

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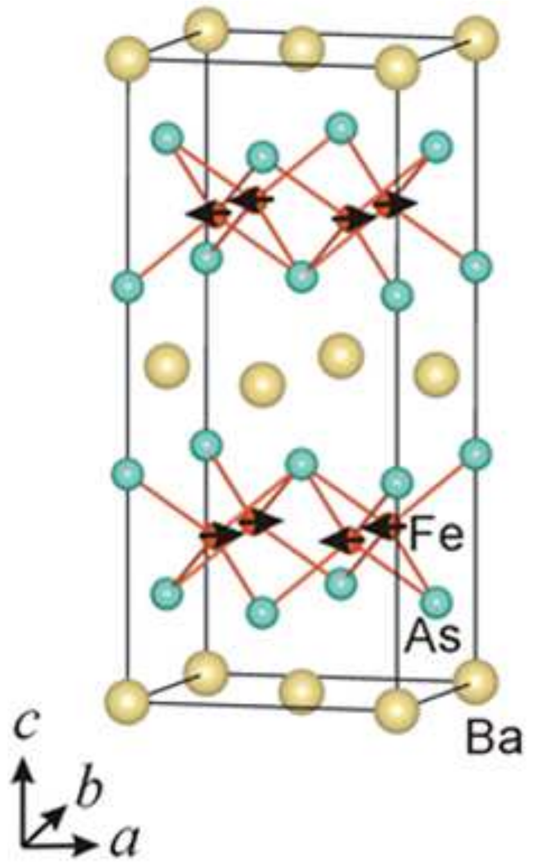

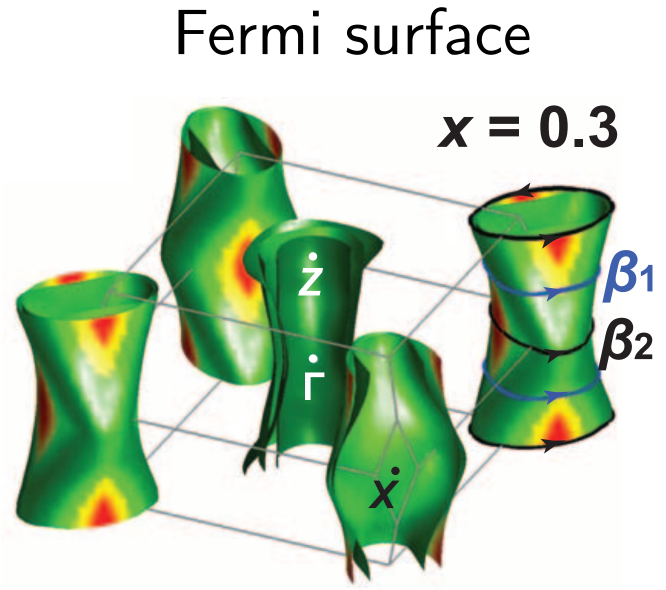

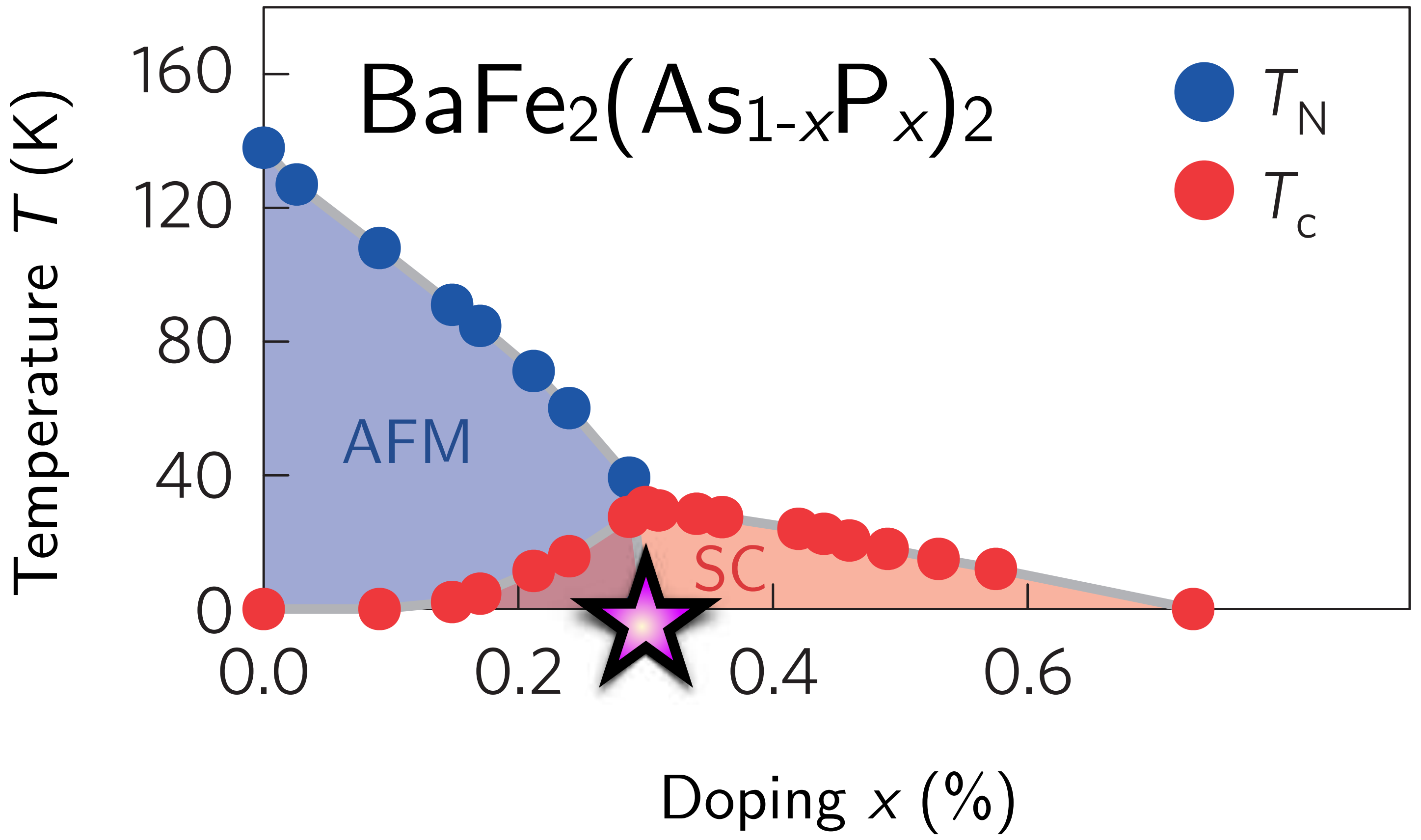

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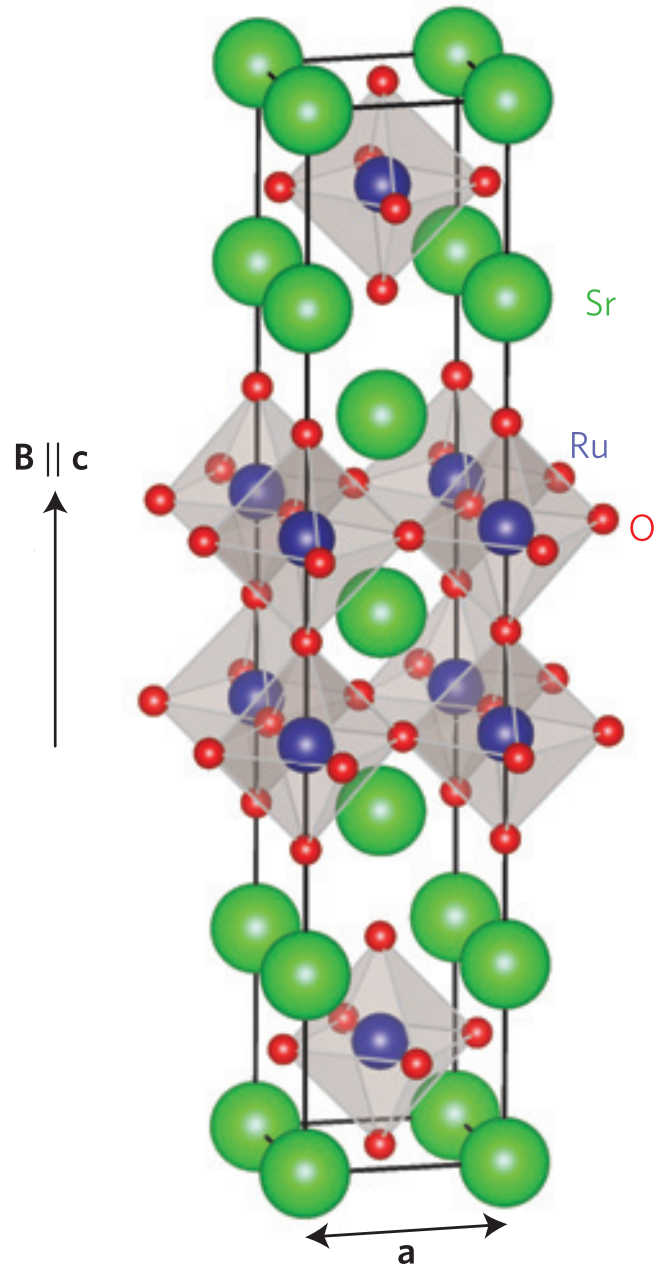

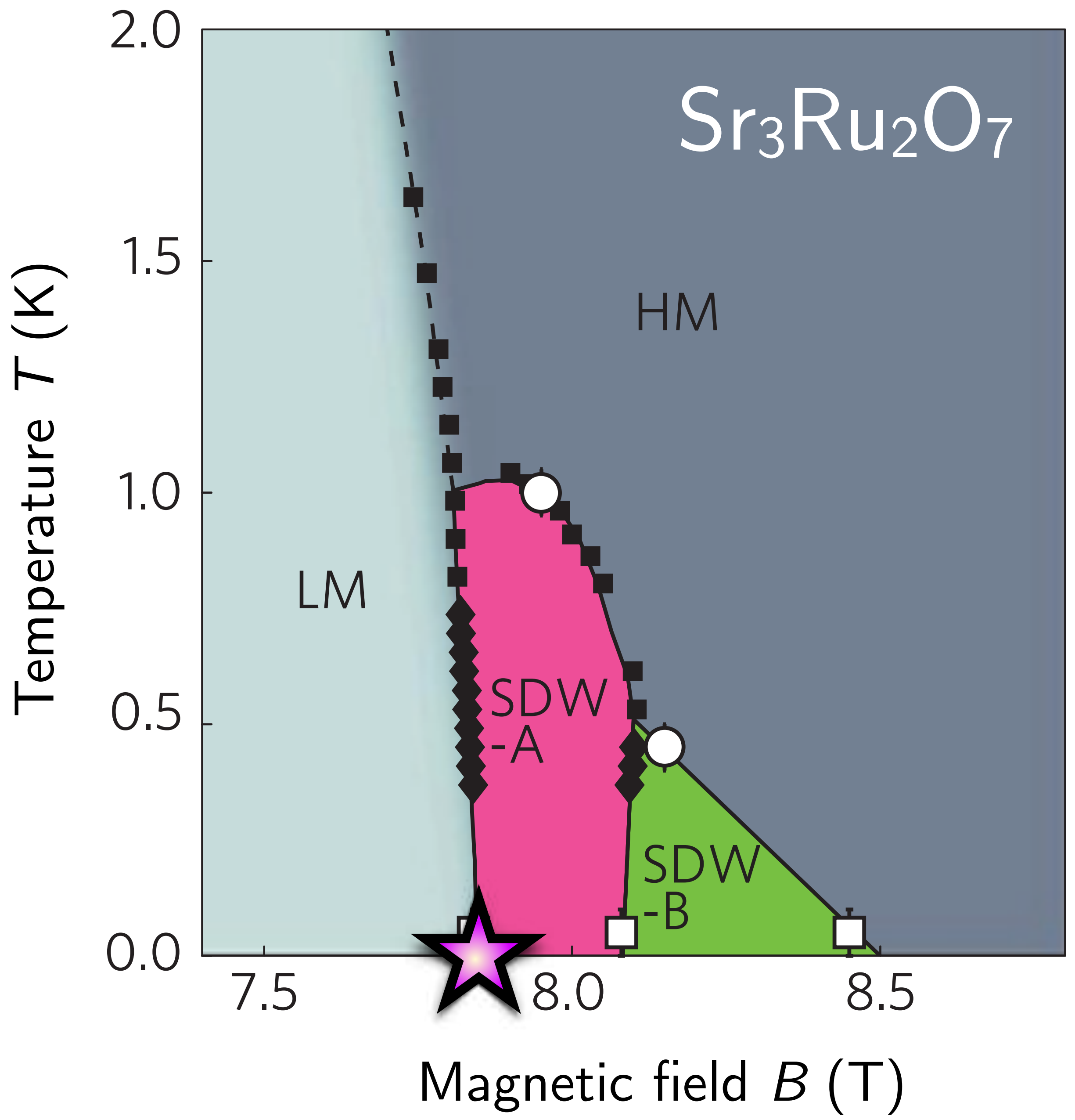

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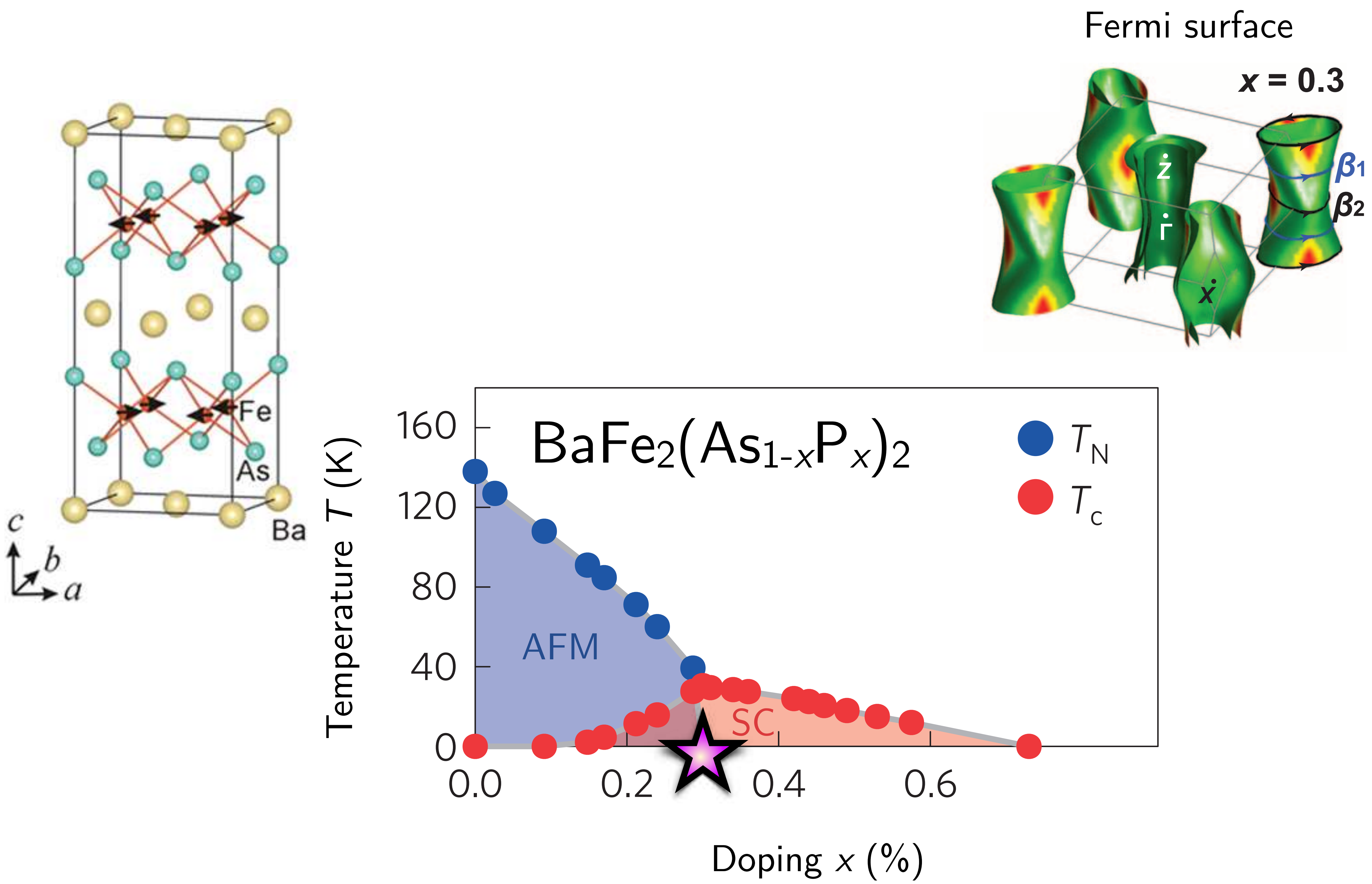

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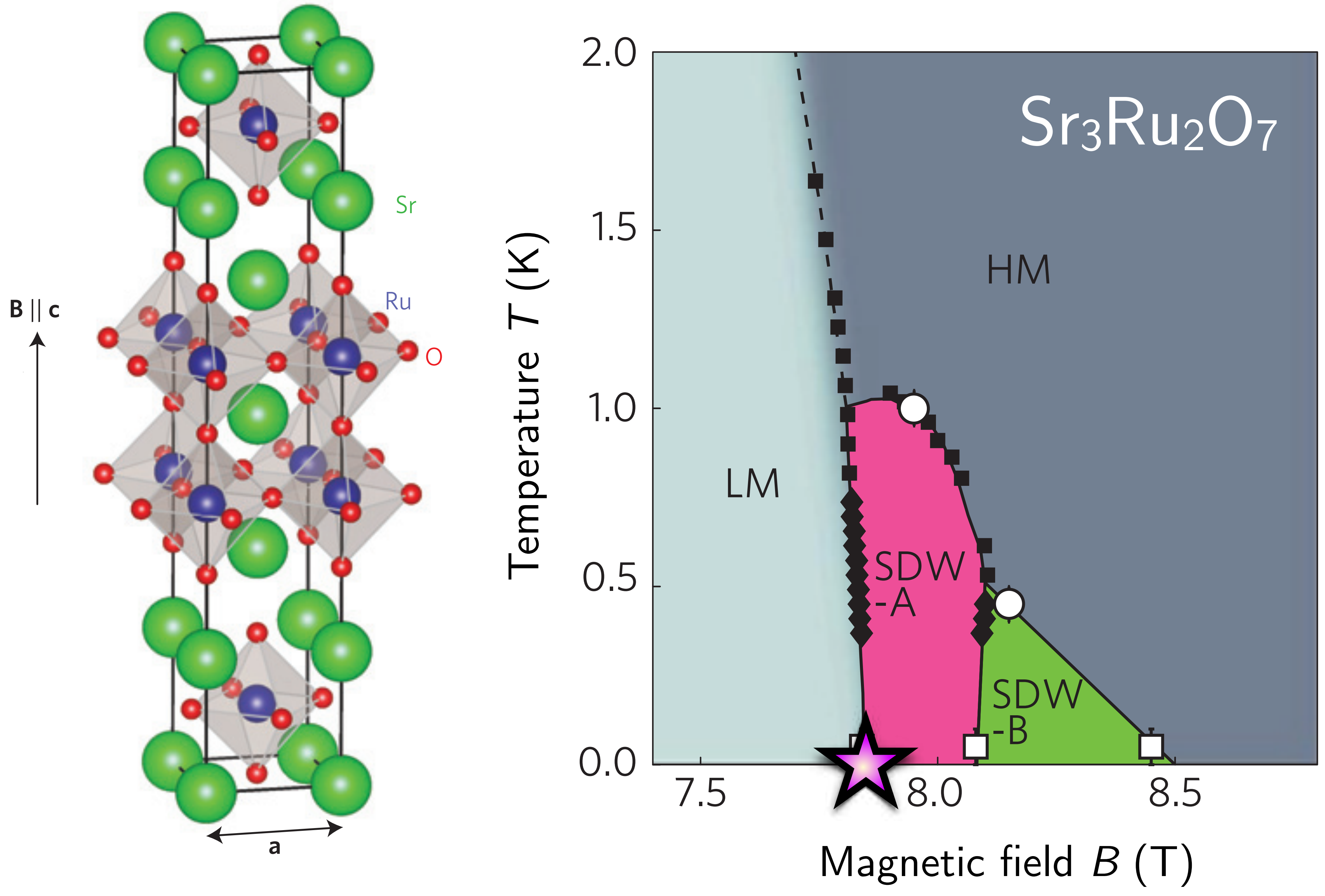


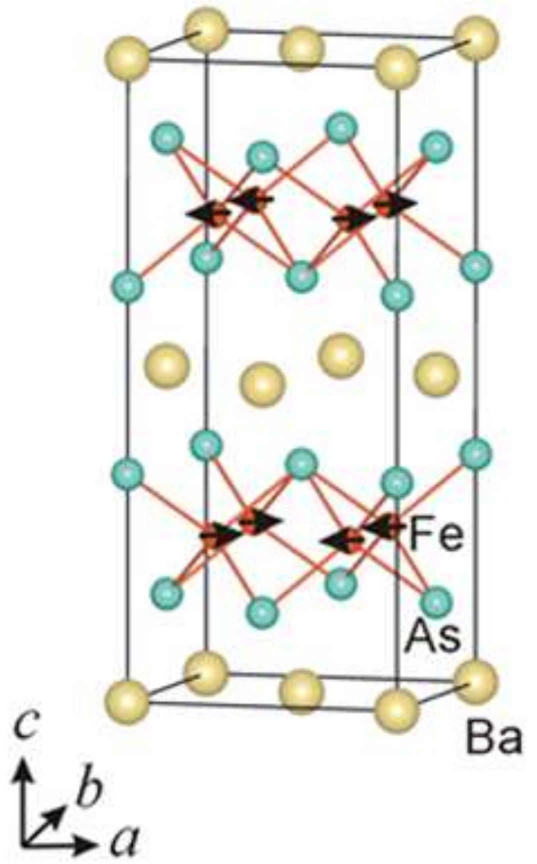
[Lester *et al.*, Nat. Mat. '15]

Universal field theory:

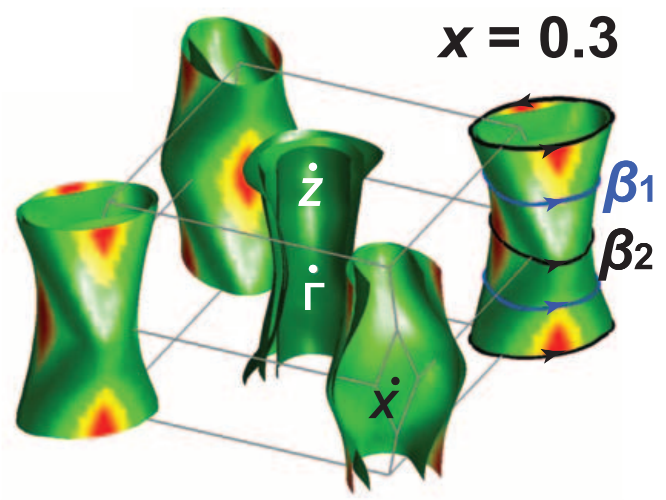
$$\mathcal{L} = \frac{1}{2}\vec{\varphi}\cdot(-\partial_\tau^2 - v_B^2\vec{\nabla}^2)\vec{\varphi} + \psi^\dagger\left(\partial_\tau + \varepsilon(-i\vec{\nabla})\right)\psi + \vec{\varphi}\cdot\psi^\dagger\vec{\sigma}\psi + \ldots$$

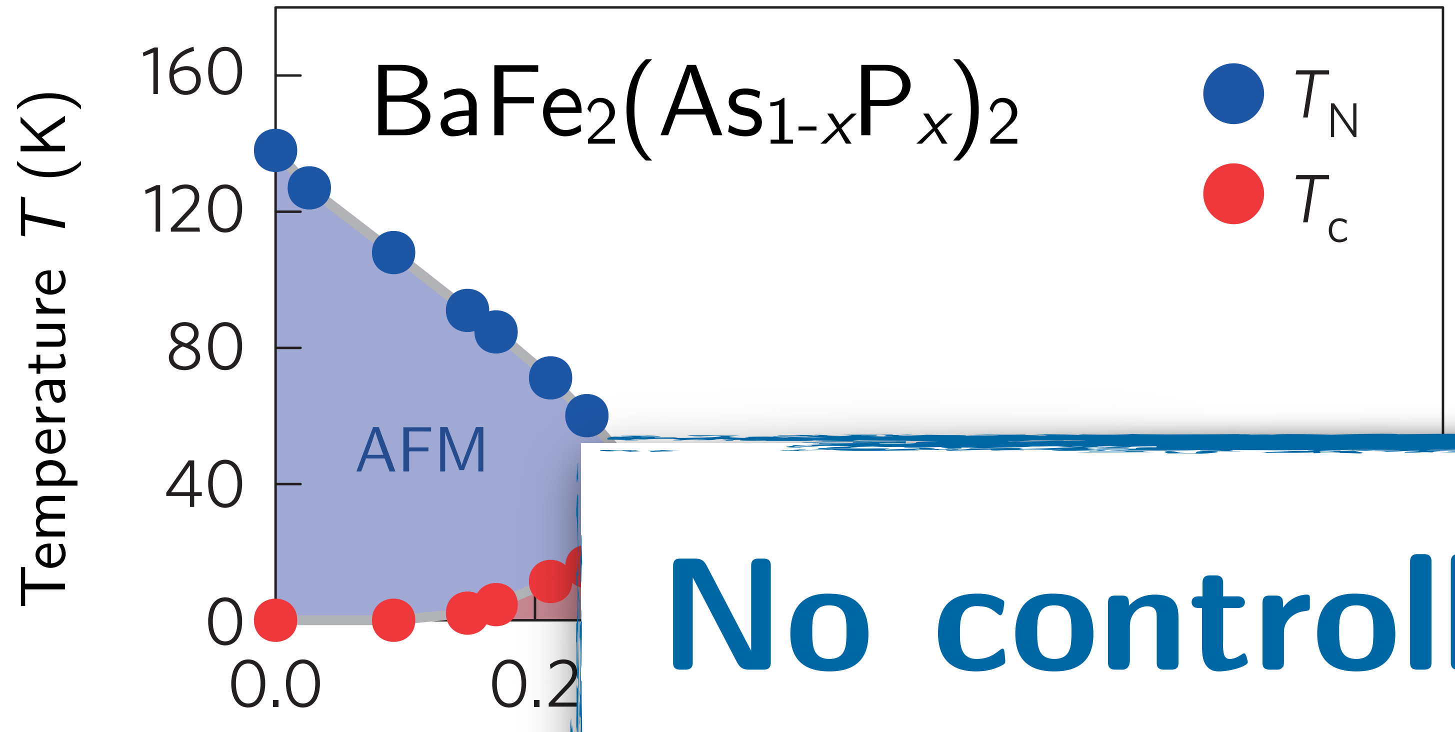
[Metlitski & Sachdev, PRB '10]

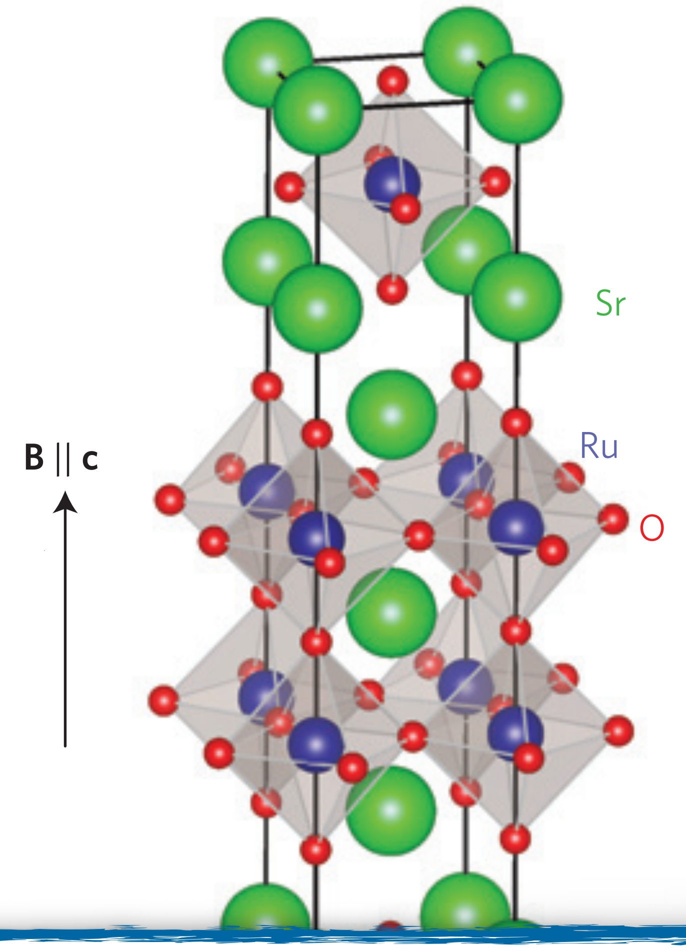
Genuine *quantum* criticality?

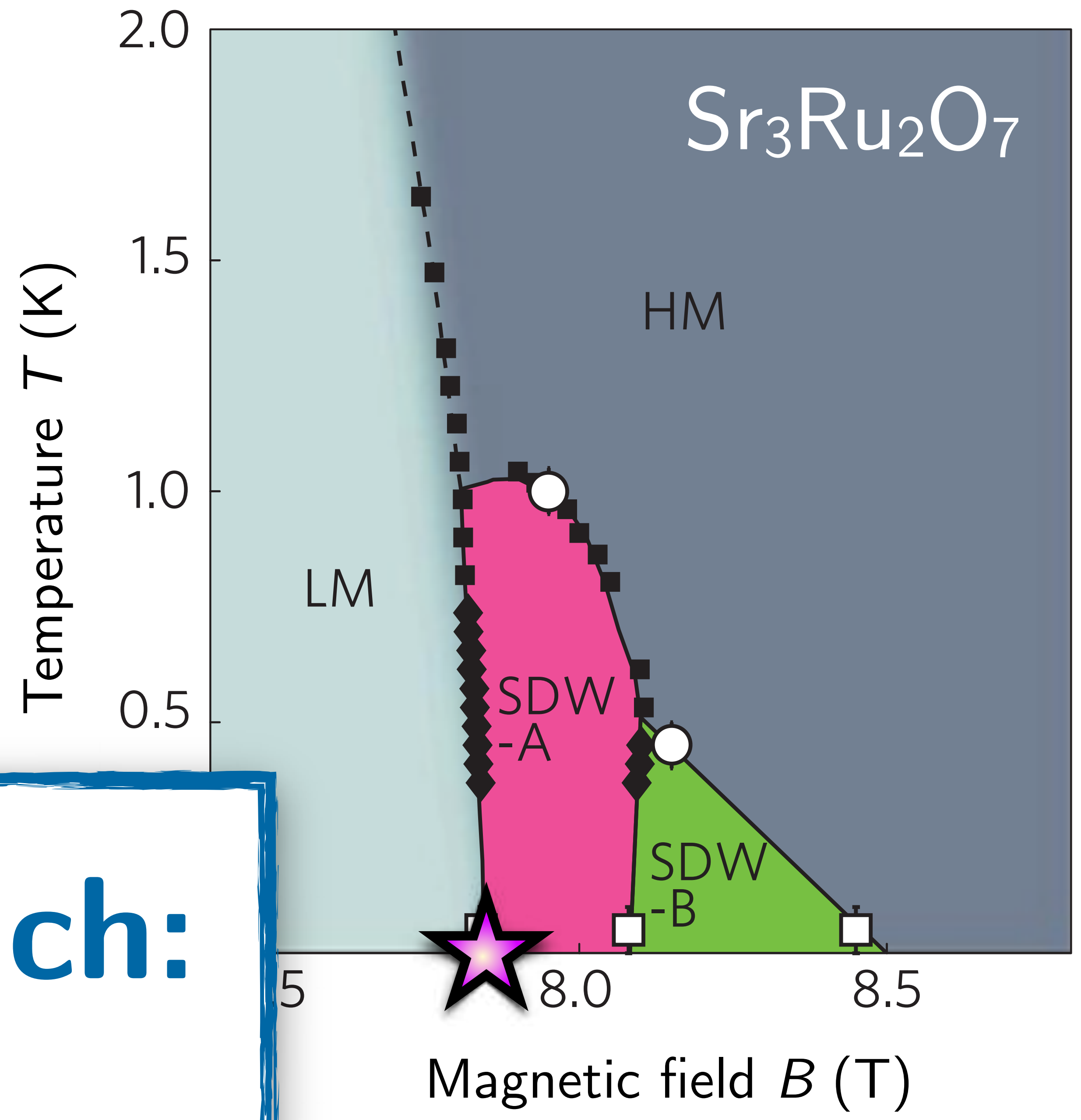


Fermi surface









[Lester *et al.*, Nat. Mat. '15]

No controlled theoretical approach:

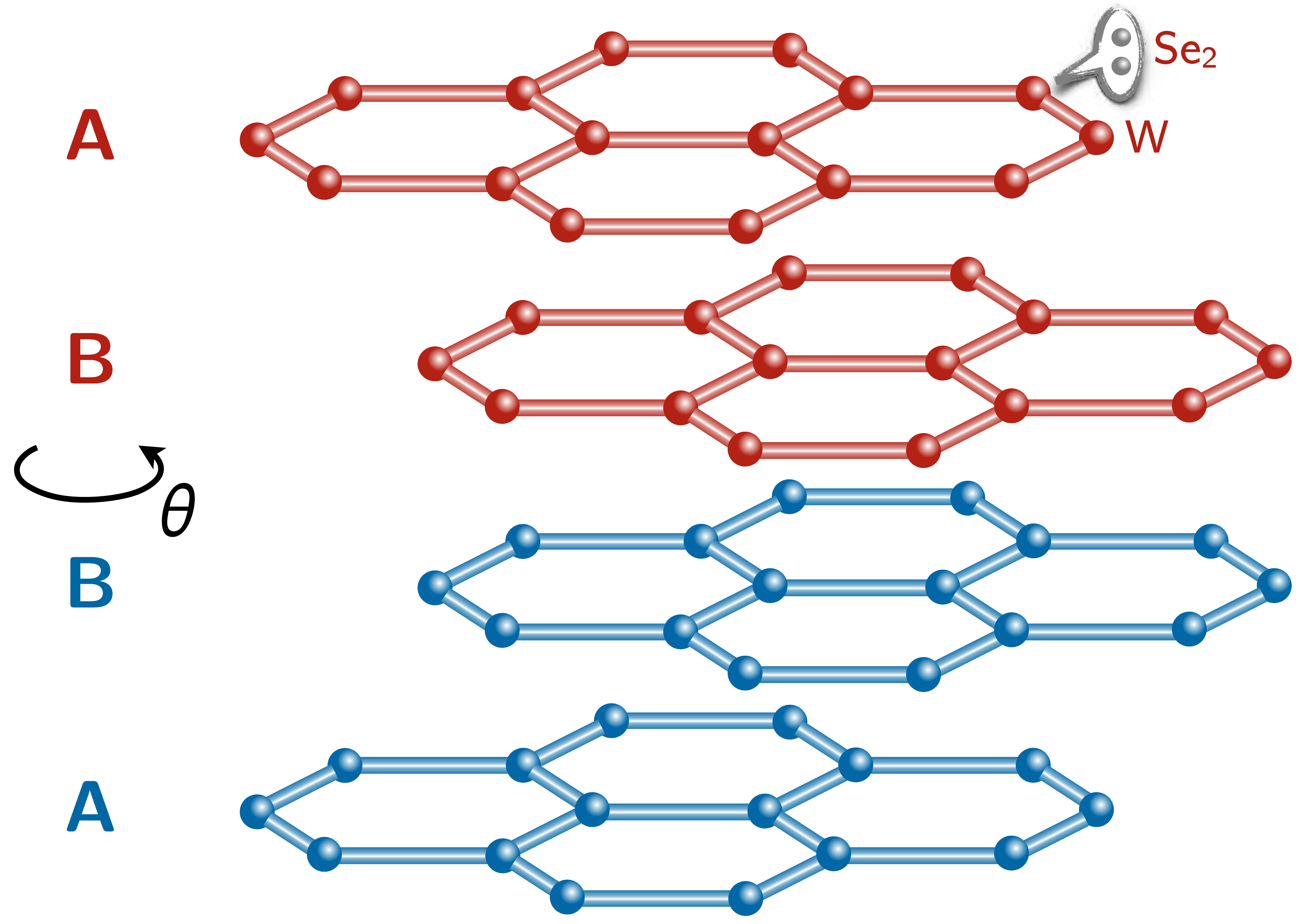
- ⚡ Too many gapless modes
- ⚡ Too small symmetry

Universal field theory:

$$\mathcal{L} = \frac{1}{2}\vec{\varphi}\cdot(-\partial_\tau^2 - v_B^2\vec{\nabla}^2)\vec{\varphi} + \psi^\dagger\left(\partial_\tau + \varepsilon(-\mathrm{i}\vec{\nabla})\right)\psi + \vec{\varphi}\cdot\psi^\dagger\vec{\sigma}\psi + \ldots$$

[Metlitski & Sachdev, PRB '10]

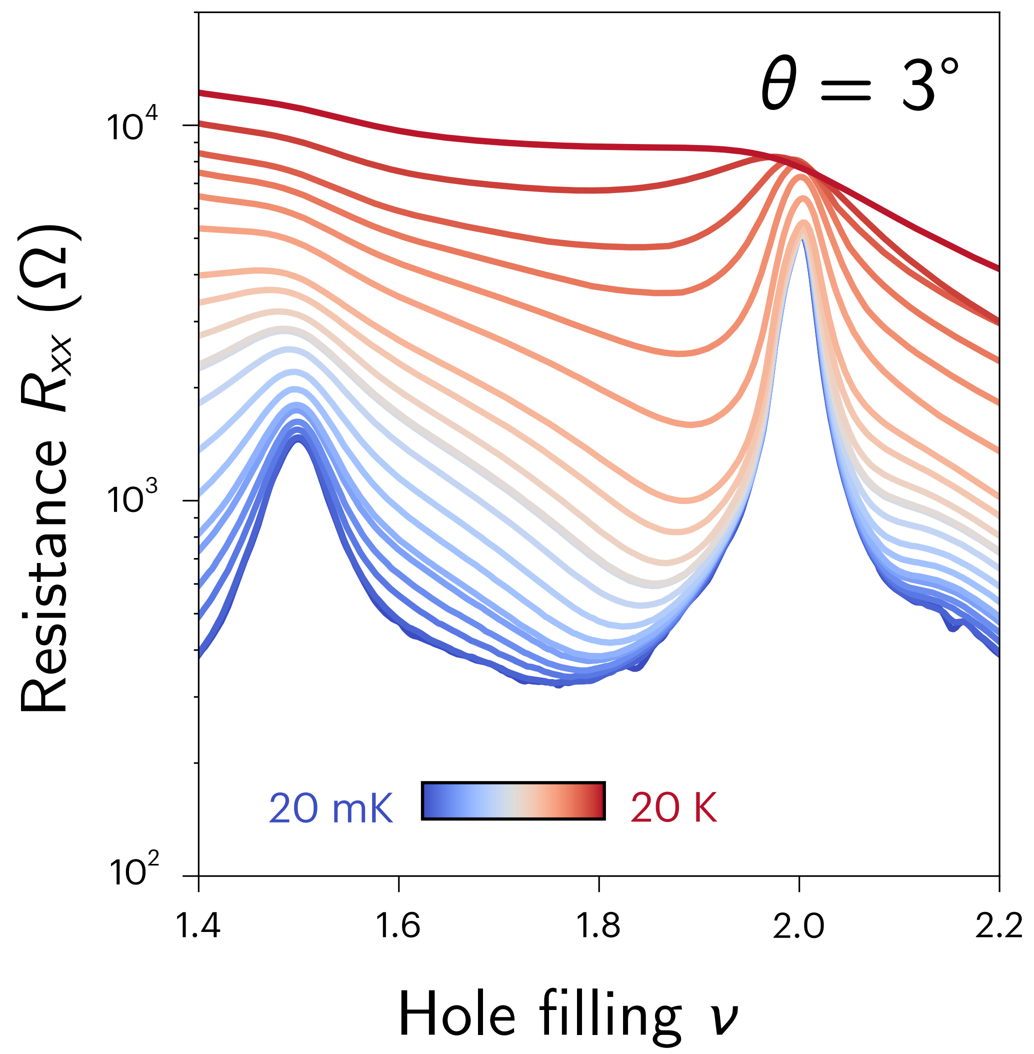
Twisted double bilayer WSe_2: Experiments

