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Earth´s Magnetic Field

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1 Tasks

To be determined:

1. The horizontal component H_h of the Earth's magnetic field according to Gauss;
2. the vertical component H_v of the Earth's magnetic field;
3. the magnetic moment m^* and the polarization J^* of a magnet.

2 General Principles

2.1 Magnetic Moment, Dipole Field, and Gauss Positions

The quotient of the magnetic moment $\vec{m}^*$ of a permanent magnet and its volume V is the polarization $\vec{J}^* = \vec{m}^*/V$. For high-quality permanent magnets, in weak external magnetic fields (e.g., in the Earth's field) and at constant temperature, $\vec{J}^*$ or $\vec{m}^*$ can be considered constant.
A small permanent magnet (magnetic moment $\vec{m}^* = m^* \vec{e}_x$) is oriented in the east-west direction. We consider its **magnetic field** at a greater distance x or y from the center. If for the bar magnet $\frac{L}{x} \ll 1$ or $\frac{L}{y} \ll 1$ (dipole approximation, see Appendix, Fig. 5) holds, then the x-components of the magnetic field strength $\vec{H}$ at two specific positions – the axial and equatorial (Gauss) positions (GP, deutsch: Gaußsche Hauptlage (GHL) Fig. 1) – are given by

$$\text{Axial GP (1. GHL)}: \ H_x^{(1)} = \frac{m^*}{2\,\pi\ \tilde{x}^3} \quad , \ \text{or Equatorial GP (2. GHL)}: \ H_x^{(2)} = -\frac{m^*}{4\,\pi\ \tilde{y}^3} \quad . \tag{1}$$

2.2 First Experiment: Superposition of Dipole Field and Earth's Field

A compass needle freely rotatable in the horizontal (xy) plane aligns itself parallel to the horizontal component H_h of the Earth's field in the y (north) direction. If the magnet is then placed horizontally with its magnetic moment $\vec{m}^*$ parallel to the east-west direction ($\vec{m}^* = m^* \vec{e}_x$) at distances $\tilde{x}$ or $\tilde{y}$ from the center of the compass needle (i.e., successively at the axial and equatorial GP, see Fig. 1), the needle turns to align with the resultant of the Earth's field (H_h) and the dipole field ($H_x^{(1)}$ or $H_x^{(2)}$).

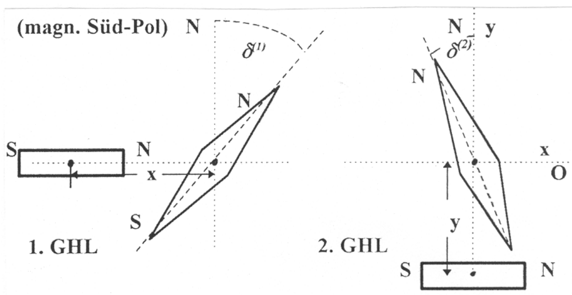

Fig. 1: Arrangement of the bar magnet relative to the compass needle for the axial and equatorial GP

The needle deflects by the respective angles $\delta^{(1)}$ or $\delta^{(2)}$ from the N-S direction (Fig. 1). The following relations apply:

$$\tan\delta^{(1)} = \frac{H_x^{(1)}}{H_\mathrm{h}}\ (a)\ ; \quad \tan\delta^{(2)} = \frac{H_x^{(2)}}{H_\mathrm{h}}\ (b)\ . \tag{2}$$

Using Eq. (1), one obtains for the quotient of the two unknowns (H_h and m^*):

$$\boxed{\frac{m^*}{H_\mathrm{h}} = 2\,\pi\,\tilde{x}^3\ \tan\delta^{(1)} = 4\,\pi\,\tilde{y}^3\ \tan\delta^{(2)} \quad .} \tag{3}$$

2.3 Second Experiment: Torsional Oscillations

The same bar magnet is now suspended by a long, thin thread so that it hangs horizontally. If it is deflected from the north-south direction by a small angle α with respect to the horizontal component of the Earth's field $\vec{H}_\mathrm{h}$, a restoring torque $\vec{M}_\mathrm{mech}$ acts on it:

$$|\vec{M}_\mathrm{mech}| = |\vec{m}^* \times \vec{B}_\mathrm{h}| = \mu_0 |\vec{m}^* \times \vec{H}_\mathrm{h}| = \mu_0 m^* H_\mathrm{h} \sin\alpha \quad . \tag{4}$$

