

Calculators (and Matlab) can calculate the values of the trigonometric functions (sine, cosine, etc...), exponentials and logarithms. To do this, they use something called the Taylor expansion (of a function). With this expansion, a function can be expressed in terms of powers of its argument. For example:

$$\begin{aligned} \sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ &= \sum_{n=1}^{\infty} \frac{(-1)^{2n-1} x^{2n-1}}{(2n-1)!} \\ \cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots \\ &= \sum_{n=0}^{\infty} \frac{(-1)^{2n} x^{2n}}{(2n)!} \end{aligned}$$

Using these expressions, we can compare against the Matlab built-in trigonometric functions  $\sin(x)$  and  $\cos(x)$ .

Exercise:  
Evaluate

$$\begin{aligned} \sin_N(x) &= \sum_{n=1}^N \frac{(-1)^{2n-1} x^{2n-1}}{(2n-1)!} \\ \cos_N(x) &= \sum_{n=0}^{\infty} \frac{(-1)^{2n} x^{2n}}{(2n)!} \end{aligned}$$

for  $x = \pi/3$ . Find the  $N$  such that you get the same result as the Matlab built-in functions give. That is, until  $\sin_N(x) - \sin(x) = 0$  and  $\cos_N(x) - \cos(x) = 0$ .