Technische Universität Dresden
Institut für Theoretische Physik

Prof. M. Vojta
M. Nayga

Theory of Frustrated Magnetism
Problem set 6

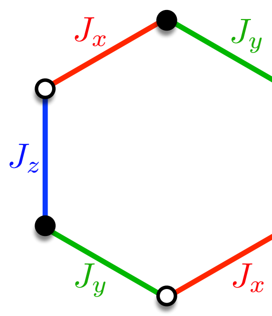
Summer term 2020

Kitaev's Honeycomb Model **18 Points**

The Kitaev honeycomb model is governed by a spin-1/2 Hamiltonian with bond-dependent exchange,

$$H_K = -\sum_{\langle ij\rangle_\gamma} J_\gamma\, \tau_i^\gamma \tau_j^\gamma = -\sum_{\langle ij\rangle_x} J_x\, \tau_i^x \tau_j^x - \sum_{\langle ij\rangle_y} J_y\, \tau_i^y \tau_j^y - \sum_{\langle ij\rangle_z} J_z\, \tau_i^z \tau_j^z \tag{1}$$

where the Ising interactions $\tau_i^\gamma \tau_j^\gamma$ ($\gamma = x, y$ or z) act on the different bonds of the honeycomb lattice as depicted in the following figure:



a) **9 Points**

Consider the anisotropic case $J_x \neq J_y \neq J_z$. Show that the excitation spectrum of the Majorana quasiparticles can become gapped for a sufficiently strong anisotropy. Determine the requirement on the exchange couplings for the spectrum to be gapless and draw the phase diagram. Now choose J_x, J_y, and J_z such that model is at the phase boundary between gapless and gapped spectrum. For this point, compute the excitation spectrum explicitly and discuss its properties.

Hint: You can assume that the ground state is flux-free. A calculation of the excitation spectrum of an equivalent tight-binding model for canonical fermions is sufficient.

b) **9 Points**

Compute the static spin-spin correlation function $\langle \tau_i^z \tau_j^z \rangle$ at $T = 0$ for the isotropic case, using the Majorana representation $\tau_i^\gamma = i b_i^\gamma c_i$. The spin-spin correlation function can eventually be expressed as a c-fermion correlator. The final integral needs to be evaluated numerically.