After being released, the bar magnet begins to oscillate about the north-south direction. The oscillation period T_0 is measured. The equation of motion (5a) simplifies for small deflections (5b for $\hat{\alpha} < 0.1$; $\approx 6°$, see Appendix)

$$J_\mathrm{T}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = -\mu_0 m^* H_\mathrm{h}\sin\alpha\ (a)\ ; \qquad \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + \mu_0\frac{m^* H_\mathrm{h}}{J_\mathrm{T}}\alpha \approx 0\ \ (b)\ . \tag{5}$$

With J_T as the moment of inertia with respect to the axis of rotation, the solution of Eq. (5b) (harmonic oscillation) is $\alpha(t) = \hat{\alpha}\cos\omega_0\, t$, where

$$\boxed{\omega_0^2 = \frac{4\pi^2}{T_0^2} = \frac{\mu_0 m^* H_\mathrm{h}}{J_\mathrm{T}} \quad (a) \quad \text{or} \quad T_0 = 2\pi\sqrt{\frac{J_\mathrm{T}}{\mu_0 m^* H_\mathrm{h}}} \quad (b) \quad .} \tag{6}$$

2.4 Horizontal Component

From Eq. (6b), the product of the two unknown quantities is obtained as

$$m^* H_\mathrm{h} = \frac{4\pi^2 J_\mathrm{T}}{\mu_0 T_0^2} \quad . \tag{7}$$

It is convenient to eliminate the magnetic moment m^* from Eqs. (7) and (3). This yields for the horizontal component:

$$\boxed{H_\mathrm{h} = \sqrt{\frac{2\pi J_\mathrm{T}}{\mu_0 \tan\delta^{(1)} x^3 T_0^2}} = \sqrt{\frac{\pi J_\mathrm{T}}{\mu_0 \tan\delta^{(2)} y^3 T_0^2}} \quad .} \tag{8}$$

2.5 Moment of Inertia

For simple geometries of homogeneous bodies, the moment of inertia J_T with respect to convenient axes (e.g., the principal axes of inertia) can be calculated. For a horizontally suspended circular cylinder of mass m, length L, and radius R rotating about its vertical symmetry axis (see Appendix), the following holds:

$$\boxed{J_\mathrm{T} = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right) \quad .} \tag{9}$$

Consider the limits $R \to 0$ or $L \to 0$!

Thus, from the two equations (3, 7 or 8), **solely from mechanical measurements**, the magnetic field strength (H_h) and the magnetic moment (m^*) can be determined.

2.6 Vertical Component

With suitable angle-measuring instruments (inclinometers), the deviation of $\vec{H}$ from the horizontal – that is, the inclination angle β – can be determined.

With $\cos\beta = H_\mathrm{h}/|\vec{H}|$ and $\tan\beta = \frac{H_\mathrm{v}}{H_\mathrm{h}}$, the vertical component follows as $H_\mathrm{v} = H_\mathrm{h}\tan\beta$.

3 Appendix

3.1 On the Earth's Magnetic Field

3.1.1 On the Origin of the Earth's Magnetic Field

There are various hypotheses and models concerning the origin of the Earth's magnetic field [3]. It appears well established that circulating convection currents of viscous plasma with a suitable topology exist between the solid inner core and the fluid outer core at roughly $R_\mathrm{E}/2$. These currents can stabilize themselves, whereby the existing field, through Lorentz forces, can increase the charge-carrier density and thereby the current and the Earth's field itself [3, 4]. The relative velocities between the solid core and the surrounding liquid layer are on the order of 1 m/year [4].
These effects are subject to fluctuations, which manifest as changes in the magnitude and direction of $\vec{H}$ (see below).
In 1991, the Earth's magnetic field could be roughly approximated by a geocentric dipole field whose axis deviates by about 11° from the geographic north-south direction [3] (see Fig. 2 a).

