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Theory of Frustrated Magnetism
Problem set 4

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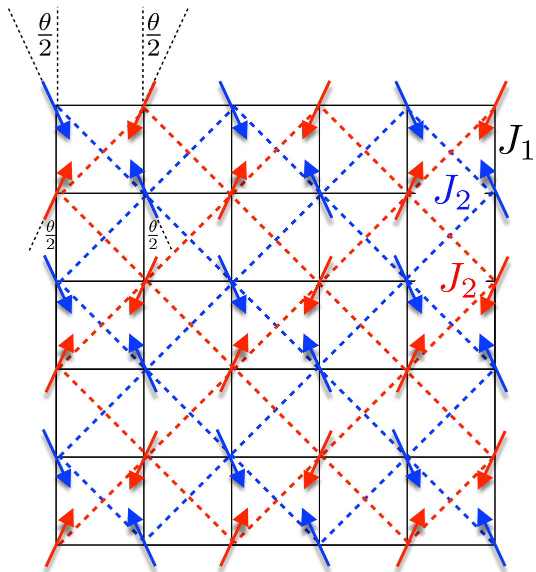
Order-by-disorder: J_1-J_2 square-lattice Heisenberg model — 18 Points



On the left the square lattice with nearest-neighbor (black solid lines) and next-nearest-neighbor bonds (dashed red/blue lines) is shown. We consider the spin-S Heisenberg Hamiltonian ("J_1–J_2 model")

$$\mathcal{H} = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j \ . \quad (1)$$

For $J_1 = 0$, lattice sites coupled by blue links are decoupled from lattice sites coupled by red links. In the following, we refer to these sites as the "blue" and the "red lattice". For $J_2 > 0$ both the red and the blue lattices possess a Néel-ordered ground state independently from each other. Their relative orientation can be parametrized by the angle θ shown in the figure.

a) **4 Points**

In the following, we assume $J_1 > 0$ and $J_2/J_1 > 1/2$. Show that the classical ground state energy E_0^{cl} does *not* depend on J_1. How many classical ground states are there?

b) **8 Points**

Now we apply linear spin-wave theory (harmonic approximation). Show that the terms linear in spin-wave operators vanish. Solve the quadratic part via Bogoliubov transformation. Determine $\omega_{\vec{k}}(\theta)$ in order to compute the quantum corrections to E_0^{cl}.

c) **6 Points**

How many quantum ground states are there? You can find them by computing the minima of $E_0(\theta)$ with respect to θ ("order-by-disorder").

Hint: The final sum over momentum can be expressed as an integral over momentum. For $J_2 \gg J_1$ one can expand the integrand such that the θ dependence can be separated from the integral: $\int d\vec{k}\, \omega_{\vec{k}}(\theta) \approx f(\theta) \int d\vec{k}\, \tilde{\omega}_{\vec{k}}$.