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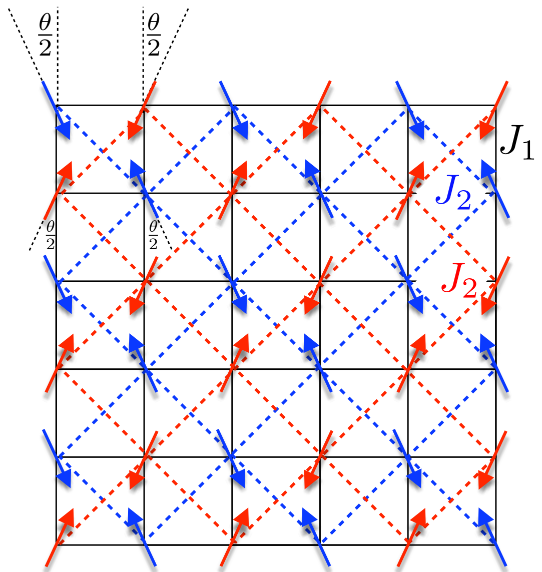
Theory of Frustrated Magnetism
Problem set 4

Summer term 2020

Order-by-disorder: J_1-J_2 square-lattice Heisenberg model — 16 Points



On the left the square lattice with nearest-neighbor (black solid lines) and next-nearest-neighbor bonds (dashed red/blue lines) is shown. We consider the spin-S Heisenberg Hamiltonian ("J_1–J_2 model")

$$\mathcal{H} = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j \ . \qquad (1)$$

For $J_1 = 0$, lattice sites coupled by blue links are decoupled from lattice sites coupled by red links. In the following we refer to these sites as the "blue" and the "red" lattice. For $J_2 > 0$ both the red and the blue lattices possess a Néel-ordered ground state independently from each other. Their relative orientation can be parametrized by the angle θ shown in the figure.

a) — 4 Points

In the following we assume $J_1 > 0$ and $J_2/J_1 > 1/2$. Show that the classical ground state energy E_0^{cl} does *not* depend on J_1. How many classical ground states are there?

b) — 8 Points

Now we apply linear spin-wave theory (harmonic approximation). Which terms vanish? Solve the quadratic part via Bogoliubov transformation. Determine $\omega_{\vec{k}}(\theta)$ in order to compute the quantum corrections to E_0^{cl}.

c) — 4 Points

How many quantum ground states are there? You can find them by computing the minima of $E_0(\theta)$ with respect to θ ("order-by-disorder"). You may check this for any value of $2 \geq J_2/J_1 > 1/2$ and you may use Mathematica or other computer program for integration.