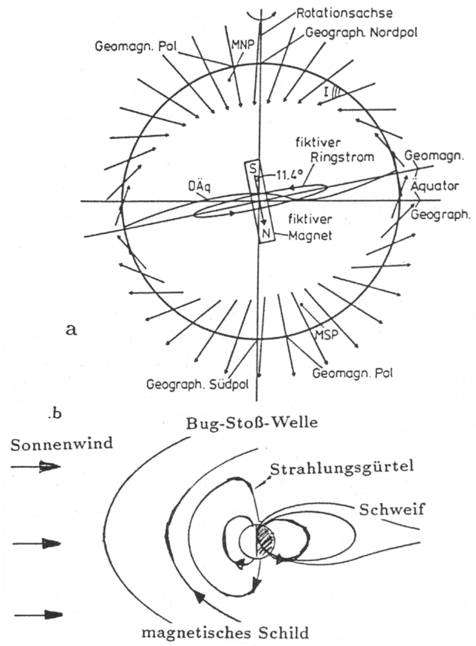

Fig. 2: a. Earth's field as a dipole field [3]; b. Distortion by the solar wind [4]

The magnitude corresponds to a dipole moment at the Earth's center produced by an equivalent current loop of $m_e^*/\mu_0 \approx 8\cdot 10^{22}$ Am^2. Due to the solar wind (predominantly electrons and protons), the Earth's field is strongly distorted at greater distances (the deflected charged particles produce additional magnetic fields), i.e., it is flattened on the dayside and stretched on the nightside (Fig. 2b). This deformation co-rotates with the Earth and causes part of the daily fluctuations in field strength (on the order of a few percent). During periods of strong solar activity (magnetic storms), much larger fluctuations occur (exceeding 10%).

3.1.2 Global View of the Earth's Magnetic Field

Figures 3a and b show, in Mercator projection, the magnitudes of $|\vec{H}|$ (a) and H_h (b) at the Earth's surface (1980, [3]). Significant deviations from the pure dipole model, as well as the presence of poles and auxiliary poles, are visible.