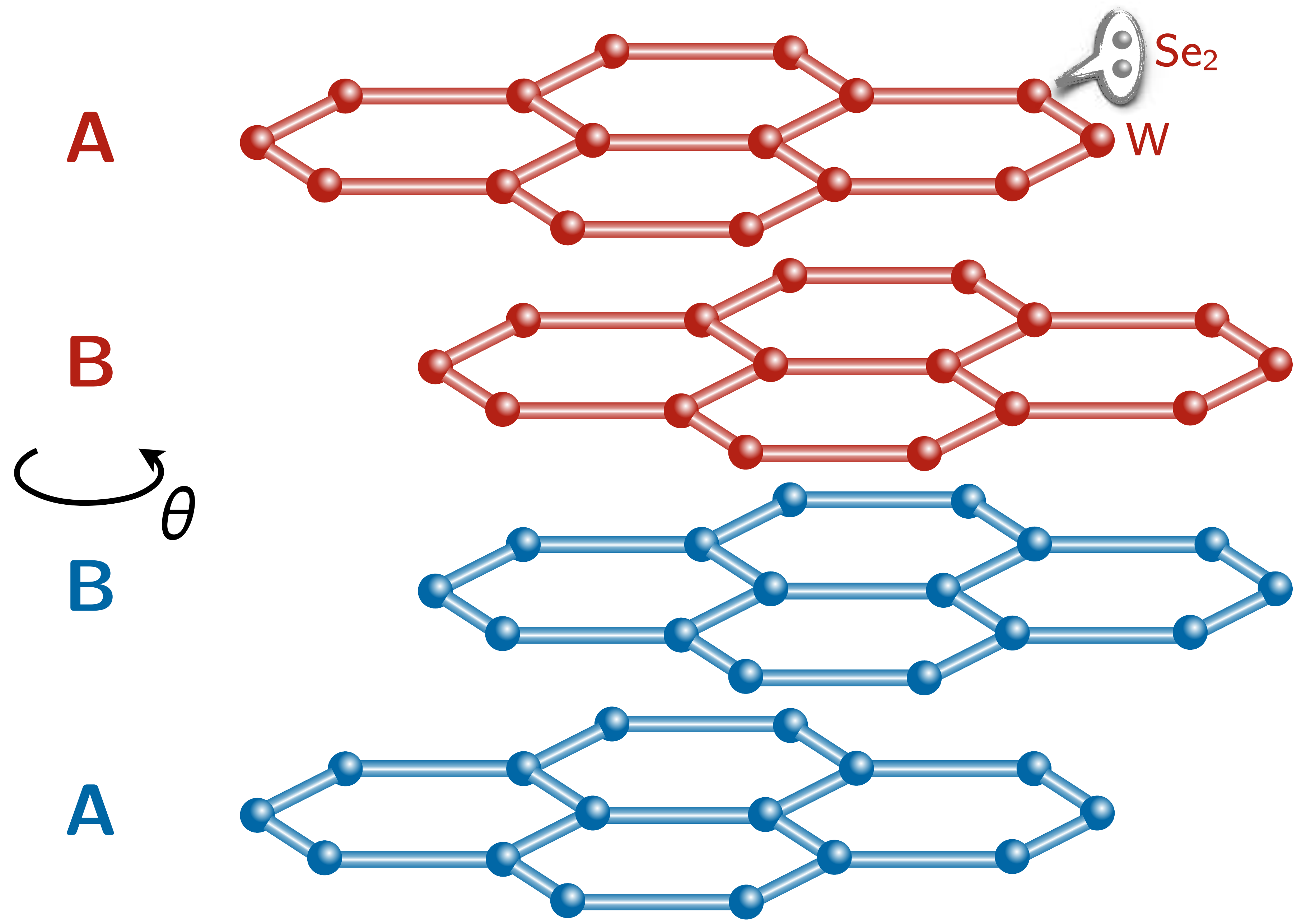


[Ma *et al.*, Nat. Mater. '25]

Twisted double bilayer WSe_2: Experiments

Transport:





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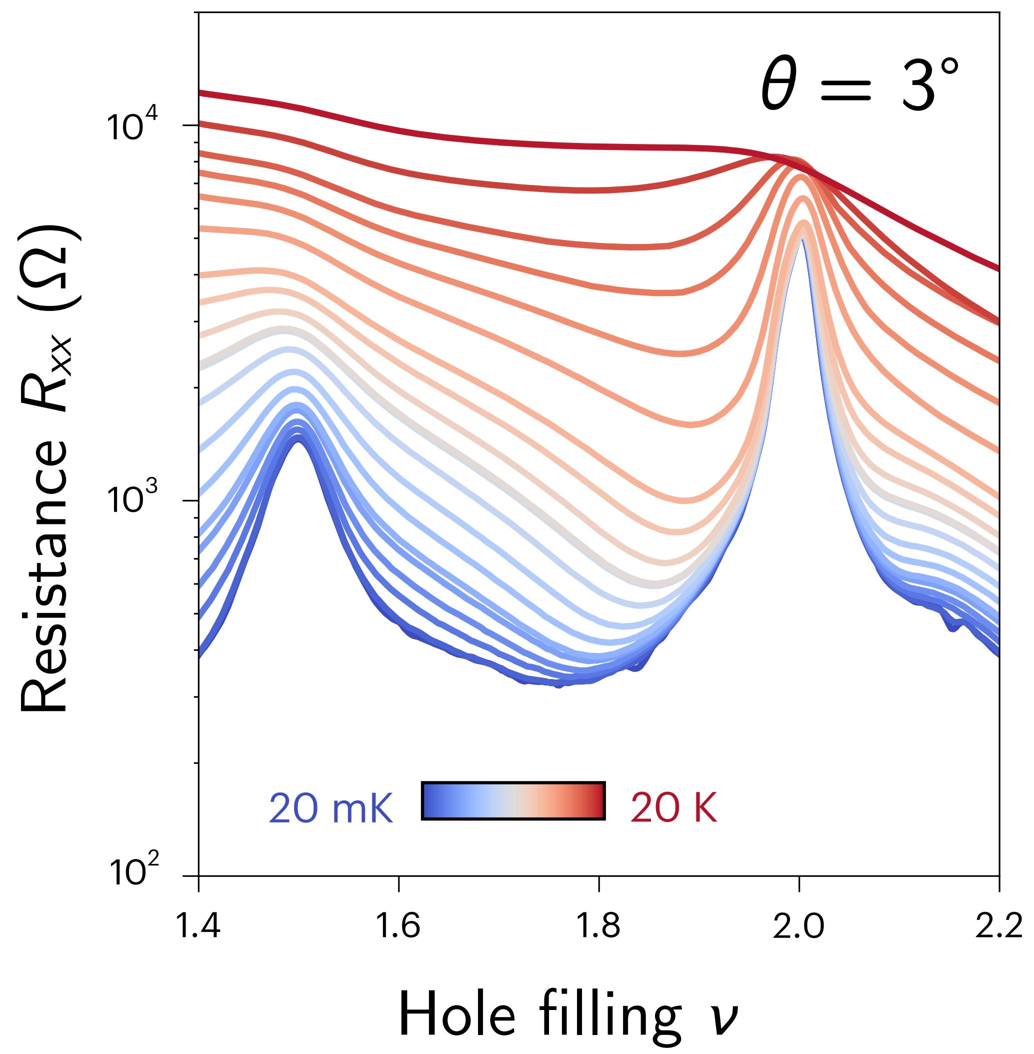

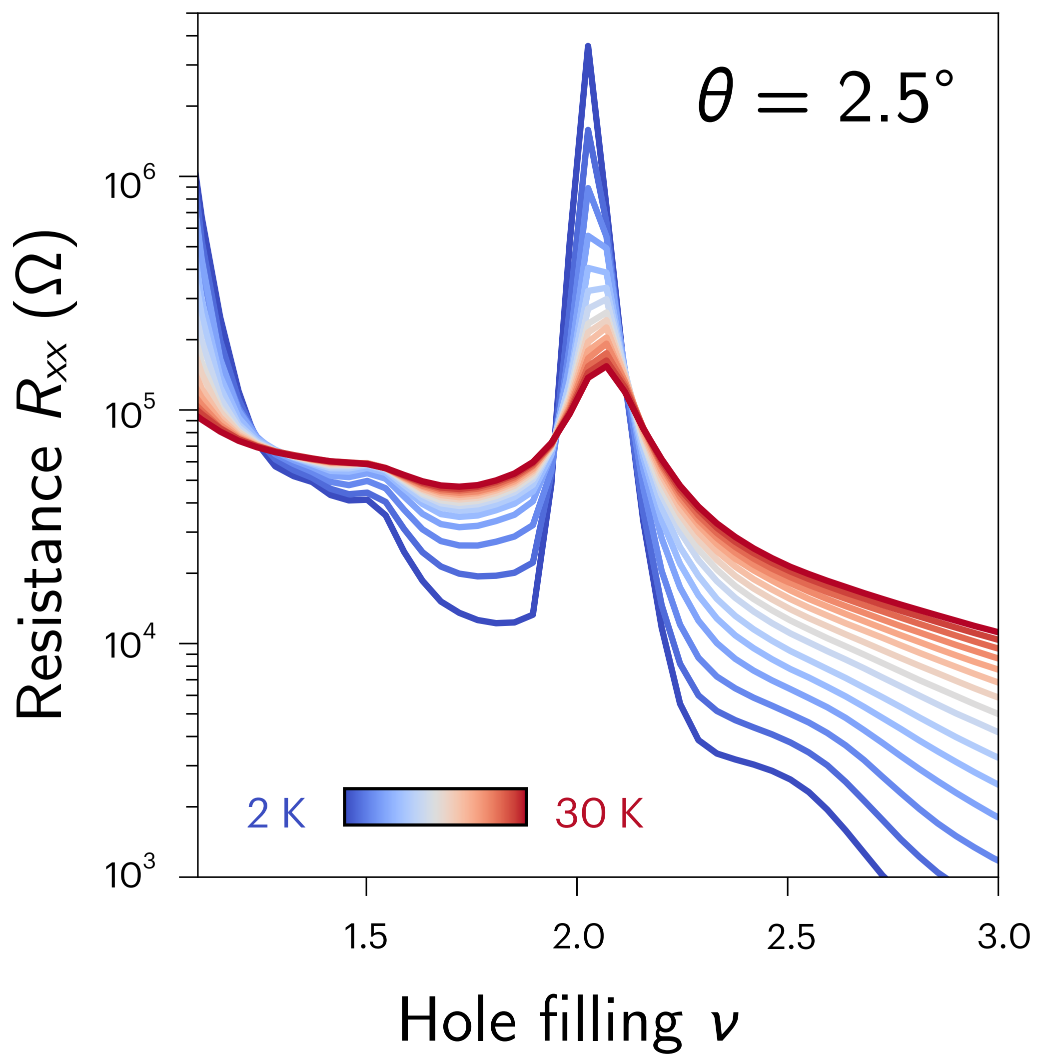

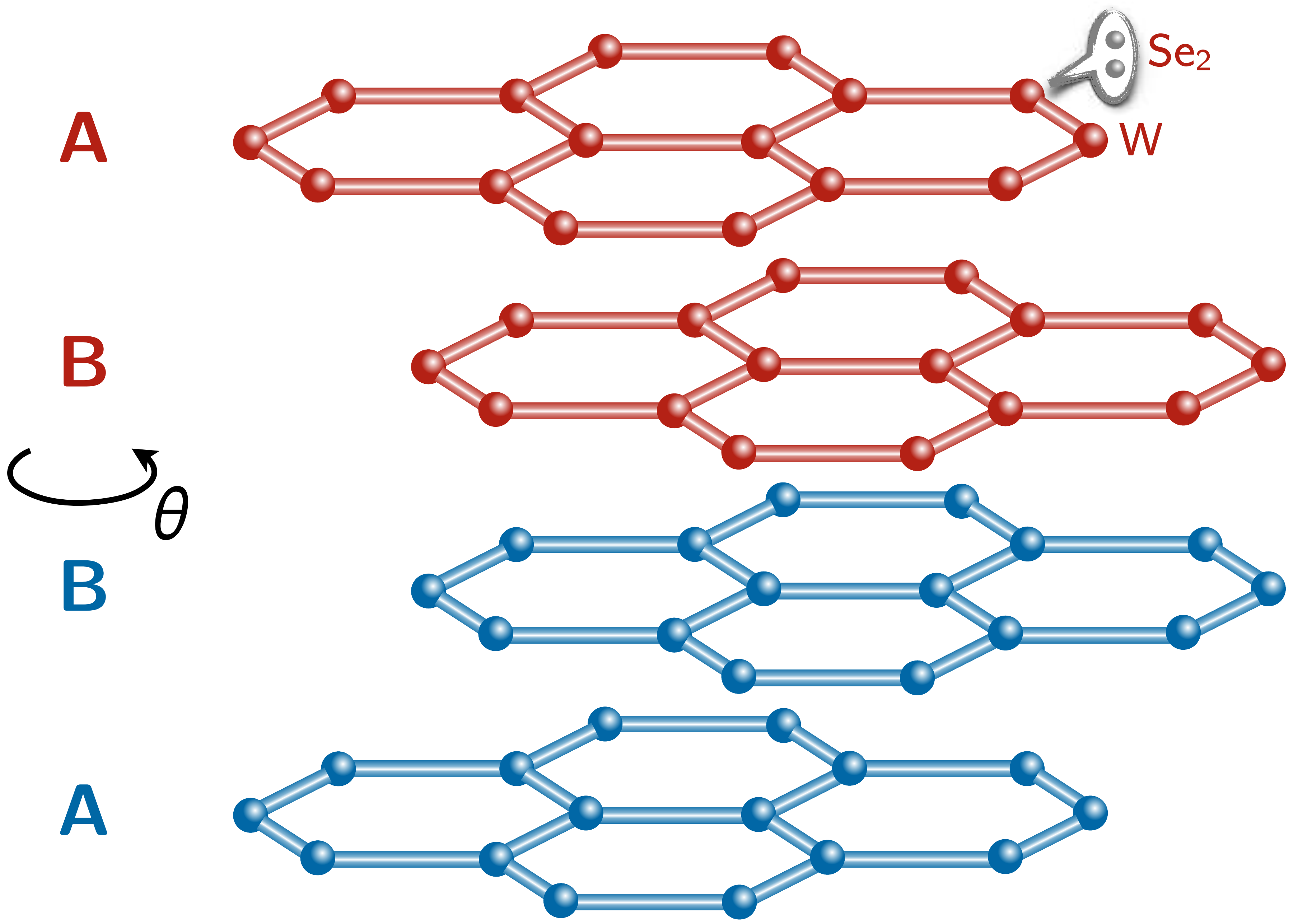

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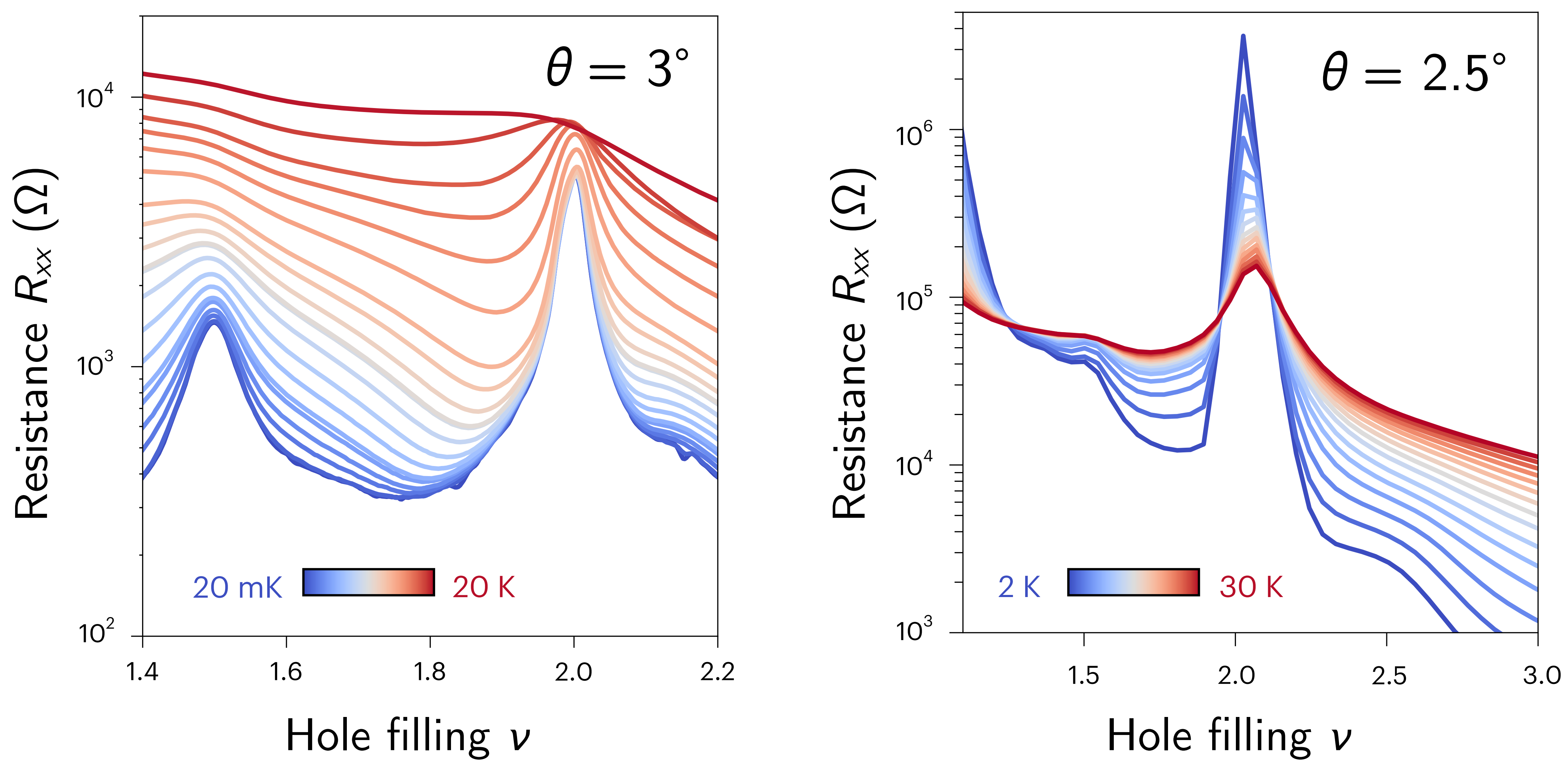
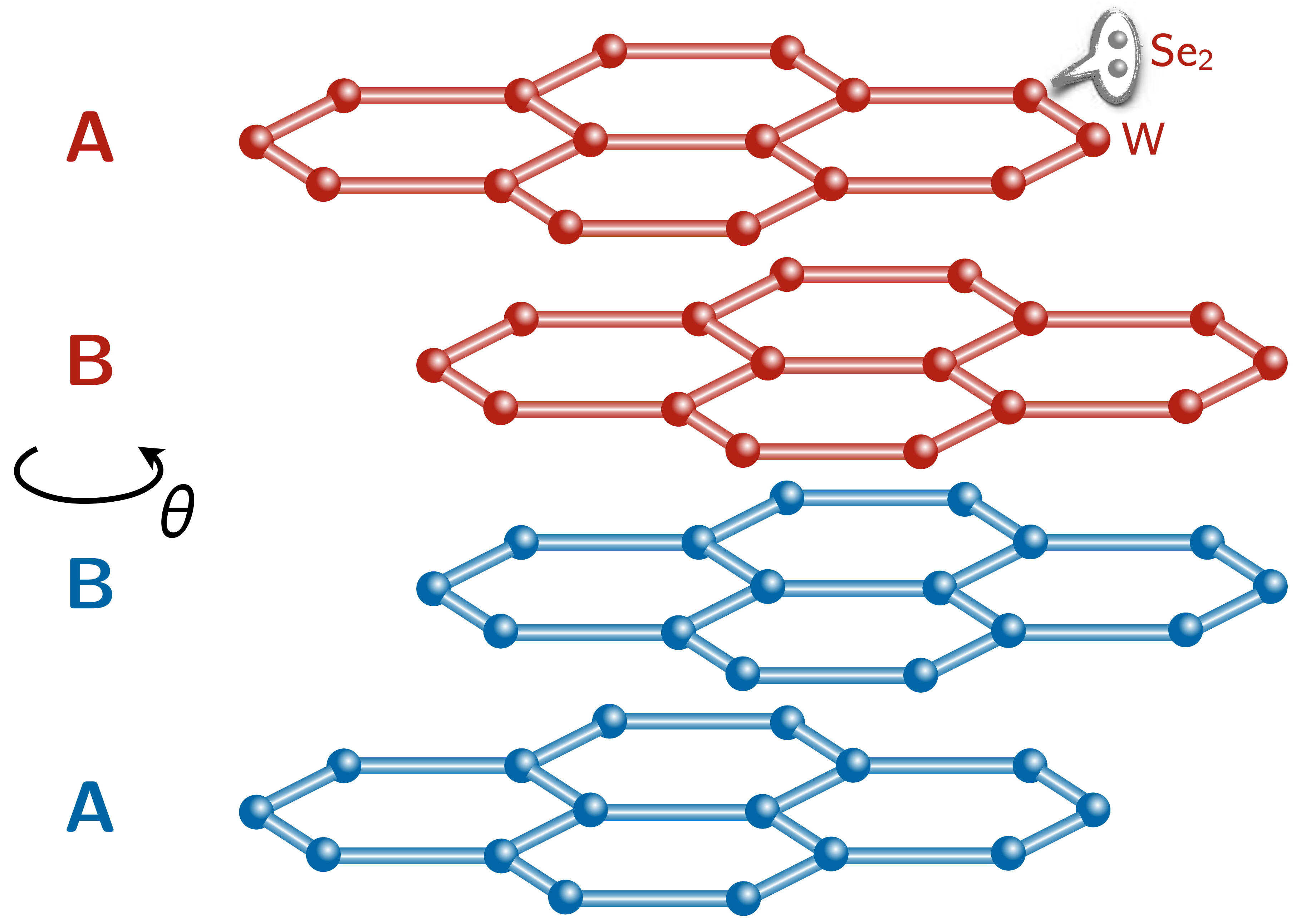
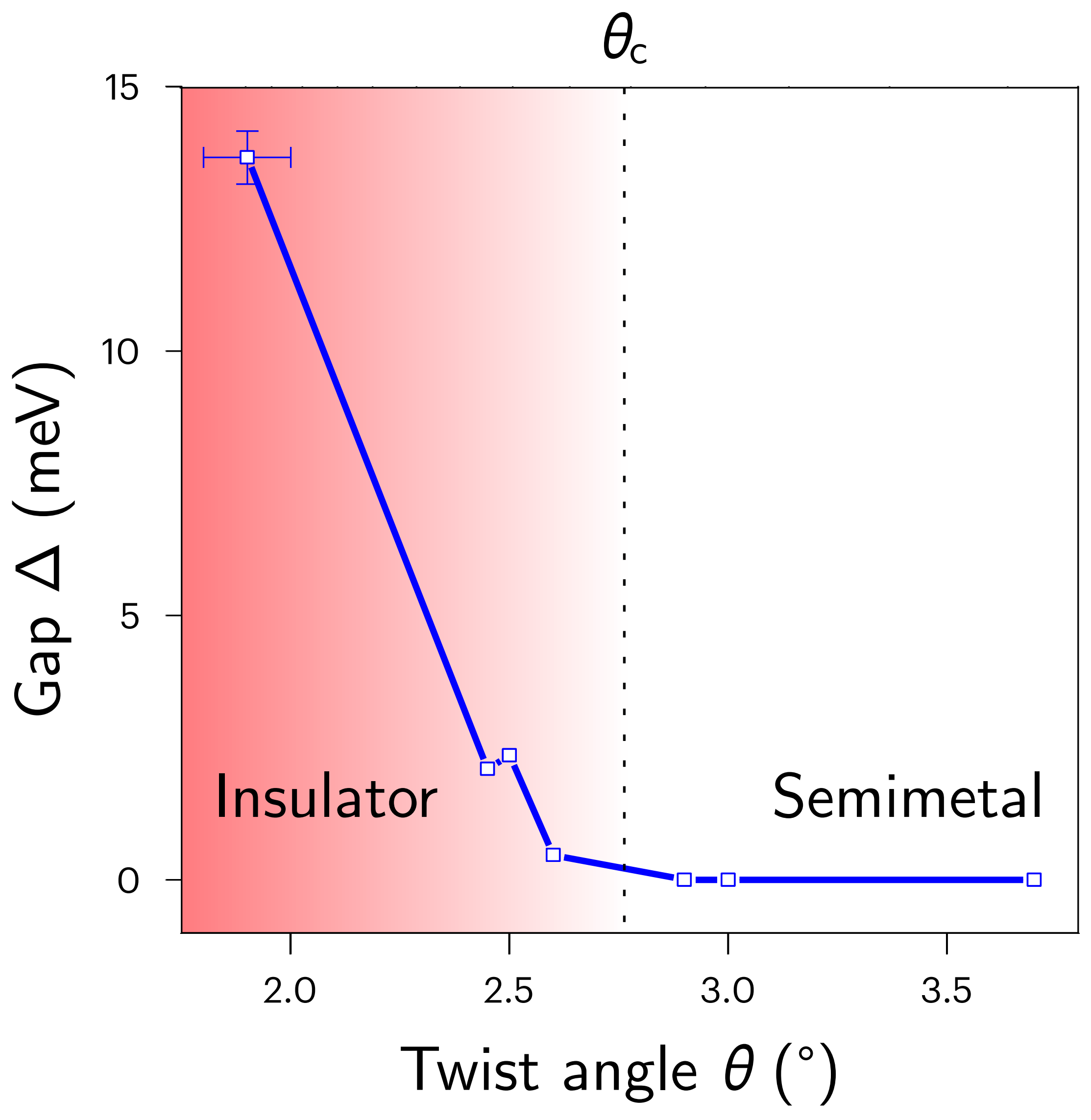




Semimetal-to-insulator transition:



… for $\nu = 2$

[Ma *et al.*, Nat. Mater. '25]

Twisted double bilayer WSe_2: Experiments

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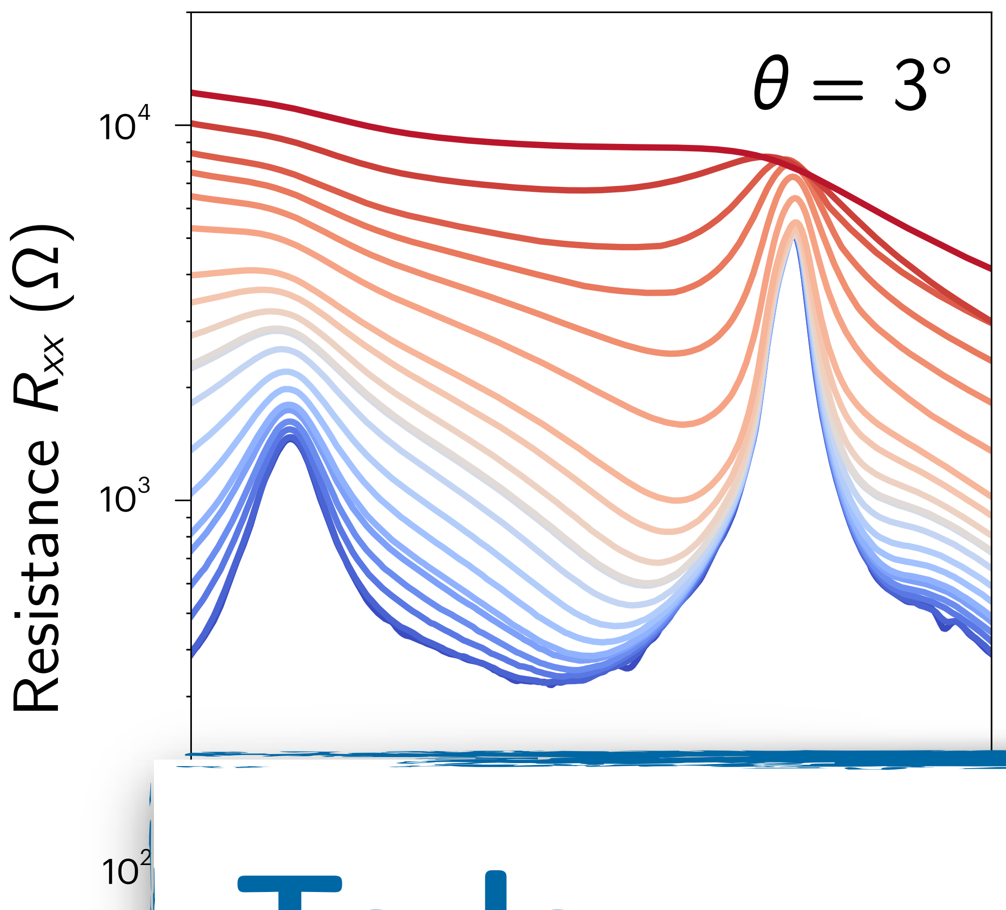
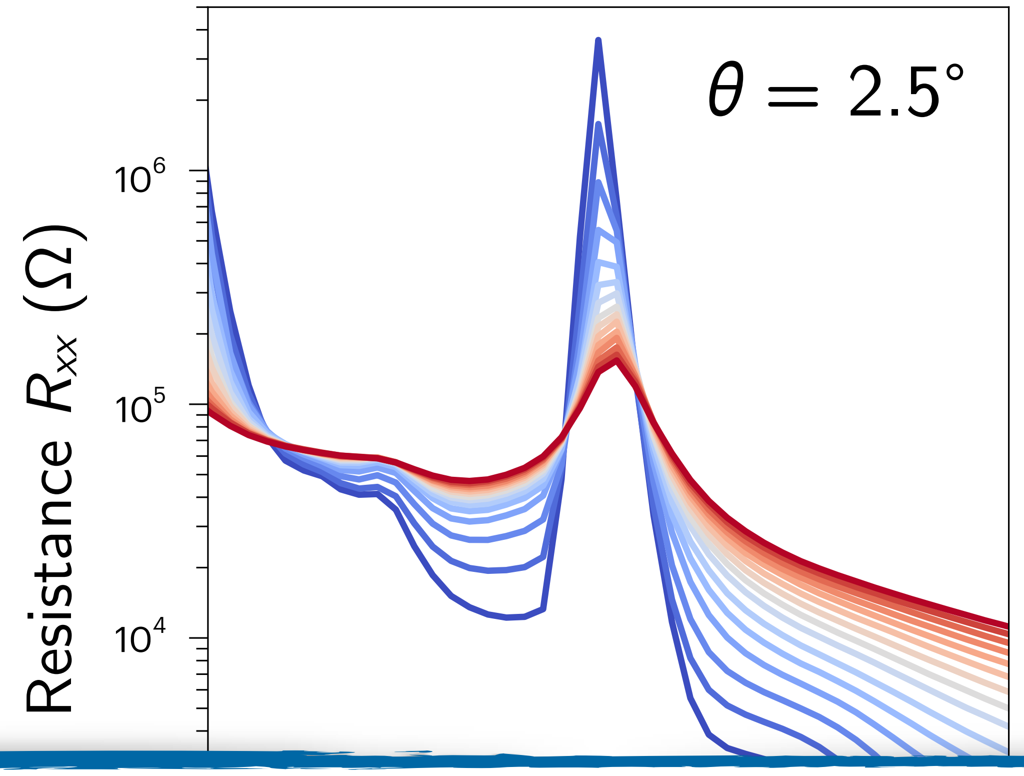
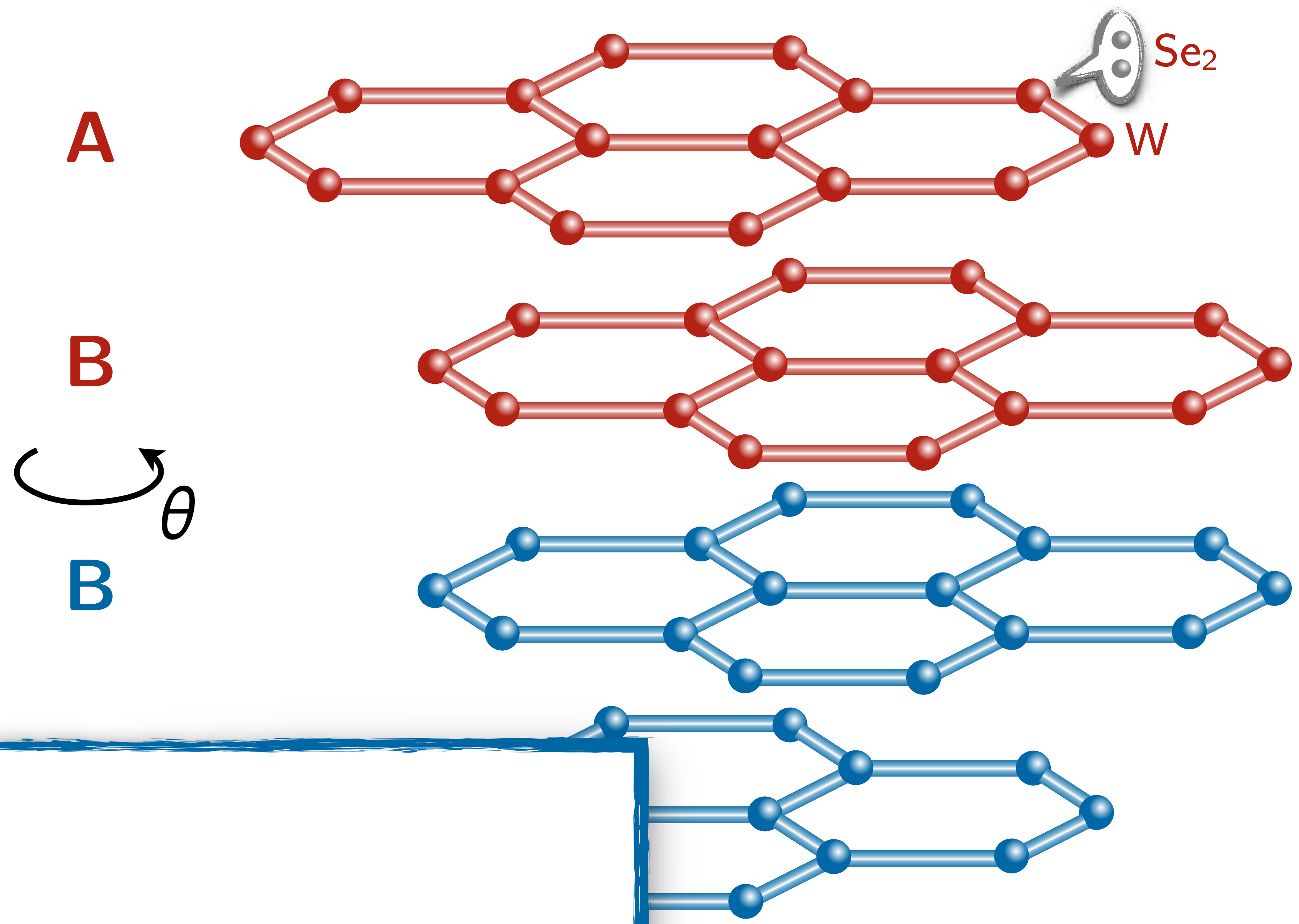






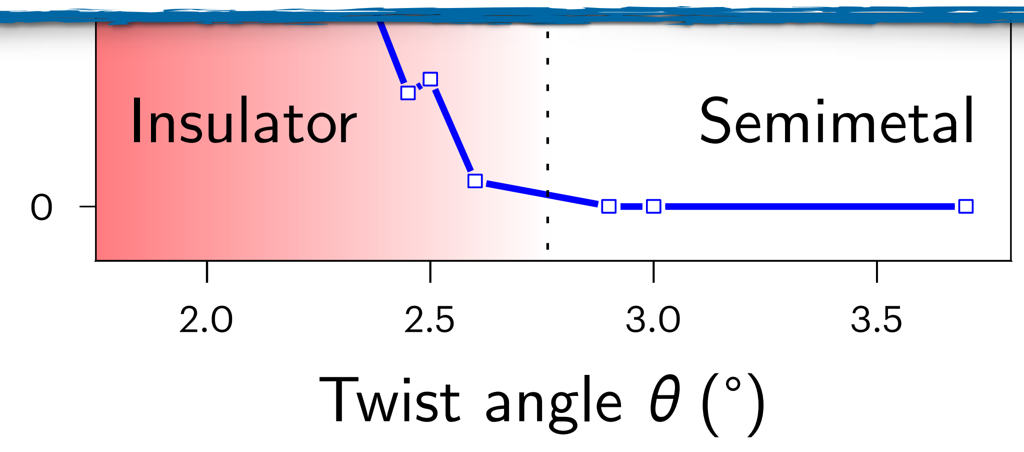
Semimetal-to-insulat

Today:

(1) Dirac semimetal ($\theta > 2.7°$)

(2) Antiferromagnetic insulator ($\theta < 2.7°$)

(3) Emergent Lorentz symmetry ($\theta \simeq 2.7°$)



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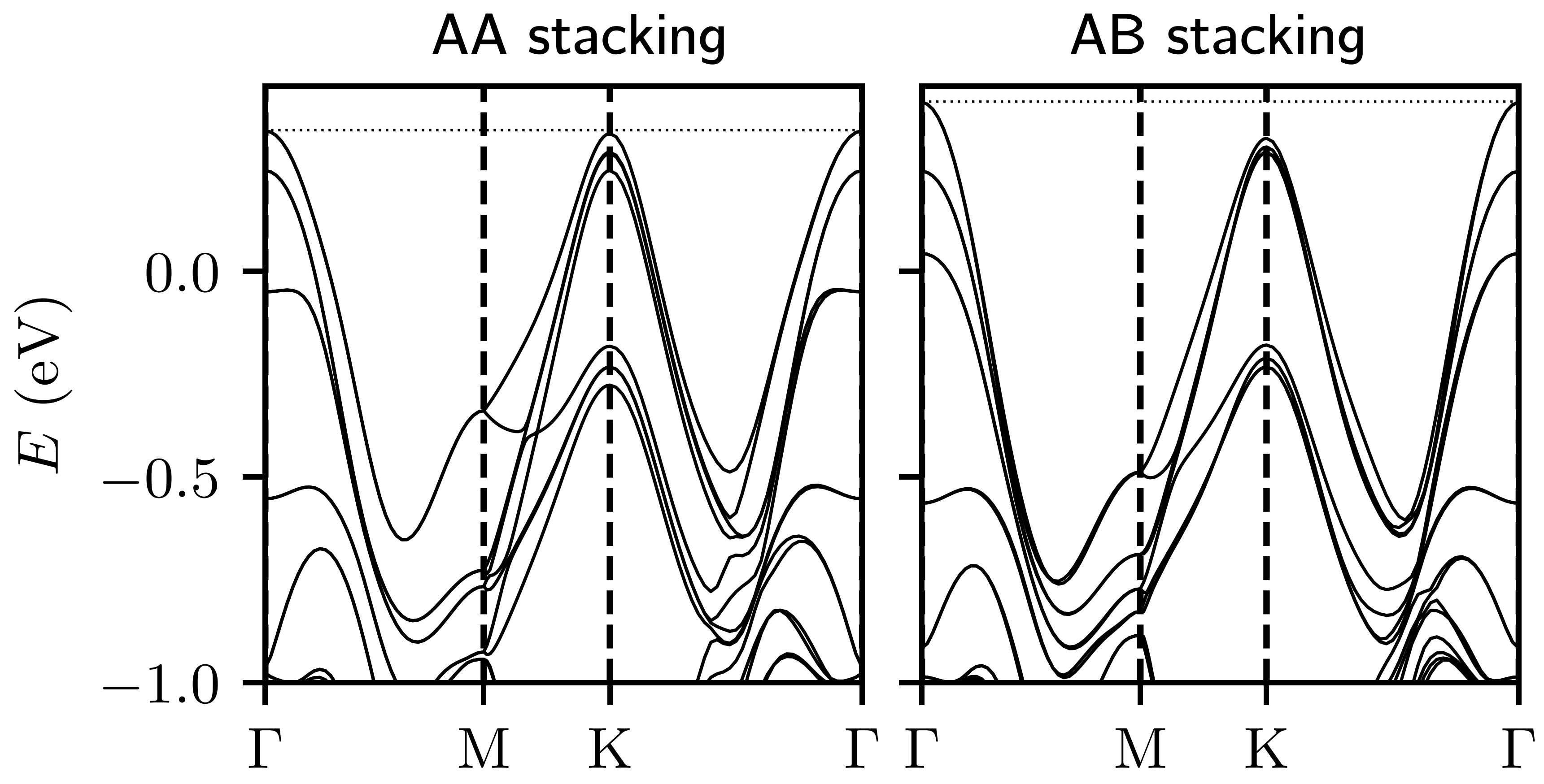
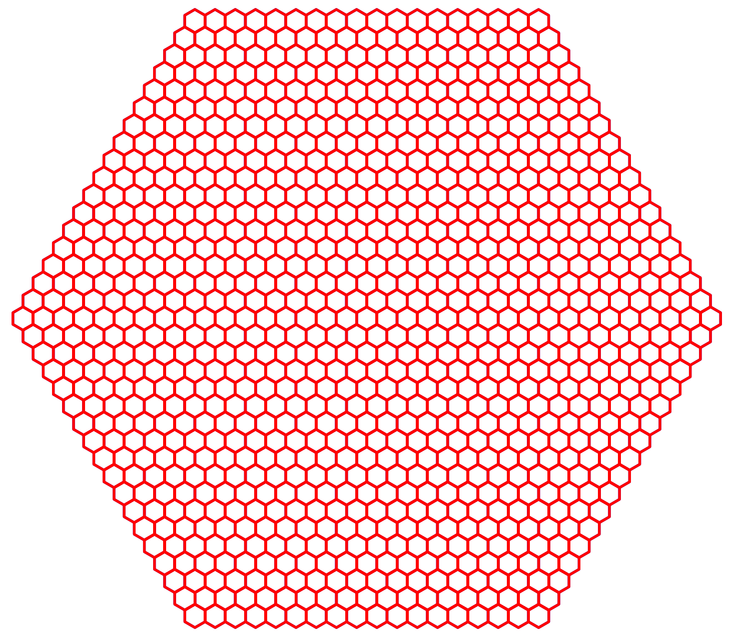
Twisted double bilayer WSe_2: Noninteracting spectrum

