8. Quenched disorder and glassiness

Defects are ubiquitous in solids (missing atoms, misplaced atoms, ...). They may change the system's properties qualitatively ! (Example: Anderson localization)

„ quenched " ≙ not in thermal equilibrium

≙ Hamiltonian parameters describing disorder do not vary (as function of time) in response to magnetic degrees of freedom (or other parameter variations)

(as opposed to „ annealed disorder ", where disorder d. o. f. equilibrate with rest of system)

Defects break translation symmetry. It is efficient to think about defect configuration as random. Then, calculating observables in systems with quenched disorder involves averaging over (all) disorder configurations.

ATTN: Disorder average must be performed for each observable separately ! *

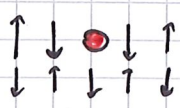
ATTN: Sometimes not only averages but also full distributions are important !

* In principle, one needs to disorder-average thermodynamic potential (or $\ln Z$) in the presence of source fields (!), and then take derivatives.

8.1. Quenched disorder in magnets

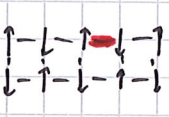
Microscopically, various types of defects can occur, which require specific modelling at the Hamiltonian level.

a) Missing magnetic atom or magnetic atom replaced by non-magnetic atom: spin vacancy



b) Extra magnetic atom:
Extra (impurity) spin

c) Lattice defect on non-magnetic sites:
Bond impurity (as exchange paths are modified)



d) Crystallographic dislocations, disclinations, domain walls, ...

⋮

Often, a particular microscopic defect causes multiple local modifications of the (exchange) Hamiltonian which are not independent. In spin-anisotropic systems, one may also get (in addition to local variations of exchange couplings) local variations of anisotropies and of g factors (due to local variations of crystalline electric field).

On the level of effective order-parameter theories (of φ^4 type), one may introduce disorder into field-theory parameters. One distinguishes

A) random mass $S = \int d^d x \, d\tau \; \left(c^2 (\partial_x \varphi)^2 + (\partial_\tau \varphi)^2 + m(x) \varphi^2 + u \varphi^4 \right)$

B) random field $S = \ldots - h(x) \varphi$

C) random Berry phase (spatially variations in Berry-phase terms encoding quantum dynamics)

Case B) applies to situations where quenched disorder couples linearly to order parameter, e.g., bond defects in a valence-bond solid.

Case A) applies if such linear coupling is forbidden, e.g., for bond defects in ordered magnets (in the absence of magnetic field: bond defect is time-reversal even, order parameter is time-reversal odd).

In frustrated systems, more complicated types of coupling may occur.

Quite generically, quenched disorder tends to weaken or destroy ordered phases: For instance, missing spins or missing bonds reduce connectivity and hence reduce stiffness of order. Defects also perturb periodically modulated states. As a result, novel disorder-driven phases may occur (but there are exceptions).

8.2. Order by quenched disorder

While quenched disorder tends to destabilize magnetically ordered states, the opposite (!) is possible in strongly frustrated magnets.

Consider a system which has a (continuously) degenerate ground-state manifold at the classical level. This degeneracy can be lifted by fluctuations, such that an ordered state is selected (order by disorder, Chap. 4). The degeneracy can also be lifted by defects, in a way that an ordered state is selected: „order by quenched disorder" (!).

Example: J_1-J_2 square-lattice Heisenberg AF (see also exercise sheet 4)

Assume $J_2 \gg J_1$.

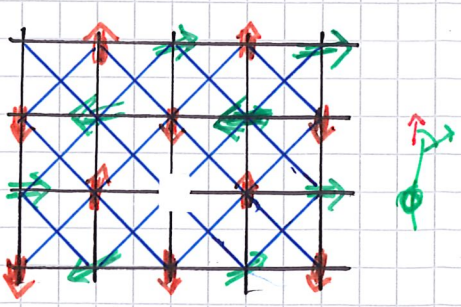
Two interpenetrating square lattices order AF; relative angle ϕ is undetermined at classical level

↝ U(1) manifold of classical ground states

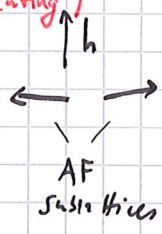
Both thermal and quantum fluct. select collinear states,

$$\Delta E_{fluct} \propto -JNS \cos^2\phi$$

Now consider effect of vacancies.
A single vacancy (on green sublattice) results in uniform moment (of size S) on this sublattice (whose direction is determined by ϕ).