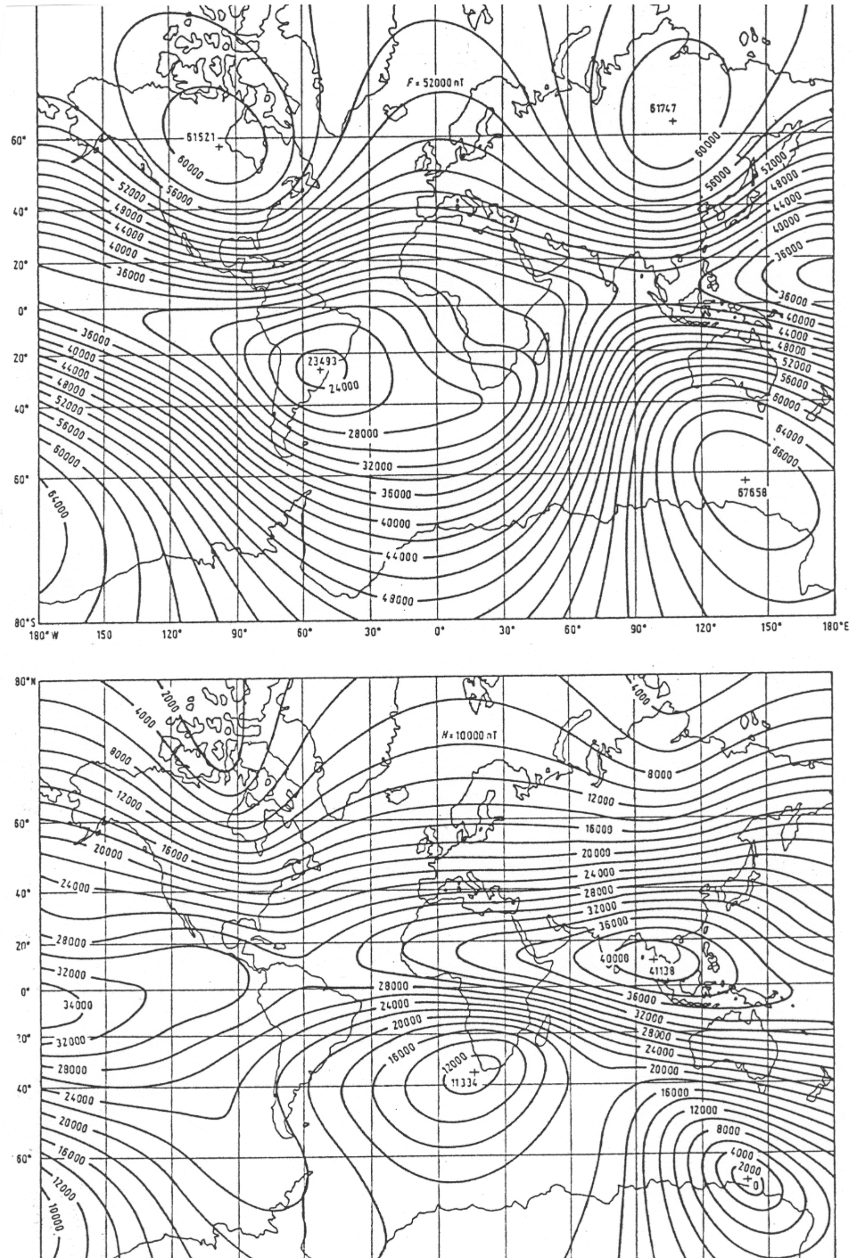

Fig. 3: Lines of equal total (a) and horizontal intensity (b) of the Earth's magnetic field in Mercator projection, measured in 1980 [3]; the unit for $\mu_0 \cdot H$ is nT

3.1.3 Long-Term Fluctuations of the Earth's Field

Since Gauss (around 1800) – and even earlier (e.g., Gilbert around 1600) – the Earth's magnetic field has been regularly measured. Its magnitude has been decreasing monotonically since 1800 (Fig. 4 a; will the field vanish in about 2000 years?). Information about earlier epochs is obtained by studying iron-bearing minerals and sediments.

