Quantum Stochastic Processes

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Problem set 2 to be discussed on Thursday, November 12th, 2009

. see http://tu-dresden.de/physik/tqo/lehre

Buffon's needle

A plane is ruled by the lines $y = n$ with $n = 0, \pm 1, \pm 2, \ldots$ and a needle of unit length is cast randomly on to the plane. What is the probability that it intersects some line?
Buffon designed the experiment in order to estimate the numerical value of π.
Try it if you have time.

Change of distributions

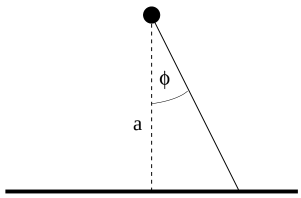
Let R_1, R_2 be two independent random variables of uniform distribution in the interval $[0, 1]$. We define two new random variables X_1, X_2 through

$$X_1 = \sqrt{-2\sigma \ln R_1}\ \cos(2\pi R_2), \qquad X_2 = \sqrt{-2\sigma \ln R_1}\ \sin(2\pi R_2).$$

Determine the probability distributions of X_1 and X_2? Are X_1 and X_2 correlated?

Particle source

A point-like source with distance a from a line emits particles with a uniformly distributed angle $\phi \in [-\frac{\pi}{2}, +\frac{\pi}{2}]$ (see figure). Determine the probability density of the distribution of the particles on the line.



Random walker 1

A walker on a line takes a step to the right with probability p or a step to the left of equal size with probability $q = (1 - p)$.

a) Determine the position of the walker and its variance after N steps.

b) Find an appropriate time-continuous limit. State the corresponding diffusion equation and its solution.

c) Write a little computer program for the walker and confirm your findings.

Random walker 2

With every step a walker on a line covers a random distance ℓ, drawn uniformly from the interval $\ell \in [L - b, L + b]$.

a) Determine the mean distance covered by the walker and its variance after N steps.

b) Again, can you find an appropriate time-continuous limit?

c) Write a little computer program for the walker and confirm your findings.