TECHNISCHE UNIVERSITÄT DRESDEN

# Introduction to Matlab

## Integration and simulations

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DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

# Differential equations

Many natural phenomena can be modelled with differential equations. The simplest equations have the form:

$$\frac{dx}{dt} = f(x, t)$$

This means that *x* changes in time, as fast as *f(x,t)* determines it. For example:

$$\frac{dx}{dt} = 5$$

In this example, the rate of change of *x* is always 5.

A simple way of computing the values of *x* for every value of *t* is to use numerical methods. Here we will use a simple tool called the Euler forward method. This is done with the following formula:

$$x(n+1) = x(n) + f\big(x(n)\big)\Delta t$$

where $\Delta t$ is the size of the time step and *n* is a time index.

For the previous example, it means:

$$x(n+1) = x(n) + 5\Delta t$$

# Differential equations

Example:

$$\frac{dx}{dt} = 5x$$

With the Euler method it turns into:

$$x(n+1) = x(n) + 5x(\mathrm{n})\Delta t$$

To start a simulation of this system, we use an initial condition and a (small) value for $\Delta t$. That is, the first value of *x*, *x(1) = 1*, for example, and $\Delta t$ = 0.1. Then we can calulate x(2):

$$x(2) = x(1) + 5x(1)\Delta t = 1 + 5 * 1 * 0.1 = 1.5$$

Then we can do this recursively:

$$x(3) = x(2) + 5x(2)\Delta t = 1.5 + 5*1.5*0.1 = 2.25$$

And we can do this for any value of n by using, for example, a for-loop.

# N-dimensional

A two dimensional system looks like this:

$$\frac{dx}{dt} = f(x, y)$$

$$\frac{dy}{dt} = g(x, y)$$

Then, with the Euler method one has:

$$x(n+1) = x(n) + f\big(x(n), y(n)\big)\Delta t$$

$$\mathrm{y}(n+1) = y(n) + g\big(x(n), y(n)\big)\Delta t$$

For this, one needs two initial conditions, *x(1)* and *y(1)*

# N-dimensional example

A two dimensional system looks like this:

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -x$$

Then, with the Euler method one has:

$$x(2) = x(1) + \mathrm{y}(1)\Delta t$$
$$\mathrm{y}(2) = y(1) - \mathrm{x}(1)\Delta t$$

For every step n, one needs to update both equations, so:

$$x(n+1) = x(n) + \mathrm{y(n)}\Delta t$$
$$\mathrm{y}(n+1) = y(n) - \mathrm{x(n)}\Delta t$$

We need two initial conditions. For example, *x(1) = 1, y(1) = 0*

# Exercises

Solve (numerically) the following systems of equations with the given parameters and plot *x* for all of them in different windows:

1. $\frac{dx}{dt} = 3x^2 - 2x$, for t from 0 to 2, dt = 0.1, x(1) = 0.1
2. Find the Hindmarsh–Rose model in wikipedia. Solve the system with the following parameters: a=1;b=3;c=1;d=5;s = 4;x_R = -8/5;r = 5e-3; I = 6; Tini = 0;Tend = 10000; dt = 0.05; Use initial conditions all in zeroes.
3. Repeat the previous exercise, but with the following variable input: I_inp = 8*sin(0.0001*( 1:(Tend/dt) ) ); This means that you need to give a different input for each step of the for-loop.