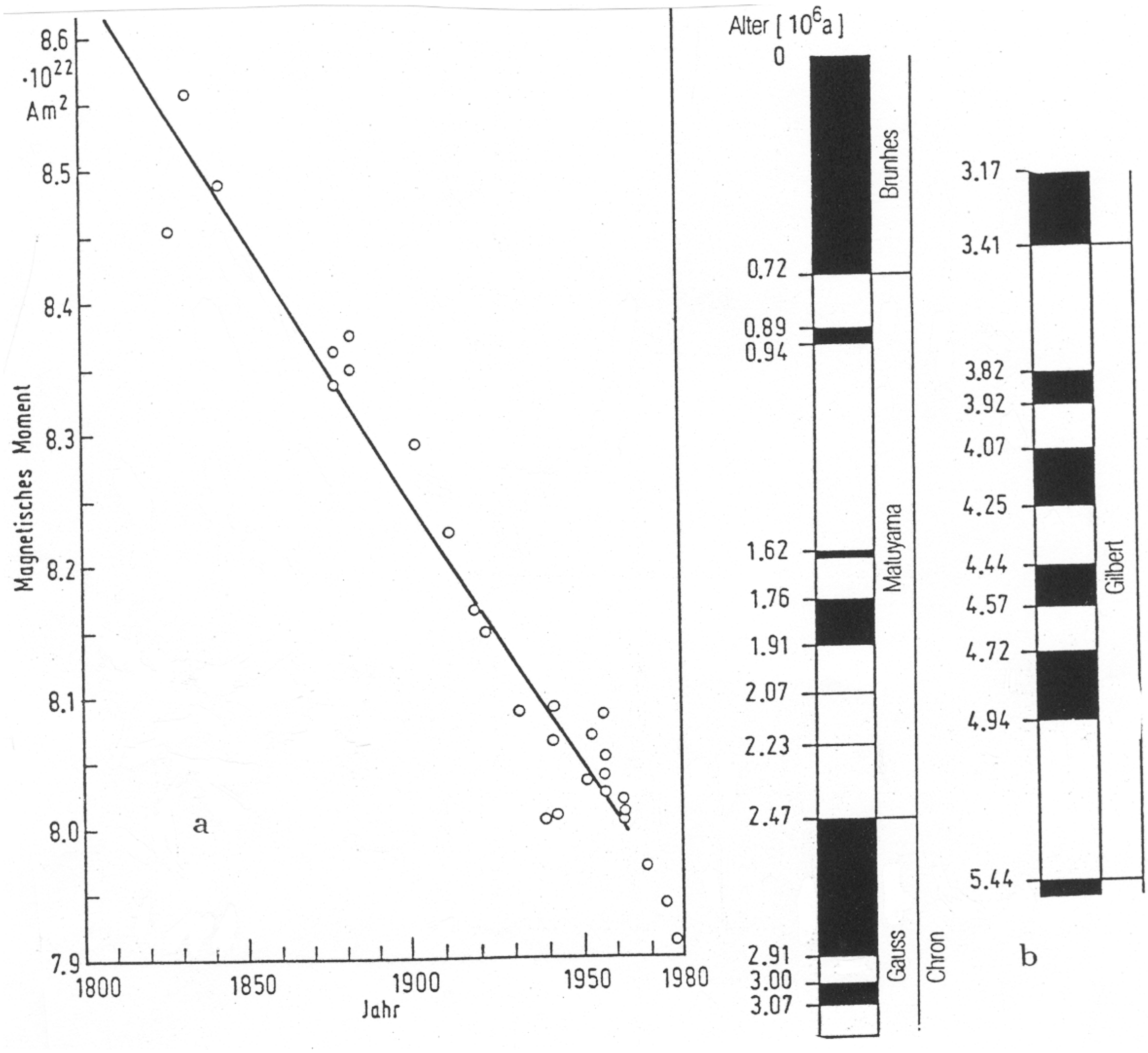

Fig. 4: Temporal variations of the Earth's magnetic field:
a. Decrease of the global moment m_e^* over the past 200 years;
b. normal (present, black) and reversed polarity (white) in the past $5 \cdot 10^6$ years, paleomagnetism, [3].

Through this **paleomagnetism**, the magnitude and direction of the Earth's magnetic field during the geological epoch in which the rock cooled below its Curie temperature (depending on rock type, between 200 and 700 °C [3]) – that is, when it became ferro- or ferrimagnetic – can be inferred. Over geological timescales, the field has undergone continuous fluctuations, also influenced by continental drift. When the dipole components fell below those of higher-order moments, a **polarity reversal** could occur (as shown by model calculations [3]). The last confirmed polarity reversal occurred about $0.7 \cdot 10^6$ years ago (Fig. 4b).

3.2 On the Dipole Approximation

The equations (1) can be approximately derived from the electric analog, even for smaller relative distances L/x or L/y. We therefore consider two point charges, $+Q$ and $-Q$, separated by $2a = L$ $(m^* \vec{e}_x = QL\vec{e}_x)$. The field strengths of the individual charges at a distance $\vec{r}$ are, according to Coulomb:

$$\vec{E} = \pm \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} \frac{\vec{r}}{r} \; ; \; E = \frac{Q}{4\pi\epsilon_0 r^2} \; . \quad (10)$$

1st Principal Axis (Axial GP):
Along the connection (x-) axis at the observation point P, $r^{(+)} = x - \frac{L}{2}$ and $r^{(-)} = x + \frac{L}{2}$ ($a = \frac{L}{2}$ = half dipole length). The total field strength produced by both point charges follows from Eq. (10) (vector sum) as

$$\begin{aligned} E^{(1)}_{\text{dipol}} &= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right] \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{4ax}{(x^4 - 2a^2x^2 + a^4)} \right] \\ &= \frac{2 \cdot 2aQ}{4\pi\epsilon_0 x^3} \cdot \frac{1}{(1 - \frac{L^2}{4x^2})^2} \\ &= \frac{m^*}{2\pi\epsilon_0 x^3} \cdot F^{(1)}_{\text{corr}}(L/x) \quad . \quad (11) \end{aligned}$$

2nd Principal Axis (Equatorial GP):
Perpendicular to the connecting axis, the relevant distance in Eq. (10) is $r = (a^2+y^2)^{1/2}$. The resulting field strength at point P follows from the vector superposition of $\vec{E}^{(+)}$ and $\vec{E}^{(-)}$ with

