

# Lecture on "Quantum Phase Transitions"

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SoSe 18  
L. Janssen

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## Contents

- 1 Introduction
- 2 Classical phase transitions and universality
- 3 Statistical mechanics and path integrals
- 4 Renormalization group
- 5 Theoretical models for quantum phase transitions
- 6 Quantum phase transitions : Primer
- 7 Magnetic quantum phase transitions
- 8 Quantum phase transitions of bosons and fermions
- 9 Applications

Mo 9:20 - 10:50 Lecture  
Fr 13:00 - 14:20 Lecture / exercise (in-turns)

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Website: <http://tinyurl.com/qpt-ss18>

Literature: see website

- Exam:
- no written exam
  - oral exam as part of rigorous/replacement exam (doctoral degree)  
or Master's exam Theoretical Physics (MSc degree) possible
  - exam prerequisite for MSc exam: written problem solutions,  $> \frac{1}{3}$  of points & presented in exercise class
  - assessment (PL) (ungraded) for Bachelor programme possible:  
written problem solutions,  $> \frac{1}{3}$  of points & presented in exercise class

# Dates (tentative!)

|    |     |         |         |  |
|----|-----|---------|---------|--|
| 1  | Mo  | 9.4.18  | (KWI) L |  |
|    | Fr  | 13.4.18 | L       |  |
|    | Mo  | 16.4.18 | L       |  |
|    | Fr  | 20.4.18 | E       | : Landau functional for 1 <sup>st</sup> ord. trans. / Two order param. |
| 5  | Mo  | 23.4.18 | L       |  |
|    | Fr  | 27.4.18 | L       |  |
|    | Mo  | 30.4.18 | L       |  |
|    | Fr  | 4.5.18  | E       | : Tricrit. point AFM / Scaling Hypothesis                              |
|    | Mo  | 7.5.18  | L       |  |
| 10 | Fr  | 11.5.18 | L       |  |
|    | Mo  | 14.5.18 | L       |  |
|    | Fr  | 18.5.18 | E       | : Part. function for free fields / Part. function and cont. limit      |
|    | Mo  | 28.5.18 | L       |  |
|    | Fr. | 1.6.18  | L       |  |
| 15 | Mo  | 4.6.18  | L       |  |
|    | Fr  | 8.6.18  | E       | : Scaling dimension of higher-order op. / Perturb. to Wilson-Fisher FP |
|    | Mo  | 11.6.18 | L       |  |
|    | Fr  | 15.6.18 | L       |  |
|    | Mo  | 18.6.18 | L       |  |
| 20 | Fr  | 22.6.18 | E       | : Classical Ising chain / Rel. between gap and corr. length            |
|    | Mo  | 25.6.18 | L       |  |
|    | Fr  | 29.6.18 | L       |  |
|    | Mo  | 2.7.18  | L       |  |
|    | Fr  | 6.7.18  | E       | : Quantum Ising chain  |
| 25 | Mo  | 9.7.18  | L       |  |
|    | Fr  | 13.7.18 | L       |  |
|    | Mo  | 16.7.18 | L       |  |
| 28 | Fr  | 20.7.18 | E       | : Shift exponent / SM-to-CDW transition in graphene                    |

7 exercises  
21 lectures

# 1 Introduction

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Phase (a definition attempt): Equilibrium state of matter whose qualitative characteristics do not change upon small change of external parameters ("stable")

⇒ thermodynamic potential varies analytically

Phases are:

- characterized by symmetry of  $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$  (density operator)
- separated by phase transitions

Phase transition: Point in parameter space at which equilibrium properties of a system change qualitatively ("unstable")

⇒ thermodynamic potential nonanalytic

Phase transitions can:

- be continuous or discontinuous
- occur at finite  $T$  ("thermal") or  $T=0$  ("quantum")

Quantum phase transition (QPT): Phase transition at  $T=0$ , which occurs upon varying non-thermal control parameter (pressure, magnetic field, chemical composition, ...)

⇒ ground-state energy nonanalytic in control parameters

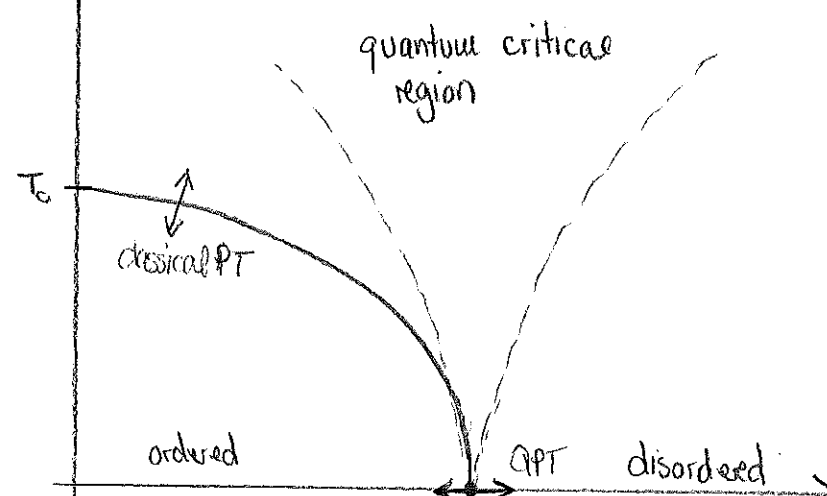
Remark: QPT apparently driven by "quantum fluctuations"

Disclaimer:  $T=0$  ⇒ quantum described by single phase-coherent (many-body) wavefunction