Untwisted spectrum:





[Pan *et al.*, PRRes ’23]

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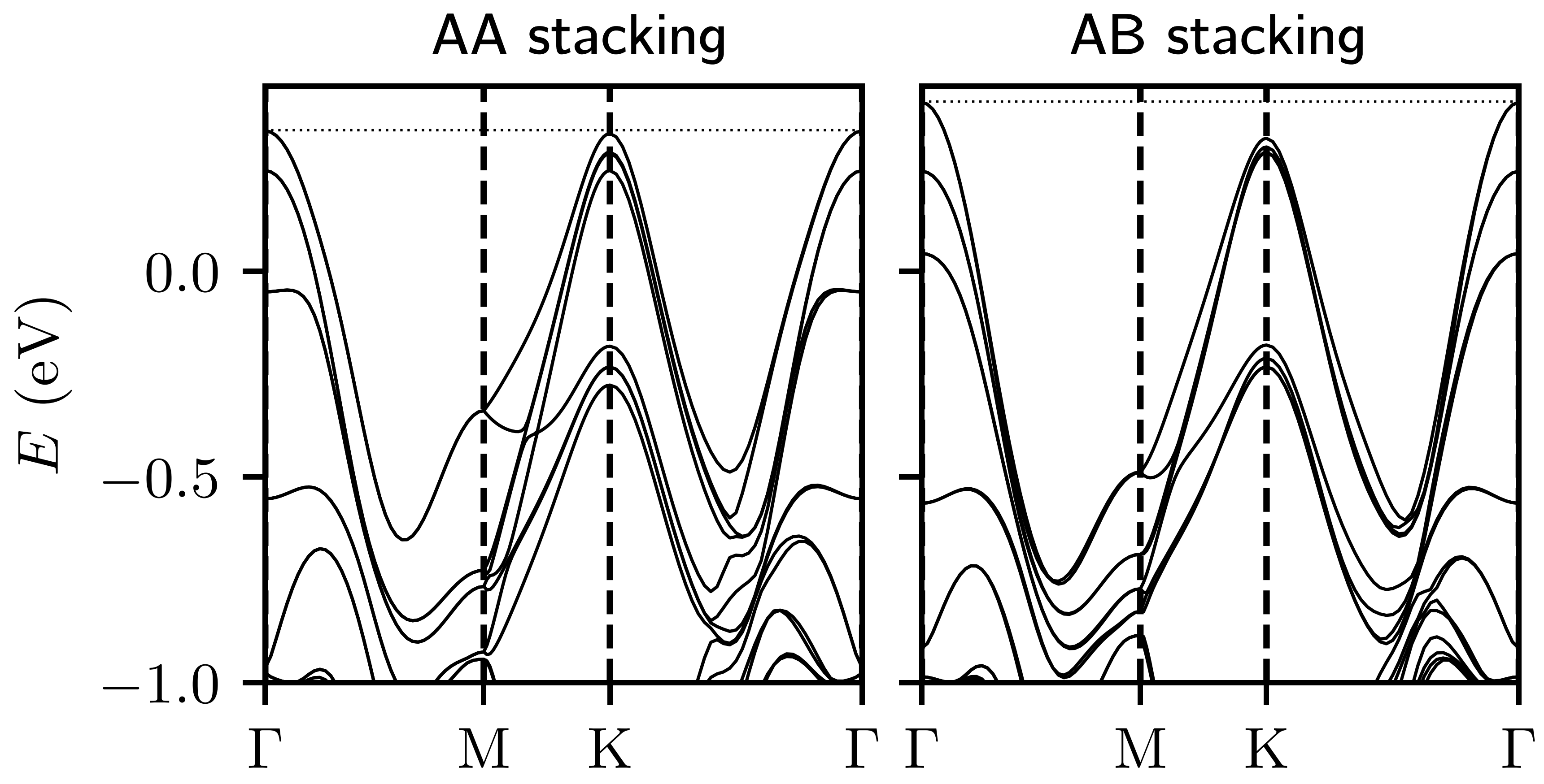

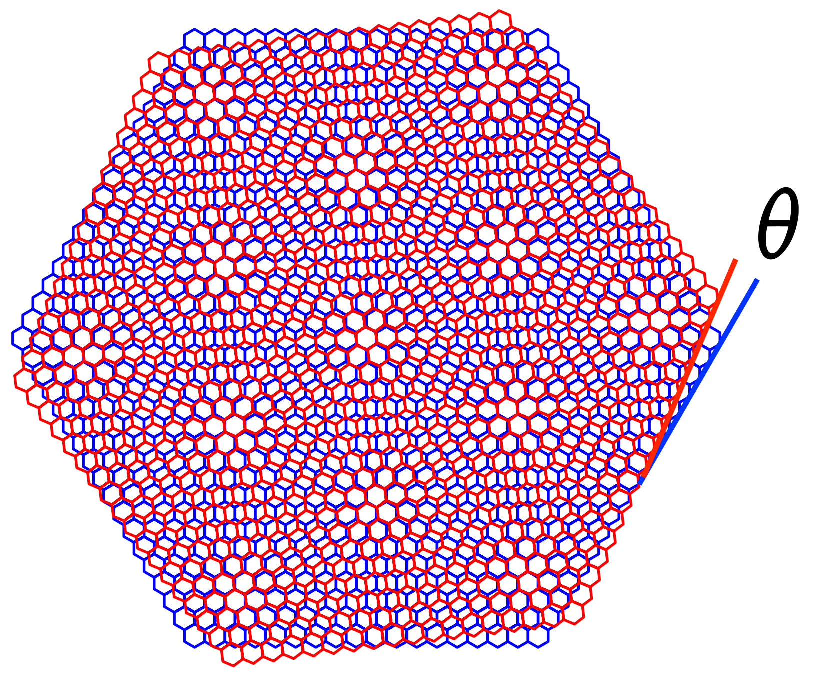

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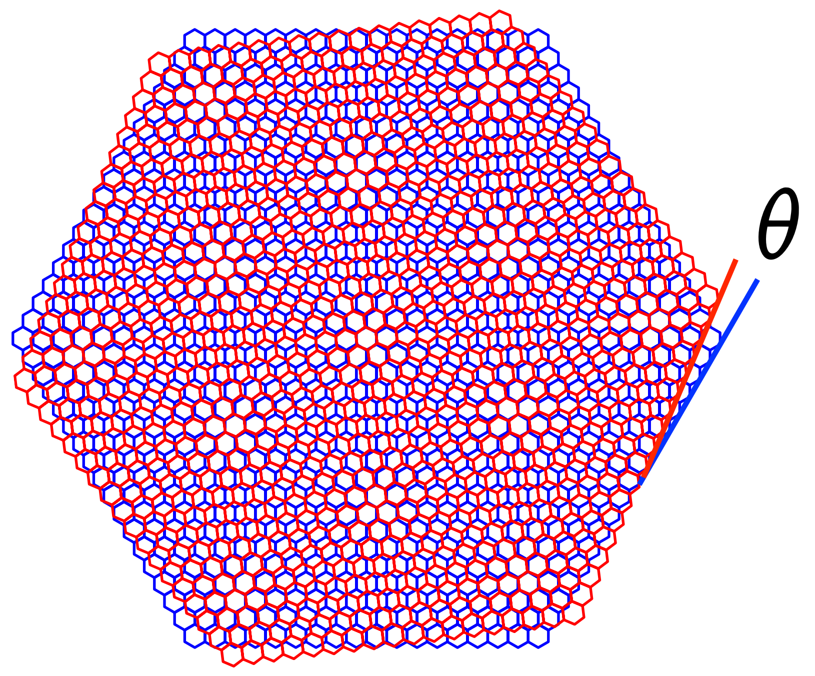

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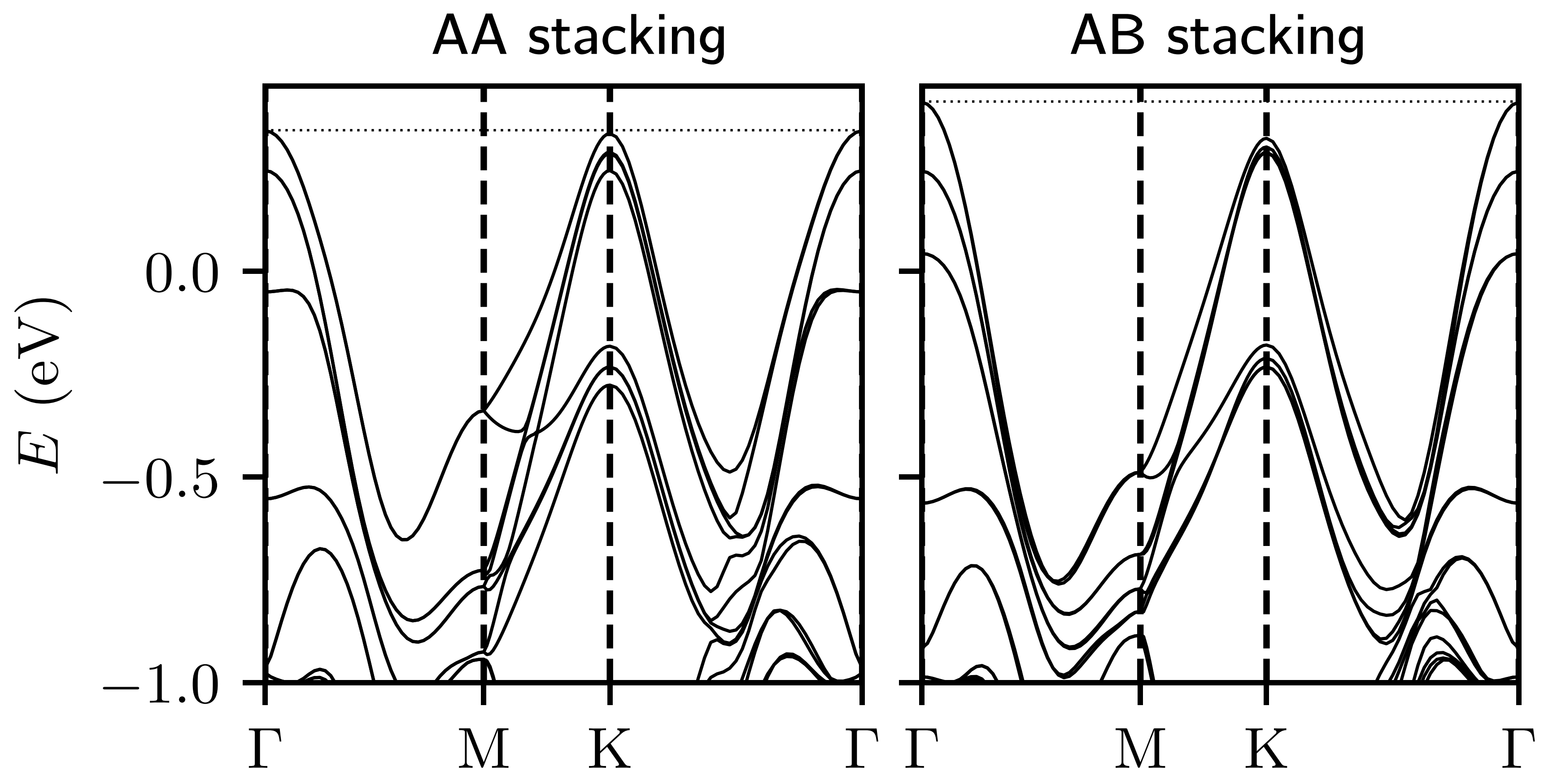

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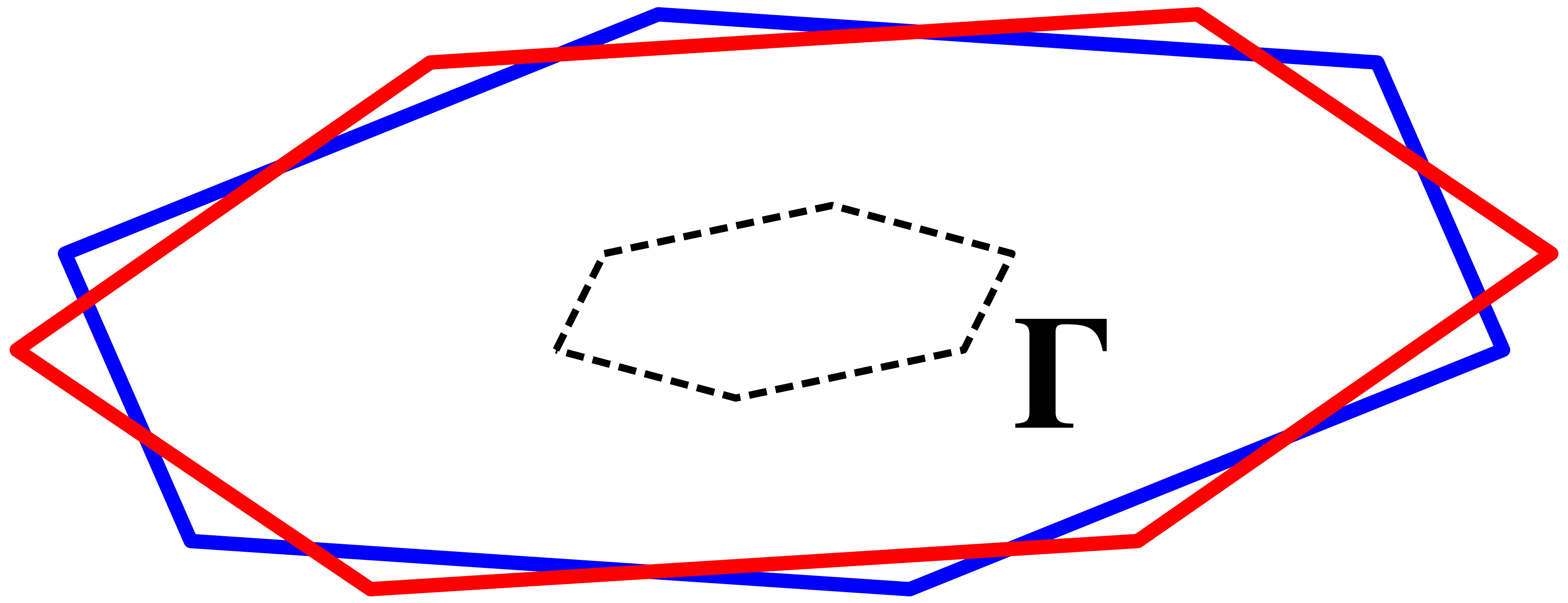
Moiré Brillouin zone:



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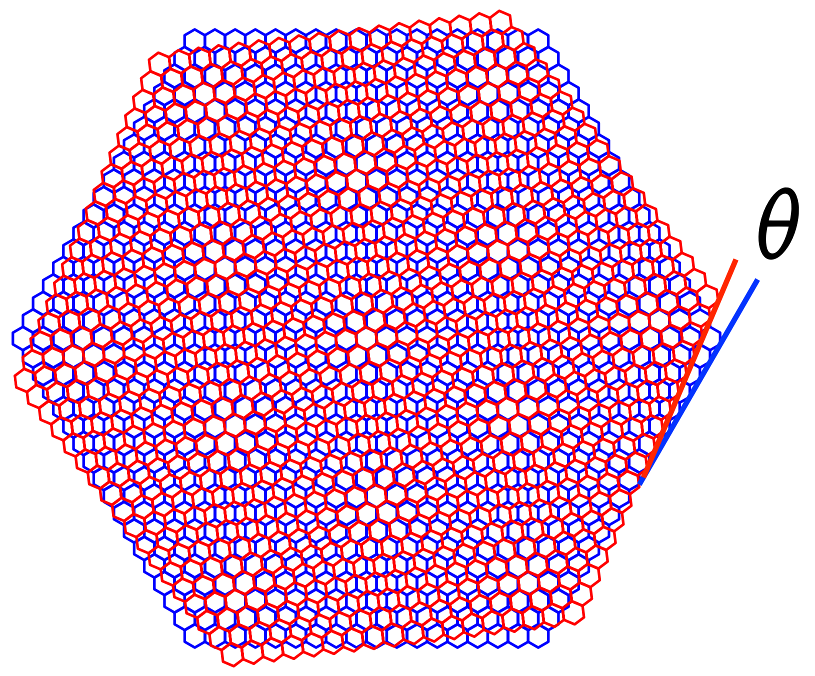

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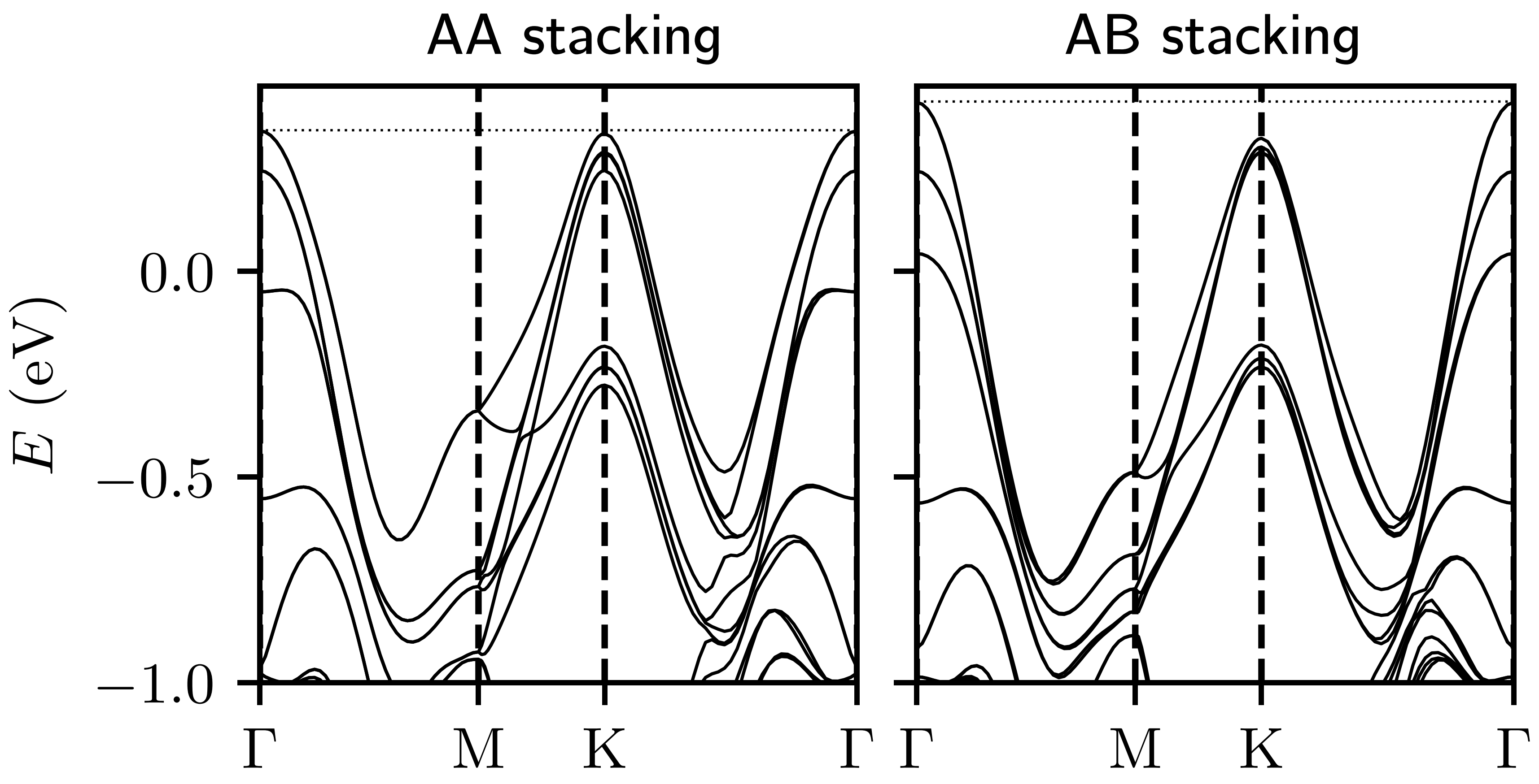

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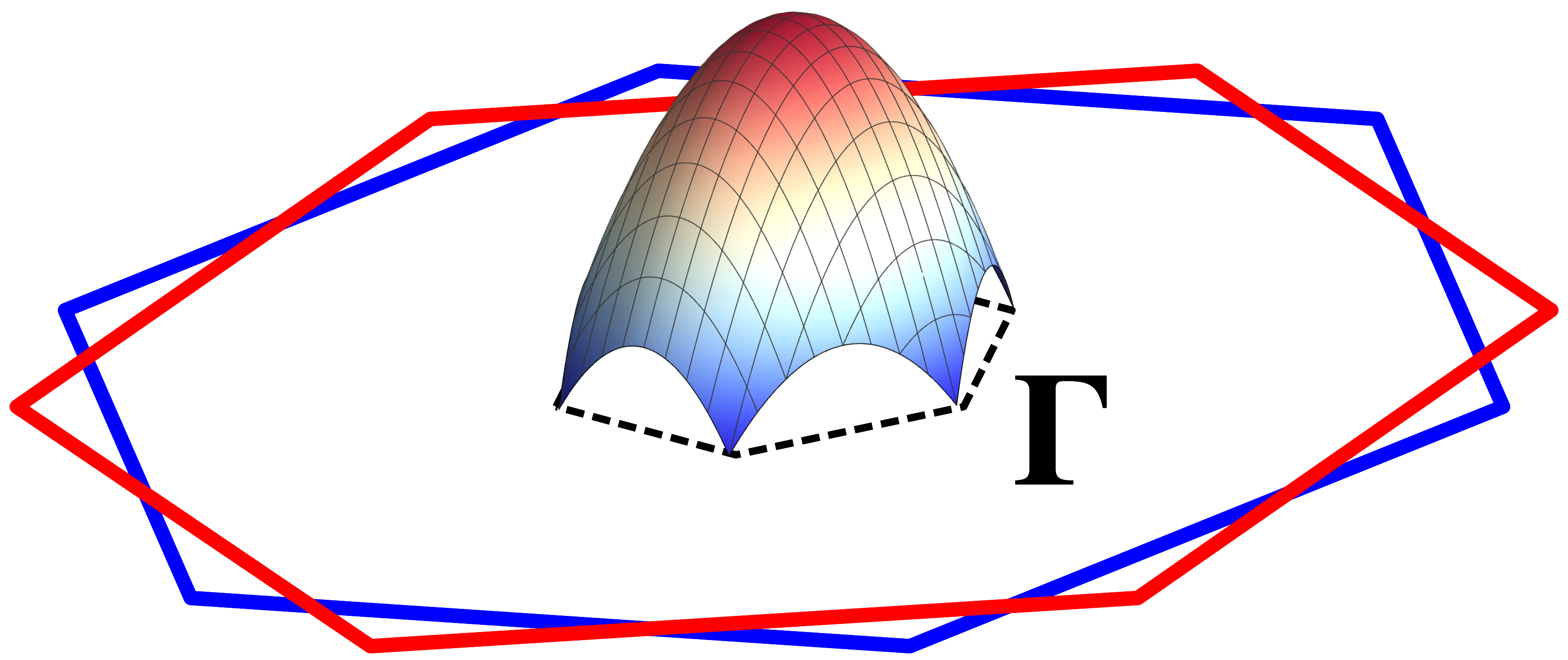
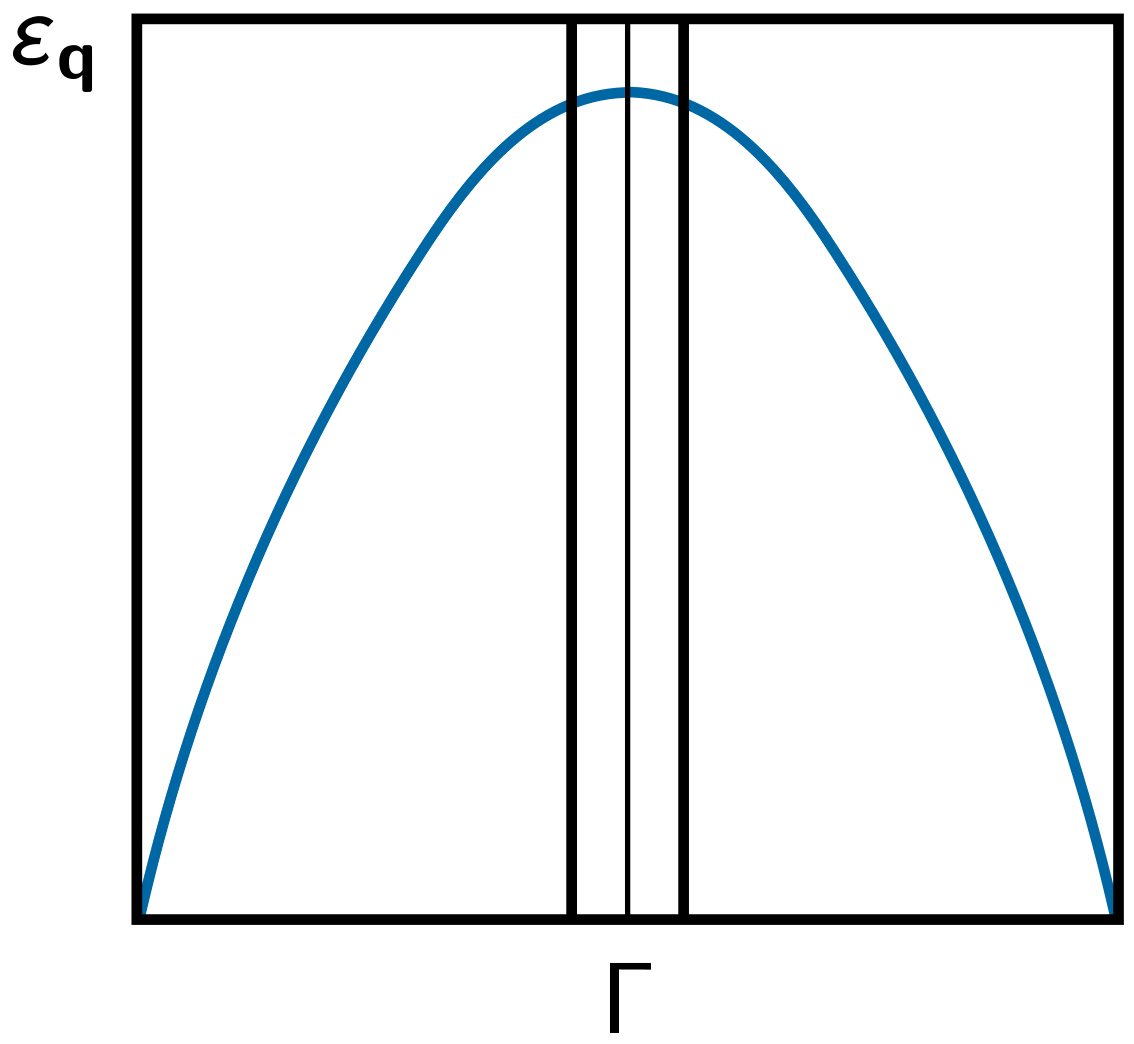
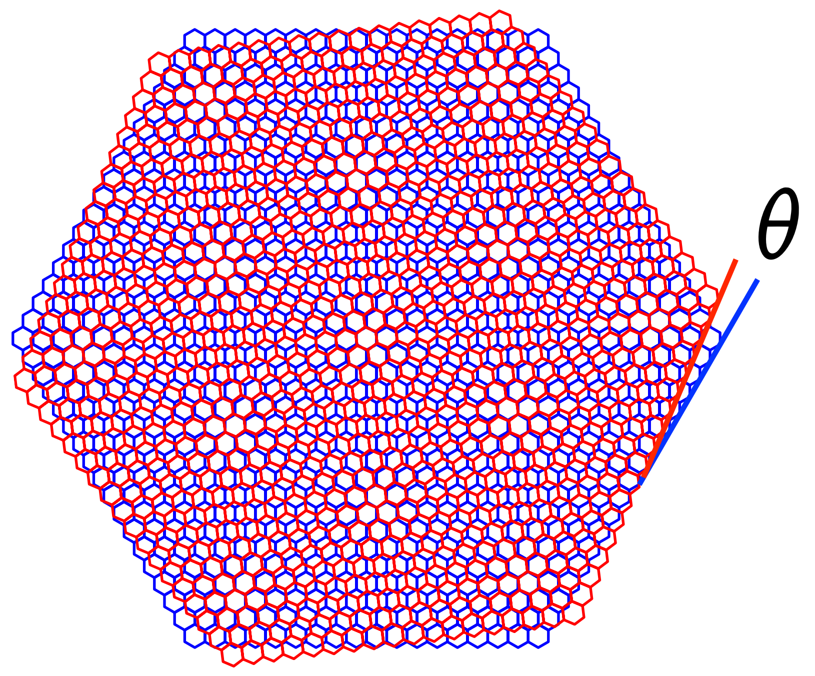


Twisted spectrum:



$$\varepsilon_{\mathbf{q}} = -\frac{\mathbf{q}^2}{2m^*} + \mathcal{O}(q^3)$$

Twisted double bilayer WSe_2: Noninteracting spectrum



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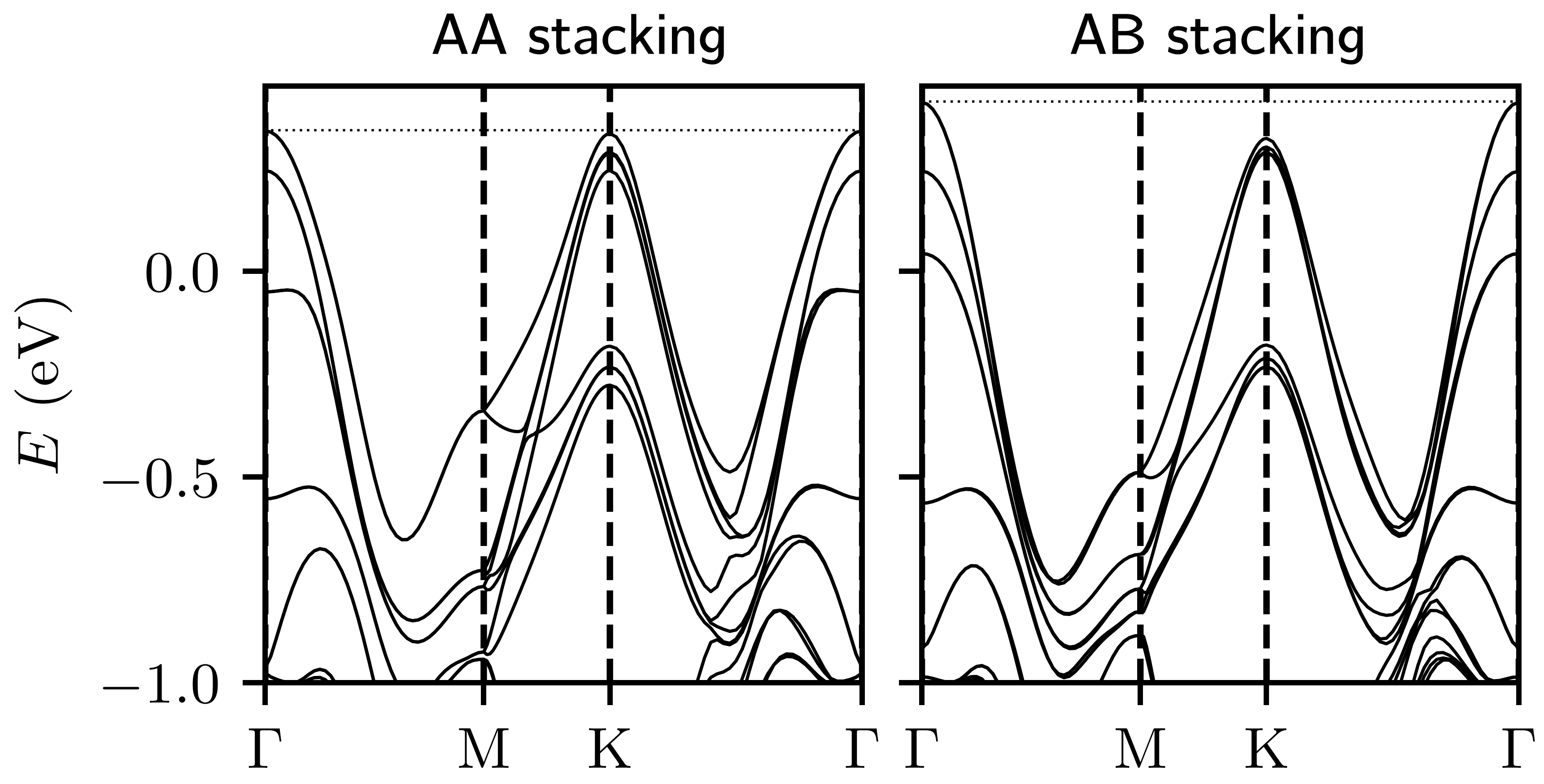

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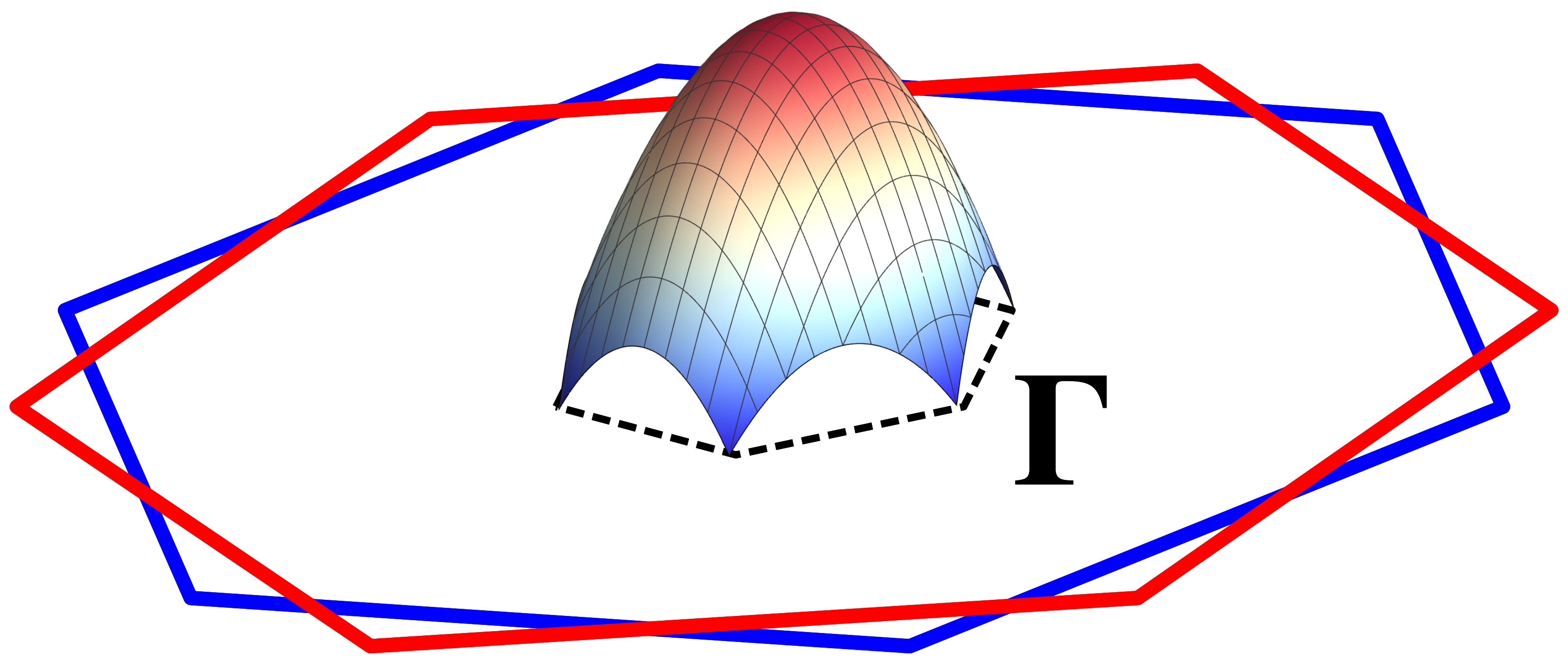

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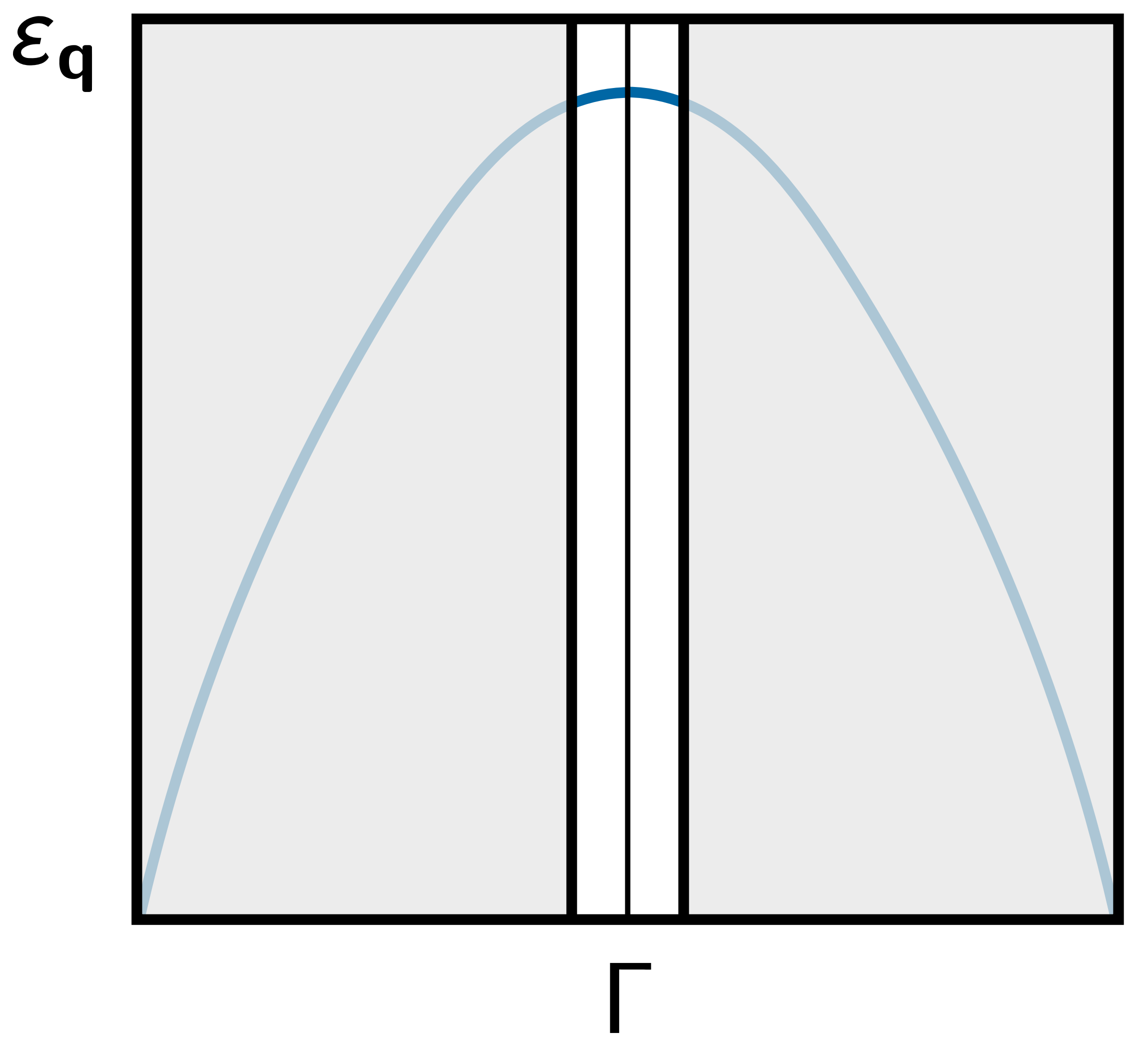

