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One-dimensional physics
Problem set 2

Summer term 2017

1. Particle-hole excitations in one and two dimensions **2 Points**

Consider free electrons with a dispersion relation

$$E(\mathbf{k}) = \frac{k^2 - k_F^2}{2m}, \tag{1}$$

where $k = |\mathbf{k}|$, and with k_F being the Fermi momentum. Analyze the allowed range of values for the energy of particle-hole excitations,

$$\omega_{\mathbf{k}}(\mathbf{q}) = E(\mathbf{k}+\mathbf{q}) - E(\mathbf{k}), \tag{2}$$

as a function of $q = |\mathbf{q}|$ in one and two dimensions (at zero temperature). Plot your results, and interpret them in terms of the quasiparticle character of density waves.

2. The chiral anomaly in three dimensions **4 Points**

The chiral anomaly is not only important in quantum wires, but also in a special class of three-dimensional materials called "Weyl semimetals", an example being TaAs. Measuring length in units such that the lattice distance $a = 1$, and energy in units such that the hopping $t = 1$, a simple tight-binding model for a Weyl semimetal is given by

$$H = \sum_{\mathbf{k}} \left(c_\uparrow^\dagger(\mathbf{k}), c_\downarrow^\dagger(\mathbf{k})\right) \mathcal{H}_{\mathbf{k}} \begin{pmatrix} c_\uparrow(\mathbf{k}) \\ c_\downarrow(\mathbf{k}) \end{pmatrix}, \tag{3}$$

$$\mathcal{H}_{\mathbf{k}} = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + \left[\cos(k_z) - \cos(k_0) + 4 - 2\cos(k_x) - 2\cos(k_y)\right]\sigma_z, \tag{4}$$

where $0 < k_0 < \pi$ is a parameter of the Hamiltonian, while $c_s^\dagger(\mathbf{k})$ creates an electron of spin $s = \uparrow, \downarrow$ and momentum $\mathbf{k}$, and with σ_j being the Pauli matrices.

a) **1 Point**

Which momenta are important for a low-energy description of the system? Expand the Hamiltonian close to these momenta to the lowest non-vanishing order in momentum.

b) **2 Points**

Now specialize to the case $k_0 = \pi/2$, and consider the Weyl semimetal described by Eq. (3) to be placed in a magnetic field along the z-axis, $\mathbf{B} = B\hat{e}_z$. Neglect the Zeeman effect, and find the energies of the Landau levels associated with the low-energy Hamiltonian derived in part a). To this end, you may find it useful to work in the Landau gauge $\mathbf{A} = Bx\hat{e}_y$, and to choose the magnetic field such that $eB > 0$, where e is the charge of the electron. You will also find it necessary to perform a Fourier transformation. To perform this transformation, you can approximate the low-energy description of the system by extensions of the low-energy Hamiltonians for the different momenta with low-energy modes to an infinite momentum range each, much like we did in the lecture for the linearized spectrum of a quantum wire. How can you estimate the Landau level degeneracy?

c) **1 Point**

After the extension to the infinite momentum range, the spectrum of one of the Landau levels is highly reminiscent of the approximate spectrum we obtained for a quantum wire. We discussed in the lecture that a quantum wire shows a chiral anomaly. Thus, what happens when an electric field $\mathbf{E}$ is applied to the Landau levels of a Weyl semimetal (at zero temperature)? Argue that the steady state current along the direction of the magnetic field that is established in the presence of backscattering between the two sets of low-energy modes is proportional to $\mathbf{E} \cdot \mathbf{B}$ (you can assume the backscattering to be described by a relaxation time approximation, and to be proportional to the population imbalance). Which relatively simple experiment can thus detect the chiral anomaly in Weyl semimetals?