AF on red sublattice and uniform moment on green sublattice interact via J_1 : optimum configuration is for $\phi = 90°, 270°$ (perpendicular staggered magnetizations) ; this is akin to behavior of collinear AF in (weak) uniform (!) field (canting)



One can show that vacancies cause an energy contribution :

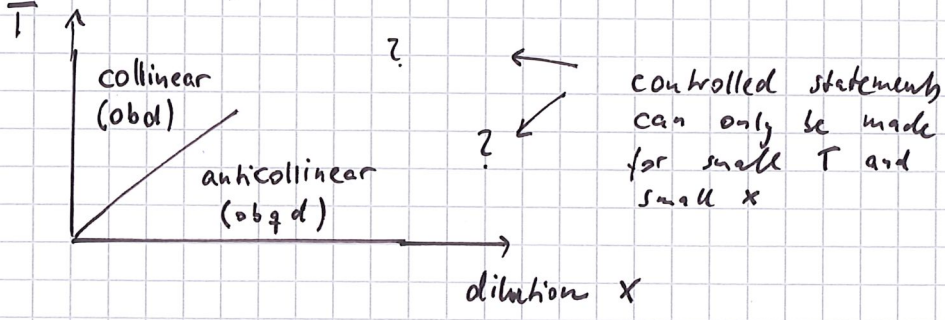
$$\Delta E_{vac} \simeq - J N_{vac} S^2 \sin^2\phi$$

(effective biquadratic interaction, sign opposite to that coming from fluct.)

$\rightsquigarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}$ preferred $\hat{=}$ anticollinear order

Apparently, this effect competes with that from thermal or quantum fluctuations !

Resulting classical phase diagram



Similar physics : Classical pyrochlore XY AF

⋮

(not many systems have been looked at carefully)

8.3. Spin glasses

A spin glass is a state (of local moments) where moments are frozen (in time) but spatially disordered (no periodic structure ⇝ no sharp magnetic Bragg peak).

This implies autocorrelations which are long-ranged in time:

$$q_{EA} = \lim_{t\to\infty} \lim_{N\to\infty} \left[\langle S_i(t_0)\, S_i(t_0+t) \rangle \right]_{av,t_0} \neq 0$$

($N \to \infty$: system size)

(Edwards-Anderson order parameter of spin glass)

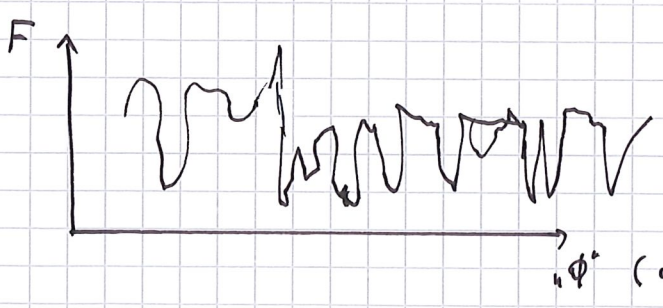
$q_{EA} \neq 0$ typically below a freezing temperature T_f.

It is debated whether or not T_f corresponds to a thermodynamic phase transition. Most people think it does ⇝ glass transition at temperature $T_g \equiv T_f$.

In Heisenberg system, the freezing of moments implies spontaneously broken SU(2) symmetry.

In contrast to non-disordered systems, a spin glass typically has many discrete (up to global symmetry transformations) locally stable states.

Free-energy landscape (in Landau sense) has many local minima which are close in energy, separated by barriers.



In the limit $N \to \infty$, some of the barriers become infinitely high, such that the system remains trapped in one (or a group of) minima $\hat{=}$ valley (instead of sampling all of phase space)

↘ Ergodicity is broken (!) below T_f.