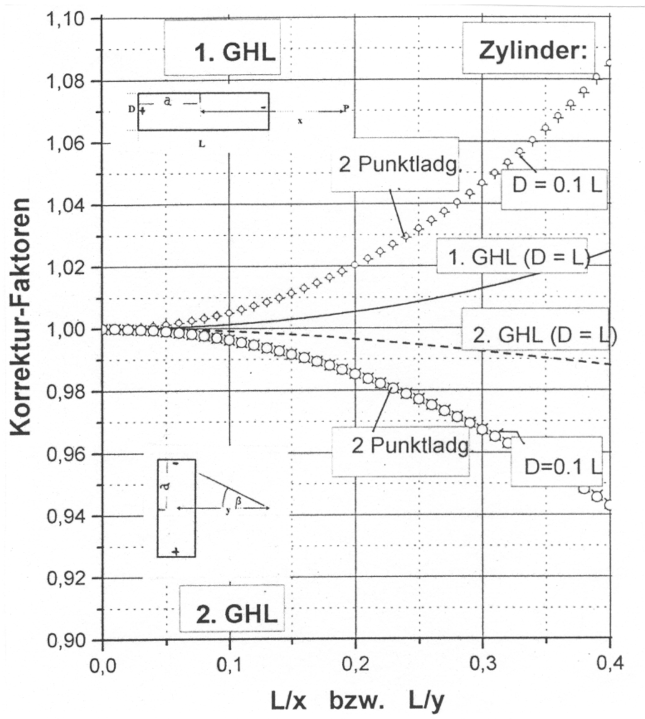

Fig. 5: Deviations from the ideal dipole field near the dipole or cylindrical magnet: corresponding correction factors $F^{(1)}$ and $F^{(2)}$ for two point charges (Eqs. (11,13)) and for real cylinders (long $L/D = 10$; short $L = D$)

$$\sin\beta = \frac{a}{\sqrt{a^2+y^2}} = \frac{E^{(2)}/2}{E_0} \; ; \quad E_0 = \frac{Q}{4\pi\epsilon_0(y^2+a^2)} \qquad (12)$$

$$\begin{aligned} \text{thus } |E^{(2)}_{\text{dipol}}| &= \frac{2Q}{4\pi\epsilon_0(y^2+a^2)} \cdot \frac{a}{(a^2+y^2)^{1/2}} = \frac{m^*}{4\pi\epsilon_0 y^3} \cdot \frac{1}{(1+\frac{L^2}{4y^2})^{1/2}} \\ &= \frac{m^*}{4\pi\epsilon_0 y^3} \cdot F^{(2)}_{\text{corr}}(L/y) \quad . \qquad (13) \end{aligned}$$

As shown in Fig. 5, the correction factors $F^{(1)}_{\text{corr}}$ and $F^{(2)}_{\text{corr}}$ derived for two point charges correspond approximately to those for a long cylinder, which can be computed by summing about 10^8 dipoles over the cylinder volume. In the magnetic case, ϵ_0 is replaced by μ_0.

Note: A spherical magnet (approximated roughly by a cube or a ”short“ cylinder) produces a pure dipole field at any distance (compare the correction factors for $D = L$ in Fig. 5). Otherwise, the **dipole approximation** requires $L/x \ll 1$ or $L/y \ll 1$, i.e. $F_{\text{corr}} \to 1$ (Fig. 5).

Example:

Let $L = 10$ cm and x or $y = 50$ cm, i.e. $L/x = L/y = 0.2$.

From Fig. 5, the field values corrected by F_{corr} would yield $\tan\vartheta^{(1)}$ about 2% too large ($F = 1.02$) and $\tan\vartheta^{(2)}$ about 1.5% too small ($F = 0.985$). Hence, the determined H_{h} value must be corrected according to Eq. (8) by approximately 1% downward (axial GP) or 0.75% upward (equatorial GP). The corrections are opposite in sign for longer cylindrical magnets. It is therefore advisable to work with both GP configurations.

