5. Exercise sheet for Theoretical Femtosecond Physics

PD Dr. Frank Großmann, Summer 2017

1*: *Volkov wavefunction*

Using Gaussian Wavepacket Dynamics calculate the wavefunction of a free electron in a laser field with the potential

$$V_{\mathrm{L}}(x,t) = ex\mathcal{E}_0\cos(\omega t)$$

in length gauge.

a) Determine the solutions (p_t, q_t) of the classical equations of motion with the initial conditions $(0, q_0)$. Then calculate the classical kinetic energy and its derivative and average the results over one period $T = 2\pi/\omega$ of the external field. Interprete the results.

b) Use the result for α_t from the free particle case (why is this possible?).

c) Employing integration by parts, show that

$$\int_0^t \mathrm{d}t' L = -\int_0^t \mathrm{d}t' \frac{p_{t'}^2}{2m_{\mathrm{e}}} + q_t p_t$$

holds. Use this result to determine the phase $\delta_t = \int_0^t \mathrm{d}t'(L - \alpha_{t'})$ and insert everything in the GWD expression. Why is the final result exact?

2: *Rotating Wave Approximation*

Consider a two-level system interacting with a monochromatic laser field.

a) Average the TDSE over times long in comparison to $1/(\omega+\omega_{21})$ in order to motivate neglecting the counter-rotating terms.

b) Using the initial conditions $d_1(0) = 1$ and $d_2(0) = 0$ give explicit expressions for C and D defined in the lectures and for $d_1(t)$ and $d_2(t)$. Depict $|d_2(t)|^2$ for resonance as well as for off-resonance.

3: *Dipole Matrix Elements*

An electron shall move in an inversion symmetric potential $V(x) = V(-x)$ in one spatial dimension.

a) Show that the eigenfunctions of the TISE fulfill

$$\psi_{2n}(x) = \psi_{2n}(-x) \qquad \text{resp.} \qquad \psi_{2n+1}(x) = -\psi_{2n+1}(-x)$$

and that diagonal dipole matrix elements $\mu_{nn} = \langle\psi_n|e\hat{x}|\psi_n\rangle$ therefore always vanish.

b) Calculate the dipole matrix element between the ground and the first excited state of the harmonic oscillator

$$V(x) = \frac{1}{2}m_{\mathrm{e}}\omega_{\mathrm{e}}^2 x^2$$

with a frequency in the visible range, $\omega_{\mathrm{e}} = 3.14 \times 10^{15}\,\mathrm{s}^{-1}$. Determine the Rabi frequency in the resonance case for 3 different field strengths $\mathcal{E}_0 = 1, 10^6, 10^{10}\,\mathrm{V\,cm}^{-1}$. Is the condition for the applicability of the RWA fulfilled for all field strengths?

4*: *Rosen-Zener model*
Consider the TDSE for the two-level Rosen-Zener model.

a) Prove the equation for c_2 that can be gained by the elimination of c_1.
b) Transform the independent variable with the help of

$$z = \frac{1}{2}(\tanh\frac{t}{T_{\mathrm{p}}} + 1).$$

What is the differential equation for $c_2(z)$?
c) Consider the special case $\epsilon = 0$ and determine $c_2(t = \infty)$ for the initial conditions $c_2(t = -\infty) = 0$ and $c_1(-\infty) = 1$.
Hint: Use the hypergeometric function (see I. S. Gradshteyn and I. M. Rhyzik, *Tables of Integrals Series and Products* (Academic Press, San Diego, 1994), Section 9.1, or http://dlmf.nist.gov/15) and
$F(a, b; c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$, $\qquad \Gamma(1-x)\Gamma(x) = \frac{\pi}{\sin(\pi x)}$, $\qquad \Gamma(3/2) = \sqrt{\pi}/2$

to be discussed on 21.06.2017
exercises with an asterisk may be handed in for examination purposes