Each valley behaves like a thermodynamic phase.

q_{EA} measures mean-square single-valley (!) local spontaneous magnetization, averaged over all valleys.

$$q_{EA} = \left[\sum_{a \leftarrow \text{valley}} P_a \, (m_i^a)^2 \right]_{av}$$

where $P_a = e^{-\beta F_a} / \sum_a' e^{-\beta F_a}$ is valley probability distribution.

Note that this is different from mean-squared local equilibrium magnetization:

$$q = [m_i^2]_{av} = \left[\left(\sum_a P_a \, m_i^a \right)^2 \right]_{av} = \left[\sum_{ab} P_a P_b \, m_i^a m_i^b \right]_{av}.$$

A spin glass below T_f is NOT in thermal equilibrium.

Spin glasses typically emerge from a combination of frustration and disorder; for concrete models see next section.

NOTE: Analogous properties apply to structural glasses, e.g. window glass.

Conceptually, a spin glass is a (semi) classical state of local moments: Spins have finite $\langle \vec{S}_i \rangle$; quantum fluct. are not important.

Broken ergodicity implies highly non-trivial dynamical behavior in spin glasses. In particular, properties depend on temperature and field history (!) of sample (because system gets trapped in different valleys) $\hat{=}$ „ageing".

Relaxation in spin glasses is typically extremely slow; processes can cover many orders of magnitude in time (up to seconds, minutes or more). Often, properties depend only logarithmically on time.

Slow relaxation manifests itself in a significant frequency dependence of (e.g.) magnetic susceptibility for small frequencies (Hz ... kHz), making ac susceptibility measurements a standard tool to detect spin-glass behavior.

8.4. Spin-glass models and beyond

„Canonical" spin-glass models combine disorder and frustration.

Edwards-Anderson model

Heisenberg model on regular lattice, with distance-dependent random interactions:

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Random J_{ij} drawn (e.g.) from Gaussian distrib

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi \Delta_{ij}}} \exp\left(-J_{ij}^2 / 2\Delta_{ij}\right)$$

with

$$[J_{ij}^2]_{av} \equiv \Delta_{ij} = \Delta(\vec{R}_i - \vec{R}_j)$$ < short-ranged / long-ranged

Simplifications: Ising (instead of Heisenberg)

$\Delta_{ij} \to \Delta$ (independent of distance $\hat{=}$ infinitely long ranged interaction)

↷ **Sherrington-Kirkpatrick model**

$$H = -\frac{1}{2} \sum_{ij} J_{ij} S_i^z S_j^z \quad , \quad [J_{ij}^2]_{av} \equiv \Delta_{ij} = \frac{J^2}{N}$$

(N = no. of spins)

Infinite-range interaction makes mean-field theory possible.

ATTN: Broken ergodicity requires m-f theory with replica symmetry breaking! (not for this lecture)

Infinite-range interactions can be employed with other interactions and other spin representations.

↷ Sachdev-Ye-Kitaev (and related) models

$$H = \frac{1}{2} \sum_{ij}' J_{ij} \vec{S}_i \cdot \vec{S}_j \qquad \text{(Heisenberg-like)}$$

with Gaussian infinite-range interaction

$$P(J_{ij}) \propto \exp\left(-J_{ij}^2 / 2J^2\right)$$

Model can be solved in different large-M limits, i.e., for spins belonging to SU(M) with different representations of SU(M). This yields:

- Semiclassical spin glass (Bray/Moore 1980)
- Gapless spin liquid (Sachdev/Ye 1993)
- Quantum spin glass (Georges/Parcollet/Sachdev 2000)

If one re-writes spins using fermionic (Majorana) representation and uses infinite-range character of interaction, one arrives at

$$H = \frac{1}{4!} \sum_{ijkl}' J_{ijkl} \chi_i \chi_j \chi_k \chi_l \qquad \text{(Kitaev 2015)}$$

again with Gaussian J_{ijkl}.

Note: Infinite-range (or all-to-all) interactions imply that system has neither geometry nor lattice!