**Quantum Stochastic Processes**

Walter Strunz, Institut für Theoretische Physik , TU Dresden, Wintersemester 2009/2010

---

## Problem set 6

to be discussed on Thursday, January 21th, 2010

.

see http://tu-dresden.de/physik/tqo/lehre

### Stratonovich stochastic differential equations

For both linear and non-linear Ito stochastic Schrödinger equation of Problem set 5, determine the corresponding Stratonovich version. Show that the nonlinear Stratonovich equation preserves the norm of the state.

### Quantum harmonic oscillator

For $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$ with $[q,p] = i\hbar$ determine the variances $\langle q^2 \rangle$, $\langle p^2 \rangle$ in the thermal state $\sim e^{-H/kT}$.

### Classical oscillator model for Brownian motion

Consider the classical Hamiltonian $H_{\rm tot} = H_{\rm sys} + H_{\rm int} + H_{\rm env}$ with

$$\begin{aligned}
H_{\rm sys} &= H_{\rm sys}(q,p) = \frac{p^2}{2m} + V(q) \\
H_{\rm int} &= H_{\rm int}(q,p,q_\lambda,p_\lambda) = \frac{1}{2}\sum_\lambda m_\lambda \omega_\lambda^2 \left( q_\lambda - \frac{g_\lambda}{m_\lambda \omega_\lambda^2} q \right)^2 \\
H_{\rm env} &= H_{\rm env}(p_\lambda) = \sum_\lambda \left\{ \frac{p_\lambda^2}{2m_\lambda} \right\},
\end{aligned}$$

where the $g_\lambda$ are some coupling constants. Show that the model leads to a generalized Langevin-type equation

$$m\ddot{q}(t) + \int_0^t ds\, \kappa(t-s)\dot{q}(s) + V'(q(t)) = F(t)$$

for the position $q(t)$ of the Brownian particle (in the derivation, one term is dropped; why?).
Next assume a thermal initial state for the environmental degrees of freedom and determine the corresponding averages $\langle F(t) \rangle$, $\langle F(t)F(s) \rangle$.
How do we recover Langevin's original equation?

### Quantum noise

Now think of $H_{\rm tot}$ of the last problem to be the Hamiltonian of a quantized system. Along the same lines, derive an expression for the quantized Langevin force $\hat{F}(t)$ and determine $\langle \hat{F}(t) \rangle$, $\langle \hat{F}(t)\hat{F}(s) \rangle$, accordingly.