Ultracold Quantum Gases

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Problem set 1

to be discussed in May, 2011

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see http://tu-dresden.de/physik/tqo/lehre

Planck 1900/1901 [*Ann. d. Phys.* **4**, 553, 1901]

Planck considers a set of N independent harmonic oscillators (frequencies ν) and aims to distribute the total energy $U_N = N \cdot U$ among those oscillators. He writes in §3: *Hierzu ist es notwendig, U_N nicht als eine stetige, unbeschränkt teilbare, sondern als eine discrete, aus einer ganzen Zahl von endlichen gleichen Teilen zusammengesetzte Grösse aufzufassen. Nennen wir einen solchen Teil ein Energieelement ϵ so ist mithin zu setzen: $U_N = P \cdot \epsilon$, wobei P eine ganze, im allgemeinen grosse Zahl bedeutet ....*

a) How many possibilities $\mathcal{R}$ are there for the distribution of P (identical) *Energieelemente* among the N oscillators?

b) Use Stirling ($n! \approx n^n$) to show that with $\epsilon = h\nu$, the entropy $S = \frac{k}{N} \log \mathcal{R}$ of one of the oscillators takes the value

$$S = k \left[\left(1 + \frac{U}{h\nu}\right) \log \left(1 + \frac{U}{h\nu}\right) - \frac{U}{h\nu} \log \left(\frac{U}{h\nu}\right) \right].$$

c) Use $\frac{1}{T} = \frac{\partial S}{\partial U}$ to determine the average energy U of one oscillator.

d) Planck's law for the energy density $\rho(\nu)$ of black body radiation follows from $\rho(\nu) = \frac{8\pi\nu^2}{c^3} U$ (see Planck, equation (8) or Bose's introduction). Discuss!

Bose 1924 [*Z. Phys.* **26**, 178, 1924]

Now study Bose's derivation of Planck's law. Keep in mind that in 1924 no "proper" quantum mechanics was available [Heisenberg (1925), Schrödinger (1926)]. What do you consider to be Bose's main contribution?

Please turn over !!!

Einstein 1924/25 [*Sitzungsber. Kgl. Preuss. Akad. Wiss.* 1924, **261** (1924); ibid. 1925, **3** (1925)]

Einstein's novel idea is to apply Bose's approach to a gas of atoms (first paper). In the second paper he elaborates on the phenomenon of Bose-Einstein condensation.

a) Discuss equ. (24) and explain why condensation follows as a consequence. What is the difference to classical condensation (gas–liquid)?

b) Why is there a problem with Nernst's theorem using "Boltzmann"- but no such problem using "Bose"-statistics?

c) Discuss which phenomena led Einstein to consider Hydrogen and Helium in §9.

d) Determine the critical density according to equ. (39) for a gas of Rubidium atoms at room temperature, $T = 1$K, $T = 1$mK und $T = 1\mu$K.