3.3 Moment of Inertia

The so-called polar moment of inertia of a thin circular disk (axis = cylinder axis) is (for infinitesimal dm) $J_P = \mathrm{d}m \cdot R^2/2$. The equatorial moment of inertia (axis = diameter), J_a, is half of that: $\mathrm{d}J_a = \mathrm{d}m \cdot R^2/4$.

For a long cylinder of length L, infinitesimal disks are arranged in series, and their equatorial moments of inertia along the z-axis must be integrated. If the infinitesimal disk is displaced by z parallel to the rotation axis, the parallel-axis theorem (Steiner?s theorem), in general,

$$J_A = J_s^* + ms^2 \quad , \tag{14}$$

here becomes with $J_s^* = \mathrm{d}m(\frac{R^2}{4})$: $\mathrm{d}J_\mathrm{T} = \mathrm{d}m(\frac{R^2}{4}) + \mathrm{d}mz^2$. Integration yields

$$\begin{aligned} J_\mathrm{T} = J_s &= (\rho\pi R^2)\int_{-L/2}^{+L/2} \mathrm{d}z(R^2/4 + z^2) \\ &= (\rho\pi R^2)(R^2/4 \cdot [z]_{-L/2}^{+L/2} + [\frac{z^3}{3}]_{-L/2}^{+L/2}) \\ &= \rho\pi R^2 L\left[\frac{R^2}{4} + \frac{L^2}{12}\right] \quad . \end{aligned} \tag{15}$$

With $m = \rho\pi R^2 L$, Eq. (9) follows.

3.4 SI Units

1. $\mu_0 = 4\pi \cdot 10^{-7}\ \frac{\mathrm{Vs}}{\mathrm{Am}}$;
2. Magnetic field strength:
   a. $[H] = \frac{\mathrm{A}}{\mathrm{m}}$;
   b. $[\mu_0 H] = \frac{\mathrm{Vs}}{\mathrm{m}^2} = \mathrm{T}$ (tesla);
3. Induction B; Magnetization M; Polarization J^*:
   $B = \mu_0 H + J^* = \mu_0(H + M)$; $[\mu_0 H] = [B] = [J^*] = \frac{\mathrm{Vs}}{\mathrm{m}^2} = \mathrm{T}$;
4. Magnetic moment:
   a. $[m^*] = [VJ] = \mathrm{Vsm}$;
   b. $[m^{**} = \frac{m^*}{\mu_0}] = [\mathrm{VM}] = \mathrm{Am}^2$.
5. Note:
   1 J = 1 VAs = 1 Nm = 1 kg·m^2/s^2

4 Questions

1. Sketch, based on Fig. 2, a field line of the Earth's magnetic field ($\vec{H}$) for a geographic latitude of about 50°, and indicate the horizontal and vertical components.
2. From Fig. 3, obtain the value of H_h for Central Europe and convert the unit nT into A/m.
3. Explain the equation of motion for torsional oscillations, including the solutions for small amplitudes. On what does the oscillation period of the magnet in the Earth's field depend?
4. What is meant by the (mass) moment of inertia? How is it calculated for a long rod about each of its three principal axes? What are the equatorial and polar moments of inertia of a thin circular disk?
5. How is the moment of inertia of a circular cylinder of length L and diameter $2R$ calculated with respect to all three principal axes?
6. What is Steiner´s theorem (parallel-axis theorem)?
7. How is the field strength calculated in electrostatics for
   (a) a point charge, and
   (b) a dipole;

   and in magnetostatics for a dipole field? In which power of distance do the field strengths of a point charge and a dipole decrease?
8. What are the axial and equatorial Gauss planes (Gausssche Hauptlagen) with respect to the field of a bar magnet?
9. How are the magnetic moment m^* and the magnetic polarization J^* related?
10. At what distance x (e.g., from a neighboring workstation in the lab) must a magnet be placed so that the magnetic field it produces changes the local horizontal component (e.g., 20 A/m) by less than 0.1%? (see Eq. (1))

    Example: Magnet volume $V = 2$ cm^3; polarization $J^* = 1$ Vs/m^2; $m^* = 2 \cdot 10^{-6}$ Vsm.

Authorship

This laboratory manual was originally written by L. Jahn and revised by M. Kreller. Current updates are made by the laboratory supervisors.

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