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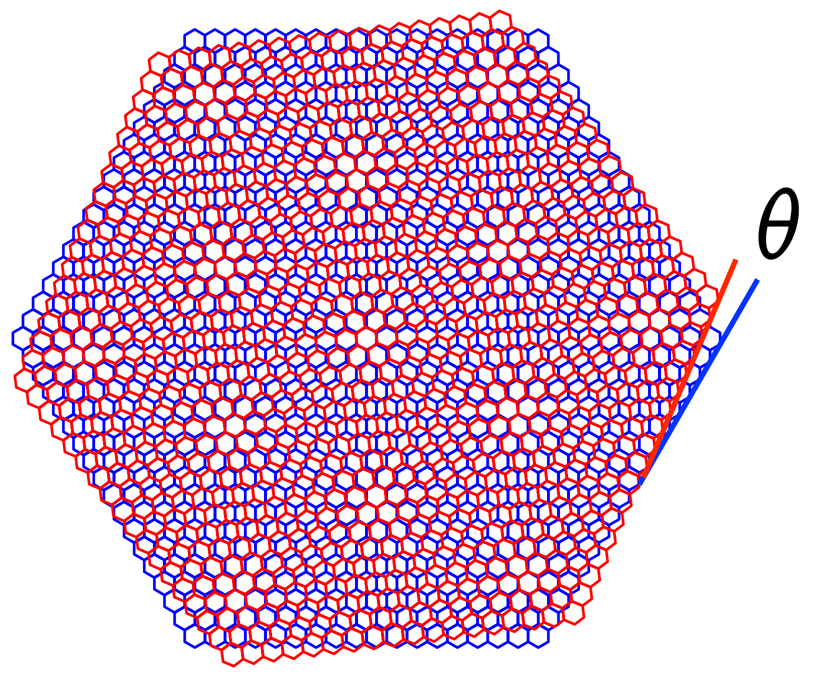

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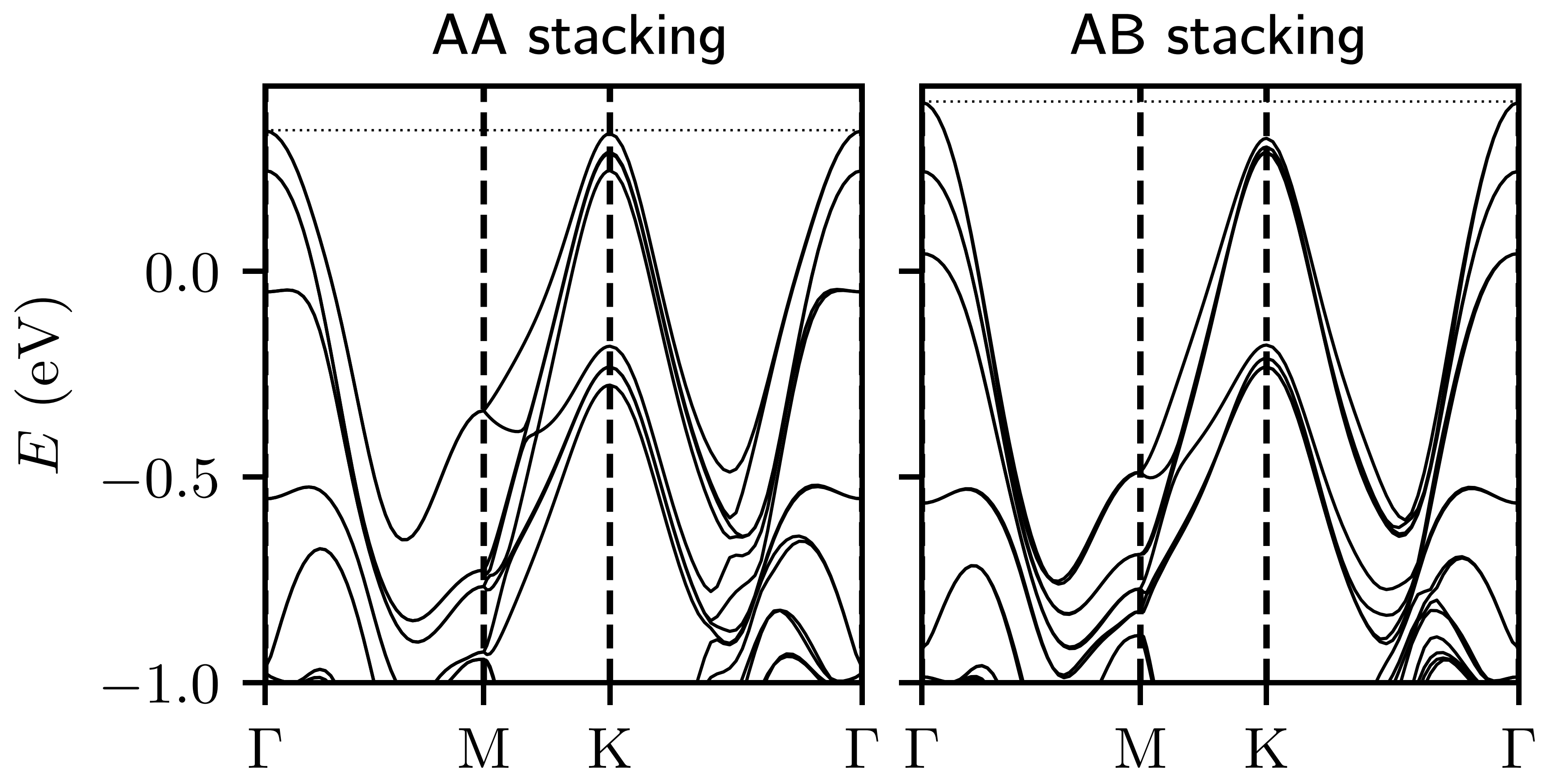

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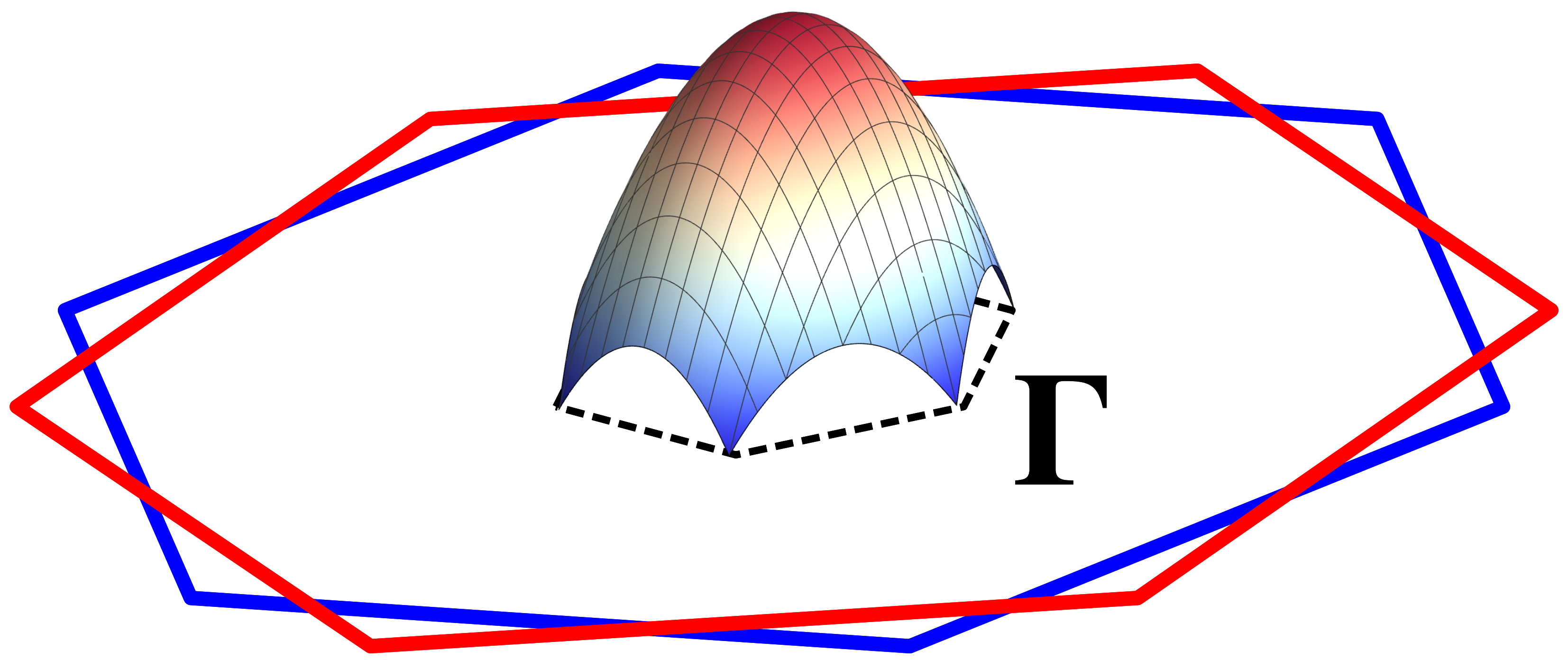

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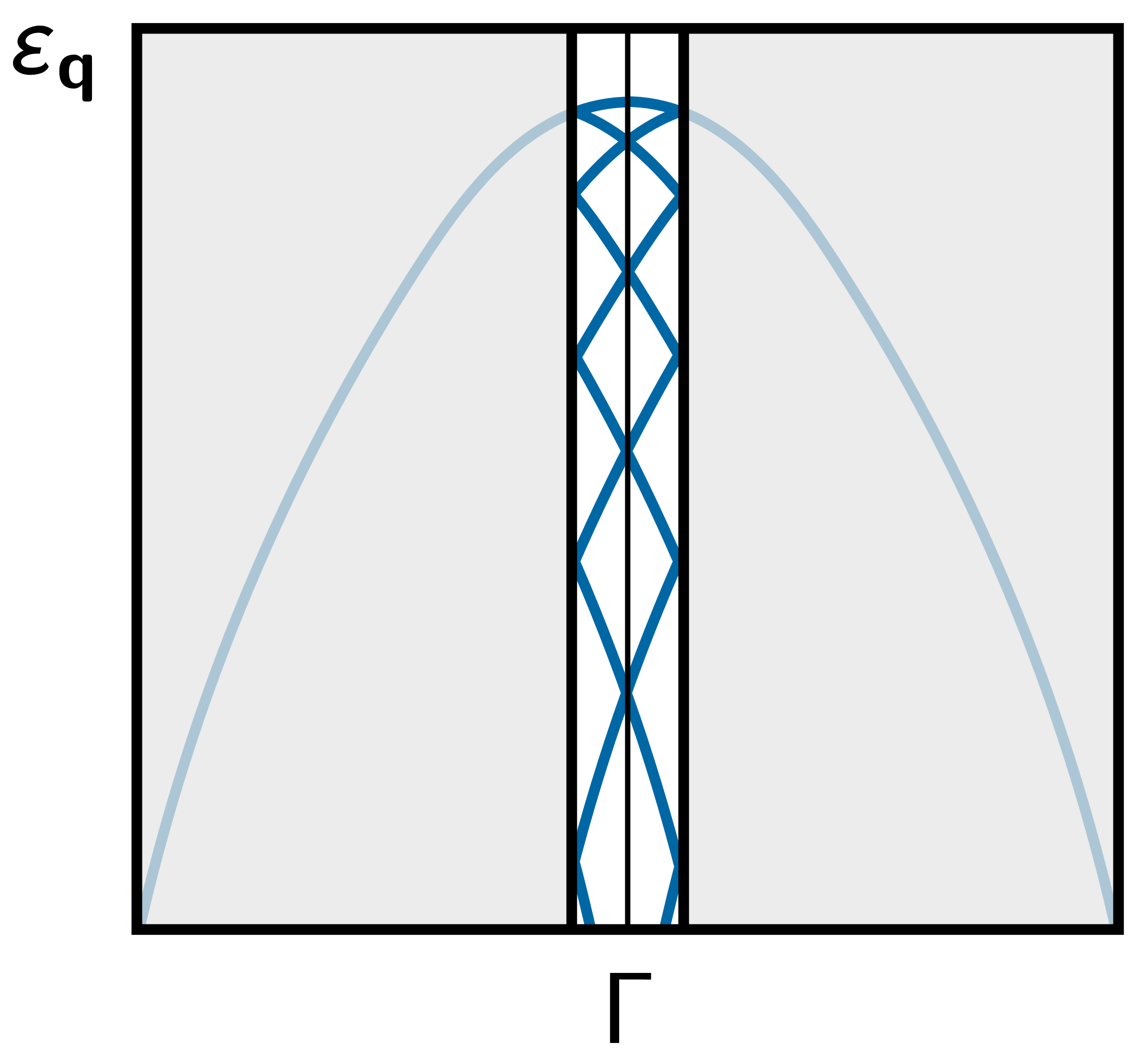

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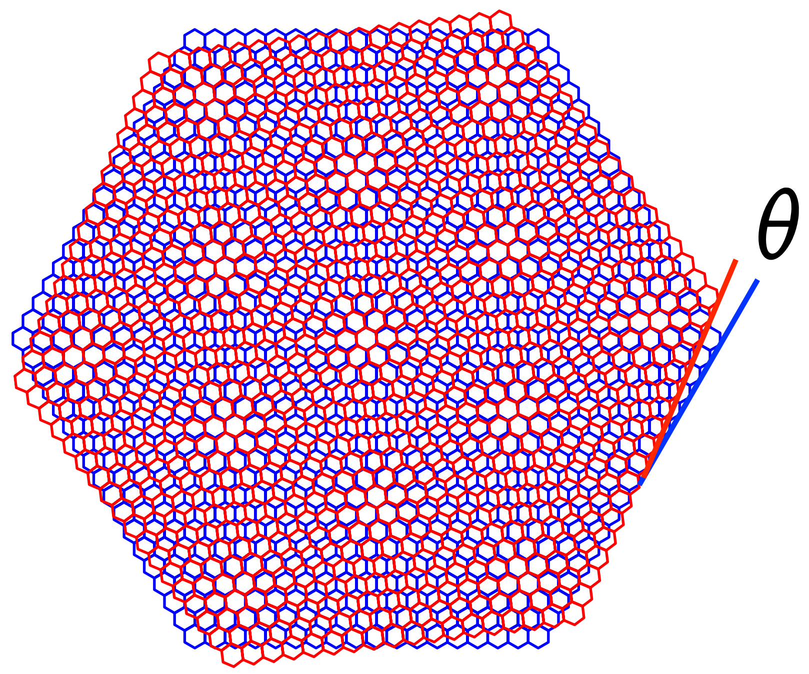
→ Talk by M. Ünzelmann

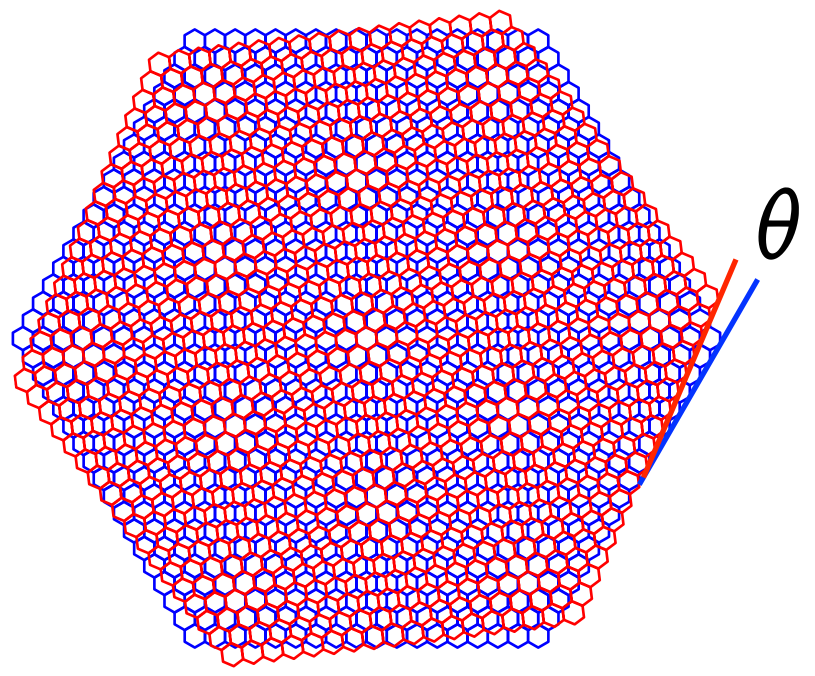
Twisted double bilayer WSe_2: Theory



Interacting Bistritzer-MacDonald model:

$$\mathcal{H} = \sum_{\mathbf{k}\in \mathrm{mBZ}} c_{\mathbf{k}}^{\dagger} h(\mathbf{k}) c_{\mathbf{k}} - \frac{1}{2A} \sum_{\mathbf{q}} V_{\mathbf{q}} :\rho_{\mathbf{q}} \rho_{-\mathbf{q}}:$$

Twisted double bilayer WSe_2: Theory



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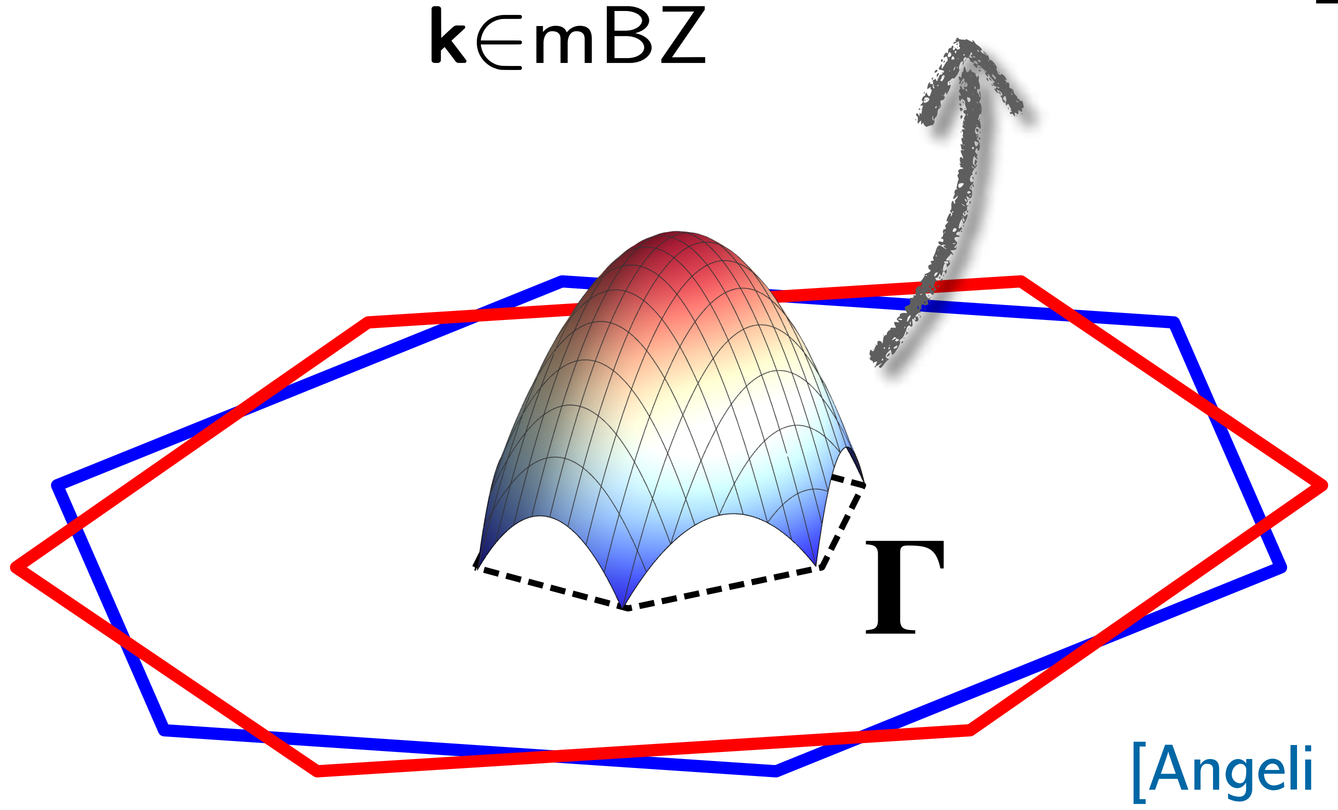
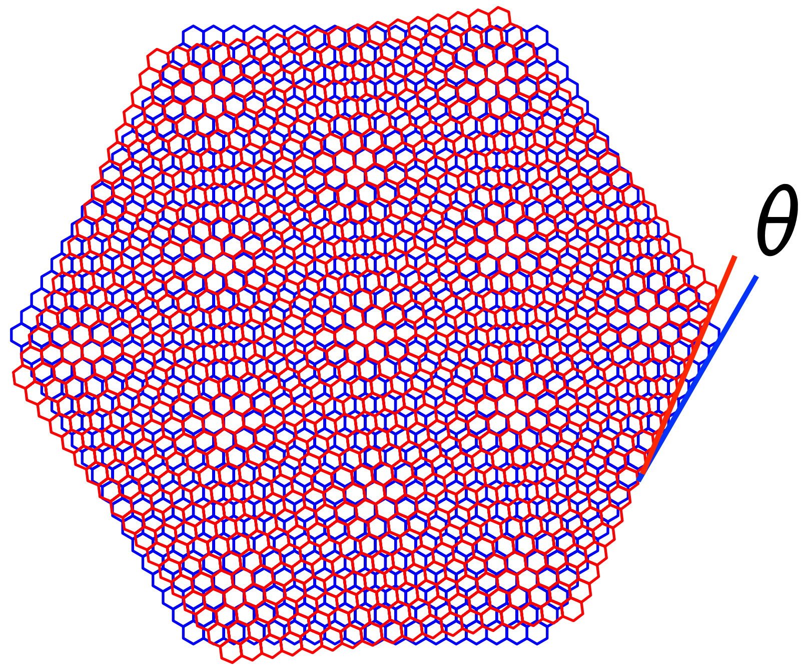


[Angeli & MacDonald, PNAS '21]

Twisted double bilayer WSe_2: Theory



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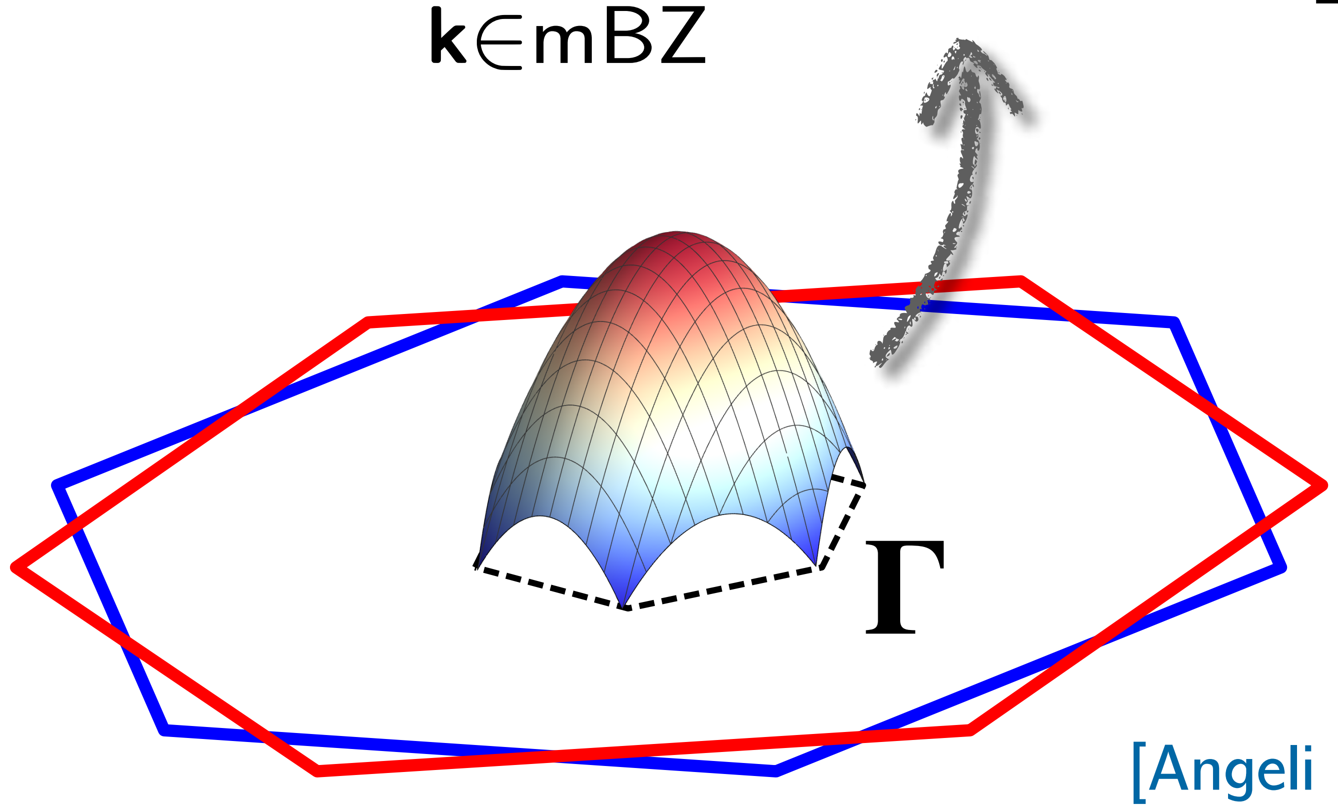
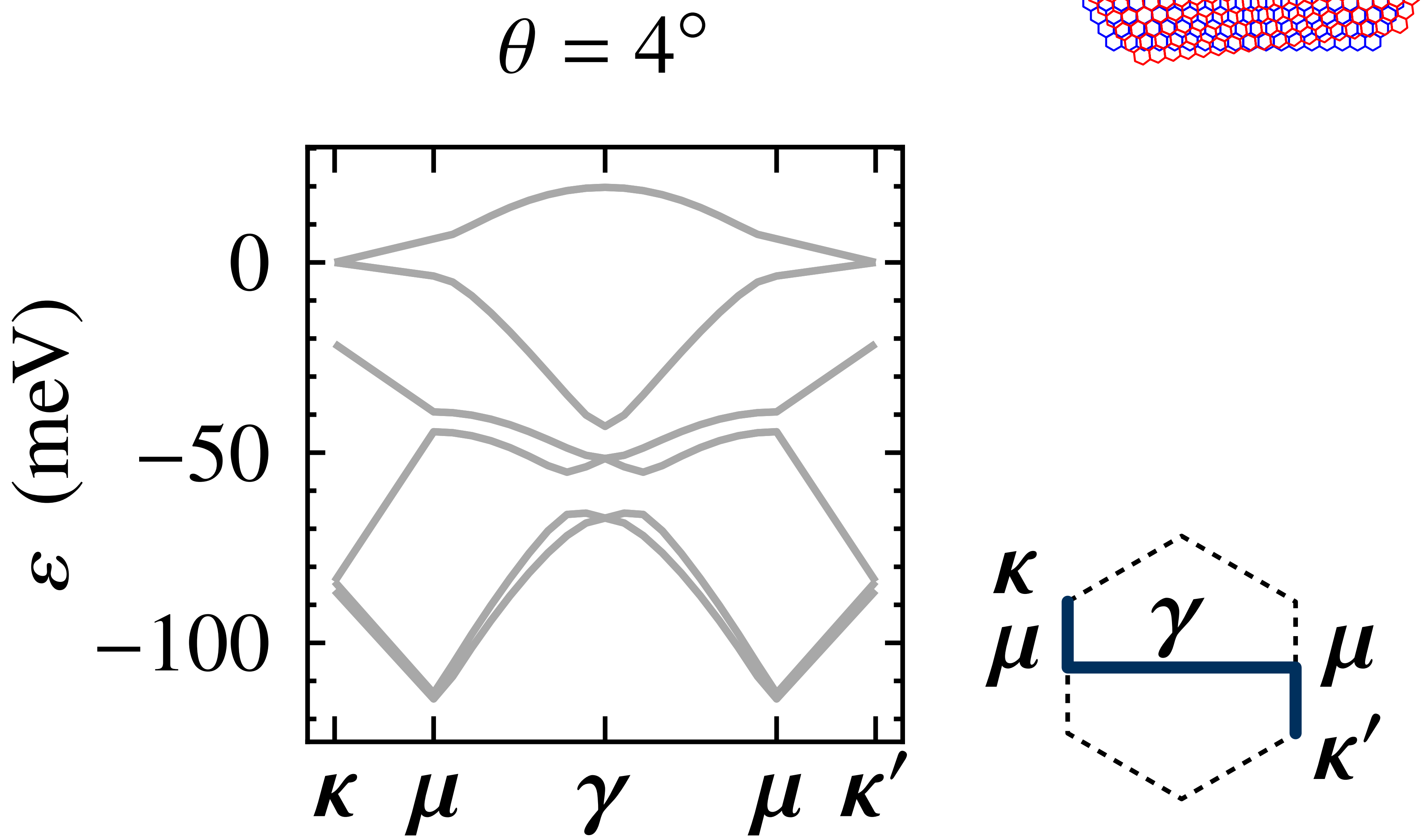
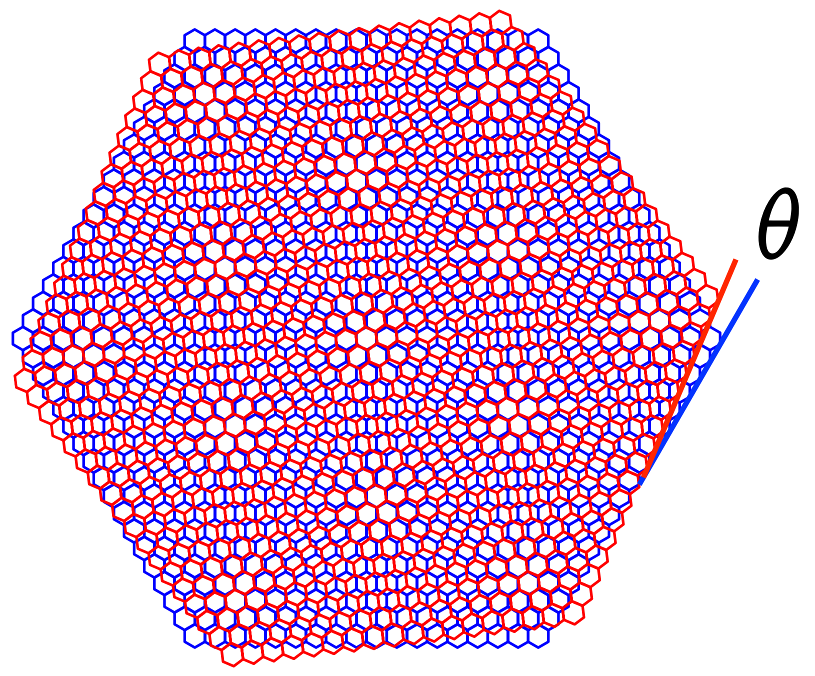


[Angeli & MacDonald, PNAS '21]

Noninteracting spectrum:

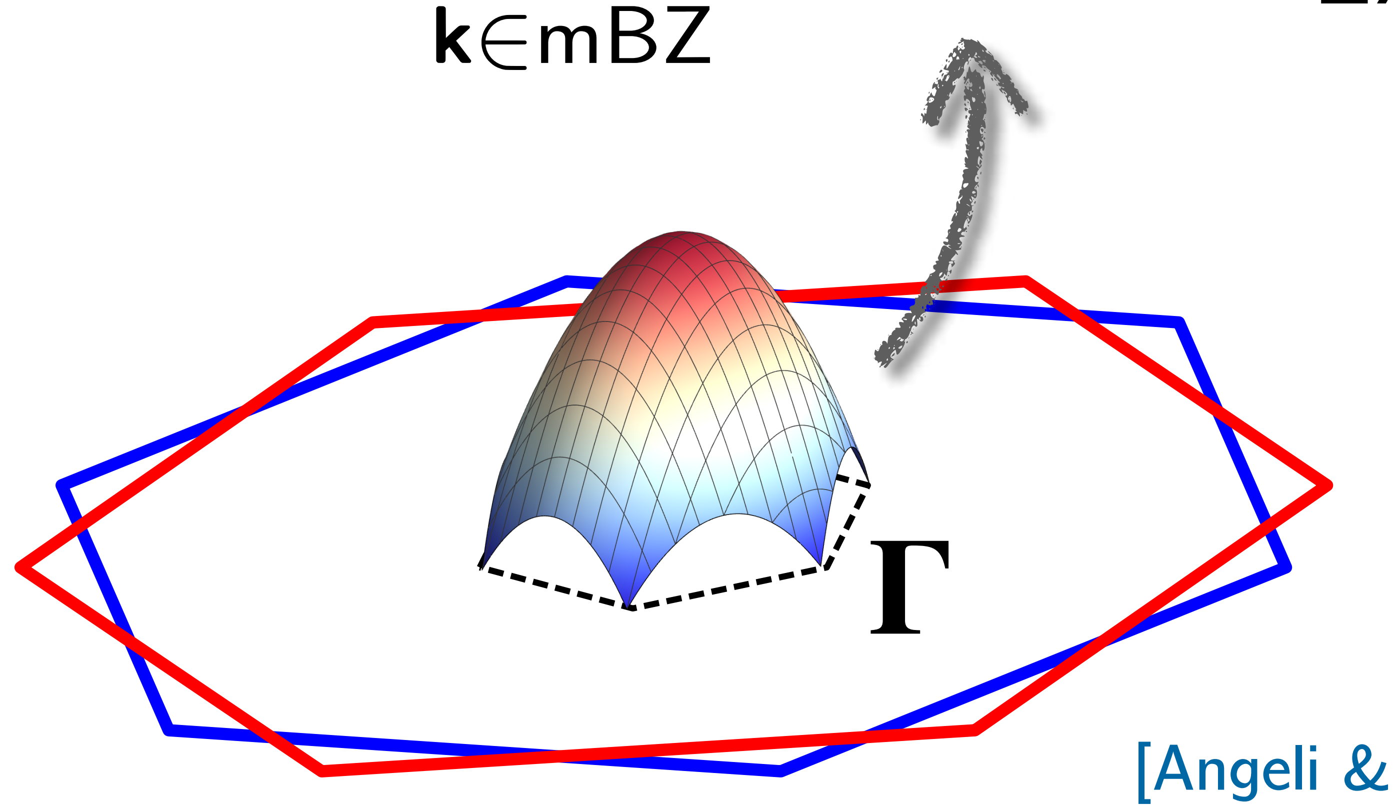


Twisted double bilayer WSe_2: Theory



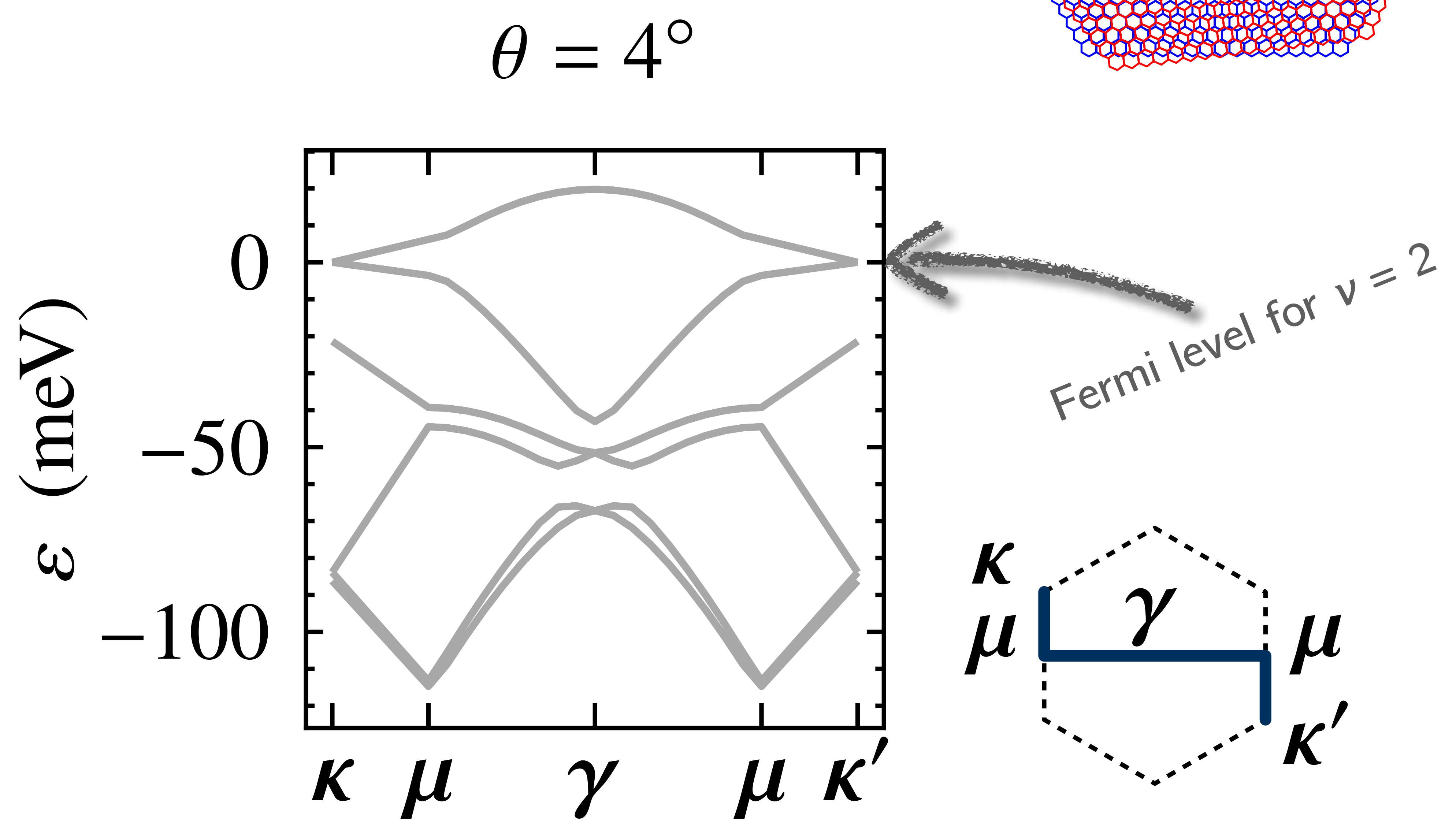
Interacting Bistritzer-MacDonald model:

$$\mathcal{H} = \sum_{\mathbf{k}\in \text{mBZ}} c^\dagger_{\mathbf{k}} h(\mathbf{k}) c_{\mathbf{k}} - \frac{1}{2A}\sum_{\mathbf{q}} V_{\mathbf{q}} :\rho_{\mathbf{q}}\rho_{-\mathbf{q}}:$$

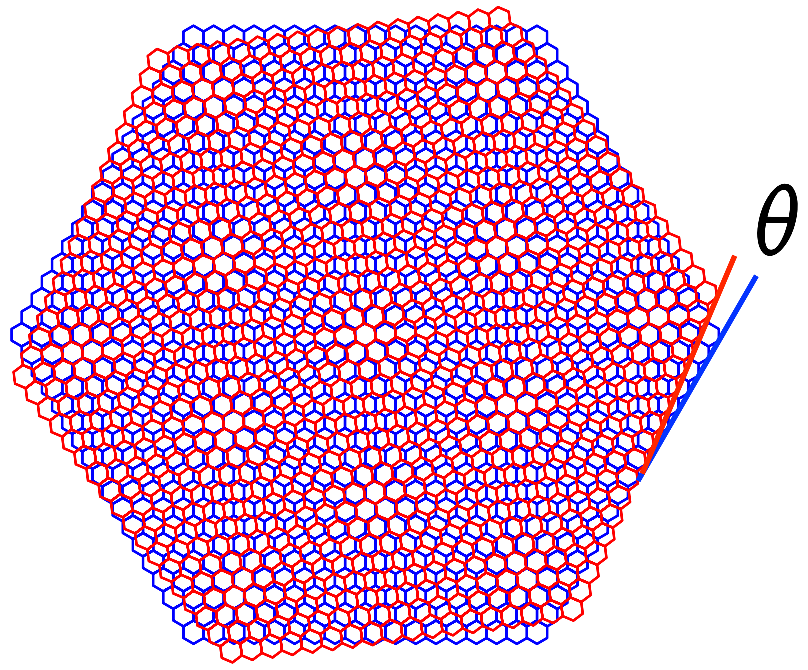


[Angeli & MacDonald, PNAS '21]

