

Theory of Frustrated Magnetism

Problem set 6

Summer term 2023

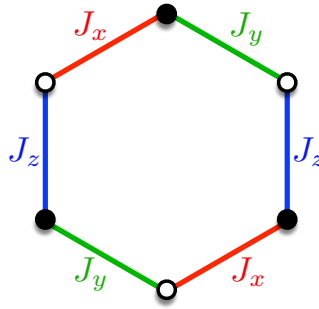
Kitaev's Honeycomb Model

18 Points

The Kitaev honeycomb model is governed by a spin-1/2 Hamiltonian with bond-dependent exchange,

$$H_K = \sum_{\langle ij \rangle_\gamma} -J_\gamma \tau_i^\gamma \tau_j^\gamma = - \sum_{\langle ij \rangle_x} J_x \tau_i^x \tau_j^x - \sum_{\langle ij \rangle_y} J_y \tau_i^y \tau_j^y - \sum_{\langle ij \rangle_z} J_z \tau_i^z \tau_j^z, \quad (1)$$

where the Ising interactions $\tau_i^\gamma \tau_j^\gamma$ ($\gamma = x, y$ or z) act on the different bonds of the honeycomb lattice as depicted in the following figure:



a)

9 Points

Consider the anisotropic case $J_x \neq J_y \neq J_z$. Show that the excitation spectrum of the Majorana quasiparticles can become gapped for a sufficiently strong anisotropy. Determine the requirement on the exchange couplings for the spectrum to be gapless and draw the phase diagram. Now choose J_x , J_y , and J_z such that model is at the phase boundary between gapless and gapped spectrum. For this point, compute the excitation spectrum explicitly and discuss its properties.

Hint: You can assume that the ground state is flux-free. A calculation of the excitation spectrum of an equivalent tight-binding model for canonical fermions is sufficient.

b)

9 Points

Compute the static spin-spin correlation function $\langle \tau_i^z \tau_j^z \rangle$ at $T = 0$ for the isotropic case, using the Majorana representation $\tau_i^\gamma = ib_i^\gamma c_i$. The spin-spin correlation function can eventually be expressed as a c -fermion correlator. The final integral needs to be evaluated numerically.