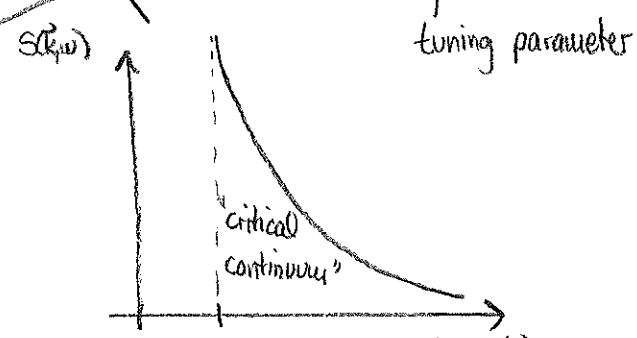
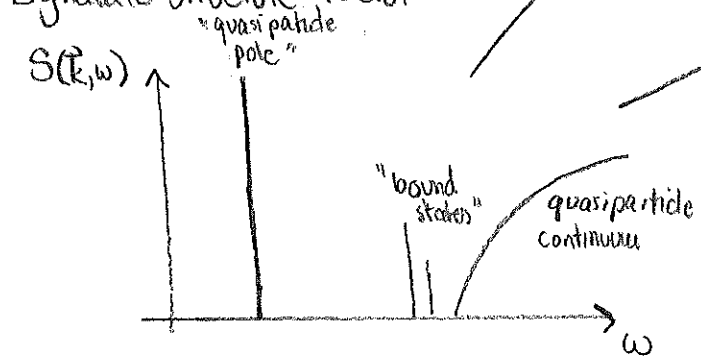
"fluctuations": deviations from a reference state (e.g., ordered magnet)

Experimental relevance:

Quantum phase diagram:  $T$



Dynamic structure factor:  $S(\vec{k}, \omega)$

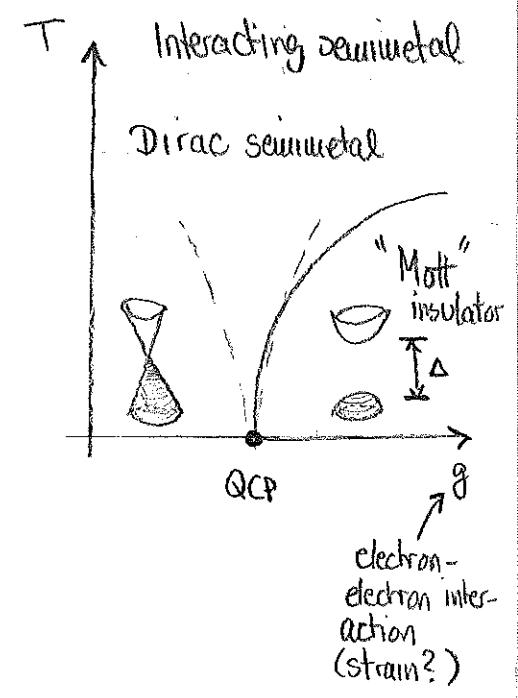
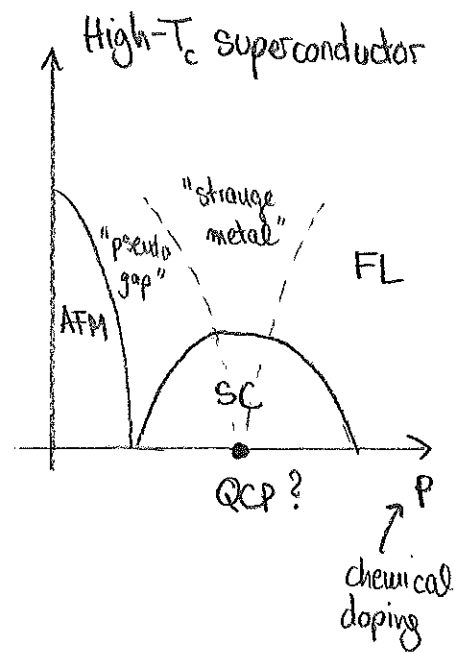
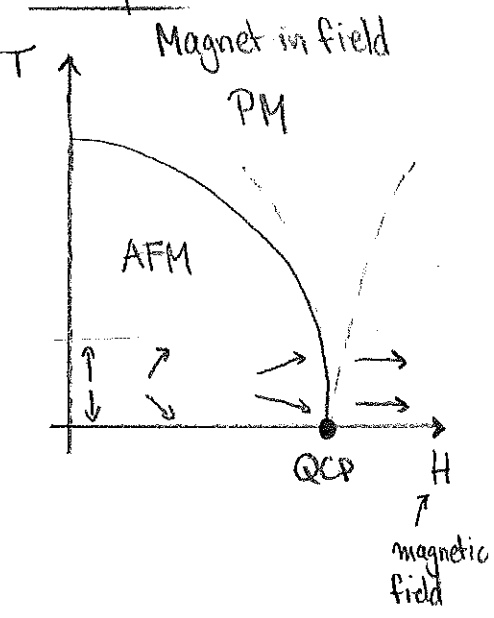


e.g., Fermi liquid:  $C_v \sim T$   
 gapped magnet:  $C_v \sim \frac{1}{T} e^{-\frac{\Delta}{k_B T}}$  (with  $\Delta$  labeled as 'gap')

e.g.,  $C_v \sim T^{\frac{d}{2}}$  (with  $d$  labeled as 'dimension')  
 "dynamical critical exponent"

"novel state of matter"

Examples:



Other examples:

- disordered system ("Anderson transition")
- cold atoms on an optical lattice
- quark matter ("chiral symmetry breaking" in QCD)