Noninteracting spectrum:



Twisted double bilayer WSe_2: Theory



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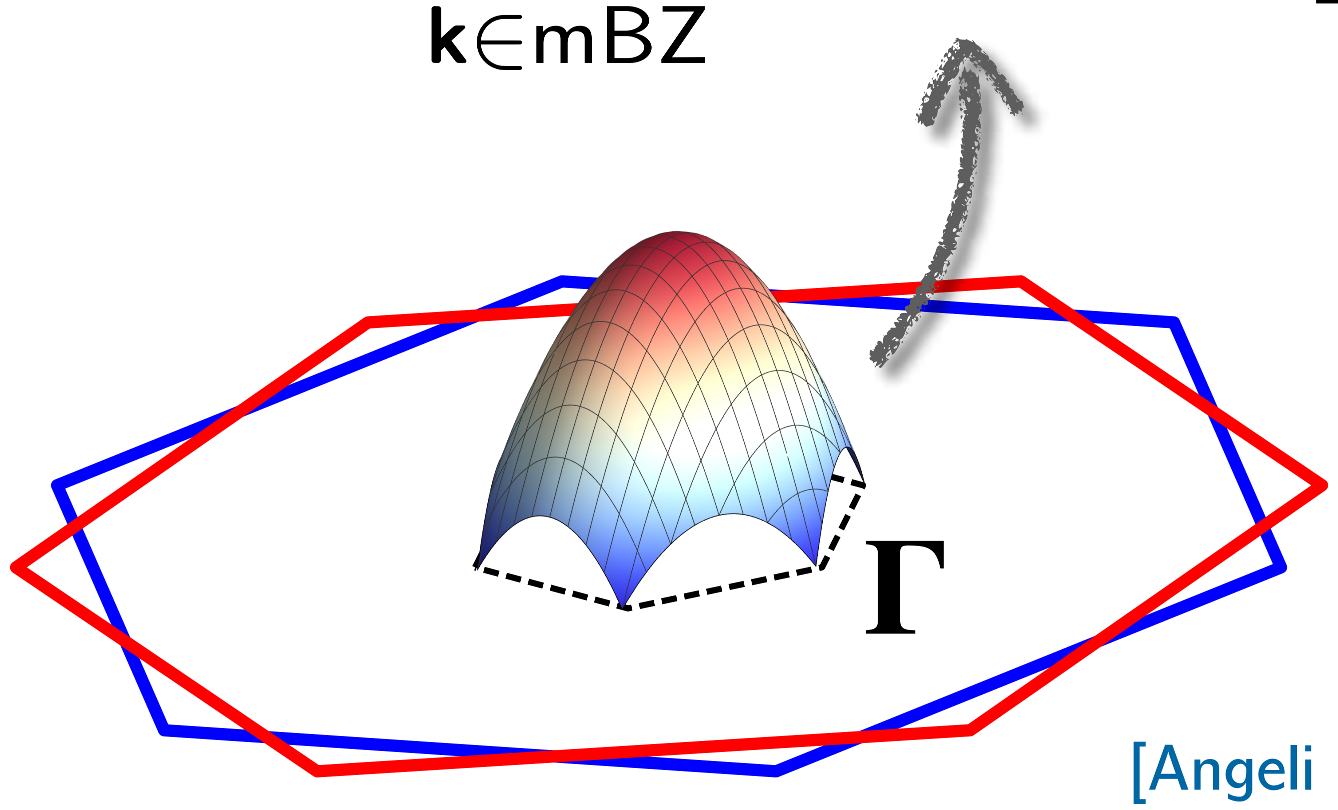

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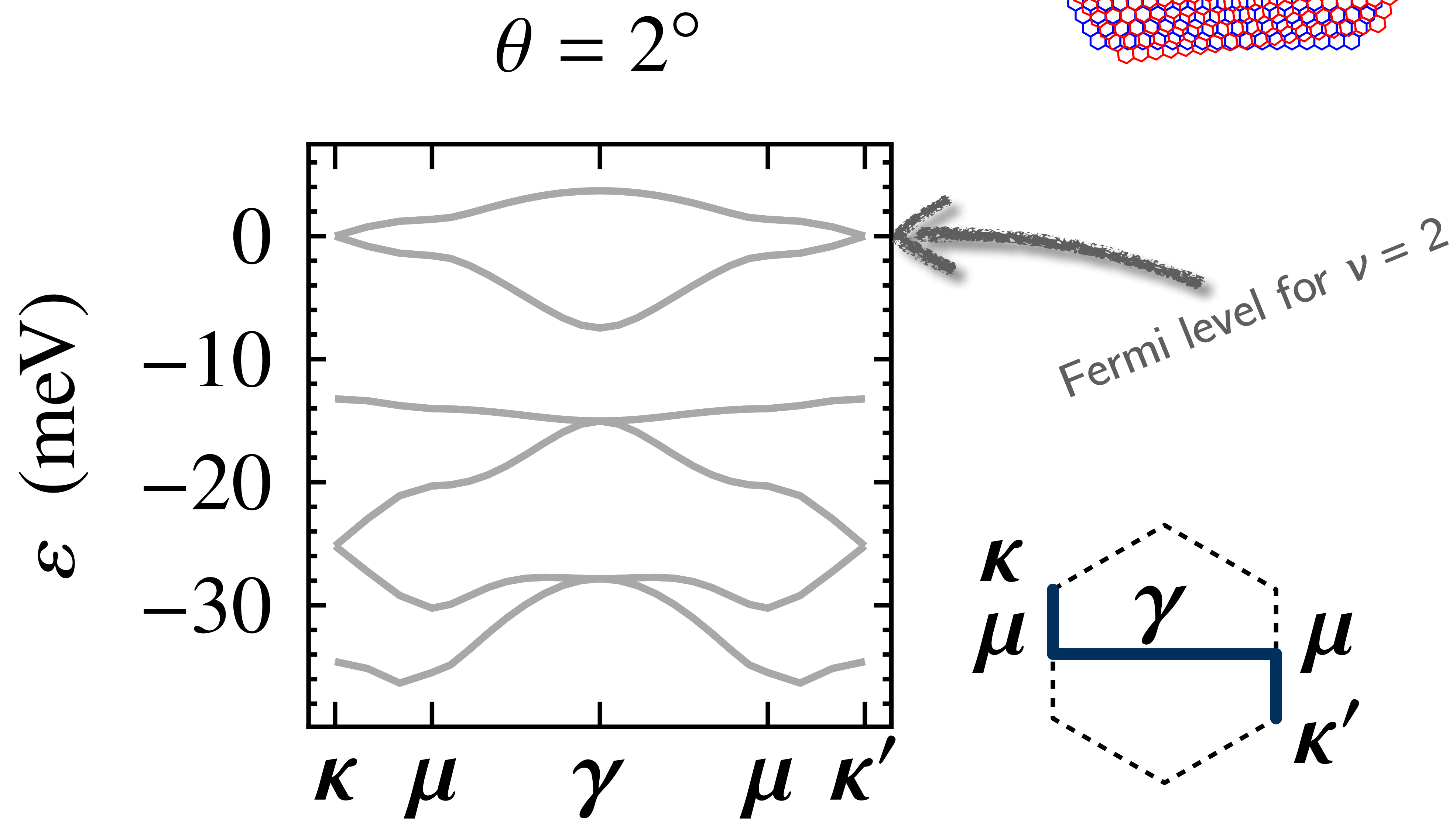

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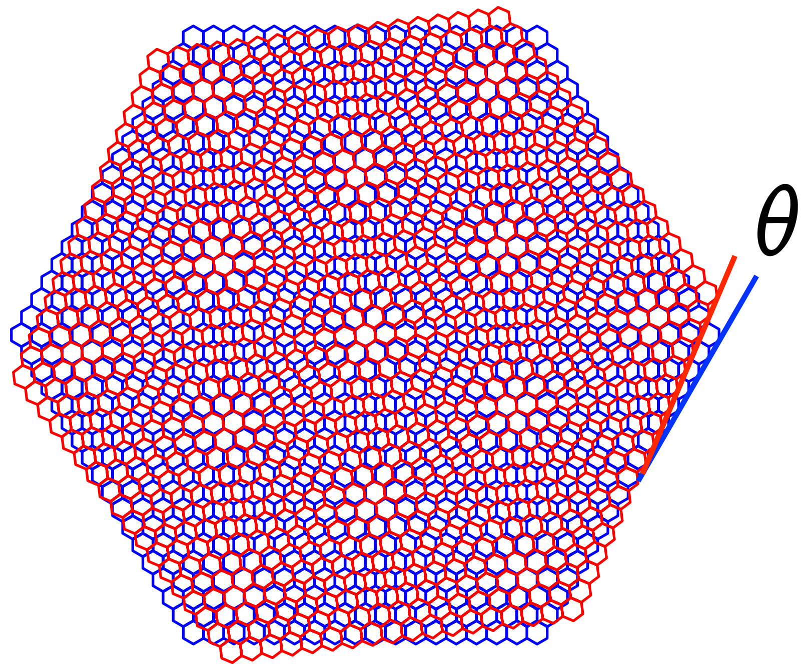

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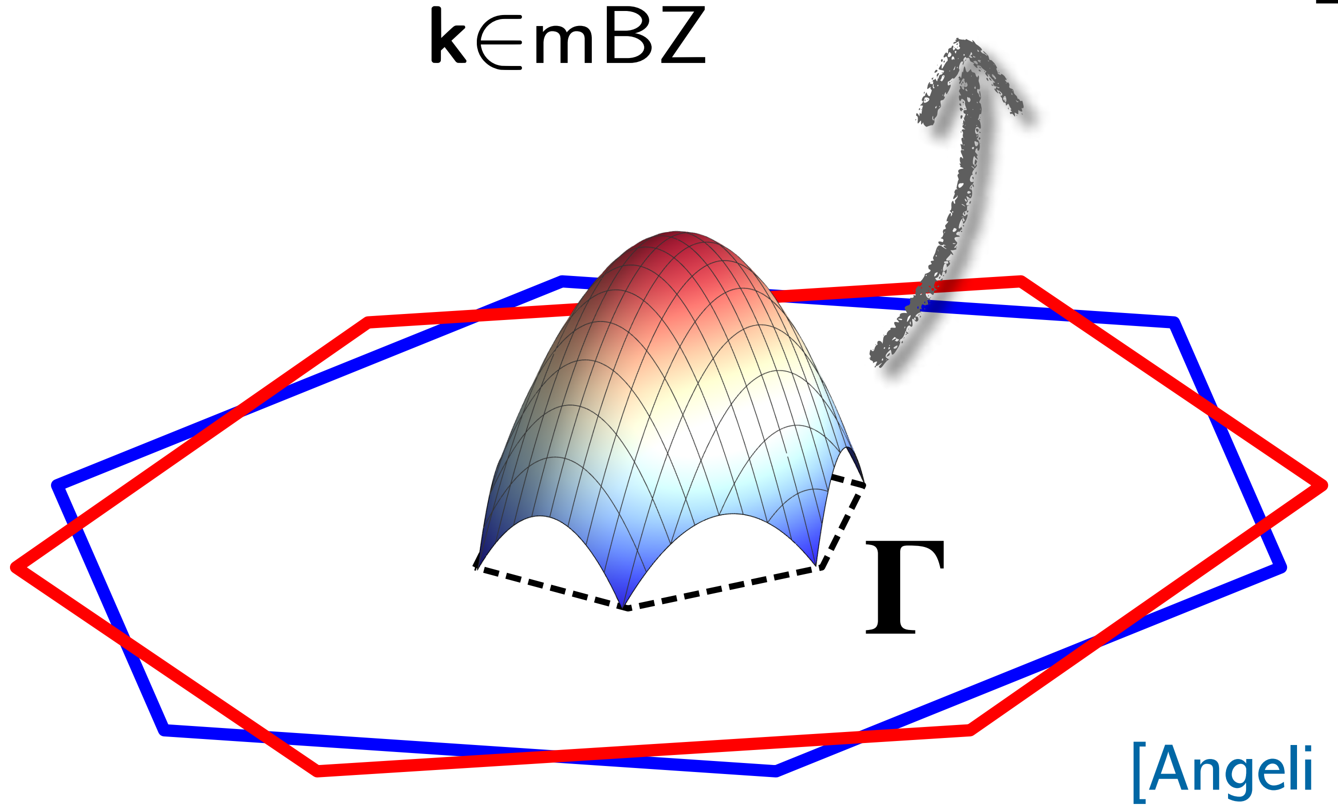

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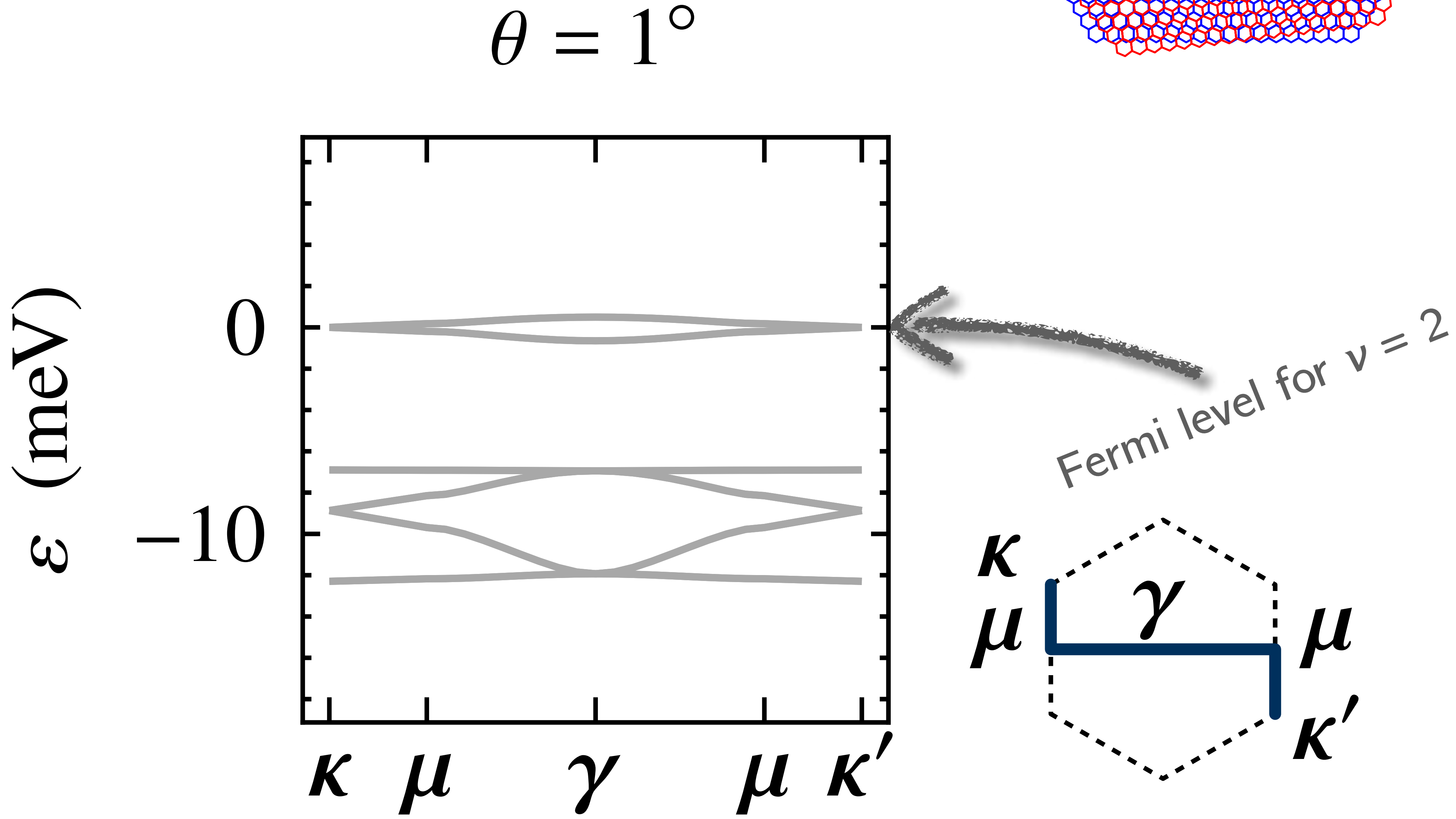

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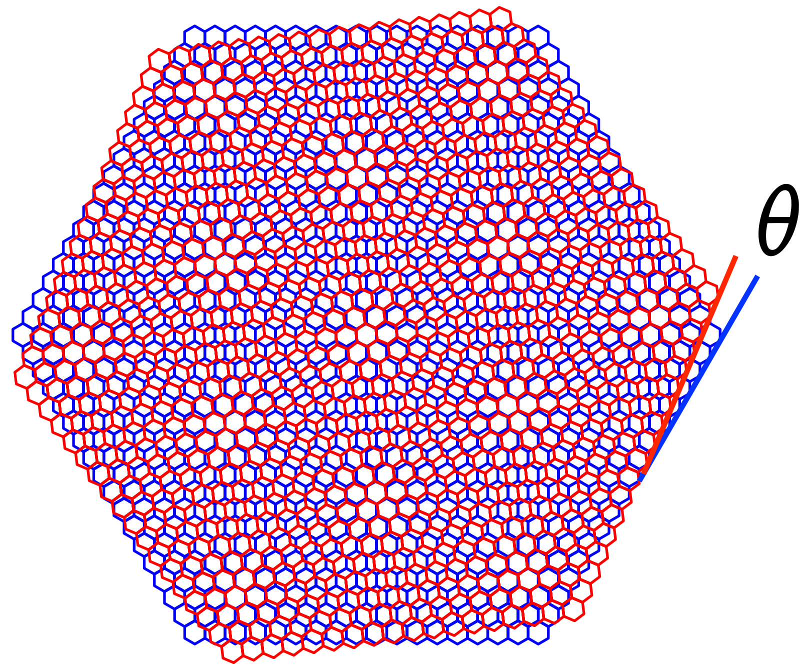

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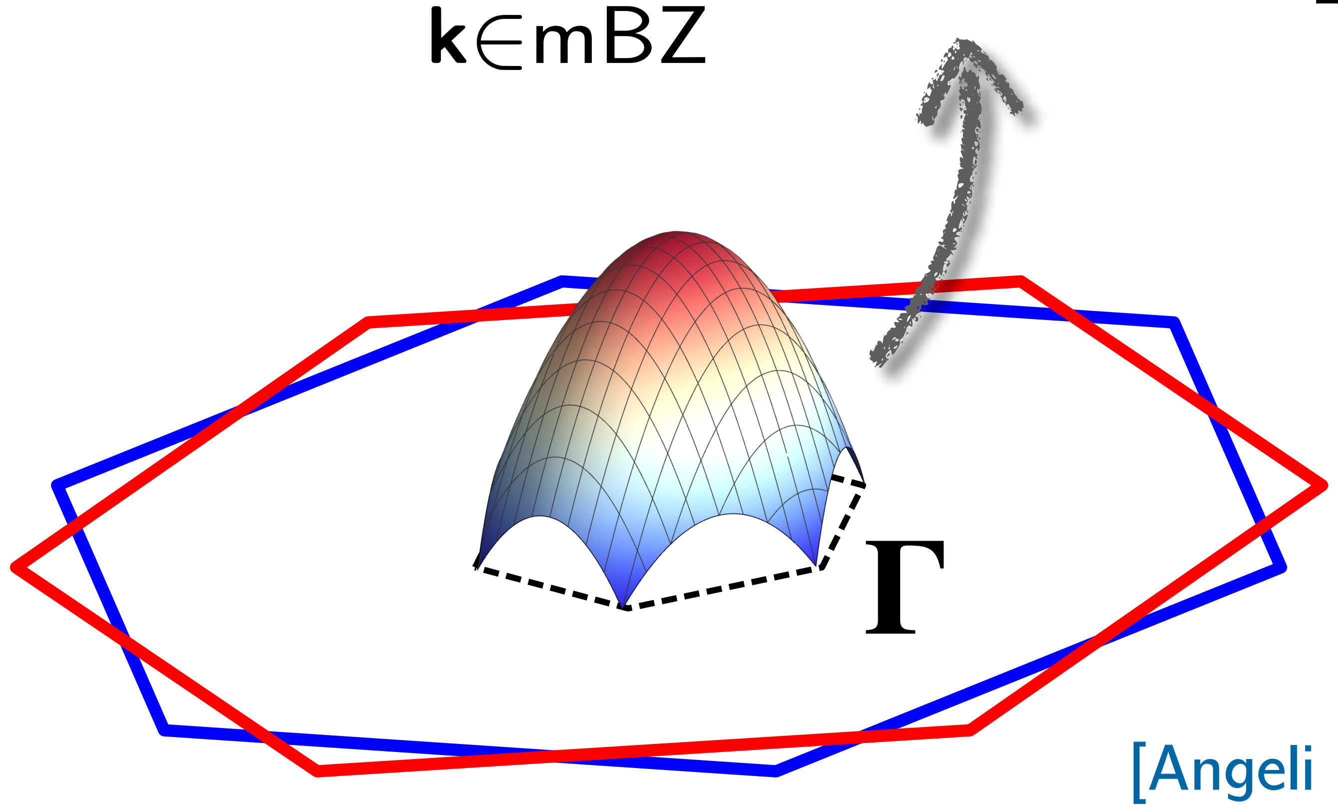

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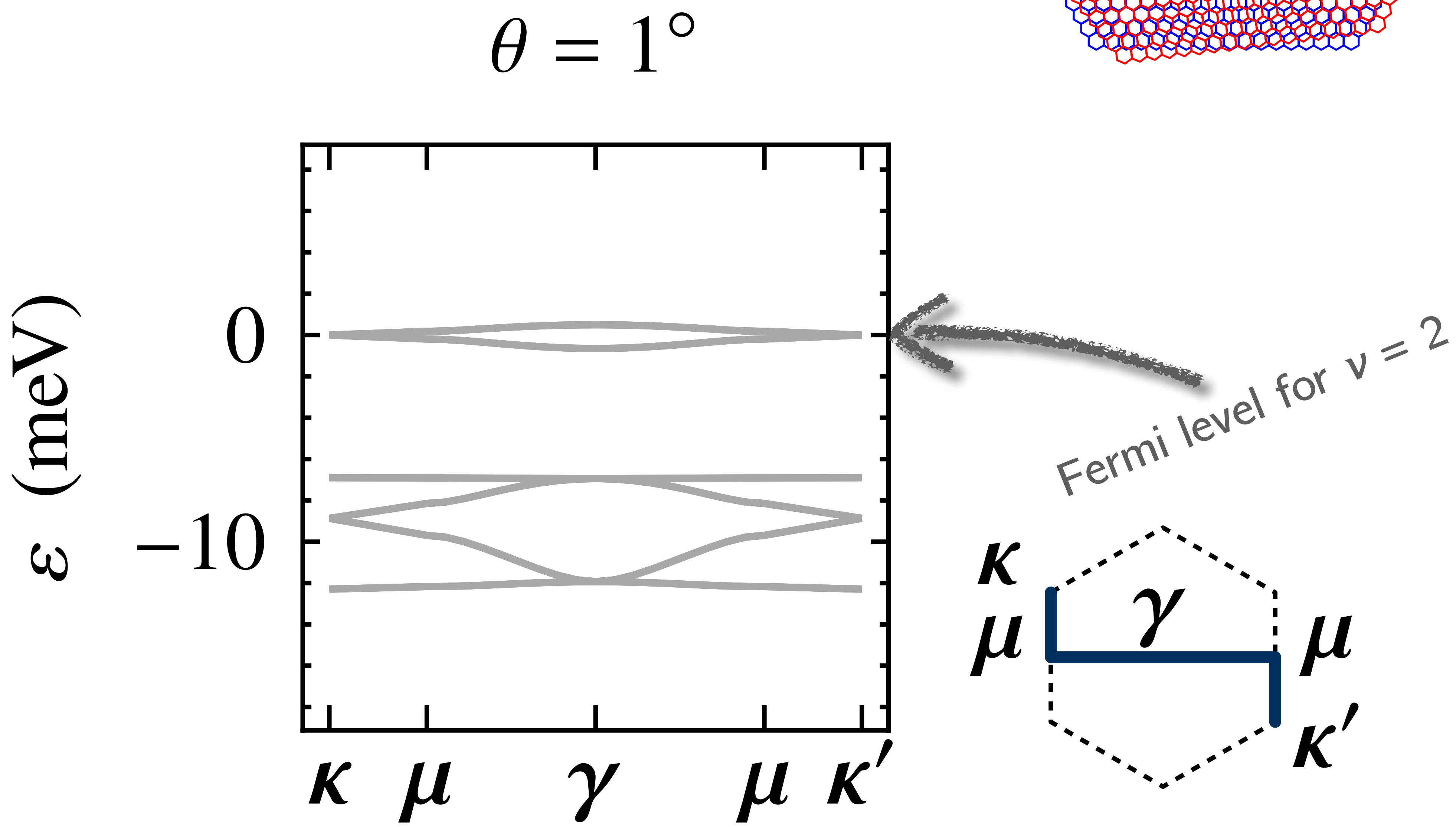
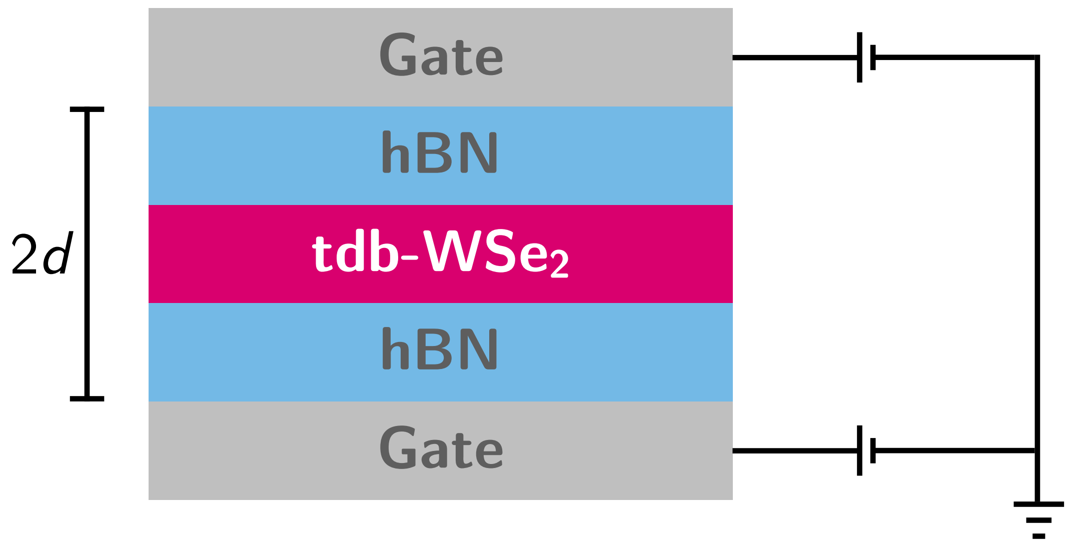


Coulomb interaction:

$$V_{\mathbf{q}} = \frac{e^2}{2\epsilon_0 \epsilon_{\text{eff}} |\mathbf{q}|} \tanh(|\mathbf{q}| d)$$



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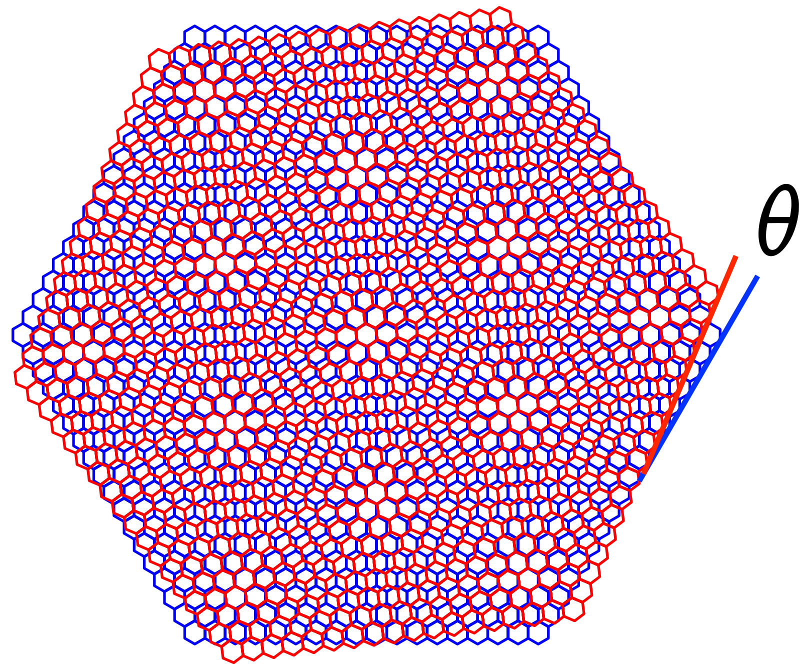

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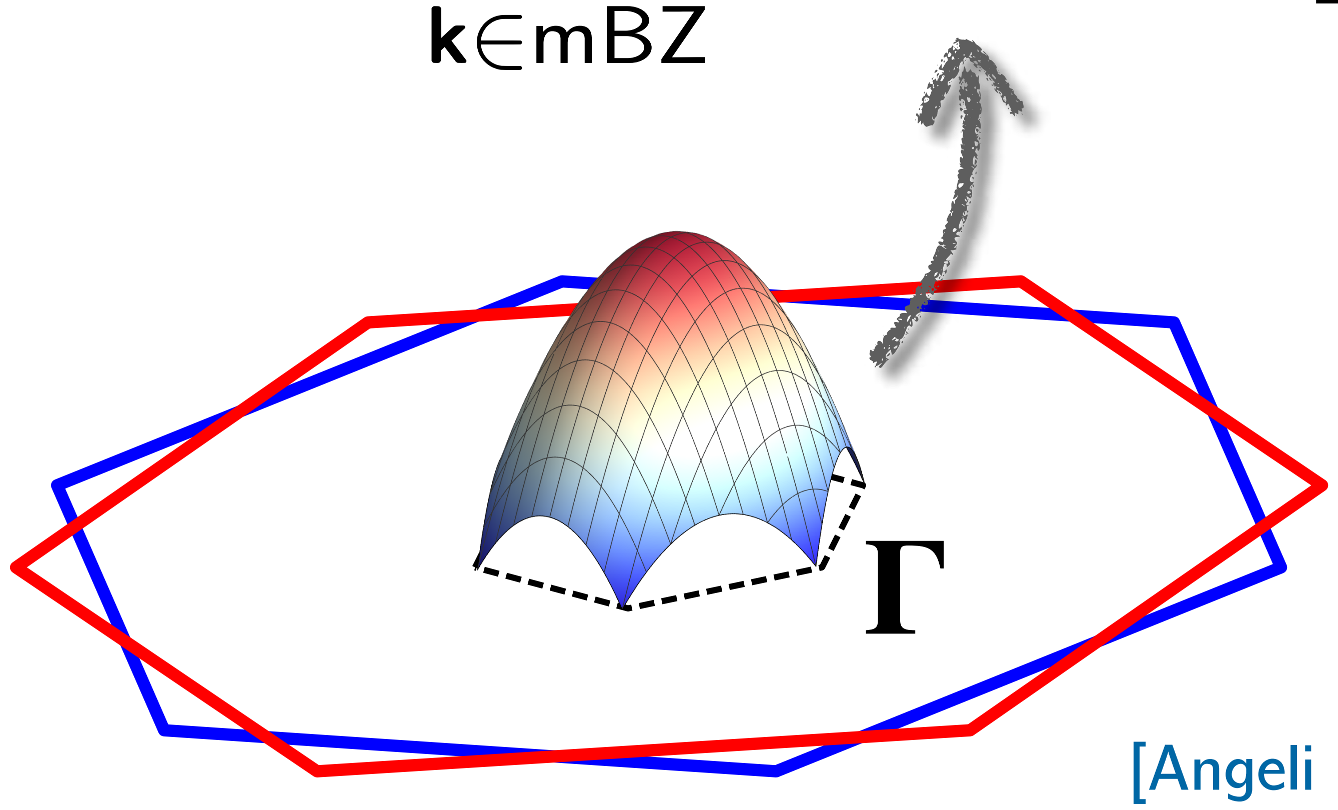

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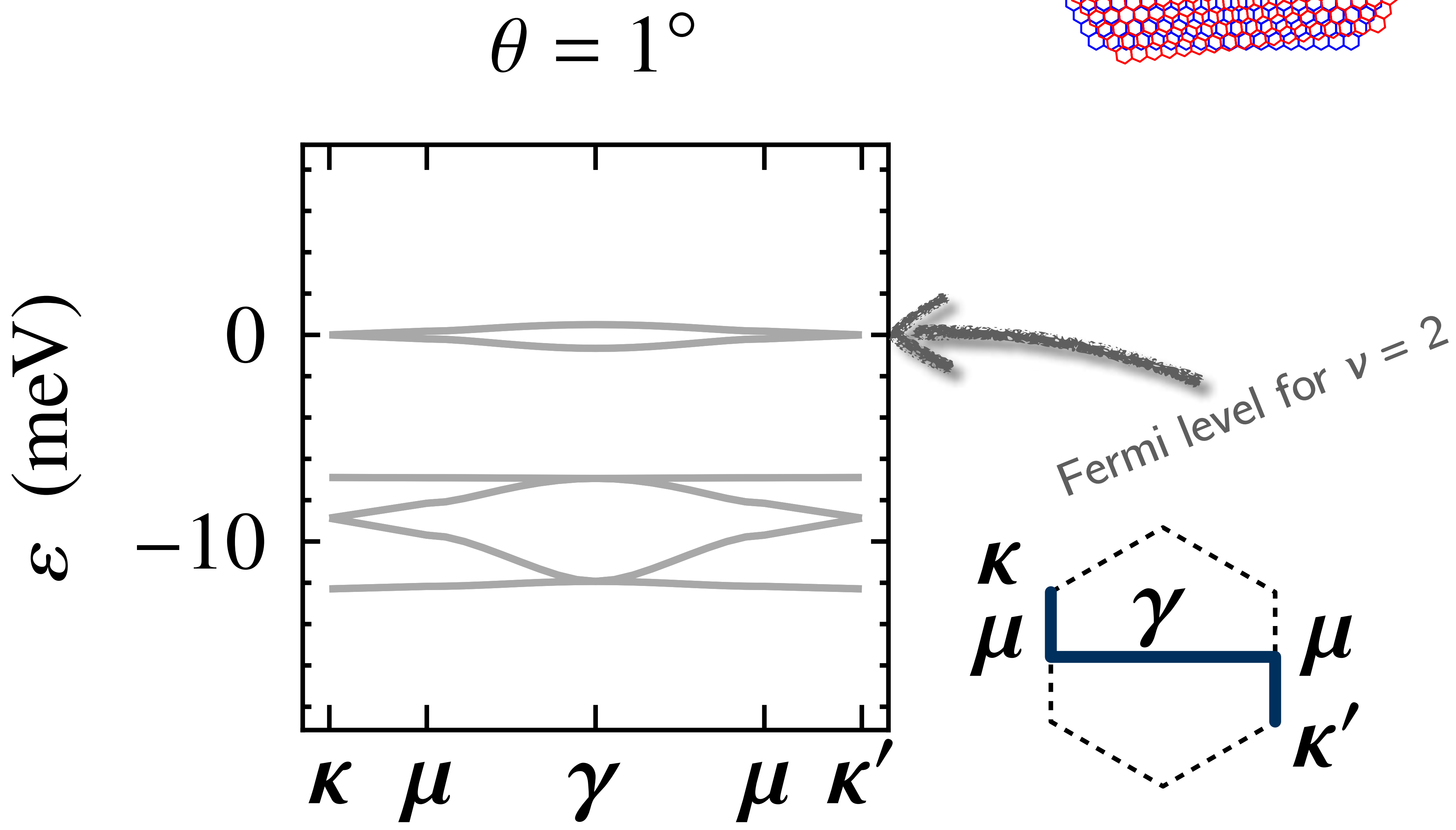

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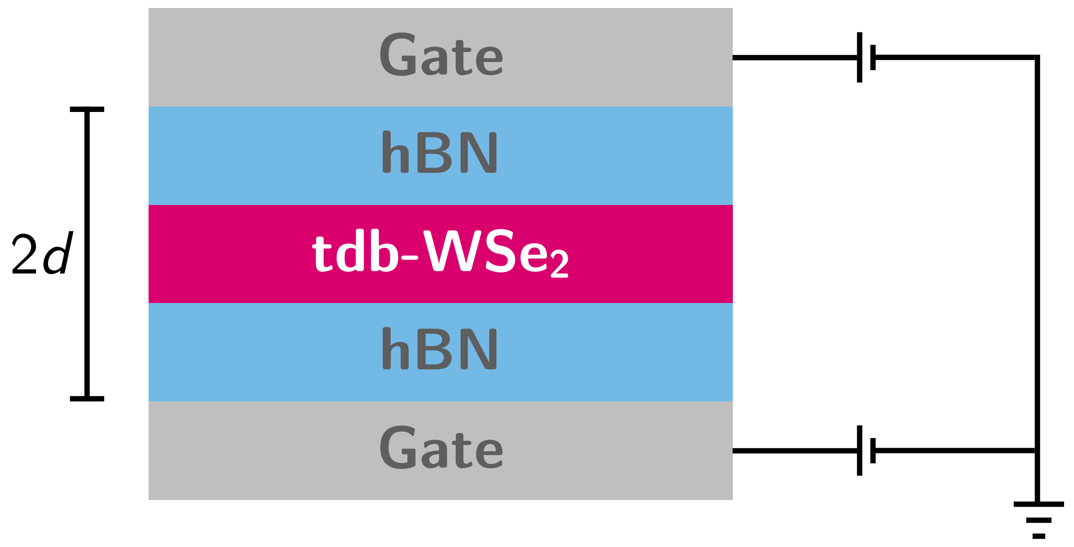
Effective permittivity

$\epsilon_{\mathrm{eff}} \lesssim \mathcal{O}(100)$



Large θ: Dirac semimetal

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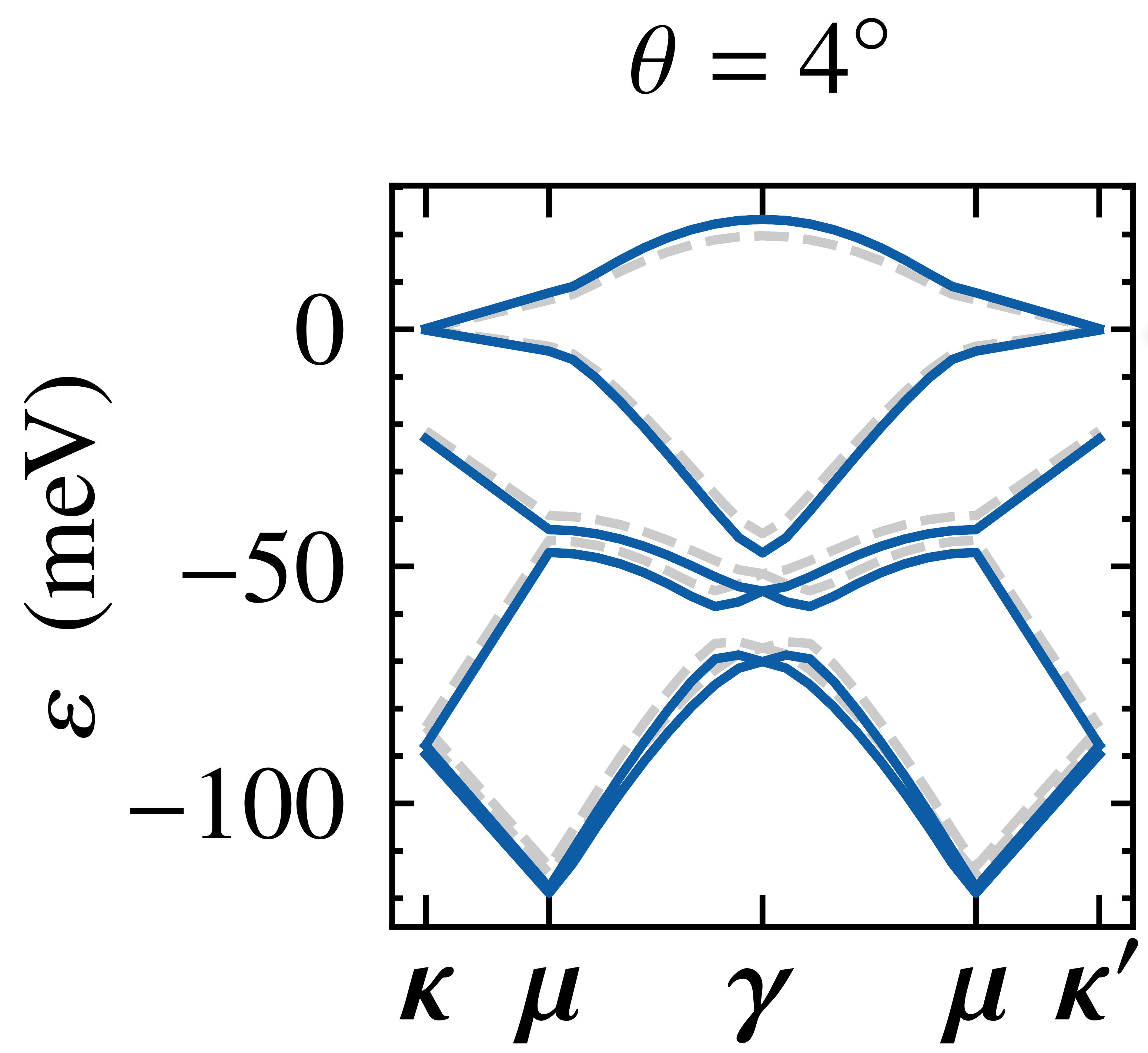

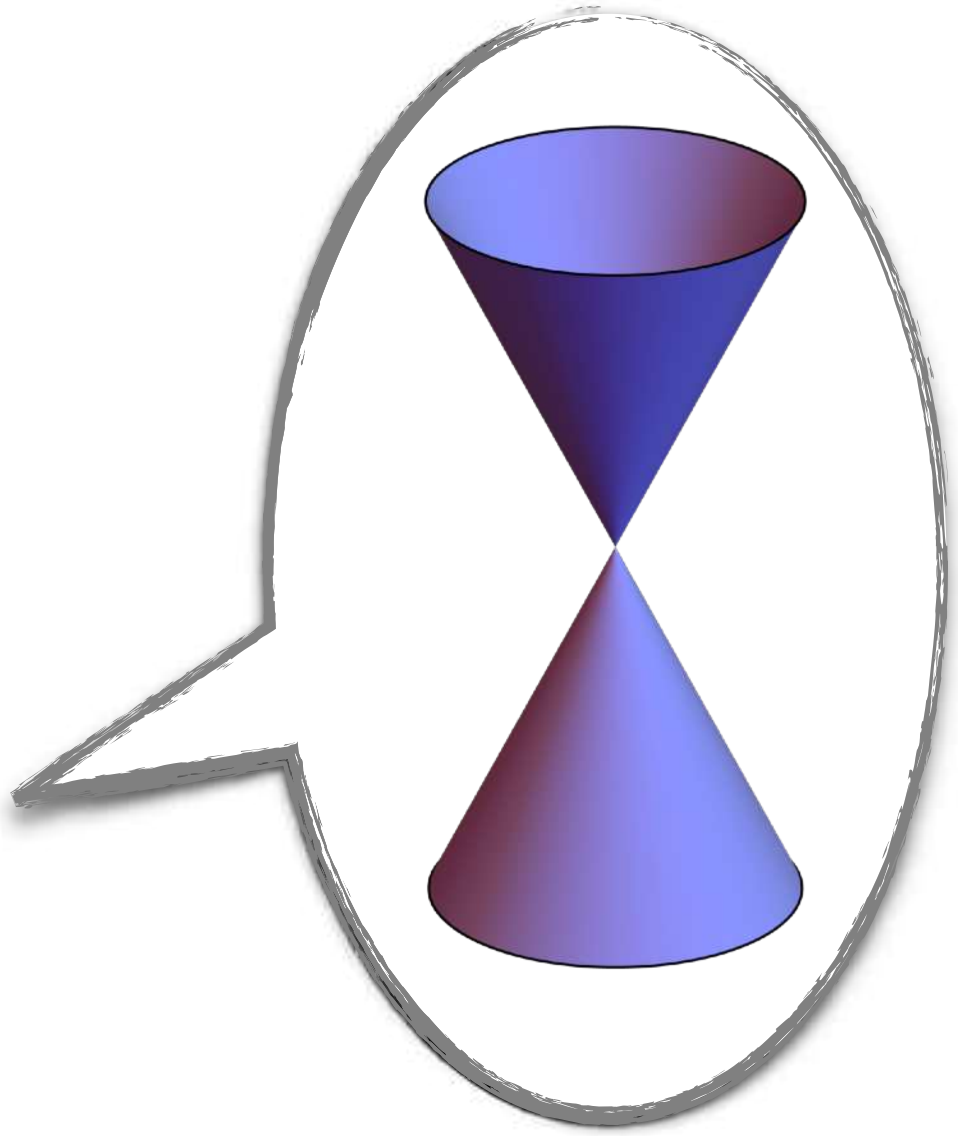

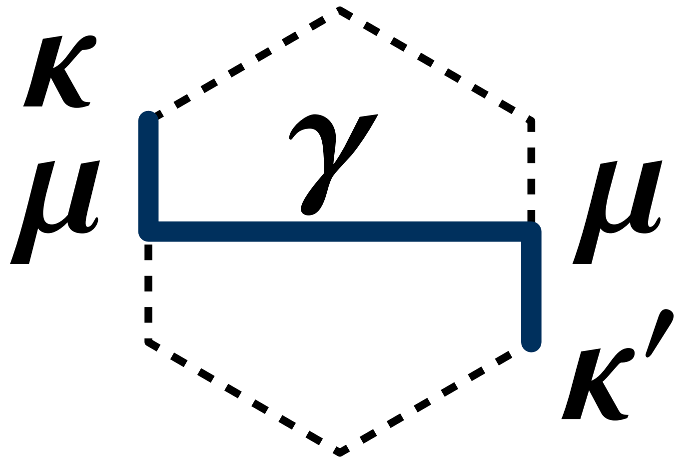

… from selfconsistent Hartree-Fock analysis

… for $\nu = 2$ holes per unit cell

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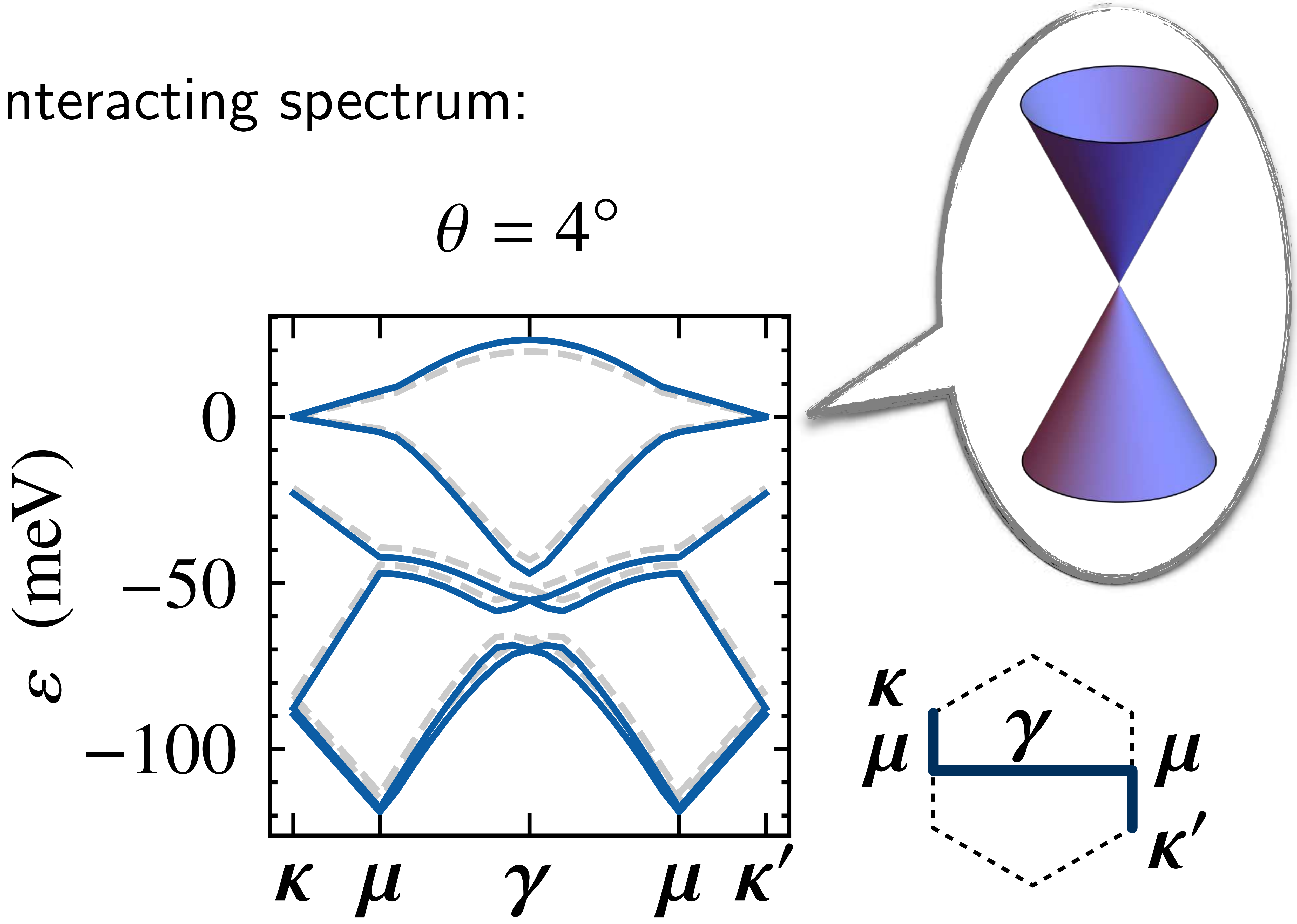

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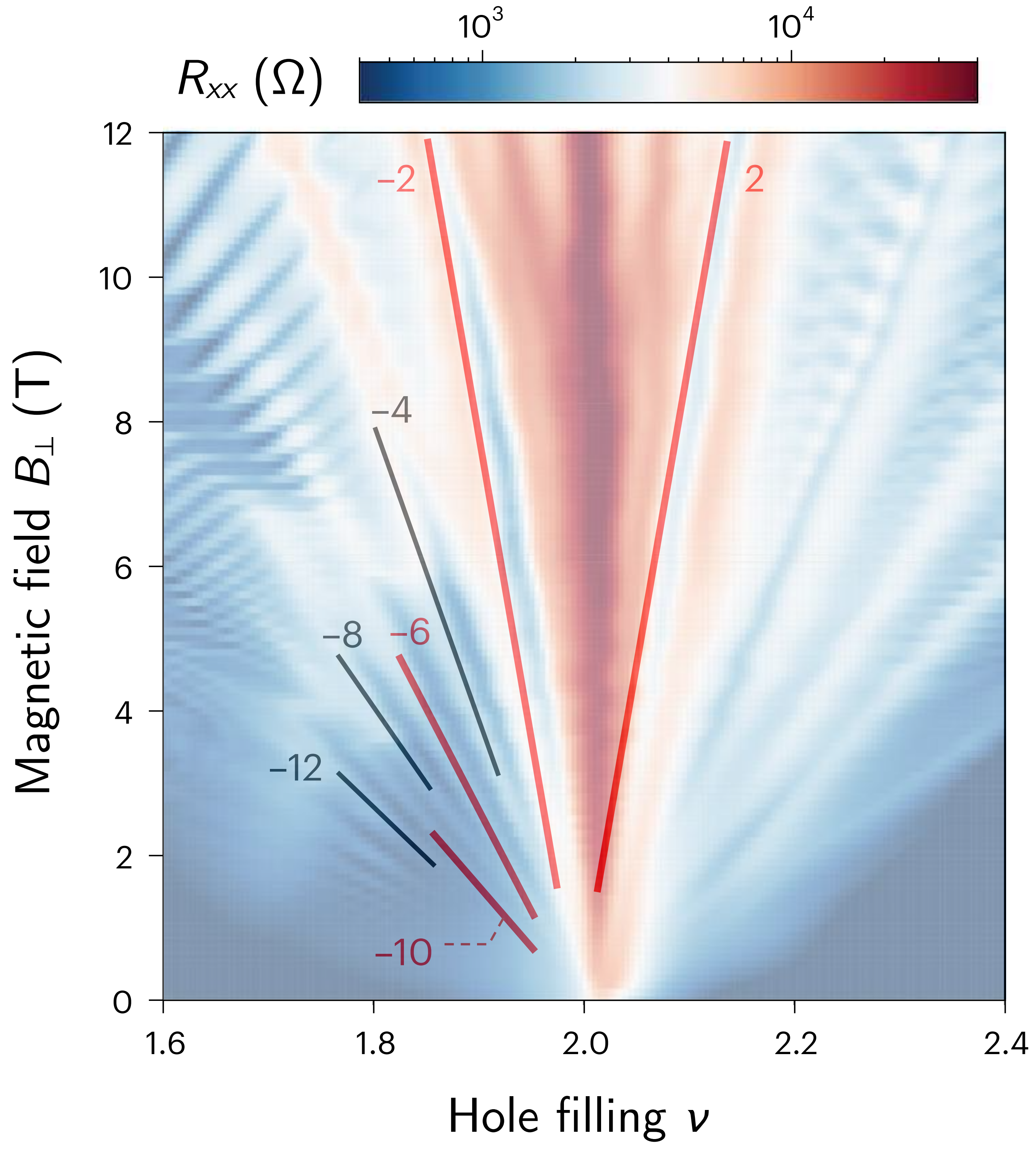
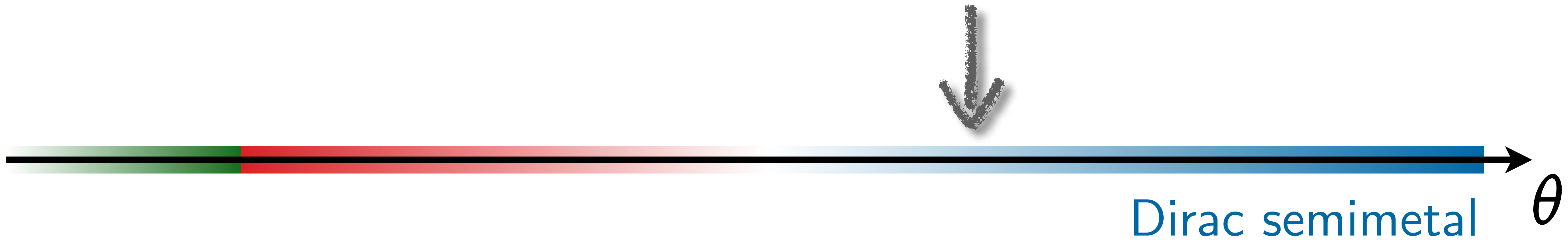
Magnetotransport:



[Ma *et al.*, Nat. Mater. ’25]



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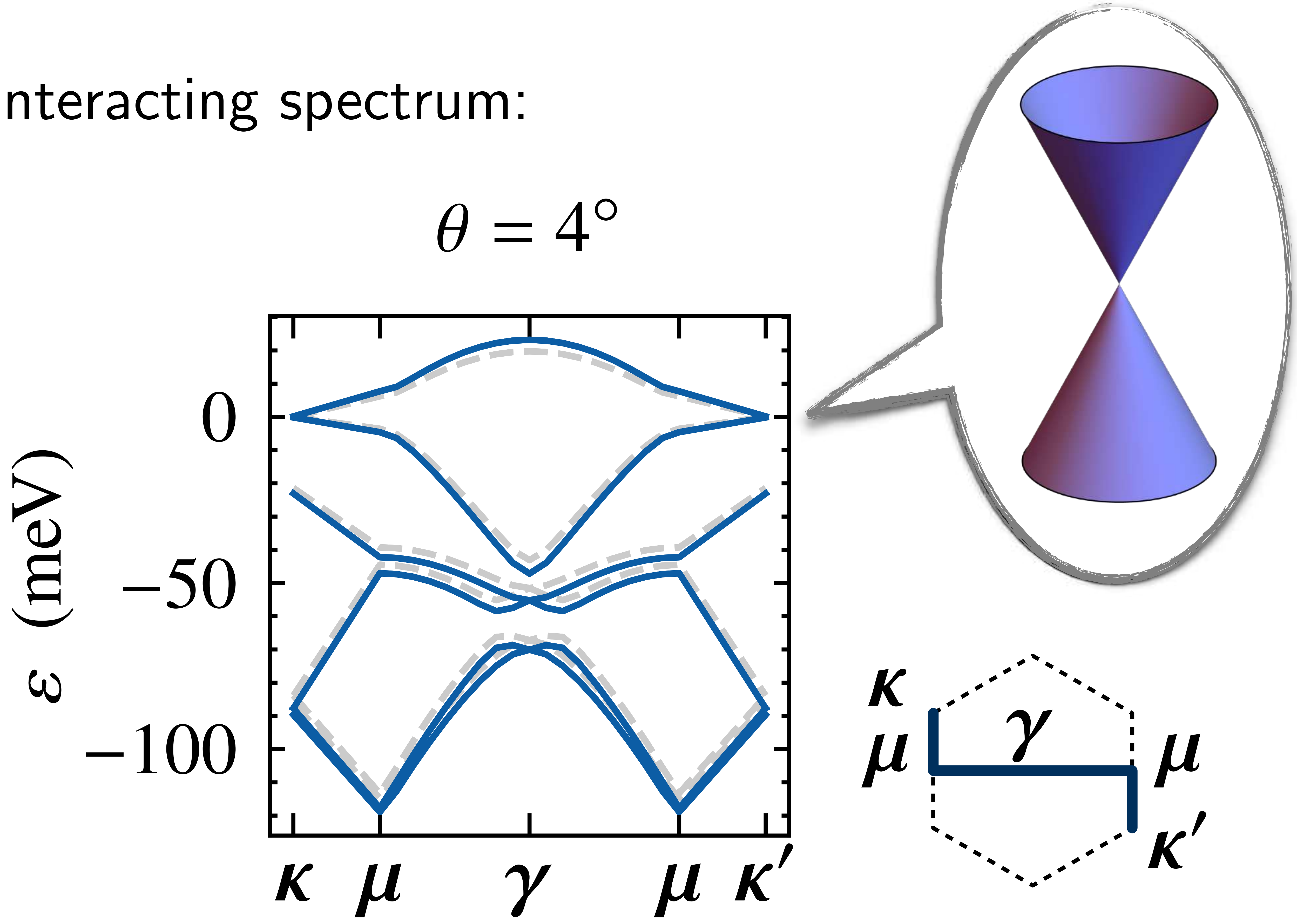
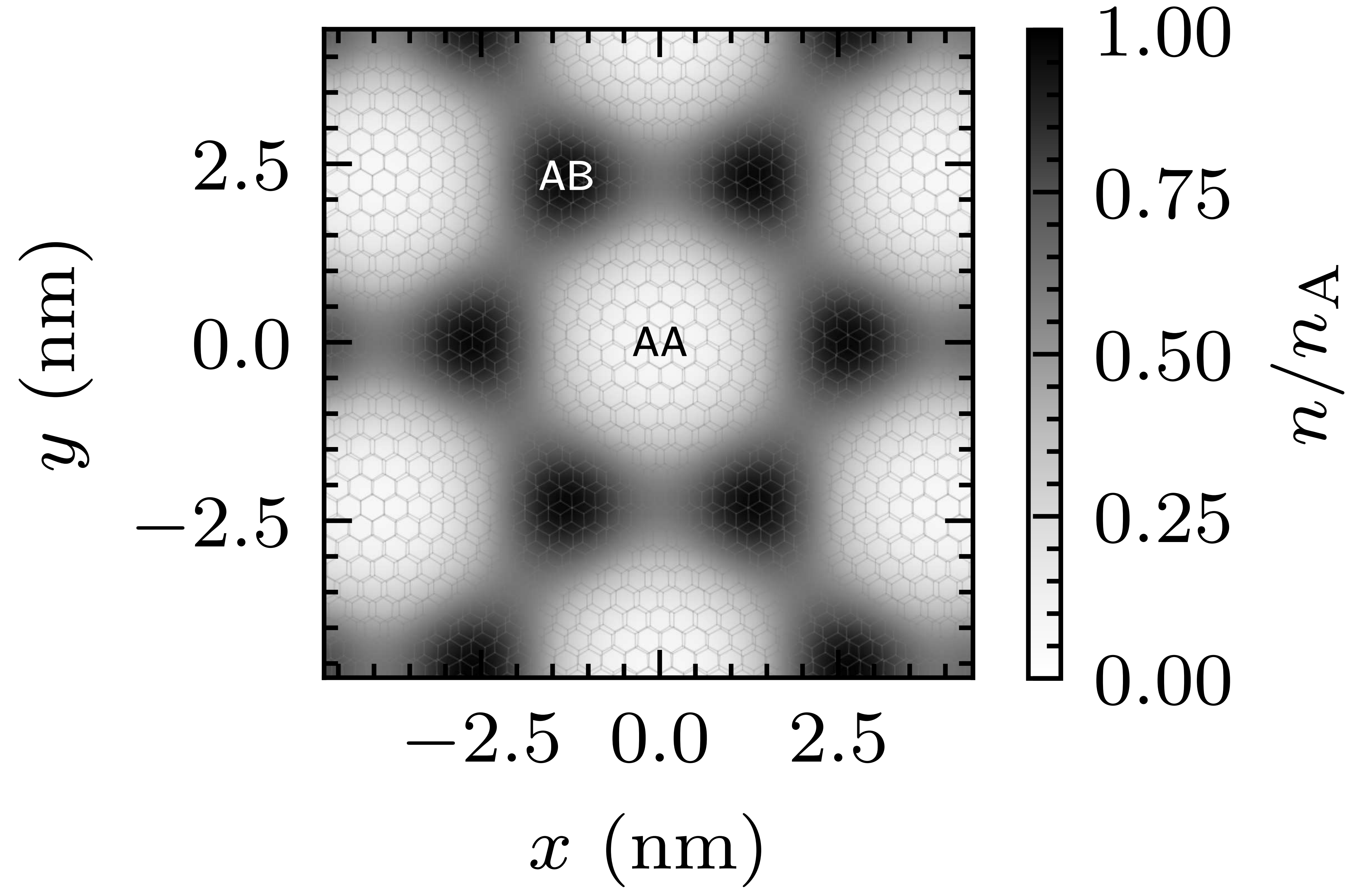


Charge density:



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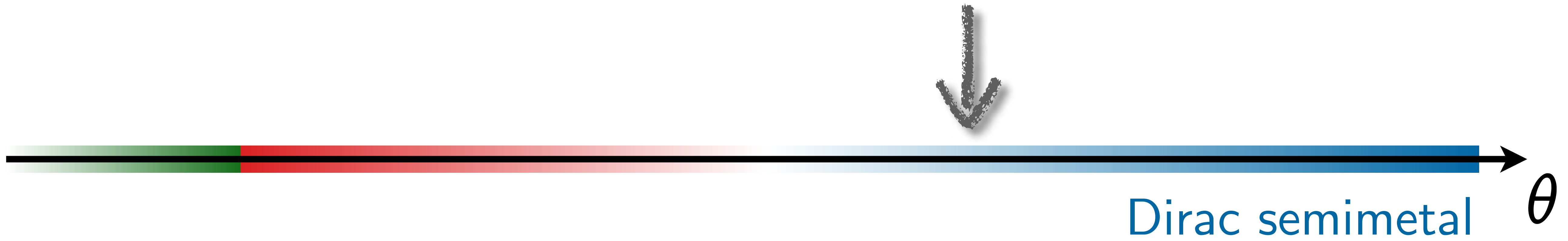

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Small θ: Antiferromagnetic insulator

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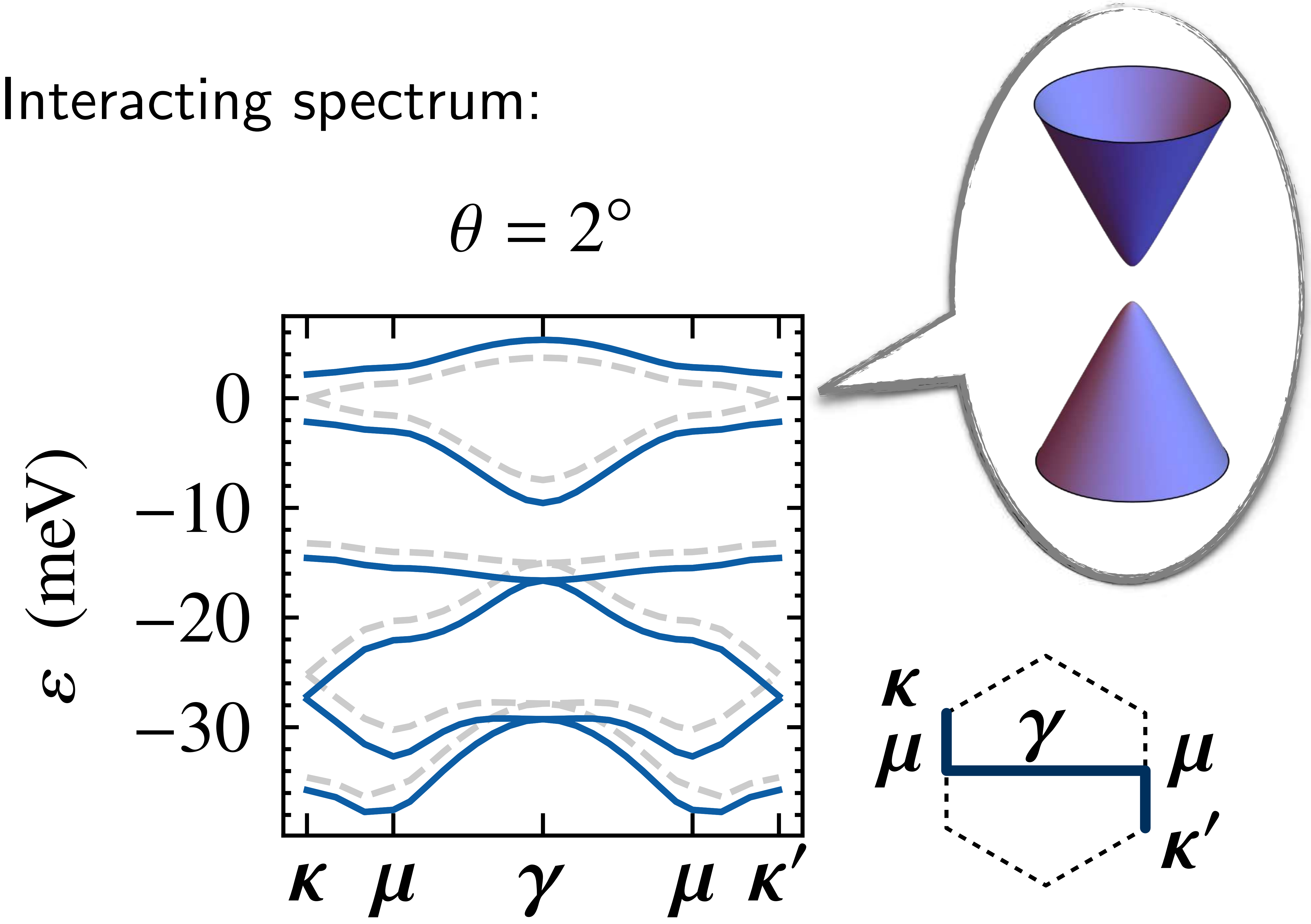

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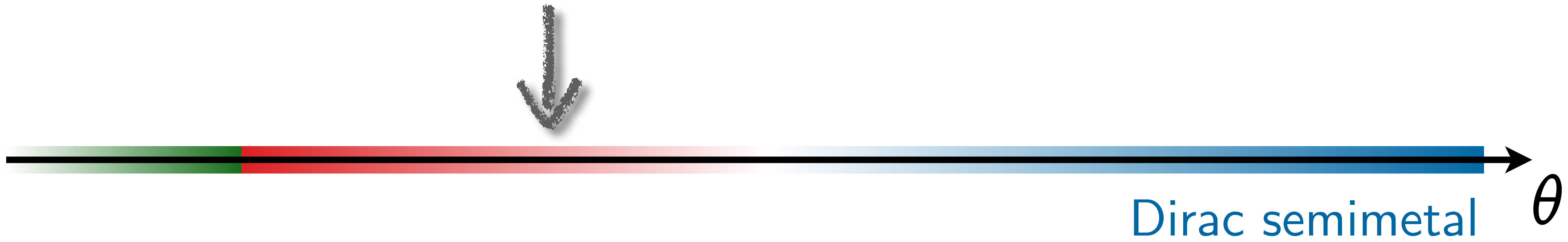

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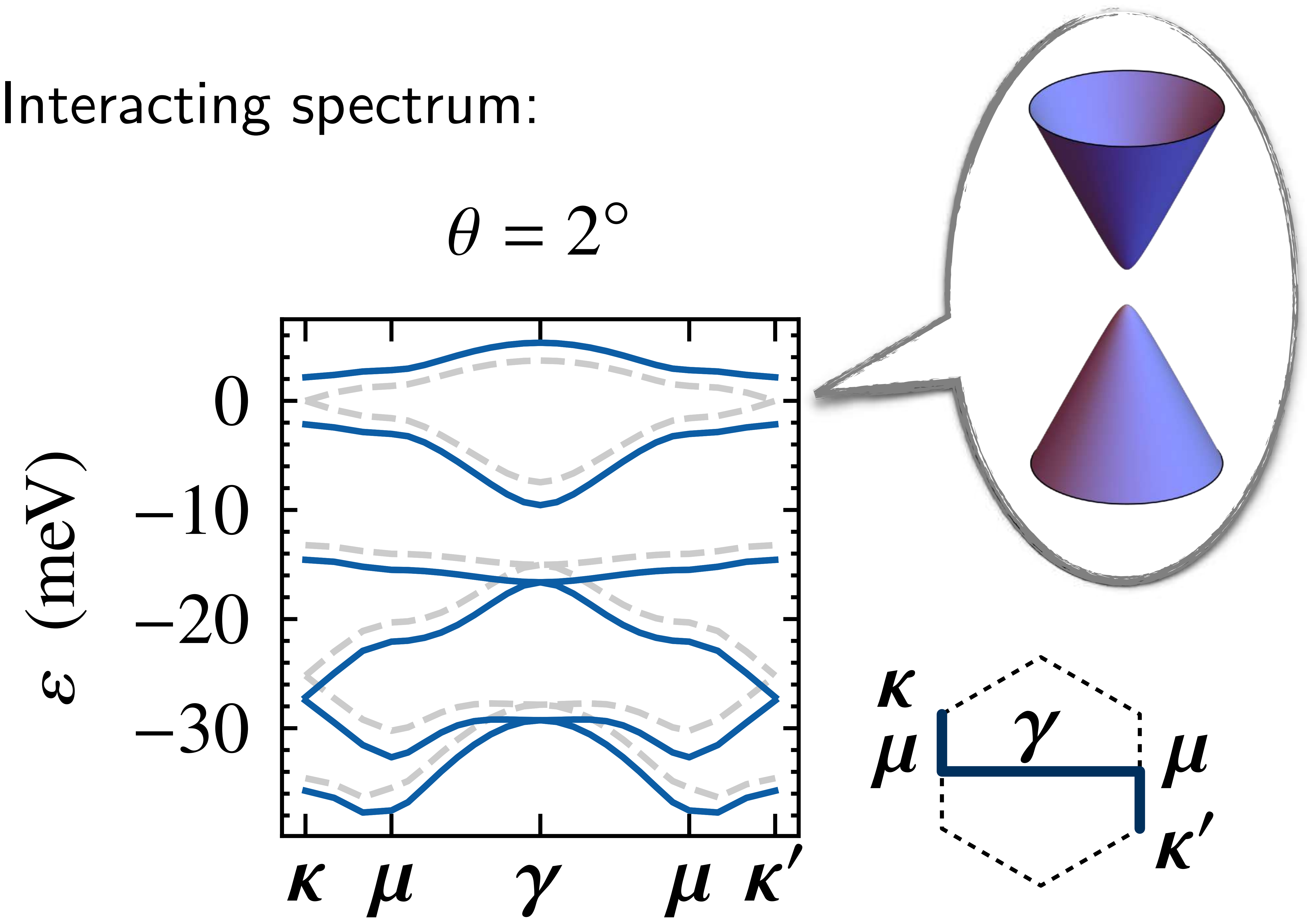
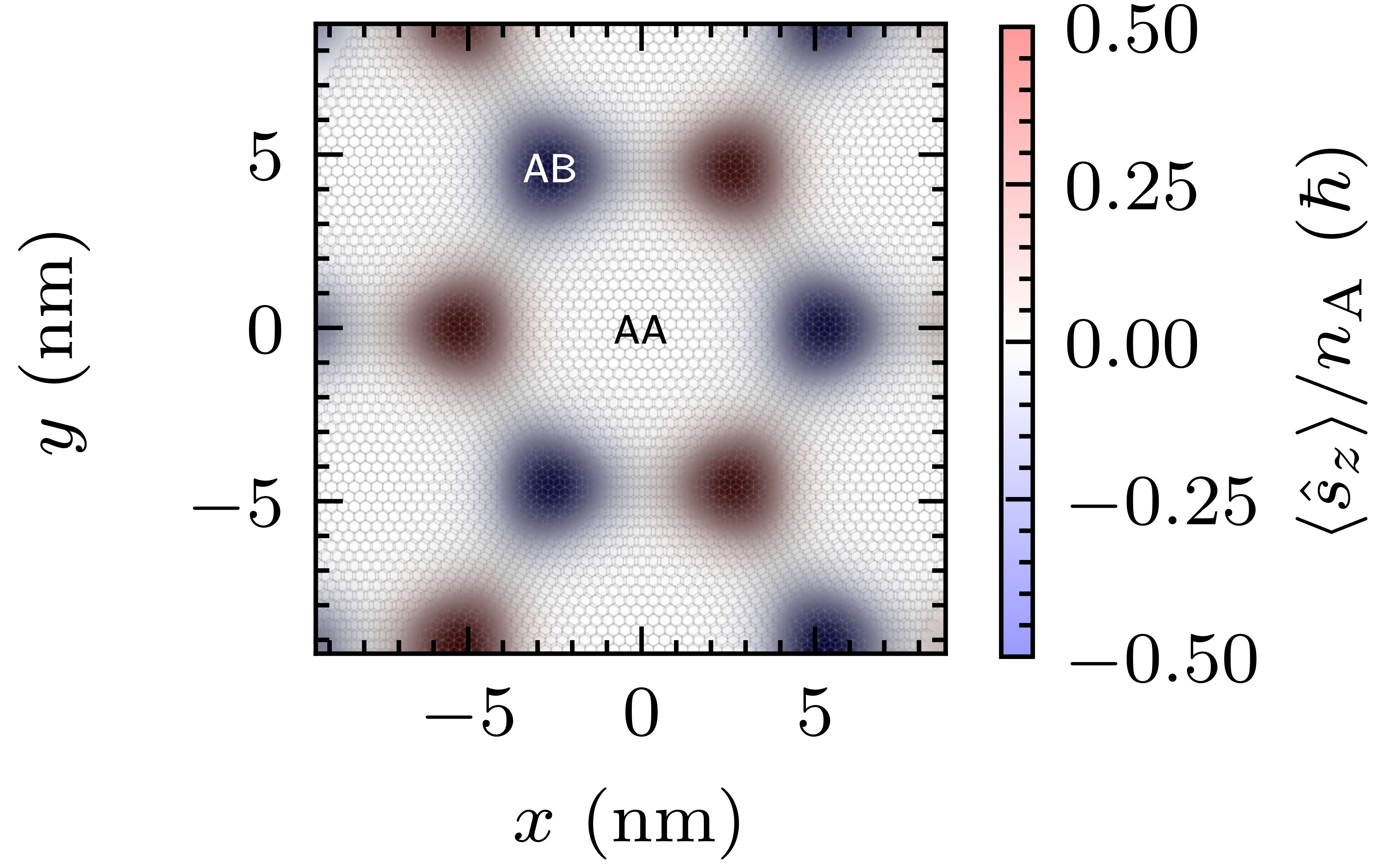


Spin density:



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Very small θ: Ferromagnetic insulator

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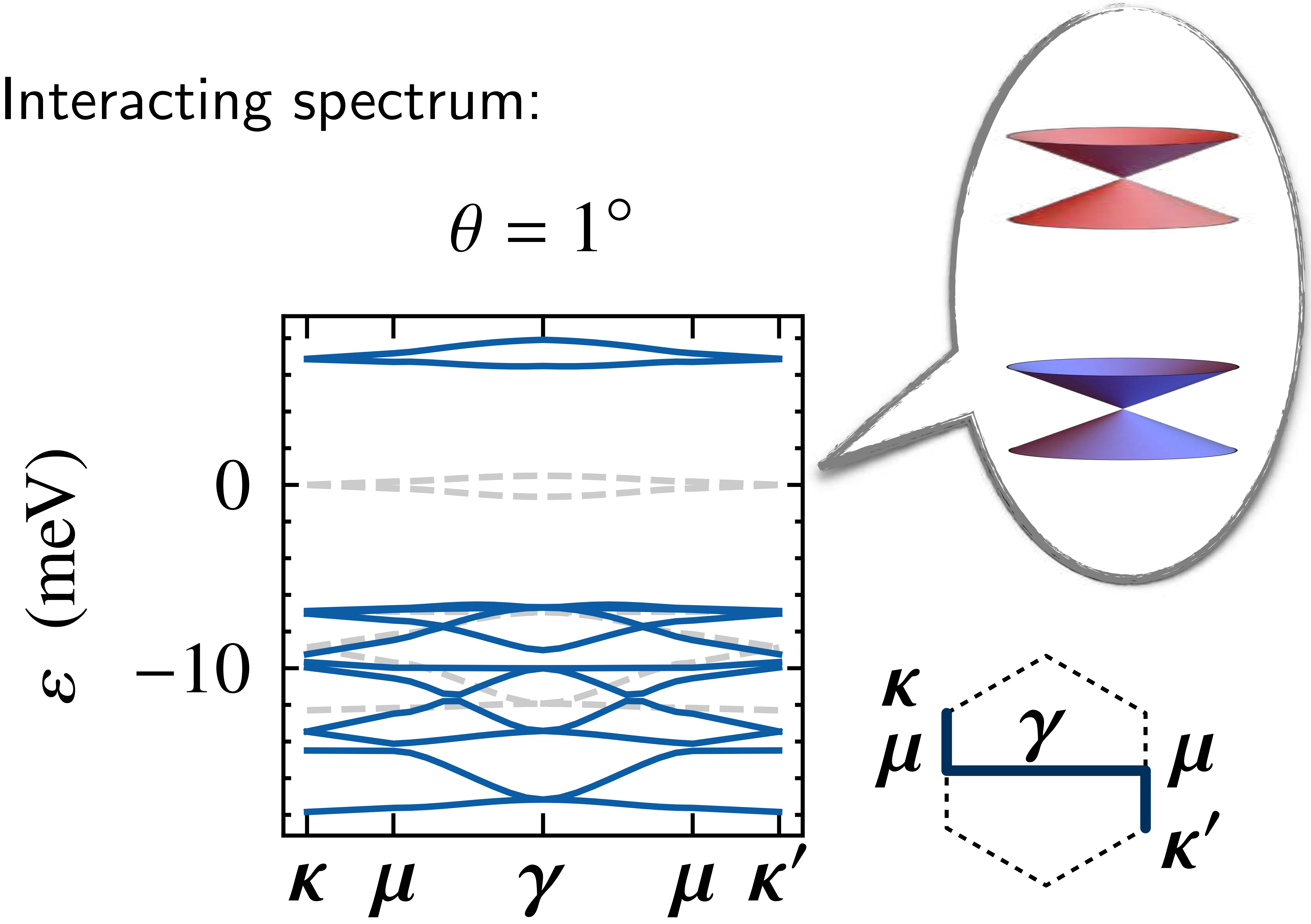

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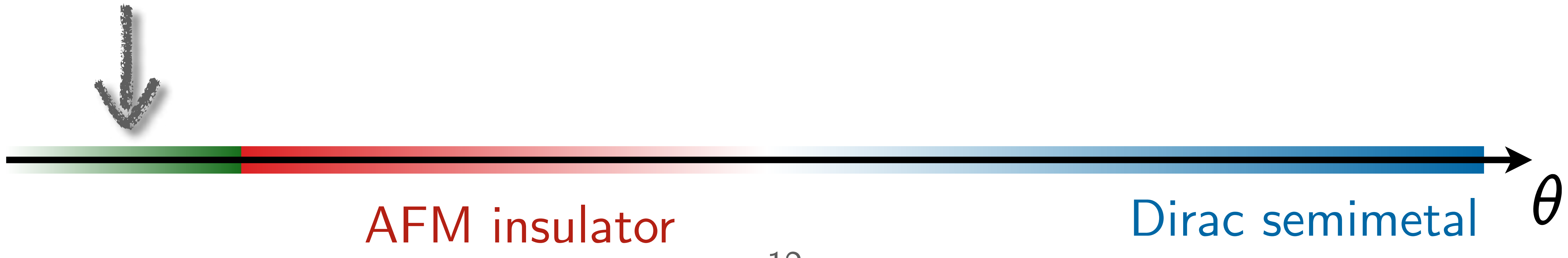

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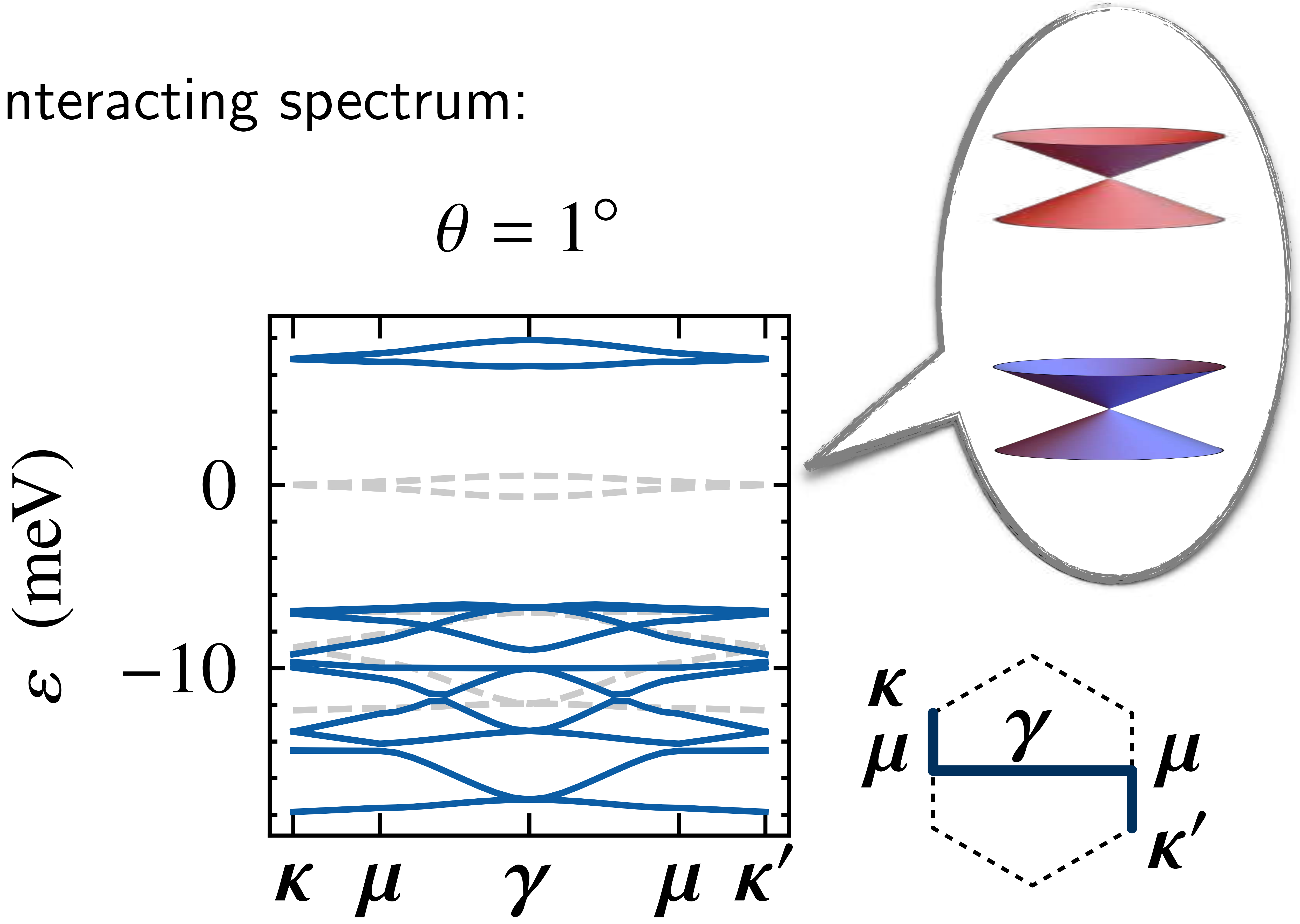

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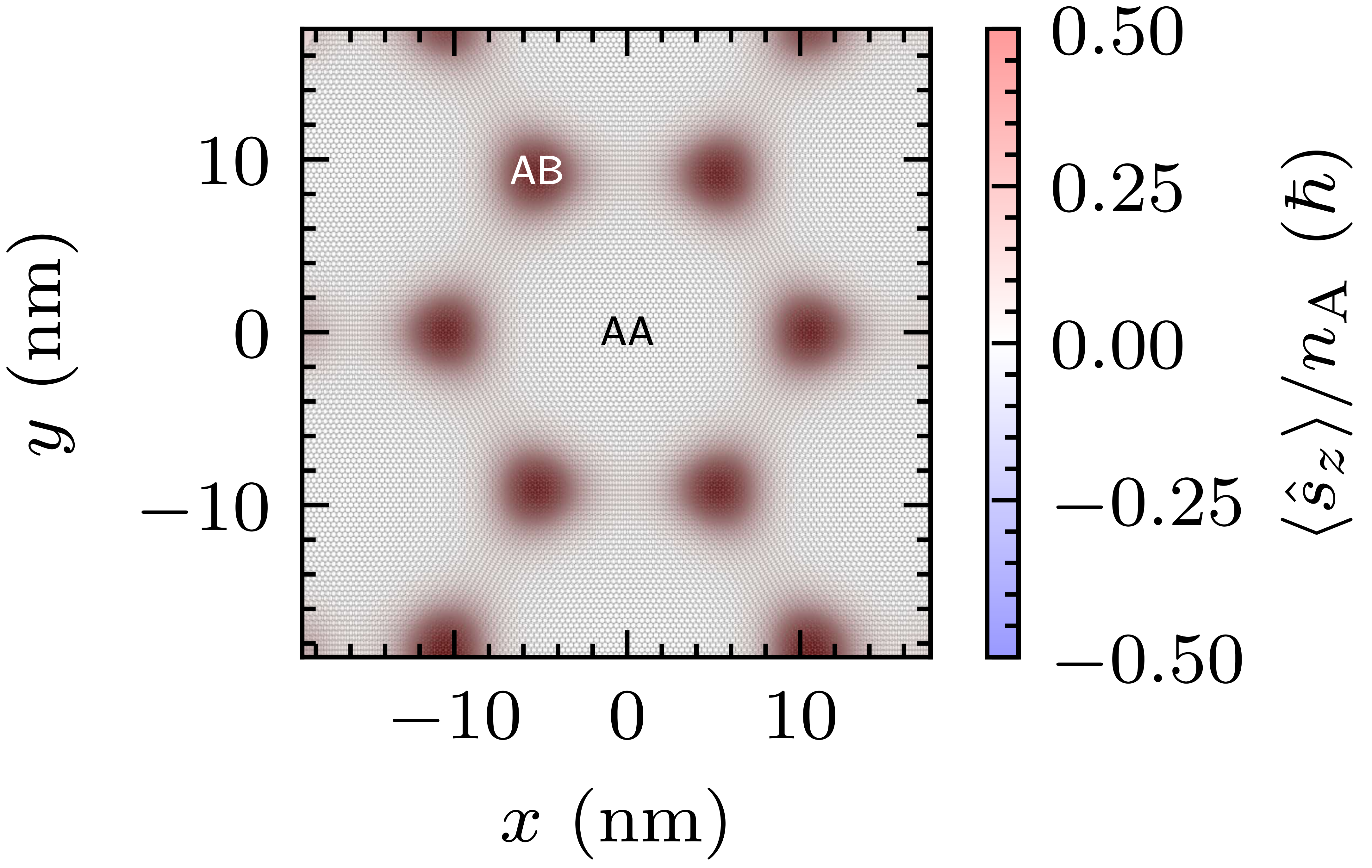

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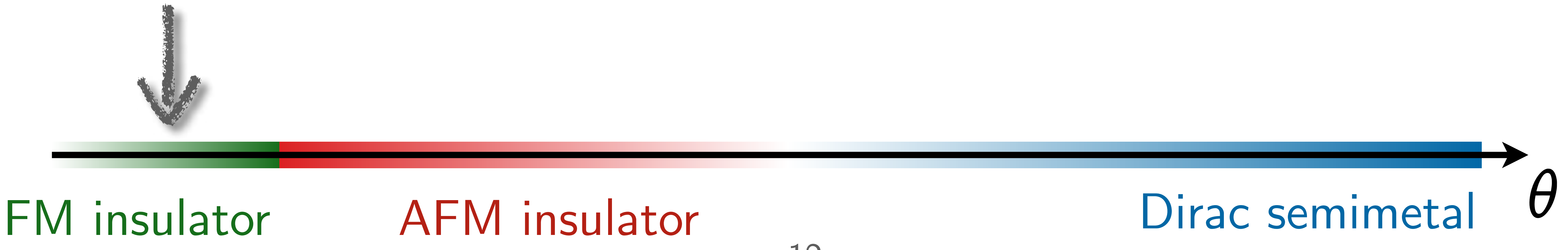

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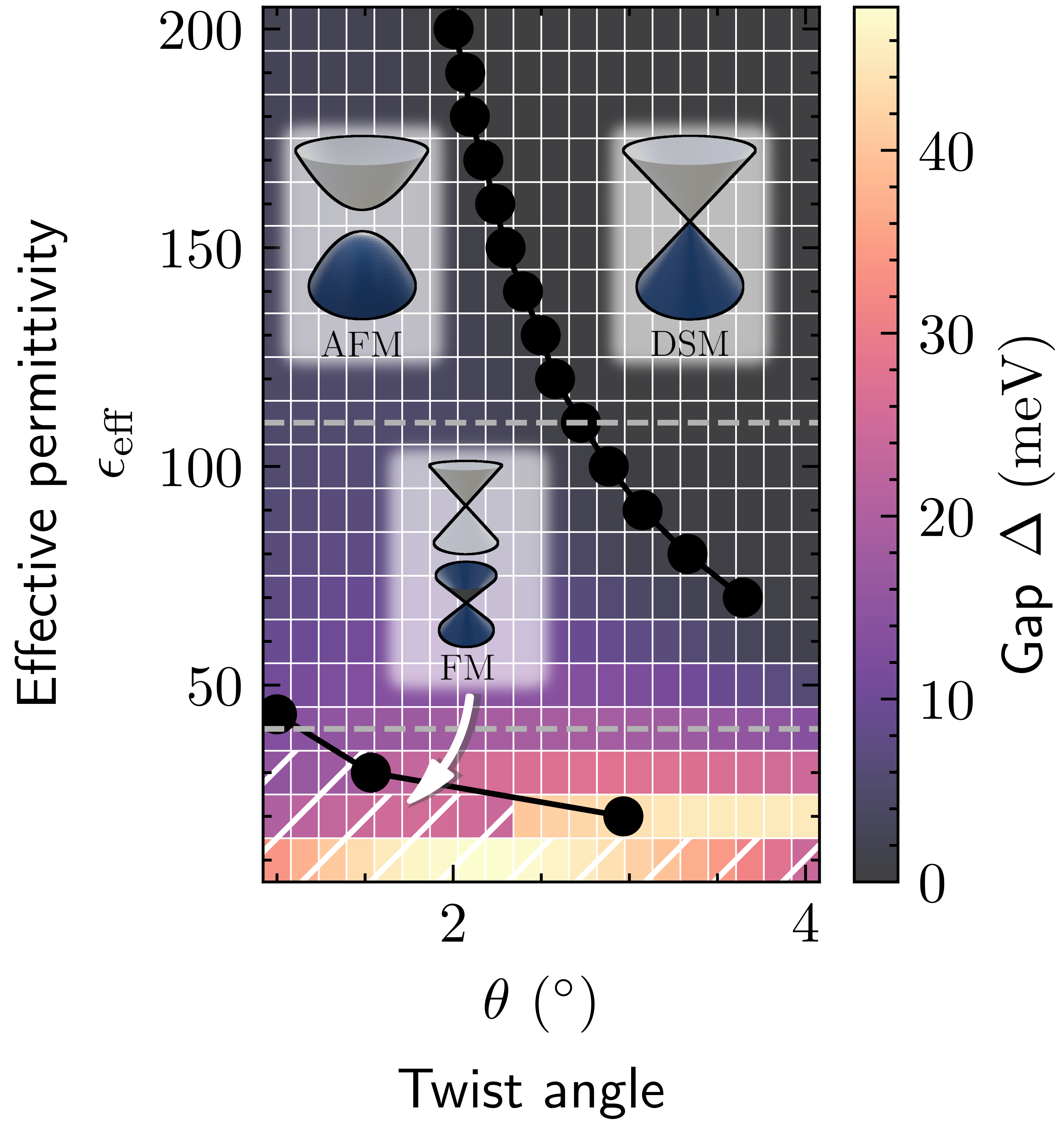
Quantum phase diagram



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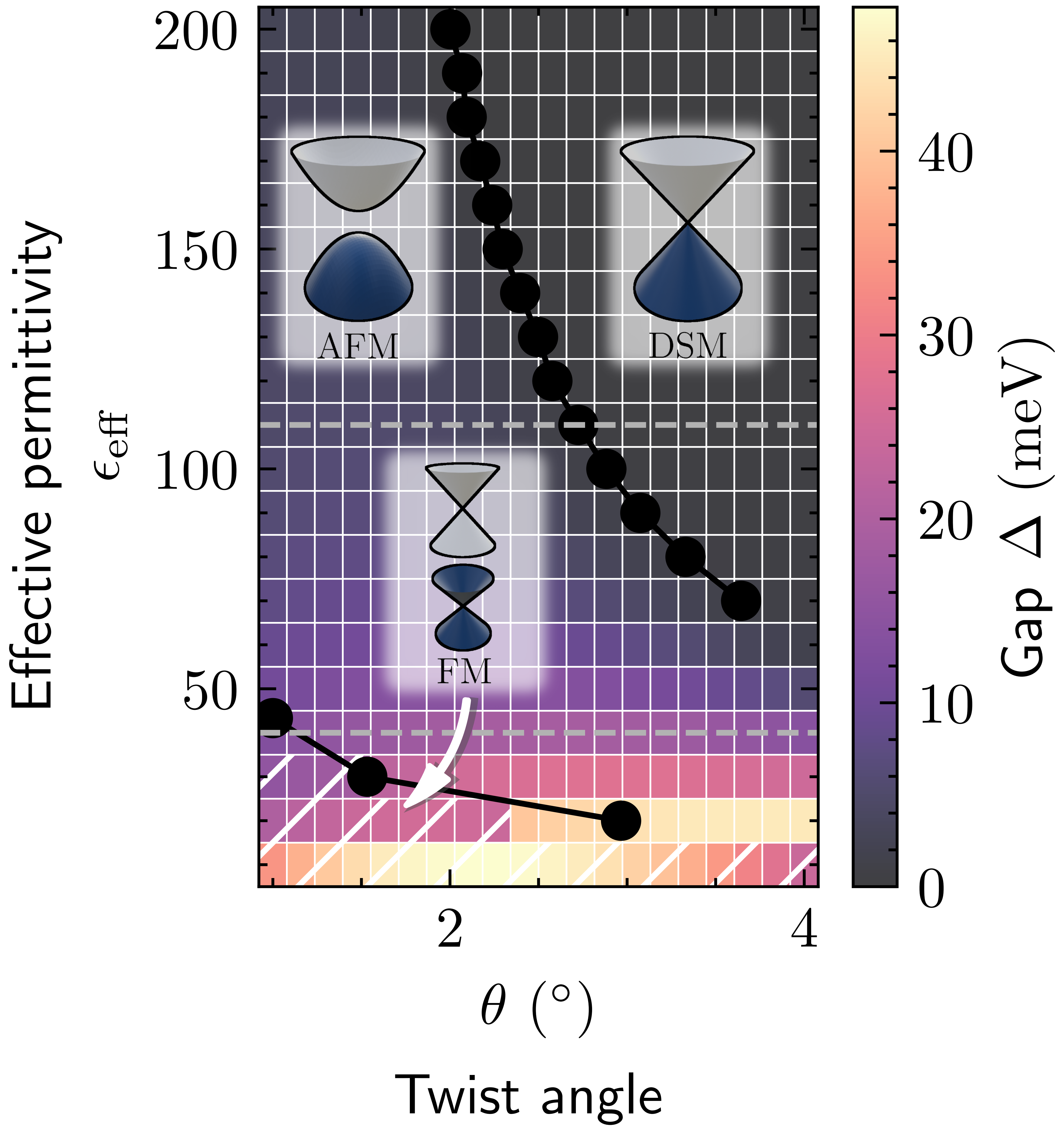

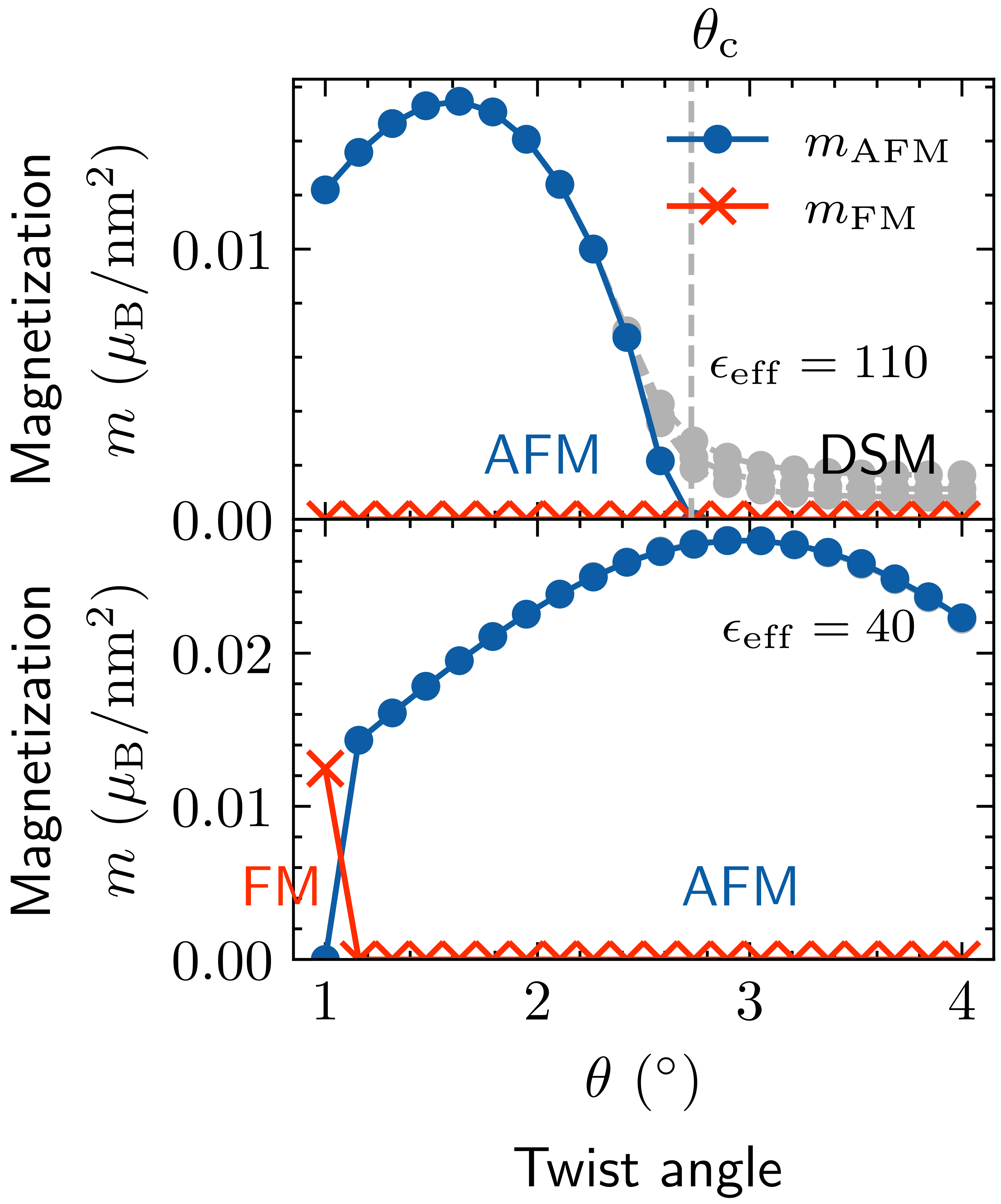

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DSM-to-AFM quantum criticality

Universal field theory:

$$\mathcal{L} = \frac{1}{2}\vec{\varphi} \cdot (-\partial_\tau^2 - v_\mathrm{B}^2 \vec{\nabla}^2)\vec{\varphi} + \psi^\dagger \left(\partial_\tau + v_\mathrm{F}\gamma_0\vec{\gamma} \cdot \vec{\nabla}\right)\psi + g\vec{\varphi} \cdot \psi^\dagger\gamma_0\vec{\sigma}\psi + \dots$$

[Biedermann & LJ, arXiv:2509.04561]

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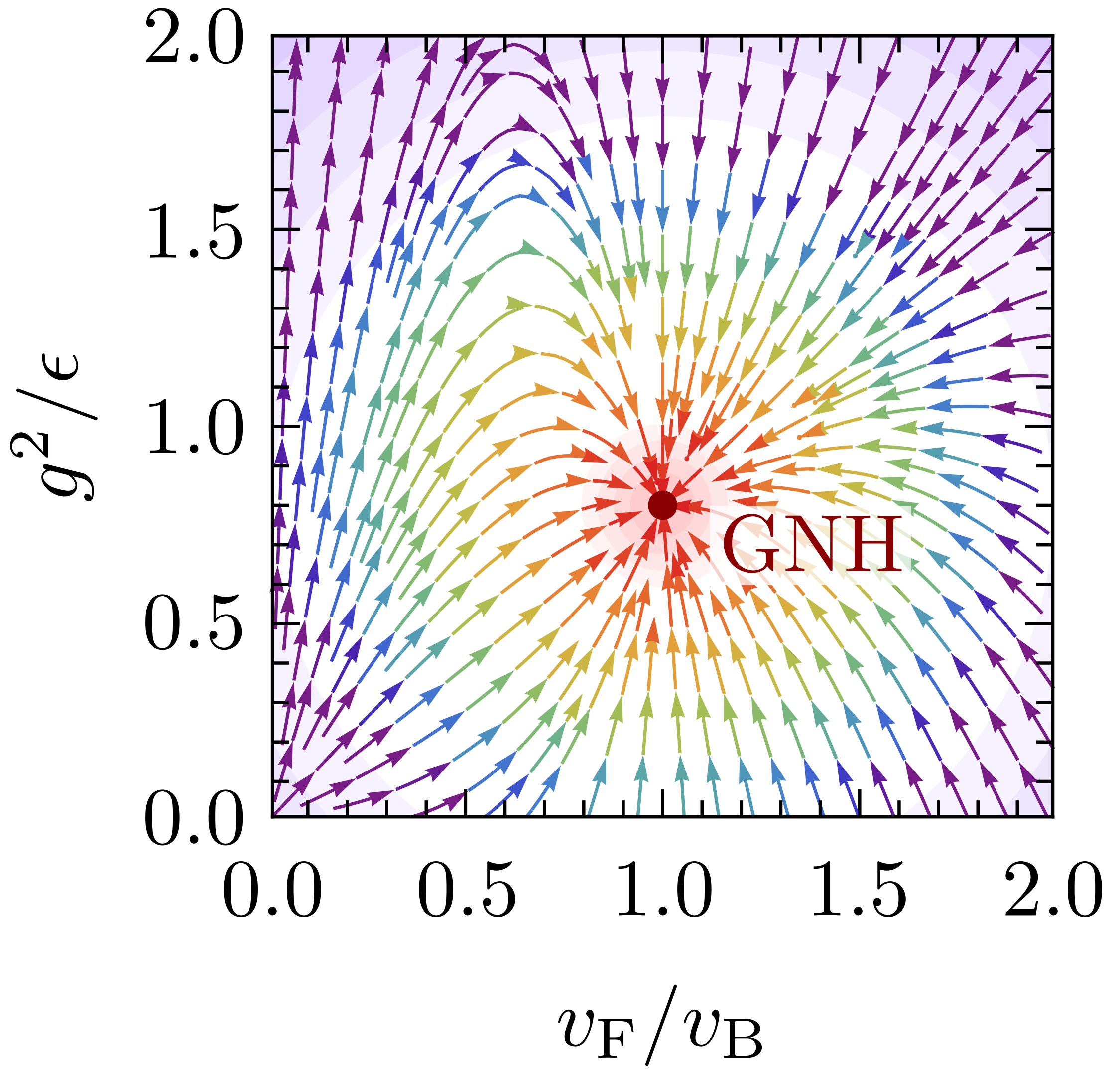
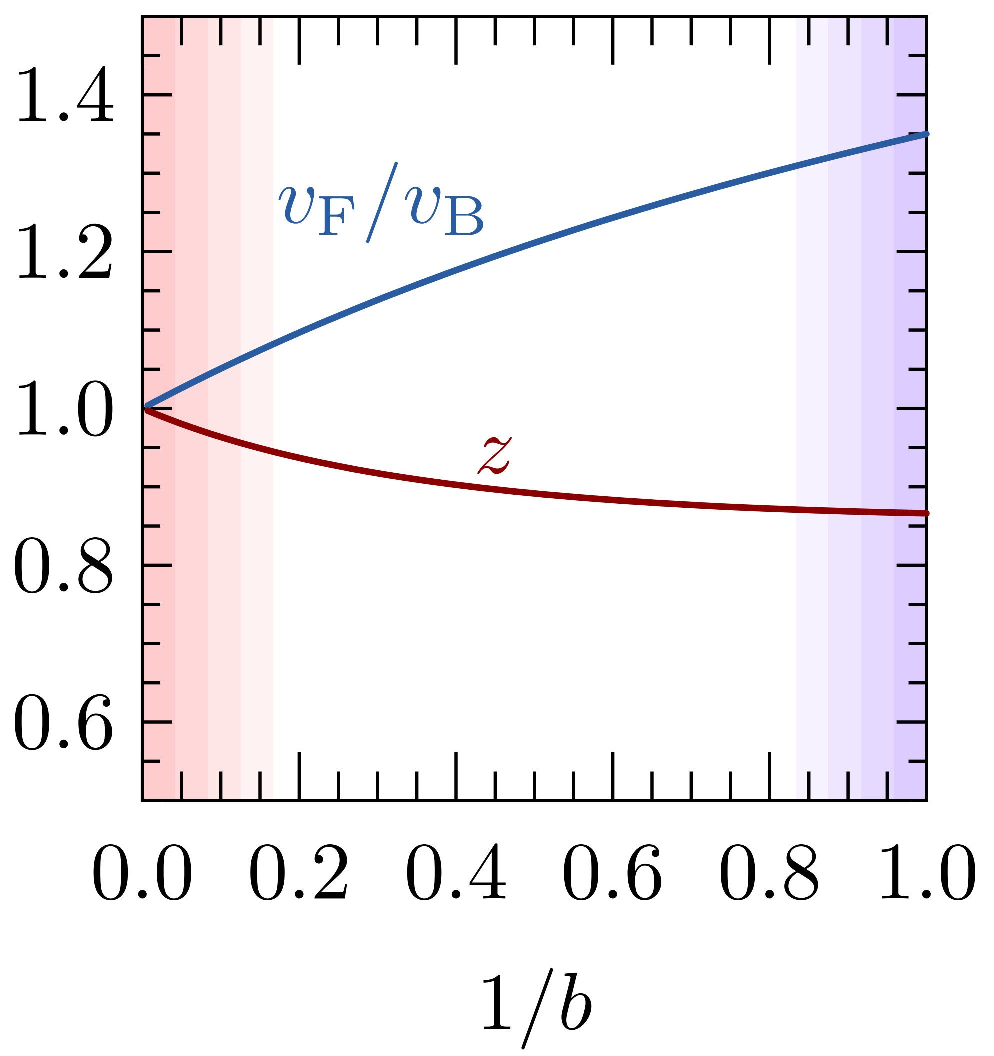
Renormalization group flow:

... in $d = 3 - \epsilon$ spatial dimensions





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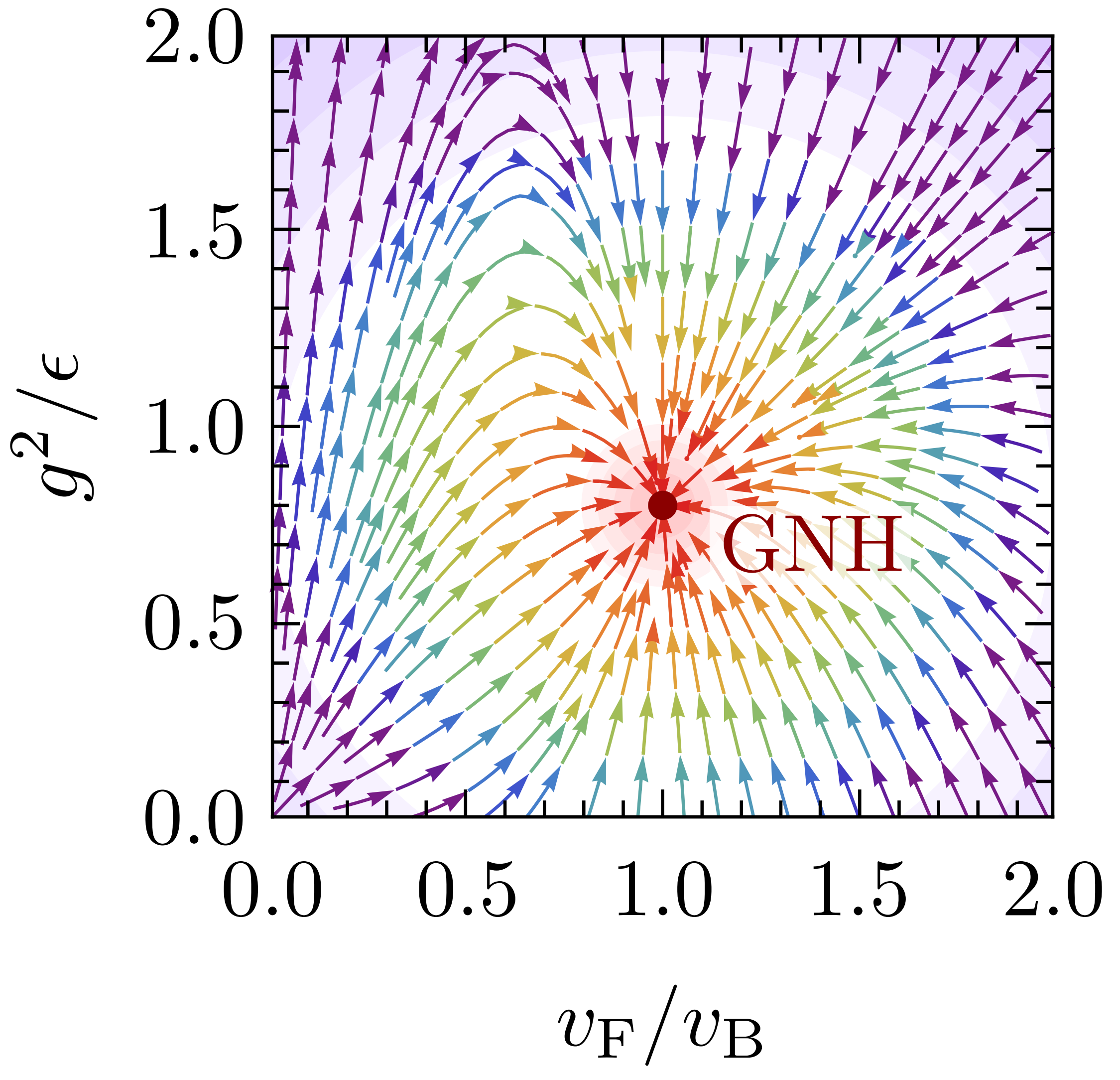

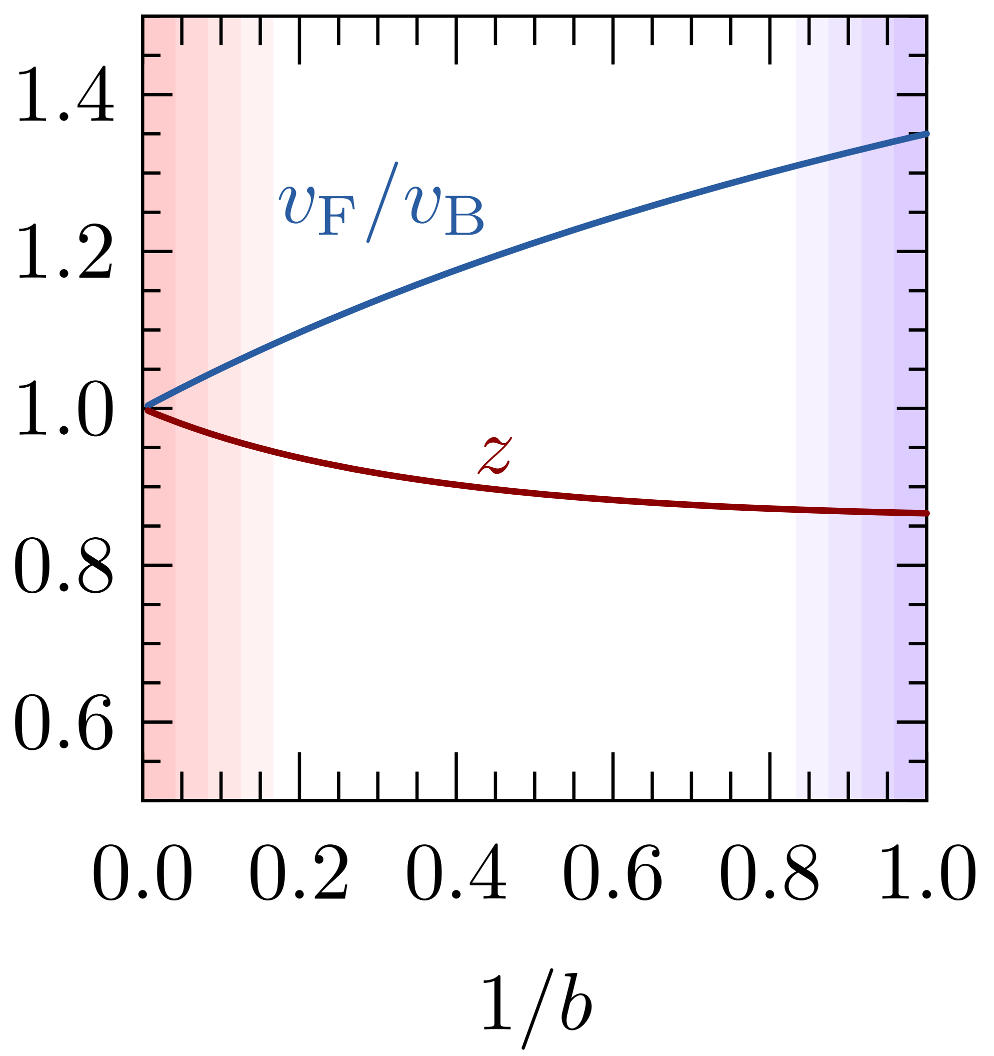

⇒ Emergent Lorentz symmetry

[Biedermann & LJ, arXiv:2509.04561]

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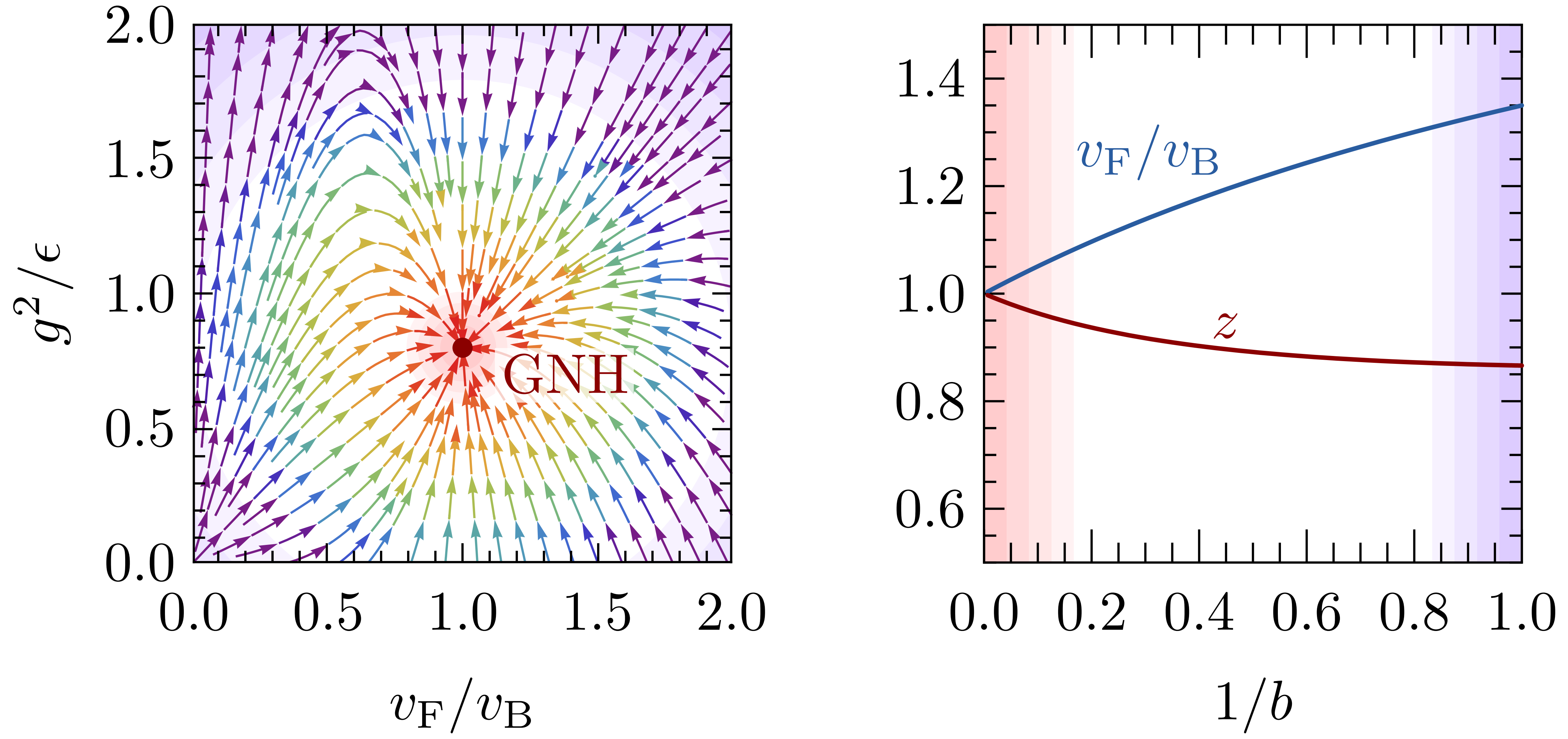


⇒ Emergent Lorentz symmetry

... characterized by set of universal critical exponents

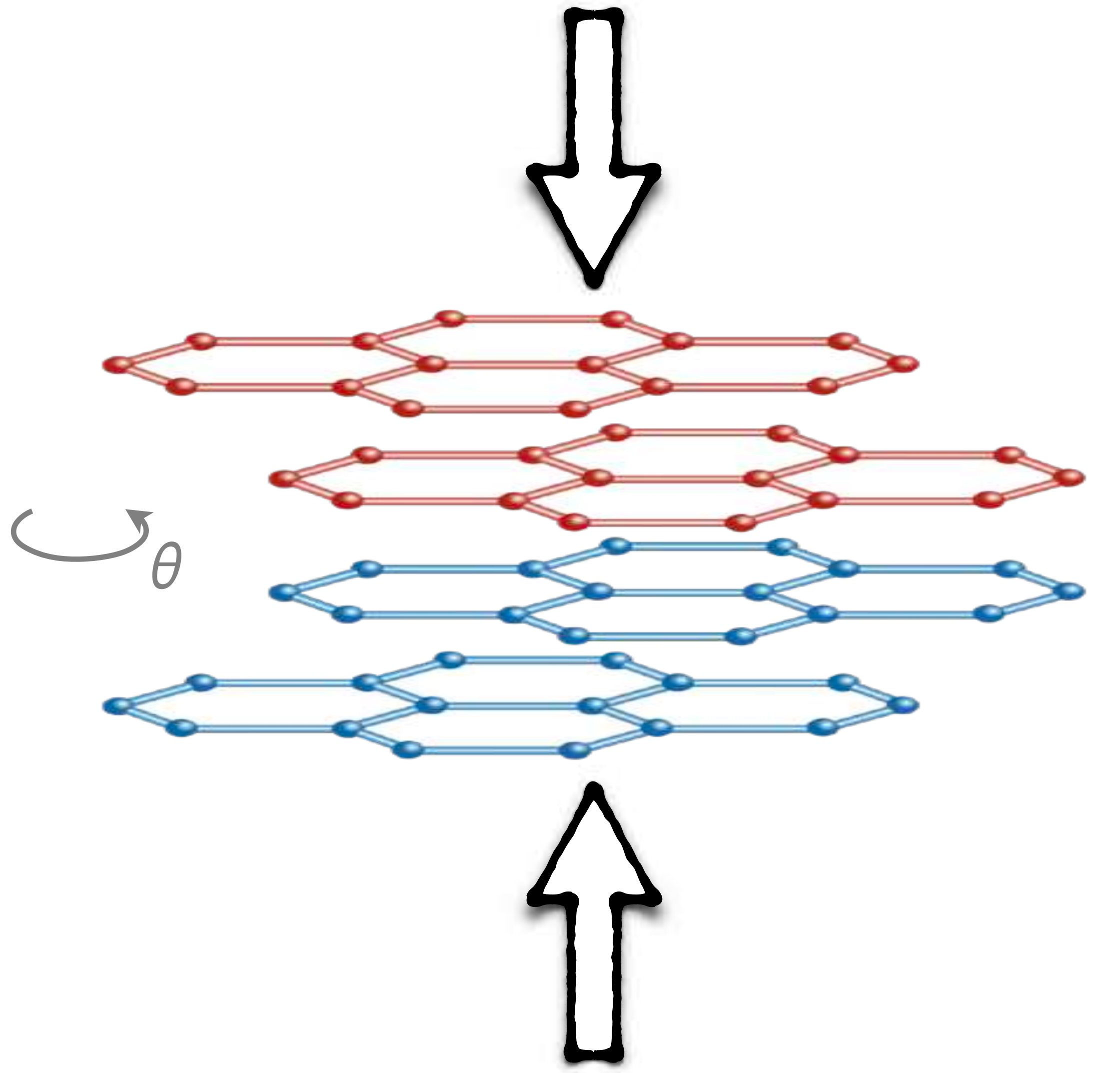
$$z = 1 \qquad \nu = \frac{1}{2} + \frac{21\epsilon}{55} + \mathcal{O}(\epsilon^2)$$

$$\eta_\psi = \frac{3\epsilon}{10} + \mathcal{O}(\epsilon^2) \qquad \eta_\phi = \frac{4\epsilon}{5} + \mathcal{O}(\epsilon^2)$$

⇒ Gross-Neveu-Heisenberg universality

[Biedermann & LJ, arXiv:2509.04561]

Pressure-tuned transition



… from selfconsistent Hartree-Fock analysis

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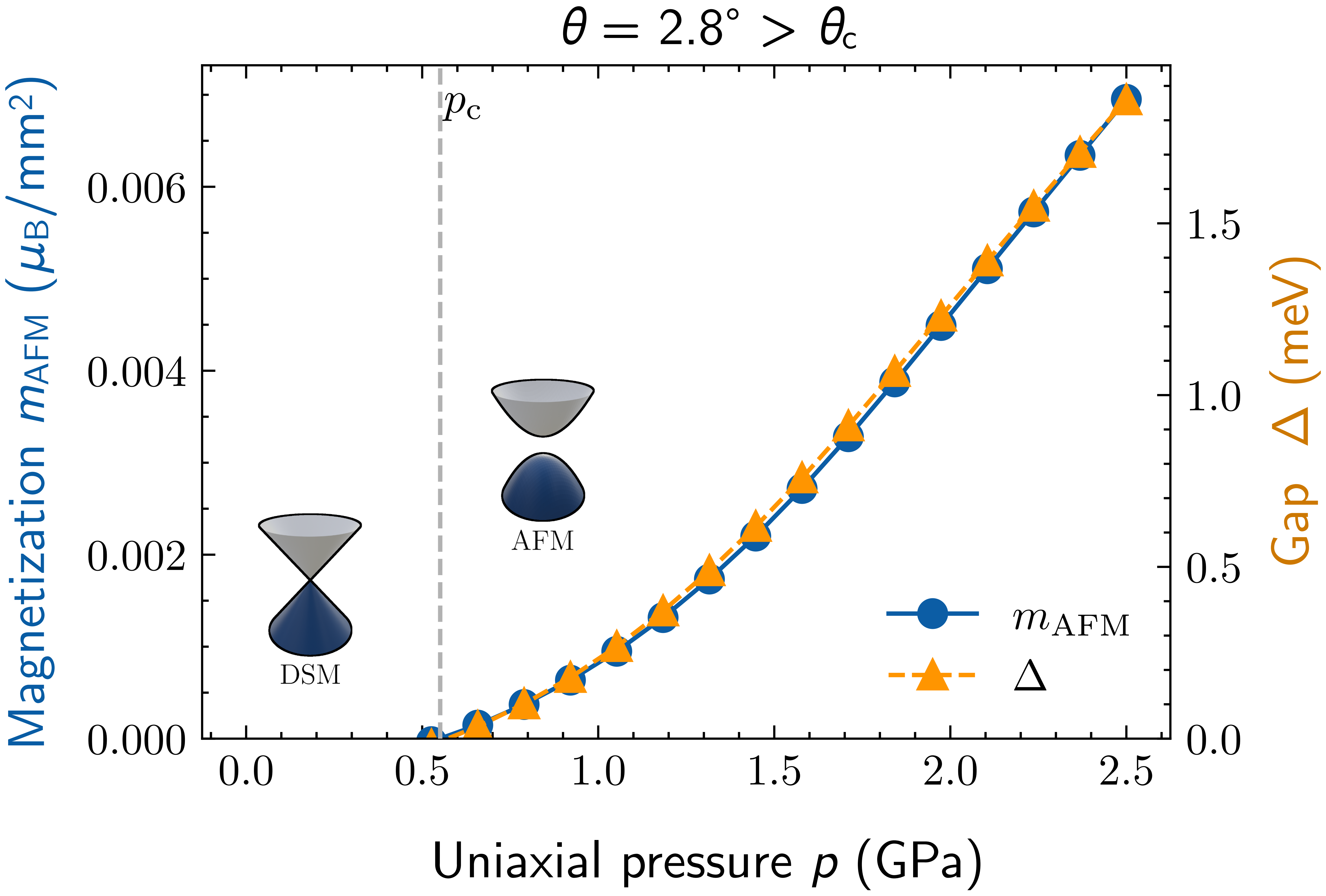
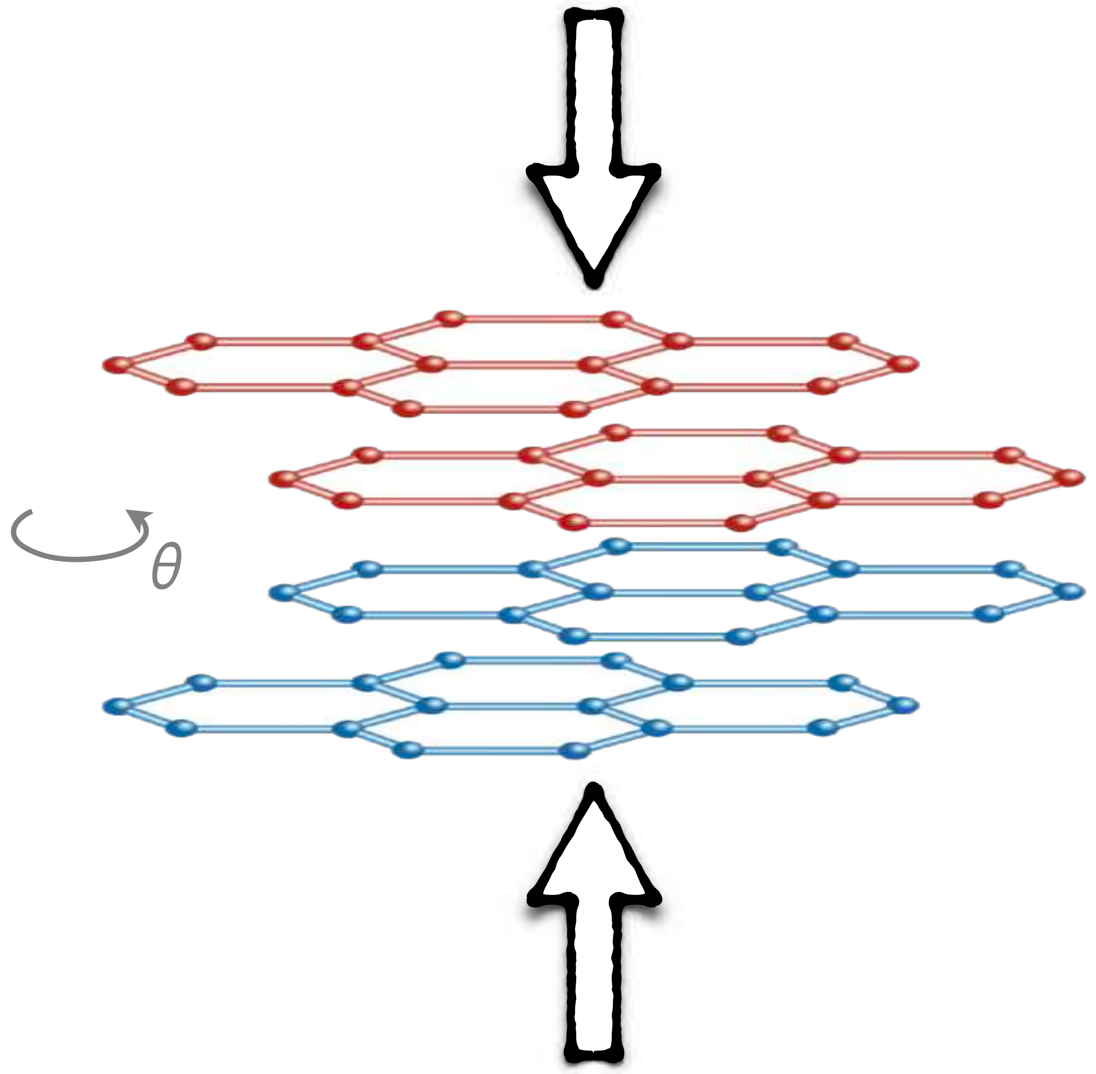
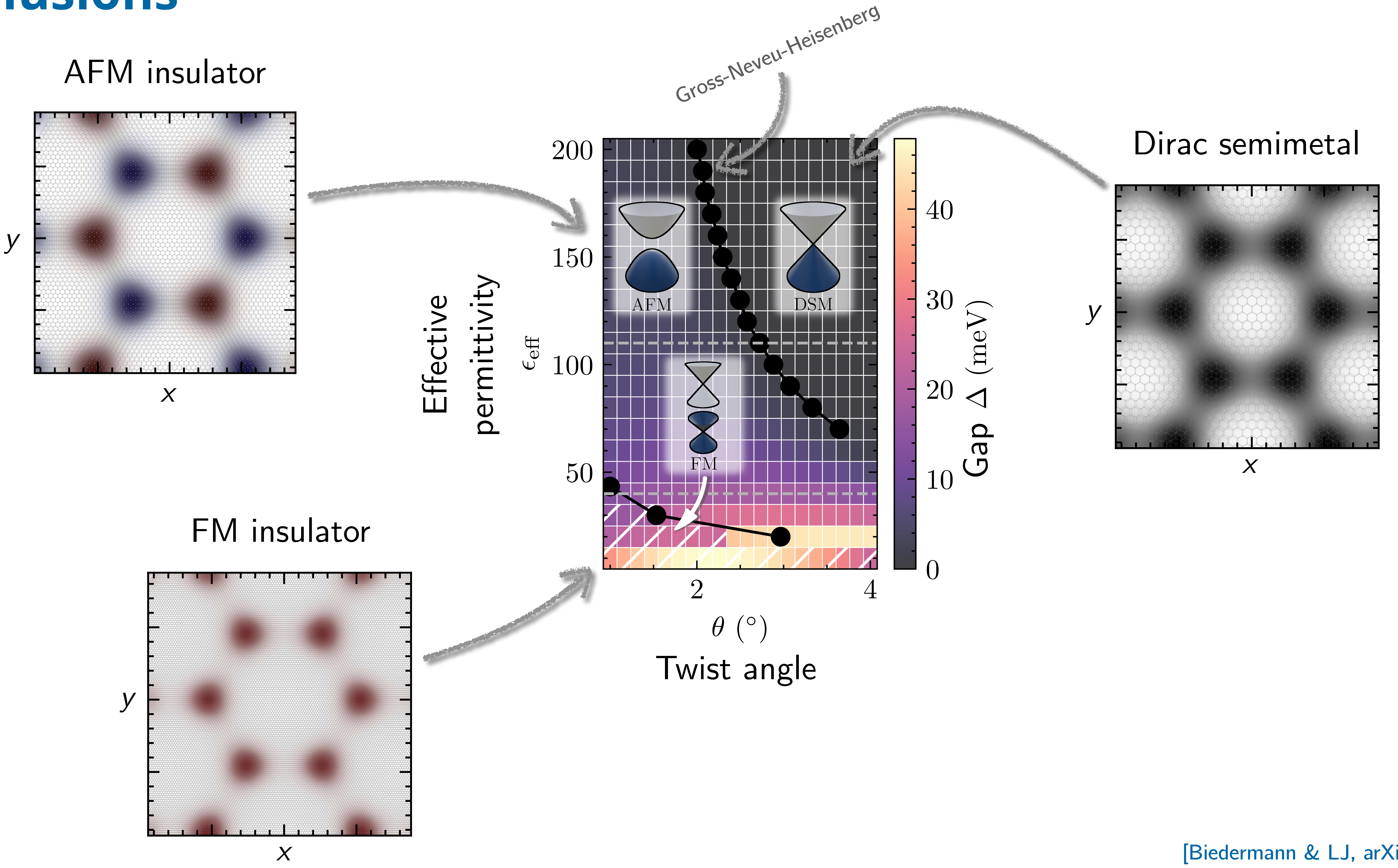




… from selfconsistent Hartree-Fock analysis

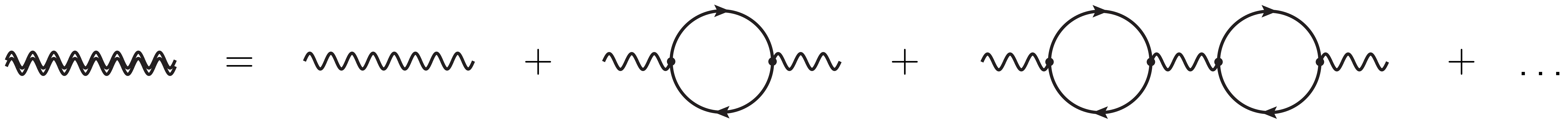
Conclusions



[Biedermann & LJ, arXiv:2509.04561]

Internal screening

Screened Coulomb interaction:



... RPA in atomistic tight-binding model

[Biedermann & LJ, arXiv:2509.04561]

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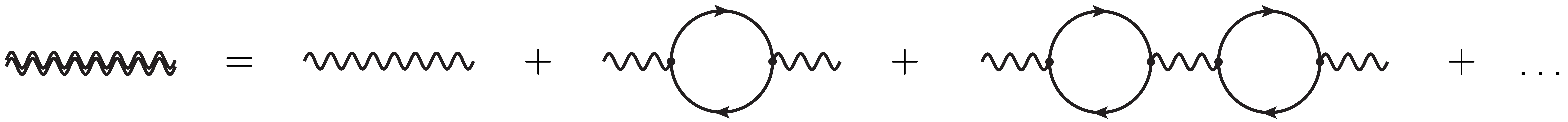
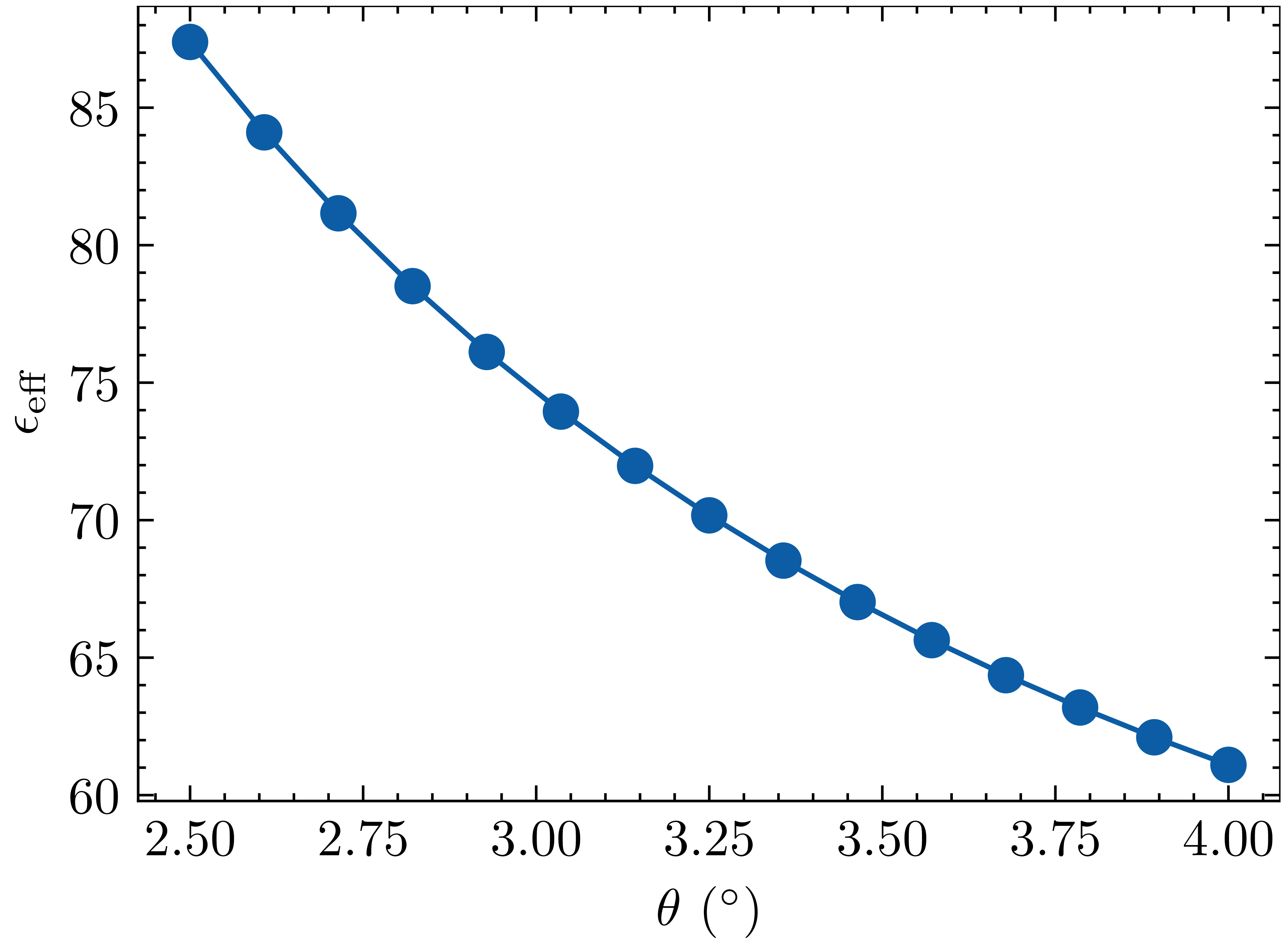


... RPA in atomistic tight-binding model

Effective dielectric permittivity:



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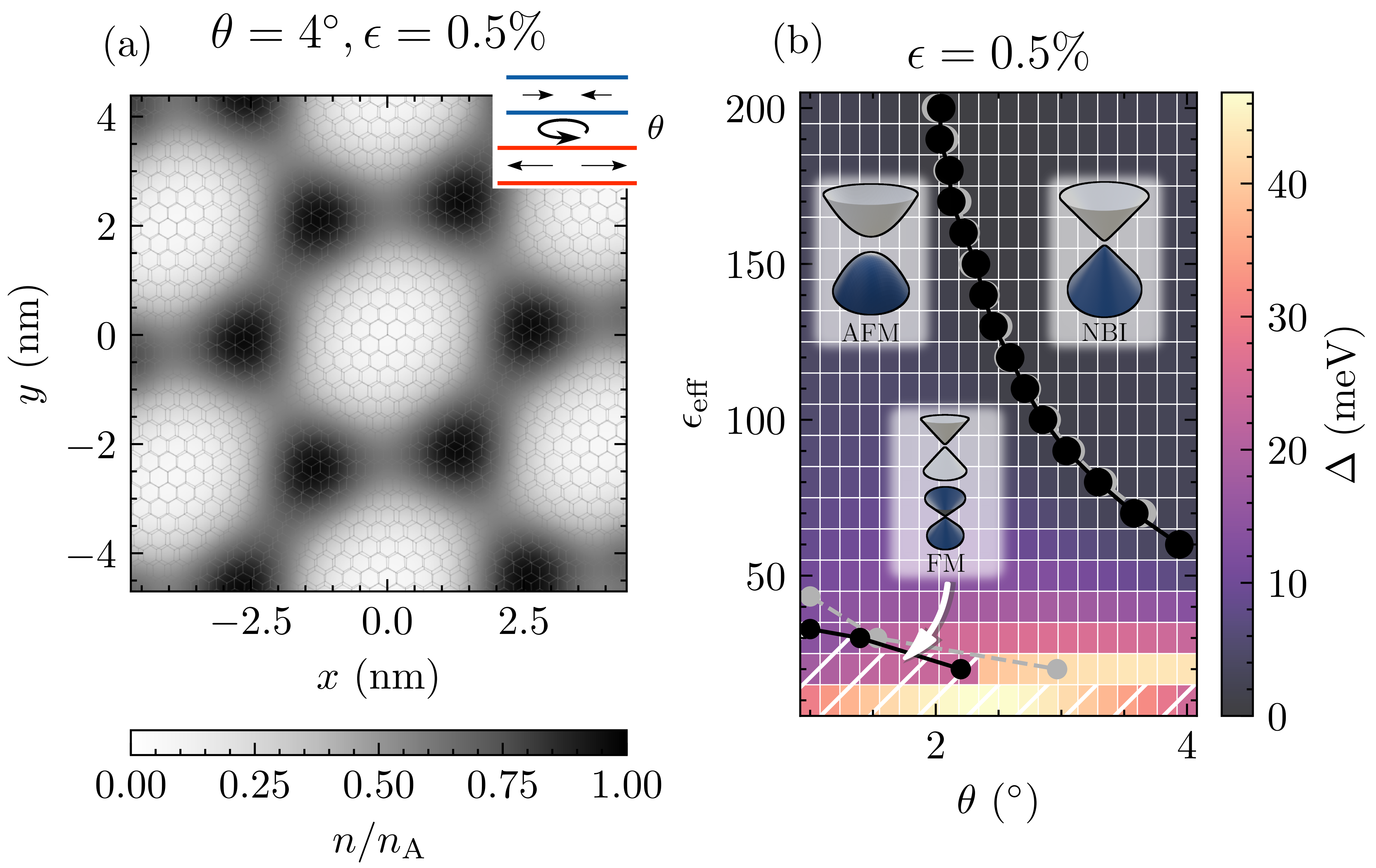
Twisted double bilayer WSe_2: Heterostrain



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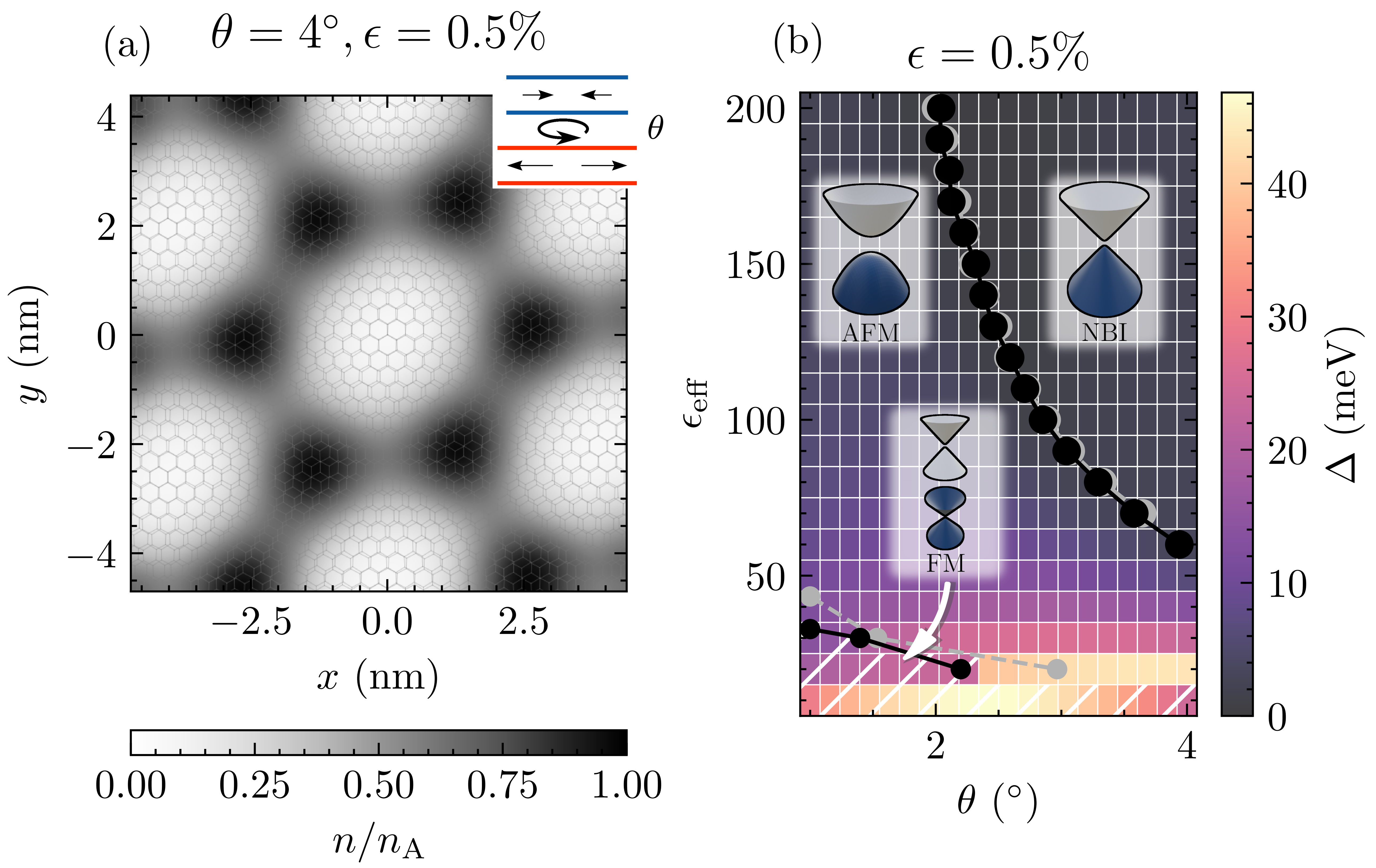
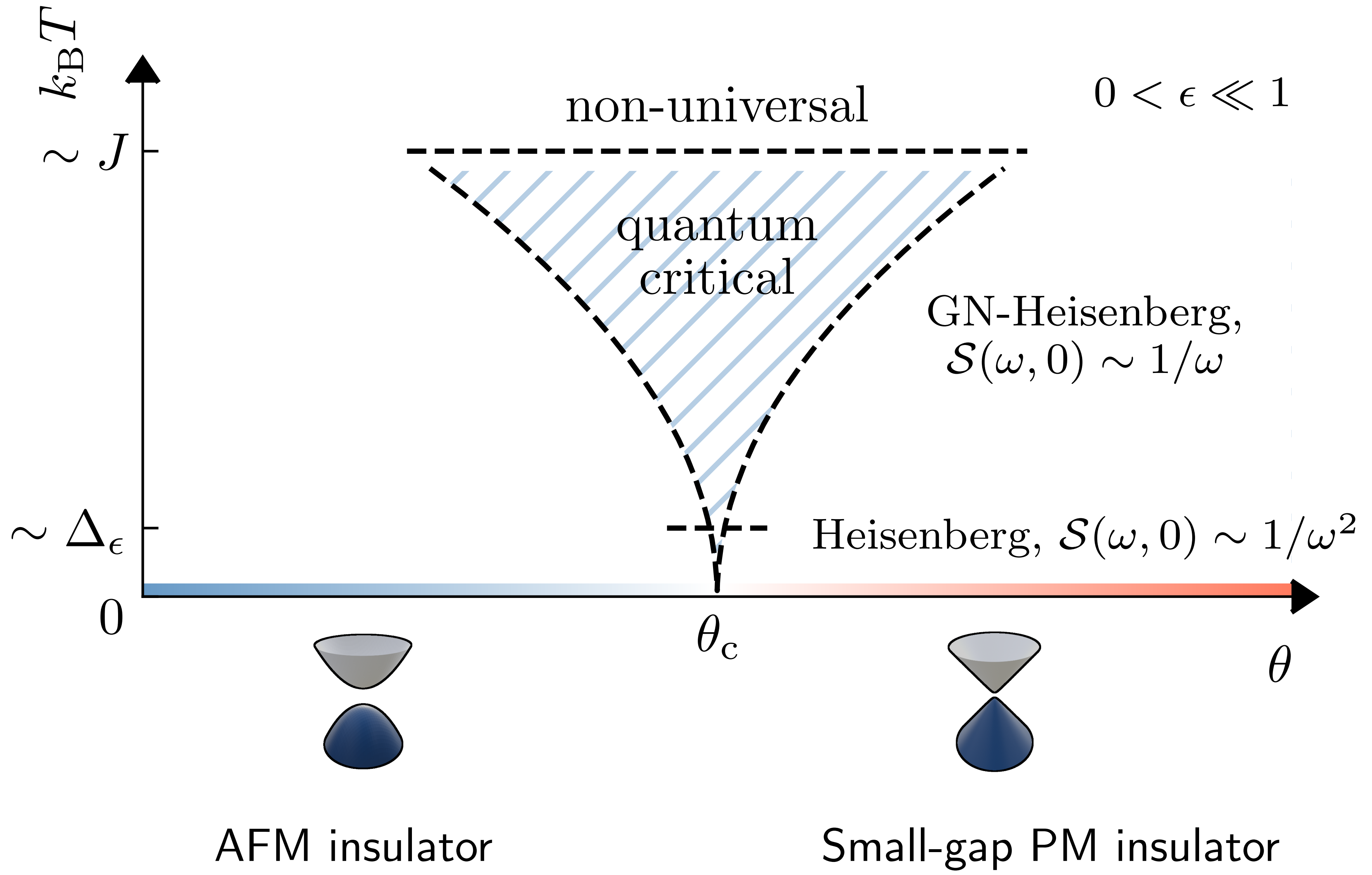
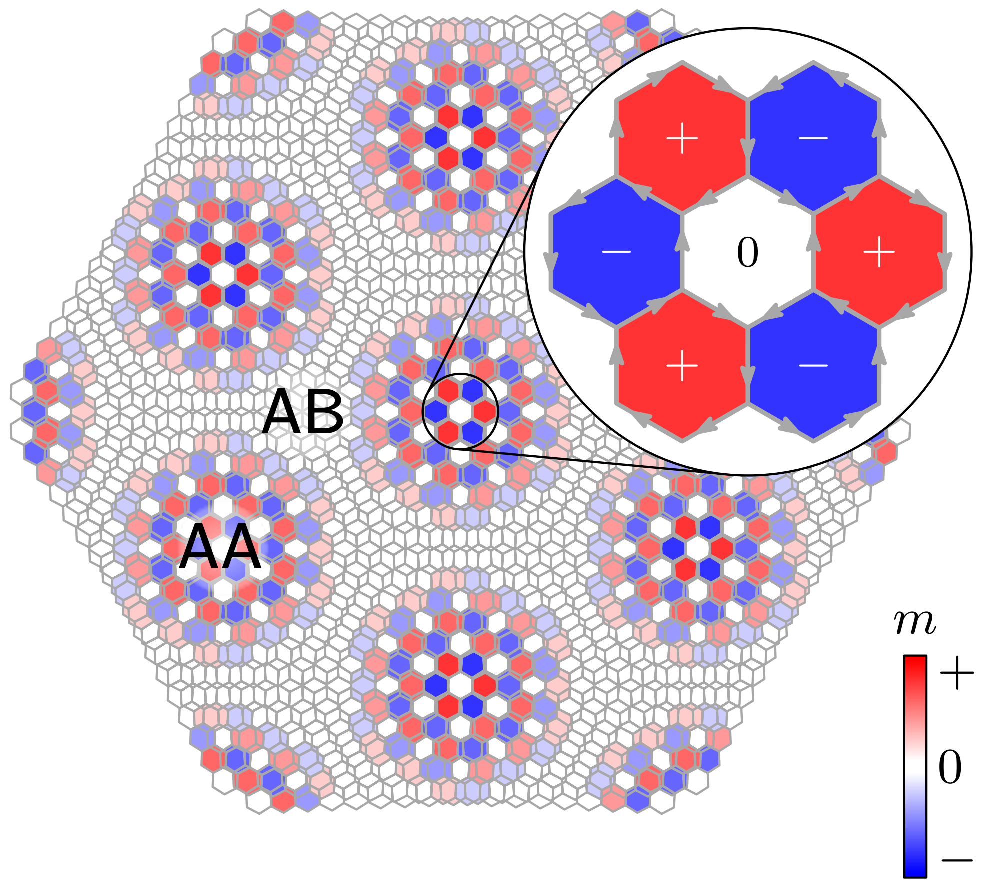


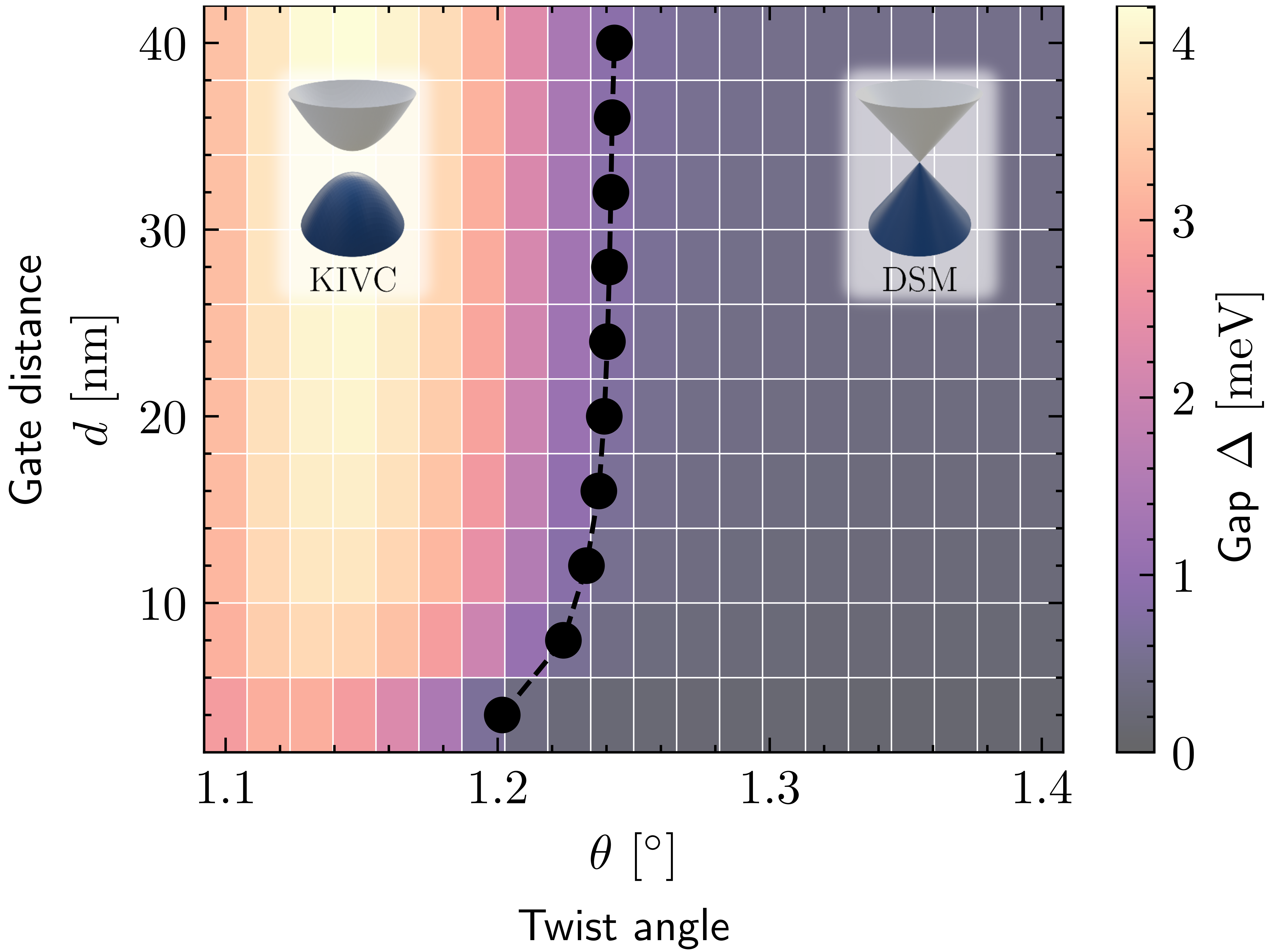


[Biedermann & LJ, arXiv:2509.04561]

Twisted bilayer graphene

Kramers intervalley-coherent insulator





[Biedermann, LJ, PRB '25]

Gross-Neveu-Heisenberg universality

Universality field theory:

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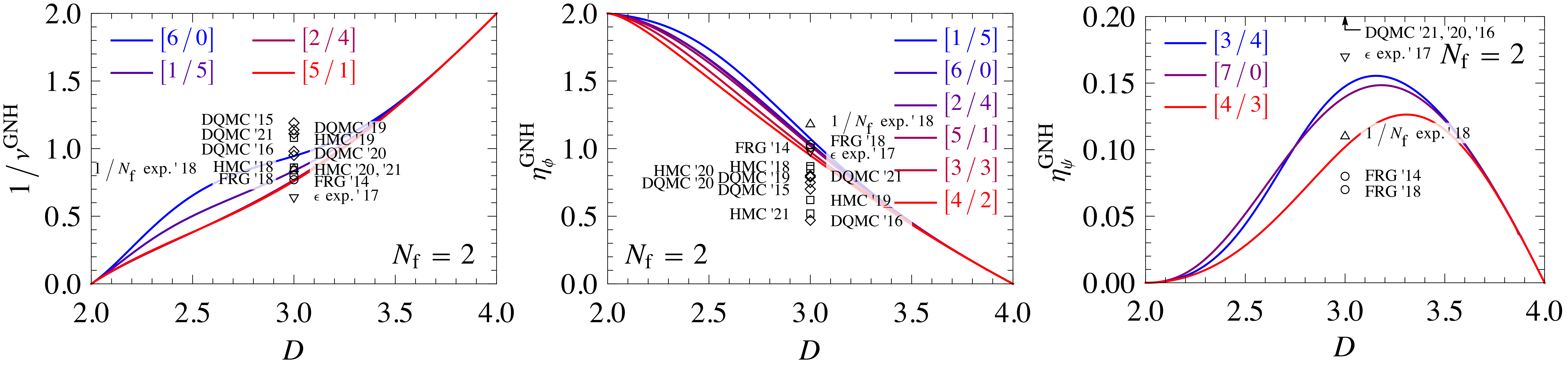
Critical exponents:



[Ladovrechis, Ray, Meng, LJ, PRB '23]